INFO-F-409
Learning dynamics

Learning, evolutionary game theory and the evolution of co-operation

T. Lenaerts and A. Nowé
MLG, Université Libre de Bruxelles and
AI-lab, Vrije Universiteit Brussel

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Summary

- What? Why?
- Rational choice
- Strategic games
- Nash Equilibrium
- Best
- Dominance
- Mixed strategies
- Mixed-strategy Nash Equilibria
- Support finding
- Lemke-Howson algorithm
- Extensive-form games
- sub-game perfect equilibrium
- Simultaneous moves
- Chance moves
- Bayesian games
- Assignment 1

2

The formation of agents' beliefs

Now that we can determine the Nash and sub-game perfect equilibria ...

How can we reach them?

Which equilibrium is preferred ?

3

The formation of agents' beliefs

Can we expect that the equilibrium will be reached ?

Players could chose their action from an introspective analysis of the game : removing dominated strategies

Learning the beliefs about the other player in response of the information she receives :

1. Best response dynamics
2. Fictitious play
3. Stimulus-response or reinforcement learning
4. Evolutionary or cultural dynamics

4

Levels of learning

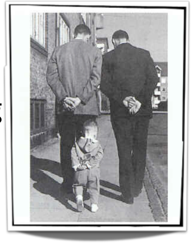
innate

reflex actions

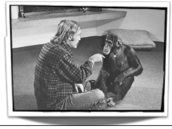
imprinting (specific and irreversible)

Conditioning

Observational and imitative learning

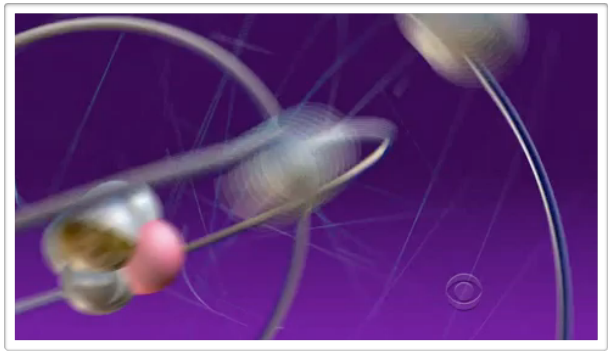


Teaching



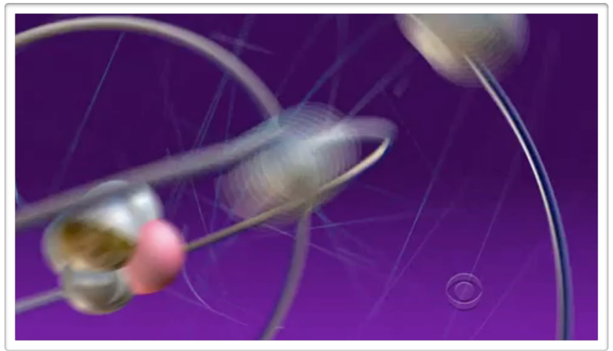
learned

Conditioning



Scene from the Big Bang Theory (S03E03, The Gothowitz Deviation)

Conditioning



Scene from the Big Bang Theory (S03E03, The Gothowitz Deviation)

Best-response dynamics

In the first period, choose a best response to an arbitrary deterministic belief about the other players' actions

In every period after the first, choose the best response to the other players' actions in the previous round

An action profile that remains the same over time is a pure Nash equilibrium of the game

Best-response dynamics

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

Depending on the prior beliefs these dynamics may not converge

Take for instance the Battle of the sexes, which has 3 equilibria ((1,0),(1,0)), ((0,1),(0,1)) and ((2/3,1/3),(1/3, 2/3))

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BELIEF	A plays	B plays
prior	B	B
1	B	B
2	B	B
...	...	...

Best-response dynamics

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BELIEF	A plays	B plays
prior	B	B
1	B	B
2	B	B
...	...	...

BELIEF	A plays	B plays
prior	S	S
1	S	S
2	S	S
...	...	...

Best-response dynamics

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BELIEF	A plays	B plays
prior	B	B
1	B	B
2	B	B
...	...	...

BELIEF	A plays	B plays
prior	S	S
1	S	S
2	S	S
...	...	...

BELIEF	A plays	B plays
prior	S	B
1	B	S
2	S	B
...	...	...

Fictitious play

Every agent starts with an arbitrary probabilistic belief about the other players actions.

In the first round she chooses a BR to this prior probabilistic belief and observes the other player's actions, say A.

she changes here belief so that A gets probability 1

In the second round, she produces a best response to this belief and observes the other player's action, say B

she changes here belief to one that assigns 1/2 to action A and 1/2 to action B

In the third round ...

Fictitious play

Consider again the Battle of the sexes:

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

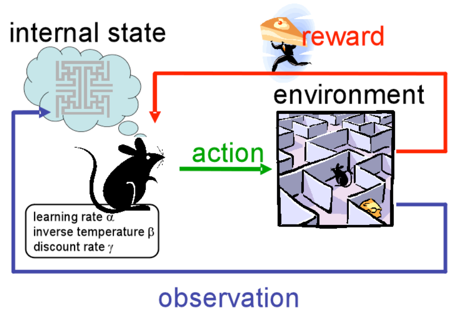
		BELIEF A plays		BELIEF B plays	
prior		(1,0)		(0,1)	
1	S	(1,1)	B	(1,1)	TOTAL = 2
2	S	(1,2)	S	(1,2)	TOTAL = 3
3	S	(1,3)	S	(1,3)	TOTAL = 4
4	S	(2,3)	B	(1,4)	TOTAL = 5
5	S	(2,4)	S	(1,5)	TOTAL = 6
6	S	(2,5)	S	(1,6)	TOTAL = 7
7	...	...	...	...	

Fictitious play

So in any period, the agent adopts the belief that her opponent is using a mixed strategy in which the probability of each action is proportional to the frequency with which her opponent has chosen that action in the previous rounds

The process converges to a mixed strategy Nash equilibrium from initial beliefs

Stimulus-response learning



Roth-Erev learning, Bush-Mosteller, ...

In CS see prof. Vranckx presentation on reinforcement learning

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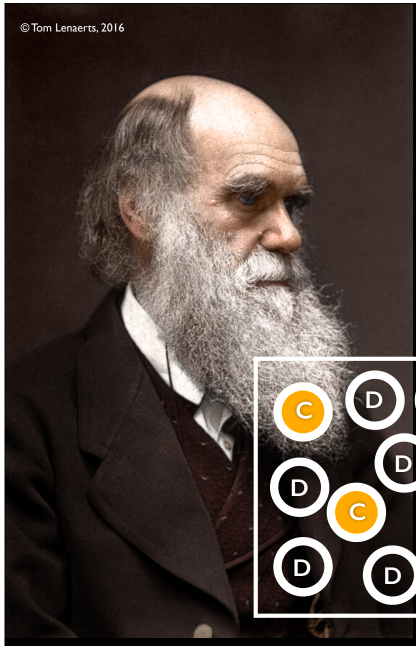
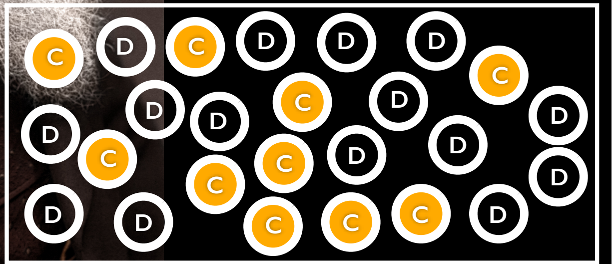


An evolutionary perspective ….

Non-rational players: preferences = actions

Darwinian competition in populations

Frequency-dependence



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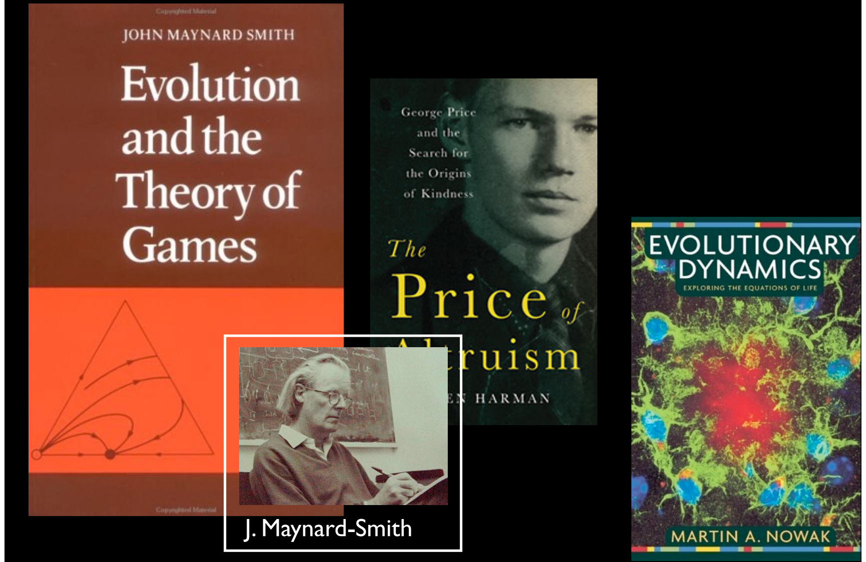

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Social dilemmas

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… attempt to define

Social dilemmas are situations in which each member of a group has a clear and unambiguous incentive to make a choice that—when made by all members—provides poorer outcomes for all than they would have received if none had made the choice. Thus, by doing what seems individually reasonable and rational, people end up doing less well than they would have done if they had acted unreasonably or irrationally. This paradoxical pos-

R.M. Dawes and D.M. Messick (2000) Social Dilemmas. International Journal of Psychology 35(2):111-116

THE QUESTION OF COOPERATION

Social dilemmas are situations in which individual rationality leads to collective irrationality. That is, individually reasonable behavior leads to a situation in which everyone is worse off than they might have been otherwise. Many of

P. Kollock (1998) Social Dilemmas: the anatomy of cooperation Ann. Rev. Sociol. 24:183-214 16

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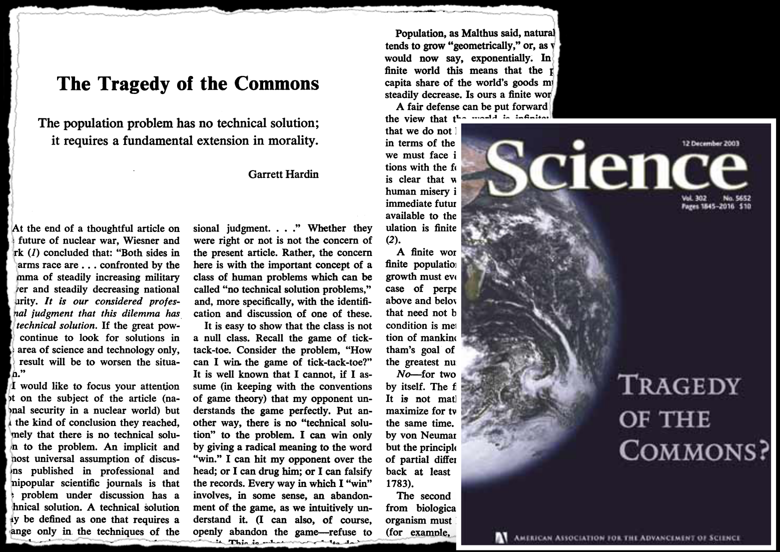

The Tragedy of the Commons

The population problem has no technical solution; it requires a fundamental extension in morality.

Garrett Hardin

sional judgment. . . ." Whether they were right or not is not the concern of the present article. Rather, the concern here is with the important concept of a class of human problems which can be called "no technical solution problems," and, more specifically, with the identification and discussion of one of these.

It is easy to show that the class is not a null class. Recall the game of tick-tack-toe. Consider the problem, "How can I win the game of tick-tack-toe?" It is well known that I cannot, if I assume (in keeping with the conventions of game theory) that my opponent understands the game perfectly. Put another way, there is no "technical solution" to the problem. I can win only by giving a radical meaning to the word "win." I can hit my opponent over the head; or I can drug him; or I can falsify the records. Every way in which I "win" involves, in some sense, an abandonment of the game, as we intuitively understand it. (I can also, of course, openly abandon the game—refuse to

Using common and public goods …

	Excludable	Non-excludable
Rivalrous	Private goods food, clothing, car, TV, house, …	Common goods fish stocks, timber, oil, coal, …
Non-rivalrous	Club goods cinemas, private parks, satelite tv, …	Public goods air, knowledge, national defense, street lightning, social welfare,..

Rivalry; whether the consumption of a good by one person precludes its consumption by another person

Excludability; whether it is possible to exclude a person from consumption of the goods

Using common and public goods …

Neither state control nor privatisation are necessary to protect common resources

Bottom-up small-scale institutions, created by communes relying on or living close to the resource, that govern access to the resource will ensure the well-being of themselves and the survival of the resource.

Using common and public goods …

BUT

Do the people in the commune really have the **capacity to organize** themselves for any common resource?

What about **individual differences in benefits**?

What about **free-riders**?

What if **others** not belonging to that commune also want access?

Central question:
How to reach cooperation in social dilemmas?

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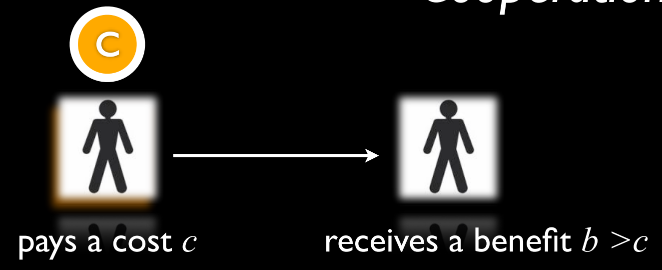
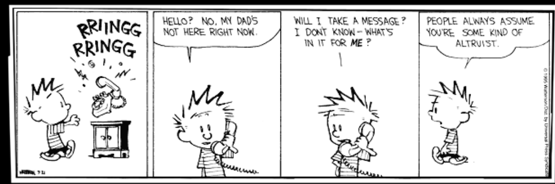
21

Cooperation?





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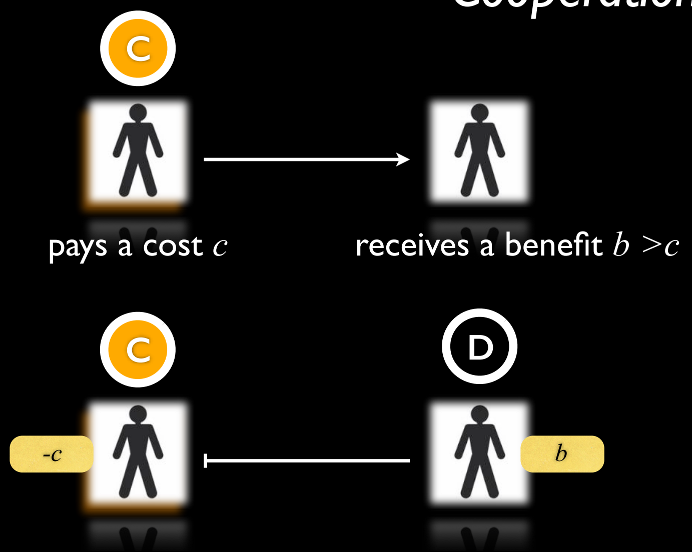
Cooperation?



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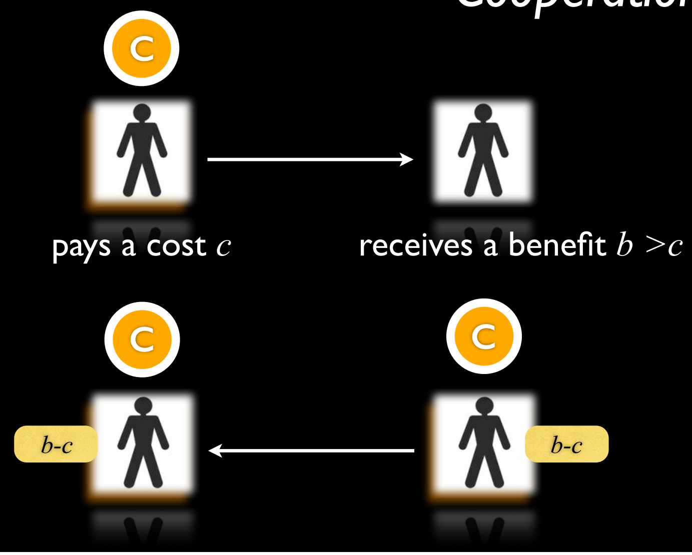
Cooperation?



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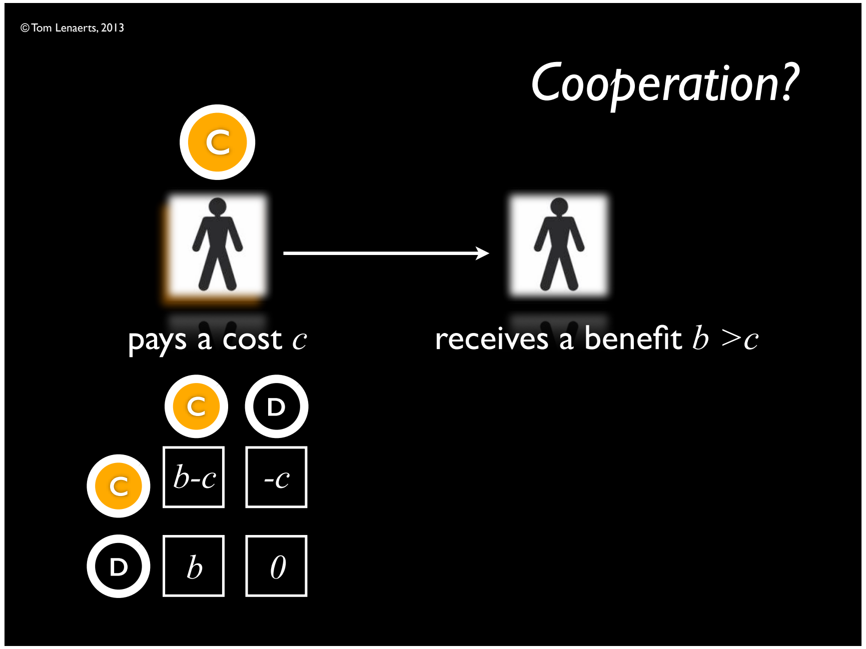

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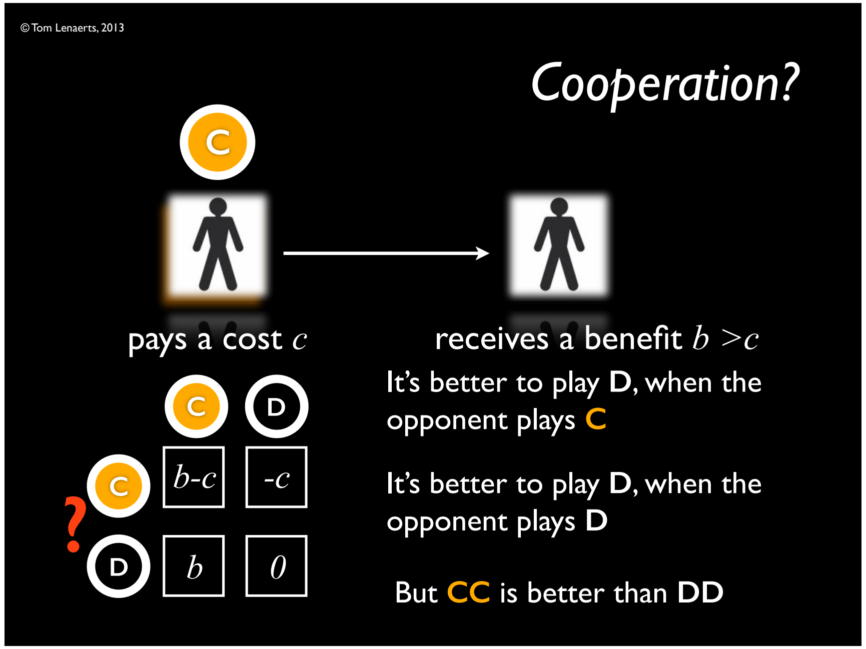

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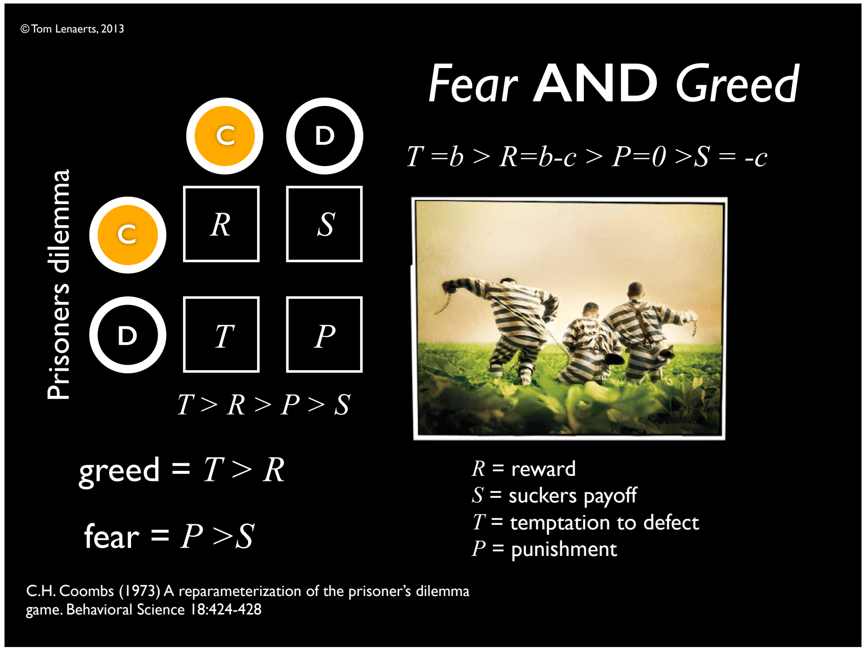

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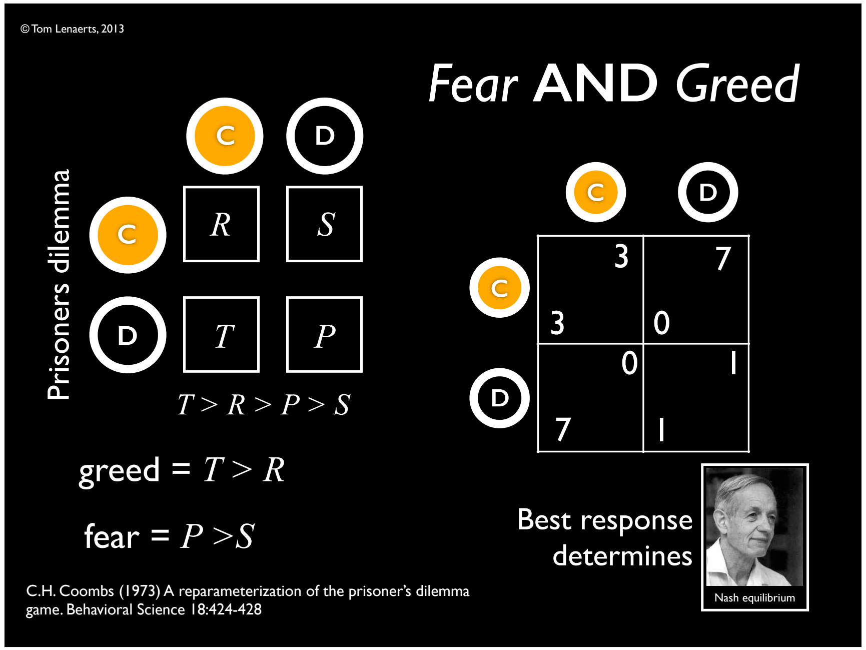

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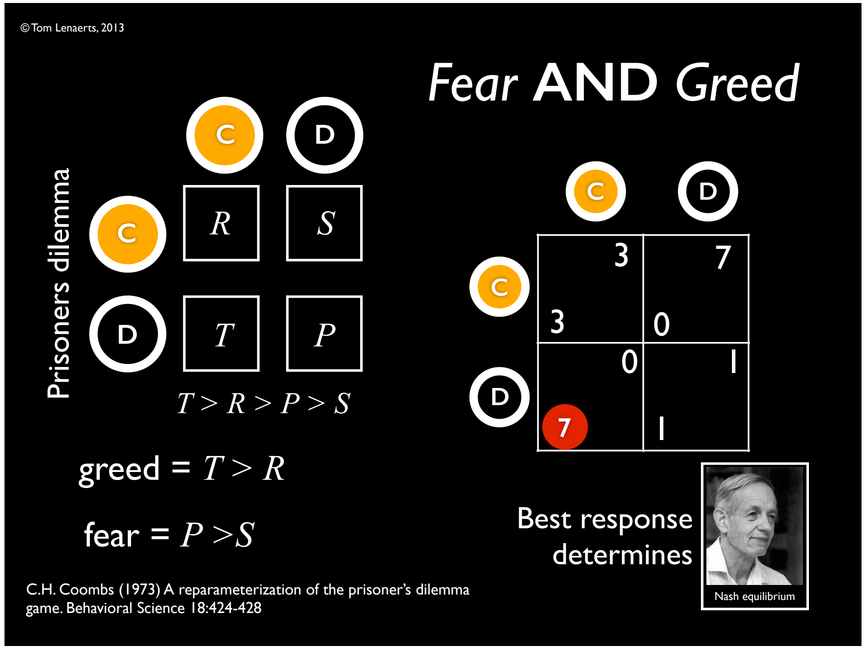

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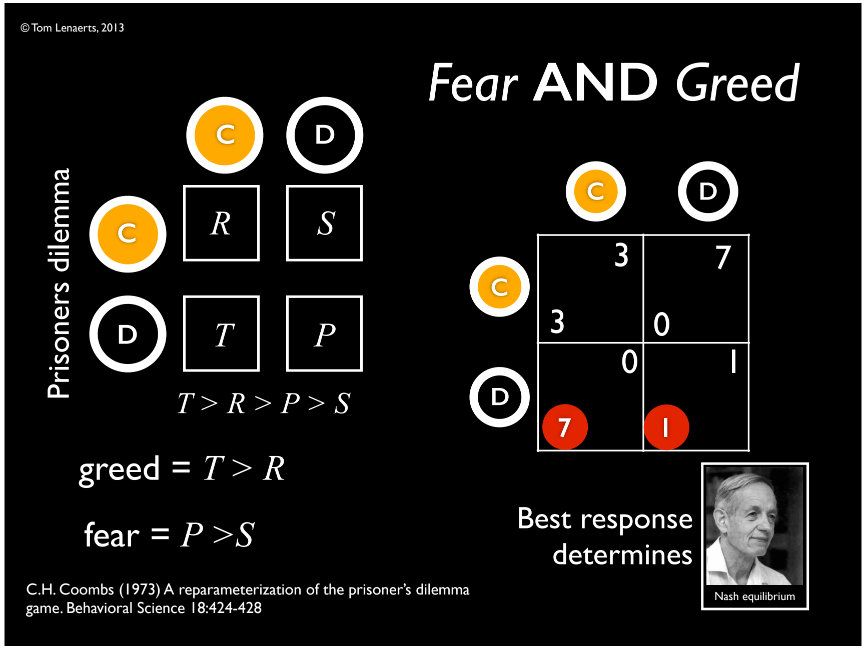

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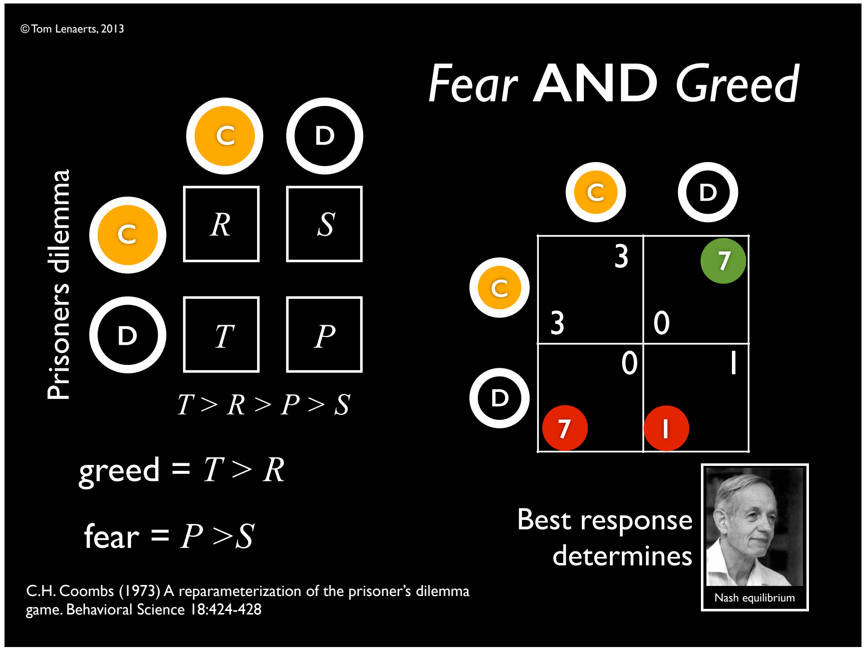

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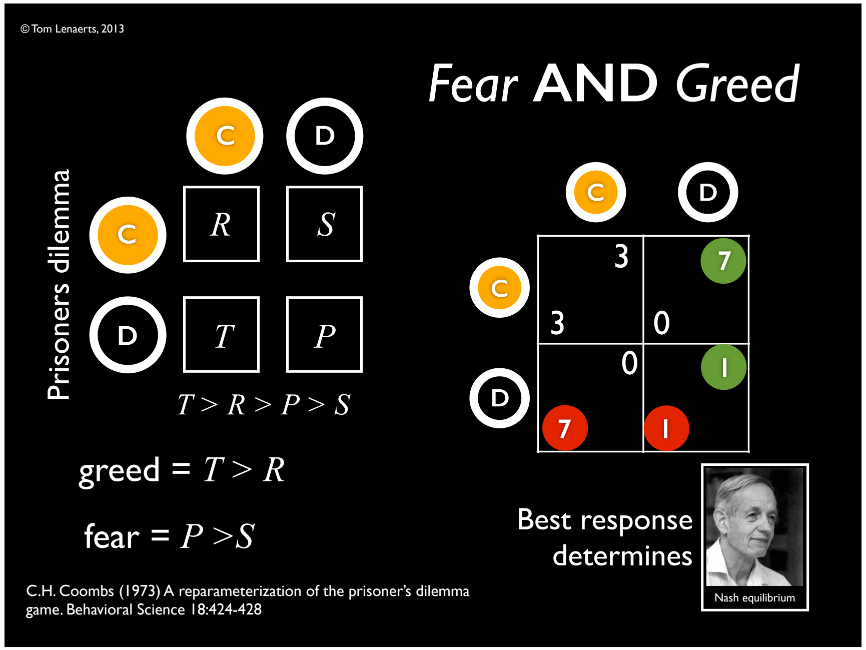

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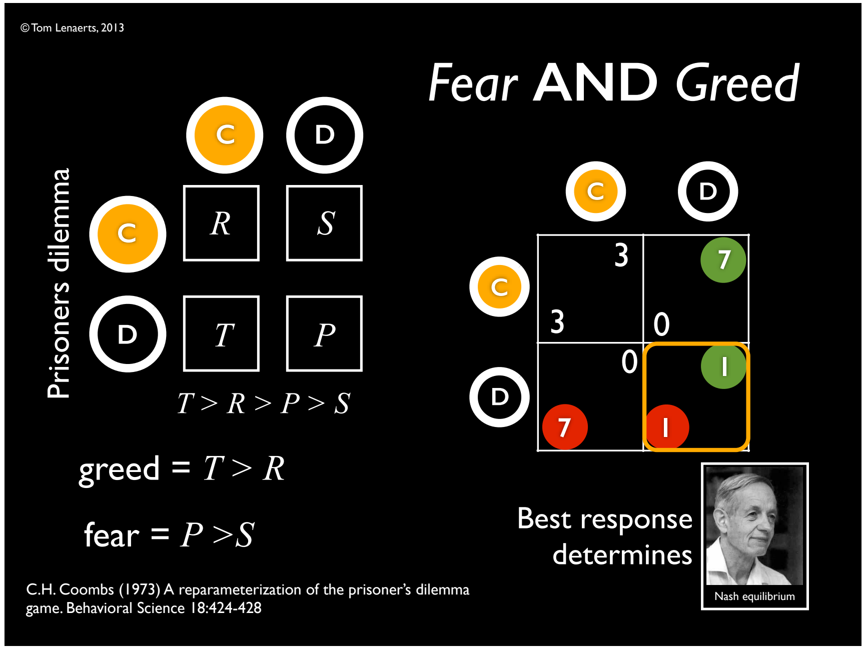

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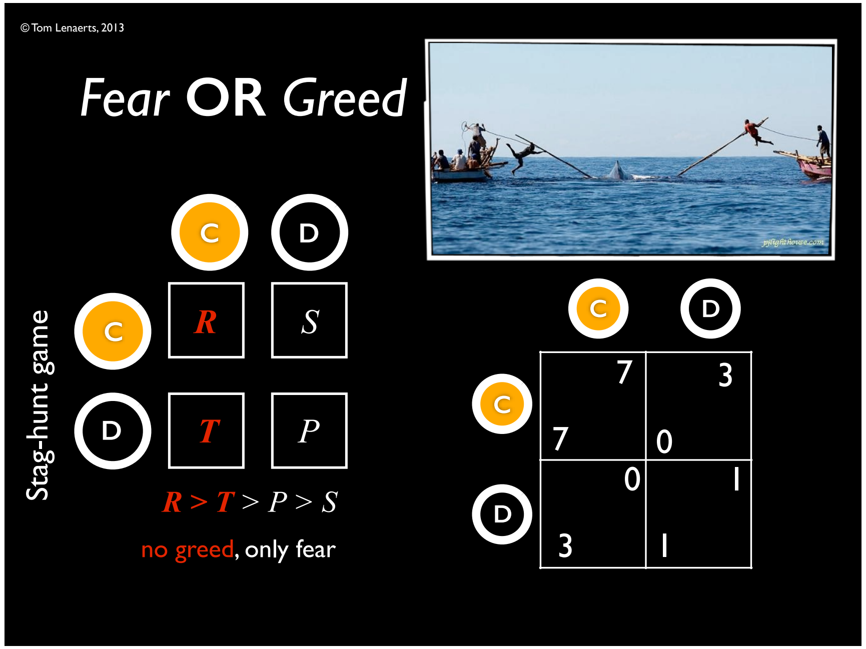

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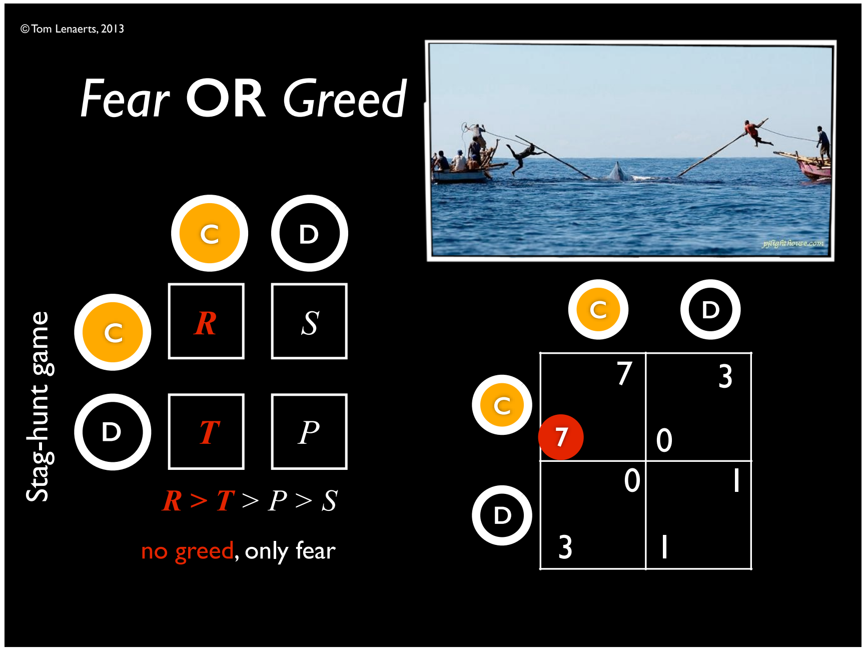

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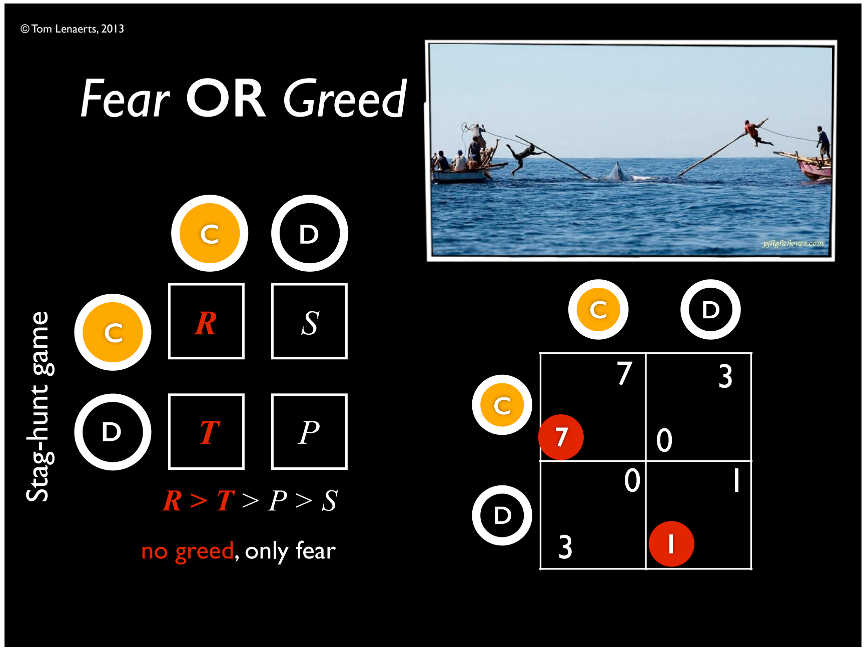

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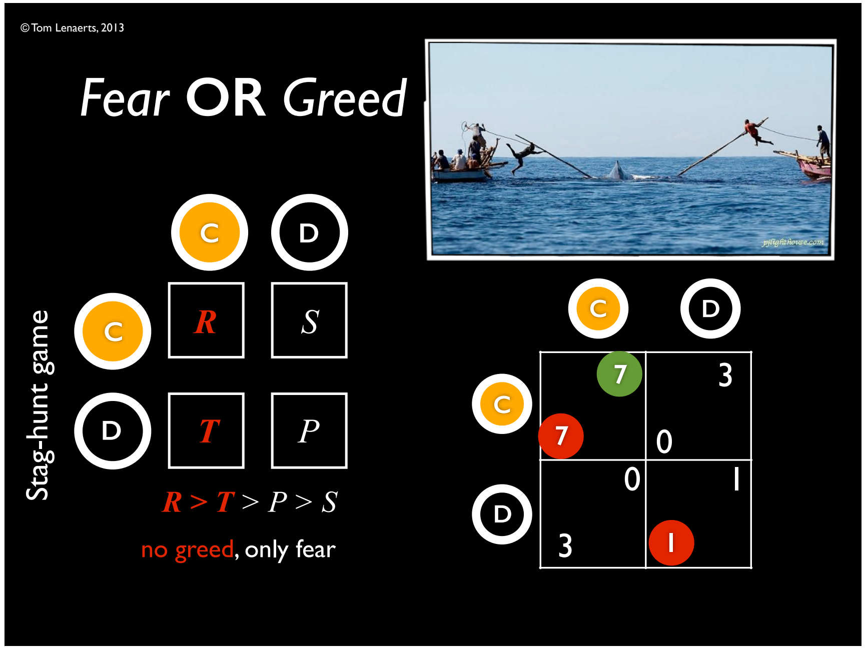

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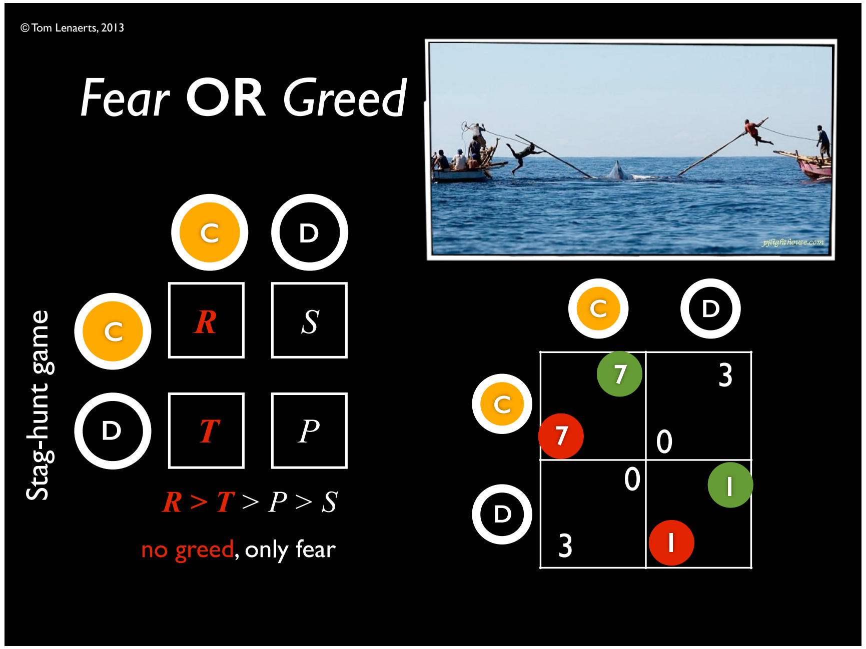

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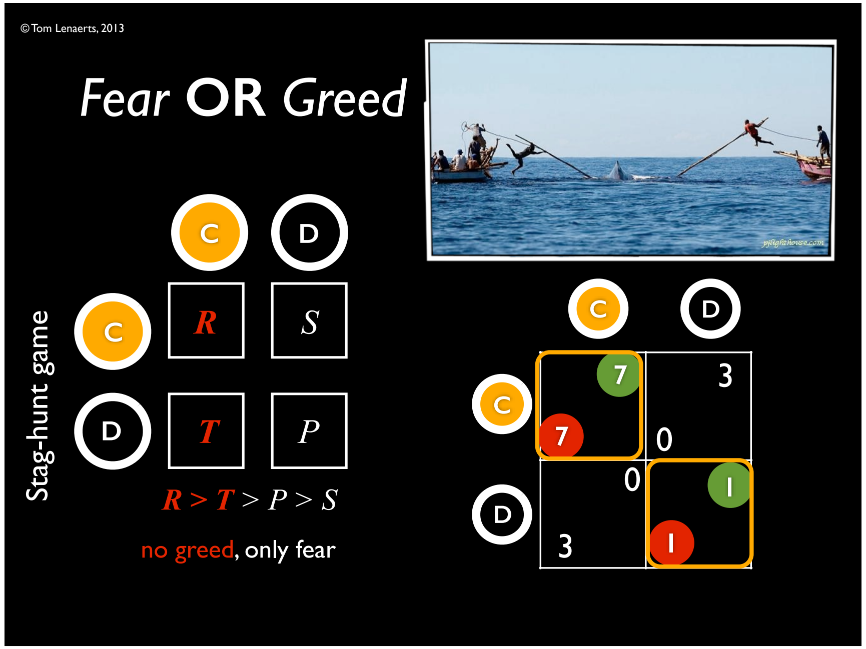

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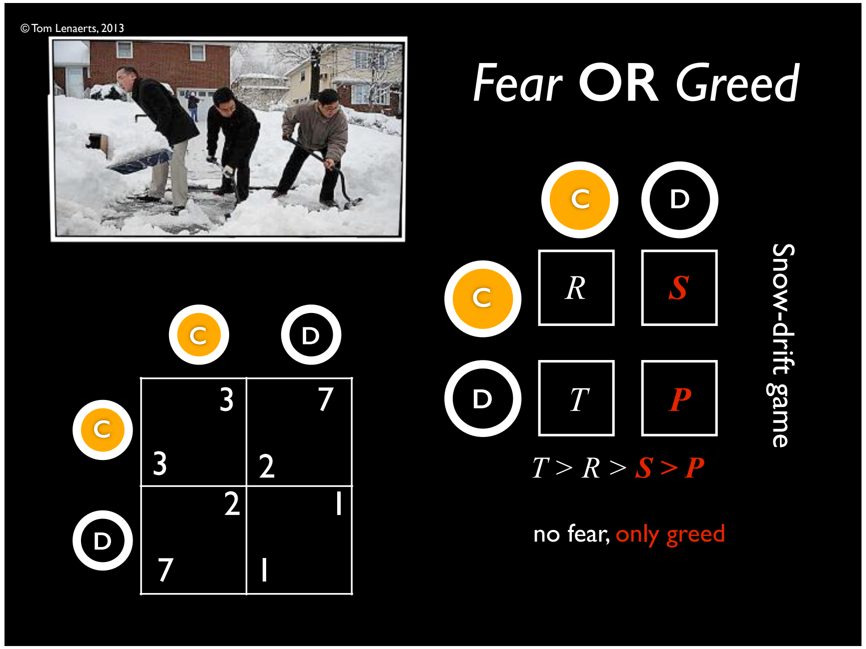

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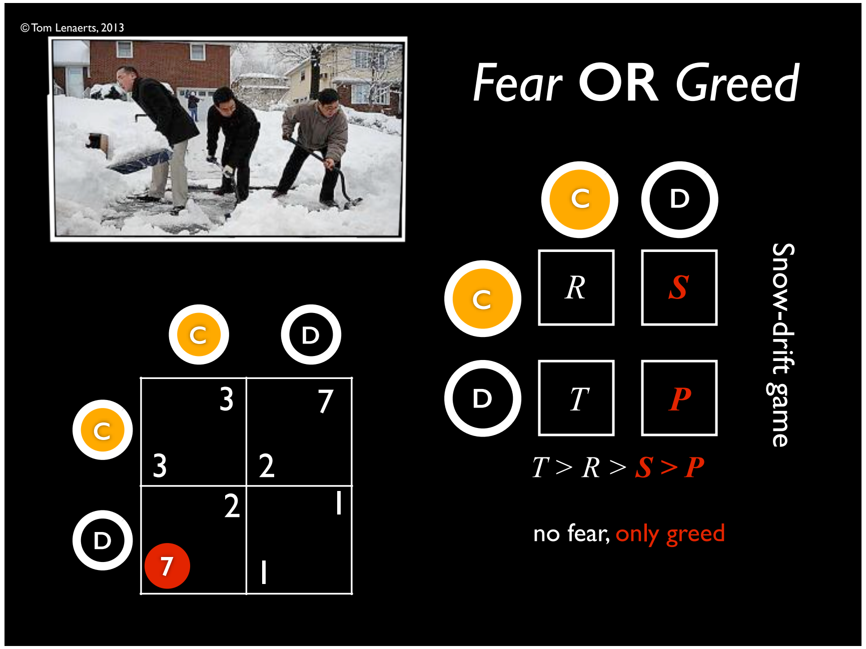

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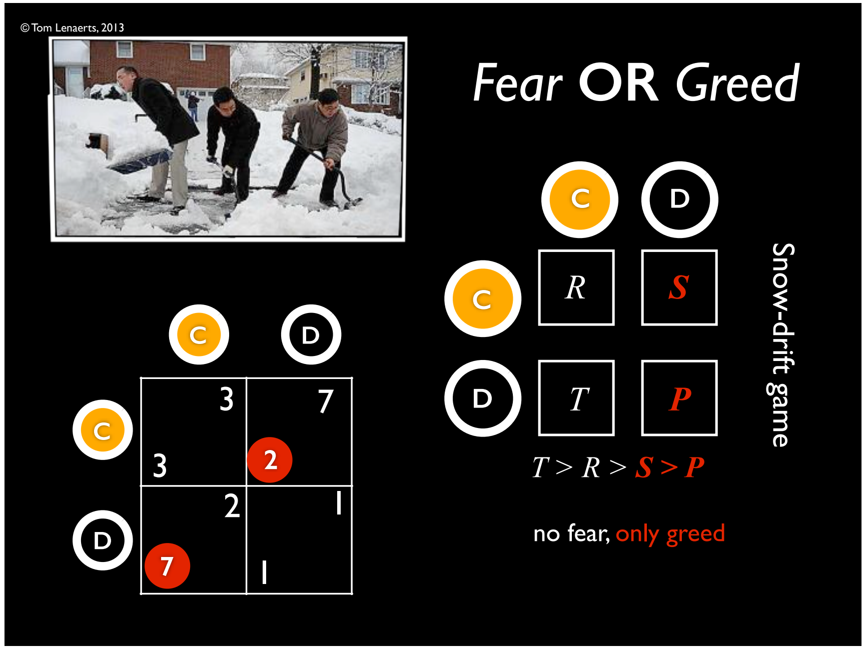

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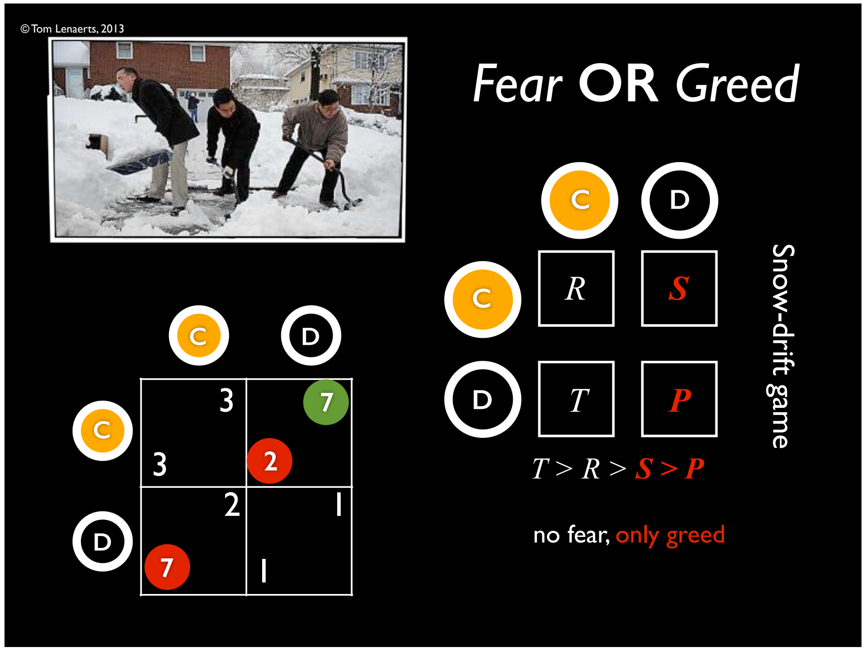

28-4

28-5


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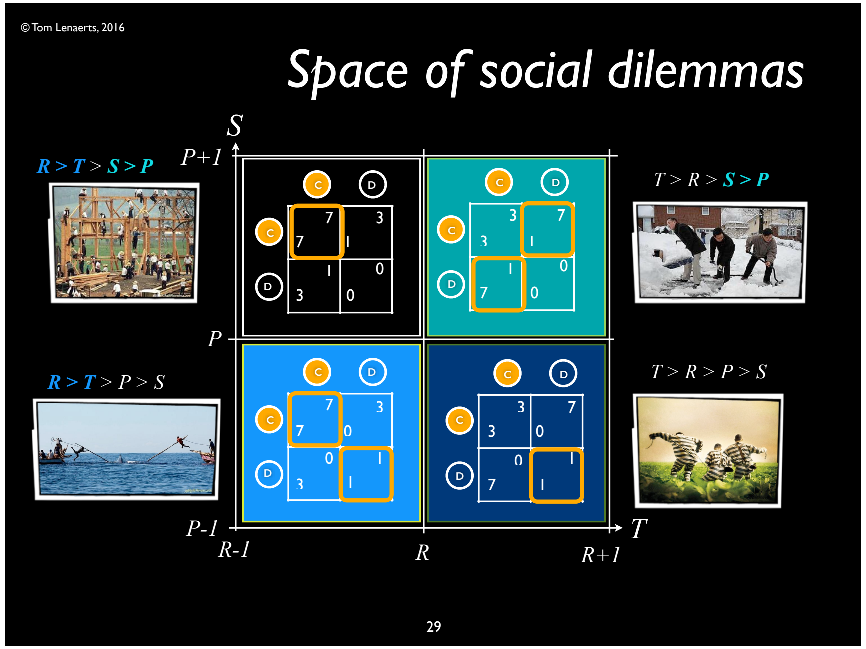
Fear OR Greed
C
D
R
S
T
P
T > R > S > P
no fear, only greed
Snow-drift game
3
7
2
1

28-6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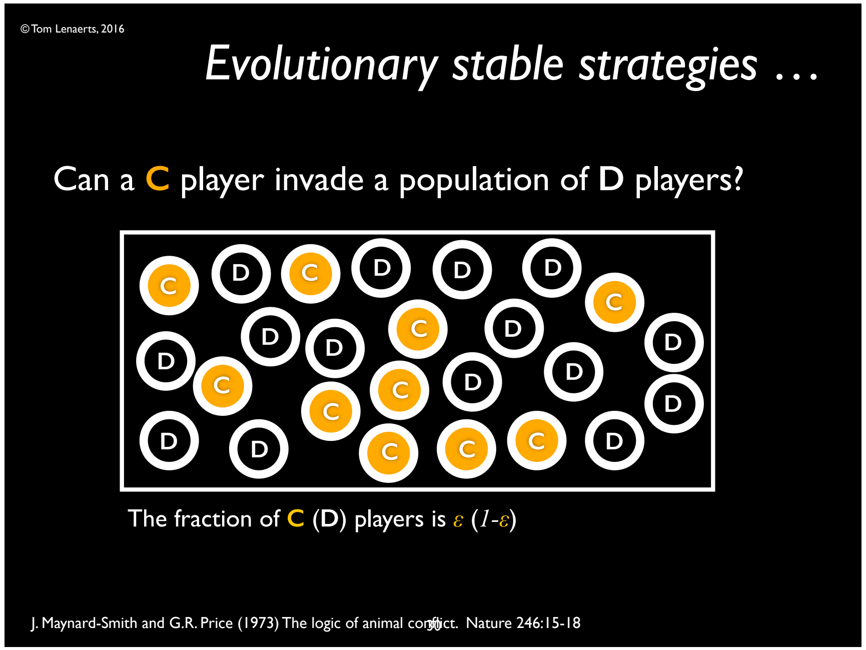
Space of social dilemmas
S
P+1
P
P-1
R-1
R
R+1
T
R > T > S > P
T > R > S > P
R > T > P > S
T > R > P > S
29

29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Evolutionary stable strategies …
Can a C player invade a population of D players?
The fraction of C (D) players is ε (1-ε)
J. Maynard-Smith and G.R. Price (1973) The logic of animal conflict. Nature 246:15-18

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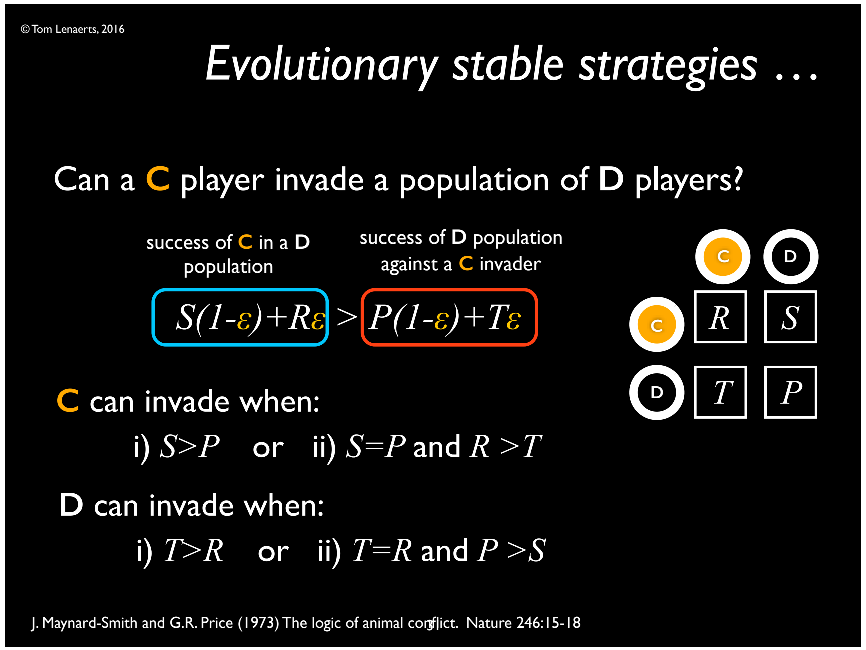
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Evolutionary stable strategies …
Can a C player invade a population of D players?
success of C in a D population
success of D population against a C invader
S(1-ε)+Rε > P(1-ε)+Tε
C can invade when:
i) S>P or ii) S=P and R >T
D can invade when:
i) T>R or ii) T=R and P >S
R
S
T
P
J. Maynard-Smith and G.R. Price (1973) The logic of animal conflict. Nature 246:15-18

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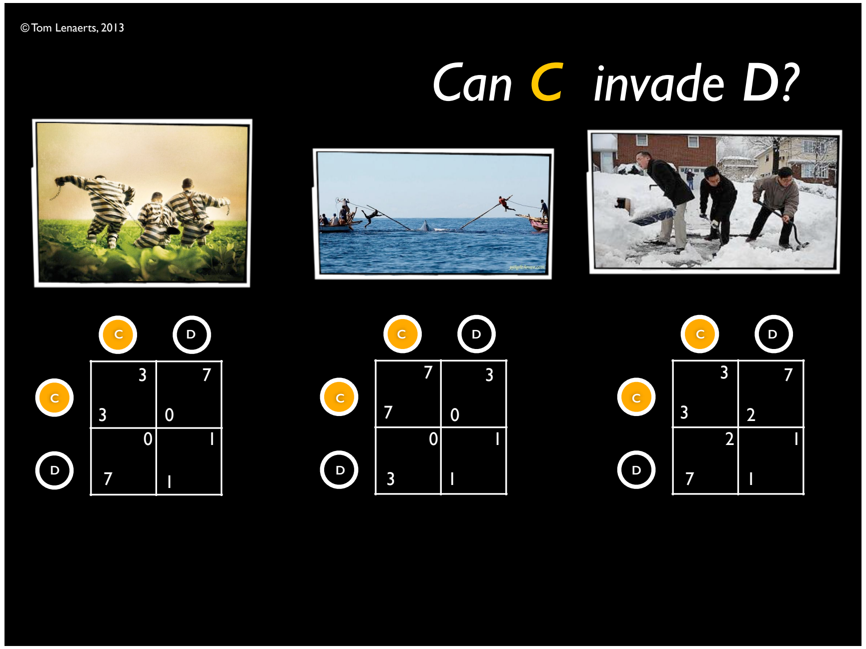

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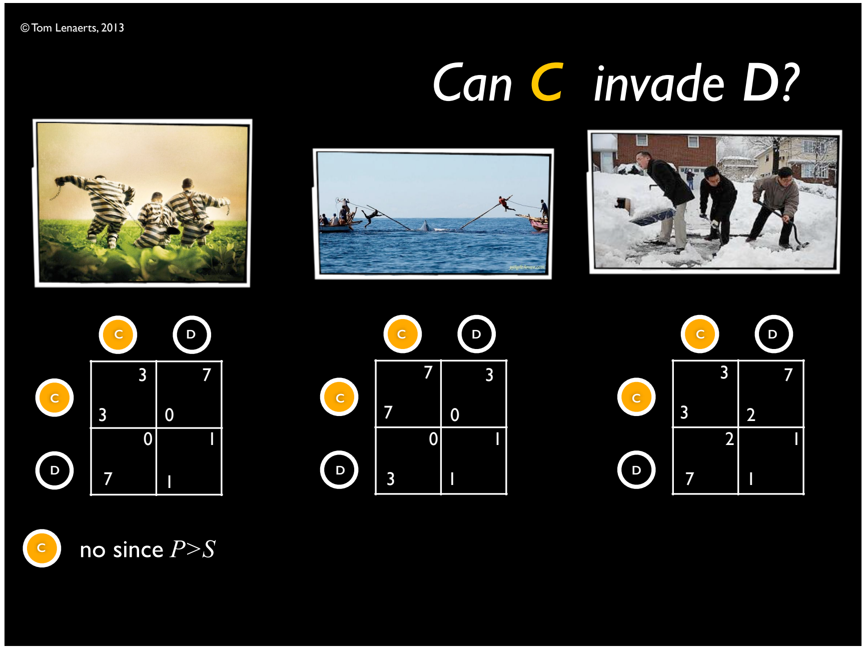

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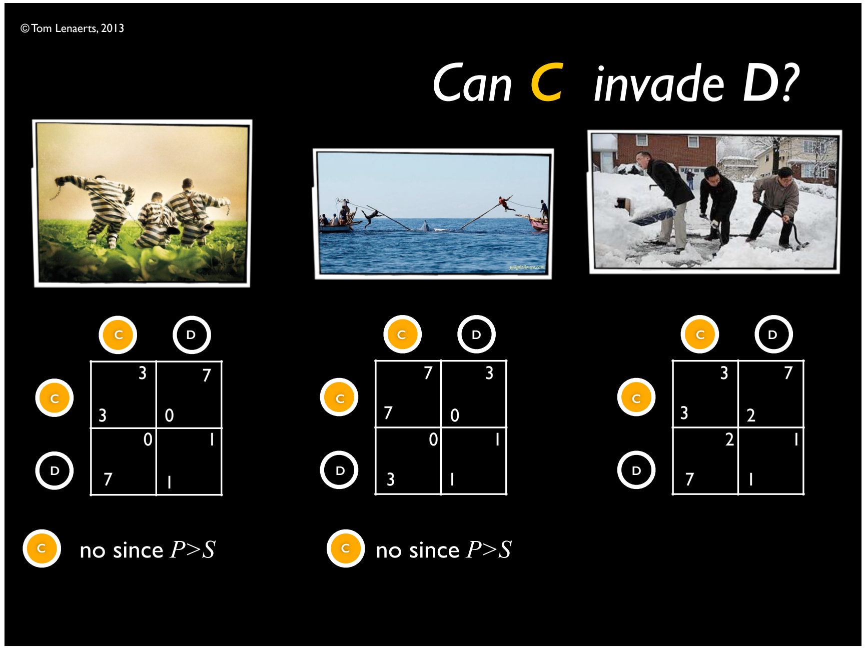

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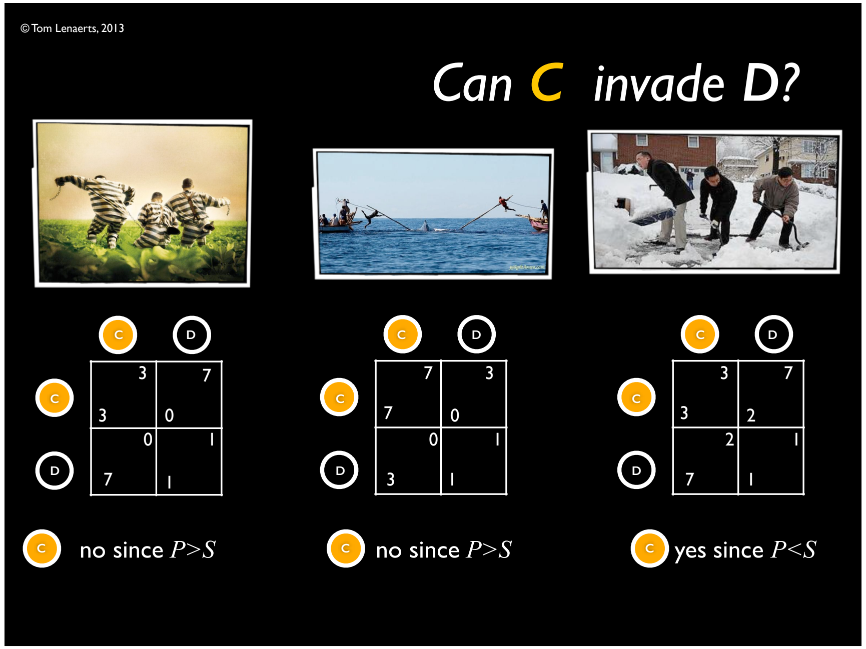

32-4

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Can D invade C?

	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

	C	D
C	3, 3	2, 7
D	7, 2	1, 1

C no since $P>S$

C no since $P>S$

C yes since $P<S$

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Can D invade C?

	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

	C	D
C	3, 3	2, 7
D	7, 2	1, 1

C no since $P>S$
D yes since $R<T$

C no since $P>S$

C yes since $P<S$

32-6

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Can D invade C?

	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

	C	D
C	3, 3	2, 7
D	7, 2	1, 1

C no since $P>S$
D yes since $R<T$

C no since $P>S$
D no since $R>T$

C yes since $P<S$

32-7

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Can D invade C?

	C	D
C	3, 3	0, 7
D	7, 0	1, 1

	C	D
C	7, 7	0, 3
D	3, 0	1, 1

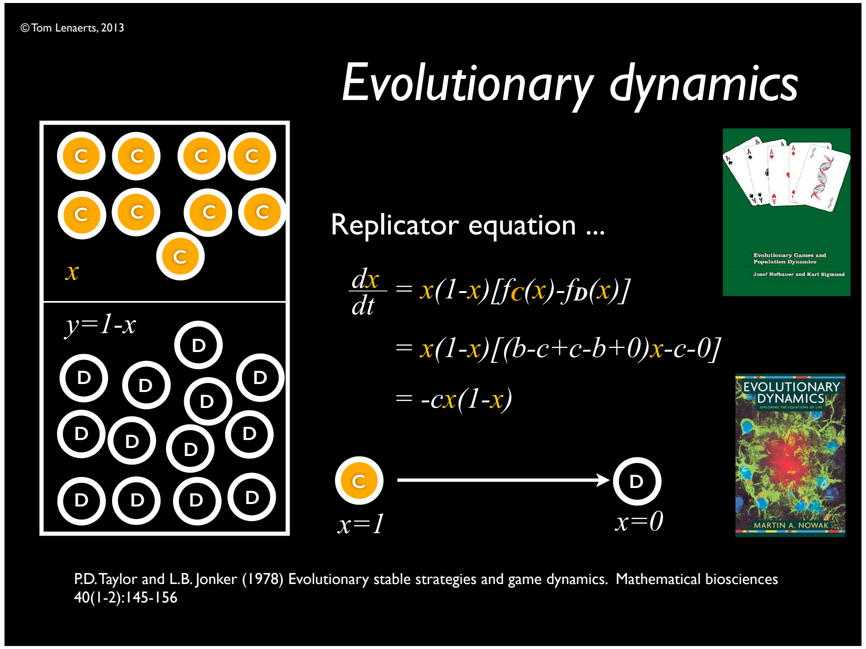
	C	D
C	3, 3	2, 7
D	7, 2	1, 1

C no since $P>S$
D yes since $R<T$

C no since $P>S$
D no since $R>T$

C yes since $P<S$
D yes since $R<T$

32-8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Evolutionary dynamics
Replicator equation …
$\frac{dx}{dt} = x(1-x)[f_C(x)-f_D(x)]$
$= x(1-x)[(b-c+c-b+0)x-c-0]$
$= -cx(1-x)$
x
y=1-x
x=1
x=0
Evolutionary Games and Population Dynamics
Josef Hofbauer and Karl Sigmund
EVOLUTIONARY DYNAMICS
MARTIN A. NOWAK
P.D. Taylor and L.B. Jonker (1978) Evolutionary stable strategies and game dynamics. Mathematical biosciences 40(1-2):145-156

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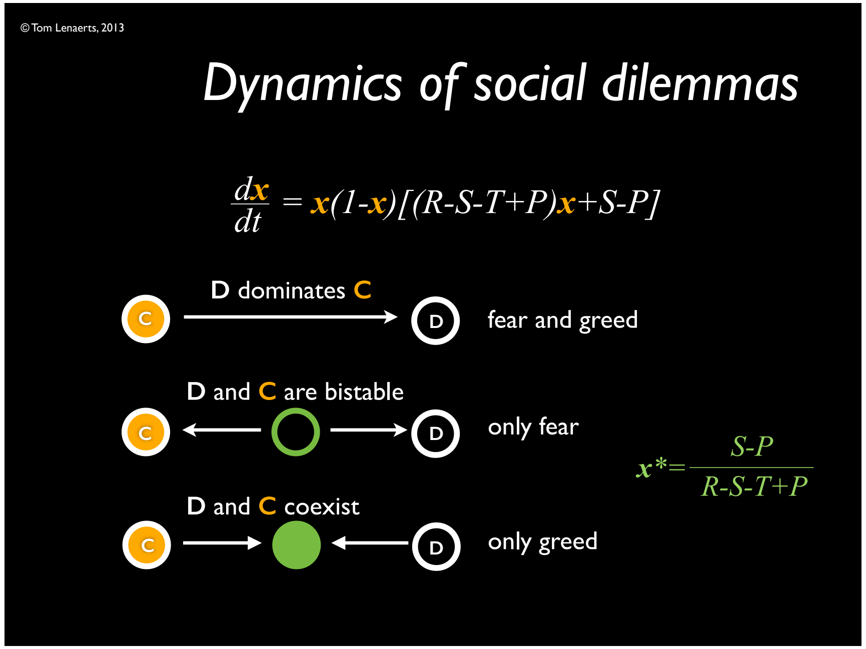
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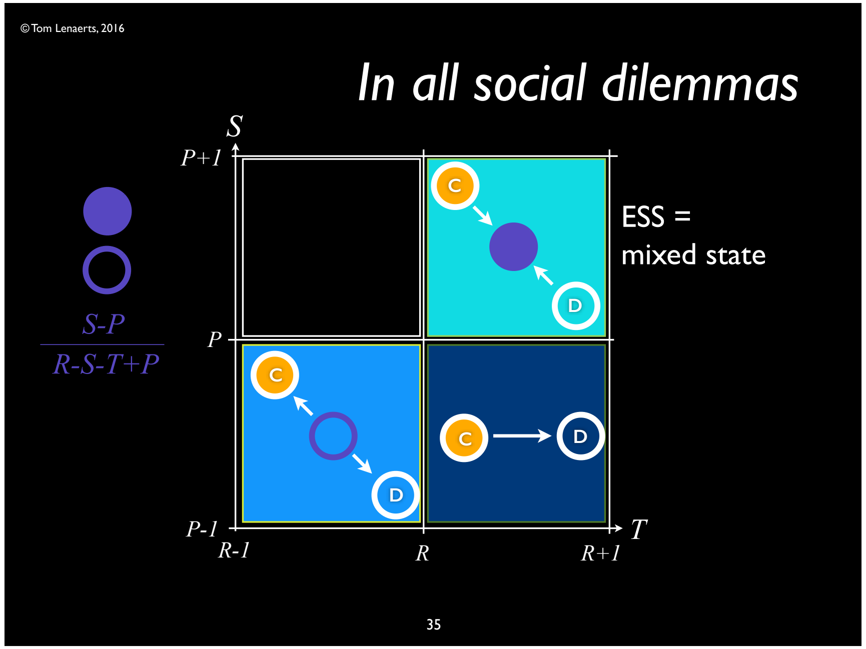
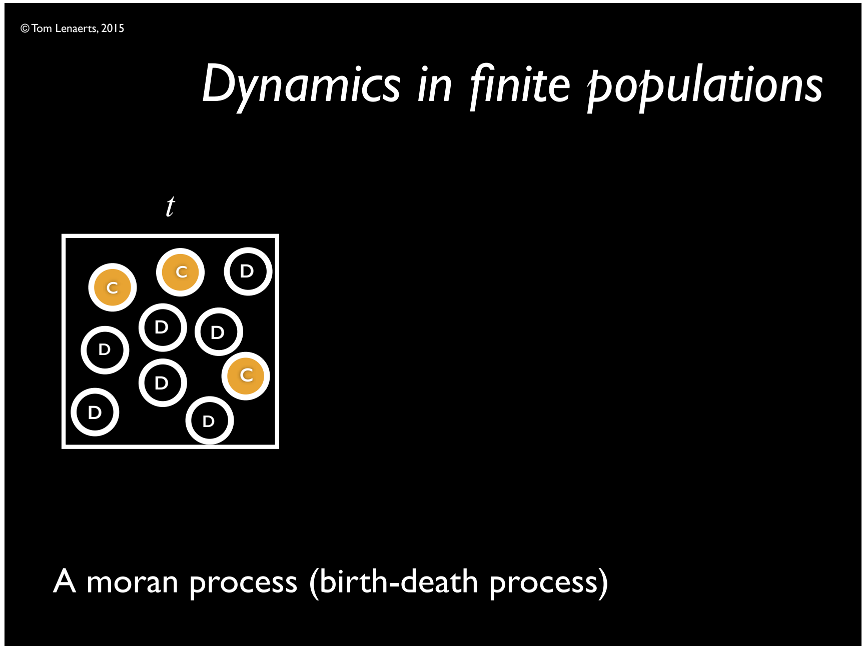
Dynamics of social dilemmas
$\frac{dx}{dt} = x(1-x)[(R-S-T+P)x+S-P]$
D dominates C
fear and greed
D and C are bistable
only fear
D and C coexist
only greed
$x^* = \frac{S-P}{R-S-T+P}$

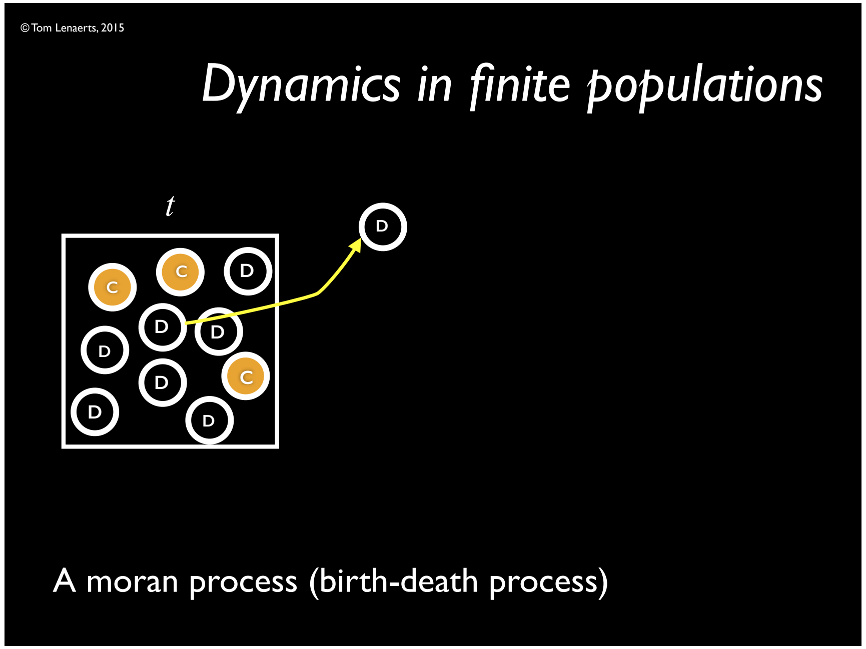
34


©Tom Lenaerts, 2016
In all social dilemmas
S
P+1
P
P-1
R-1
R
R+1
T
ESS =
mixed state
$\frac{S-P}{R-S-T+P}$
35

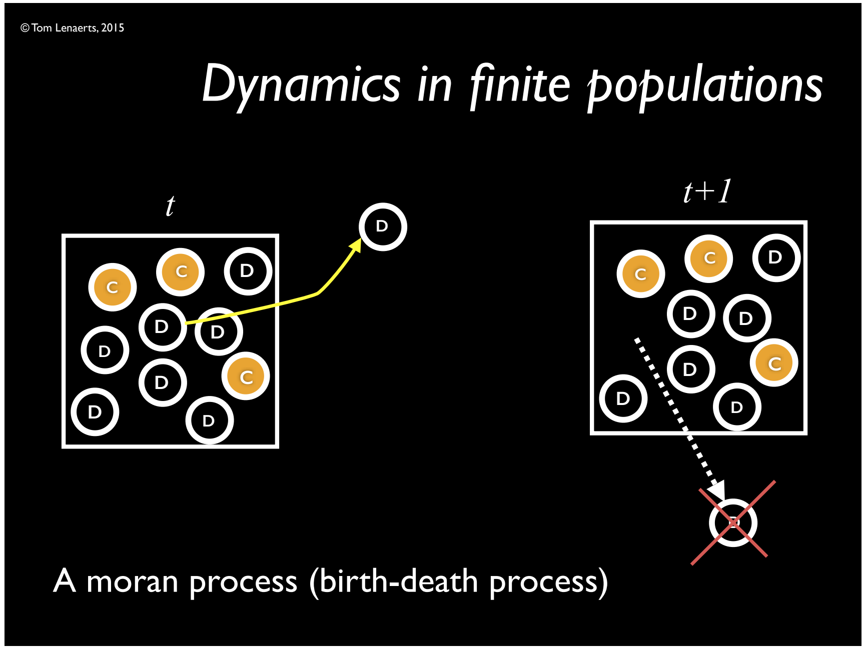
35


©Tom Lenaerts, 2015
Dynamics in finite populations
t
A moran process (birth-death process)

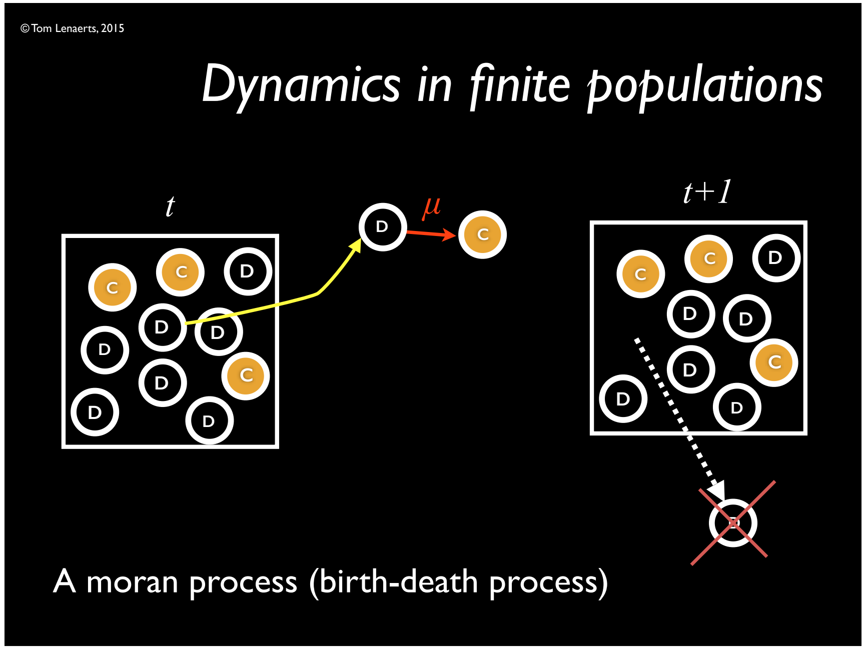
36-1



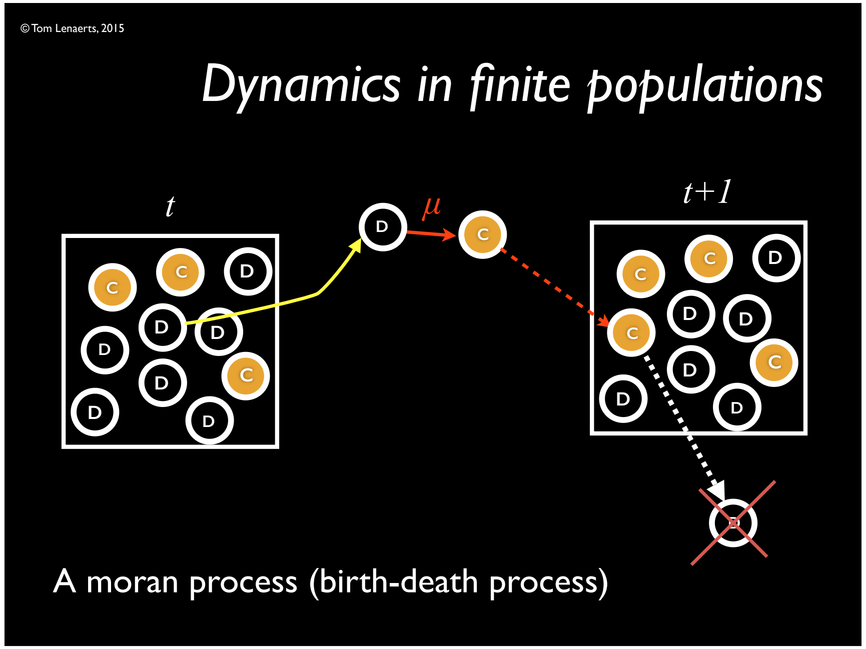
36-2



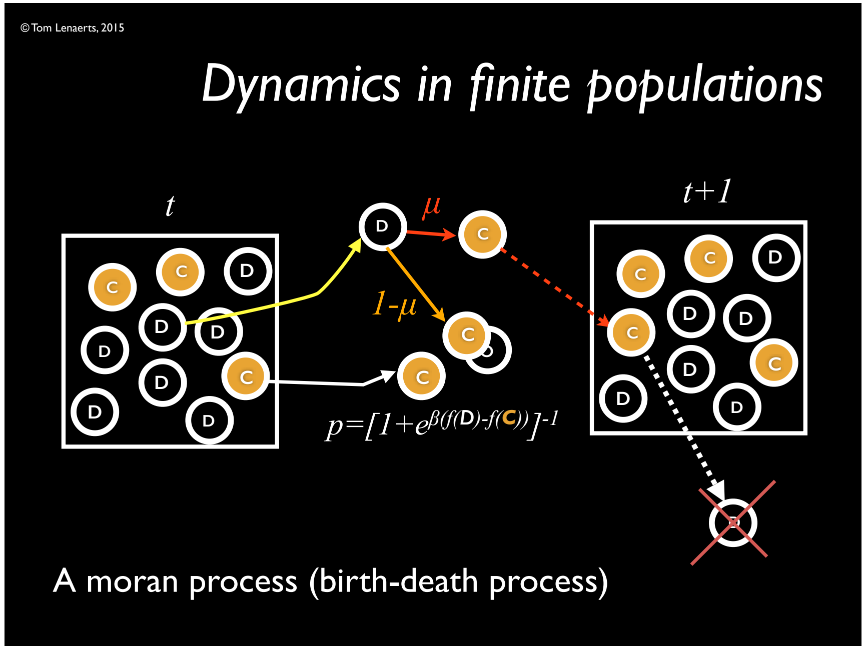
36-3



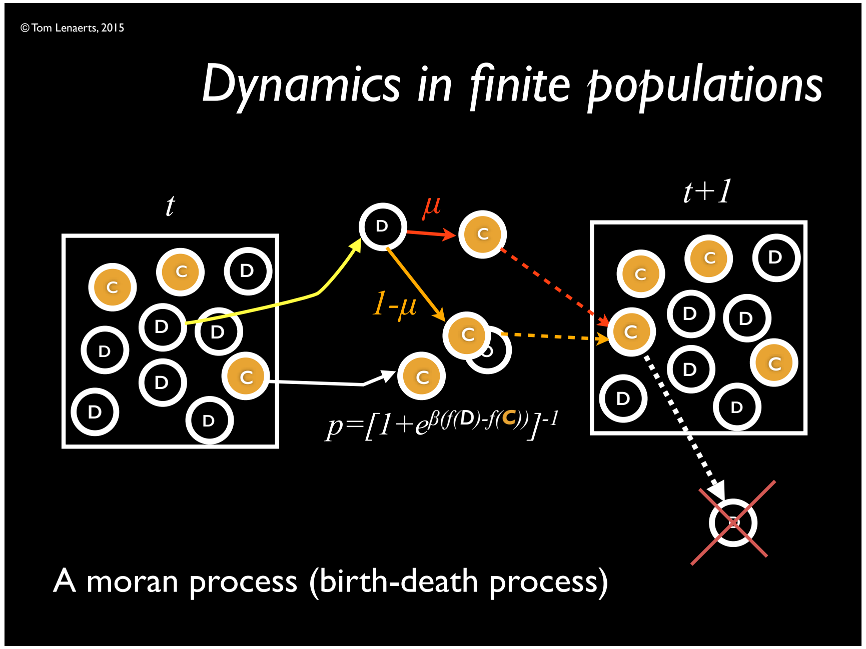
36-4



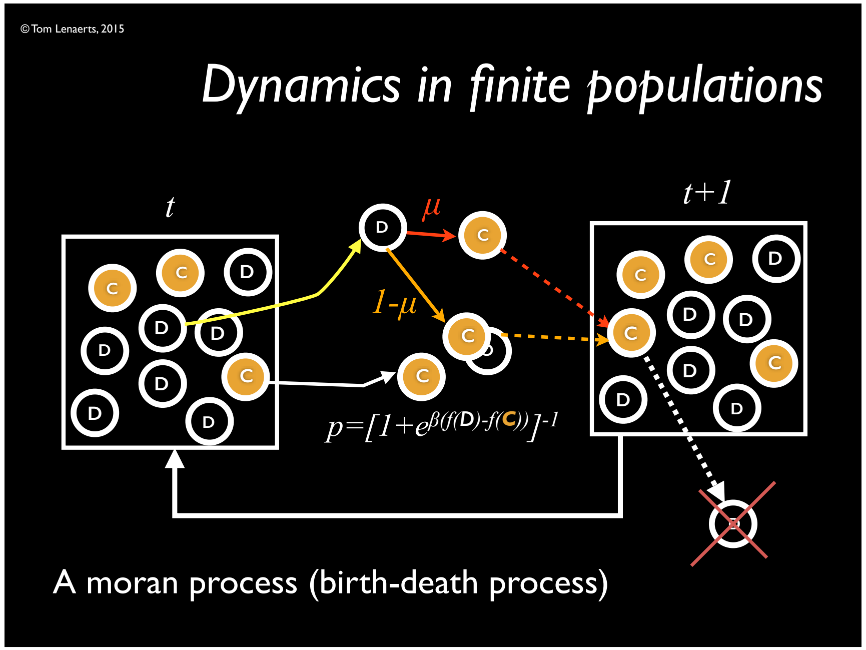
36-5



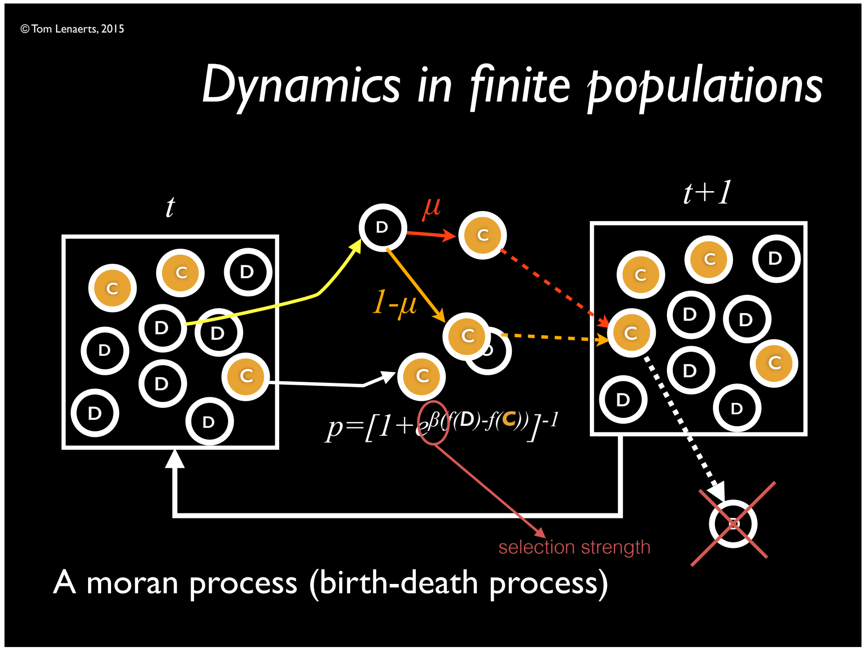
36-6



36-7



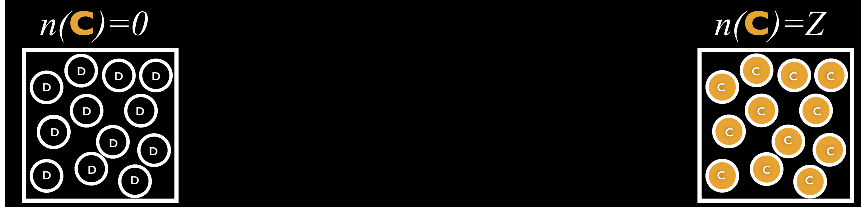
36-8



36-9

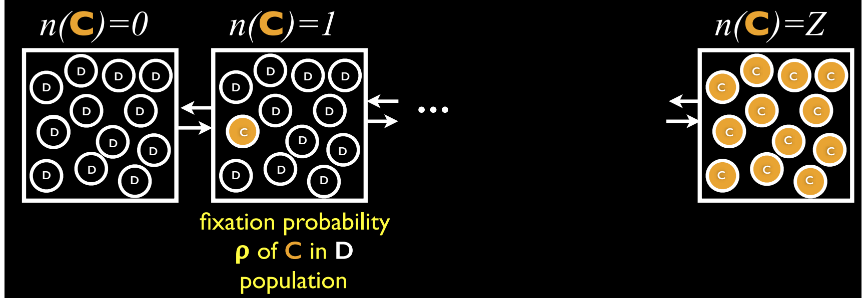
Dynamics in finite populations

Under the assumption that mutations are rare, we either end up with



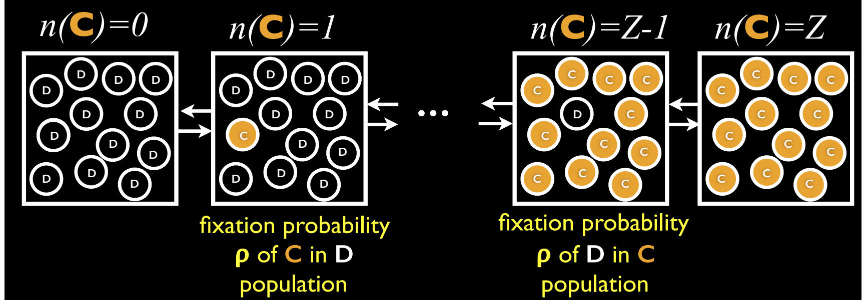
Dynamics in finite populations

Under the assumption that mutations are rare, we either end up with



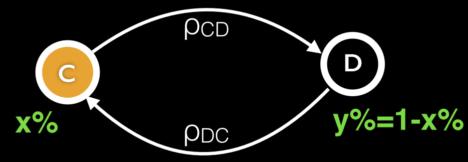
Dynamics in finite populations

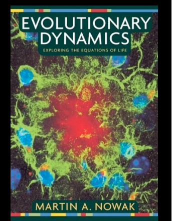
Under the assumption that mutations are rare, we either end up with



Dynamics in finite populations

Which produces a **reduced Markov chain**

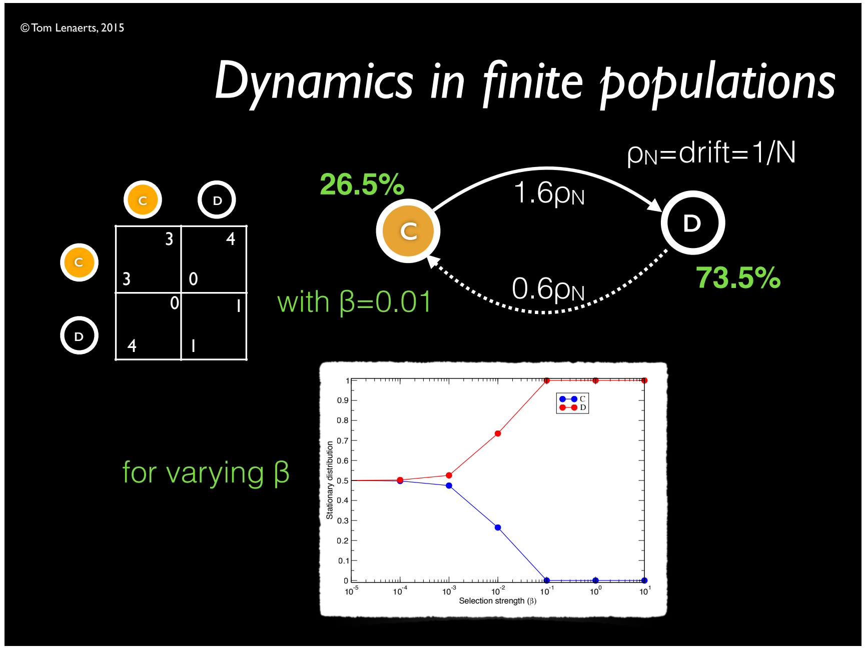




For which the **stationary distributions** can be calculated = *how likely it is to end up in either monomorphic state*

Fudenberg, D., & Imhof, L.A. (2006). Imitation processes with small mutations. J. Econ.Theo, 131,251–262.

Imhof, L.A., Fudenberg, D., & Nowak, M.A. (2005). Evolutionary cycles of cooperation and defection. Proc Nat Acad Sci USA, 102(31), 10797–10800.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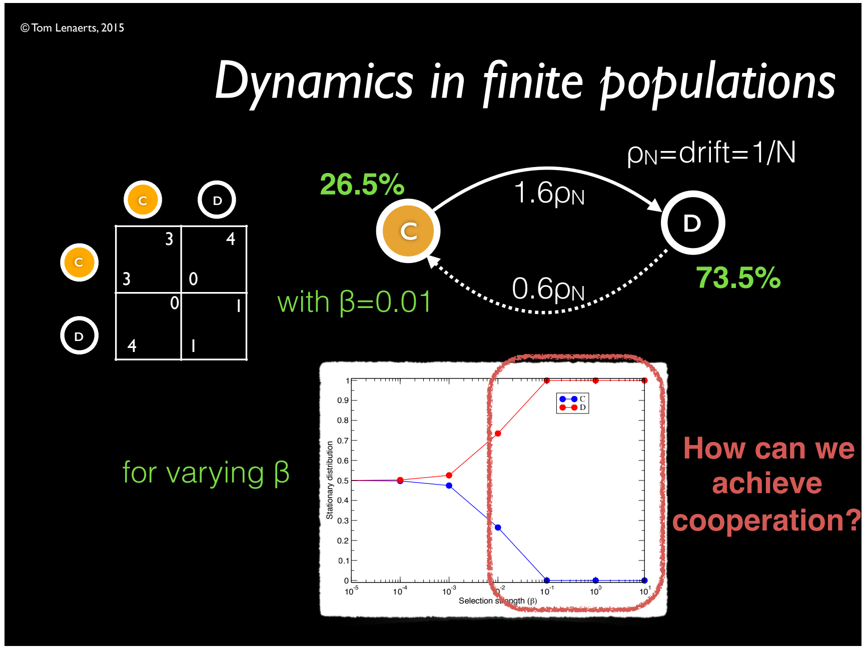
Dynamics in finite populations
ρN=drift=1/N
26.5%
1.6ρN
C
D
0.6ρN
73.5%
with β=0.01
for varying β
Stationary distribution
Selection strength (β)

39-1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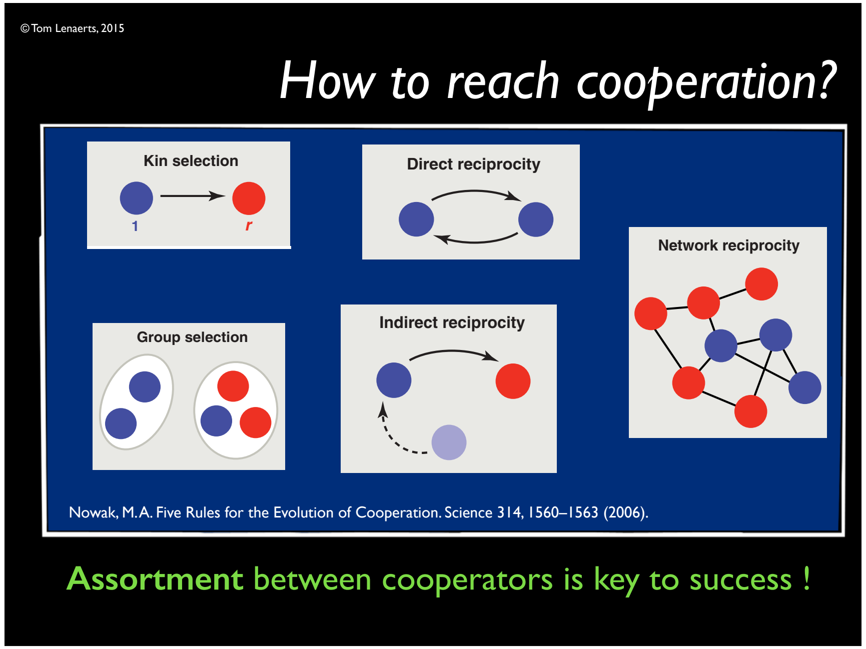
Dynamics in finite populations
ρN=drift=1/N
26.5%
1.6ρN
C
D
0.6ρN
73.5%
with β=0.01
for varying β
How can we achieve cooperation?

39-2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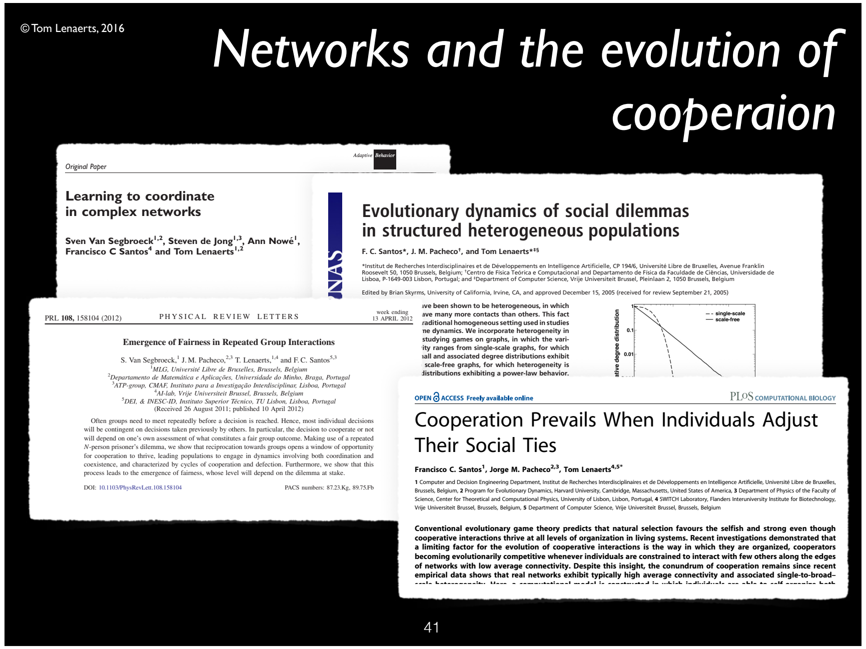
How to reach cooperation?
Kin selection
Direct reciprocity
Network reciprocity
Group selection
Indirect reciprocity
Nowak, M.A. Five Rules for the Evolution of Cooperation. Science 314, 1560–1563 (2006).
Assortment between cooperators is key to success !

40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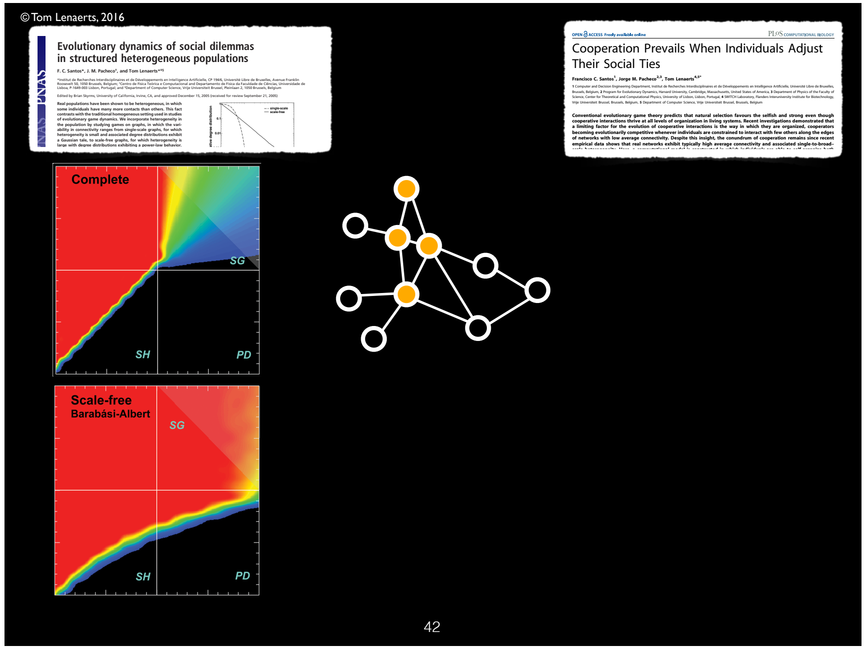
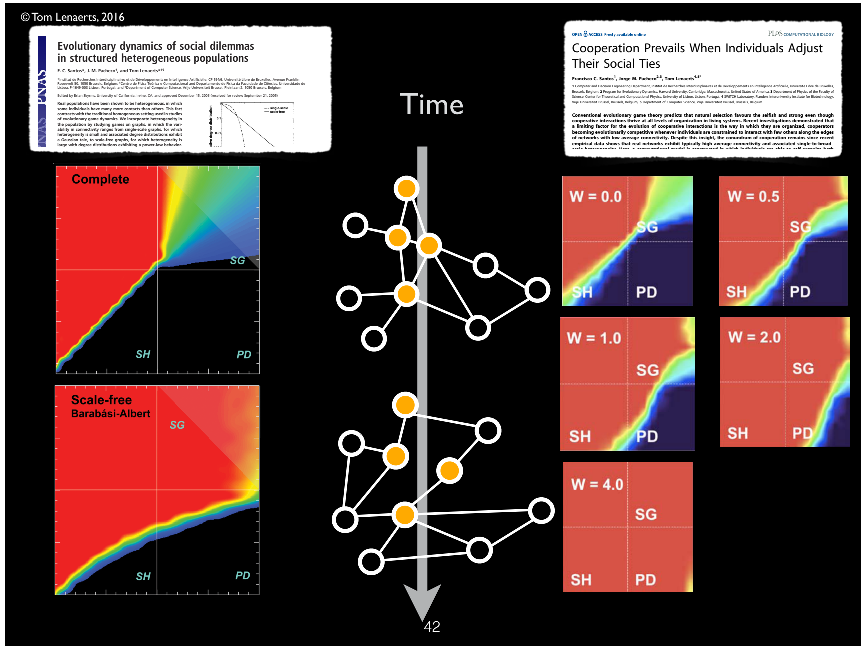
Networks and the evolution of cooperaion
Learning to coordinate in complex networks
Evolutionary dynamics of social dilemmas in structured heterogeneous populations
Emergence of Fairness in Repeated Group Interactions
Cooperation Prevails When Individuals Adjust Their Social Ties
41

41

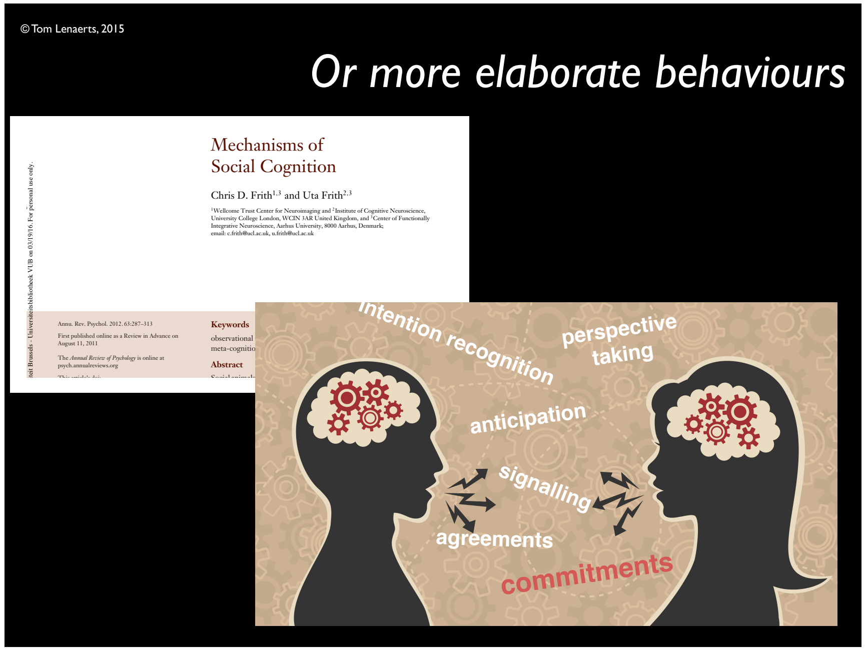


42-1



42-2

42-3



43

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Trust drives social interactions …
integrity
sincerity
reliability
Trust
commitment
consistency
competence
How to make sure we can trust a partner?
44

44

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"Commitment is giving up options to change incentives in a situation"
"specialised capacities to make commitments may have evolved by natural selection"
EVOLUTION and the CAPACITY for COMMITMENT
RANDOLPH M. NESSE

45

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INTERFACE
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Research
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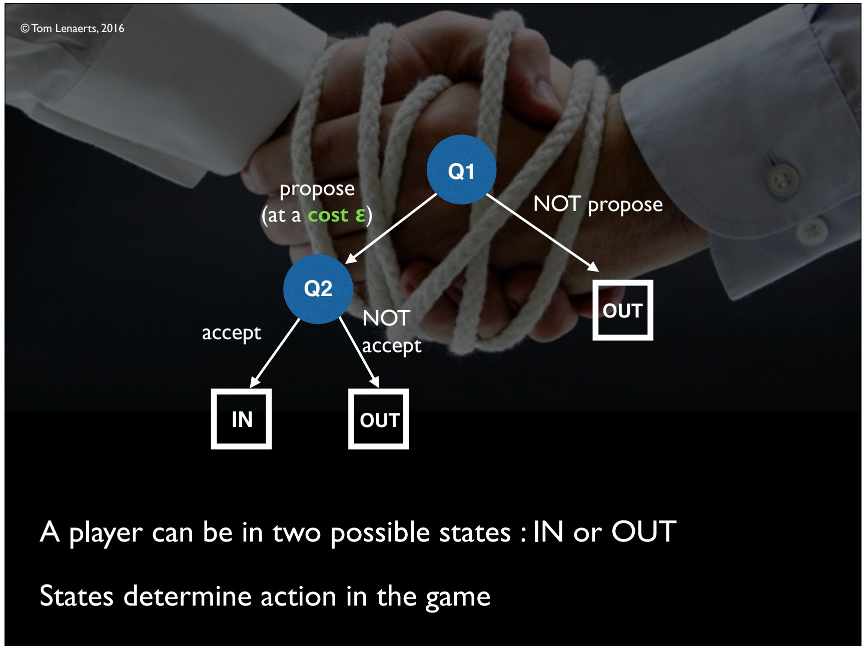
http://dx.doi.org/10.1098/rsif.2014.1203
Avoiding or restricting defectors in public goods games?
The Anh Han[1,2,3,†], Luís Moniz Pereira[4] and Tom Lenaerts[1,2]
[1]AI lab, Computer Science Department, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium
[2]MLG, Département d'Informatique, Université Libre de Bruxelles, Boulevard du Triomphe CP212, 1050 Brussels, Belgium
[3]School of Computing, Teesside University, Borough Road, Middlesbrough TS1 3BA, UK
[4]Centro de Inteligência Artificial (CENTRIA), Departamento de Informática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal
TAH, 0000-0002-3095-7714; LMP, 0000-0001-7880-4322; TL, 0000-0003-3645-1455
SCIENTIFIC REPORTS
OPEN
Good Agreements Make Good Friends
The Anh Han[1,2], Luís Moniz Pereira[3], Francisco C. Santos[4,5] & Tom Lenaerts[1,2]
SUBJECT AREAS:
BIOLOGICAL PHYSICS
BEHAVIOURAL METHODS
EVOLUTIONARY THEORY
SOCIAL EVOLUTION
[1]AI lab, Computer Science Department, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium, [2]MLG, Département d'Informatique, Université Libre de Bruxelles, Boulevard du Triomphe CP212, 1050 Brussels, Belgium, [3]Centro de Inteligência Artificial (CENTRIA), Departamento de Informática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal, [4]INESC-ID and Instituto Superior Ténico, Universidade de Lisboa, IST-Taguspark, 2744-016 Porto Salvo, Portugal, [5]ATP-group, CMAF, Instituto para a Investigação Interdisciplinar, P-1649-003 Lisboa Codex, Portugal.
Received 2 July 2013
Accepted 30 August 2013
Published 18 September 2013
Correspondence and
When starting a new collaborative endeavor, it pays to establish upfront how strongly your partner commits to the common goal and what compensation can be expected in case the collaboration is violated. Diverse examples in biological and social contexts have demonstrated the pervasiveness of making prior agreements on posterior compensations, suggesting that this behavior could have been shaped by natural selection. Here, we analyze the evolutionary relevance of such a commitment strategy and relate it to the costly punishment strategy, where no prior agreements are made. We show that when the cost of arranging a commitment deal lies within certain limits, substantial levels of cooperation can be achieved. Moreover, these levels are higher than that achieved by simple costly punishment, especially when one insists on sharing the arrangement cost. Not only do we show that good agreements make good friends, agreements based on shared costs result in even better outcomes.

46



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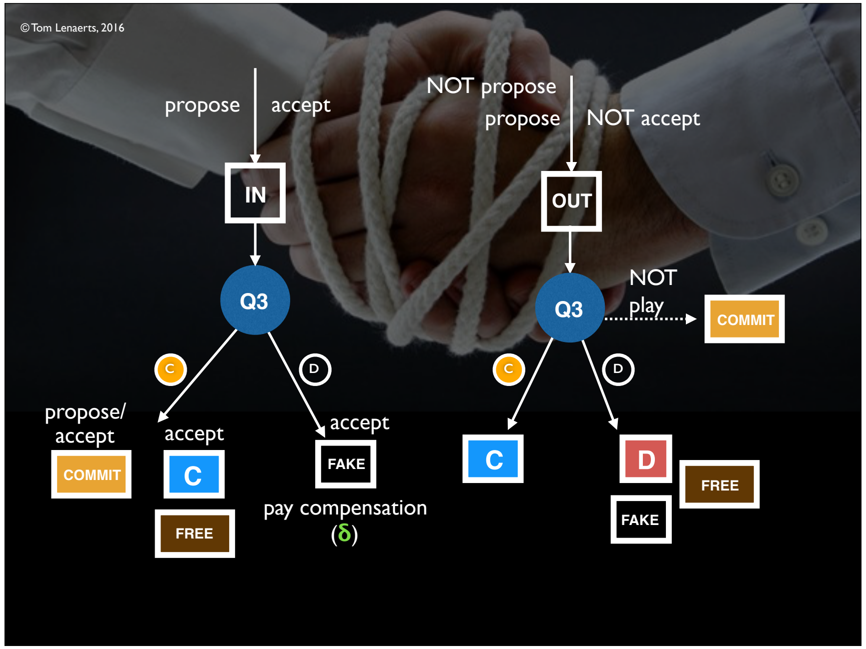
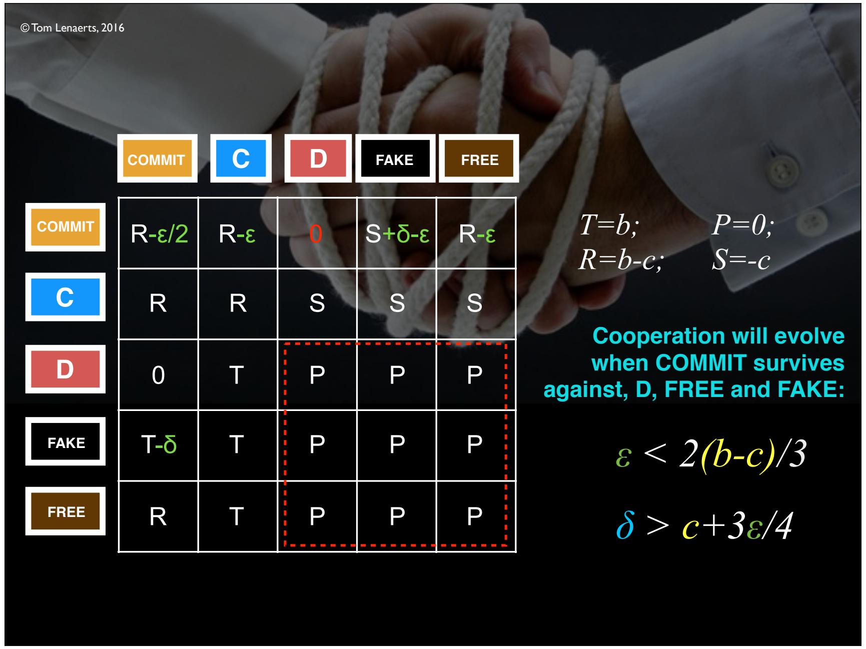
Q1
propose (at a cost ε)
NOT propose
Q2
OUT
accept
NOT accept
IN
OUT
A player can be in two possible states : IN or OUT
States determine action in the game

47

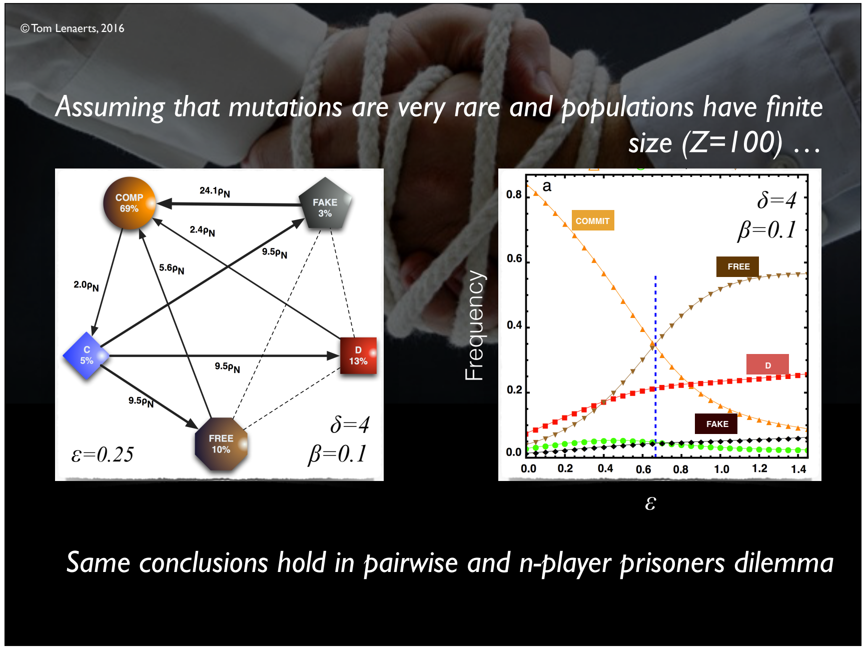


48



	COMMIT	C	D	FAKE	FREE
COMMIT	R-ε/2	R-ε	0	S+δ-ε	R-ε
C	R	R	S	S	S
D	0	T	P	P	P
FAKE	T-δ	T	P	P	P
FREE	R	T	P	P	P

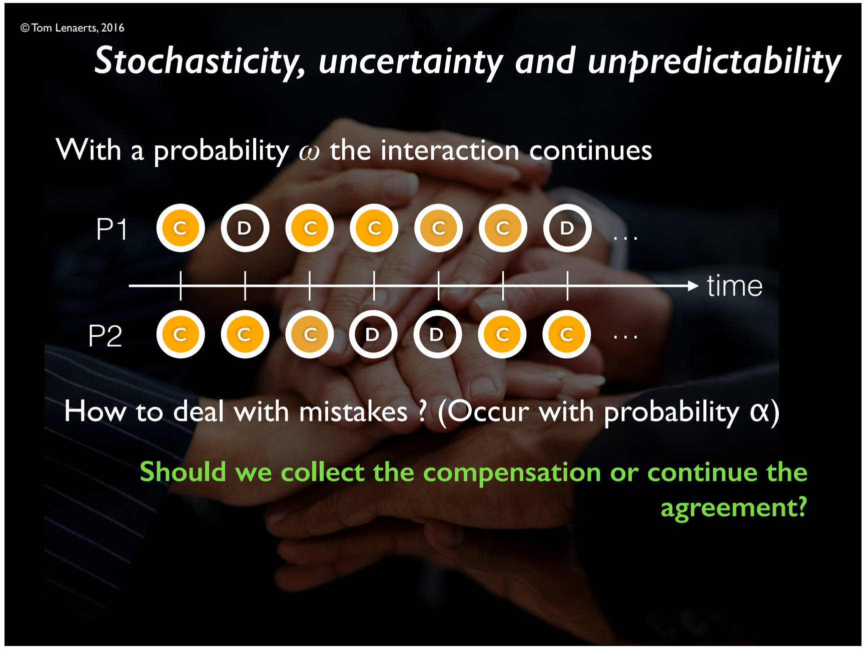
49



50

51

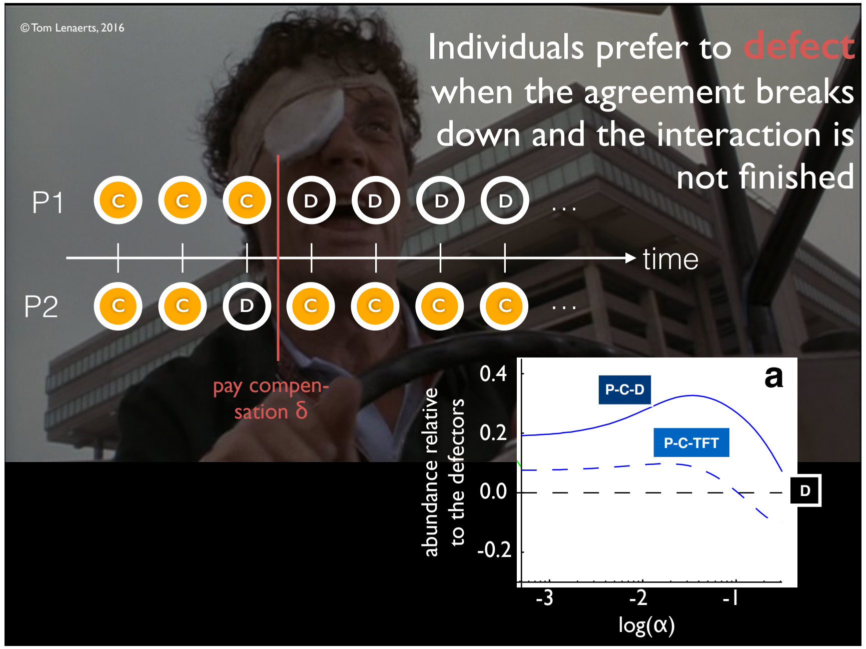
52-1



52-2

52-3

53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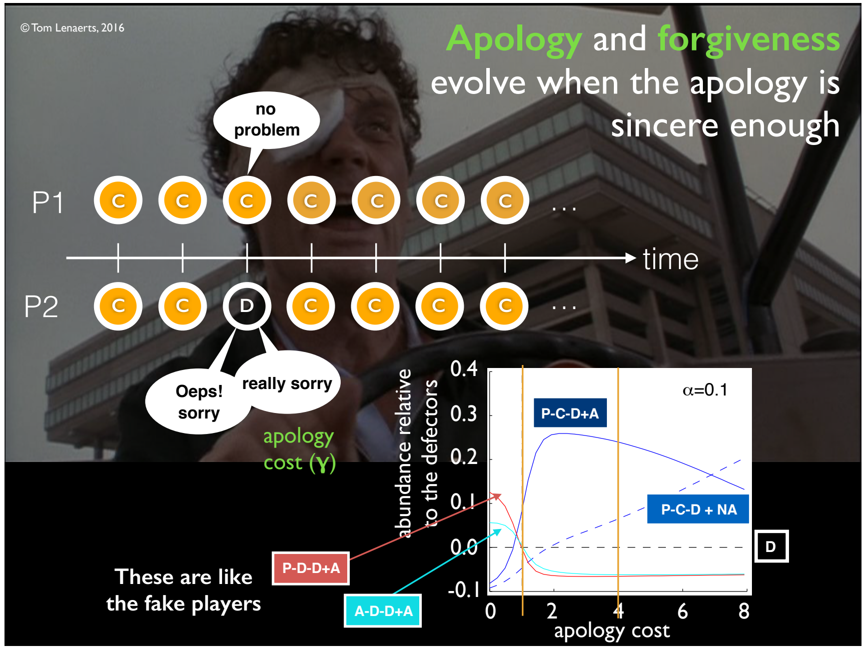
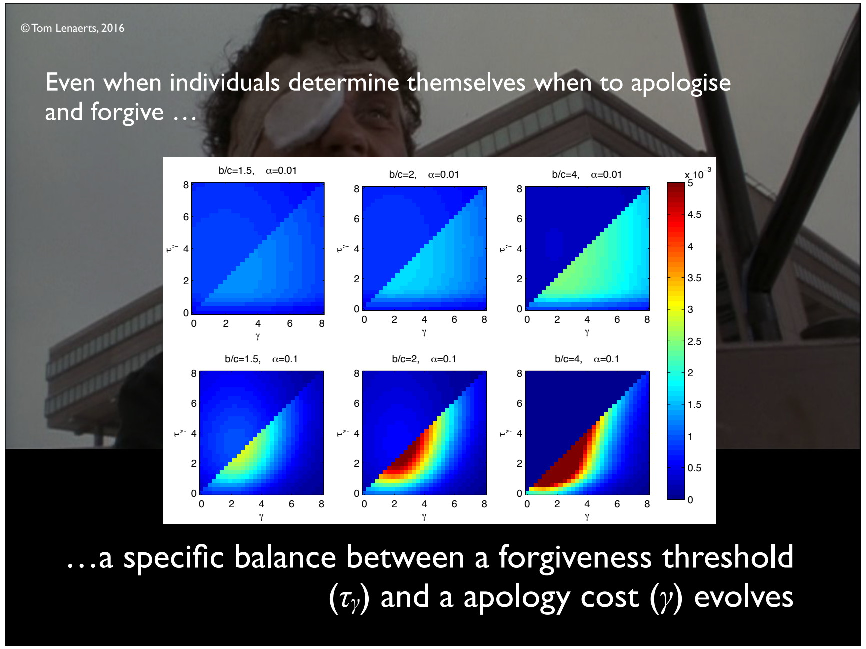
Individuals prefer to defect when the agreement breaks down and the interaction is not finished
P1
C C C D D D D …
time
P2
C C D C C C C …
pay compen-
sation δ
abundance relative to the defectors
0.4
0.2
0.0
-0.2
a
P-C-D
P-C-TFT
D
-3
-2
-1
log(α)

54


©Tom Lenaerts, 2016
Apology and forgiveness evolve when the apology is sincere enough
no problem
P1
C C C C C C C …
time
P2
C C D C C C C …
Oeps! sorry
really sorry
apology cost (γ)
These are like the fake players
abundance relative to the defectors
0.4
0.3
0.2
0.1
0.0
-0.1
α=0.1
P-C-D+A
P-C-D + NA
P-D-D+A
A-D-D+A
D
0
2
4
6
8
apology cost

55


©Tom Lenaerts, 2016
Even when individuals determine themselves when to apologise and forgive …
b/c=1.5, α=0.01
b/c=2, α=0.01
b/c=4, α=0.01
b/c=1.5, α=0.1
b/c=2, α=0.1
b/c=4, α=0.1
…a specific balance between a forgiveness threshold (τ_γ) and a apology cost (γ) evolves

56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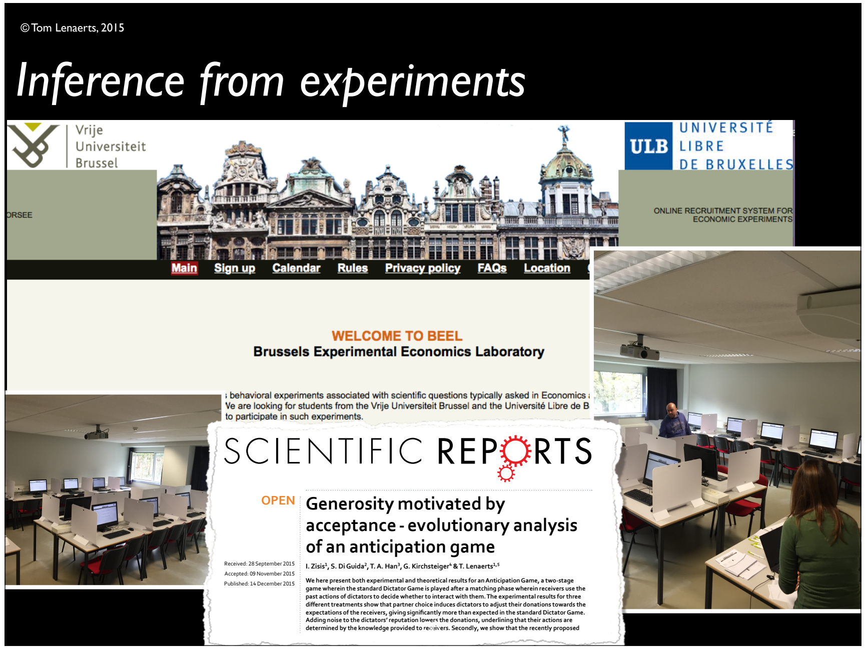
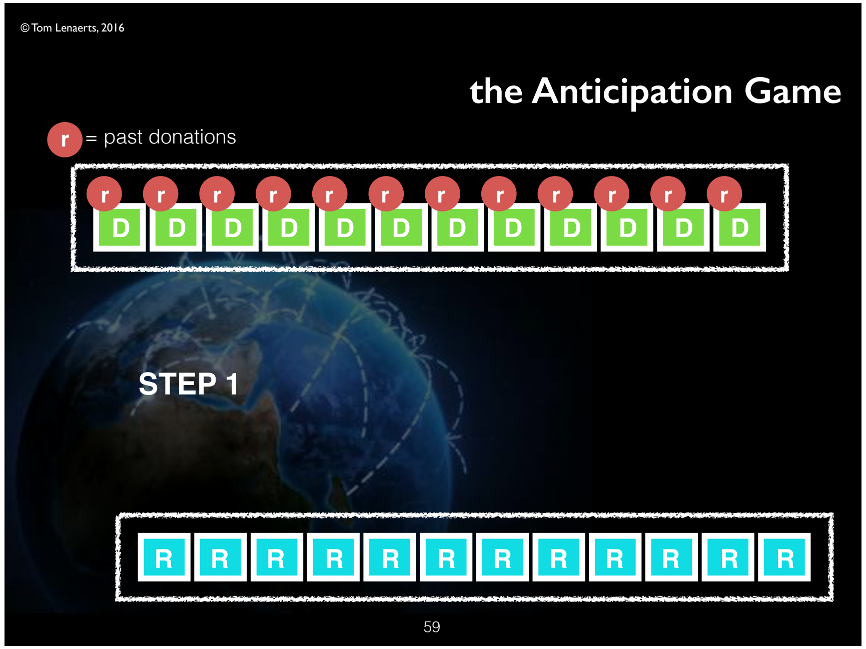
How do people really behave in games?
57

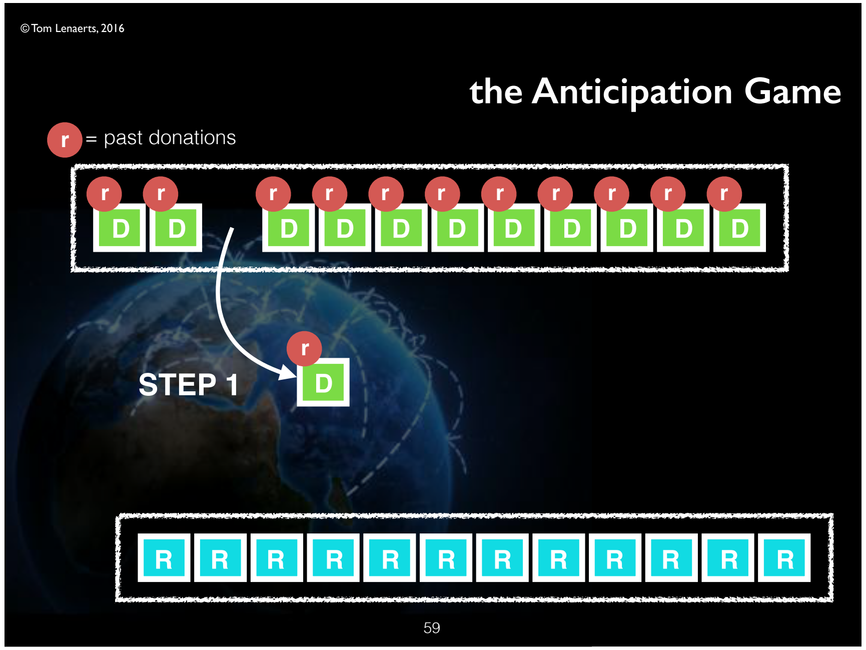
57



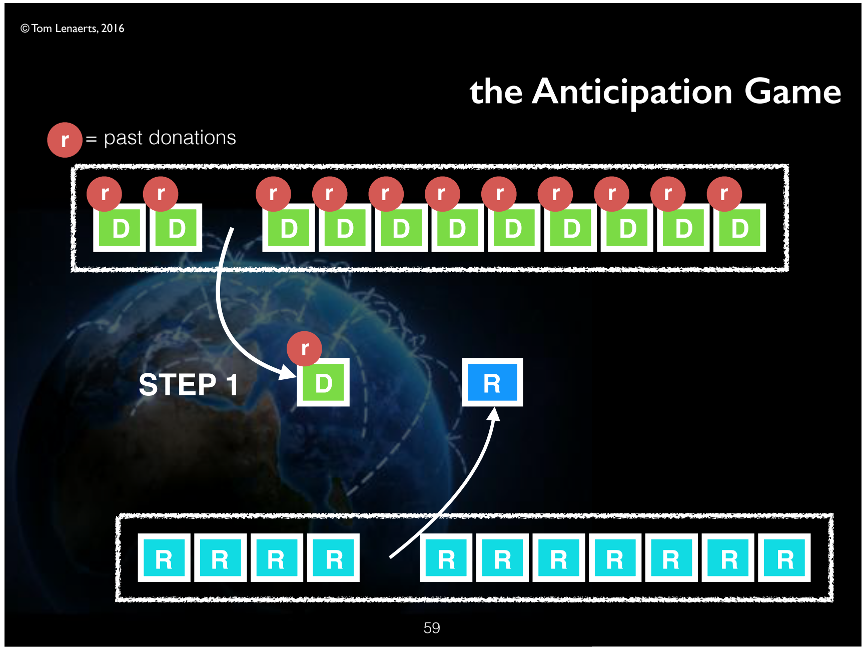
58



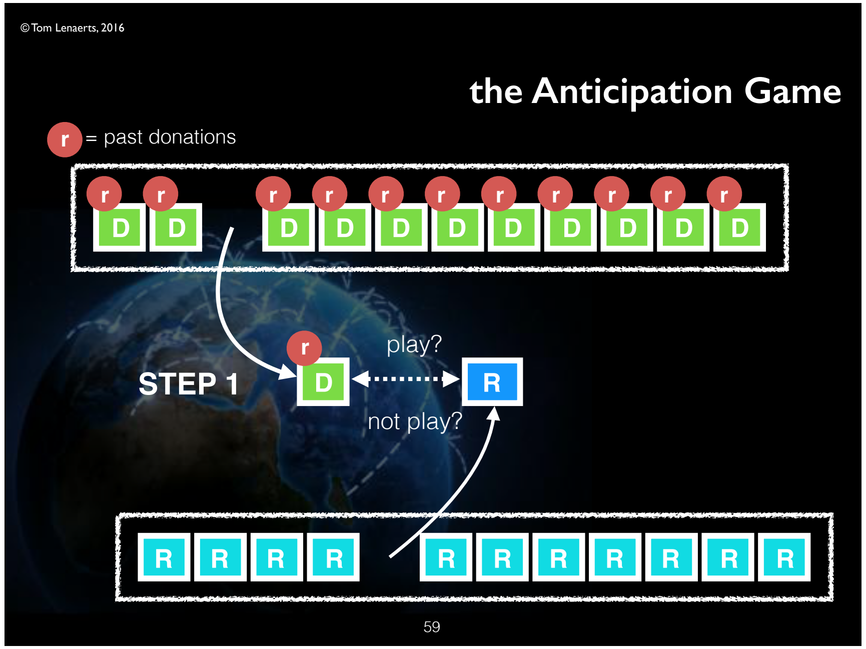
59-1



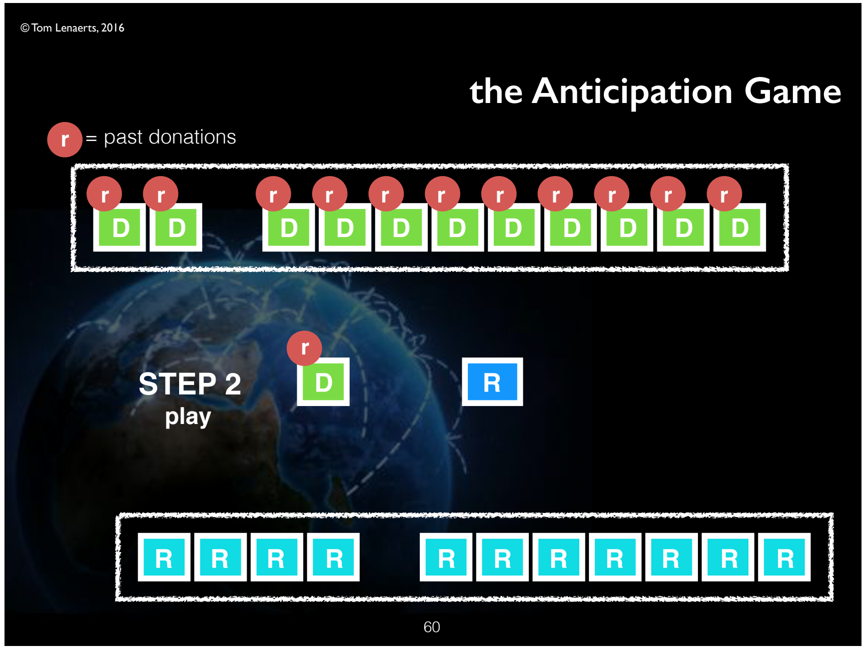
59-2



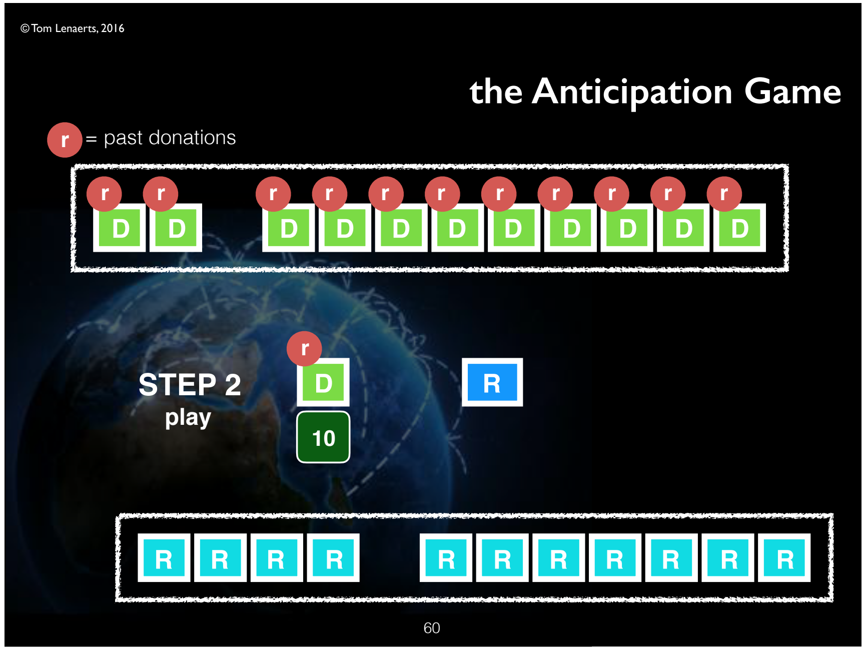
59-3



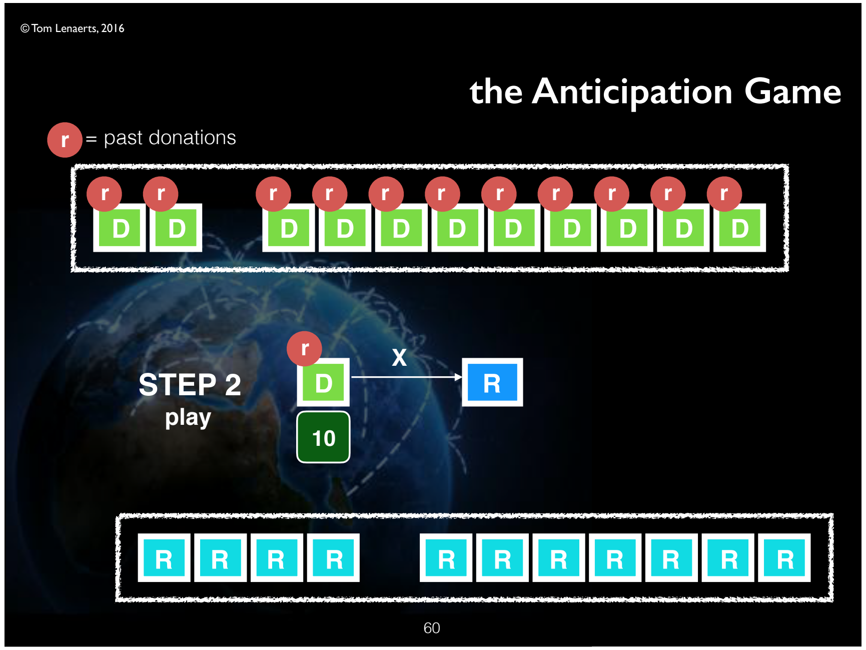
59-4



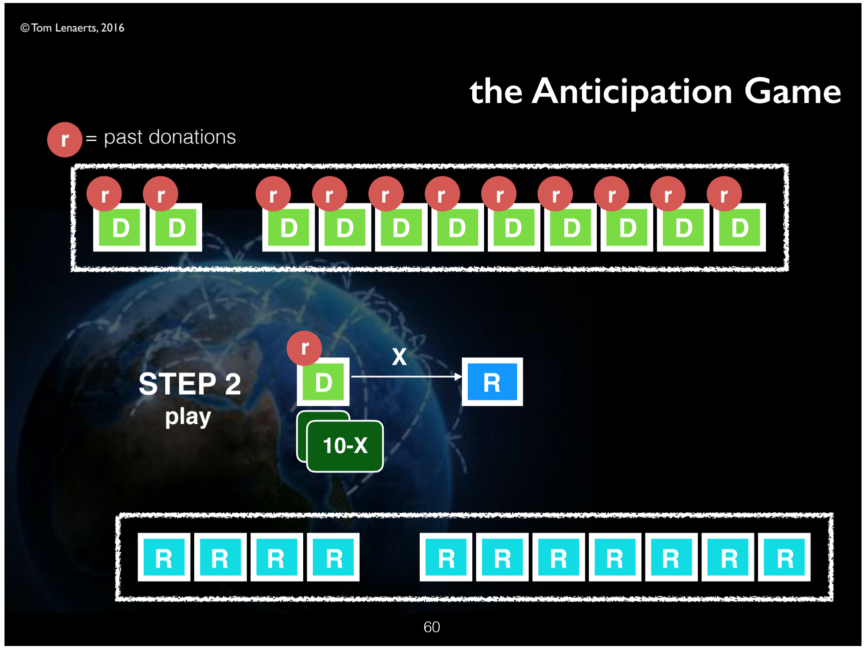
60-1



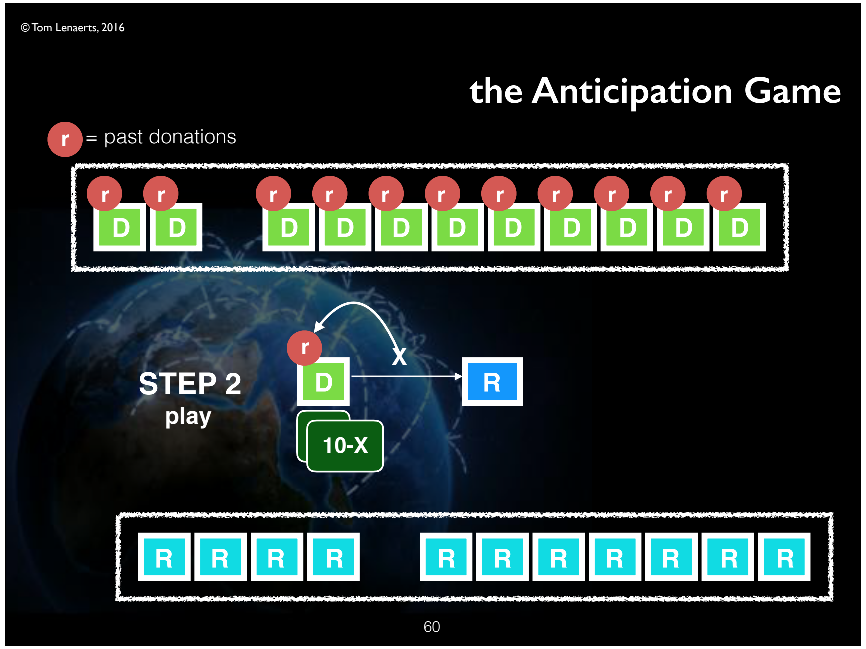
60-2



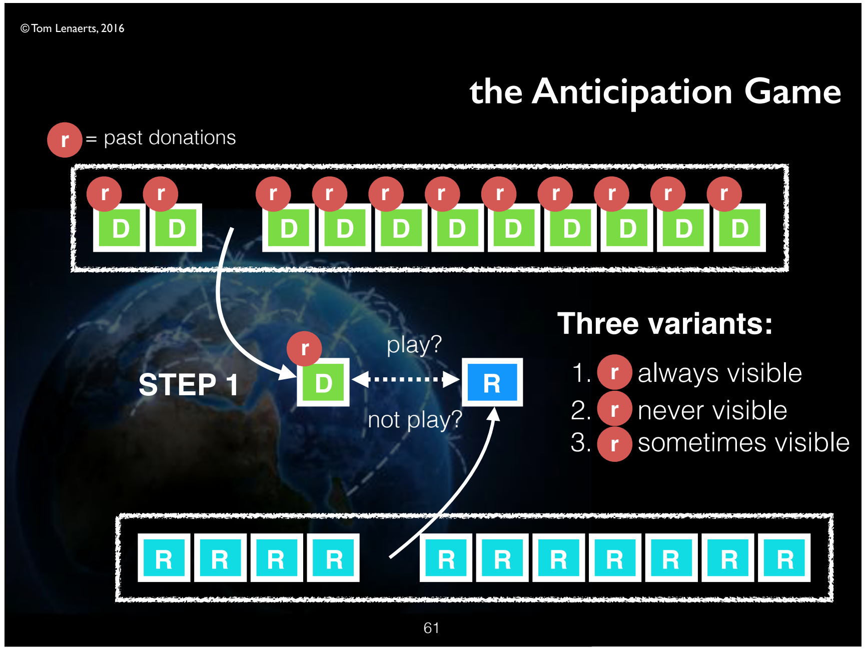
60-3


©Tom Lenaerts, 2016
the Anticipation Game
r = past donations
STEP 2
play
X
10-X
60

60-4


©Tom Lenaerts, 2016
the Anticipation Game
r = past donations
STEP 2
play
X
10-X
60

60-5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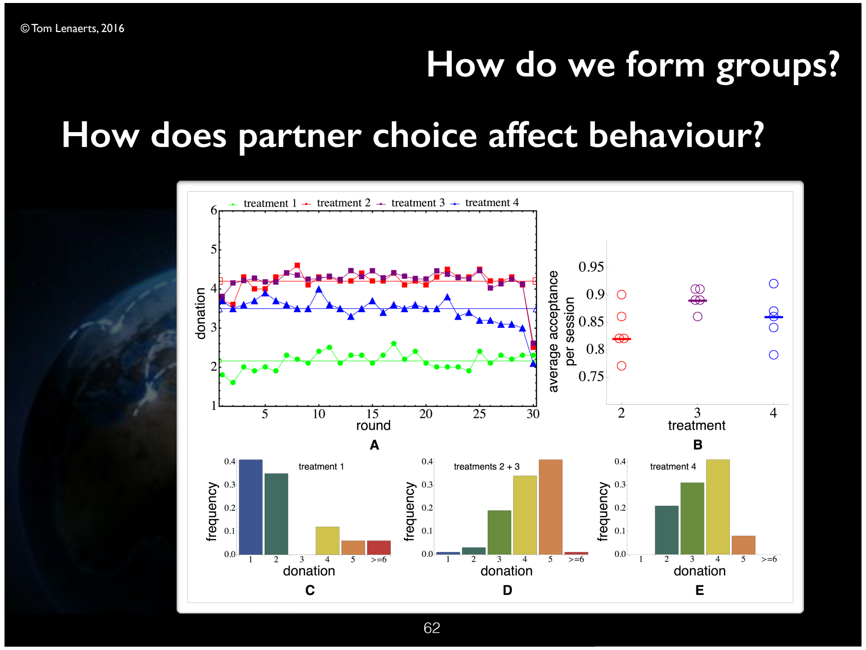
the Anticipation Game
r = past donations
STEP 1
play?
not play?
Three variants:
1. r always visible
2. r never visible
3. r sometimes visible
61

61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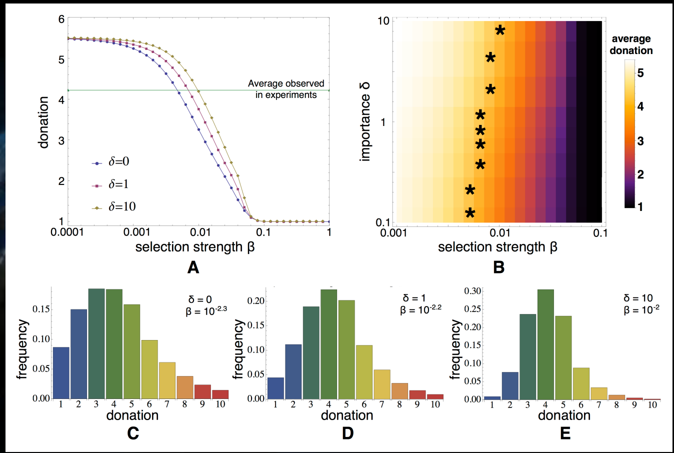
How do we form groups?
How does partner choice affect behaviour?
treatment 1 treatment 2 treatment 3 treatment 4
donation
round
A
average acceptance per session
treatment
B
frequency
donation
C
D
E
62

62

Dictator success depends on their capacity to anticipate



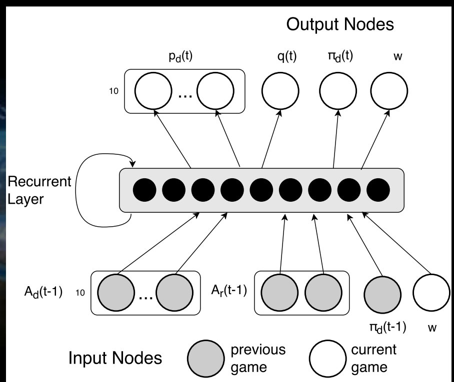
63

Psychological mechanisms for the evolution of cooperation?

How to model these cognitive mechanisms?

64

Recurrent Neural Networks



Elman, J. L. 1990. Finding structure in time. Cognitive Science 14(2):179–211

Lalev, E., and Grinberg, M. 2006. Backward vs. forward- oriented decision making in the iterated prisoners dilemma: A comparison between two connectionist models. In Workshop on Anticipatory Behavior in Adaptive Learning Systems, 345–364. Springer.

65-1

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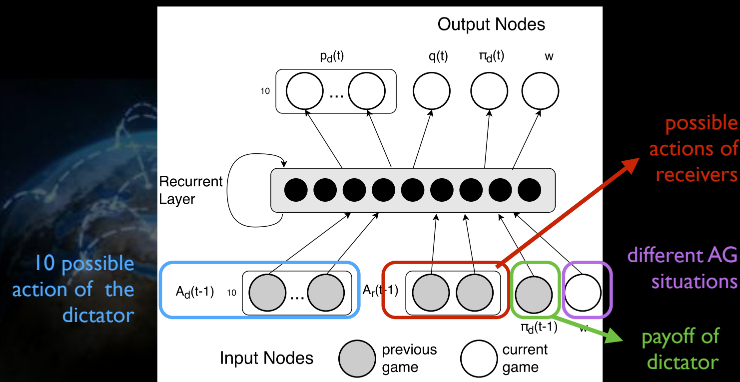
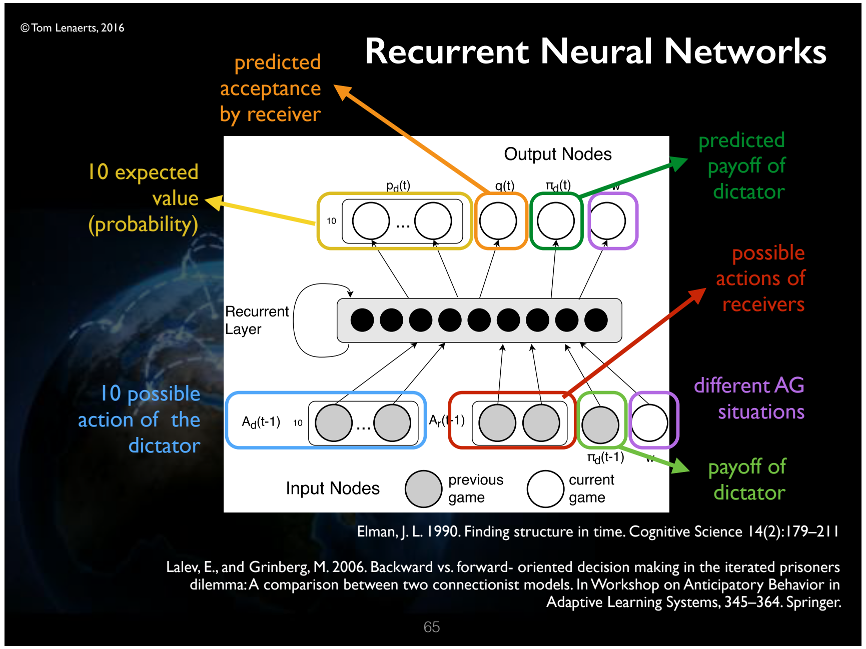
Recurrent Neural Networks



Elman, J. L. 1990. Finding structure in time. Cognitive Science 14(2):179–211

Lalev, E., and Grinberg, M. 2006. Backward vs. forward- oriented decision making in the iterated prisoners dilemma: A comparison between two connectionist models. In Workshop on Anticipatory Behavior in Adaptive Learning Systems, 345–364. Springer.

65-2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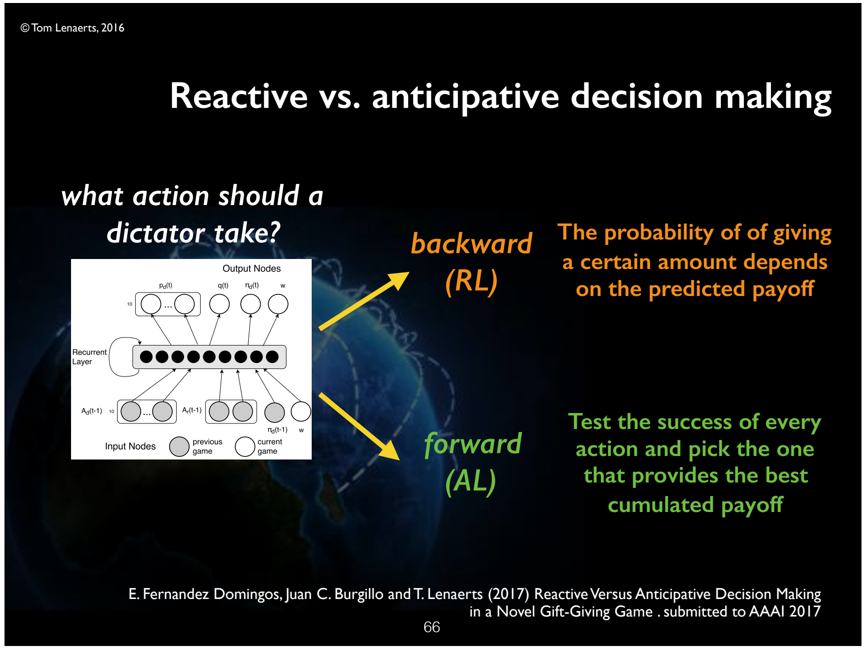
Recurrent Neural Networks
predicted acceptance by receiver
10 expected value (probability)
predicted payoff of dictator
possible actions of receivers
different AG situations
10 possible action of the dictator
payoff of dictator
Output Nodes
Recurrent Layer
Input Nodes
previous game
current game
Elman, J. L. 1990. Finding structure in time. Cognitive Science 14(2):179–211
Lalev, E., and Grinberg, M. 2006. Backward vs. forward- oriented decision making in the iterated prisoners dilemma: A comparison between two connectionist models. In Workshop on Anticipatory Behavior in Adaptive Learning Systems, 345–364. Springer.
65

65-3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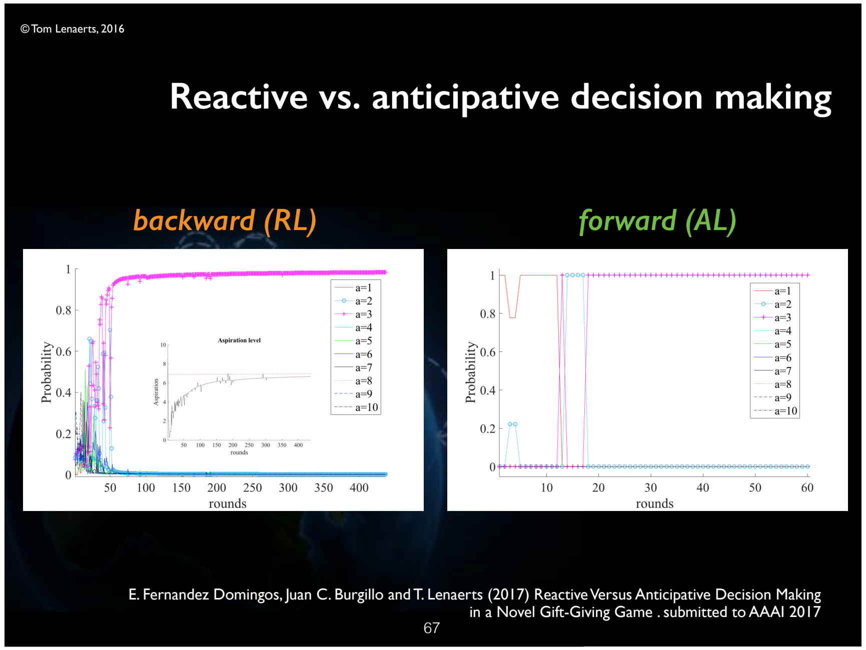
Reactive vs. anticipative decision making
what action should a dictator take?
backward (RL)
The probability of of giving a certain amount depends on the predicted payoff
forward (AL)
Test the success of every action and pick the one that provides the best cumulated payoff
Output Nodes
Recurrent Layer
Input Nodes
E. Fernandez Domingos, Juan C. Burgillo and T. Lenaerts (2017) Reactive Versus Anticipative Decision Making in a Novel Gift-Giving Game . submitted to AAAI 2017
66

66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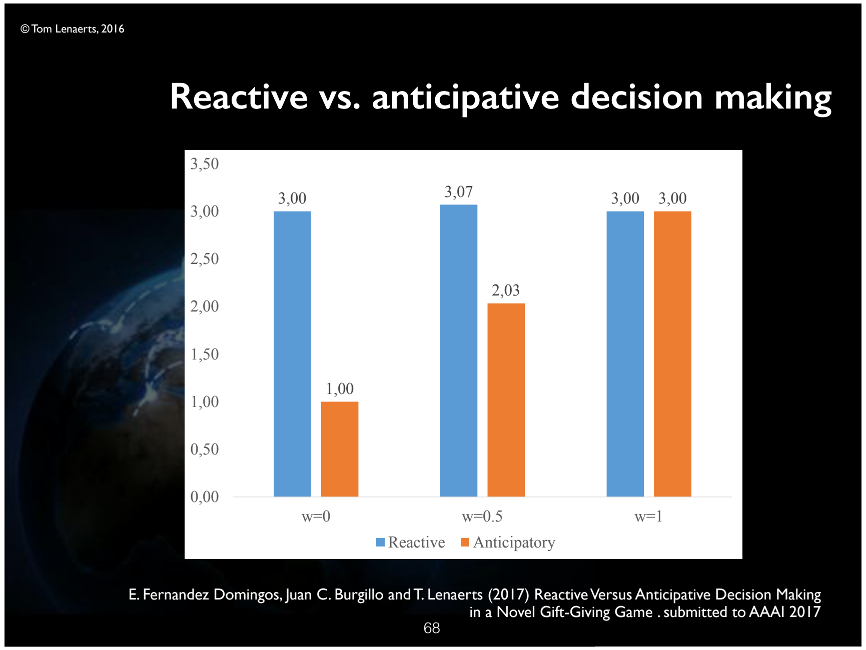
Reactive vs. anticipative decision making
backward (RL)
forward (AL)
Probability
rounds
Aspiration level
E. Fernandez Domingos, Juan C. Burgillo and T. Lenaerts (2017) Reactive Versus Anticipative Decision Making in a Novel Gift-Giving Game . submitted to AAAI 2017
67

67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Reactive vs. anticipative decision making
3,00
1,00
3,07
2,03
3,00
3,00
w=0
w=0.5
w=1
Reactive
Anticipatory
E. Fernandez Domingos, Juan C. Burgillo and T. Lenaerts (2017) Reactive Versus Anticipative Decision Making in a Novel Gift-Giving Game . submitted to AAAI 2017
68

68

What to conclude?

Changes in r changes the environment for decision making

To reproduce individual behaviour → introduce payoff received for future gains

Anticipatory decision making produces correct response to a changing environment

"Every living systems is an anticipating system"
(R. Rosen 1985)

69

69

Questions

70

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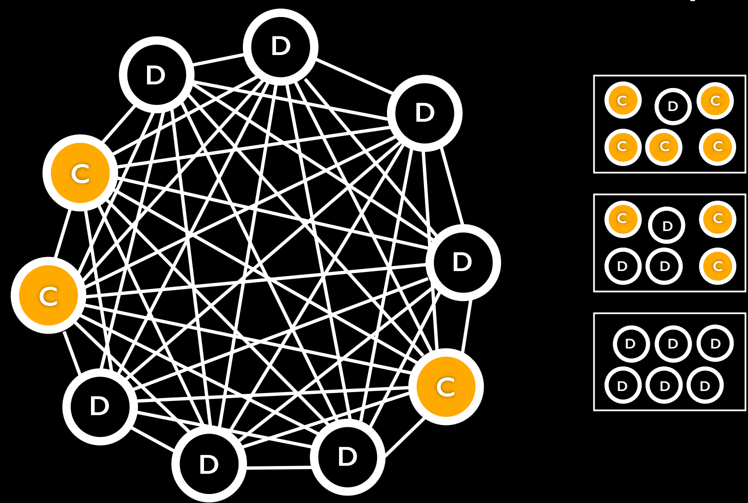
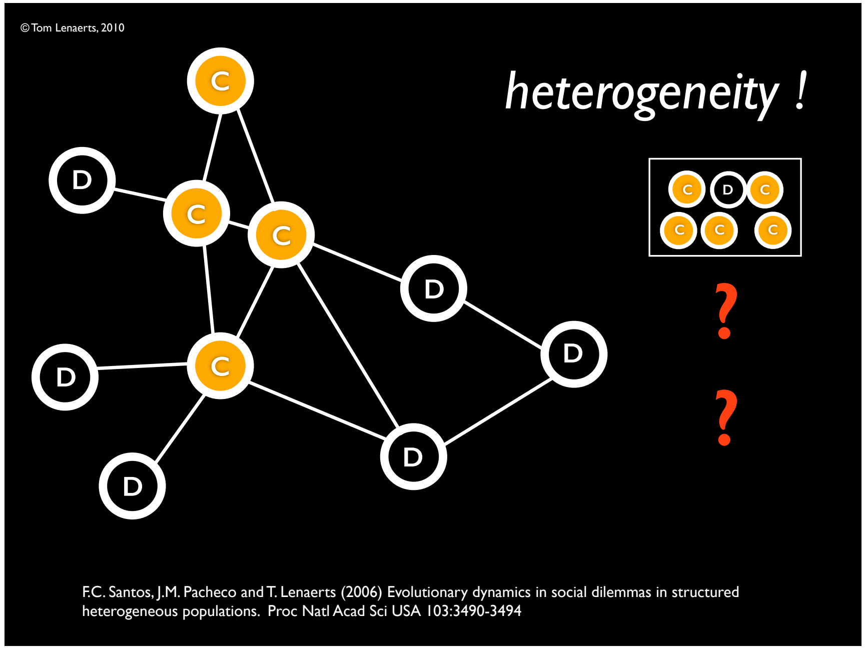
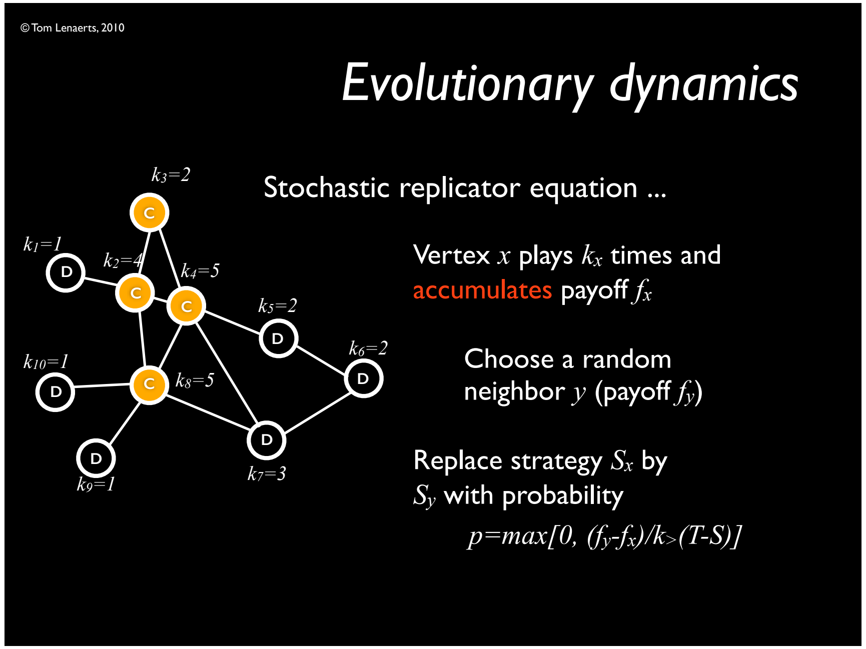
The well-mixed assumption



72


©Tom Lenaerts, 2010
heterogeneity !
C
D
?
?
F.C. Santos, J.M. Pacheco and T. Lenaerts (2006) Evolutionary dynamics in social dilemmas in structured heterogeneous populations. Proc Natl Acad Sci USA 103:3490-3494

73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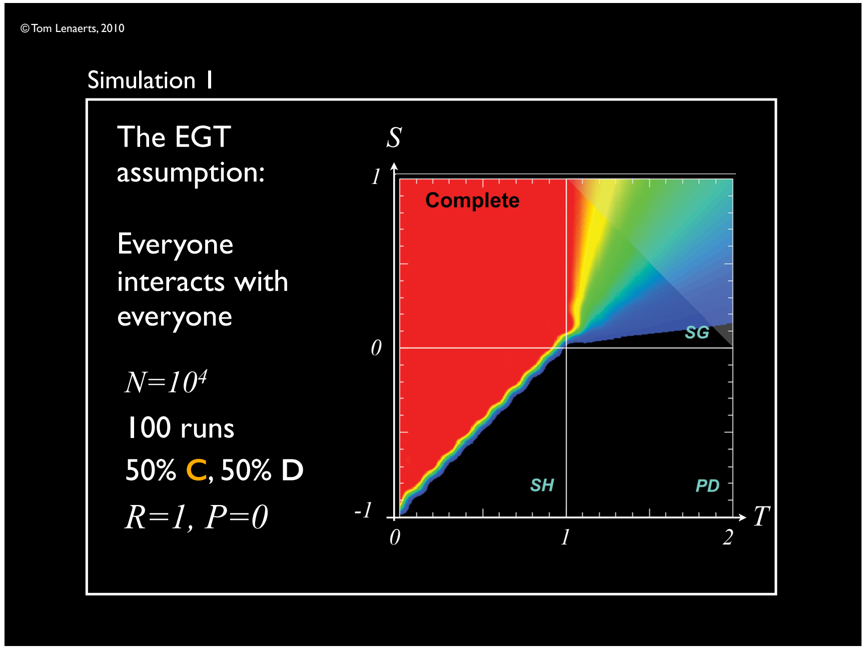
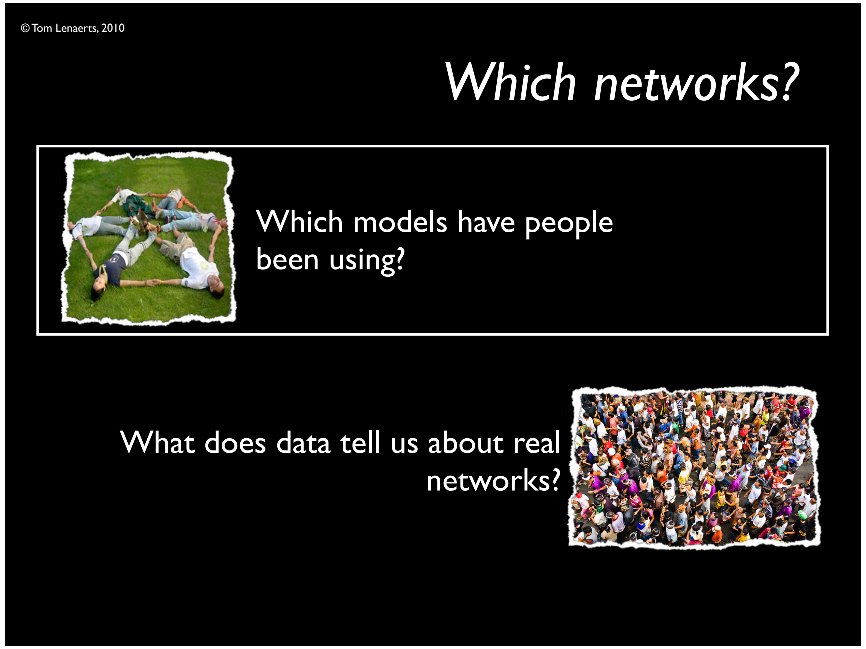
Evolutionary dynamics
Stochastic replicator equation ...
$k_1=1$ $k_2=4$ $k_3=2$ $k_4=5$ $k_5=2$ $k_6=2$ $k_7=3$ $k_8=5$ $k_9=1$ $k_{10}=1$
Vertex x plays k_x times and accumulates payoff f_x
Choose a random neighbor y (payoff f_y)
Replace strategy S_x by S_y with probability
$p=max[0, (f_y-f_x)/k_>(T-S)]$

74


©Tom Lenaerts, 2010
Simulation I
The EGT assumption:
Everyone interacts with everyone
$N=10^4$
100 runs
50% C, 50% D
$R=1, P=0$
S
1
0
-1
Complete
SG
SH
PD
0
1
2
T

75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Which networks?
Which models have people been using?
What does data tell us about real networks?

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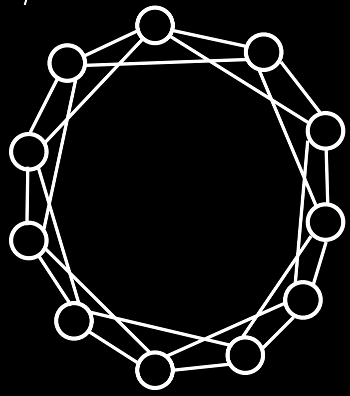
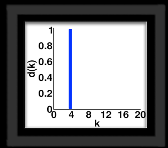
Regular graphs

Every node has exactly the same degree $<k>=4$



regular and democratic network



77

Simulation II

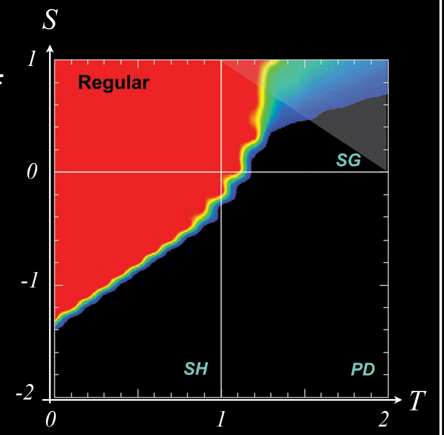
There is a limit to the number of interactions but its democratic

$N=10^4$ $<k>=4$

100 runs

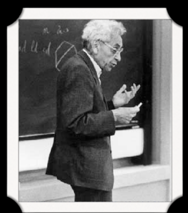
50% C, 50% D

$R=1, P=0$



78

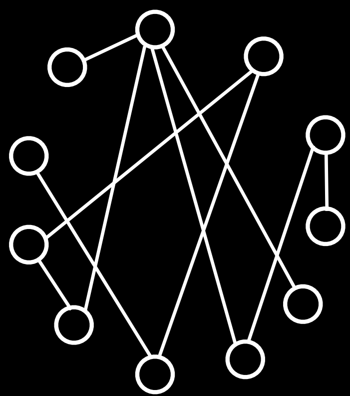
Random graphs



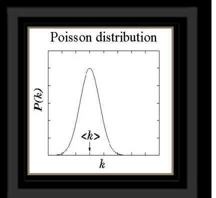
P. Erdős (1913-1996)

Connect with probability p

$p=1/6,\ N=10$
$\Rightarrow <k>\approx 1.5$



random and democratic network



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Simulation III

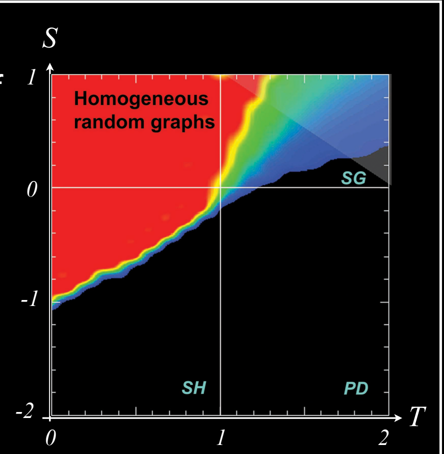
There is a limit to the number of interactions but its democratic and random

$N=10^4$ $<k>=4$

100 runs

50% C, 50% D

$R=1, P=0$



80

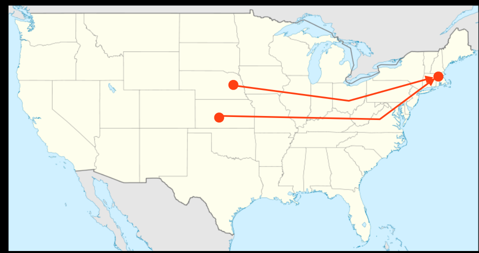
Which networks?

Which models have people been using?

What does data tell us about real networks?

Small world experiment

What is the average number of connections between any two people?

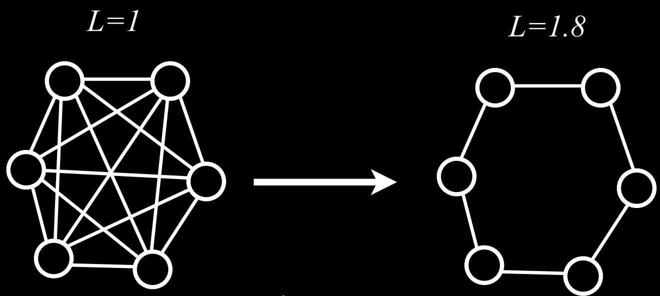


S. Milgram (1933-1984)

“Six degrees of separation“ (J. Guare, 1990)

J. Travers and S. Milgram (1969) An experimental study of the small-world problem. Sociometry 32(4):425-443

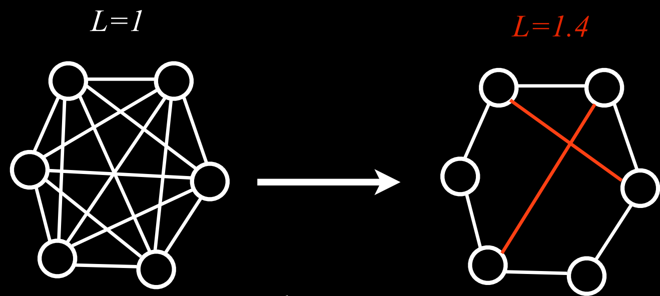
Average path length



$$L=\frac{1}{N(N-1)}\sum_{i\neq j} d_{min}(i,j)$$

The average path length (L) is a measure of proximity between nodes

Average path length

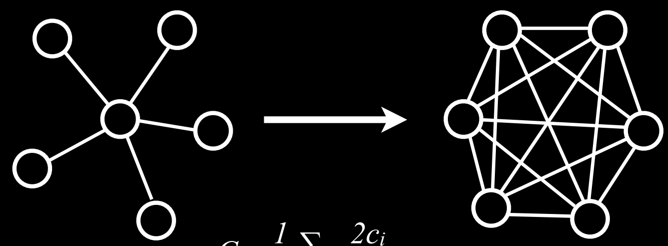


$$L=\frac{1}{N(N-1)}\sum_{i\neq j} d_{min}(i,j)$$

The average path length (L) is a measure of proximity between nodes

Cluster coefficient

C=0 C=1



$$C=\frac{1}{N}\sum_i \frac{2c_i}{k_i(k_i-1)}$$

The cluster coefficient (C) is a measure for cliquishness

84-1

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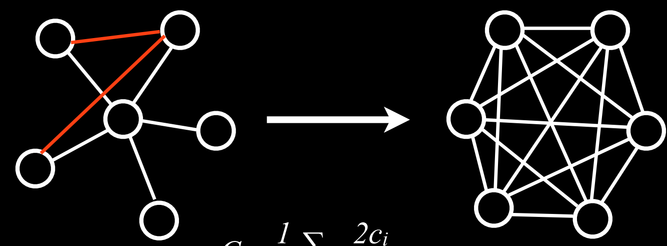
Cluster coefficient

C≈0.5 C=1



$$C=\frac{1}{N}\sum_i \frac{2c_i}{k_i(k_i-1)}$$

The cluster coefficient (C) is a measure for cliquishness

84-2

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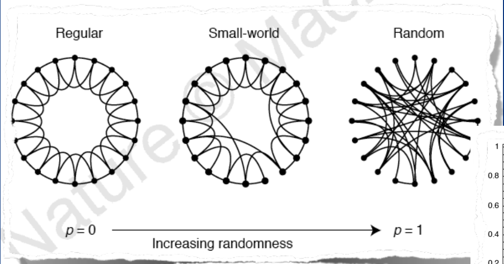
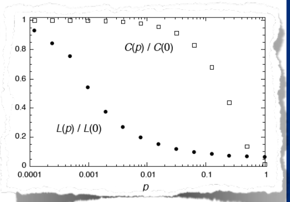
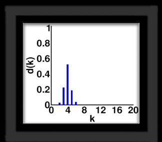
Small world networks



electrical power grid of South California, network of world airports, movie-actor network, the neuronal network of the worm C.elegans



D.Watts and S. Strogatz (1998) Collective dynamics of 'small-world' networks. Nature 393:440-442



Mechanism:

1. take a regular graph
2. randomly rewire every edge with probability p
3. avoid loops and double edges

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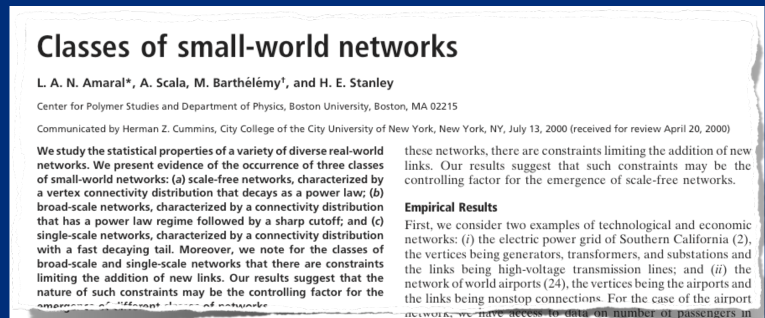
Network classes



Classes of small-world networks

L. A. N. Amaral*, A. Scala, M. Barthélémy†, and H. E. Stanley

Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215

Communicated by Herman Z. Cummins, City College of the City University of New York, New York, NY, July 13, 2000 (received for review April 20, 2000)

We study the statistical properties of a variety of diverse real-world networks. We present evidence of the occurrence of three classes of small-world networks: (a) scale-free networks, characterized by a vertex connectivity distribution that decays as a power law; (b) broad-scale networks, characterized by a connectivity distribution that has a power law regime followed by a sharp cutoff; and (c) single-scale networks, characterized by a connectivity distribution with a fast decaying tail. Moreover, we note for the classes of broad-scale and single-scale networks that there are constraints limiting the addition of new links. Our results suggest that the nature of such constraints may be the controlling factor for the emergence of different classes of networks.

these networks, there are constraints limiting the addition of new links. Our results suggest that such constraints may be the controlling factor for the emergence of scale-free networks.

Empirical Results

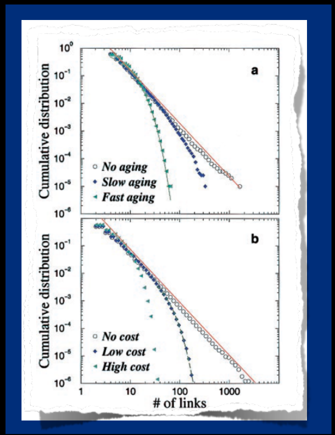
First, we consider two examples of technological and economic networks: (i) the electric power grid of Southern California (2), the vertices being generators, transformers, and substations and the links being high-voltage transmission lines; and (ii) the network of world airports (24), the vertices being the airports and the links being nonstop connections. For the case of the airport network, we have access to data on number of passengers in

L.A.N. Amaral, A. Scala, M. Barthelemy and H.E. Stanley (2000) Classes of small-world networks. Proc Natl Acad Sci USA 97(21): 11149-11152

electrical power grid of South California, network of world airports, movie-actor network, acquintance network of mormons, friendship network of 417 Madison Junior High school students, the neuronal network of the worm C.elegans, the conformational space of a lattice polymer chain

86

Network classes



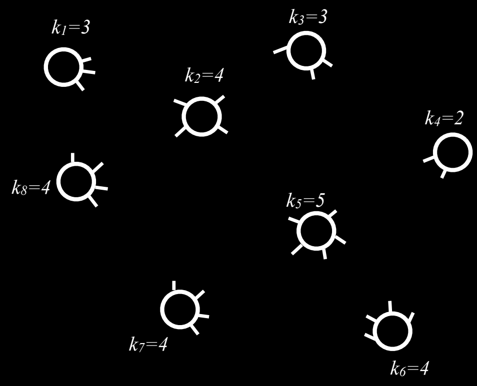
Aging of vertices as in the movie-actor network

cost of adding links or the limited capacity of vertices as in the airport network

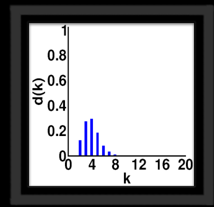
L.A.N. Amaral, A. Scala, M. Barthelemy and H.E. Stanley (2000) Classes of small-world networks. Proc Natl Acad Sci USA 97(21): 11149-11152

87

Configuration model



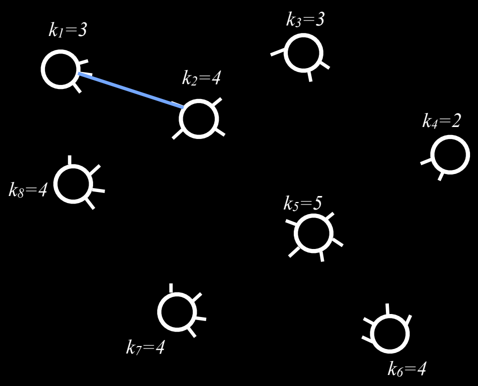
create random networks with a particular degree distribution



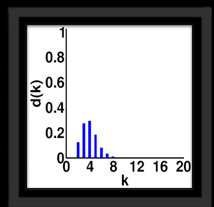
M. Molloy and B. Reed (1995) A critical point for random graphs with a given degree sequence. Random Struct. Algorithms 6:161-180

88-1

Configuration model



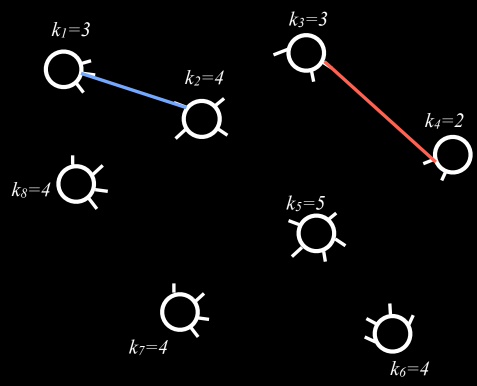
create random networks with a particular degree distribution



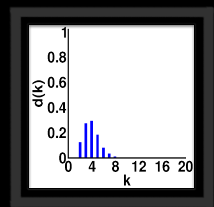
M. Molloy and B. Reed (1995) A critical point for random graphs with a given degree sequence. Random Struct. Algorithms 6:161-180

88-2

Configuration model

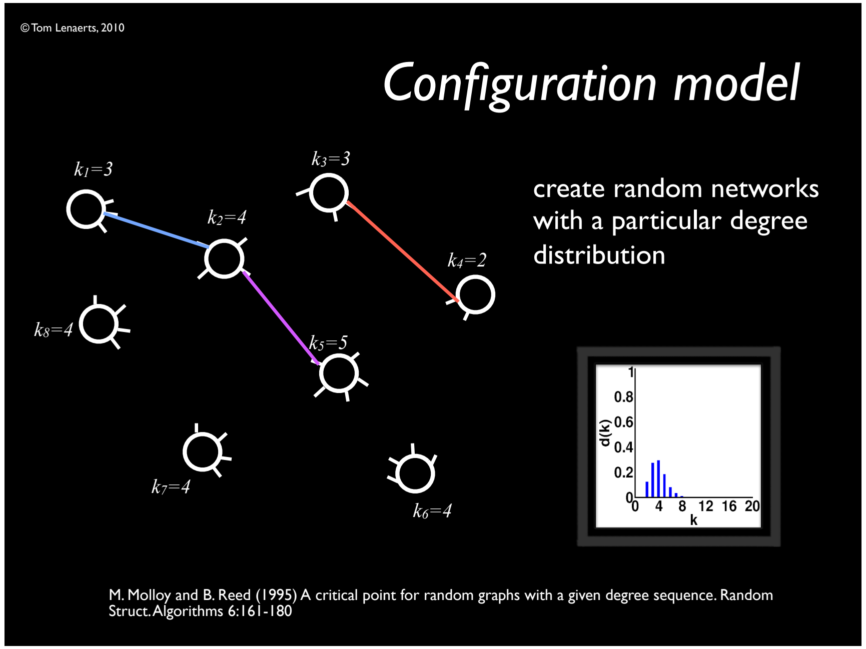


create random networks with a particular degree distribution



M. Molloy and B. Reed (1995) A critical point for random graphs with a given degree sequence. Random Struct. Algorithms 6:161-180

88-3



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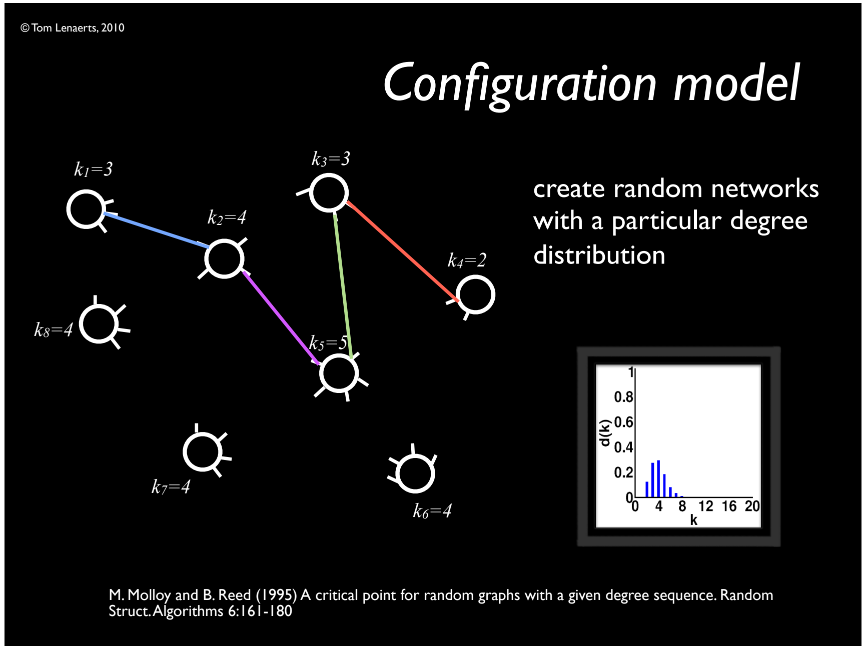

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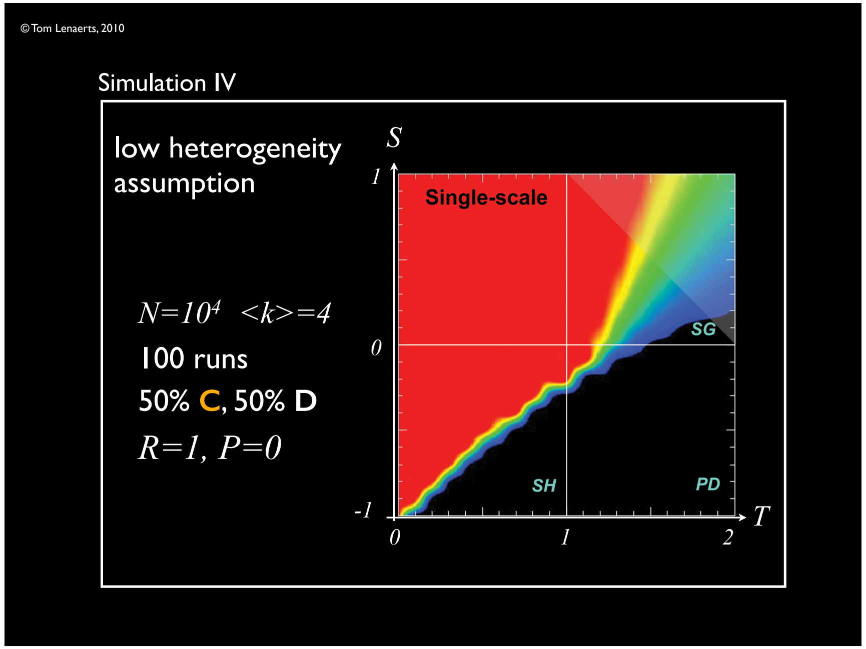
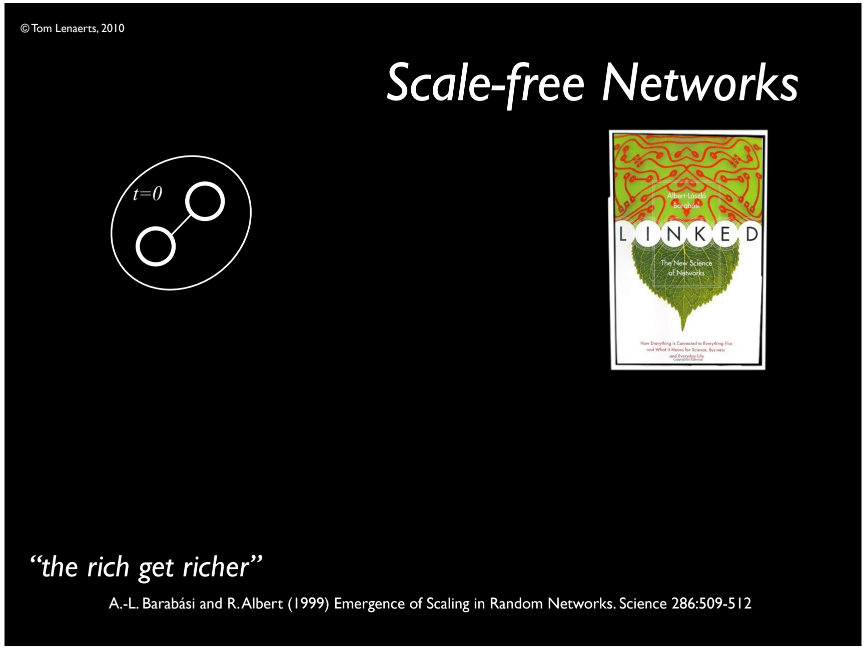
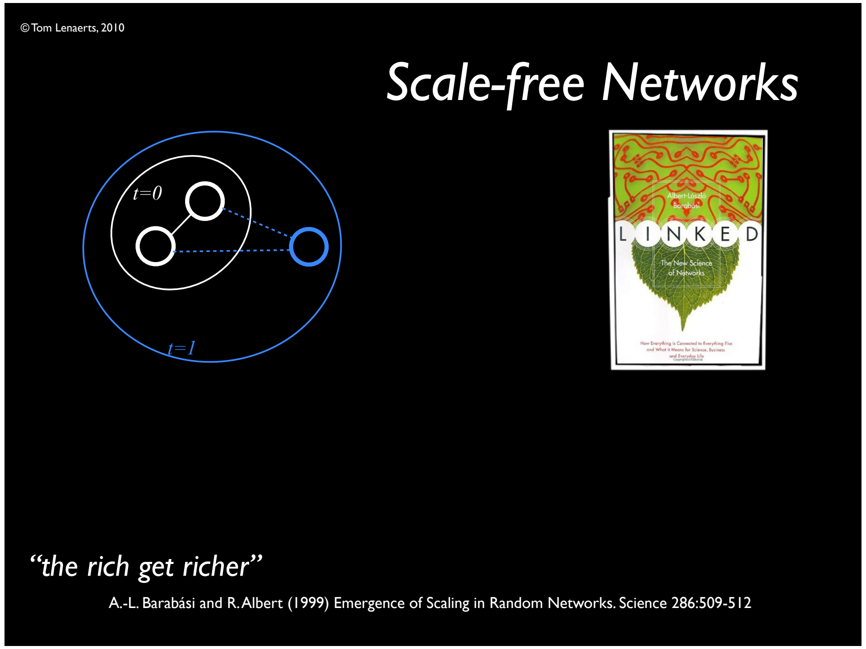


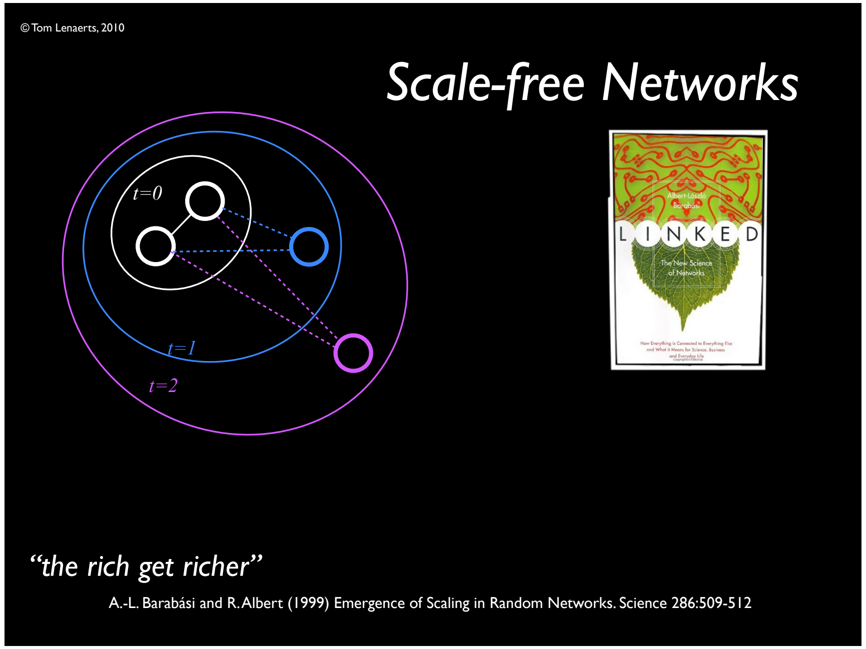
89



90-1



90-2



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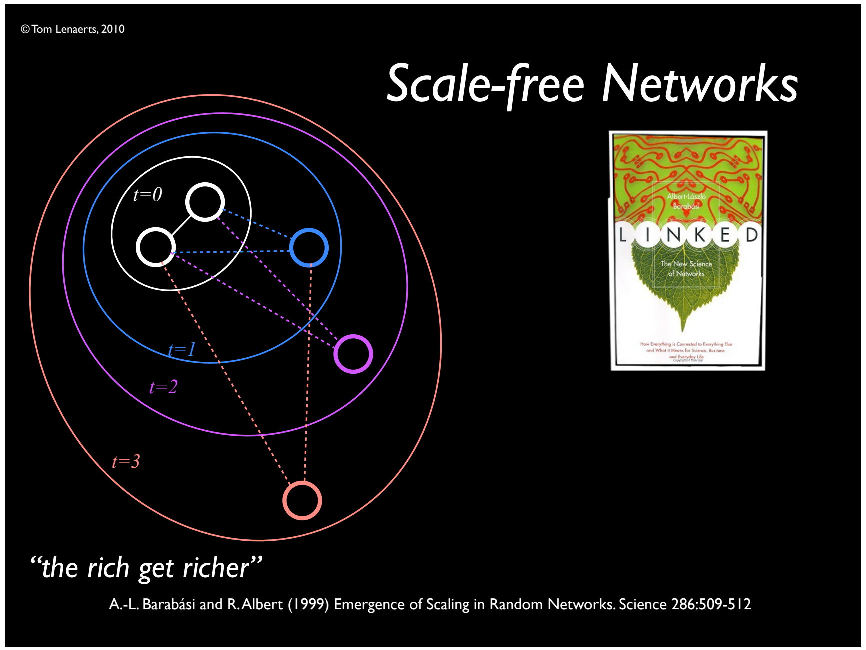

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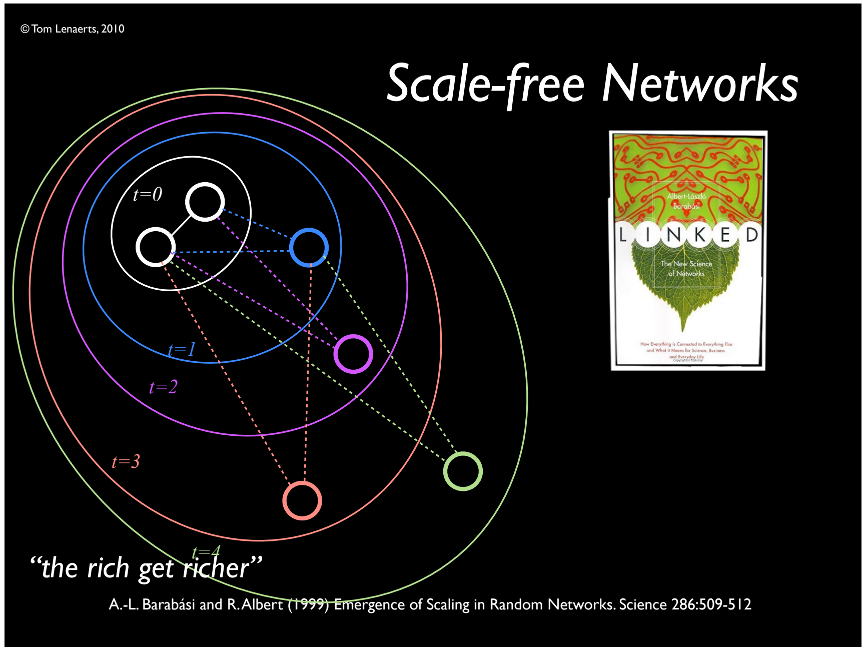

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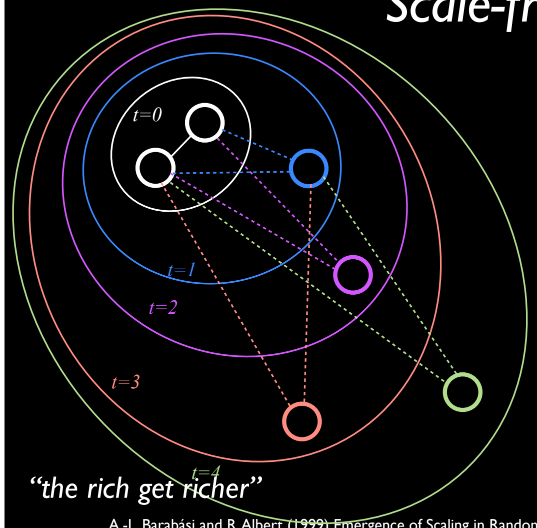
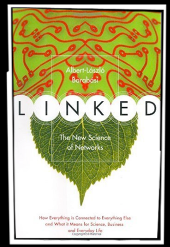
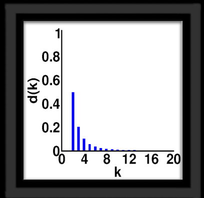
Scale-free Networks







"the rich get richer"

A.-L. Barabási and R. Albert (1999) Emergence of Scaling in Random Networks. Science 286:509-512

90-6

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Simulation V

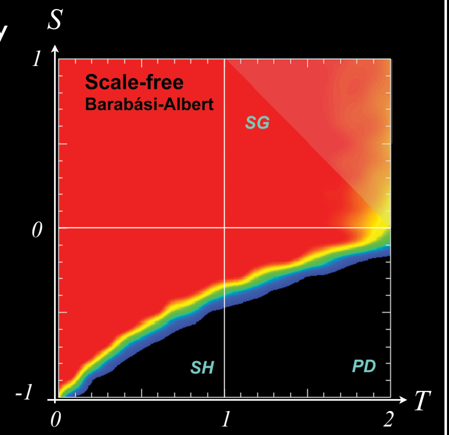
high heterogeneity assumption

$N=10^4$ $<k>=4$

100 runs

50% C, 50% D

$R=1,\ P=0$



91

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Simulation VI

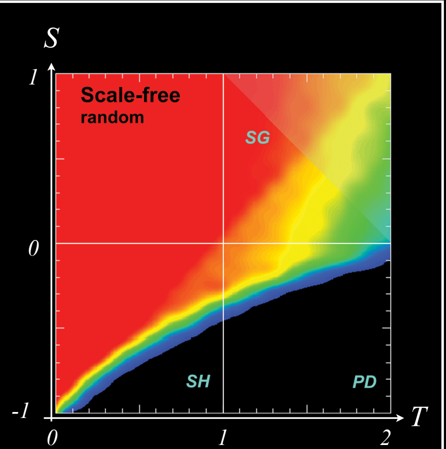
the rich are no longer friends

$N=10^4$ $<k>=4$

100 runs

50% C, 50% D

$R=1,\ P=0$



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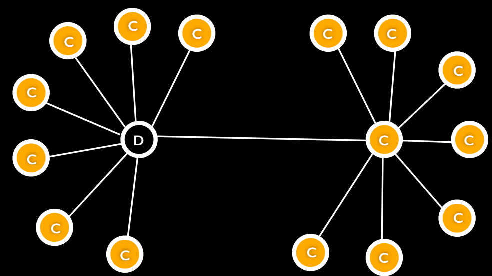
intuition

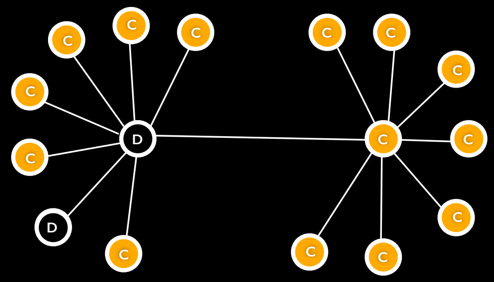
Defectors are victims of their own success …



93-1

intuition

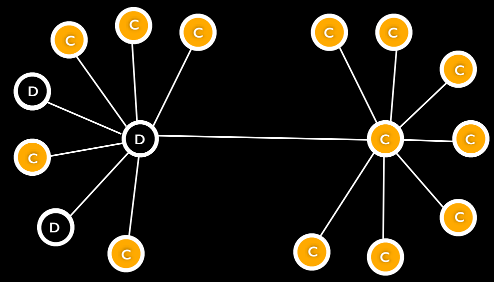
Defectors are victims of their own success ...



93-2

intuition

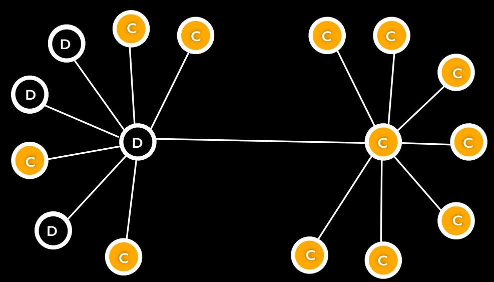
Defectors are victims of their own success ...



93-3

intuition

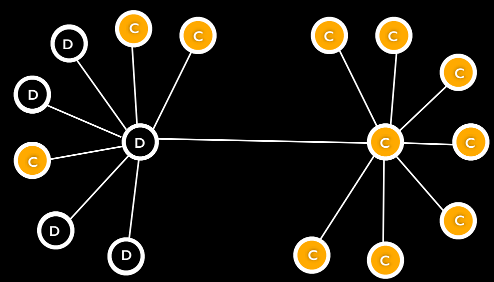
Defectors are victims of their own success ...



93-4

intuition

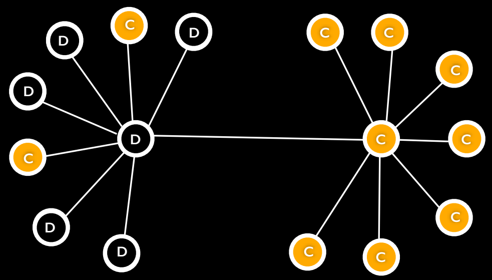
Defectors are victims of their own success ...



93-5

intuition

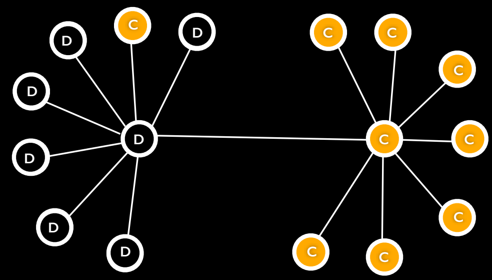
Defectors are victims of their own success ...



93-6

intuition

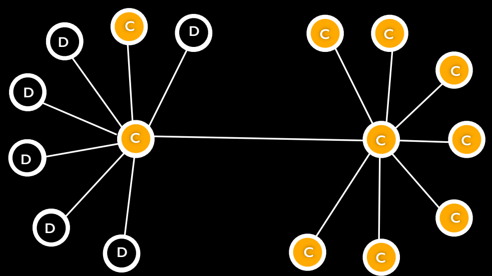
Defectors are victims of their own success ...



93-7

intuition

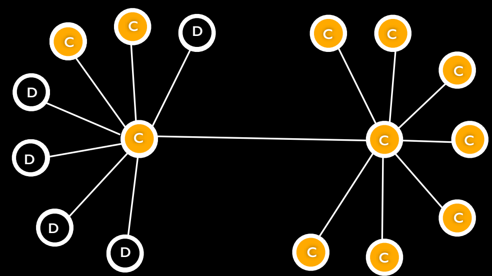
Defectors are victims of their own success ...



93-8

intuition

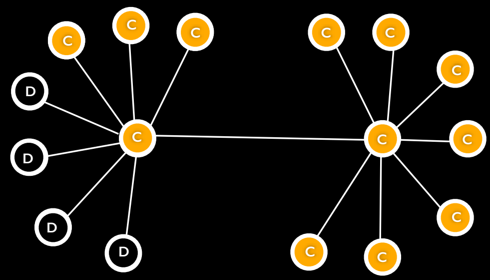
Defectors are victims of their own success ...



93-9

intuition

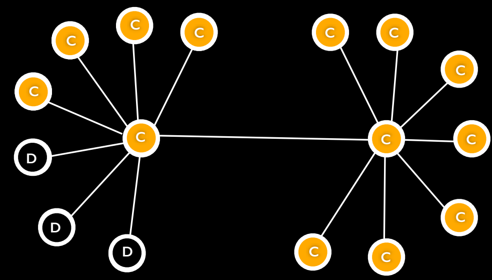
Defectors are victims of their own success ...



93-10

intuition

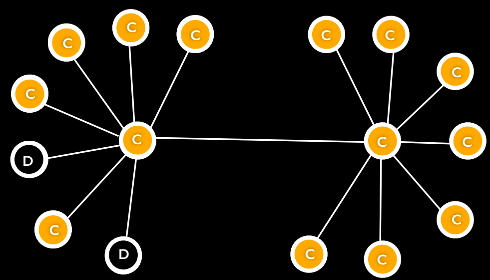
Defectors are victims of their own success ...



93-11

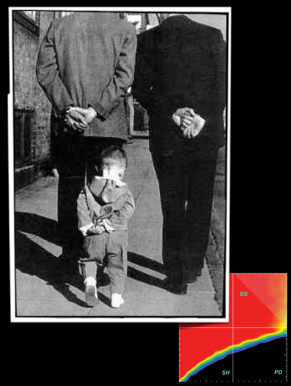
intuition

Defectors are victims of their own success ...



93-12

social versus individual learning



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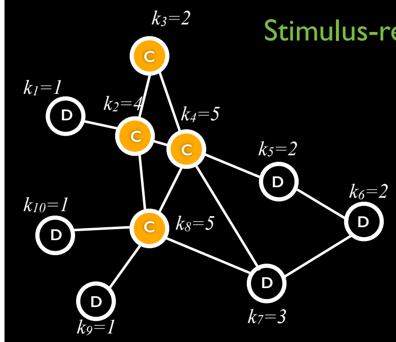
social versus individual learning



Stimulus-response learning ...

Vertex x plays **once** with a random neighbor y and **both receive a payoff** f_x (and f_y)

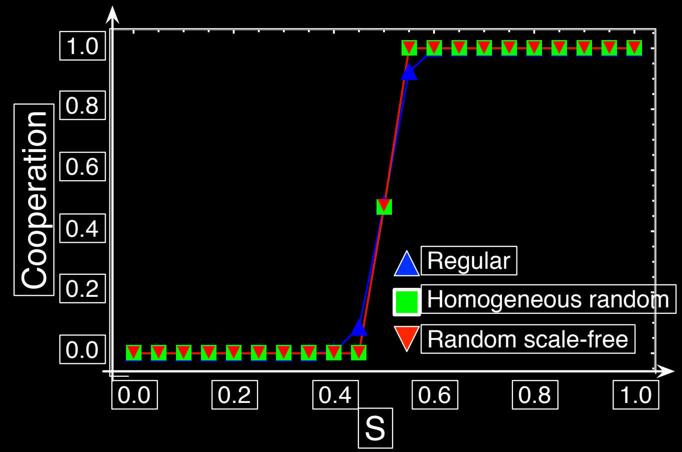
update strategy using the following model $i \in \{x,y\}$:

$p_i(t+1) = p_i(t) + \lambda\beta_i(t)*(1-p_i(t))$
when i played **C** at time t

$p_i(t+1) = p_i(t) - \lambda\beta_i(t)*p_i(t)$
when i played **D** at time t

95

social versus individual learning



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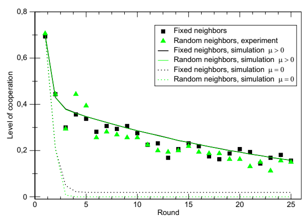
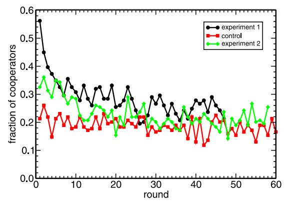
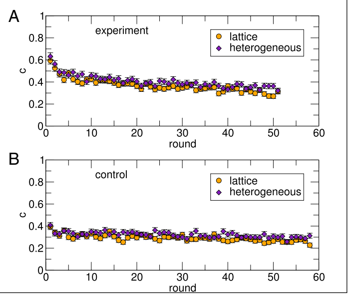
Presentation next week!







Traulsen, A., Semmann, D., Sommerfeld, R. D., Krambeck, H.-J., & Milinski, M. (2010). **Human strategy updating in evolutionary games.** Proceedings of the National Academy of Sciences, 107(7), 2962–2966.

Grujić, J., Fosco, C., Araujo, L., Cuesta, J. A., & Sánchez, A. (2010). **Social Experiments in the Mesoscale: Humans Playing a Spatial Prisoner's Dilemma.** PLoS ONE, 5(11), e13749.

Gracia-Lazaroa, C., Ferrera, A., Ruiza, G., Alfonso Tarancona, B., Jose A Cuestaa, C., Angel Sancheza, C., & Moreno, Y. (2012). **Heterogeneous networks do not promote cooperation when humans play a Prisoner's Dilemma.** Proceedings of the National Academy of Sciences, 109(32), 12922–12926.

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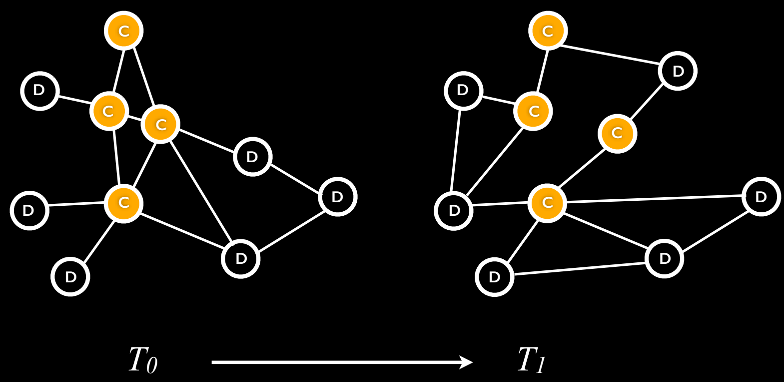
Networks are dynamic



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Networks are dynamic

Agent-based simulations

F.C. Santos, J.M. Pacheco and T. Lenaerts (2006) Cooperation previals when individuals adjust their social ties. PLoS Comp Biol 2(12):e178

S. Van Segbroeck, F.C. Santos, A. Nowé, J.M. Pacheco and T. Lenaerts (2008) The evolution of prompt reactions to adverse ties. BMC Evol Biol 8:287

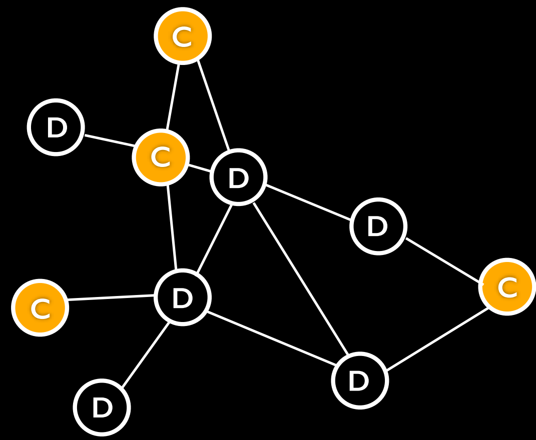
Analytics and numerical approximations

J.M Pacheco, A. Traulsen and M. Nowak (2006) Coevolution of strategy and structure in complex networks with dynamical linking. Phys Rev Lett 97:258103

S. Van Segbroeck, F.C. Santos, T. Lenaerts and J.M. Pacheco (2009) Reacting differently to adverse ties promotes the evolution of cooperation. Phys Rev Lett 102:058105

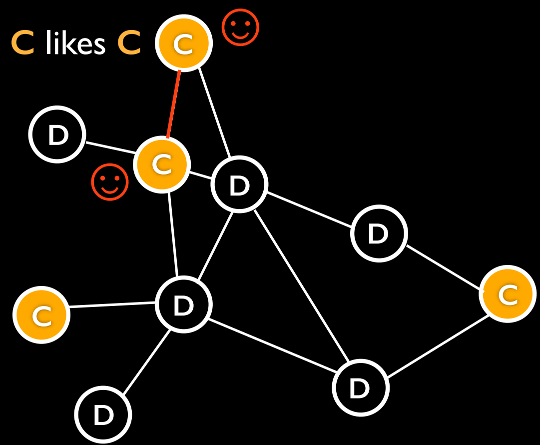
99

rewiring strategy



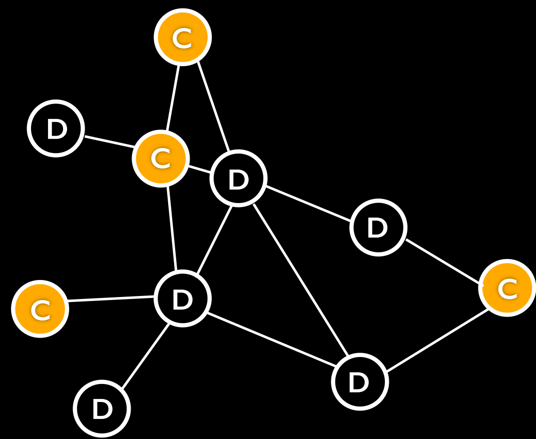
100-1

rewiring strategy

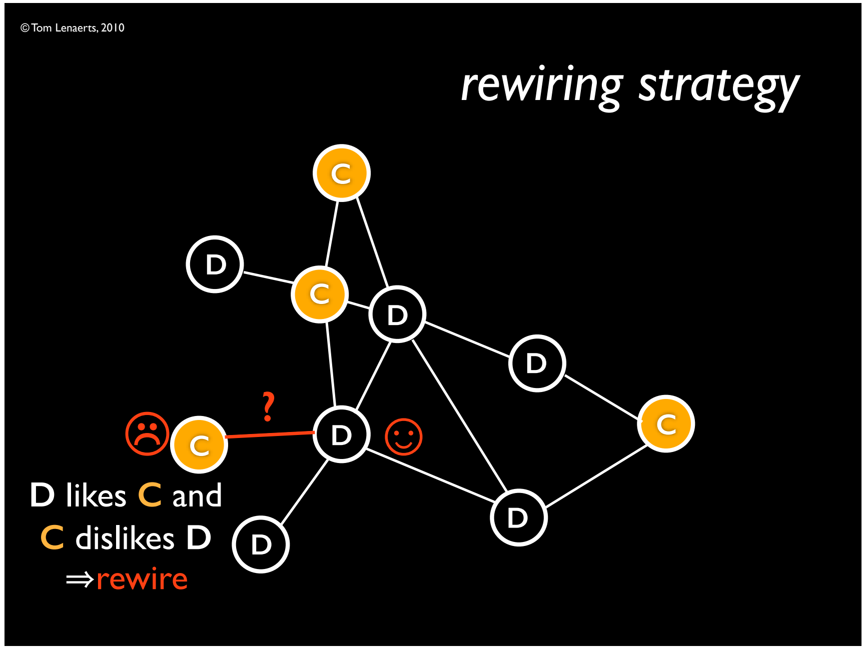


100-2

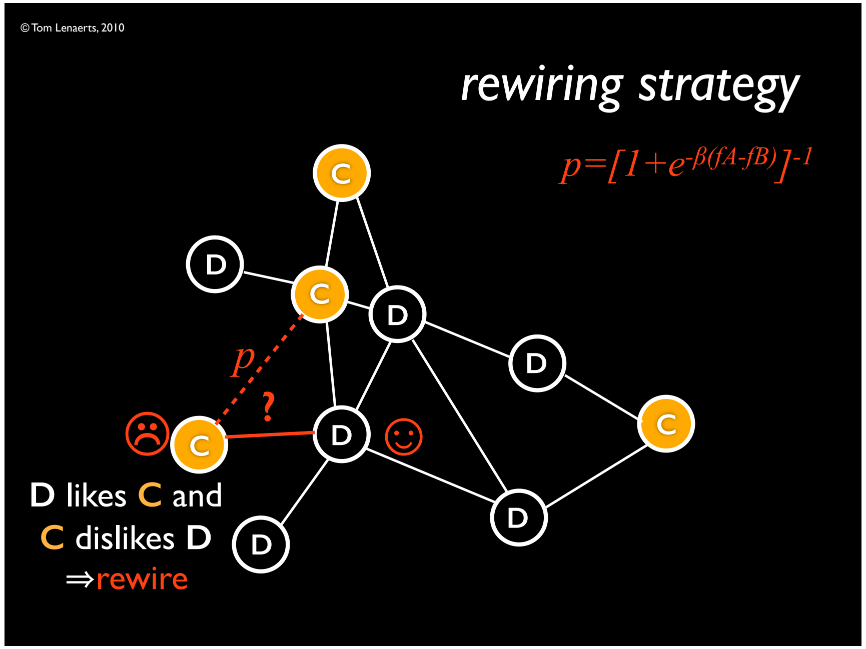
rewiring strategy



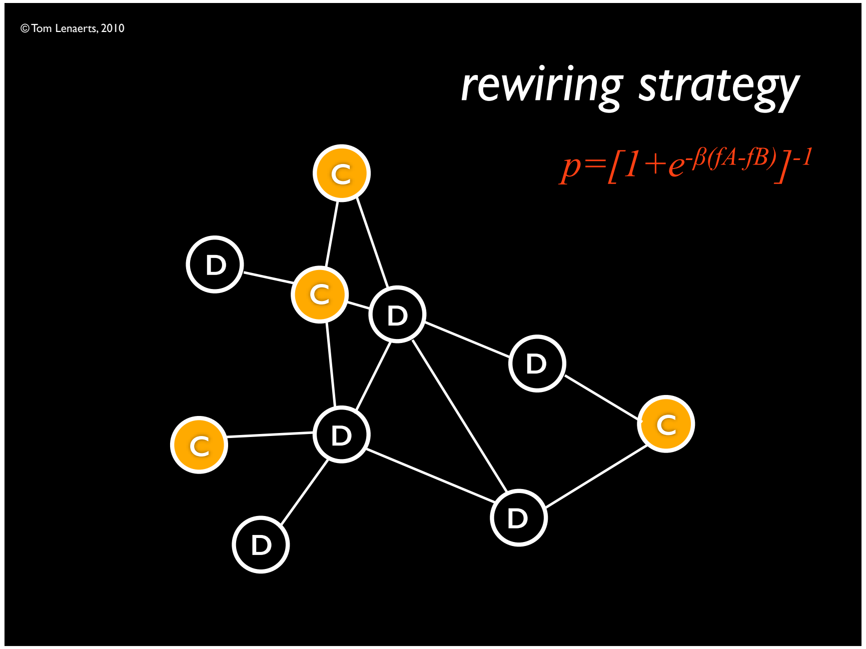
100-3



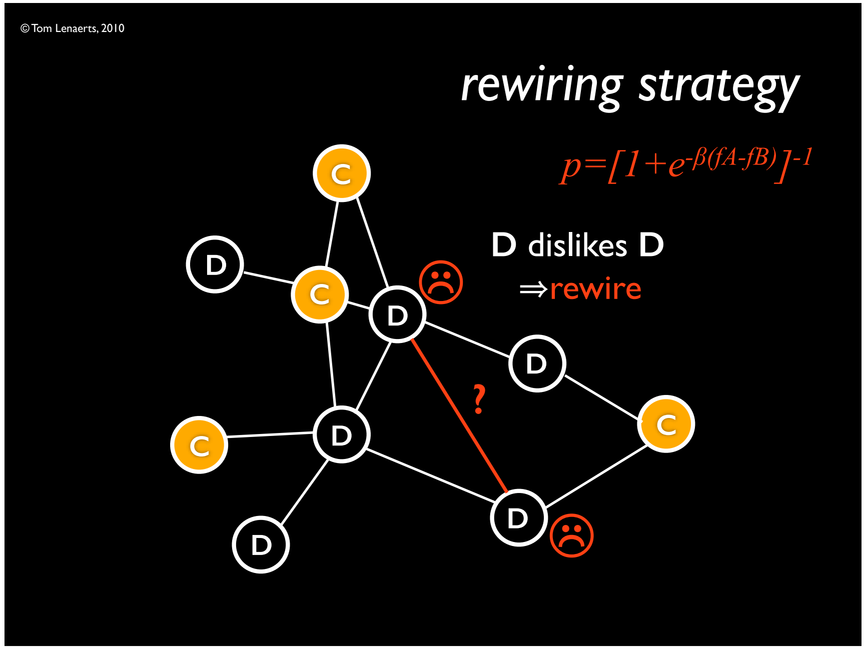
100-4



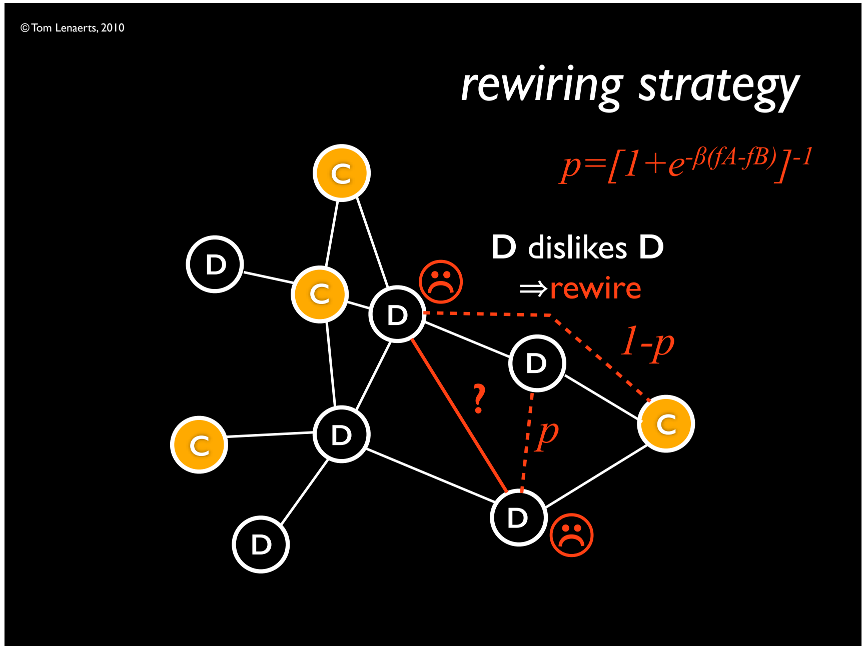
100-5



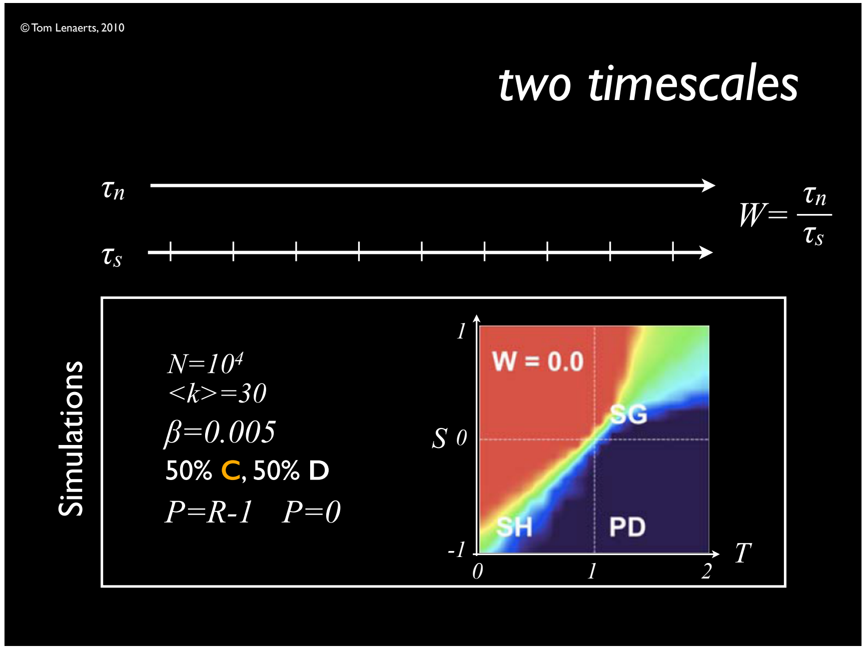
100-6



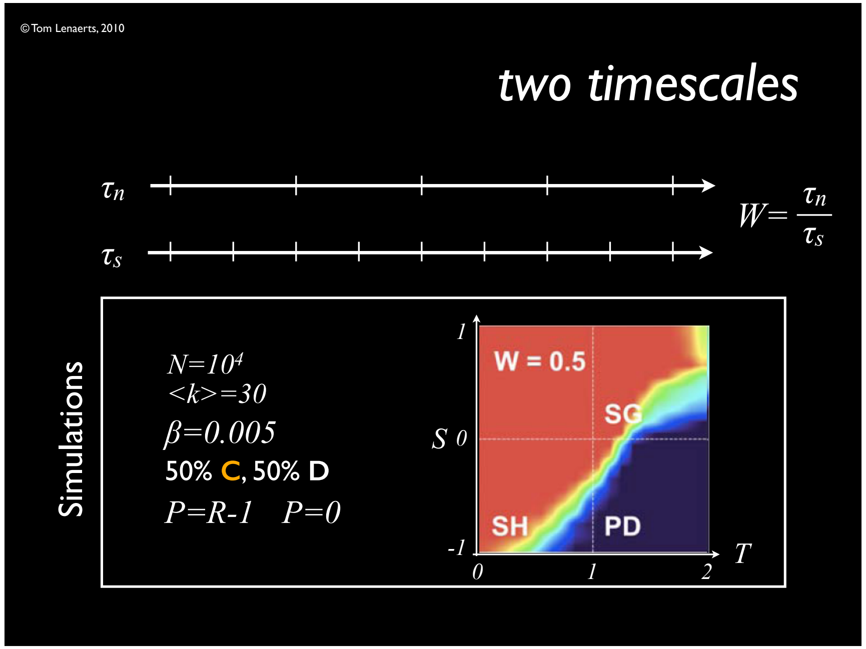
100-7



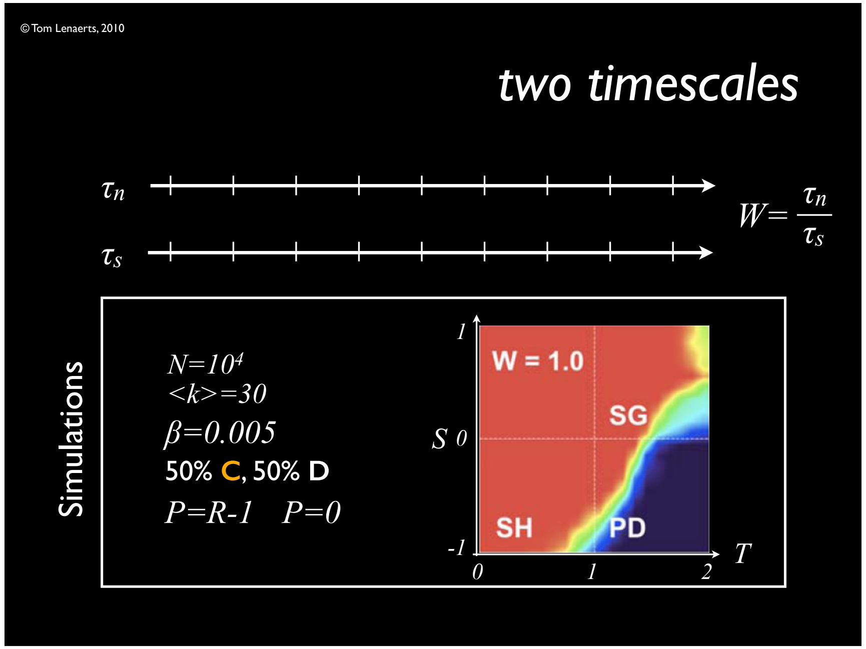
100-8



101-1



101-2



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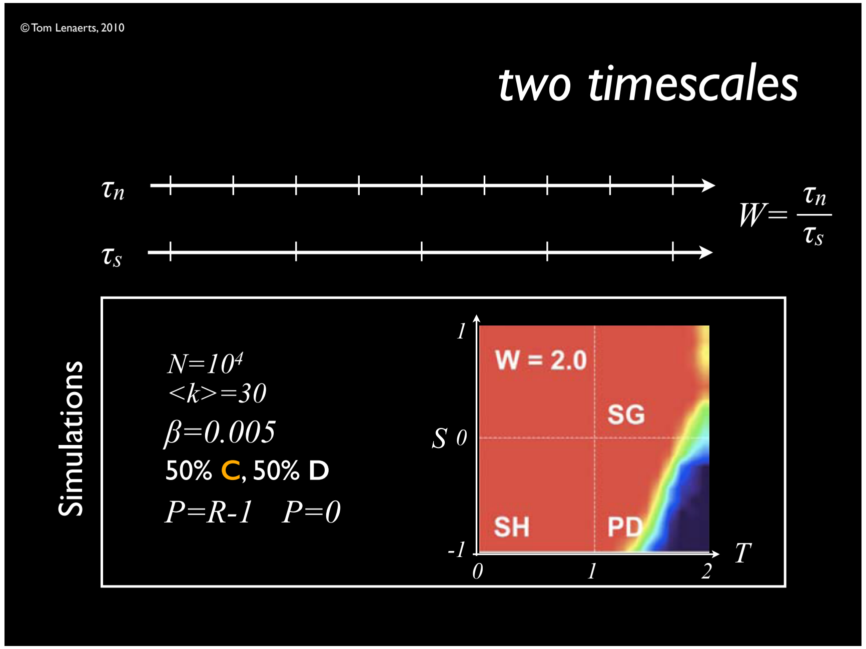

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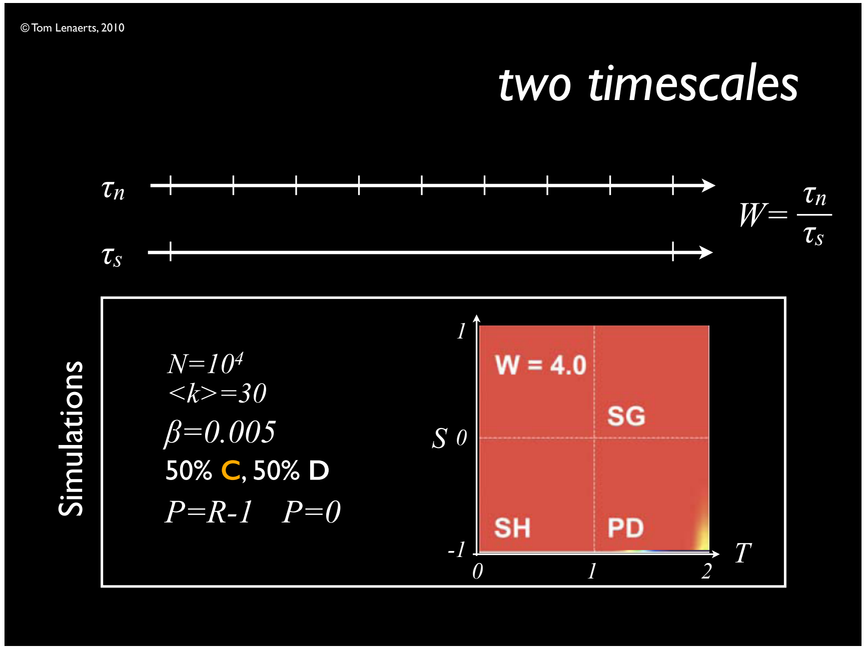

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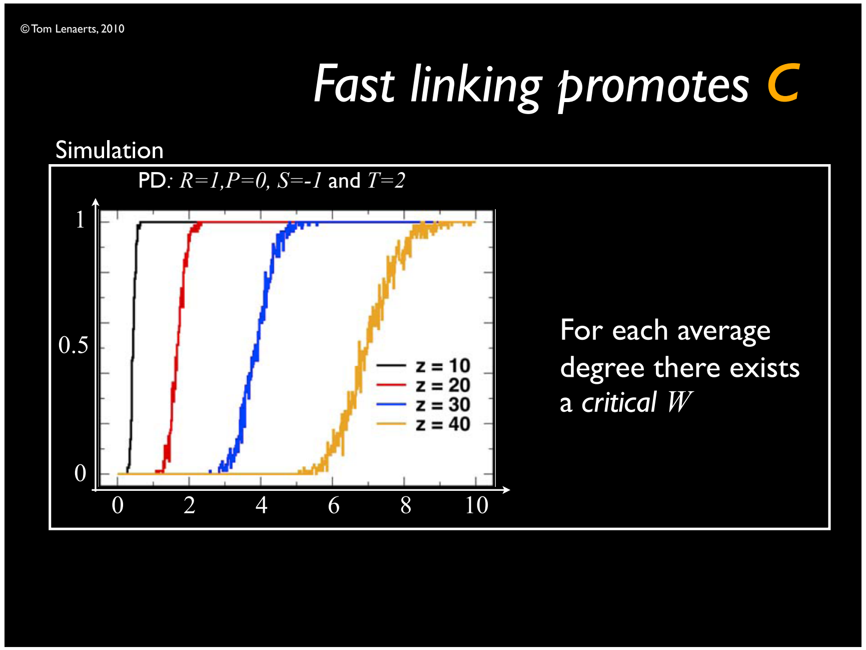

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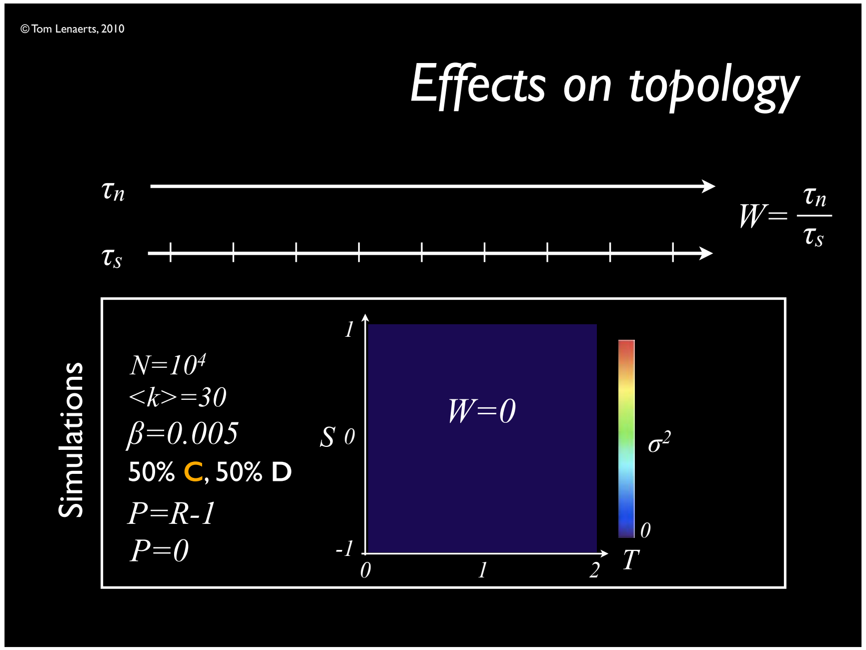
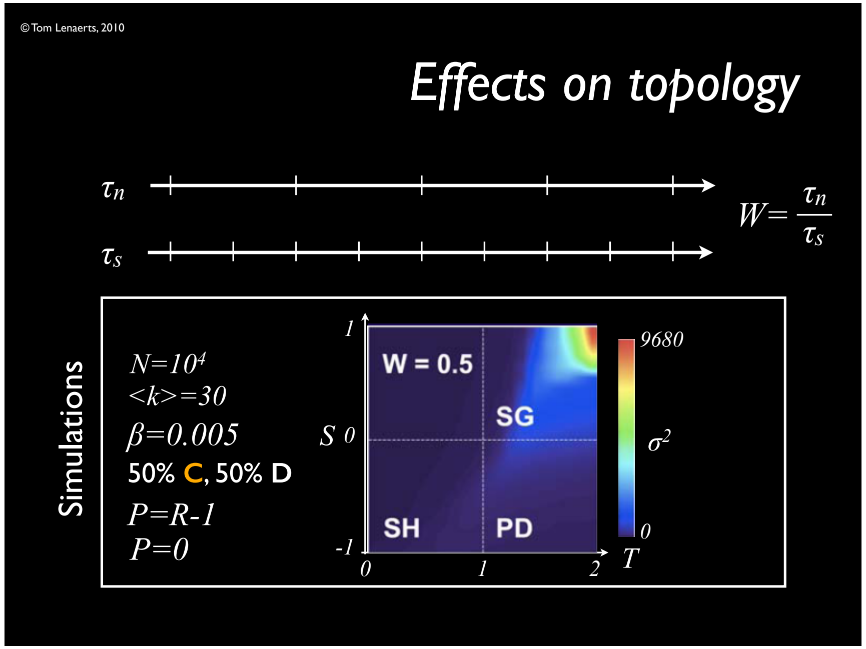
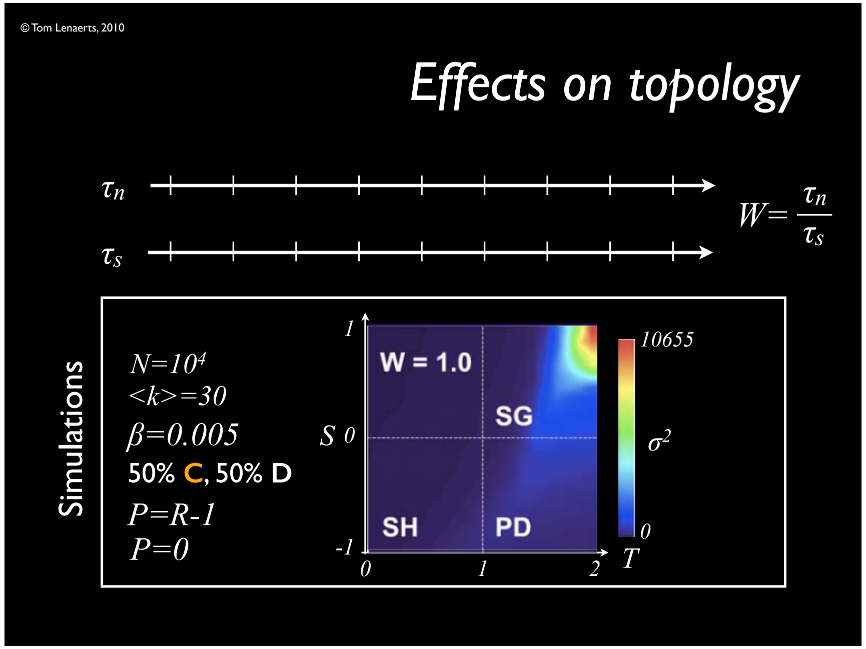


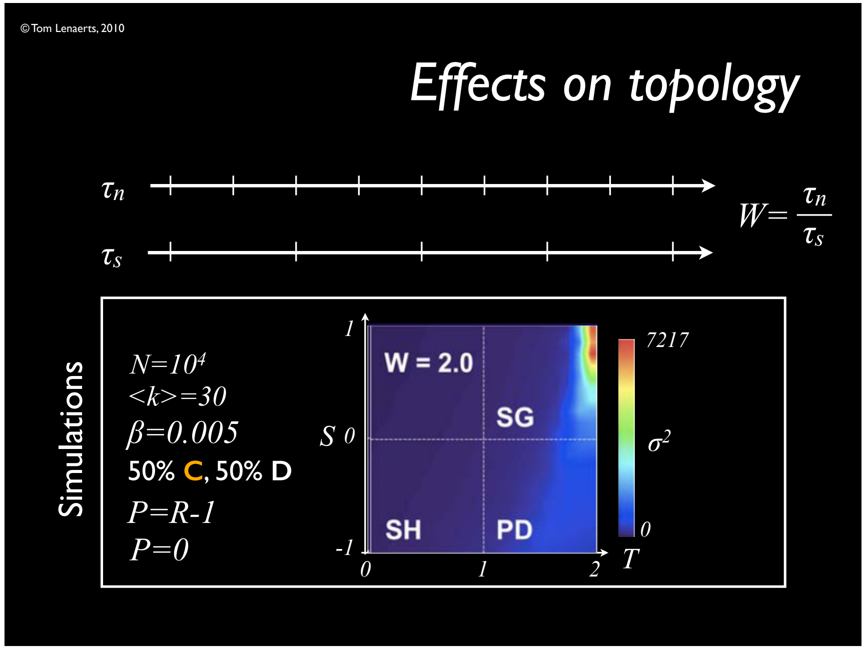
103-1



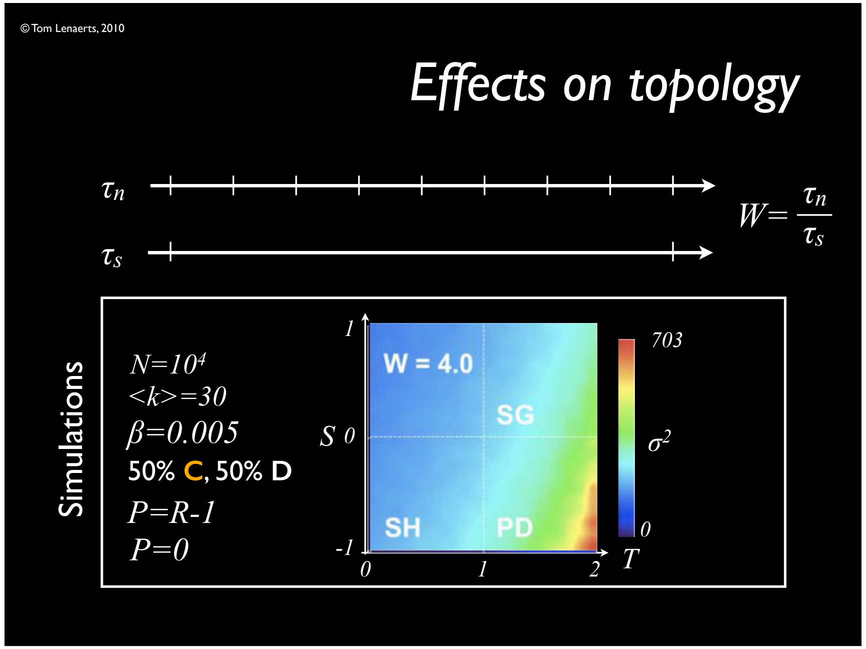
103-2



103-3

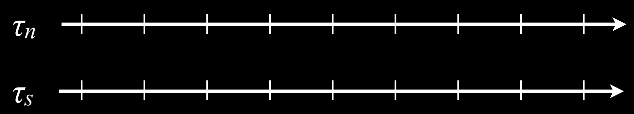


103-4



103-5

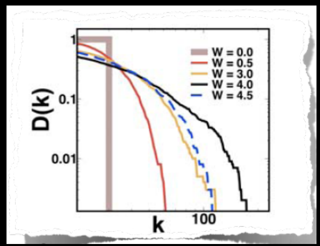
Effects on topology



$$W = \frac{\tau_n}{\tau_s}$$

Simulations

$N=10^4$
$<k>=30$
$\beta=0.005$
50% C, 50% D
$T=2,\ R=1,$
$P=0,\ S=1$



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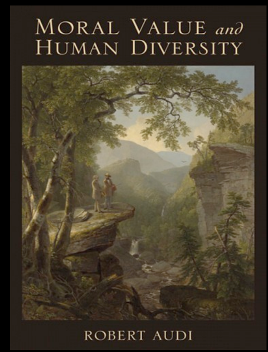
Everyone reacts differently



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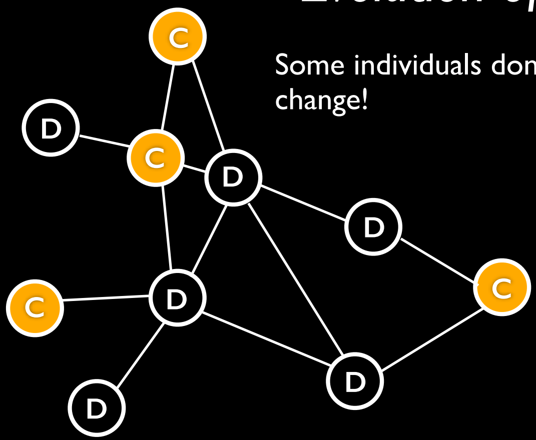
Evolution of rewiring



Some individuals don't like change!

Change introduces uncertainty!

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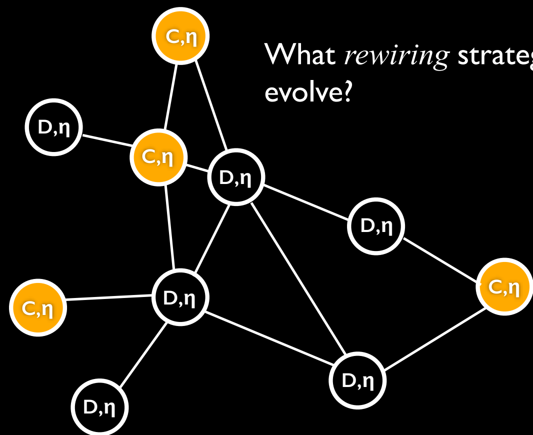
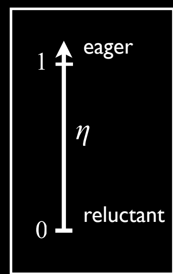
Evolution of rewiring



What *rewiring* strategy will evolve?



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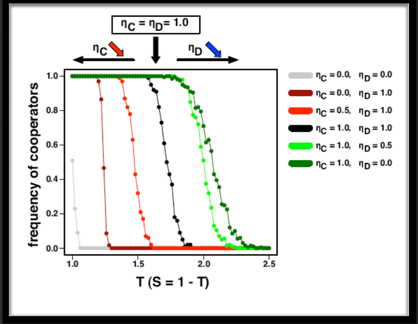
Fast versus slow

Simulation I



Assume fixed η for C and D

PD game

$N=10^3 \quad z=30$

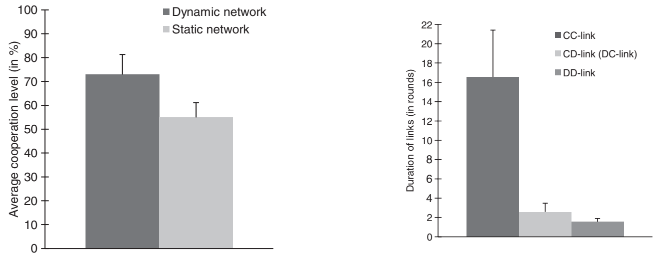
100 runs

50% C, 50% D

$W=2.5 \quad \beta=0.005$

108

Recent experiments



Fehl, K., Van Der Post, D. J., & Semmann, D. (2011). **Co-evolution of behaviour and social network structure promotes human cooperation.** Ecology Letters, 14(6), 546–551.

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Summary

C

110-1

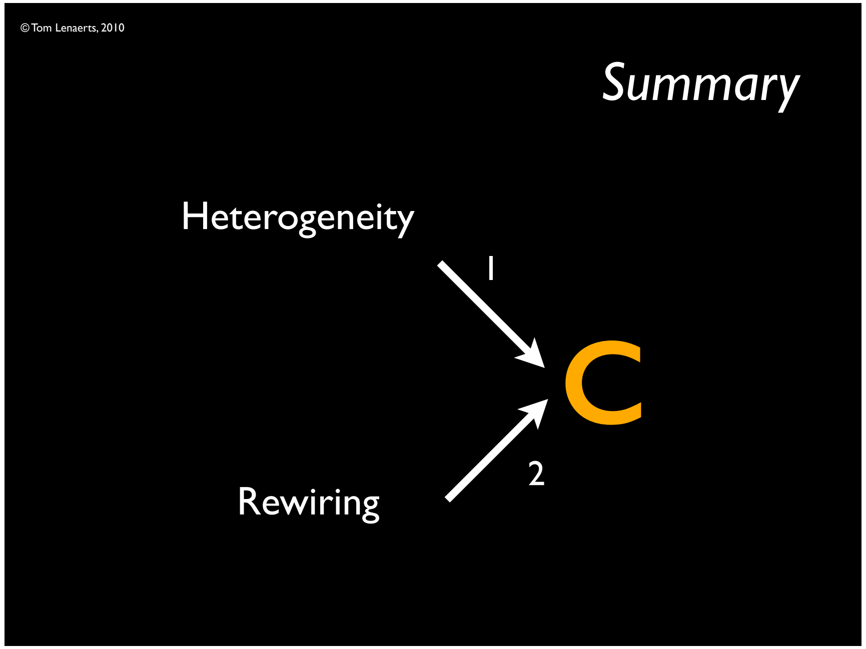
Summary

Heterogeneity

I

C

110-2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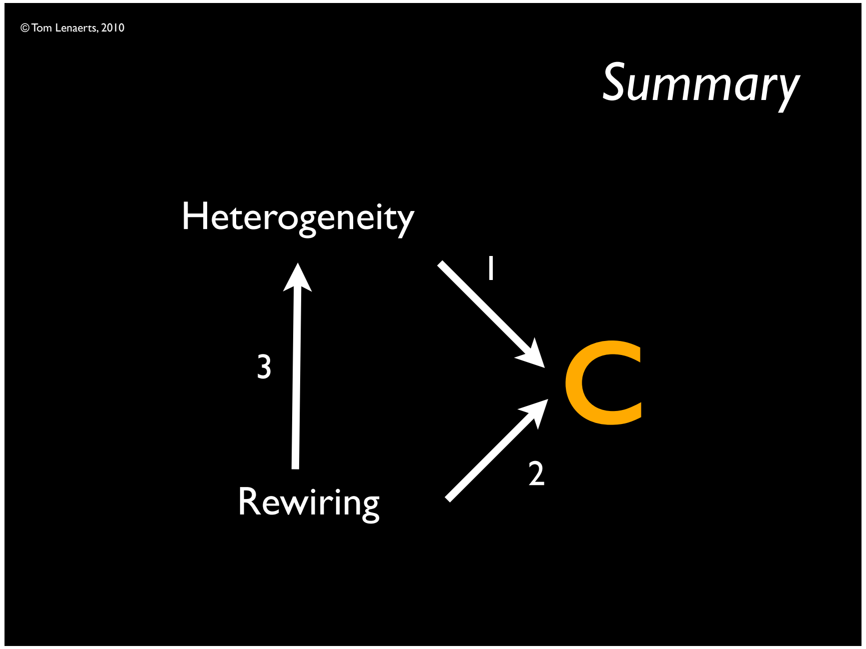
Summary
Heterogeneity
1
C
2
Rewiring

110-3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Summary
Heterogeneity
1
3
C
2
Rewiring

110-4