INFO-F-409 Learning dynamics

An introduction to Game Theory

ULB

T. Lenaerts and A. Nowé
MLG, Université Libre de Bruxelles and
AI-lab, Vrije Universiteit Brussel

VUB

Computational Game Theory

An introduction to Game Theory

ULB

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AI-lab, Vrije Universiteit Brussel

VUB

My coordinates

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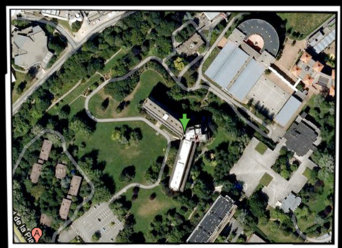


Schedule

Date (thursdays)	Description
22/09/2016	No course this day
29/09/2016	Game theory basics
6/10/2016	Mixed strategies and Nash algorithms
13/10/2016	Extensive form games and their equilibria
20/10/2016	N-armed bandits (stateless reinforcement learning)
27/10/2016	Evolutionary Game Theory and the evolution of cooperation
3/11/2016	Networks and their influence on cooperation
10/11/2016	No course this week
17/11/2016	Reinforcement learning and MDPS
24/11/2016	Sparse Interactions
1/12/2016	Selfish load balancing
8/12/2016	Graphical games
15/12/2016	Project preparation time
22/12/2016	Project preparation time
29/12/2016	Winter break
5/01/2016	
Exam: Article + presentation of group project	

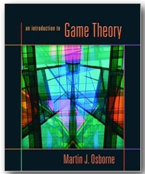
Practical things

- **~3 Assignments during the course**
  - They are taken into account (50%) for the final grade.
  - Assignments are personal (NO TEAMWORK), this will be checked !
  - Mail your solutions
    - NO paper copies !!!
    - Please provide a single (self-contained) *.PDF file.
  - Schedule (temporary)
    - Assignment 1 Game theory basics
    - Assignment 2 Evolutionary game theory
    - Assignment 3 Reinforcement learning

Practical things

- **Exam = scientific project**
  - study a topic related to the course (some possibilities will be provided)
    - Look for something **YOU** like on for instance google scholar
  - Formulate a question you want to study
  - implement a software that allows you to answer that question
  - Write a scientific article (The unofficial guide for authors)
  - Present and **discuss** articles in January

Bibliography



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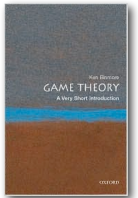

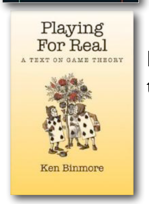

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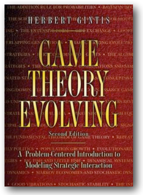
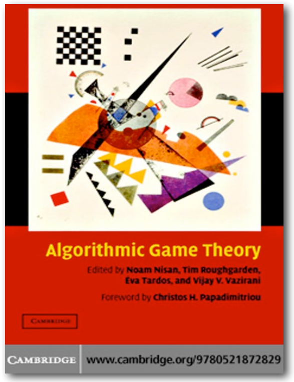
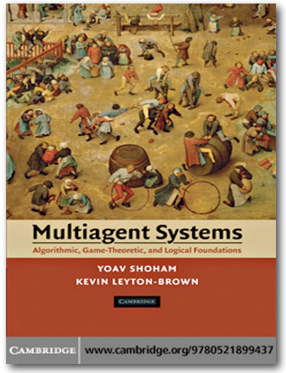


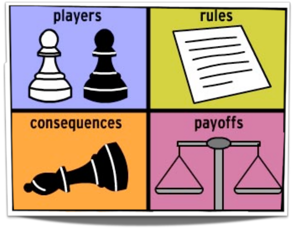
for computer science





What?

[...] *A game is a competitive activity in which players contend with each other according to a set of rules* [...]



[...] *Game theory is a theory/tool that helps us understand situations in which decision-makers interact* [...]

What ?

games

What ?

economy

What ?

economy

What ?

economy

What ?

gam

politics

What ?

Game shows

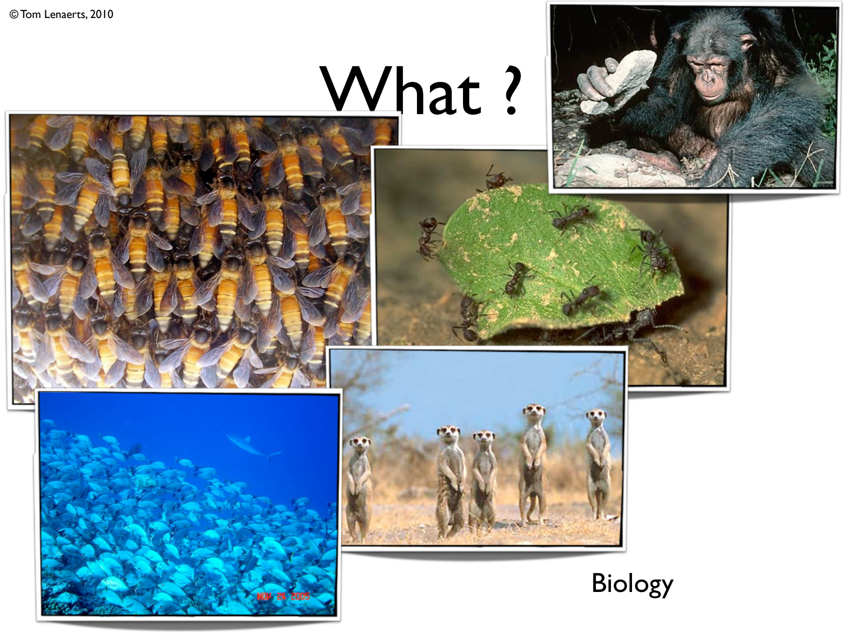
What?

Fragment from Golden Balls (ITV1)

What?

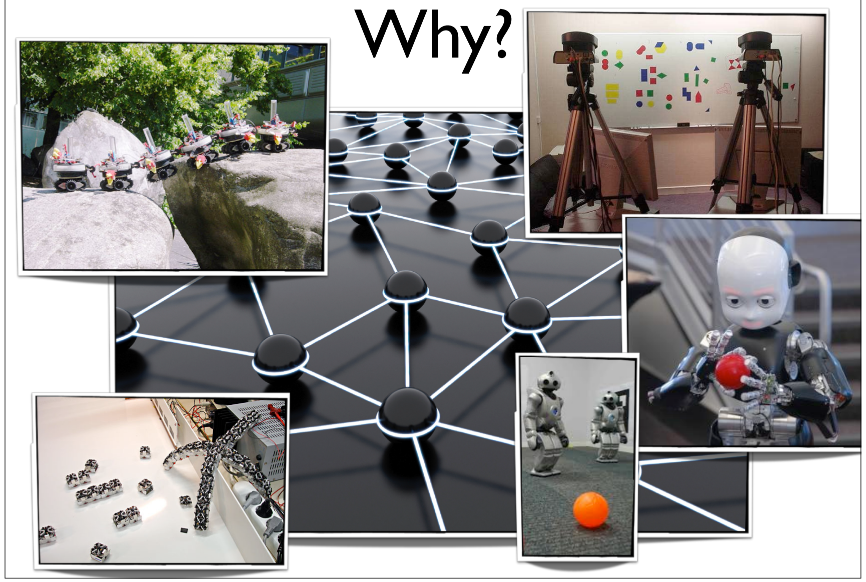
Fragment from Golden Balls (ITV1)

What ?



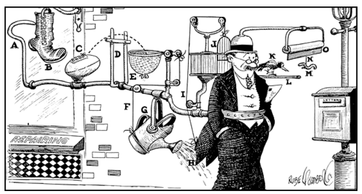
Biology

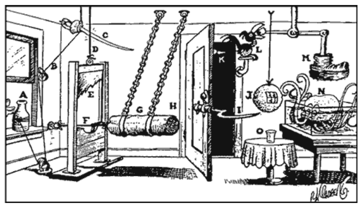
Why?



Model building

[...] *Game-theoretic modeling starts with an idea related to some aspect of interacting decision-makers. We express this idea precisely in a model, **incorporating features** of the situation that appear to be **relevant**.* [...] *We wish to put enough ingredients into the model to obtain **nontrivial insights**,* [...] *we wish to **lay bare the underlying structure** of the situation as opposed to describing its every detail. The next step is to analyze the model - to discover its **implications*** [...] *Our analysis may **confirm our idea, or suggest it is wrong**. If it is wrong the analysis should help us understand why* [...]





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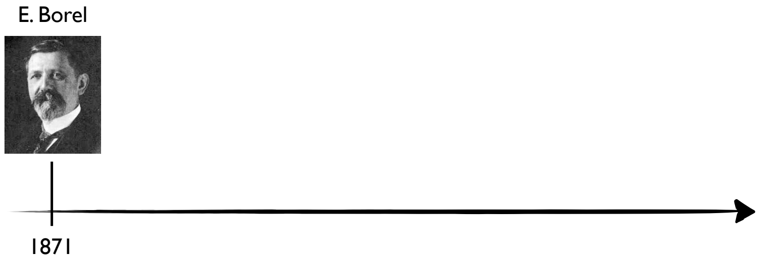

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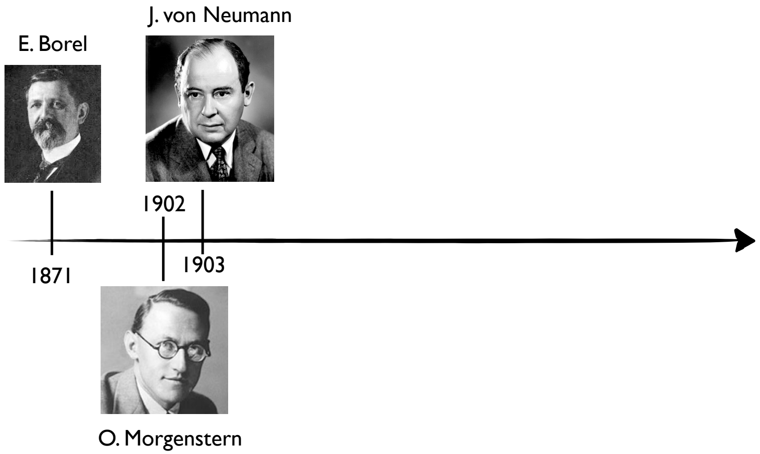

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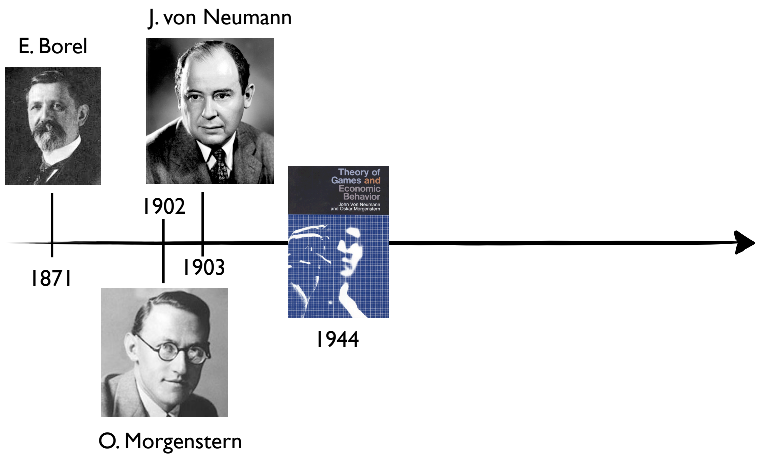

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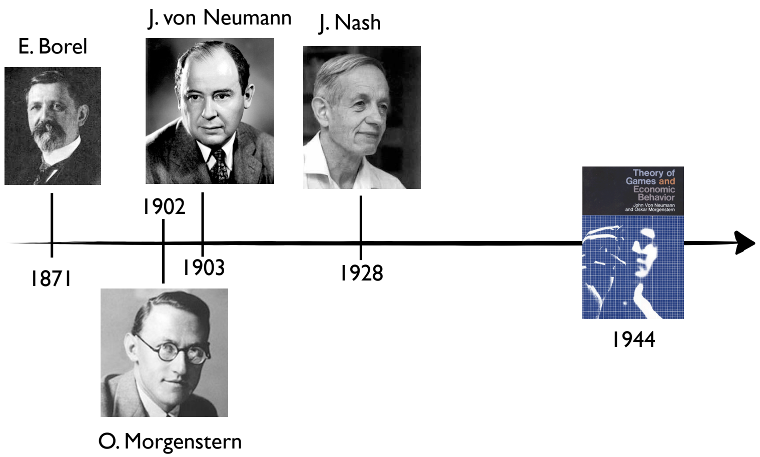

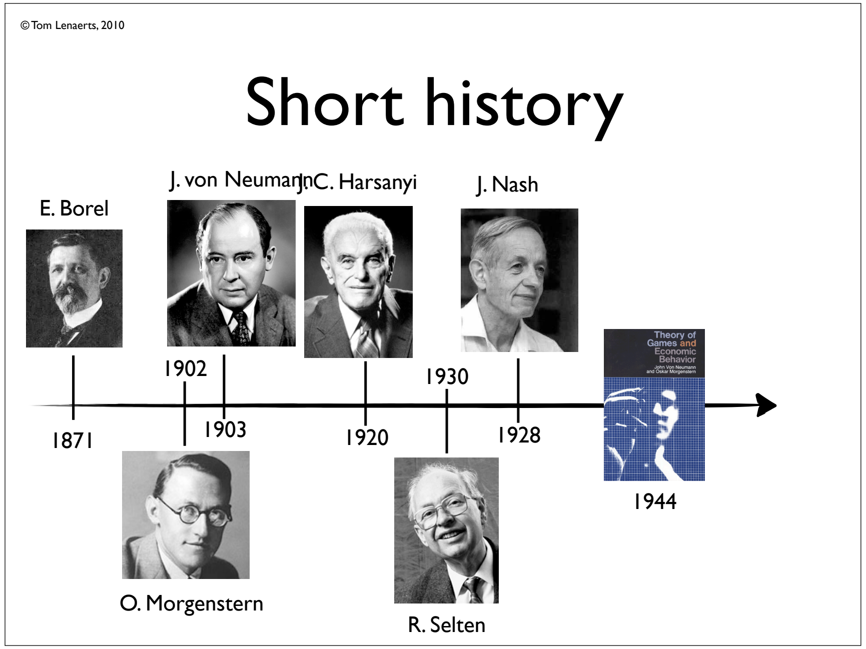
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Short history
E. Borel
J. von Neumann
J.C. Harsanyi
J. Nash
1902
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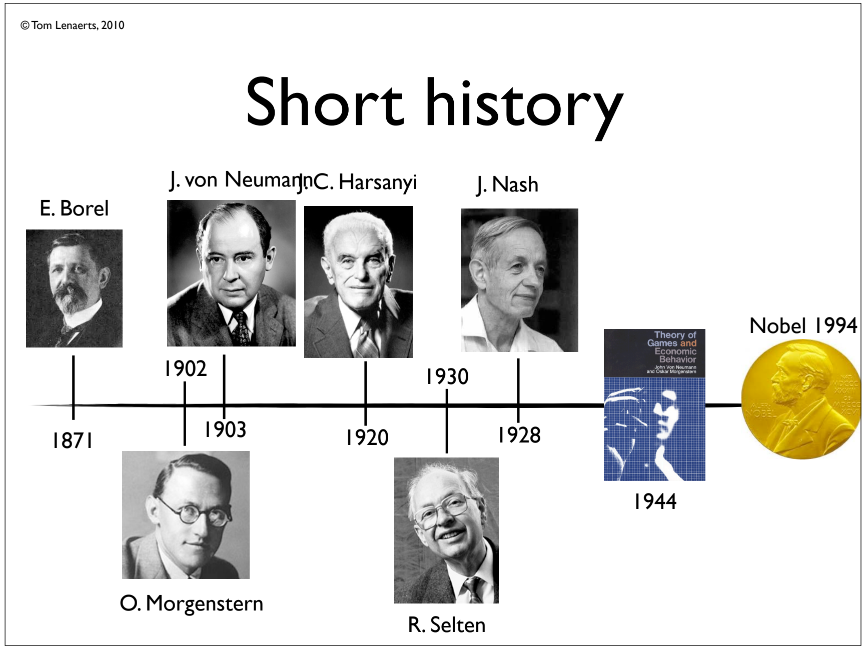
1871
1903
1920
1928
Theory of Games and Economic Behavior
1944
O. Morgenstern
R. Selten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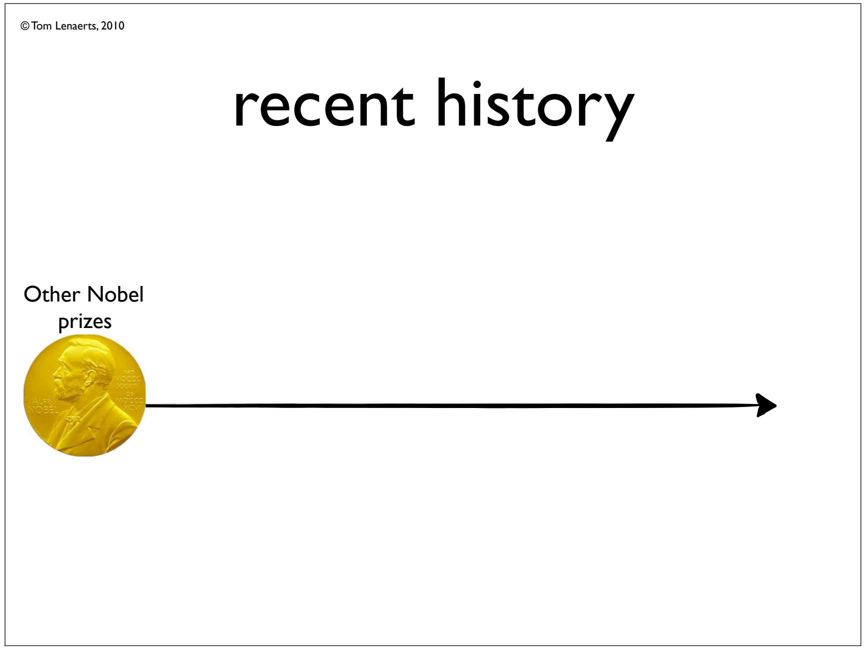
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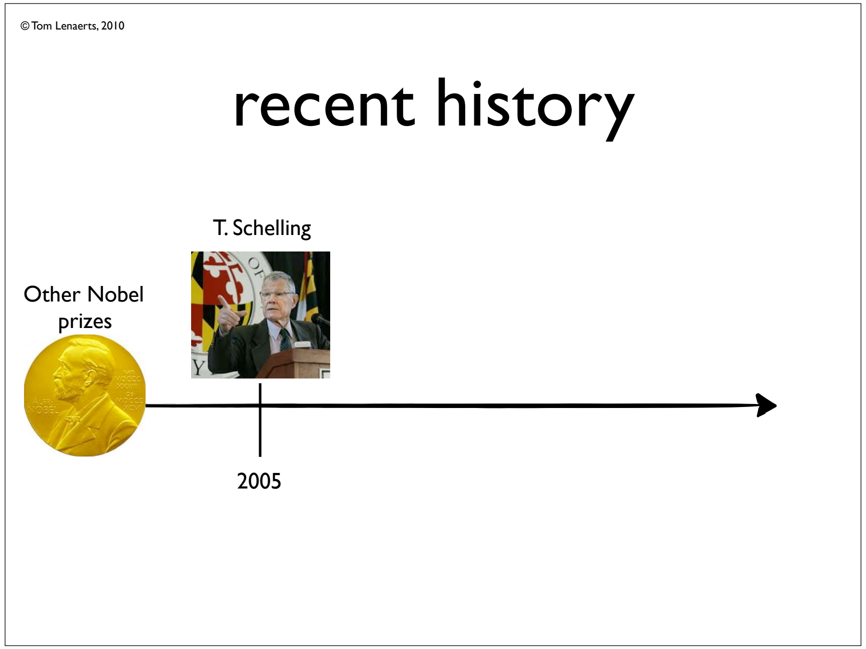
Other Nobel prizes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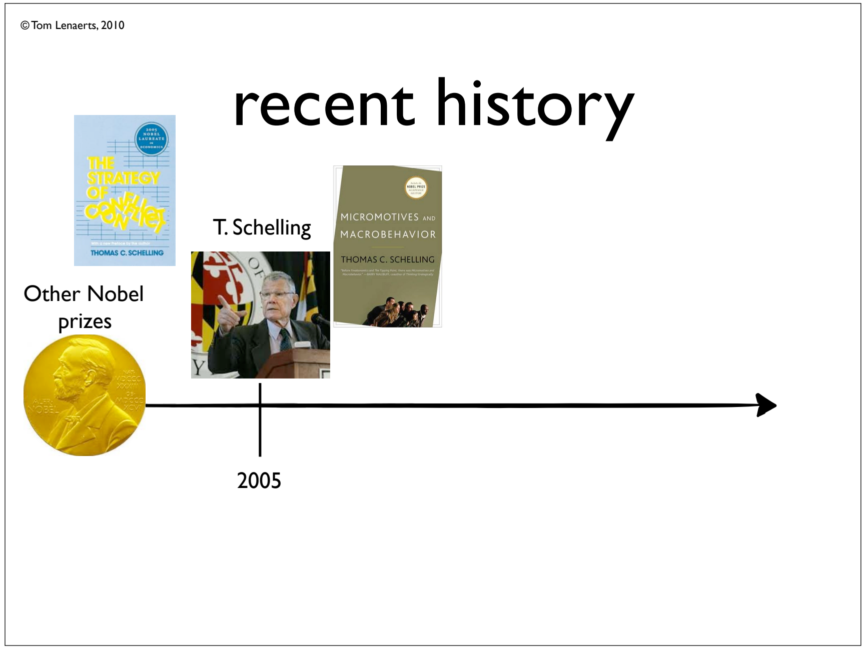
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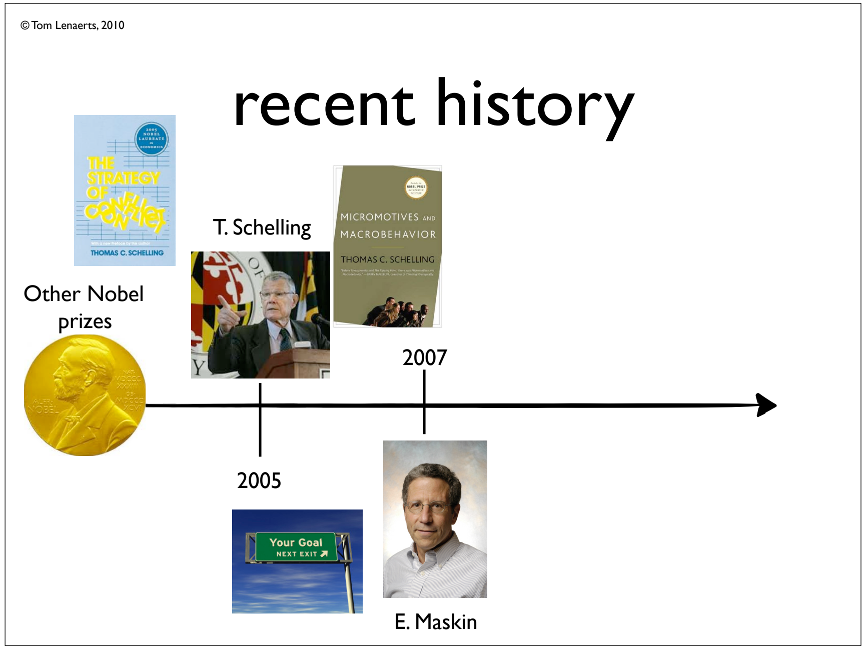
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2005
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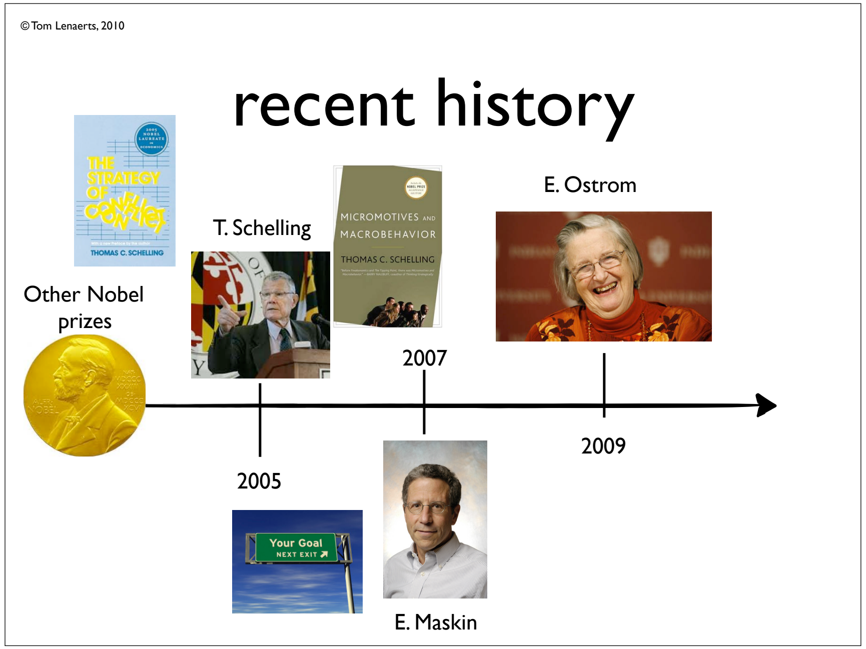
E. Maskin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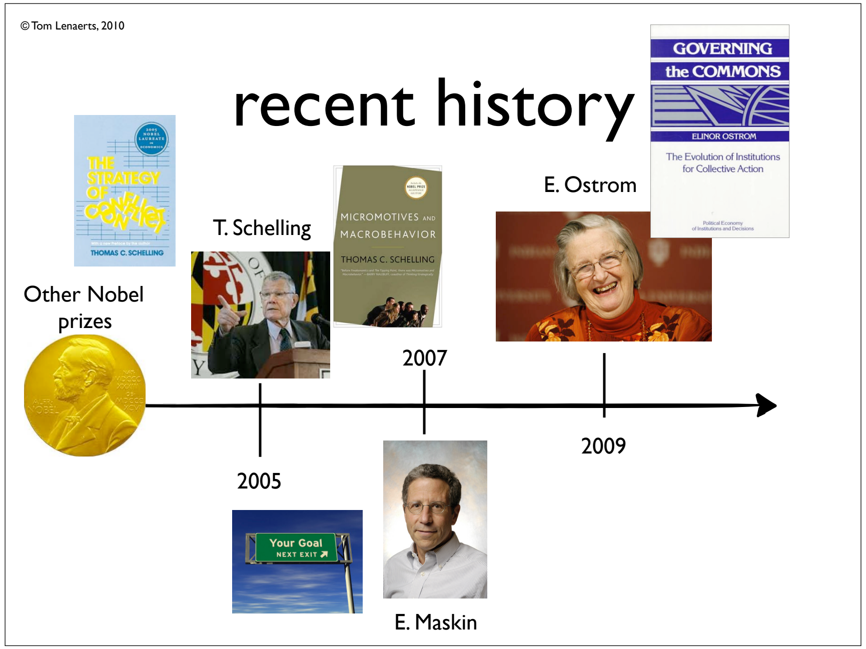
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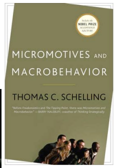
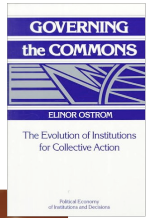


E. Ostrom



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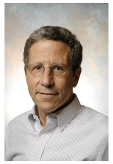

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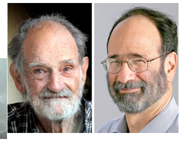
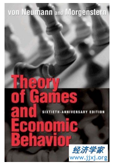


L. Shapley and A.E Roth

Evolutionary game theory

J. Von Neumann & O. Morgenstern

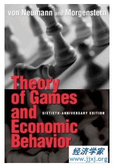
1944

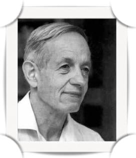


Evolutionary game theory

J. Von Neumann & O. Morgenstern

1944 1950



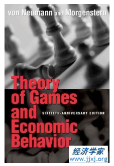


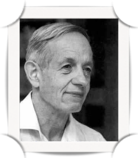
J. Nash

Evolutionary game theory

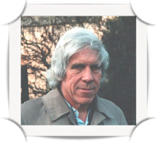
J. Von Neumann & O. Morgenstern

1944 1950 1964



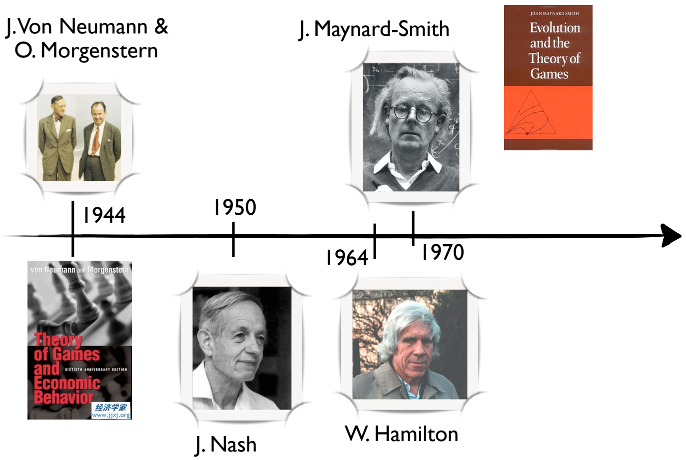


J. Nash

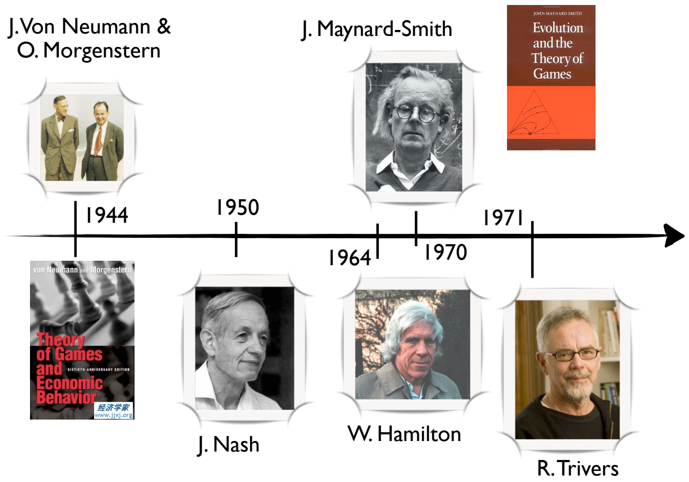


W. Hamilton

Evolutionary game theory



Evolutionary game theory



Rational choice

A decision-maker chooses the best **action** according to her **preferences**, among all the actions available

Actions in Golden Balls game : $A=\{split,steal\}$

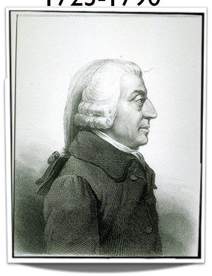
Preferences should be consistent and can be represented by a function $u(x)$

In Golden Balls game : $u(steal) > u(split)$

The scale of he numbers in this function do not relate to the importance of a preference

The theory of rational choice

A. Smith
1723-1790



[...] *The action chosen by a decision-maker is at least as good, according to her preferences, as every other available action* [...]

This theory pervades economic theory !

Is not always applicable !

Rational choice according to Nash

Fragment from A Beautiful mind (2001)

Rational choice according to Nash

Fragment from A Beautiful mind (2001)

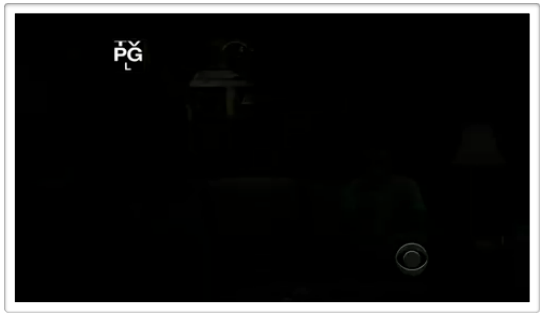
Other decision-makers

A decision-maker preferences' are affected by the preferred actions of other decision-makers

Such situations are modeled as games !

Strategic games

Fragment fromThe Big-Bang Theory (2008)



Consists of :

- a set of players
- for each player a set of actions
- for each player, **preferences over the set of actions**

Strategic games

Fragment fromThe Big-Bang Theory (2008)

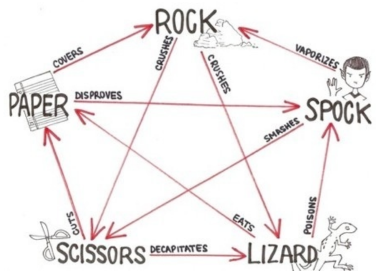
Consists of :
- a set of players
- for each player a set of actions
- for each player, **preferences over the set of actions**

Strategic games

Players : Sheldon and Rajesh

Actions : {rock, paper, scissors, lizard,spock}

Preferences :



Strategic games

Players : Sheldon and Rajesh

Actions : {rock, paper, scissors, lizard,spock}

Preferences :

The Golden Balls dilemma

Players : Sarah and Steve

Actions : {split, steal}

Preferences :

	split	steal
split	50075 50075	100150 0
steal	0 100150	0 0

The prisoner's dilemma

Players : Two thiefs

Actions : {Quiet, Fink}

Preferences :

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

u(Fink,Quiet) > u(Quiet,Quiet) > u(Fink,Fink) > u(Quiet,Fink)

The prisoner's dilemma

This game extends to a variety of situations

- working on a joint project,
- duopoly
- arms race
- use of a common property

The chicken game

Fragment from Footloose (1984)

The chicken game

Fragment from Footloose (1984)

The chicken game

Players : Kevin Bacon and Chuck

Actions : {swerve, straight}

a.k.a. snowdrift game

	swerve	straight
swerve	0 0	+1 -1
straight	-1 +1	-10 -10

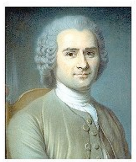
u(straight,swerve) > u(swerve,swerve) > ***u(swerve,straight) > u(straight,straight)***

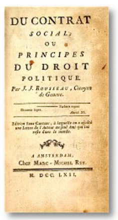
The stag-hunt game

The stag-hunt game

Players : two hunters

Actions : {whale, fish}





a.k.a. coordination game

	whale	fish
whale	2 2	1 0
fish	0 1	1 1

u(whale,whale) > u(fish,whale) *> u(fish, fish) > u(whale, fish)*

arms race?

	Refrain	Arm
Refrain	2 2	3 0
Arm	0 3	1 1

prisoner's dilemma

	Refrain	Arm
Refrain	3 3	2 0
Arm	0 2	1 1

stag-hunt game
a.k.a. security dilemma

Matching pennies

Strictly competitive

Zero-sum game

Players : two players

Actions : {head, tail}

	head	tail
head	-1 +1	+1 -1
tail	+1 -1	-1 +1

Matching pennies

Example : IPad and look-a-likes

A newcomer will prefer that his ipad-clone looks and feels like the original

The established producer wants to ensure the difference

Matching pennies

Example : IPad and look-a-likes

A newcomer will prefer that his ipad-clone looks and feels like the original

The established producer wants to ensure the difference

Bach-Stravinsky game

a.k.a. Battle of the sexes

Players : two players

Actions : {Bach, Stravinsky}

	Bach	Strav.
Bach	1 2	0 0
Strav.	0 0	2 1

Asymmetric games

A game is called symmetric when the row and column player have the same preferences over the same actions

... when they have the same payoff matrix ($A=B^T$)

Symmetric games : prisoners dilemma, the chicken game, the stag-hunt game, ...

Asymmetric games : Bach-Stravinsky, inspection game, ...

Inspection game

"A tax authority wants taxpayers to truthfully report income, an employer wants an employee to work hard, a regulator wants a factory to comply with pollution regulations, police want motorists to observe speed limits, etc.

A fundamental problem for authorities is how to induce compliance with desired behavior when individuals have incentives to deviate from such behavior. A standard approach is to monitor a proportion of individuals and penalize those caught misbehaving." (Quote from D. Nosenzo et al 2010 Discussion Paper 2010--

	Comply	Cheat
Don't Inspect	60, 25	0, 40
Inspect	52, 25	12, 20

Symmetricalization

An asymmetric game can be transformed into a symmetrical version of the game either by

1. Assuming that each player can act as row and column player 50% of the time

DC meets DC
i) D agains C = 60
ii) C against D = 25

average = 42.5

	DC	DH	IC	IH
DC	42.5, 42.5	12.5, 50	42.5, 37.5	12.5, 46
DH	50, 12.5	20, 20	40, 18.5	10, 26
IC	37.5, 42.5	18.5, 40	37.5, 37.5	18.5, 36
IH	46, 12.5	26, 10	36, 18.5	16, 16

Symmetricalization

An asymmetric game can be transformed into a symmetrical version of the game either by

2. Assuming that both players have the same actions but only receives payoff when playing against the correct ones

When playing D against D, I against I, C against C and H against H, there is no payoff

	D	I	C	H
D	0, 0	0, 0	60, 25	0, 40
I	0, 0	0, 0	52, 25	12, 20
C	25, 60	25, 52	0, 0	0, 0
H	40, 0	20, 12	0, 0	0, 0

Nash equilibrium

Which action will be chosen by each player?

Nash equilibrium

Which action will be chosen by each player?

Theory of rational choice states that each player chooses the **best available action**

Nash equilibrium

Which action will be chosen by each player?

Theory of rational choice states that each player chooses the **best available action**

Since this choice depends on the actions of the other player, each player must form a **belief** about the other players' actions and preferences

Nash equilibrium

Which action will be chosen by each player?

Theory of rational choice states that each player chooses the **best available action**

Since this choice depends on the actions of the other player, each player must form a **belief** about the other players' actions and preferences

This belief is formed based on the **knowledge of the game and past experiences**

Nash equilibrium

Which action will be chosen by each player?

Theory of rational choice states that each player chooses the **best available action**

Since this choice depends on the actions of the other player, each player must form a **belief** about the other players' actions and preferences

This belief is formed based on the **knowledge of the game and past experiences**

BUT ! each play is considered in isolation (players do not know each other)

Nash equilibrium

Definition :

A Nash Equilibrium (NE) is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^* given that every other player j adheres to a_j^*

A NE corresponds to a stable "social norm": *if everyone follows it, no person will wish to deviate from this*

Note that the solution proposed in the bar game in the movie a beautiful mind does not correspond to a Nash equilibrium (Anderson and Enger, 2002)

Nash equilibrium

Assume that (a_i', a_{-i}) is the action profile in which every player j **except i** chooses her action a_j as specified by a, whereas player i deviates to a_i'

Definition :

The action profile a^* in a strategic game is a Nash Equilibrium if for every player i and for every action a_i of player i, a^* is at least as good according to player i's preferences as the action profile (a_i, a_{-i}^*) in which player i chooses a_i while every other player j chooses action a_j^*. Equivalently, for every player i,

$u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for every action a_i of player i

where u_i is the payoff function that represents player i's preferences

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1

Assume the profile :

(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

Assume the profile :

(Quiet, Quiet) $u_1 \to 3$ $u_2 \to 3$

(Fink, Quiet) $u_1 \to 7$

(Fink, Fink) $u_1 \to 1$ $u_2 \to 1$

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

Assume the profile :

(Quiet, Quiet) $u_1 \to 3$ $u_2 \to 3$

(Fink, Quiet) $u_1 \to 7$

(Quiet, Fink) $u_2 \to 7$

(Fink, Fink) $u_1 \to 1$ $u_2 \to 1$

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 3	7 0
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Assume the profile :

(Quiet, Quiet) $u_1 \to 3$ $u_2 \to 3$

(Fink, Quiet) $u_1 \to 7$

(Quiet, Fink) $u_2 \to 7$

(Fink, Fink) $u_1 \to 1$ $u_2 \to 1$

(Quiet, Fink) $u_1 \to 0$

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

Assume the profile :

(Quiet, Quiet) $u_1 \to 3$ $u_2 \to 3$

(Fink, Quiet) $u_1 \to 7$

(Quiet, Fink) $u_2 \to 7$

(Fink, Fink) $u_1 \to 1$ $u_2 \to 1$

(Quiet, Fink) $u_1 \to 0$

(Fink, Quiet) $u_2 \to 0$

Examples

1. The prisoner's dilemma

Assume the profile :

	Quiet	Fink
Quiet	3 / 3	7 / 0
Fink	0 / 7	1 / 1

(Quiet, Quiet) $u_1 \rightarrow 3$ $u_2 \rightarrow 3$

(Fink, Quiet) $u_1 \rightarrow 7$

(Quiet, Fink) $u_2 \rightarrow 7$

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

(Fink, Quiet) $u_2 \rightarrow 0$

Examples

1. The prisoner's dilemma

	Quiet	Fink
Quiet	3 / 3	7 / 0
Fink	0 / 7	1 / 1

Note that any deviation from this NE results in a worse outcome.
This NE is therefore also a *strict* NE : for every player i, $u_i(a^*) > u_i(a_i, a_{-i}^*)$ for every action a_i of player i

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

(Fink, Quiet) $u_2 \rightarrow 0$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, swerve) $u_1 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, swerve) $u_1 \rightarrow +1$

(swerve, straight) $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, swerve) $u_1 \rightarrow +1$

(swerve, straight) $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(swerve, swerve) $u_1 \rightarrow 0$ $u_2 \rightarrow 0$

(straight, swerve) $u_1 \rightarrow +1$

(swerve, straight) $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$ $u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$

(straight, swerve) $u_2 \rightarrow -1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -
Straight	- / +1	-10 / -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, swerve) $u_1 \rightarrow$ 0

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, swerve) $u_1 \rightarrow$ 0

(straight, straight) $u_2 \rightarrow$ -10

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, swerve) $u_1 \rightarrow$ 0

(straight, straight) $u_2 \rightarrow$ -10

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow$ -10

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, swerve) $u_1 \rightarrow$ 0

(straight, straight) $u_2 \rightarrow$ -10

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow$ -10

(swerve, swerve) $u_2 \rightarrow$ 0

Examples

2. The chicken game

Assume the profile :

	Swerve	Straight
Swerve	0 / 0	+1 / -1
Straight	-1 / +1	-10 / -10

(straight, swerve) $u_1 \to +1$ $u_2 \to -1$
(swerve, swerve) $u_1 \to 0$
(straight, straight) $u_2 \to -10$

(swerve, straight) $u_1 \to -1$ $u_2 \to +1$
(straight, straight) $u_1 \to -10$
(swerve, swerve) $u_2 \to 0$

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Examples

2. The chicken game

Assume the profile :

Both NE are also strict NE

	Swerve	Straight
Swerve	0 / 0	+1 / -1
Straight	-1 / +1	-10 / -10

1 $u_2 \to -1$
werve, swerve) $u_1 \to 0$
(straight, straight) $u_2 \to -10$

(swerve, straight) $u_1 \to -1$ $u_2 \to +1$
(straight, straight) $u_1 \to -10$
(swerve, swerve) $u_2 \to 0$

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Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2 / 2	1 / 0
fish	0 / 1	1 / 1

(whale, whale) $u_1 \to 2$ $u_2 \to 2$

(fish, fish) $u_1 \to 1$ $u_2 \to 1$

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Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2 / 2	1 / 0
fish	0 / 1	1 / 1

(whale, whale) $u_1 \to 2$ $u_2 \to 2$
(fish, whale) $u_1 \to 1$

(fish, fish) $u_1 \to 1$ $u_2 \to 1$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

(fish, whale) $u_2 \rightarrow 0$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

(fish, whale) $u_2 \rightarrow 0$

Examples

3. The stag-hunt or whale-fish game

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

Both NE are also a *strict* NE

$u_2 \rightarrow 2$

1

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

(fish, whale) $u_2 \rightarrow 0$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \rightarrow 2$ $u_2 \rightarrow 1$

(Strav., Strav.) $u_1 \rightarrow 1$ $u_2 \rightarrow 2$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \rightarrow 2$ $u_2 \rightarrow 1$

(Strav., Bach) $u_1 \rightarrow 0$

(Strav., Strav.) $u_1 \rightarrow 1$ $u_2 \rightarrow 2$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

(Bach, Bach) $u_1 \rightarrow 2$ $u_2 \rightarrow 1$

(Strav., Bach) $u_1 \rightarrow 0$

(Bach, Strav.) $u_2 \rightarrow 0$

(Strav., Strav.) $u_1 \rightarrow 1$ $u_2 \rightarrow 2$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	1 / 2	0 / 0
Strav.	0 / 0	2 / 1

(Bach, Bach) $u_1 \to 2$ $u_2 \to 1$
(Strav., Bach) $u_1 \to 0$
(Bach, Strav.) $u_2 \to 0$

(Strav., Strav.) $u_1 \to 1$ $u_2 \to 2$
(Bach, Strav.) $u_1 \to 0$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	1 / 2	0 / 0
Strav.	0 / 0	2 / 1

(Bach, Bach) $u_1 \to 2$ $u_2 \to 1$
(Strav., Bach) $u_1 \to 0$
(Bach, Strav.) $u_2 \to 0$

(Strav., Strav.) $u_1 \to 1$ $u_2 \to 2$
(Bach, Strav.) $u_1 \to 0$
(Strav., Bach) $u_2 \to 0$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	1 / 2	0 / 0
Strav.	0 / 0	2 / 1

(Bach, Bach) $u_1 \to 2$ $u_2 \to 1$
(Strav., Bach) $u_1 \to 0$
(Bach, Strav.) $u_2 \to 0$

(Strav., Strav.) $u_1 \to 1$ $u_2 \to 2$
(Bach, Strav.) $u_1 \to 0$
(Strav., Bach) $u_2 \to 0$

Examples

4. The Bach-Stravinsky or Battle of the sexes game

Assume the profile :

	Bach	Strav.
Bach	1 / 2	0 / 0
Strav.	0 / 0	2 / 1

Both NE are also a *strict* NE

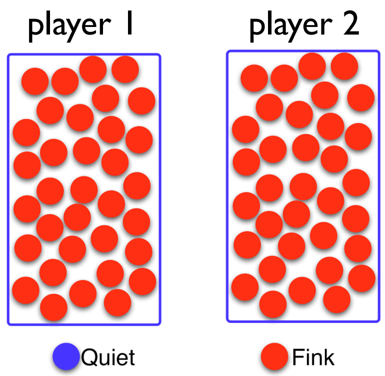
$u_2 \to 1$
0
(Bach, Strav.) $u_2 \to 0$

(Strav., Strav.) $u_1 \to 1$ $u_2 \to 2$
(Bach, Strav.) $u_1 \to 0$
(Strav., Bach) $u_2 \to 0$

Population steady state

A NE corresponds to a *steady state* of an interaction between the members of several populations, one for each player in the game

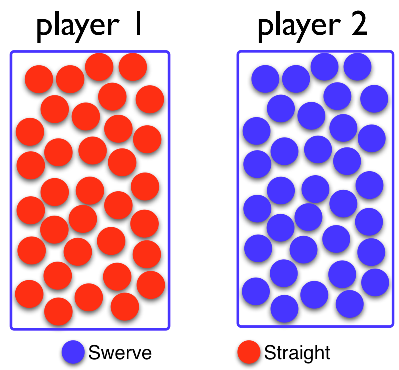
	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1



Population steady state

A NE corresponds to a *steady state* of an interaction between the members of several populations, one for each player in the game

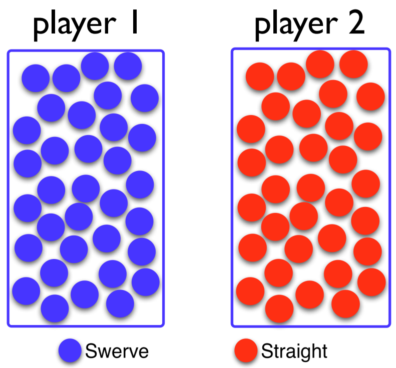
	Swerve	Straight
Swerve	0, 0	-1, +1
Straight	+1, -1	-10, -10



Population steady state

A NE corresponds to a *steady state* of an interaction between the members of several populations, one for each player in the game

	Swerve	Straight
Swerve	0, 0	-1, +1
Straight	+1, -1	-10, -10



Best-response

How to find the Nash Equilibrium in bigger games?

	Bach	Strav.
Bach	2, 1	0, 0
Strav.	0, 0	1, 2

For every action a column player chooses there is a subset of best responses of the row player

$B_r(\text{Bach})=\{\text{Bach}\}$ *and* $B_r(\text{Strav})=\{\text{Strav}\}$

best response function of r player

$$B_i(a_{-i})=\{a_i \in A_i : u_i(a_i,a_{-i}) \geq u_i(a_i',a_{-i}) \ \forall\ a_i' \in A_i\}$$

Best-response

Best response function can thus be used to define NE

Definition :

The action profile a^* in a strategic game is a Nash Equilibrium if and only if every player's action is a best response to the other player's actions

a_i^* is in $B_i(a_{-i}^*)$ for every player i

Best-response

Best-response functions can be used to find the NE

METHOD :

STEP 1: find the best-response function for each player

STEP 2: find the action profiles that satisfy : a_i^* is in $B_i(a_{-i}^*)$ for every player i

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

Best response of player 2 to player 1

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$

Best response of player 2 to player 1

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$

Best response of player 2 to player 1

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

$B_2(T)=\{L\}$

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

$B_2(T)=\{L\}$ $B_2(M)=\{L,C\}$

Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	2, 1	0, 1	0, 0
B	0, 1	0, 0	1, 2

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

$B_2(T)=\{L\}$ $B_2(M)=\{L,C\}$ $B_2(B)=\{R\}$

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Best-response

Best-response functions can be used to find the NE

Example : STEP 1: find the best-response function for each player

	L	C	R
T	1, 2	2, 1	1, 0
M	$\boxed{2, 1}$	0, 1	0, 0
B	0, 1	0, 0	$\boxed{1, 2}$

Best response of player 1 to player 2

$B_1(L)=\{M\}$ $B_1(C)=\{T\}$ $B_1(R)=\{T,B\}$

Best response of player 2 to player 1

$B_2(T)=\{L\}$ $B_2(M)=\{L,C\}$ $B_2(B)=\{R\}$

STEP 2: boxes with two coloured payoffs are NE

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Best-response

	Split	Steal
Split	3, 3	0, 7
Steal	7, 0	$\boxed{1, 1}$

	Swerve	Straight
Swerve	0, 0	$\boxed{-, +1}$
Straight	$\boxed{+1, -}$	-10, -10

	whale	fish
whale	$\boxed{2, 2}$	0, 1
fish	1, 0	$\boxed{1, 1}$

	Bach	Strav.
Bach	$\boxed{2, 1}$	0, 0
Strav.	0, 0	$\boxed{1, 2}$

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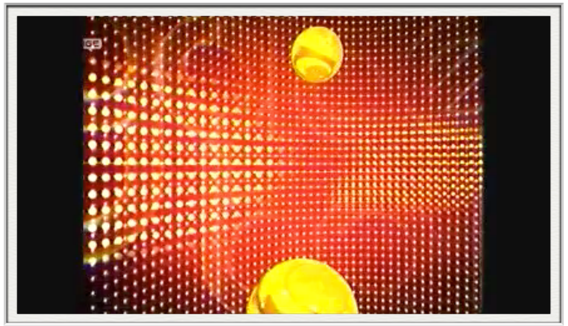
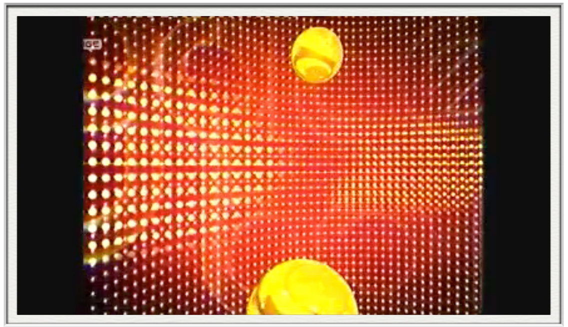
How to beat the game?



Fragment from Golden Balls (ITV1)

How to beat the game?



Fragment from Golden Balls (ITV1)

Dominance

In any game, a player's action *strictly dominates* another action if it is superior, no matter what the other player does

	Split	Steal
Split	3, 3	0, 7
Steal	7, 0	1, 1

Steal strictly dominates Split

If player 2 plays Split, then player 1 prefers Steal

If player 2 plays Steal, then player 1 also prefers Steal

Dominance

Definition :

In a strategic game player i's action a_i'' **strictly** dominates her action a_i' if

$u_i(a_i'',a_{-i}) > u_i(a_i',a_{-i})$ for every list a_{-i} of the other player's action

We say that a_i' is strictly dominated

Strictly dominated actions can never be part of a NE since they are not part of a best response to any actions

Dominance

Definition :

In a strategic game player i's action a_i'' **weakly** dominates her action a_i' if

$u_i(a_i'',a_{-i}) \geq u_i(a_i',a_{-i})$ for every list a_{-i} of the other player's action and

$u_i(a_i'',a_{-i}) > u_i(a_i',a_{-i})$ for **some** list a_{-i} of the other player's action

We say that a_i' is weakly dominated

Dominance

	L	R
T	1	0
M	2	0
B	2	1

No matter what the column player does...

M weakly dominates T

B weakly dominates M

BUT: B strictly dominates T

Dominance

	whale	fish
whale	2, 2	1, 0
fish	0, 1	1, 1

Neither *whale* nor *fish* strictly or weakly dominates the other player's action

	Swerve	Straight
Swerve	0, 0	+1, -
Straight	-, +1	-10, -10

Neither *swerve* nor *straight* strictly or weakly dominates the other player's action

Pareto efficiency

Pareto optimality is a measure of efficiency.

"An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player."

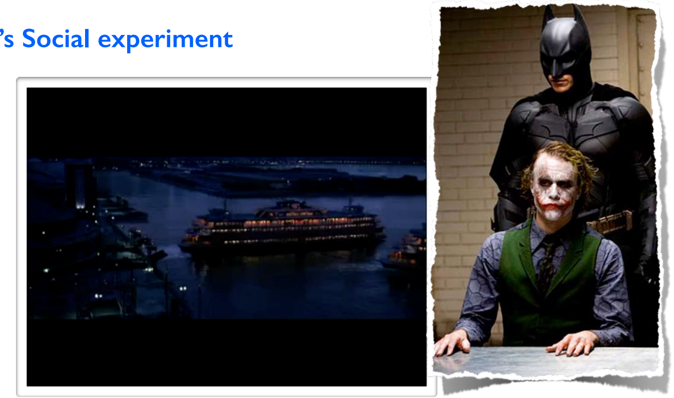
	whale	fish
whale	2, 2 (NE and PO)	1, 0
fish	0, 1	1, 1 (NE)

	Split	Steal
Split	3, 3 (PO)	7, 0
Steal	0, 7	1, 1 (NE)

Shor, Mikhael, "Pareto Optimal," Dictionary of Game Theory Terms, Game Theory .net, <http://www.gametheory.net/dictionary/ParetoOptimal.html> Web accessed: 11/09/2012

Game theory in popular culture

The Joker's Social experiment



What does the payoff matrix look like? Are there any pure Nash equilibria?

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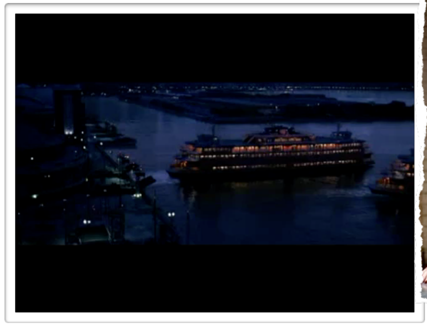

What does the payoff matrix look like? Are there any pure Nash equilibria?