Introduction to Computational Models of Cognition

Understanding human cognition has been one of the main driving forces behind over a century of research in psychology. Mathematical approaches in the study of cognition date from as early as the 19th century, when researchers like Ernst Heinrich Weber developed mathematical models describing the so-called "just-noticeable difference" effect, the process by which humans can perceive differences between objects (Raymond & Rutherford, 2012). However, it was not until the advent of theoretical computer science and digital computers in the 20th century that *computational psychology* rose as a field. Amid the creation of the digital computer, cognitive psychologist developed mathematical formalisms and computational models describing cognition as an information processing phenomena, in close resemblance to how digital computers work. Researchers like George Miller, Allen Newell, Herbert Simon, and Frank Rosenblatt, applied computational methods to the study of perception, language, and problem-solving, effectively establishing the field with their collective efforts (Boden, 2008a). The rise of computational psychology offered an alternative approach to the study of the mind, one based on algorithms and computer simulations, rather than on correlations and laboratory-based experimentation, the dominant paradigms according to the, at the time, president of the American Psychological Association Lee Cronbach (Cronbach, 1957).

Nowadays, in the first quarter of the 21st century, the field enjoys a fast-paced growth in a world where nations and tech-giants race to discover the keys to human and artificial intelligence. In this introduction, I offer a brief overview of the history of computational approaches in cognitive science, along with a few thoughts about its importance and unique perspective in the study of human thinking and behavior. I take a historical perspective, focusing on articulating a high-level narrative of the evolution of the field, rather than in the many individual examples of computational models. For a more exhaustive review, refer to the excellent work of Boden about the subject (2008a, 2008b).

The rise, fall, and resurgence of computational cognitive science

Alas, computational cognitive science took a long time to establish itself among the main paradigms in the study of the mind and behavior. During the first half of the 20th century, the study of the mind was considered problematic by many, given our incapacity of directly accessing its contents (Skinner, 196; Watson, 1913). Constructs like "consciousness" or "memory", are things that researchers can *indirectly* measure by observing people's behavior, but there is no way to "touch" and "see" a piece of memory in someone else's mind. Those are private events, subjective experiences, inaccessible entities of ethereal constitution. True, at the time, we were able to observe and measure patterns of electrical activity in the brain (Haas, 2003), but finding a direct correspondence between such patterns and a particular chunk of memory was ambiguous at best. This is one of the main reasons why the invention of the digital computer was so impactful for cognitive psychologists. It provided a physical exemplar of something that could do many of the things that the human mind does: arithmetic, logic, storage information, and more. After all, you can *build*, *touch*, and *see* a digital computer. You know exactly where a piece of information is processed and saved in memory. It is hard witnessing a digital computer in action and ignore the resemblance to the inner workings of the human mind.

There isn't an exact date, event, or piece of research signaling the foundation of computational cognitive science as a field. Yet, there are a few works that had a lasting influence on the development of the field worth mentioning. In 1956, Allen Newell and Herbert Simon introduced a computer program that they called "*The Logic Theorist*" (Newell & Simon, 1956). This program had the remarkable capacity of discovering proofs for theorems in propositional logic, employing *heuristics* considered to be similar to those observed in human problem-solving. Three years later, in 1959, Newell, Shaw, and Simon introduced a new program called GPS-I for "*General Problem Solving I*" (Newell et al., 1959). This time, the authors were more explicit about the resemblance to human cognition, as the GPS-I had the explicit intention of helping to "...understand the information processes that underlie human intellectual, adaptive, and creative abilities" (p. 0). These early works are exemplars of the *symbolic approach* to artificial intelligence and cognitive science, approach based on the idea that intelligence is essentially the result of *symbol manipulation*, as in the Logic Theorist and GPS-I.

Newell, Shaw, and Simon's work made remarkably progress on emulating human problem-solving skills, by working under the assumption that the human mind operates by manipulating symbols as in logic and mathematics. We can think of this approach as a *top-down approach* in cognitive science. From this perspective, the existence of symbolic representations in the mind is taken for granted as a starting point, aiming to explain how such entities operate to generate human thinking and behavior. Contrasting with this line of work, other researchers adopted a perspective that we can qualify as *bottom-up*. This approach drew inspiration from how the nervous system works: via the collective interplay of complex ensembles of neurons. In 1943, Warren McCulloch and Walter Pitts published "*A logical calculus of the ideas immanent in nervous activity*" (McCulloch & Pitts, 1943), introducing what is considered the first computational model of an artificial neuron. Although the McCulloch and Pitts approach were significantly different from the one taken by Newell, Shaw, and Simon, their artificial neuron was able to perform some remarkable feats as well, like emulating the behavior of boolean functions like the AND, OR, and NOR logic gates.

It is important to highlight the McCulloch and Pitts architecture was modeling a *single neuron*, with multiple binary input values (zeros or ones), and a single binary outcome. One of the first and best well-known models utilizing *multiple* neurons to produce an output is the "*Perceptron*", introduced by Frank Rosenblatt in 1958 (Rosenblatt, 1958). One of the main differences between the McCulloch-Pitts' and Rosenblatt's architectures, aside from the number of neurons, was that Rosenblatt's model incorporated a *learning algorithm*, that allowed the network to adjust its parameters to make better predictions over time. The learning part was of crucial importance for later developments in the field because it enabled the possibility of creating artificial systems capable to "discover" solutions instead of forcing humans to figure out the solution beforehand, and then building the solution *into* the system. This became part of a larger corpus of thinking in cognitive science known as *connectionism*, which in a nutshell, postulates that human intelligence emerges from the coordinated action of billions of neurons in the brain. This may not sound like a particularly bold statement today, but proposing that complex intelligent behavior can emerge *entirely* from the interplay of *non-intelligent* processing units wasn't entirely obvious at the time.

The works of Newell, Shaw, Simon, McCulloch, Pitts, Rosenblatt, and many others, sparked a great amount of interest in cognitive science and artificial intelligence. Unfortunately, the expectations surrounding the field were too high to be fulfilled. Researchers' claims did not help to temper expectations either. In 1965, Herbert Simon famously predicted that machines would be able to do what humans can do in 10 years, which as we know today, it never happened. The many limitations of the early symbolic and connectionist approaches to the study of the mind became clear, as researchers unsuccessfully attempted to applied such ideas to problems that were trivially easy to solve for humans, like image recognition or language understanding. In 1969, Marvin Minsky and Seymour Papert famously demonstrated that Rosenblatt's perceptron wasn't able to learn simple patterns like the exclusive-or (XOR) rule, which had a devastating effect on the academic interest on artificial neural networks. The symbolic approach would later receive its fair share of criticism as well. Douglas Hofstadter (1979) would criticize symbolism by its reliance on a "rigid" notion of concepts and categories inherited from mathematical logic, which he argued could not capture the "fluid" nature of how humans use concepts and categories. Researchers in the connectionist tradition (McClelland et al., 1986) would repeatedly point out to the disconnection between symbolic-cognition and how the brain works (parallelism, noisiness, graceful degradation, etc.). The failure of "expert-systems" to deliver technological solutions in the industrial sector would damage the symbolic approach reputation as well. In the artificial intelligence community, these failures and unfulfilled promises would cause a series of "winters", for both connectionist and symbolic approaches, where research funds and interest in the field faded away. In the great scheme of things, the cognitive psychology community continued to be dominated by the more traditional correlational and lab-based experimentation approaches, relegating the role of computational modeling to the background.

In the aftermath of Minsky and Papert's critique, the enthusiasm surrounding connectionist models of cognition almost completely disappeared. Nonetheless, the lack of enthusiasm did not deter all researchers from pursuing the "connectionist way" of thinking. In particular, a group of researchers led by David Rumelhart and James McClelland), the so-called "*Parallel Distributed Processing*

Research Group" (PDP research group), would work together exploring the implications of neural network models for the understanding of human intelligence (McClelland et al., 1986). Their work would be crystallized in a two volumes book titled "*Parallel Distributed Processing: Explorations in the Microstructure of Cognition*". The PDP research group work would help to revitalize the interest in neural network models of cognition in the 80s. In particular, the introduction of today's well-known "*backpropagation algorithm*" by Rumelhart, Hinton, and Williams, would be the key to overcome Minsky and Papert's criticism. In the book, the trio would tackle Minsky and Papert's challenges one by one, showing how to solve problems like learning the exclusive-or (XOR) rule by combining multiple layers of non-linear processing units, trained with the backpropagation algorithm.

By the late 80s, the future seemed bright for connectionist models. Nonetheless, they would rapidly collide with the limitations of data availability and computational power. It turned out that training large neural networks to learn anything resembling human-level intelligence, required large amounts of data and computation, that were scarce and expensive at the time. Interest in artificial neural networks would diminish again, yet, a few things were different this time: many of the early limitations were overcome; several models tackling language, memory, and perception were introduced as "proof of concepts"; a method to efficiently train deep neural networks had been established; and a new generation of researchers had been trained in this line of thought, researchers who would carry on with the connectionist agenda in the following decades.

At this point, it may be useful to pause and ponder about the relationship between symbolic and connectionist approaches. In the early 50, the fierce competition and mutual criticism weren't nearly as heated as it later became. They also were researchers attempting to combine both approaches. For instance, Donald Norman and Tim Shallice (1981) introduced models of attentional control of executive functioning combining symbolic-like planning and connectionist-like pattern recognition. This line of thought has extended until today, whit many researchers attempting to take advantage of the strengths of both approaches (Sun & Alexandre, 2013, Moreno et al., 2019). At present, it is hard to evaluate the role and the future of hybrid approaches, but it is clear that still is an active area of research.

As in most things in society, computational cognitive science also has its own "third way", challenging the dominance of the symbolic and connectionist duality in the field. *Probabilistic models of cognition* describe a formal framework of human reasoning under uncertainty, grounded on probability theory (Chater et al., 2006, Griffiths et al., 2008). Even though Bernoulli and Bayes did theorize about human reasoning, their reflections were largely ignored by the computational cognitive science community until the late 80s and early 90s. The works of Roger Shepard on generalization theory (1987), Judea Pearl on belief networks and causality (1988), and Ulf Grenander on pattern theory (1993), were foundational for the probabilistic perspective. Why probabilistic models took so long to break into the field it is unclear. According to Chater, Tenenbaum, and Yuille

(Chater et al., 2006), there are at least three major reasons: the historical focus on architectures over content within those architectures; formal approaches to reason under uncertainty had been historically studied using non-probabilistic approaches, grounded in logic or heuristics; and finally, probabilistic methods have been perceived as too restricted in scope compared to linguistic description, logical representations, and neural network architectures. Although not mentioned by Chater and colleagues, probabilistic modeling entails a fair amount of mathematical formalism and computational resources to be implemented, things that most cognitive scientists did not have access to, either by lack of training or resources. Regardless of the reason for the late arrival, the fact is that the probabilistic perspective was established during the early 90s, grew in popularity over the next few decades, and today has become one the major focus of attention in computational cognitive science.

In the early 21st century, the availability of computational resources and data has exploded. Researchers in cognitive science and artificial intelligence have greatly benefited from this explosion. One of the best well-known examples of the impact of data and computational availability in the field is "*The ImageNet Large Scale Visual Recognition Challenge*" (ILSVRC; Deng et al., 2009). Since 2010, the ImageNet project runs a large scale competition on object recognition and image classification, where researchers from all over the world compete to produce models as accurate as possible. By 2011, solutions based on models fed with carefully crafted image features were achieving top-5 error rates (i.e., classifications where the correct label was among the 5 most likely answers produced by the model) of ~25%. The surprise came in 2012, when Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton (2012), introduced a convolutional neural network model with a top-5 error rate of ~17%. An 8% reduction in the error rate may not sound very impressive, but keep in mind that the field had struggled during decades to reduce the error rate, often with little success. The impact Krizhevsky, Sutskever, and Hinton's were enormous, effectively reviving the interest in artificial neural networks, in both the artificial intelligence and cognitive science communities.

How does the computational cognitive science field look today? The truth is that the correlational and lab-based experimental approaches still dominate cognitive psychology as a whole, at least by the numbers (just try a Google Scholar search or counting the number of Journsal by area). However, it is fair to say that in the last couple of years psychology has witnessed an increasing presence interest in computational models. More articles are published every year at major journals and conferences, and more students are being trained in computational methods than ever before. Cognitive scientists still debate about the merits of connectionist and symbolic approaches, but the landscape seems to be more open, with probabilistic and hybrid approaches joining the conversation. It is also fair to say that the prominence of deep learning in artificial intelligence has greatly contributed to the cause of connectionism, placing this approach as the center of attention, and making it the one producing the highest volume of research. Yet, as history demonstrates, scientists have been remarkably bad at predicting the future of the field, repeatedly overlooking its limitations and miscalculating its potential.

What are computational models of cognition?

To explain what computational models of cognition are, It is useful to state first what they are not: *computational models of cognition are not exact replicas of the human mind*, in the same manner, that city-maps are not exact replicas of real cities. The only way to create an exact map of a city is by rebuilding the city all over again until you get an inch by inch match between the city and your map, point at which the map becomes useless, as famously exposed by Jorge Luis Borges on "Del Rigor de la Ciencia" ("On Exactitude of Science", 1998):

> …In that Empire, the Art of Cartography attained such Perfection that the map of a single Province occupied the entirety of a City, and the map of the Empire, the entirety of a Province. In time, those Unconscionable Maps no longer satisfied, and the Cartographers Guilds struck a Map of the Empire whose size was that of the Empire, and which coincided point for point with it. The following Generations, who were not so fond of the Study of Cartography as their Forebears had been, saw that that vast Map was Useless, and not without some Pitilessness was it, that they delivered it up to the Inclemencies of Sun and Winters. In the Deserts of the West, still today, there are Tattered Ruins of that Map, inhabited by Animals and Beggars; in all the Land there is no other Relic of the Disciplines of Geography.
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> —Suarez Miranda,Viajes devarones prudentes, Libro IV,Cap. XLV, Lerida, 1658

Therefore, computational models of cognition are, in a sense, maps of cognition, or more academically, *simplified abstract representation of the mind*. Now, maps can help to understand multiple aspects of a city: the weather, political boundaries, economic regions, roads, topography, etc. Again, the same can be said about models of cognition. Some models help to understand memory, others language production, others visual perception, and so on. True, a model of language production does not need to be computational, it can be stated verbally, or graphically in a diagram. Informally, *to compute* means to take a set of inputs, manipulate those inputs using a set of operations following a sequence of instructions (or algorithm), to then produce an output. From here, we derive that computational models of cognition can be defined as *simplified abstract representations of the mind, that describe how some aspects of the mind process information in an algorithmic fashion, to produce some output* (e.g., language, inference, perception, etc). This is both more convoluted and more humble than "a replica of the mind".

What are computational models of cognition good for?

Examining the history of computational models in cognitive science says a lot about the importance of this approach to the study of the mind and behavior. Still, is important to clarify how computational modeling is different from other approaches, and what advantages and disadvantages entail.

Any attempt to classify the multiple approaches to the study of cognition and behavior will inevitably misrepresent their "true identity", exaggerating some aspects and neglecting others. Nonetheless, this exercise will help us to illustrate what is unique to computational approaches. Following Cronbach's perspective (1957), the two main traditions in scientific psychology are the *experimental* and the *correlational*. By experimental, he referred to laboratory-based quantitative studies. Today, we may want to add to that tradition any controlled experimental study aimed to establish causation, regardless of the setting. The correlational tradition is a bit fuzzier. About this Cronbach said: "*The correlational method, for its part, can study what man has not learned to control or can never hope to control*" (p. 672), which is a different way to say non-experimental and non-controlled studies, in any setting, using qualitative and quantitative methods. Very broad.

To begin, it is important to remember that cognition is not something you can directly observe. If we could, the chances are that many of today's challenges and controversies would be solved and that I wouldn't be writing this document. Any research study about cognition, either correlational or experimental, will proceed by measuring behavior, things like reaction time or eye-movements, and then making inferences about cognitive processes. Even if you take the perspective that cognitive processes are *literally* patterns of neural activity in the brain, studying such patterns is very limited in scope, very complicated, and very expensive. Let's say that you want to study language acquisition in early childhood. How would you approach the study of the cognitive processes operating when a child learns language? Techniques like functional magnetic resonance imaging (fMRI) are really hard to perform on children, since requires staying still in a giant noisy tube for an extended period of time. Even if you manage to introduce a child in an fMRI machine and stay still for 20 minutes (as many very clever researchers do), now you have to figure out a task that the child can perform, with some sort of remote control. True, there are some task that can be used with young children, but the question now is *what* and *how much* can you learn from having a child performing one task for 20 min in such setting. You have also to consider the fact that language acquisition is a developmental process, so multiple sessions over an extended period of time are required to gain better insight. Other techniques like electroencephalogram are less invasive and easier to perform on children, but many of the fMRI limitations persist. Does this mean that studies based on measuring behavior and brain activity are useless? Absolutely not. They both have their strengths and weaknesses. Our point was to illustrate that they have many limitations that make desirable to have an additional method to study human cognition.

Computational modeling has a lot to offer to enhance our understanding of human cognition, as a complement, not a supplement, of behavioral and brain-based studies. I want to highlight the following aspects: *isolation*, *simulation*, *simplification*, *quantification*, *practicality*, and *theory exploration*.

- **Isolation**: in studies with human subjects, it is really hard to isolate the effect of a specific cognitive process. In correlational studies, this is simply impossible, as anyone trying to account for "confounding" factors have experienced. In experimental studies, you can isolate the "treatment" theoretically impacting a cognitive process, but the process itself. Computational models do allow for such isolation of processes, as the processes are built into the model by the modeler, and can be altered at will without touching other aspects of the model.
- **Simulation**: computational models allow answering "what if" questions. What if I accelerate the speed at which the model learns in this particular task? Easy, just increase the "learning rate" parameter. What if I introduce "noise" in the communication among neurons in my neural network? Easy, too, just add some random noise to the weights matrix. Artificially manipulating cognitive processes in humans are possible too, for instance, by using transcranial magnetic stimulation or TMS. Unfortunately, you have little to no control over which processes are affected, unless you use very invasive methods, which usually require brain surgery. Additionally, the results of a simulation can be compared to human data, to further assess the validity of the model.
- **Simplification**: each element in a computational model needs to be explicitly defined. This fact forces the modeler to carefully select a subset of cognitive mechanisms to be incorporated into the model. As a consequence, the simplification of highly complex and interactive mental processes is achieved. Simplifying a complex system may help to better understand the role of each component.
- **Quantification**: by its very nature, computational models allow for fine-grained quantification of the role of each mechanism in a model. This is not unique to computational models, but still is one of its main advantages.
- **Practically**: in most instances, computational models require only two things: a computer and a modeler. More complex scenarios may require a cluster of computers and multiple modelers, but still, that is considerably simpler and cheaper, than most experimental psychology protocols. Although having human-generated data is desirable in many instances, this is not strictly required, and even if you use human data, it is common practice to use secondary sources of data (data collected in previous studies, sometimes by other research groups). In many cases, computational models can be tested with synthetic data, or with no data at all.
- **Theory exploration**: truth to be told, any research approach would allow for theory exploration. However, the computational modeling approach has the advantage of access to secondary data, and synthetic data, lowering the bar for examining the implications of hypothesized cognitive mechanism. In a way, it allows for rapid iteration without having to necessarily design a whole data collection process from scratch every time.

What are the limitations of computational models of cognition?

By now, you may have devised many objections and weaknesses of the computational modeling approach. I'll limit myself to mention a few that are particularly important to mention in my opinion: *oversimplification*, *overcomplexity*, *falsifiability*, and *technical complexity*:

- **Oversimplification**: this may sound contradicting, because before I highlighted *simplification* as one of the main advantages of computational models. I see simplification as a double-edged sword: on the one hand, it can help to see the bigger picture, but in the other, it can convey a distorted image of a phenomenon, to the point that hinders comprehension rather the enhancing it.
- **Overcomplexity**: related to the previous point, I also argued that models help to handle complexity. But they also allow the modeler to make the model as complex as it wants. Very complex, with lots of free parameters, can fit or approximate almost any human-generated data, giving the impression that they are a good representation of the cognitive mechanism at play. This may or may not be the truth, but the issues are that now understanding the model becomes almost impossible.
- **Falsifiability**: Popper's falsifiability criteria (2005), refers to the idea that for a scientific hypothesis to be valid, it has to be formulated in a manner in which it can be proved wrong. In other words, you have specified in advance what piece of evidence would prove your model wrong. It turns out that proving computational models wrong is quite hard, precisely because you can make it increasingly complex. This does not mean that it is impossible. For example, Palminteri, Wyart, & Koechlin (2017) have proposed utilizing the *generative performance criteria* for model falsification, this is: "*the ability of a given model to generate the data. The generative performance is evaluated by comparing model simulations to the actual data. For this comparison both frequentist and Bayesian statistics can be used*" (p. 426).
- **Technical complexity**: there is a significant barrier to computational modeling: mathematical and programming skills. Building computational models require a firm understanding of the mathematics involved in the model definition and non-trivial programming skills. Historically, mathematical and computational skills have not been part of the core training of researchers in psychological sciences. If you examine the background of most well-known cognitive modelers, you will find that the vast majority of them had additional training in mathematics and computer science, and some were straight mathematicians or computer scientists. At present, there are reasons to be optimistic, as tools that simplify the building of computational models becomes more available, and computational skills are added to the formative curriculum of young researchers.

Conclusions

Computational models of cognition have a long tradition that began in the first half of the 20th century, as a blend of computational theory and cognitive psychology. Since, four major approaches have emerged within this perspective: symbolic-based models, connectionist-based models, hybrids of connectionist and symbolic models, and the probability-based models. The computational approach has arguably contributed to improve and expand our understanding of human cognition and behavior. Among its many strengths, we have identified: isolating mechanism, simulation studies, simplification of complex systems, quantification of mechanism, practicality, and ease of theory exploration. Among its weaknesses, we have mentioned: oversimplification, overcomplexity, the difficulty of falsification, and technical complexity of implementation. In our perspective, recent advances in computation and data availability, along with the public and private of developing artificial intelligence technologies, provide a fertile context for the growth and consolidation of the computational perspective in cognitive science.

In the next sections, I provide a series of tutorials covering the theory, mathematical background, code implementation, and application, of canonical models in the connectionist tradition of cognitive modeling.

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