Meaning and Dialogue Coherence: A Proof-theoretic Investigation

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AIMS

- Relate *dialogue coherence* to *logical inference*.
- Show that certain *structural properties* of coherent dialogue can viewed as traces of *multi-agent inference*.
- Model dialogue coherence as emerging from the *meaning* of the *logical connectives* (*if* … *then* …, *and*, *or*). For that purpose, the meaning of the connectives is characterized in terms of their inferential role.

BACKGROUND

- We focus on coherence at the *dialogue structure level*.
  - *Pairing*: Question followed by an answer, Command followed by acknowledgement, ...
  - *Nesting*: E.g., question–answer pair inside another question–answer pair:

A:	How do you do, good lady. I am Arthur, King of the Britons. Who's castle is that?
W:	King of the who?
A:	The Britons.
W:	Who are the Britons?
	[...]
W:	No one lives there.

[From Monthy Python and the Holy Grail]

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A: The Britons.
W: Who are the Britons?
[...]
W: No one lives there.

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Background

- Dialogue type: *Cooperative Information-oriented dialogue*
  - *Main purpose*: exchange of information.
  - *Participants' aims*: cooperate with each others' requests for information (no persuasion, negotiation or coercion).

STRATEGY

1. We start with a theory of meaning for the logical connectives in terms of their inferential role;
2. This theory is explicated in terms of the Natural Deduction calculus (*cf.* Sundholm, 1986);
3. We extend the calculus to a calculus for *multiple situated reasoners*;
4. We show how proof (search) trees that are generated with the calculus can be systematically mapped to multi-agent dialogues;
5. We highlight some of the structural properties of these dialogues which correspond with those found in naturally occurring dialogues (by conversation analysts).

Natural Deduction

Ich wollte zunächst einmal einen Formalismus aufstellen, der dem wirklichen Schliessen möglichst nahe kommt. So ergab sich ein Kalkül des natürlichen Schliessens.

First I wished to construct a formalism that comes as close as possible to actual reasoning. Thus arose a "calculus of natural deduction". (Gentzen, 1935)

NATURAL DEDUCTION

- Language $\mathcal{L}$
- $At \subset \mathcal{L}$ $(a, b, c, \ldots)$
- if $A, B \in \mathcal{L}$, then $(A \rightarrow B) \in \mathcal{L}$ and $(A\&B) \in \mathcal{L}$
- Judgement: $H \vdash A$
- Persistent assumptions: Γ

  Can be updated, e.g., $\Gamma := \Gamma \cup \{a\}$

- (member) $$\frac{A \in \Gamma \cup H}{H \vdash A}$$

Natural Deduction

- (arrow intro) $$\frac{H \cup \{A\} \vdash B}{H \vdash A \to B}$$
- (arrow elim) $$\frac{H \vdash A \to B \quad H \vdash A}{H \vdash B}$$
- (conj. intro) $$\frac{H \vdash A \quad H \vdash B}{H \vdash A \& B}$$
- (conj. elim) $$\frac{H \vdash A \& B}{H \vdash A} \qquad \frac{H \vdash A \& B}{H \vdash B}$$

MULTI-AGENT NATURAL DEDUCTION

- Set of agents $\mathcal{A}$ with α, β, γ, $\ldots \in \mathcal{A}$
- Channels of communication $\mathcal{C} \subseteq \mathcal{A} \times \mathcal{A}$
- Judgement: $[\alpha]\ H \vdash A$
- (tr) $\dfrac{[\beta]\ H \vdash A}{[\alpha]\ H \vdash A}$ $\quad \Gamma_\alpha := \Gamma_\alpha \cup \{\bigwedge H \rightarrow A\}$
  and $\langle \alpha, \beta \rangle \in \mathcal{C}$
- $\bigwedge H$ stands for the conjunction $A_1 \& A_2 \& \ldots$ of the formulae $A_1, A_2, \ldots$ that are members of H. If H is empty, $\bigwedge H \rightarrow A = A$.

Example of a Proof Tree

- Agents α, β and γ. With $\Gamma_\alpha = \emptyset$, $\Gamma_\beta = \{a\}$, and $\Gamma_\gamma = \{b\}$.
- Goal: α wants to build a proof for $a\&b$.
- $$\dfrac{\dfrac{\dfrac{[\beta]\ a \in \Gamma_\beta \cup \emptyset}{[\beta]\ \emptyset \vdash a}(mem.)}{[\alpha]\ \emptyset \vdash a}(1)(tr) \qquad \dfrac{\dfrac{[\gamma]\ b \in \Gamma_\gamma \cup \emptyset}{[\gamma]\ \emptyset \vdash b}(mem.)}{[\alpha]\ \emptyset \vdash b}(2)(tr)}{[\alpha]\ \emptyset \vdash a\&b}(conj.intro)$$
- Side conditions: (1) $\Gamma_\alpha := \Gamma_\alpha \cup \{a\}$; (2) $\Gamma_\alpha := \Gamma_\alpha \cup \{b\}$.
- Result: $\Gamma_\alpha = \{a, b\}$

FROM PROOF TREE TO DIALOGUE STRUCTURE (I)

1.	$\alpha:$	goal-know-if($a\&b$)
2.	$\alpha:$	(transfer) goal-know-if(a)
3.	$\beta:$	goal-know-if(a)
4.	$\beta:$	in-assumptions(a)
5.	$\beta:$	confirmed(a)
6.	$\alpha:$	confirmed(a)
7.	$\alpha:$	(transfer) goal-know-if(b)
8.	$\gamma:$	goal-know-if(b)
9.	$\gamma:$	in-assumptions(b)
10	$\gamma:$	confirmed(b)
11.	$\alpha:$	confirmed(b)
12.	$\alpha:$	confirmed($a\&b$)

FROM PROOF TREE TO DIALOGUE STRUCTURE (II)

- α_i : goal-know-if(A) $\mapsto$ α_i : I am wondering whether A.
- α_i : (transfer) goal-know-if(A), α_j : I am wondering whether A $\mapsto$ α_i : Tell me α_j, A?
- α_i : confirmed(A), α_j : confirmed(A) $\mapsto$ α_i : confirmed(A).
- α_i : in-assumptions(A), α_i : confirmed(A) $\mapsto$ α_i : A.
- α_i : confirmed(A) $\mapsto$ α_i : That confirms A.

From Proof Tree to Dialogue Structure (II)

1. α : I am wondering whether $a\&b$.
2. α : Tell me β, a?
3. β : a.
4. α : Tell me γ, b?
5. γ : b.
6. α : That confirms $a\&b$.

$$\dfrac{\dfrac{\dfrac{[\beta]\ a \in \Gamma_\beta \cup \emptyset}{[\beta]\ \emptyset \vdash a}(mem.)}{[\alpha]\ \emptyset \vdash a}(1)(tr) \qquad \dfrac{\dfrac{[\gamma]\ b \in \Gamma_\gamma \cup \emptyset}{[\gamma]\ \emptyset \vdash b}(mem.)}{[\alpha]\ \emptyset \vdash b}(2)(tr)}{[\alpha]\ \emptyset \vdash a\&b}(conj.intro)$$

Generative Systems for Dialogue Modelling

1. A hybrid inference system $\mathcal{I}$ consisting of:
   (a) A language $\mathcal{L}$;
   (b) A set of agents $\mathcal{A}$, each with a set of assumptions Γ_{α_i};
   (c) A communication channel $\mathcal{C}$;
   (d) A set of hybrid inference rules $\mathcal{R}$ for the language and the agents.
2. A specification of the set of potential dialogues $\mathcal{D}_P$ between the agents, given the language $\mathcal{L}$.
3. A mapping m from proof trees, generated with $\mathcal{I}$, to coherent dialogues $\mathcal{D}$.

$\mathcal{S}_2$: From Proof Trees to Proof Search Trees

- Add to expressions of $\mathcal{D}_P$: I don't know whether $\langle Prop \rangle$.
- Initial situation: $\Gamma_\alpha = \Gamma_\beta = \emptyset$ and $\Gamma_\gamma = \{a, b\}$
-

$$\frac{\dfrac{\dfrac{}{(\star_1)\ [\beta]\,\emptyset \vdash a} \qquad \dfrac{[\gamma]\, a \in \Gamma_\gamma \cup \emptyset}{(\star_2)\ [\gamma]\,\emptyset \vdash a}}{[\alpha]\,\emptyset \vdash a} \qquad \dfrac{\dfrac{[\gamma]\, a \in \Gamma_\gamma \cup \emptyset}{[\gamma]\,\emptyset \vdash a}}{[\alpha]\,\emptyset \vdash b}}{[\alpha]\,\emptyset \vdash a \& b}$$

Dialogue Structure

1. $\alpha :$ I am wondering whether $a \& b$.
2. $\alpha :$ Tell me β, a?
3. $\beta :$ I don't know whether a.
4. $\alpha :$ Tell me γ, a?
5. $\gamma :$ a.
6. $\alpha :$ Tell me γ, b?
7. $\gamma :$ b.
8. $\alpha :$ That confirms $a \& b$.

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Dialogue Structure

1. α : I am wondering whether $a\&b$.
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$\mathcal{S}_3$: Adding Observation

- Proposition A is an observable proposition for agent α is written as $A \in \mathcal{O}_\alpha$.
- α actually observes that A is written $obs(\alpha, A)$.

- (obs.) $\dfrac{A \in \mathcal{O}_\alpha \;\; obs(\alpha, A)}{[\alpha]H \;\vdash\; A}$ $\quad \Gamma_\alpha := \Gamma_\alpha \cup \{A\}$

$\mathcal{S}_3$ Example

- $\Gamma_\pi = \emptyset$;
- $\Gamma_\delta = \{ht \rightarrow sd\}$ (have tempature entails see doctor);
- π's goal is to find out whether sd (see doctor).
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$$
\dfrac{\dfrac{\dfrac{ht \in \mathcal{O}_\pi \quad obs(\pi, ht)}{[\pi]\ \emptyset \vdash ht}\,(3)\ (obs.)}{[\delta]\ \emptyset \vdash ht}\,(2)\ (tr) \qquad \dfrac{[\delta]\ ht \rightarrow sd \in \Gamma_\delta}{[\delta]\ \emptyset \vdash ht \rightarrow sd}}{\dfrac{[\delta]\ \emptyset \vdash sd}{[\pi]\ \emptyset \vdash sd}\,(1)\ (tr)}
$$

Side conditions: (1) $\Gamma_\pi := \Gamma_\pi \cup \{sd\}$, (2) $\Gamma_\delta := \Gamma_\delta \cup \{ht\}$, (3) $\Gamma_\pi := \Gamma_\pi \cup \{ht\}$.

$\mathcal{S}_3$ Dialogue

1.	π:	Do I need to see a doctor?
2.	δ:	Do you have a temperature?
3.	π:	Wait a minute [π checks her temperature], yes, I do.
4.	δ:	Then you do need to see a doctor.

$\mathcal{S}_3$ Dialogue

1. π: Do I need to see a doctor?
2. δ: Do you have a temperature?
3. π: Wait a minute [π checks her temperature], yes, I do.
4. δ: Then you do need to see a doctor.

Open Issues

- Addition of negation and consistency maintenance;
- Unsuccessful speech acts;
- Gricean implicatures (omitting inference steps);
- Adding imperatives;
- More powerful logics;
- Towards a model for autonomous dialogue agents.
- ...

Comparisons and Conclusions

- Extended Standard Natural Deduction with multiple agents and observation;
- Rather than special purpose update and generation rules (Beun 2001, Ginzburg 1996, Traum & Larsson 2003, ...) we derive dialogue structure from the meaning of the logical connectives.
- Dialogue game approaches have concentrated on explaining logically valid inference in terms of formal winning strategies for dialogue (Lorenzen, Hintikka). These have focussed on adversarial (proponent versus opponent) rather than cooperative dialogue.

Comparisons and Conclusions

Motivating project: *What is the role/place of logic and sentence semantics in models of dialogue coherence?*