Solutions Manual
for
Microwave Engineering 3/e

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Solutions Manual

for

Microwave Engineering
Third Edition

Contained here are solutions for all of the end-of-chapter problems in the third edition of Microwave Engineering. Some of these problems require the derivation of theoretical results, but many are design oriented. Some of these problems are easy, while others are lengthy and challenging. Many of the matching, coupler, filter, and amplifier design problems ask for a CAD analysis of the final circuit, where it is presumed that the student has access to a microwave CAD software tool, such as Ansoft's Serenade, or similar. There are several such packages that are available for free download on the Internet. The Wiley web site contains Serenade files for the problems and examples amenable to CAD analysis.

Working problems is a critical part of the learning process for engineering students, and these problems have been developed to give students practice in applying the basic concepts of microwave engineering, as well as practice in the analysis and design of practical microwave circuits and components. These problems can be used as assigned homework problems, exam problems, or as supplemental problems for students to work out on their own. The present edition features many new and revised problems, but if additional problems are needed, it should be easy for the instructor to derive new problems from those given in the text. Also new to this edition is the inclusion of short answers to many of the problems at the back of the text.

The majority of these solutions have been checked with known results, compared with independent solutions by others or, in the case of design problems, verified by computer simulation. Such results usually have a check mark to indicate that they have a high (but not perfect!) likelihood of correctness. Nevertheless, there are undoubtedly some errors that remain, and the author will be grateful if such mistakes are brought to his attention.

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Chapter 1

(1.1) As in Example 1.3, assume outgoing plane wave fields in each region. To get J_{sx}, we need H_y, since $\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s$ ($\hat{n} = \hat{z}$). Then we must have E_x to get $\bar{S} = \bar{E} \times \bar{H}^* = \pm S \hat{z}$. So the form of the fields must be,

for $z < 0$, $\bar{E}_1 = \hat{x} A e^{jk_0 z}$ $\qquad$ for $z > 0$, $\bar{E}_2 = \hat{x} B e^{-jkz}$

$\bar{H}_1 = \dfrac{-\hat{y}}{\eta_0} A e^{jk_0 z}$ $\qquad$ $\bar{H}_2 = \hat{y} \dfrac{B}{\eta} e^{-jkz}$

with $k_0 = \omega\sqrt{\mu_0 \epsilon_0}$, $k = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r}$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, $\eta = \sqrt{\mu_0/\epsilon_0 \epsilon_r}$, and A and B are unknown amplitudes to be determined.

The boundary conditions at $z=0$ are, from (1.36) and (1.37),

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = 0 \quad \Rightarrow \quad A = B$$

$$\hat{z} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s \quad \Rightarrow \quad -\left(\frac{B}{\eta} + \frac{A}{\eta_0}\right) = J_0$$

$$\therefore \quad A = B = \frac{-J_0 \eta \eta_0}{\eta + \eta_0}$$

(1.2) $$\nabla \times \bar{E} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}\right) + \hat{z}\,\frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi}\right)$$

$$\nabla \times \nabla \times \bar{E} = \hat{\rho}\left[\frac{-1}{\rho^2}\frac{\partial^2 E_\rho}{\partial \phi^2} - \frac{\partial^2 E_\rho}{\partial z^2} + \frac{\partial^2 E_z}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \rho \partial \phi} + \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial \phi}\right]$$

$$+\hat{\phi}\left[-\frac{\partial^2 E_\phi}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_z}{\partial \phi \partial z} - \frac{\partial^2 E_\phi}{\partial \rho^2} - \frac{1}{\rho}\frac{\partial E_\phi}{\partial \rho} + \frac{E_\phi}{\rho^2} - \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial \phi} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial \phi \partial \rho}\right]$$

$$+\hat{z}\left[\frac{-\partial^2 E_z}{\partial \rho^2} - \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_\rho}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \phi \partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z} - \frac{1}{\rho}\frac{\partial E_z}{\partial \rho}\right]$$

$$\nabla(\nabla \cdot \bar{E}) = \hat{\rho}\left[\frac{\partial^2 E_\rho}{\partial \rho^2} + \frac{\partial^2 E_z}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \rho \partial \phi} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial \rho} - \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial \phi} - \frac{E_\rho}{\rho^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial^2 E_z}{\partial \phi \partial z} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial \rho \partial \phi} + \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial \phi}\right]$$

$$+\hat{z}\left[\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial \phi \partial z} + \frac{\partial^2 E_\rho}{\partial \rho \partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z}\right]$$

If we apply ∇^2 to the cylindrical components of $\bar{E}$ we get:

$$\nabla^2 \bar{E} \stackrel{?}{=} \hat{\rho}\nabla^2 E_\rho + \hat{\phi}\nabla^2 E_\phi + \hat{z}\nabla^2 E_z \qquad \text{(THIS IS NOT A VALID STEP!)}$$

$$= \hat{\rho}\left[\frac{1}{\rho}\frac{\partial E_\rho}{\partial \rho} + \frac{\partial^2 E_\rho}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\rho}{\partial \phi^2} + \frac{\partial^2 E_\rho}{\partial z^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial E_\phi}{\partial \rho} + \frac{\partial^2 E_\phi}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{\partial^2 E_\phi}{\partial z^2}\right]$$

$$+\hat{z}\left[\frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2}\right]$$

Note that the $\hat{\rho}$ and $\hat{\phi}$ components of $\nabla\times\nabla\times\bar{E}$ and $\nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E}$ do not agree. This is because $\hat{\rho}$ and $\hat{\phi}$ are not constant vectors, so $\nabla^2\bar{E} \neq \hat{\rho}\nabla^2 E_\rho + \hat{\phi}\nabla^2 E_\phi + \hat{z}\nabla^2 E_z$. The $\hat{z}$ components are equal.

(1.3) $\bar{S} = \bar{E} \times \bar{H}^* = E_0 H_0 \hat{z} \neq 0$

The problem here is that Poynting's theorem requires a closed surface integral for a meaningful interpretation in terms of power flow. If we calculate $\oint \bar{S} \cdot d\bar{s}$ over the closed surface of a cube bounded by the magnet faces and the capacitor plates, we will get zero, since $\hat{n} = \hat{z}$ on one side of the cube, while $\hat{n} = -\hat{z}$ on the opposite side. Since S is a constant, these terms cancel.

(1.4) $E_y = E_0 \cos(\omega t - kz)$, $f = 2.4\,\text{GHz}$, $E_0 = 30\,\text{V/m}$

a) $\eta = 377/\sqrt{2.55} = 236\,\Omega$

$H_x = -E_y/\eta = -0.127 \cos(\omega t - kz)$

b) $v_p = c/\sqrt{\epsilon_r} = 1.88 \times 10^8$ m/sec

c) $k = \omega / v_p = 80.2\ \text{m}^{-1}$

$\Delta\phi = k \Delta z = 5514° = 114°$

(1.5) Let $\bar{E} = A(\hat{x} - j\hat{y}) e^{-jk_0 z} + B(\hat{x} + j\hat{y}) e^{-jk_0 z}$

where A is the amplitude of the RHCP component, and B is the amplitude of the LHCP component. Equating this expression to the given linearly polarized field gives,

$\hat{x}: \quad A + B = E_0$

$\hat{y}: \quad -jA + jB = 2E_0$

Solving for A, B gives

$$A = \left(\tfrac{1}{2} + j\right) E_0$$

$$B = \left(\tfrac{1}{2} - j\right) E_0$$

Any linearly polarized wave (in any direction) can be decomposed into the sum of two circularly polarized waves.

(1.6) From eq. (1.76),

$$\bar{H} = \frac{1}{\eta_0} \hat{n} \times \bar{E} \quad , \quad \bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}$$

$$\bar{S} = \bar{E} \times \bar{H}^* = \frac{1}{\eta_0} \bar{E} \times \hat{n} \times \bar{E}^*$$

$$= \frac{1}{\eta_0} \left[(\bar{E}\cdot\bar{E}^*)\hat{n} - (\bar{E}\cdot\hat{n})\bar{E}^* \right] \qquad \text{(from B.5)}$$

Since $\bar{k}\cdot\bar{E}_0 = k_0 \hat{n}\cdot\bar{E}_0 = 0$ from (1.69) and (1.74), we have

$$\bar{S} = \frac{\hat{n}}{\eta_0} \bar{E}\cdot\bar{E}^* = \frac{\hat{n}}{\eta_0} |E_0|^2 \quad \text{W/m}^2 \quad ✓$$

(1.7) Writing general plane wave fields in each region:

$\bar{E}^i = \hat{x} e^{-jk_0 z}$	$\bar{H}^i = \frac{\hat{y}}{\eta_0} e^{-jk_0 z}$	for $z<0$
$\bar{E}^r = \hat{x} \Gamma e^{jk_0 z}$	$\bar{H}^r = \frac{-\hat{y}}{\eta_0} \Gamma e^{jk_0 z}$	for $z<0$
$\bar{E}^s = \hat{x}(A e^{-jkz} + B e^{jkz})$	$\bar{H}^s = \frac{\hat{y}}{\eta}(A e^{-jkz} - B e^{jkz})$	for $0<z<d$
$\bar{E}^t = \hat{x} T e^{-jk_0(z-d)}$	$\bar{H}^t = \frac{\hat{y}}{\eta_0} T e^{-jk_0(z-d)}$	for $z>d$

Now match E_x and H_y at $z=0$ and $z=d$ to obtain four equations for Γ, T, A, B:

$z=0$: $\quad 1+\Gamma = A+B \qquad \frac{1}{\eta_0}(1-\Gamma) = \frac{1}{\eta}(A-B)$

$z=d$: $\quad j(-A+B) = T \qquad \frac{j}{\eta}(-A-B) = \frac{T}{\eta_0} \qquad$ (since $d = \lambda_0/4\sqrt{\epsilon_r}$)

Solving for Γ gives

$$\Gamma = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2} \quad ✓$$

CHECK:

$\lambda/4$ TRANSFORMER $\Rightarrow Z_{in} = \eta^2/\eta_0$, $\Gamma = \dfrac{\eta^2/\eta_0 - \eta_0}{\eta^2/\eta_0 + \eta_0} = \dfrac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2}$.

(1.8) The incident, reflected, and transmitted fields can be written as,

$$\bar{E}^i = E_0(\hat{x} - j\hat{y})e^{-jk_0 z} \qquad \bar{H}^i = j\frac{E_0}{\eta_0}(\hat{x} - j\hat{y})e^{-jk_0 z} \qquad \text{(RHCP)}$$

$$\bar{E}^r = E_0\Gamma(\hat{x} - j\hat{y})e^{jk_0 z} \qquad \bar{H}^r = -j\frac{E_0}{\eta_0}\Gamma(\hat{x} - j\hat{y})e^{jk_0 z} \qquad \text{(LHCP)}$$

$$\bar{E}^t = E_0 T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad \bar{H}^t = j\frac{E_0}{\eta}T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad \text{(RHCP)}$$

Matching fields at $z=0$ gives

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}, \qquad T = \frac{2\eta}{\eta + \eta_0}$$

The Poynting vectors are: $\quad (\hat{x} - j\hat{y}) \times (\hat{x} - j\hat{y})^* = 2j\hat{z}$

For $z<0$: $\bar{S}^- = (\bar{E}^i + \bar{E}^r) \times (\bar{H}^i + \bar{H}^r)^* = \dfrac{2\hat{z}|E_0|^2}{\eta_0}\left(1 - |\Gamma|^2 + \Gamma e^{2jk_0 z} + \Gamma^* e^{-2jk_0 z}\right)$ ✓

For $z>0$: $\bar{S}^+ = \bar{E}^t \times \bar{H}^{t*} = \dfrac{2\hat{z}|E_0|^2|T|^2}{\eta^*}e^{-2\alpha z}$ ✓

at $z=0$,

$$\bar{S}^- = \frac{2\hat{z}|E_0|^2}{\eta_0}\left(1 - |\Gamma|^2 + \Gamma - \Gamma^*\right) = \frac{2\hat{z}|E_0|^2}{\eta_0}(1+\Gamma)(1-\Gamma^*) \checkmark$$

$$\bar{S}^+ = 2\hat{z}|E_0|^2 \frac{4\eta}{|\eta + \eta_0|^2} \qquad \left(\text{using } T = \frac{2\eta}{\eta + \eta_0}\right)$$

$$= \frac{2\hat{z}|E_0|^2}{\eta_0}\left(\frac{2\eta}{\eta + \eta_0}\right)\left(\frac{2\eta_0}{\eta + \eta_0}\right)^* = \frac{2\hat{z}|E_0|^2}{\eta_0}(1+\Gamma)(1-\Gamma^*) \checkmark$$

Thus $\bar{S}^- = \bar{S}^+$ at $z=0$, and power is conserved.

(1.9) From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = 2\pi j f\sqrt{\mu_0\epsilon_0}\sqrt{5-j2} = j\frac{2\pi(1000)}{300}\sqrt{5.385\angle{-22^\circ}}$$

$$= 48.5\angle{79^\circ} = 9.25 + j47.6 = \alpha + j\beta \quad (\text{neper/m, rad/m})$$

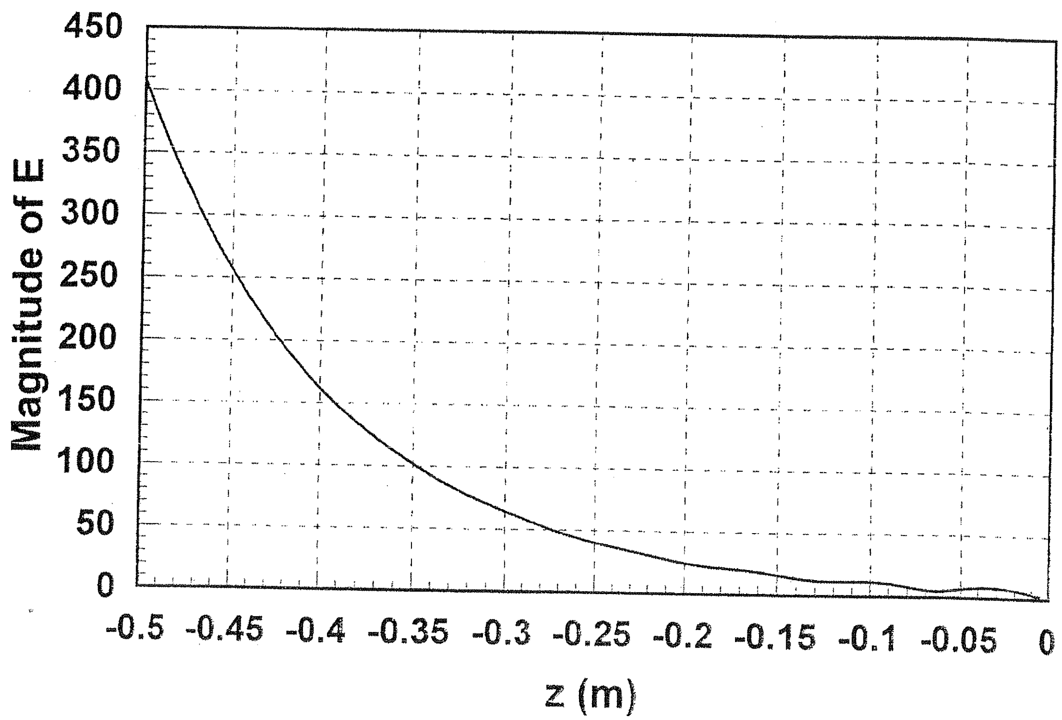
$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\sqrt{\mu_0\epsilon_0}}{j\omega\sqrt{\mu_0\epsilon}}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{\sqrt{5-j2}} = \frac{377}{2.32\angle{-11^\circ}} = 163\angle{11^\circ}\ \Omega$$

$$\Gamma = -1$$

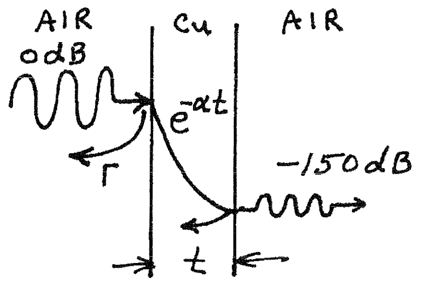
For $z<0$, $\quad \bar{E} = \bar{E}^i + \bar{E}^r = 4\hat{x}\left(e^{-\gamma z} - e^{\gamma z}\right)$

$$|\bar{E}| = 4\left|e^{-\alpha z}e^{-j\beta z} - e^{\alpha z}e^{j\beta z}\right|$$

$|\bar{E}|$ vs z is plotted below.



(1.10) The total loss through the sheet is the product of the transmission losses at the air-copper and copper-air interfaces, and the exponential loss through the sheet.



$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = 2.09\times10^{-6}\,m = \frac{1}{\alpha}$$

$$\eta_c = \frac{(1+j)}{\sigma\delta_s} = 8.2\times10^{-3}(1+j)\,\Omega$$

a) Power transfer from air into copper is given by,

$$1-|\Gamma|^2 \quad , \quad \Gamma = \frac{\eta_c-\eta_o}{\eta_c+\eta_o} \simeq \frac{8.2\times10^{-3}(1+j)-377}{377} = -0.999956+j\,4.35E-5$$

This yields a power transfer of -40.6 dB into the copper. By symmetry, the same transfer occurs for the copper-air interface.

b) the attenuation within the copper sheet is,

$$\text{copper att.} = 150\,dB - 40.6\,dB - 40.6\,dB = 68.8\,dB$$

$$= -20\log e^{-t/\delta_s} \Rightarrow \underline{t = 0.017\ mm} \checkmark$$

(J. Mead provided this correction on 9/04)

(1.11) From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = j\frac{2\pi(3000)}{300}\sqrt{3(1-j.1)} = 5.435 + j108.964 = \alpha + j\beta \quad m^{-1}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r(1-j.1)}} = 217.121\angle 2.855°$$

(a) $$P_i = Re\left\{\frac{|\bar{E}_i(z=0)|^2}{\eta^*}\right\} = 46.000 \quad W/m^2 \quad ✓$$

$\Gamma = -1$ at $z = l = 20$ cm

$$\bar{E}_r = \Gamma \bar{E}_i(z=l)\,e^{\gamma(z-l)} = -100\hat{x}\,e^{-2\gamma l}e^{\gamma z}$$

$$P_r = Re\left\{\frac{|\bar{E}r(z=0)|^2}{\eta^*}\right\} = 0.595 \quad W/m^2 \quad ✓$$

(b) $\bar{E}_t = \bar{E}_i + \bar{E}r$

$$\bar{E}_t\,(z=0) = 100\hat{x}\,(1 - e^{-2\gamma l}) \qquad H_t\,(z=0) = \frac{100\hat{y}}{\eta}(1 + e^{-2\gamma l})$$

$$P_{in} = Re\left\{\bar{E}_t \times \bar{H}_t^* \cdot \hat{z}\right\} = 45.584 \quad W/m^2$$

But $P_i - P_r = 45.405\ W/m^2 \neq P_{in}$. This is because P_i and P_r individually are not physically meaningful in a lossy medium.

The above values were computed entirely using a FORTRAN program, with 6-digit precision. The error between $P_i - P_r$ and P_{in} is only about 0.4% – this could be made more significant if the loss were increased.

(1.12) This current sheet will generate obliquely propagating plane waves. From (1.132)–(1.133), assume

$$\left.\begin{aligned}\bar{E}_1 &= A(\hat{x}\cos\theta_1 + \hat{z}\sin\theta_1)\, e^{-jk_o(x\sin\theta_1 - z\cos\theta_1)} \\ \bar{H}_1 &= \frac{-A}{\eta_o}\,\hat{y}\, e^{-jk_o(x\sin\theta_1 - z\cos\theta_1)}\end{aligned}\right\}\text{ for } z<0$$

$$\left.\begin{aligned}\bar{E}_2 &= B(\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)} \\ \bar{H}_2 &= \frac{B}{\eta}\,\hat{y}\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)}\end{aligned}\right\}\text{ for } z>0$$

with $k_o = \omega\sqrt{\mu_o\epsilon_o}$, $k = \sqrt{\epsilon_r}\,k_o$, $\eta_o = \sqrt{\mu_o/\epsilon_o}$, $\eta = \eta_o/\sqrt{\epsilon_r}$.

Apply boundary conditions at $z=0$:

$$\hat{z}\times(\bar{E}_2 - \bar{E}_1) = 0 \implies A\cos\theta_1 e^{-jk_o x\sin\theta_1} - B\cos\theta_2 e^{-jkx\sin\theta_2} = 0$$

$$\hat{z}\times(\bar{H}_2 - \bar{H}_1) = J_s \implies \frac{A}{\eta_o} e^{-jk_o x\sin\theta_1} + \frac{B}{\eta} e^{-jkx\sin\theta_2} = -J_o e^{-j\beta x}$$

For phase matching we must have $k_o\sin\theta_1 = k\sin\theta_2 = \beta$

$\therefore\ \theta_1 = \sin^{-1}\beta/k_o$ ✓ $\quad \theta_2 = \sin^{-1}\beta/k$ (must have $\beta < k_o$)

Then, $\quad A\cos\theta_1 = B\cos\theta_2$, $\quad \dfrac{A}{\eta_o} + \dfrac{B}{\eta} = -J_o$

$$A = \frac{-J_o\eta\eta_o\cos\theta_2}{\eta\cos\theta_2 + \eta_o\cos\theta_1}, \quad B = \frac{-J_o\eta\eta_o\cos\theta_1}{\eta\cos\theta_2 + \eta_o\cos\theta_1}$$

Check: If $\beta = 0$, then $\theta_1 = \theta_2 = 0$, and $A = B = \dfrac{-J_o\eta\eta_o}{\eta + \eta_o}$,

which agrees with Problem 1.1 ✓

(1.13) This solution is identical to the parallel polarized dielectric case of Section 1.8, except for the definitions of k_1, k_2, η_1, and η_2. Thus,

$$k_0 \sin\theta_i = k_0 \sin\theta_r = k \sin\theta_t \qquad ; \; k = k_0\sqrt{\mu_r}$$

$$\Gamma = \frac{\eta\cos\theta_t - \eta_0 \cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i} \qquad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i}$$

$$\eta = \eta_0\sqrt{\mu_r}$$

There will be a Brewster angle if $\Gamma = 0$. This requires that,

$$\eta\cos\theta_t = \eta_0\cos\theta_i$$

$$\sqrt{\mu_r}\sqrt{1-\left(\frac{k_0}{k}\right)^2\sin^2\theta_i} = \cos\theta_i = \sqrt{1-\sin^2\theta_i}$$

$$\mu_r\left(1-\frac{1}{\mu_r}\cancel{\sin^2\theta_i}\right) = 1-\cancel{\sin^2\theta_i}$$

or, $\mu_r = 1$. This implies a uniform region, so there is no Brewster angle for $\mu_r \neq 1$. ✓

(1.14) Again, this solution is similar to the perpendicular polarized case of Section 1.8, except for the definition of k_1, k_2, η_1, η_2. Thus,

$$\Gamma = \frac{\eta\cos\theta_i - \eta_0\cos\theta_t}{\eta\cos\theta_i + \eta_0\cos\theta_t} \quad , \quad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_i + \eta_0\cos\theta_t}$$

A Brewster angle exists if

$$\eta\cos\theta_i = \eta_0\cos\theta_t$$

$$\sqrt{\mu_r}\sqrt{1-\sin^2\theta_i} = \sqrt{1-\frac{1}{\mu_r}\sin^2\theta_i}$$

$$\mu_r^2 - \mu_r^2\sin^2\theta_i = \mu_r - \sin^2\theta_i$$

$$\mu_r = (\mu_r+1)\sin^2\theta_i$$

$$\sin\theta_i = \sin\theta_b = \sqrt{\frac{\mu_r}{1+\mu_r}} < 1 \quad \checkmark$$

Thus, a Brewster angle does exist for this case.

(1.15)

$$\bar{E} = 2\hat{x} + 3\hat{y} + 4\hat{z}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} 1 & -2j & 0 \\ 2j & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2-6j \\ 9+4j \\ 16 \end{bmatrix}$$

(1.16)

$$D_x = \epsilon_0(\epsilon_r E_x + jKE_y)$$

$$D_y = \epsilon_0(-jKE_x + \epsilon_r E_y)$$

$$D_z = \epsilon_0 E_z$$

Then,

$$D_+ = D_x - jD_y = \epsilon_0(\epsilon_r - K)E_x - j\epsilon_0(\epsilon_r - K)E_y = \epsilon_0(\epsilon_r - K)E_+$$

$$D_- = D_x + jD_y = \epsilon_0(\epsilon_r + K)E_x + j\epsilon_0(\epsilon_r + K)E_y = \epsilon_0(\epsilon_r + K)E_-$$

OR,

$$\begin{bmatrix} D_+ \\ D_- \\ D_z \end{bmatrix} = \begin{bmatrix} (\epsilon_r - K) & 0 & 0 \\ 0 & (\epsilon_r + K) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} E_+ \\ E_- \\ E_z \end{bmatrix}$$

From Maxwell's equations,

$$\left.\begin{aligned} \nabla\times\bar{E} &= -j\omega\mu\bar{H} \\ \nabla\times\bar{H} &= j\omega[\epsilon]\bar{E} \end{aligned}\right\} \quad \begin{aligned} \nabla\times\nabla\times\bar{E} &= -j\omega\mu\,\nabla\times\bar{H} = \omega^2\mu[\epsilon]\bar{E} \\ \nabla^2\bar{E} &+ \omega^2\mu[\epsilon]\bar{E} = 0 \quad \text{(CARTESIAN)} \end{aligned}$$

Expanding this wave equation gives,

$$\nabla^2 E_x + \omega^2\mu\epsilon_0(\epsilon_r E_x + jKE_y) = 0 \qquad (1)$$

$$\nabla^2 E_y + \omega^2\mu\epsilon_0(-jKE_x + \epsilon_r E_y) = 0 \qquad (2)$$

$$\nabla^2 E_z + k_0^2 E_z = 0 \qquad (3)$$

adding (1) $+j$(2) gives $\nabla^2(E_x + jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r+K)E_x + j(\epsilon_r+K)E_y] = 0$

$$\nabla^2 E^+ + \omega^2\mu\epsilon_0(\epsilon_r+K)E^+ = 0$$

$$\therefore \beta_+ = k_0\sqrt{\epsilon_r+K} \quad \checkmark$$

adding (1) $-j$(2) gives $\nabla^2(E_x - jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r - \kappa)E_x - j(\epsilon_r-\kappa)E_y] = 0$

$$\nabla^2 E^- + \omega^2\mu\epsilon_0(\epsilon_r - \kappa)E^- = 0$$

$$\therefore\ \beta_- = k_0\sqrt{\epsilon_r - \kappa}\ ✓$$

Note that the wave equations for E^+, E^- must be satisfied simultaneously. Thus, for E^+ we must have $E^- = 0$. This implies that $E_y = jE_x = jE_0$. The actual electric field is then, $\bar{E}^+ = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} + j\hat{y})e^{-j\beta_+ z}$ (LHCP)

This is a LHCP wave. Similarly for $\bar{E}^-$ we must have $E^+ = 0$: $\bar{E}^- = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} - j\hat{y})e^{-j\beta_- z}$ (RHCP)

(1.17) Comparing (1.118), (1.125), and (1.129) shows that

$$E_t = \frac{J_t}{\sigma} = \frac{J_s}{\sigma\delta_s} = R_s J_s.$$

Thus $\bar{E}_t = R_s\bar{J}_s = R_s\,\hat{n}\times\bar{H}$ is the desired surface impedance relation. Applying this to the surface integral of (1.155) gives, on S,

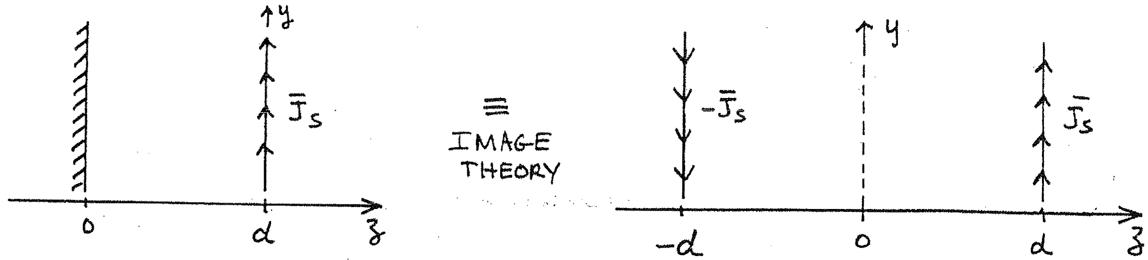
$$[(\bar{E}_1\times\bar{H}_2) - (\bar{E}_2\times\bar{H}_1)]\cdot\hat{n} = R_s[(\hat{n}\times\bar{H}_{1t})\times\bar{H}_{2t} - (\hat{n}\times\bar{H}_{2t})\times\bar{H}_{1t}]$$

(USING B.5) $$= R_s[\cancel{(\bar{H}_{2t}\cdot\hat{n})}^{0}\bar{H}_{1t} - \cancel{(\bar{H}_{2t}\cdot\bar{H}_{1t})\hat{n}} - \cancel{(\bar{H}_{1t}\cdot\hat{n})}^{0}H_{2t} + \cancel{(\bar{H}_{1t}\cdot\bar{H}_{2t})\hat{n}}]$$

$$= 0$$

So (1.157) is obtained.

(1.18)



First find the fields due to the source at $z=d$. From (1.139)–(1.140),

For $z<d$,
$$\bar{E}_1 = A\hat{y}\, e^{-jk_o(x\sin\theta - z\cos\theta)}$$
$$\bar{H}_1 = \frac{A}{\eta_o}(\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta - z\cos\theta)}$$

For $z>d$,
$$\bar{E}_2 = B\hat{y}\, e^{-jk_o(x\sin\theta + z\cos\theta)}$$
$$\bar{H}_2 = \frac{B}{\eta_o}(-\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta + z\cos\theta)}$$

Apply boundary conditions at $z=d$:

$$\hat{z}\times[\bar{E}(d^+) - \bar{E}(d^-)] = 0 \Rightarrow A e^{jk_o d\cos\theta} = B e^{-jk_o d\cos\theta}$$

$$\hat{z}\times[\bar{H}(d^+) - \bar{H}(d^-)] = \bar{J}_s \Rightarrow [-B\cos\theta\, e^{-jk_o d\cos\theta} - A\cos\theta\, e^{jk_o d\cos\theta}] \cdot e^{-jk_o x\sin\theta} = \eta_o J_o e^{-j\beta x}$$

For phase matching, $k_o\sin\theta = \beta$

Then, $A = \dfrac{-\eta_o J_o}{2\cos\theta} e^{-jk_o d\cos\theta}$ $\quad B = \dfrac{-\eta_o J_o}{2\cos\theta} e^{jk_o d\cos\theta}$

$$\bar{E} = \frac{-\eta_o J_o \hat{y}}{2\cos\theta}\begin{cases} e^{-jk_o[x\sin\theta - (z-d)\cos\theta]} & z<d \\ e^{-jk_o[x\sin\theta + (z-d)\cos\theta]} & z>d \end{cases}$$

The fields due to the source at $z=-d$ can then be found by replacing d with $-d$, and J_o with $-J_o$:

$$\bar{E} = \frac{\eta_0 J_0 \hat{y}}{2\cos\theta} \begin{cases} e^{-jk_0[x\sin\theta - (z+d)\cos\theta]} & z < -d \\ e^{-jk_0[x\sin\theta + (z+d)\cos\theta]} & z > -d \end{cases}$$

Combining these results gives the total fields:

$$\bar{E} = \frac{-j\eta_0 J_0 \hat{y}}{\cos\theta} \begin{cases} e^{-jk_0 x\sin\theta}\, e^{-jk_0 d} \sin(k_0 z\cos\theta) & 0 < z < d \\ e^{-jk_0 x\sin\theta}\, e^{-jk_0 z} \sin(k_0 d\cos\theta) & z > d \end{cases}$$

CHECK: If $\beta = 0$, then $\theta = 0$ and we have,

$$\bar{E} = -j\eta_0 J_0 \hat{y} \begin{cases} e^{-jk_0 d} \sin k_0 z & \text{for } 0 < z < d \\ e^{-jk_0 z} \sin k_0 d & \text{for } z > d \end{cases}$$

This agrees with the results in (1.161)–(1.162).

Chapter 2

(2.1) $i(t) = 1.2\cos(1.51\times10^{10}t - 80.3z)$

a) $\omega = 2\pi f = 1.51\times10^{10} \Rightarrow f = 2.4\text{ GHz}$

b) $k = 80.3\text{ m}^{-1} = 2\pi/\lambda \rightarrow \lambda = 0.0782\text{ m}$

c) $v_p = \omega/k = 1.88\times10^{8}\text{ m/sec} \Rightarrow \epsilon_r = \sqrt{c/v_p} = 2.55$

d) $I = 1.2\underline{/-80.3z}$ (rad)

(2.2) $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(5+j628)(0.01+j0.94)} = \sqrt{(628\underline{/89.54^\circ})(.94\underline{/89.39^\circ})}$

$= 24.3\underline{/89.465^\circ} = 0.23+j24.3 = \alpha+j\beta$ np/m, rad/m ✓

$Z_o = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}} = \sqrt{\dfrac{628\underline{/89.54^\circ}}{.94\underline{/89.39^\circ}}} = 25.8\underline{/0.08^\circ} = 25.8+j0.03\,\Omega$ ✓

If $R=G=0$, $\alpha = 0$

$\beta = \omega\sqrt{LC} = 24.3$ rad/m ✓

$Z_0 = \sqrt{L/C} = 25.8\,\Omega$ ✓

Note that β and Z_0 for the lossless case are very close to the corresponding values for the lossy (low-loss) case.

(2.3) Using KVL:

$$-v(z) + R\frac{\Delta z}{2}i(z) + L\frac{\Delta z}{2}\frac{\partial i(z)}{\partial t} + R\frac{\Delta z}{2}i(z+\Delta z) + L\frac{\Delta z}{2}\frac{\partial i(z+\Delta z)}{\partial t} + v(z+\Delta z) = 0$$

divide by Δz and let $\Delta z\to 0$:

$$\frac{\partial v(z)}{\partial z} = -R\,i(z) - L\frac{\partial i(z)}{\partial t} \quad ✓$$

Using KCL:

$$i(z) - \Delta z\left[G + C\frac{\partial}{\partial t}\right]\left[v(z) - \frac{\Delta z}{2}\left(R + L\frac{\partial}{\partial t}\right)i(z)\right] - i(z+\Delta z) = 0$$

divide by Δz and let $\Delta z\to 0$:

$$\frac{\partial i(z)}{\partial z} = -G\,v(z) - C\frac{\partial v(z)}{\partial t} \quad ✓$$

These results agree with (2.2a,b).

(2.4) Ignoring fringing fields, $\bar{E}$ and $\bar{H}$ can be assumed as,

$$E_y = \frac{-V_o}{d} \ V/m \ , \quad H_x = \frac{V_o}{d\eta} = \frac{I_o}{W} \ A/m \ , \quad \eta = \sqrt{\mu/\epsilon} \ .$$

Then $\bar{E} \times \bar{H}^* = \hat{z}\,|S|$ ✓ and $I_o = V_o\left(\frac{W}{\eta d}\right)$.

From (2.17) - (2.20),

$$L = \frac{\mu_o}{I_o^2}\int_S |\bar{H}|^2 ds = \frac{\mu_o}{I_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{I_o}{W}\right)^2 dx\,dy = \frac{\mu_o d}{W} \quad H/m \quad ✓$$

$$C = \frac{\epsilon}{V_o^2}\int_S |\bar{E}|^2 ds = \frac{\epsilon}{V_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{-V_o}{d}\right)^2 dx\,dy = \frac{\epsilon W}{d} \quad Fd/m \quad ✓$$

$$R = \frac{R_s}{I_o^2}\int_{C_1+C_2} |\bar{H}|^2 dl = \frac{2R_s}{I_o^2}\int_{x=0}^{W}\left(\frac{I_o}{W}\right)^2 dx = \frac{2R_s}{W} \quad \Omega/m \quad ✓$$

$$G = \frac{\omega\epsilon''}{V_o^2}\int_S |\bar{E}|^2 ds = \frac{\omega\epsilon''}{V_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{-V_o}{d}\right)^2 dx\,dy = \frac{\omega\epsilon'' W}{d} \quad S/m \quad ✓$$

These results agree with those in Table 2.1

(2.5) Assume $E_z = H_z = 0$, $\partial/\partial x = \partial/\partial y = 0$.

Then Maxwell's curl equations reduce to,

$$\frac{-\partial E_y}{\partial z} = -j\omega\mu H_x \quad (1) \qquad \frac{-\partial H_y}{\partial z} = j\omega\epsilon E_x \quad (3)$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y \quad (2) \qquad \frac{\partial H_x}{\partial z} = j\omega\epsilon E_y \quad (4)$$

Since $E_x = 0$ at $y=0$ and $y=d$, and $\partial/\partial y = 0$, we must have $E_x \equiv 0$. Then (3) implies $H_y \equiv 0$. So we have,

$$\frac{\partial E_y}{\partial z} = j\omega\mu H_x \qquad \frac{\partial H_x}{\partial z} = j\omega\epsilon E_y$$

Now let $E_y = \frac{1}{d} V(z)$ and $H_x = \frac{-1}{W} I(z)$.
Then the voltage and current are,

$$V(z) = \int_{y=0}^{d} E_y \, dy \qquad I(z) = \int_{x=0}^{W} (\hat{y} \times \bar{H}) \cdot \hat{z} \, dx = -\int_{x=0}^{W} H_x \, dx$$

Then,

$$\left.\begin{aligned} \frac{\partial V}{\partial z} &= -j \frac{\omega \mu d}{W} I(z) \implies L = \frac{\mu d}{W} \\ \frac{\partial I(z)}{\partial z} &= -j \frac{\omega \epsilon W}{d} V(z) \implies C = \frac{\epsilon W}{d} \end{aligned}\right\} \text{ agree with Table 2.1}$$

(2.6) From Table 2.1:

$$R_s = \sqrt{\frac{\omega \mu}{2\sigma}} = 0.00825 \, \Omega$$

$$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a} = 2.40 \times 10^{-7} \text{ H/m}$$

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln b/a} = 9.64 \times 10^{-11} \text{ Fd/m}$$

$$R = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right) = 3.76 \, \Omega/\text{m}$$

$$G = \frac{2\pi \omega \epsilon_0 \epsilon_r \tan \delta}{\ln b/a} = 2.42 \times 10^{-4} \text{ S/m}$$

For small loss, $Z_0 = \sqrt{L/C} = 49.9 \, \Omega$ ✓

From (2.85a), $\alpha \simeq \frac{1}{2}\left(\frac{R}{Z_0} + G Z_0 \right) = 0.044 \text{ np/m} = 0.38 \text{ dB/m}$ ✓

From RG-402U cable data: $Z_0 = 50 \, \Omega$, $\alpha = 13 \text{dB}/100\text{ft}$
$= 0.426 \text{ dB/m}$ (difference due to braided outer conductor, not solid)

Check using formulas from Example 2.7:

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{\ln b/a}{2\pi} = 49.9 \, \Omega \checkmark$$

$$\alpha_c = \frac{R_s}{2\eta \ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right) = 0.0376 \text{ np/m} = 0.326 \text{ dB/m} \checkmark$$

$$\alpha_d = \frac{\omega \epsilon_0 \epsilon_r}{2} \eta \tan \delta = 0.00605 \text{ np/m} = 0.052 \text{ dB/m}$$

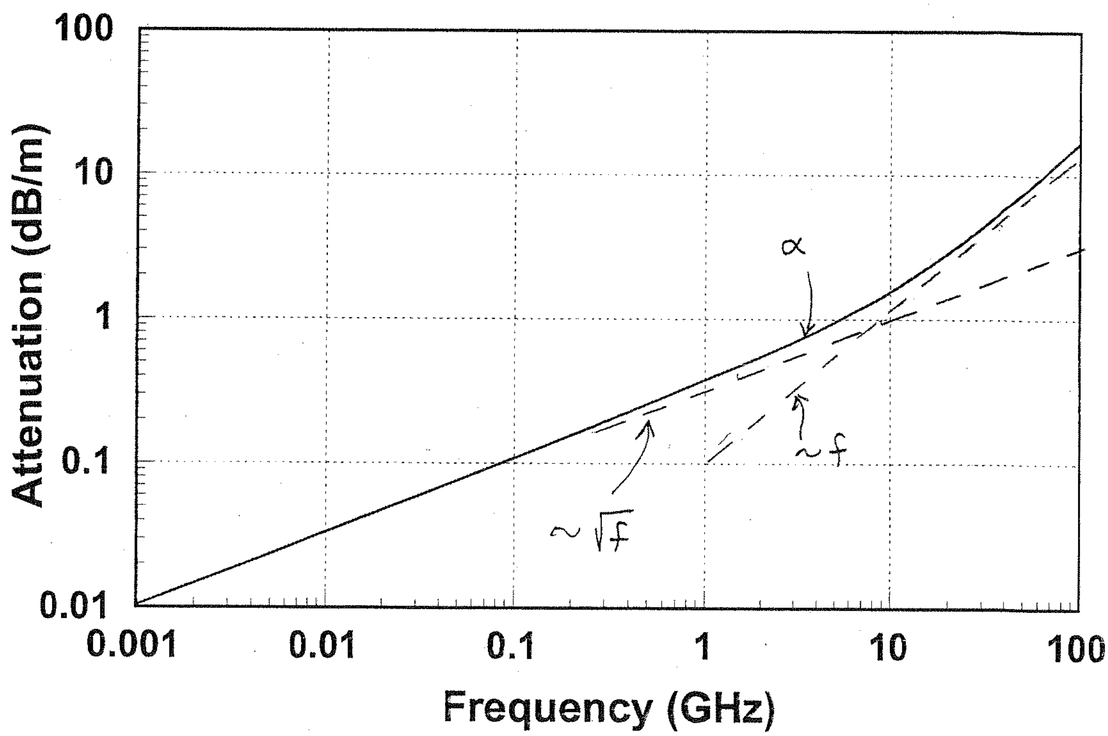
$$\alpha_T = 0.378 \text{ dB/m} \checkmark$$

also verified with Serenade.

(2.7) Using the formulas of Problem 2.6, with $\alpha \simeq \frac{1}{2}(R/Z_0 + G Z_0)$:

f	$R_s(\Omega)$	$R(\Omega)$	G (S)	α (Np/m)	α (dB/m)
1 MHz	2.6×10^{-4}	0.118	2.42×10^{-7}	1.19×10^{-3}	0.0103
10 MHz	8.25×10^{-4}	0.376	2.42×10^{-6}	3.82×10^{-3}	0.0332
100 MHz	2.6×10^{-3}	1.18	2.42×10^{-5}	1.24×10^{-3}	0.1078
1 GHz	8.25×10^{-3}	3.76	2.42×10^{-4}	4.365×10^{-2}	0.379
10 GHz	2.6×10^{-2}	11.8	2.42×10^{-3}	1.785×10^{-1}	1.55
100 GHz	8.25×10^{-3}	37.6	2.42×10^{-2}	1.96	17.0

Results are plotted below (with additional data points). Note that the frequency dependence is between $\sqrt{f}$ ($R \sim \sqrt{f}$), and f ($G \sim f$), at low and high frequencies.



(2.8) $z_L = Z_L/Z_0 = 0.400 - j0.267$

From Smith chart, $\Gamma_L = 0.461\angle 215°$

$SWR = 2.71$

$\Gamma_{in} = 0.461\angle 359°$ ✓

$Z_{in} = 203 - j5.2\ \Omega$

(2.9) $|\Gamma| = \frac{S-1}{S+1} = \frac{0.5}{2.5} = 0.2$

$|\Gamma| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \left|\frac{100 - Z_0}{100 + Z_0}\right|$ (Z_0 real)

So either, $\frac{100 - Z_0}{100 + Z_0} = 0.2 \Rightarrow Z_0 = Z_L\frac{1-\Gamma}{1+\Gamma} = 100\left(\frac{.8}{1.2}\right) = 66.7\ \Omega$ ✓

or

$\frac{100 - Z_0}{100 + Z_0} = -0.2 \Rightarrow Z_0 = Z_L\frac{1-\Gamma}{1+\Gamma} = 100\left(\frac{1.2}{.8}\right) = 150\ \Omega$ ✓

(2.10) $Z_{sc} = jZ_0 \tan\beta l$, $Z_{oc} = -jZ_0 \cot\beta l$

$Z_{sc} \cdot Z_{oc} = Z_0^2 \Rightarrow Z_0 = \sqrt{Z_{sc} Z_{oc}}$

(2.11)

C: $Z_{oc} = -j/\omega C = -j12.73\Omega = -jZ_0 \cot\beta l$ $C = 5\,pF$

$\tan\beta l = 100/12.73 \Rightarrow \beta l = 82.74°$ ✓

$\lambda_0 = 0.12\,m$, $\beta = 2\pi\sqrt{\epsilon_e}/\lambda_0 = 3854°/m \Rightarrow l = \underline{2.147\,cm}$ ✓

L: $Z_{oc} = j\omega L = +j78.5\Omega = -jZ_0 \cot\beta l$ $L = 5\,nH$

$\tan\beta l = -100/78.5 \Rightarrow \beta l = 128.1°$ ✓ $\Rightarrow \underline{l = 3.324\,cm}$ ✓

These results were verified with Serenade.

(2.12) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{30 + j40}{130 + j40} = \dfrac{50\angle 53^\circ}{136\angle 17^\circ} = 0.367\angle 36^\circ$ ✓

$P_{LOAD} = P_{INC} - P_{REF} = P_{INC}(1 - |\Gamma|^2) = 30[1 - (.367)^2] = 25.9\ W$ ✓

(2.13) $\lambda_g = \lambda_0/\sqrt{\epsilon_r} = \dfrac{300}{3000\sqrt{2.56}} = 6.25\ cm$

$\ell = \dfrac{2.0\,cm}{6.25\,cm/\lambda_g} = 0.320\lambda_g$ $\beta\ell = \frac{2\pi}{\lambda_g}(.32\lambda_g) = 115.2^\circ$

Smith chart solution: $Z_{in} = 18.98 - j20.55\,\Omega$ ✓
$\Gamma_{in} = 0.62\angle 212^\circ$ ✓
$\Gamma_L = 0.62\angle 83^\circ$ ✓
$SWR = 4.27$ ✓

These results were verified with the analytical formulas.

(2.14)

$RL = -20\log|\Gamma|$

$SWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

$|\Gamma| = 10^{-RL/20}$

$|\Gamma| = \dfrac{SWR - 1}{SWR + 1}$

SWR	$\lvert\Gamma\rvert$	RL(dB)
1.00	0.0	∞
1.01	.005	46.0
1.02	.01	40.0
1.05	.024	32.3
1.07	.0316	30.0
1.10	.0476	26.4
1.20	.091	20.8
1.22	.100	20.0
1.50	.200	14.0
1.92	.316	10.0
2.00	.333	9.5
2.50	.429	7.4

(2.15) $V_g = 15\text{v RMS},\ Z_g = 75\Omega,\ Z_0 = 75\Omega,\ Z_L = 60 - j40\Omega,\ \ell = 0.7\lambda.$

a) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-15 - j40}{135 - j40} = \frac{42.7\angle -110.6^\circ}{140.8\angle -16.5^\circ} = 0.303\angle -94^\circ = -0.021 - j0.302$$

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}\left(1 - |\Gamma|^2\right) = 0.681\text{ w} \checkmark$$

This method is actually based on $P_L = P_{inc}(1-|\Gamma|^2)$. It is the simplest method, but only applies to lossless lines.

b) $$Z_{in} = Z_0 \frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell} = 75\frac{60 + j190.8}{198.1 + j184.7} = 75\frac{200\angle 72.5^\circ}{270.8\angle 43^\circ}$$

$$= 55.4\angle 29.5^\circ = 48.2 + j27.3\ \Omega$$

$$P_L = \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 Re(Z_{in}) = \left|\frac{15}{123.2 + j27.3}\right|^2 (48.2) = 0.681\text{ w} \checkmark$$

This method computes $P_L = P_{in} = |I_{in}|^2 R_{in}$, and also applies only to lossless lines.

c) $$V(z) = V^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$V_L = V(0) = V^+(1+\Gamma) \qquad V^+ = \frac{V_g}{2} = 7.5\text{v}$$

$$= 7.5(1 - .021 - j.302)$$
$$= 7.68\angle -17^\circ$$

$$P_L = \left|\frac{V_L}{Z_L}\right|^2 Re(Z_L) = \left(\frac{7.68}{72.1}\right)^2 (60) = 0.681\text{ w} \checkmark$$

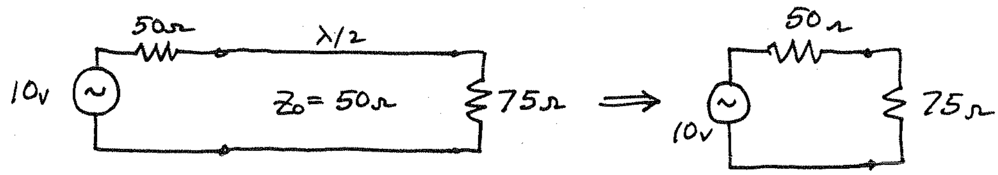
This method computes $P_L = |I_L|^2 R_L$, and applies to lossy as well as lossless lines. Note the concept that $V^+ = V_g/2$ requires a good understanding of the transmission line equations, and only applies here because $Z_g = Z_0$.

(2.16) $Z_L = jX$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$$|\Gamma|^2 = \Gamma\Gamma^* = \frac{(jX - Z_0)}{(jX + Z_0)}\,\frac{(-jX - Z_0)}{(-jX + Z_0)} = \frac{X^2 - j\cancel{Z_0}X + j\cancel{Z_0}X + Z_0^2}{X^2 + Z_0^2} = 1 \checkmark$$

(2.17)



POWER DELIVERED BY SOURCE $= \frac{1}{2}\frac{(10)^2}{50+75} = 0.400\ \text{W}$ ✓

POWER DISSIPATED IN 50Ω LOAD $= \frac{1}{2}(50)\left(\frac{10}{50+75}\right)^2 = 0.160\ \text{W}$ ✓

POWER TRANSMITTED DOWN LINE $= \frac{1}{2}(75)\left(\frac{10}{50+75}\right)^2 = 0.240\ \text{W}$ ✓

INCIDENT POWER $= \frac{1}{2}(50)\left(\frac{10}{50+50}\right)^2 = 0.250\ \text{W}$ ✓

REFLECTED POWER $= P_{INC}\,|\Gamma|^2 = .250\left|\frac{75-50}{75+50}\right|^2 = 0.010\ \text{W}$ ✓

$P_{INC} - P_{REF} = .250 - .010 = 0.240 = P_{TRANS}$ ✓

$P_{DISS} + P_{TRANS} = .160 + .240 = 0.400 = P_{SOURCE}$ ✓

(2.18)

$$\Gamma = \frac{-20 - j40}{180 - j40} = \frac{44.7\angle{-116.6^\circ}}{184.4\angle{-12.5^\circ}} = 0.24\angle{-104^\circ} = -0.058 - j\,0.233 \checkmark$$

$$V_L = 10\,\frac{80 - j40}{180 - j40} = 10\,\frac{89.4\angle{-26^\circ}}{184\angle{-12.5^\circ}} = 4.86\angle{-13.5^\circ}$$

$$V(z) = V^+\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right] \qquad V^+ = 10\,\frac{100}{100+100} = 5\,\text{v} \checkmark$$

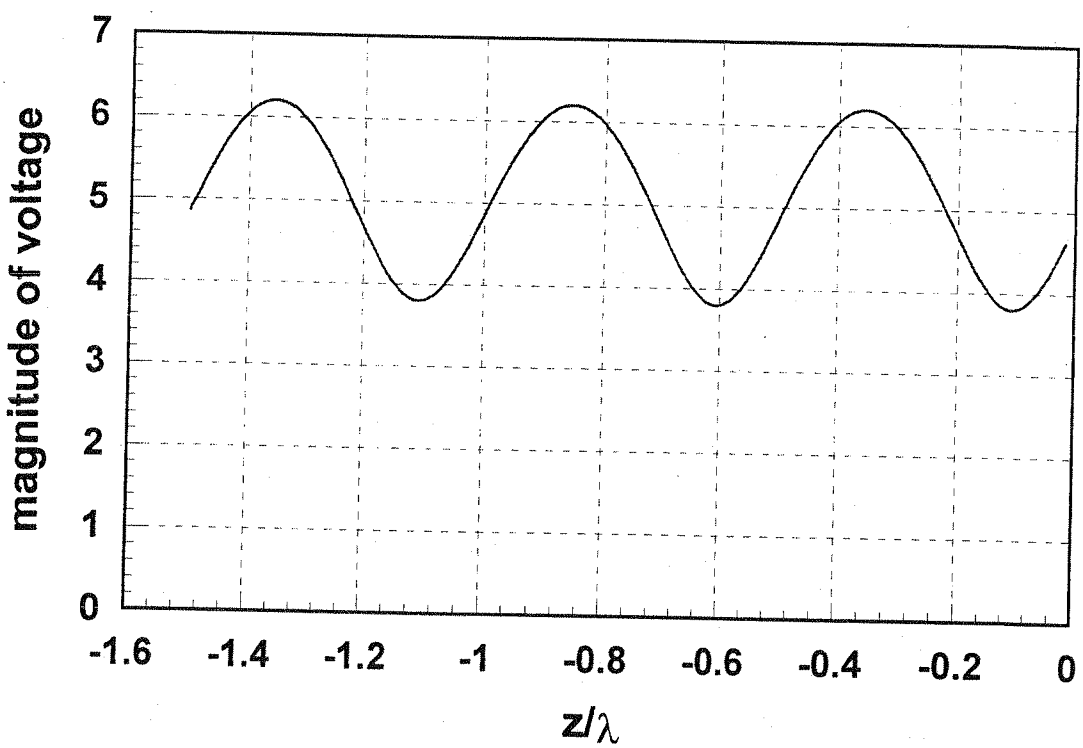
So $V(z) = 5\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right]$

$V_{MAX} = 5(1+|\Gamma|) = 5(1.24) = 6.2$ at $z = -0.355\lambda$

$V_{MIN} = 5(1-|\Gamma|) = 5(.76) = 3.8$ at $z = -0.105\lambda$

These results repeat every $\lambda/2$.

$|V(z)|$ is plotted below:



2.19 $\quad Z_0 = 50\Omega$, $Z_L = 60 + j50\Omega$, $l = 0.4\lambda$

From Smith chart, ($z_L = 1.2 + j1.0$)

a) SWR = 2.46 ✓

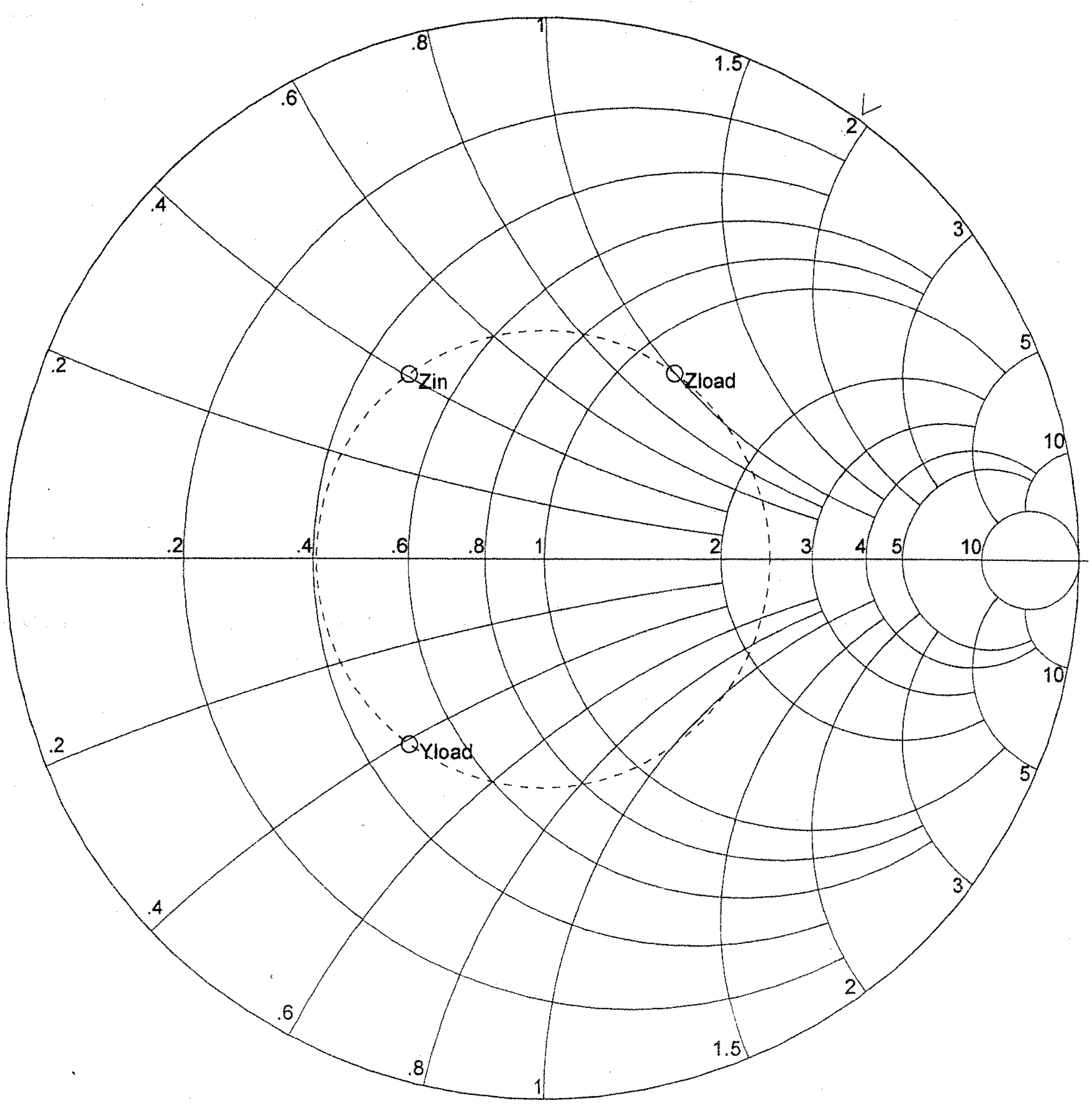
b) $\Gamma = 0.422 \angle 54°$ ✓

c) $Y_L = (.492 - j.410)/50 = 9.84 - j8.2$ mS ✓

d) $Z_{in} = 24.5 + j20.3\ \Omega$

e) $l_{min} = 0.325\lambda$

f) $l_{max} = 0.075\lambda$



2.20 $Z_0 = 50\,\Omega$, $Z_L = 40 - j30\,\Omega$, $\ell = 0.4\lambda$

From Smith chart, ($z_L = 0.80 - j0.60$)

a) SWR = 2.00

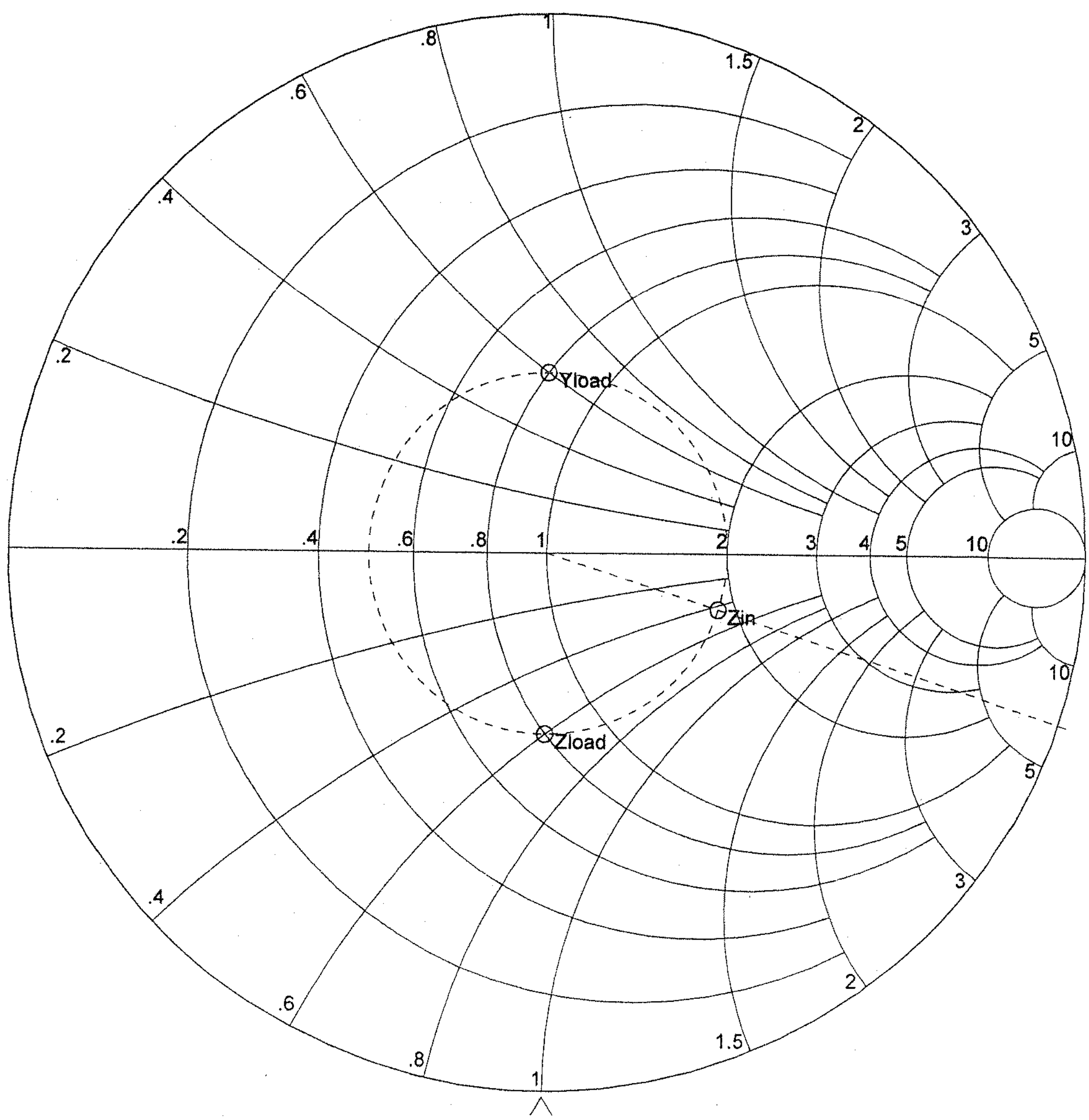
b) $\Gamma = 0.333\angle 270°$

c) $Y_L = (.800 + j.600)/50 = 16.0 + j12.0$ mS ✓

d) $Z_{in} = 93.2 - j21.6\,\Omega$

e) $\ell_{MIN} = 0.125\lambda$

f) $\ell_{MAX} = 0.375\lambda$



(2.21) $Z_0 = 50\Omega$, $Z_L = 60+j50\Omega$, $\ell = 1.8\lambda$

From Smith chart, $(z_L = 1.2+j1.0)$

a) SWR = 2.46

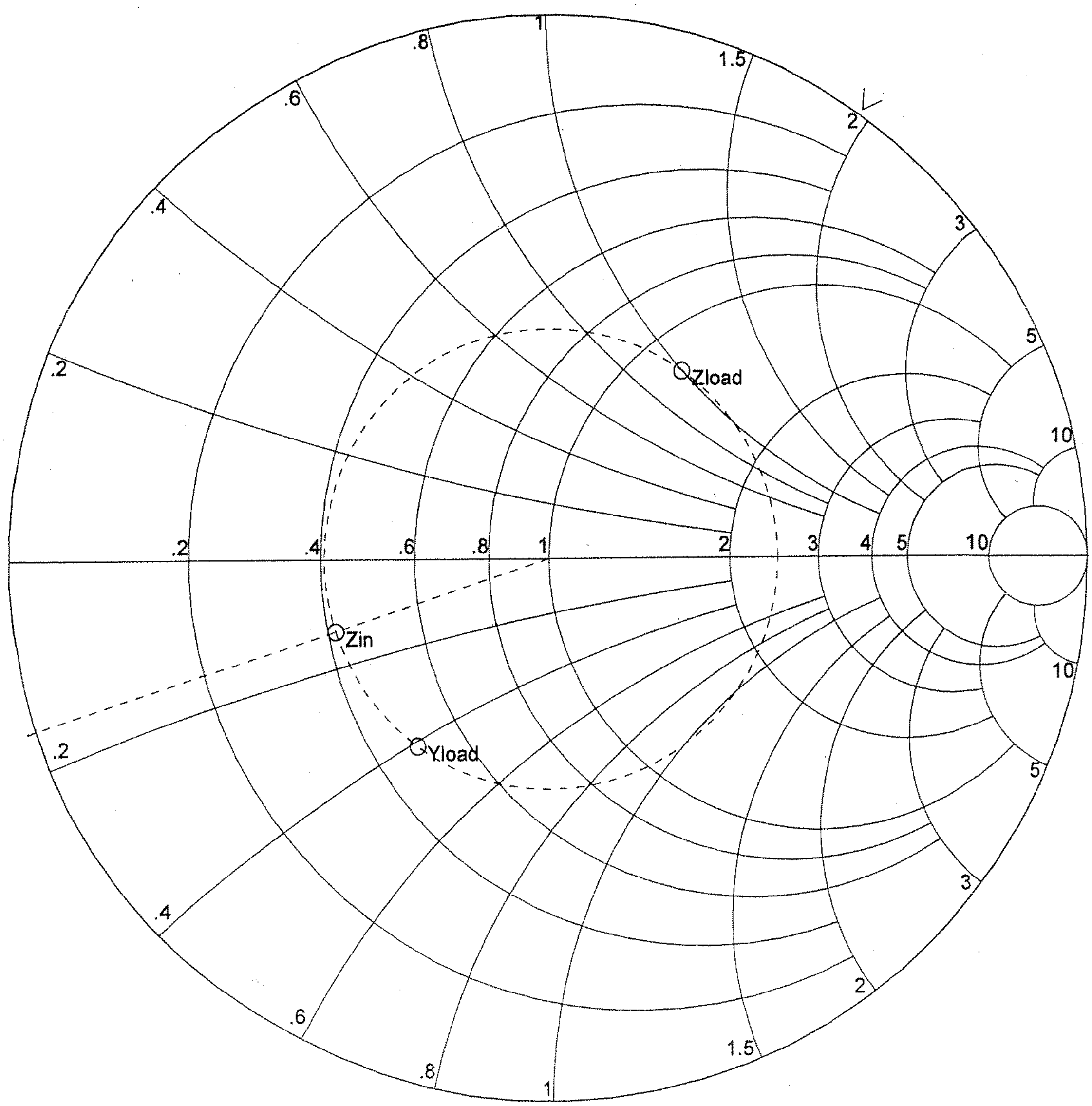
b) $\Gamma = 0.422 \angle 54°$

c) $Y_L = (.492 - j.410)/50 = 9.84 - j8.2$ mS ✓

d) $Z_{in} = 20.8 - j6.7\Omega$

e) $\ell_{MIN} = 0.325\lambda$

f) $\ell_{MAX} = 0.075\lambda$



(2.22)

a) $\ell = 0$ or $\ell = 0.5\lambda$ ✓

b) $\ell = 0.25\lambda$ ✓

c) $\ell = 0.125\lambda$ ✓

d) $\ell = 0.406\lambda$ ✓

e) $\ell = 0.021\lambda$ ✓

These results check with $Z_{in} = jZ_0 \tan\beta\ell$.

(2.23)

a) $\ell = 0.25\lambda$ ✓

b) $\ell = 0\lambda$ or 0.5λ ✓

c) $\ell = 0.375\lambda$ ✓

d) $\ell = 0.656\lambda - 0.5\lambda = 0.156\lambda$ ✓

e) $\ell = 0.271\lambda$ ✓

(add $\lambda/4$ to results of P.2.22)

(also check with $Z_{in} = -jZ_0 \cot\beta\ell$)

(2.24)

$\lambda = 4.2$ cm. From the Smith chart, $\ell_{MIN} = .9/4.2 = 0.214\lambda$ from the load, So $z_L = 2 - j.9 \Rightarrow \underline{Z_L = 100 - j45\,\Omega}$ ✓

Analytically, using (2.58)-(2.60),

$$\Gamma = |\Gamma| e^{j\theta}, \quad |\Gamma| = \frac{2.5-1}{2.5+1} = 0.428$$

$$\theta = \pi + 2\beta\ell_{MIN} = 180 + 2(360)(.214) = -26^\circ \checkmark$$

Then,

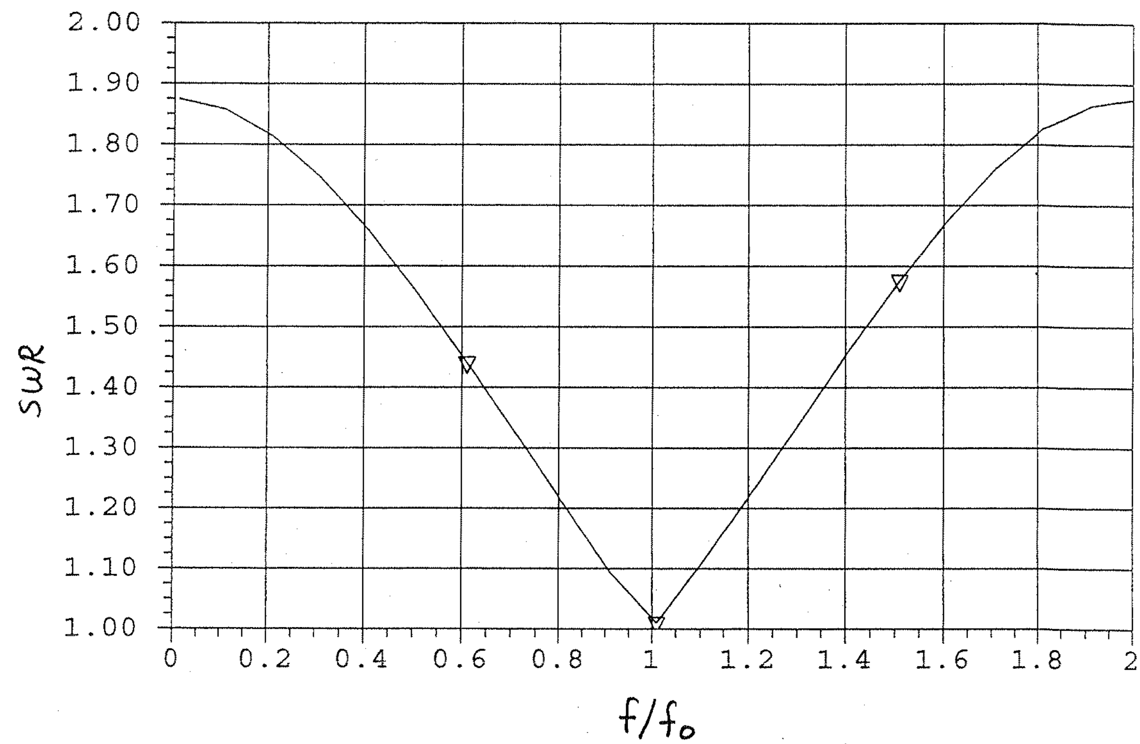
$$Z_L = \frac{1 + .428\angle -26^\circ}{1 - .428\angle -26^\circ}(50) = 50\,\frac{1.4\angle -7.7^\circ}{.643\angle 17^\circ} = 109\angle -25^\circ$$

$$= \underline{\underline{99 - j46\,\Omega}}$$

(2.25)

$$Z_t = \sqrt{40(75)} = 54.77\ \Omega$$

The VSWR is plotted vs f/f_0 below:



(2.26) On the $\lambda/4$ transformer, the voltage can be expressed as,

$$V(z) = V^+ e^{-j\beta z} + \Gamma V^+ e^{j\beta z} \quad , \quad \Gamma = \frac{R_L - \sqrt{Z_0 R_L}}{R_L + \sqrt{Z_0 R_L}}$$

at $z = -\ell$, $\quad V(-\ell) = V^i = V^+\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]$

$$V^+ = \frac{V^i}{\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]} \quad , \quad V^- = \Gamma V^+$$

(assuming V^i with a phase reference at $z = -\ell$.)

(2.27) From (2.70),

$$V_0^+ = V_g \frac{Z_{in}}{Z_{in}+Z_g} \frac{1}{(e^{j\beta l} + \Gamma_l e^{-j\beta l})}$$

From (2.67),

$$Z_{in} = Z_0 \frac{1+\Gamma_l e^{-2j\beta l}}{1-\Gamma_l e^{-2j\beta l}}$$

Then,

$$\frac{Z_{in}}{Z_{in}+Z_g} = \frac{Z_0(1+\Gamma_l e^{-2j\beta l})}{Z_0(1+\Gamma_l e^{-2j\beta l}) + Z_g(1-\Gamma_l e^{-2j\beta l})}$$

$$= \frac{Z_0(e^{j\beta l}+\Gamma_l e^{-j\beta l})\, e^{-j\beta l}}{(Z_0+Z_g) + \Gamma_l (Z_0 - Z_g)\, e^{-2j\beta l}}$$

$$= \frac{Z_0(e^{j\beta l}+\Gamma_l e^{-j\beta l})\, e^{-j\beta l}}{(Z_0+Z_g)\left[1+\Gamma_l \frac{Z_0-Z_g}{Z_0+Z_g} e^{-2j\beta l}\right]}$$

Thus,

$$V_0^+ = V_g \frac{Z_0 e^{-j\beta l}}{(Z_0+Z_g)(1-\Gamma_l \Gamma_g e^{-2j\beta l})}, \quad \text{since } \Gamma_g = \frac{Z_g - Z_0}{Z_g+Z_0}.$$

(2.28)

$$\frac{\partial \alpha_c}{\partial a} = \frac{R_s}{2\eta}\left[\frac{1}{a}\left(\frac{1}{\ln b/a}\right)^2\left(\frac{1}{a}+\frac{1}{b}\right) + \frac{1}{\ln b/a}\left(\frac{-1}{a^2}\right)\right] = 0$$

$$a\left(\frac{1}{a}+\frac{1}{b}\right) = \ln b/a$$

$$(1+b/a) = b/a \ln b/a$$

If $x = b/a$, then $1+x = x \ln x$.

(If $\frac{\partial \alpha_c}{\partial b}$ is taken, the same result is obtained if $x = a/b$)

Now solve this equation for x:

Using interval-halving method:

x	$x \ln x - x - 1$
1	-2.0
2	-1.6
3	$-.704$
4	$.545$
3.5	$-.115$
3.6	$.011$
3.55	$-.052$
$\rightarrow$ 3.59	$-.01$

For $x = \frac{b}{a} = 3.59$,

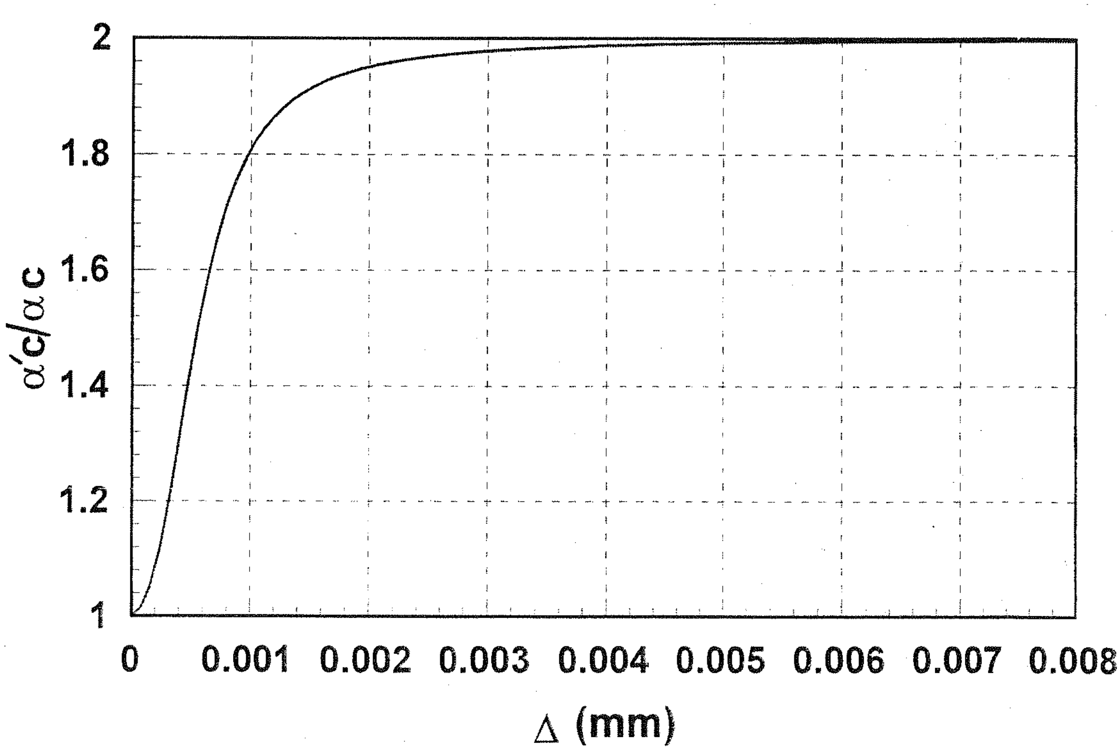
$$Z_0 = \frac{\eta}{2\pi} \ln \frac{b}{a} = \frac{377}{\sqrt{\epsilon_r}} \ln(3.59) = \frac{76.7}{\sqrt{\epsilon_r}} \simeq 77\Omega \text{ for } \epsilon_r = 1.$$

Thus, for an air dielectric, minimum attenuation occurs for a characteristic impedance near 77Ω.

(2.29) The skin depth of copper at 10 GHz is $\delta_s = 6.60 \times 10^{-7}$ m.

Then, compute $\frac{\alpha_c'}{\alpha_c} = 1 + \frac{2}{\pi} \tan^{-1} 1.4 \left(\frac{\Delta}{\delta_s}\right)^2$ (2.107)

The results are plotted below.



(2.30) Since the generator is matched to the line,

$$V_o^+ = \frac{V_g}{2} e^{-\gamma \ell} \quad \text{(phase reference at } z=0)$$

$\alpha = 0.5\,dB/\lambda = 0.0576$ neper/λ

$\gamma \ell = (\alpha + j\beta)\ell = 0.1325 + j\,108°$ ✓

Thus $|V_o^+| = \frac{10}{2} e^{-\alpha \ell} = 4.38\,v.$

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100-50}{100+50} = 0.333 \quad , \quad \Gamma(\ell) = \Gamma e^{-2\gamma \ell}$$

From (2.92)-(2.94) we then have,

$$P_{in} = \frac{|V_o^+|^2}{2Z_o}\left[1 - |\Gamma(\ell)|^2\right] e^{2\alpha \ell} = \frac{(4.38)^2}{100}\left[e^{2(.1325)} - (.333)^2 e^{-2(.1325)}\right]$$

$= \underline{0.2337\ w}$ (power delivered to line)

$$P_L = \frac{|V_o^+|^2}{2Z_o}\left(1 - |\Gamma|^2\right) = \frac{(4.38)^2}{100}\left[1 - (.333)^2\right] = \underline{0.1706\,w} \quad \text{(power to load)}$$

$$P_{Loss} = P_{in} - P_L = 0.2337 - 0.1706 = \underline{0.0631\ w}$$

The input impedance is,

$$Z_{in} = Z_o \frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} = 50\,\frac{100 + 50(.845 + j2.19)}{50 + 100(.845 + j2.19)} = 32.5 - j\,12.4\ \Omega$$

The input current is,

$$I_{in} = \frac{V_g}{R_g + Z_{in}} = \frac{10}{82.5 - j12.4} = 0.1199\angle 8.5°\ A$$

The generator power is,

$$P_S = \tfrac{1}{2} V_g |I_{in}| = 5(.1199) = \underline{0.600}\ w$$

Power lost in R_g is,

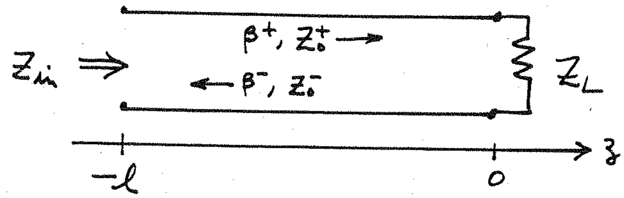
$$P_{R_g} = \tfrac{1}{2}|I_{in}|^2 R_g = \tfrac{1}{2}(.1199)^2(50) = \underline{0.3594\ w}$$

CHECK:

$P_L + P_{Loss} + P_{R_g} = .1706 + .0631 + .3594 = 0.5931\ w \simeq P_S$ ✓

$P_{in} + P_{R_g} = .2337 + .3594 = 0.5931\ w \simeq P_S$ ✓

(2.31)



$$V(z) = V_0^+ e^{-j\beta^+ z} + V_0^- e^{j\beta^- z}$$

$$I(z) = \frac{V_0^+}{Z_0^+} e^{-j\beta^+ z} - \frac{V_0^-}{Z_0^-} e^{j\beta^- z}$$

at $z=0$ (load), $\quad V(0) = V_0^+ + V_0^-$

$$I(0) = \frac{V_0^+}{Z_0^+} - \frac{V_0^-}{Z_0^-}$$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+/Z_0^+ - V_0^-/Z_0^-} = \frac{1 + V_0^-/V_0^+}{\frac{1}{Z_0^+} - \frac{V_0^-}{V_0^+}\frac{1}{Z_0^-}}$$

as usual, let $\Gamma(0) = V_0^-/V_0^+$. Then,

$$Z_L\left(\frac{1}{Z_0^+} - \Gamma\frac{1}{Z_0^-}\right) = 1+\Gamma$$

$$\frac{Z_L}{Z_0^+} - 1 = \Gamma\left(1 + \frac{Z_L}{Z_0^-}\right)$$

$$\Gamma = \Gamma(0) = \frac{Z_L - Z_0^-}{Z_L + Z_0^+} \qquad \text{(at load)}$$

The input impedance is,

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = \frac{\cancel{V_0^+}\left[e^{j\beta^+\ell} + \Gamma e^{-j\beta^-\ell}\right]}{\cancel{V_0^+}\left[\frac{1}{Z_0^+}e^{j\beta^+\ell} - \Gamma\frac{1}{Z_0^-}e^{-j\beta^-\ell}\right]}$$

$$= \frac{(Z_L+Z_0^+)e^{j\beta^+\ell} + (Z_L - Z_0^-)e^{-j\beta^-\ell}}{\frac{1}{Z_0^+}(Z_L+Z_0^+)e^{j\beta^+\ell} - \frac{1}{Z_0^-}(Z_L-Z_0^-)e^{-j\beta^-\ell}}$$

This result does not simplify much further. From (2.42),

$\Gamma(-\ell) = \Gamma(0)\, e^{-j(\beta^- + \beta^+)\ell}$ (reflection coefficient at the input)

Chapter 3

(3.1) Let $k_c^2 = k^2 - \beta^2$

H_x: multiply (3.3a) by $\omega\epsilon$, multiply (3.4b) by β, and add:

$$\omega\epsilon \frac{\partial E_z}{\partial y} - j\beta^2 H_x - \beta \frac{\partial H_z}{\partial x} = -j\omega^2\mu\epsilon H_x$$

$$H_x = \frac{j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x}\right] \checkmark$$

H_y: multiply (3.3b) by $-\omega\epsilon$, multiply (3.4a) by β, and add:

$$\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} + j\beta^2 H_y = j\omega^2\mu\epsilon H_y$$

$$H_y = \frac{-j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y}\right] \checkmark$$

E_x: multiply (3.3b) by $-\beta$, multiply (3.4a) by $\omega\mu$, and add:

$$j\beta^2 E_x + \beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y} = j\omega^2\mu\epsilon E_x$$

$$E_x = \frac{-j}{k_c^2}\left[\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y}\right] \checkmark$$

E_y: multiply (3.3a) by β, multiply (3.4b) by $\omega\mu$, and add:

$$\beta \frac{\partial E_z}{\partial y} + j\beta^2 E_y - \omega\mu \frac{\partial H_z}{\partial x} = j\omega^2\mu\epsilon E_y$$

$$E_y = \frac{j}{k_c^2}\left[\beta \frac{\partial E_z}{\partial y} - \omega\mu \frac{\partial H_z}{\partial x}\right] \checkmark$$

(3.2) From (3.66)-(3.67),

$$H_z = B_n \cos\frac{n\pi y}{d} e^{-j\beta z}$$

$$H_y = \frac{j\beta}{k_c} B_n \sin\frac{n\pi y}{d} e^{-j\beta z}$$

From (3.71), $$P_o = \frac{\omega\mu d W \beta}{4 k_c^2} |B_n|^2 \quad \text{for } n>0,\ \beta \text{ real}$$

From (2.97), the power lost in both plates is,

$$P_\ell = 2\left(\frac{R_s}{2}\right)\int_s |\bar{H}_t|^2 ds = R_s \int_{z=0}^{l} \int_{x=0}^{W} \left[|H_y(y=0)|^2 + |\overset{\nearrow 0}{H_z(y=0)}|^2\right] dx\, dz$$

$$= R_s W |B_n|^2$$

Then, $\alpha_c = \frac{P_\ell}{2P_0} = \frac{2R_s k_c^2}{k d \eta \beta}$. (agrees with (3.72)) ✓

(3.3) From appendix I, $a = 1.07$ cm, $b = 0.43$ cm.

$$\left.\begin{aligned} f_{c10} &= \frac{c}{2a} = 14.02 \text{ GHz} ✓ \\ f_{c20} &= \frac{c}{a} = 28.04 \text{ GHz} ✓ \end{aligned}\right\} \text{ LOWEST ORDER MODES}$$

$$f_{c01} = \frac{c}{2b} = 34.88 \text{ GHz} ✓$$

The fractional BW from f_{c10} to f_{c20} is $\frac{(28-14)}{(28+14)/2} = 67\%$

The fractional BW of the recommended operating range of 18.0 – 26.5 GHz is $\frac{(26.5-18.0)}{(26.5+18.0)/2} = 38\%$ (reduction of 29%)

(3.4) $k = \sqrt{\epsilon_r}\, k_0 = \sqrt{2.2}\ \frac{2\pi\,(20{,}000)}{300} = 621.3 \text{ m}^{-1}$ $\qquad R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = 0.0555\,\Omega$

$\beta = \sqrt{k^2 - (\pi/a)^2} = 547.5 \text{ m}^{-1}$ ✓ $\qquad \eta = 377/\sqrt{\epsilon_r} = 254\,\Omega$

From (3.29), $\alpha_d = \frac{k^2 \tan\delta}{2\beta} = 0.705$ Np/m $= 6.12$ dB/m ✓

From (3.96), $\alpha_c = \frac{R_s}{a^3 b \beta k \eta}(2b\pi^2 + a^3 k^2) = 0.068$ Np/m $= 0.591$ dB/m ✓

$\alpha_T = \alpha_d + \alpha_c = 6.71$ dB/m (agrees with PCAAD)

(3.5) In the section of guide of width $a/2$, the TE_{10} mode is below cutoff (evanescent), with an attenuation constant α:

$$k = \frac{2\pi(12{,}000)}{300} = 251.3 \text{ m}^{-1} \checkmark$$

$$\alpha = \sqrt{\left(\frac{\pi}{a/2}\right)^2 - k^2} = \sqrt{\left(\frac{2\pi}{.02286}\right)^2 - (251.3)^2} = 111.3 \text{ neper/m} \checkmark$$

To obtain 100 dB attenuation (ignoring reflections),

$$-100 \text{ dB} = 20 \log e^{-\alpha l}$$

$$10^{-5} = e^{-\alpha l}$$

$$l = \frac{11.5}{111.3} = 0.103 \text{ m} = \underline{10.3 \text{ cm}} \checkmark$$

(3.6) The TE_{10} H-fields from (3.89) are:

$$H_x = \frac{j\beta a A}{\pi} \sin \frac{\pi x}{a} e^{-j\beta x}$$

$$H_y = 0$$

$$H_z = A \cos \frac{\pi x}{a} e^{-j\beta x}$$

$\bar{J}_s = \hat{n} \times \bar{H}$, so the surface currents are,

ON BOTTOM WALL: $\hat{n} = \hat{y}$; $\bar{J}_s = -\hat{z} \frac{j\beta a A}{\pi} \sin \frac{\pi x}{a} e^{-j\beta z} + \hat{x} A \cos \frac{\pi x}{a} e^{-j\beta z}$

ON TOP WALL: $\hat{n} = -\hat{y}$; $\bar{J}_s = \hat{z} \frac{j\beta a A}{\pi} \sin \frac{\pi x}{a} e^{-j\beta z} - \hat{x} A \cos \frac{\pi x}{a} e^{-j\beta z}$

ON LEFT SIDE WALL: $\hat{n} = \hat{x}$, $x = 0$; $\bar{J}_s = -\hat{y} A e^{-j\beta z}$

ON RIGHT SIDE WALL: $\hat{n} = -\hat{x}$, $x = a$; $\bar{J}_s = -\hat{y} A e^{-j\beta z}$

Note that the top and bottom currents are the negative of each other.

Along the centerline of the top or bottom (broad) walls, $x = a/2$, so the surface currents can be reduced to,

$$\bar{J}_s = \pm \hat{z} \frac{j\beta a A}{\pi} e^{-j\beta z},$$

which shows that current flow is only in the longitudinal direction. Thus a narrow longitudinal slot will not break any current lines, and will have a negligible effect on the operation of the waveguide.

3.7 From (3.101),

$$\bar{E} \times \bar{H}^* \cdot \hat{z} = E_x H_y^* - E_y H_x^*$$

$$= \frac{\omega \epsilon \beta m^2 \pi^2}{a^2 k_c^4} |B|^2 \cos^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b}$$

$$+ \frac{\omega \epsilon \beta n^2 \pi^2}{b^2 k_c^4} |B|^2 \sin^2 \frac{m\pi x}{a} \cos^2 \frac{n\pi y}{b}$$

So the power flow down the guide is,

$$P_o = \frac{1}{2} \int_{x=0}^{a} \int_{y=0}^{b} \bar{E} \times \bar{H}^* \cdot \hat{z} \, dx\, dy = \frac{\omega \epsilon \beta \pi^2 |B|^2}{2 k_c^4} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \frac{ab}{4} = \frac{\omega \epsilon \beta a b}{8 k_c^2} |B|^2$$

The power loss in the walls is,

$$P_\ell = \frac{R_s}{2} \int_s |\bar{H}_t|^2 ds = R_s \left\{ \int_{x=0}^{a} |H_x(y=0)|^2 dx + \int_{y=0}^{b} |H_y(x=0)|^2 dy \right\}$$

$$= R_s \left\{ \frac{\omega^2 \epsilon^2 n^2 \pi^2}{b^2 k_c^4} |B|^2 \frac{a}{2} + \frac{\omega^2 \epsilon^2 m^2 \pi^2}{a^2 k_c^4} |B|^2 \frac{b}{2} \right\}$$

$$= R_s \frac{\omega^2 \epsilon^2 \pi^2}{2 k_c^4} |B|^2 \left(\frac{n^2 a}{b^2} + \frac{m^2 b}{a^2} \right)$$

So the attenuation is,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{R_s \omega^2 \epsilon^2 \pi^2 4 k_c^2}{2 k_c^4 \omega \epsilon \beta a b} \left(\frac{n^2 a}{b^2} + \frac{m^2 b}{a^2} \right)$$

$$= \frac{2 R_s k \pi^2}{k_c^2 \beta \eta} \left(\frac{n^2}{b^3} + \frac{m^2}{a^3} \right) \quad \text{neper/m} \checkmark$$

(3.8) From (3.109), the propagation constant is a solution of,

$$k_a \tan k_d t + k_d \tan k_a (a-t) = 0,$$

where

$$\beta = \sqrt{k_0^2 - k_a^2} = \sqrt{\epsilon_r k_0^2 - k_d^2} \qquad (3.106)$$

Since $\beta = 0$ at cutoff, we have that $k_a = k_0$, and $k_d = \sqrt{\epsilon_r}\, k_0$. Thus we must find the root of the following equation:

$$f(k_0) = k_0 \tan \sqrt{\epsilon_r}\, k_0 t + \sqrt{\epsilon_r}\, k_0 \tan k_0 t = 0 \quad (\text{since } t = a/2)$$

We know that $k_c = k_0$ must be between k_c of the empty guide, and k_c for the completely filled guide:

$$k_c(\text{EMPTY}) = \frac{\pi}{a} = 137.\ m^{-1}$$

$$k_c(\text{FILLED}) = \frac{\pi}{\sqrt{\epsilon_r}\, a} = 92.\ m^{-1}$$

k_0	$f(k_0)$
95	-1366
100	-362
105	-44
110	171
106	2.3
105.9	-2.25
✓ → 105.95	.017

This result is accurate to at least four figures, and agrees with a result given in the Waveguide Handbook.

The cutoff frequency is,

$$f_c = \frac{k_c c}{2\pi} = 5.06 \text{ GHz}$$

(3.9) The lowest order mode will have an H_z component which is even in x, and no variation in y. Thus, h_z can be written as,

$$h_z(x,y) = \begin{cases} A \cos k_d x & \text{for } |x| < W/2 \quad (k_c = k_d) \\ B e^{-k_a |x|} & \text{for } |x| > W/2 \quad (k_c = j k_a) \end{cases}$$

where k_d and k_a are the cutoff wavenumbers in the dielectric and air regions, respectively, satisfying

$$\beta = \sqrt{\epsilon_r k_0^2 - k_d^2} = \sqrt{k_0^2 - k_a^2} \quad \text{(phase matching)}$$

Next, we need e_y, from (3.19d):

$$e_y(x,y) = \frac{j\omega\mu}{k_c^2} \frac{\partial h_z}{\partial x} = \begin{cases} -j\frac{\omega\mu A}{k_d} \sin k_d x & \text{for } |x| < W/2 \\ \frac{j\omega\mu B}{k_a} e^{-k_a x} & \text{for } x > W/2 \end{cases}$$

Matching h_z and e_y at $x = W/2$ gives,

$$A \cos k_d W/2 = B e^{-k_a W/2}$$

$$-\frac{A}{k_d} \sin k_d W/2 = \frac{B}{k_a} e^{-k_a W/2}$$

Setting the determinant of these equations to zero gives,

$$k_a \tan k_d W/2 + k_d = 0.$$

A TEM mode cannot exist by itself because of the impossibility of phase matching at $x = W/2$. (For a TEM mode, $\beta = k$ in both regions, which is not possible.)

(3.10)

$$h_z(y) = \begin{cases} A\cos k_d y & \text{for } 0<y<t \quad (k_c = k_d) \\ B\cos k_a(d-y) & \text{for } t<y<d \quad (k_c = k_a) \end{cases}$$

where k_d and k_c are the cutoff wavenumbers in the dielectric and air regions, satisfying,

$$\beta = \sqrt{\epsilon_r k_o^2 - k_d^2} = \sqrt{k_o^2 - k_a^2}$$

Also, h_z has been chosen to satisfy $E_x = 0$ at $y = 0, d$. (since $E_x \sim \partial h_z/\partial y$). From (3.19),

$$e_x(y) = \frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y} = \begin{cases} \frac{j\omega\mu A}{k_d}\sin k_d y & \text{for } 0<y<t \\ \frac{-j\omega\mu B}{k_a}\sin k_a(d-y) & \text{for } t<y<d \end{cases}$$

Enforcing continuity of h_z and e_x at $y=t$ yields,

$$A\cos k_d t - B\cos k_a(d-t) = 0$$

$$\frac{A}{k_d}\sin k_d t + \frac{B}{k_a}\sin k_a(d-t) = 0$$

Setting the determinant of this set of equations to zero gives,

$$k_d \tan k_a(d-t) + k_a \tan k_d t = 0 \quad ✓$$

(This result agrees with a result on p. 160 of Harrington)

The TEM mode cannot exist on this structure.

(3.11) Maxwell's curl equations are,

$$\nabla \times \bar{E} = -j\omega\mu \bar{H} \quad , \quad \nabla \times \bar{H} = j\omega\epsilon \bar{E}$$

The ρ and ϕ components in cylindrical form are,

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_\rho \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\epsilon E_\rho$$

$$\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \qquad \frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} = j\omega\epsilon E_\phi$$

Now assume $\bar{E}(\rho,\phi,z) = \bar{e}(\rho,\phi)e^{-j\beta z}$

$$\bar{H}(\rho,\phi,z) = \bar{h}(\rho,\phi)e^{-j\beta z}$$

Then $\partial/\partial z \to -j\beta$, and the above equations reduce to:

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + j\beta E_\phi = -j\omega\mu H_\rho \quad (1) \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + j\beta H_\phi = j\omega\epsilon E_\rho \quad (3)$$

$$-j\beta E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \quad (2) \qquad -j\beta H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\epsilon E_\phi \quad (4)$$

Multiply (2) by $-\beta$, multiply (3) by $\omega\mu$, and add:

$$j\beta^2 E_\rho + \beta\frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi} = j\omega^2\mu\epsilon E_\rho$$

$$E_\rho = \frac{-j}{k_c^2}\left[\beta\frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

multiply (1) by β, multiply (4) by $\omega\mu$, and add:

$$\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} + j\beta^2 E_\phi - \omega\mu\frac{\partial H_z}{\partial \rho} = j\omega^2\mu\epsilon E_\phi$$

$$E_\phi = \frac{-j}{k_c^2}\left[\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} - \omega\mu\frac{\partial H_z}{\partial \rho}\right]$$

multiply (1) by $\omega\epsilon$, multiply (4) by β, and add:

$$\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial \phi} - j\beta^2 H_\rho - \beta\frac{\partial H_z}{\partial \rho} = -j\omega^2\mu\epsilon H_\rho$$

$$H_\rho = \frac{j}{k_c^2}\left[\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial \phi} - \beta\frac{\partial H_z}{\partial \rho}\right]$$

$$\omega\epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi} + j\beta^2 H_\phi = j\omega^2\mu\epsilon H_\phi$$

$$H_\phi = \frac{-j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

with $k_c^2 = k^2 - \beta^2$.

These results agree with those of (3.110). ✓

(3.12) Let $A=1$, $B=0$ in (3.141). Then the transverse fields are,

$$E_\rho = \frac{-j\beta}{k_c} \sin n\phi \, J_n'(k_c\rho)\, e^{-j\beta z}$$

$$E_\phi = \frac{-j\beta n}{k_c^2 \rho} \cos n\phi \, J_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\rho = \frac{j\omega\epsilon n}{k_c^2 \rho} \cos n\phi \, J_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\phi = \frac{-j\omega\epsilon}{k_c} \sin n\phi \, J_n'(k_c\rho)\, e^{-j\beta z}$$

$\bar{E}\times\bar{H}^*\cdot\hat{z} = E_\rho H_\phi^* - E_\phi H_\rho^*$.

The power flow down the guide is, for $n>0$,

$$P_o = \frac{1}{2}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\left[\frac{\beta\omega\epsilon}{k_c^2}\sin^2 n\phi\, J_n'^2(k_c\rho) + \frac{\beta\omega\epsilon n^2}{k_c^4\rho^2}\cos^2 n\phi\, J_n^2(k_c\rho)\right]\rho\, d\phi\, d\rho$$

$$= \frac{\beta\omega\epsilon\pi}{2k_c^2}\int_{\rho=0}^{a}\left[J_n'^2(k_c\rho) + \frac{n^2}{k_c^2\rho^2}J_n^2(k_c\rho)\right]\rho\, d\rho$$

Let $x = k_c\rho$
$dx = k_c d\rho$
$k_c a = p_{nm}$

$$= \frac{\beta\omega\epsilon\pi}{2k_c^4}\int_{x=0}^{p_{nm}}\left[J_n'^2(x) + \frac{n^2}{x^2}J_n^2(x)\right]x\, dx = \frac{\beta\omega\epsilon\pi}{4k_c^4}\, p_{nm}^2\, J_n'^2(p_{nm}) \quad \text{(SEE C.16)}$$

The power lost in the conducting wall is,

$$P_\ell = \frac{R_s}{2}\int_{z=0}^{1}\int_{\phi=0}^{2\pi}|H_\phi(\rho=a)|^2 a\, d\phi\, dz = \frac{aR_s}{2}\frac{\omega^2\epsilon^2}{k_c^2}J_n'^2(p_{nm})\int_{\phi=0}^{2\pi}\sin^2 n\phi\, d\phi$$

$$= \frac{aR_s\omega^2\epsilon^2\pi}{2k_c^2}J_n'^2(p_{nm})$$

The attenuation is then,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{aR_s\omega^2\epsilon^2\pi\, 4k_c^4}{4k_c^2\beta\omega\epsilon\pi p_{nm}^2} = \frac{aR_s\omega\epsilon k_c^2}{\beta p_{nm}^2} = \frac{kR_s}{\beta\eta a} \text{ neper/m} ✓$$

3.13 From Figure 3.13 the first four modes to propagate are the TE_{11}, TM_{01}, TE_{21}, and TE_{01}. (TM_{11} has same f_c as TE_{01})

$$TE_{11}: \quad f_c = \frac{p'_{11} c}{2\pi a\sqrt{\epsilon_r}} = \frac{1.841\,(300)}{2\pi(.008)\sqrt{2.3}} = 7.245 \text{ GHz} \checkmark$$

$$TM_{01}: \quad f_c = \frac{p_{01} c}{2\pi a\sqrt{\epsilon_r}} = \frac{2.405\,(300)}{2\pi(.008)\sqrt{2.3}} = 9.465 \text{ GHz} \checkmark$$

$$TE_{21}: \quad f_c = \frac{p'_{21} c}{2\pi a\sqrt{\epsilon_r}} = \frac{3.054\,(300)}{2\pi(.008)\sqrt{2.3}} = 12.019 \text{ GHz} \checkmark$$

$$TE_{01}: \quad f_c = \frac{p'_{01} c}{2\pi a\sqrt{\epsilon_r}} = \frac{3.832\,(300)}{2\pi(.008)\sqrt{2.3}} = 15.080 \text{ GHz} \checkmark$$

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3.14 From (3.153), $\Phi(\rho,\phi) = \dfrac{V_o \ln b/\rho}{\ln b/a}$

From (3.13) and appendix,

$$\bar{e}(\rho,\phi) = -\nabla_t \Phi(\rho,\phi) = -\left(+\hat{\rho}\frac{\partial \Phi}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\right) = \frac{V_o \hat{\rho}}{\rho \ln b/a}$$

Then,

$$\bar{E}(\rho,\phi,z) = \bar{e}(\rho,\phi)e^{-j\beta z} = \frac{V_o \hat{\rho}\, e^{-j\beta z}}{\rho \ln b/a} \quad \checkmark \qquad (4.155) \text{ of 1st Ed.}$$

From (3.18),

$$\bar{h}(\rho,\phi) = \frac{1}{\eta}\hat{z}\times\bar{e}(\rho,\phi) = \frac{V_o \hat{\phi}}{\eta \rho \ln b/a}$$

Then,

$$\bar{H}(\rho,\phi,z) = \frac{V_o \hat{\phi}\, e^{-j\beta z}}{\eta \rho \ln b/a} \quad \checkmark \qquad (4.157) \text{ of 1st Ed.}$$

The potential between the two conductors is,

$$V_{ab} = \int_{\rho=a}^{b} E_\rho(\rho,\phi,z)\,d\rho = V_o e^{-j\beta z} \quad \checkmark \qquad (4.158) \text{ of 1st Ed.}$$

The current on the inner conductor is,

$$I_a = \int_{\phi=0}^{2\pi} H_\phi(a,\phi,z)\,a\,d\phi = \frac{2\pi V_o e^{-j\beta z}}{\eta \ln b/a} \quad \checkmark \qquad (4.159) \text{ of 1st Ed.}$$

The characteristic impedance is,

$$Z_o = \frac{V_{ab}}{I_o} = \frac{\eta \ln b/a}{2\pi} \quad \checkmark \qquad (4.162) \text{ of 1st Ed.}$$

(3.15) The solution is similar to the TE mode case for the coax, but with e_z in place of h_z:

$$e_z(\rho,\phi) = (A \sin n\phi + B \cos n\phi)\left[C J_n(k_c \rho) + D Y_n(k_c \rho)\right]$$

Then the boundary condition that $e_z = 0$ at $\rho = a$ and at $\rho = b$ yields two equations:

$$C J_n(k_c a) + D Y_n(k_c a) = 0$$
$$C J_n(k_c b) + D Y_n(k_c b) = 0$$

or,

$$J_n(k_c a) Y_n(k_c b) = J_n(k_c b) Y_n(k_c a)$$

For the TM_{01} mode, $n=0$. Let $x = k_c a$. Then for $b = 2a$, we have that $k_c b = 2x$, and so the above equation can be written as,

$$f(x) = J_0(x) Y_0(2x) - J_0(2x) Y_0(x) = 0$$

We know that k_c should be greater than k_c for a circular waveguide of radius b, for which $k_{c01} = p_{01}/b = 2.405/2a$, which implies that $x = 1.2$. So we can begin the root search at $x = 1.2$. Using a table of Bessel functions gives the following results in only a few minutes:

x	$J_0(x)$	$Y_0(x)$	$J_0(2x)$	$Y_0(2x)$	$f(x)$
1.2	.671	.228	.003	.510	.342
1.5	.512	.382	−.260	.377	.292
2.0	.224	.510	−.397	−.017	.198
3.1	−.292	.343	.202	−.248	.003
3.2	−.320	.307	.243	−.200	−.011

Linear interpolation between $x = 3.1$ and 3.2 gives a more accurate value for the root:

$$f(x) \simeq .003 + \frac{.003 - (-.011)}{3.1 - 3.2}(x - 3.1)$$

$$\simeq .437 - .14x = 0$$

$$x = \frac{.437}{.14} = \underline{\underline{3.12 = k_c a}}$$

3.16 From (3.175),

$$\bar{E}\times\bar{H}^*\cdot\hat{z} = -E_yH_x^* = \begin{cases} \dfrac{\omega\mu_0\beta|B|^2}{k_c^2}\sin^2 k_c x & \text{for } 0\le x\le d \\ \dfrac{\omega\mu_0\beta|B|^2}{h^2}\cos^2 k_c d\, e^{-2h(x-d)} & \text{for } d\le x<\infty \end{cases}$$

The power flow is,

$$P_o = \frac{1}{2}\int_{x=0}^{\infty}\int_{y=0}^{1}\bar{E}\times\bar{H}^*\cdot\hat{z}\,dy\,dx$$

$$= \frac{\omega\mu_0\beta|B|^2}{2k_c^2}\int_{x=0}^{d}\sin^2 k_c x\,dx + \frac{\omega\mu_0\beta|B|^2}{2h^2}\cos^2 k_c d\int_{x=d}^{\infty}e^{-2h(x-d)}dx$$

$$= \frac{\omega\mu_0\beta|B|^2}{2}\left[\frac{1}{k_c^2}\left(\frac{x}{2}-\frac{\sin 2k_c x}{4k_c}\right)\Big|_0^d + \frac{\cos^2 k_c d}{h^2}\left(\frac{e^{-2h(x-d)}}{-2h}\right)\Big|_d^\infty\right]$$

$$= \frac{\omega\mu_0\beta|B|^2}{2}\left[\frac{1}{k_c^2}\left(\frac{d}{2}-\frac{\sin 2k_c d}{4k_c}\right)+\frac{\cos^2 k_c d}{2h^3}\right]$$

The power loss is,

$$P_\ell = \frac{R_s}{2}\int_s|\bar{H}_t|^2ds = \frac{R_s}{2}\int_{y=0}^{1}\int_{z=0}^{1}\left[\overbrace{|H_x(x=0)|^2}^{0}+|H_z(x=0)|^2\right]dz\,dy$$

$$= \frac{R_s}{2}|B|^2$$

So the attenuation is,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{2R_s}{4\omega\mu_0\beta\left[\frac{1}{k_c^2}\left(\frac{d}{2}-\frac{\sin 2k_c d}{4k_c}\right)+\frac{\cos^2 k_c d}{2h^3}\right]}$$

$$= \frac{R_s}{k_0\eta_0\beta\left[\frac{d}{k_c^2}-\frac{\sin 2k_c d}{2k_c^3}+\frac{\cos^2 k_c d}{h^3}\right]}$$ ✓

(3.17) Following the derivation in Section 3.6 for the TM surface waves of a dielectric slab:

$$k_c^2 = \mu_r k_0^2 - \beta^2 \qquad \text{for } 0 \le y \le d$$
$$h^2 = \beta^2 - k_0^2 \qquad \text{for } y \ge d$$

Then,

$$e_z(x,y) = \begin{cases} A \sin k_c y & \text{for } 0 \le y \le d \\ B e^{-hy} & \text{for } y > d \end{cases}$$

This form of e_z is selected to satisfy $e_z = 0$ at $y=0$, and to have exponential decay for $y \to \infty$ (radiation condition). Next, we need H_x ($H_y = E_x = H_z = 0$): From (3.23a),

$$H_x = \frac{j\omega\epsilon_0}{k_c^2}\frac{\partial E_z}{\partial y} = \begin{cases} \frac{j\omega\epsilon_0}{k_c} A \cos k_c y & \text{for } 0 \le y \le d \\ \frac{j\omega\epsilon_0}{h} B e^{-hy} & \text{for } y > d \end{cases}$$

at $y = d$:

E_z continuous $\Rightarrow$ $A \sin k_c d = B e^{-hd}$

H_x continuous $\Rightarrow$ $\frac{A}{k_c} \cos k_c d = \frac{B}{h} e^{-hd}$

or,

$$h \cos k_c d = k_c \sin k_c d$$
$$\underline{h = k_c \tan k_c d} \quad ✓$$

and,

$$\underline{h^2 + k_c^2 = (\mu_r - 1) k_0^2} \quad ✓$$

These two equations must be solved simultaneously to find h and k_c.

(3.18) TM_{0m} mode. $H_z = 0$ $\quad E_z(\rho,\phi,z) = e_z(\rho,\phi)\, e^{-j\beta z}$

(No TEM mode can be supported by this line because of the impossibility of phase matching at $\rho = b$)

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k_c^2\right) e_z(\rho,\phi) = 0$$

$\partial/\partial\phi = 0$ for $n=0$ modes $\Rightarrow$ $E_\phi = H_\rho = 0$.

Thus,

$$e_z(\rho,\phi) = \begin{cases} A\,J_0(k_d\rho) + B\,Y_0(k_d\rho) & \text{for } a \le \rho \le b \\ C\,J_0(k_a\rho) + D\,Y_0(k_a\rho) & \text{for } b \le \rho \le c \end{cases}$$

where $\quad \beta^2 = \epsilon_r k_0^2 - k_d^2 = k_0^2 - k_a^2$.

The boundary conditions are:

$e_z = 0$ at $\rho = a$ and $\rho = c$,

e_z and H_ϕ are continuous at $\rho = b$.

From (3.110d),

$$H_\phi = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial\rho},$$

So we get the following four equations:

$$A\,J_0(k_d a) + B\,Y_0(k_d a) = 0$$

$$C\,J_0(k_a c) + D\,Y_0(k_a c) = 0$$

$$A\,J_0(k_d b) + B\,Y_0(k_d b) = C\,J_0(k_a b) + D\,Y_0(k_a b)$$

$$\epsilon_r k_d\left[A\,J_0'(k_d b) + B\,Y_0'(k_d b)\right] = k_a\left[C\,J_0'(k_a b) + D\,Y_0'(k_a b)\right]$$

k_a and k_d can be expressed in terms of β, and β can be found so that the determinant of the above system of equations vanishes. This is as far as we can go without actual values for a, b, c, and ϵ_r.

3.19 To use (3.180) we compute $\sqrt{\epsilon_r}Z_o = \sqrt{2.2}\,(70) = 103.8 < 120$.

Then, $$\frac{W}{b} = x = \frac{30\pi}{\sqrt{\epsilon_r}\,Z_o} - 0.441 = 0.467 \Rightarrow W = \underline{\underline{0.147\text{ cm}}} \checkmark$$

$$\lambda_g = \frac{c}{\sqrt{\epsilon_r}\,f} = \frac{300}{\sqrt{2.2}\,(3000)} = \underline{\underline{6.74\text{ cm}}}$$

3.20 First try $W/d < 2$:

From (3.197), $$A = \frac{Z_o}{60}\sqrt{\frac{\epsilon_r+1}{2}} + \frac{\epsilon_r-1}{\epsilon_r+1}(.23 + .11/\epsilon_r) = 2.21 \checkmark$$

Then, $$W/d = \frac{8e^A}{e^{2A}-2} = 0.90 < 2 \quad \text{OK} \checkmark$$

$$W = .9(.158\text{ cm}) = \underline{0.142\text{ cm}} \checkmark$$

From (3.195) the effective permittivity is,

$$\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\frac{1}{\sqrt{1+12d/W}} = \underline{1.758} \checkmark$$

$$\lambda_g = \frac{c}{\sqrt{\epsilon_e}\,f} = \frac{300}{\sqrt{1.758}\,(4000)} = \underline{5.656\text{ cm}} \checkmark$$

3.21 $\lambda_o = 0.12\text{m}$, $\beta = 2\pi\sqrt{\epsilon_e}/\lambda_o = 3986.7°/\text{m}$ $(f = 2.5\text{ GHz})$

$C = 5\text{ pF}$: From P2.11, $\beta\ell = 82.74° \Rightarrow \ell = 2.0754\text{ cm}$ $(Z_{in} = -j12.73\Omega)$

$L = 5\text{ nH}$: From P2.11, $\beta\ell = 128.1° \Rightarrow \ell = 3.2132\text{ cm}$ $(Z_{in} = +j78.5\Omega)$

From SERENADE:

LOSSLESS: C: $Z_{in} = -j12.63\Omega$ ✓
L: $Z_{in} = +j78.7\Omega$ ✓

LOSSY: C: $Z_{in} = 0.27 - j12.82\Omega$
(t=0.5 mil) L: $Z_{in} = 0.66 + j76.7\Omega$

3.22 $k_0 = \frac{2\pi f}{c} = 104.7 \ m^{-1}$ ✓ ; $R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{2\pi(5\times10^9)(4\pi\times10^{-7})}{2(5.813\times10^7)}} = 0.018\,\Omega$ ✓

MICROSTRIP CASE:

First try $W/d > 2$: From (3.197), $B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}} = 8.0$

$$W/d = \frac{2}{\pi}\left[B-1-\ln(2B-1)+\frac{\epsilon_r-1}{2\epsilon_r}\left\{\ln(B-1)+.39-\frac{.61}{\epsilon_r}\right\}\right] = 3.09 > 2 \ ✓$$

$W = 3.09(.16\text{ cm}) = \underline{0.494 \text{ cm}}$

From (3.195), $\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\frac{1}{\sqrt{1+12d/W}} = 1.87$ ✓ $\Rightarrow \lambda_g = \frac{c}{\sqrt{\epsilon_e}\,f} = \underline{4.38\text{ cm}}$ ✓

From (3.198), $\alpha_d = \frac{k_0\epsilon_r(\epsilon_e-1)}{2\sqrt{\epsilon_e}(\epsilon_r-1)}\tan\delta = \underline{0.061}$ neper/m ✓

From (3.199), $\alpha_c = \frac{R_s}{Z_0 W} = \underline{0.073}$ neper/m ✓

Total MS Loss:

$$\text{LOSS} = (.061+.073)\left(\frac{np}{m}\right)(16\lambda_g)\left(.0438\frac{m}{\lambda_g}\right)\left(\frac{8.686\text{ dB}}{\text{neper}}\right)$$
$$= 0.82\text{ dB} \ ✓$$

STRIPLINE CASE:

From (3.180), $\sqrt{\epsilon_r}Z_0 = \sqrt{2.2}(50) = 74 < 120.$ $x = \frac{30\pi}{\sqrt{\epsilon_r}Z_0} - .441 = 0.833$

$W/b = x = 0.833 \Rightarrow W = .833(.32\text{ cm}) = \underline{0.267\text{ cm}}$ ✓

$\lambda_g = \frac{c}{\sqrt{\epsilon_r}\,f} = \underline{4.045\text{ cm}}$ ✓ $A = 1 + \frac{2W}{b-t} + \frac{1}{\pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right) = 4.73$

From (3.181), $\alpha_c = \frac{2.7\times10^{-3}R_s\epsilon_r Z_0}{30\pi b}A = \underline{0.084}$ neper/m ✓

From (3.30), $\alpha_d = \frac{k\tan\delta}{2} = \frac{\sqrt{2.2}(104.7)(.001)}{2} = \underline{0.078}$ neper/m ✓

Total S.L. Loss:

$$\text{LOSS} = (.084+.078)\left(\frac{np}{m}\right)(16\lambda_g)\left(.04045\frac{m}{\lambda_g}\right)\left(\frac{8.686\text{ dB}}{\text{neper}}\right)$$
$$= \underline{0.91\text{ dB}} \ ✓$$

Thus the microstrip line should be used.

(3.23) $H_z(x,y,z) = h_z(x,y)e^{-j\beta z}$; h_z real, β real

From (3.19),

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial x}e^{-j\beta z} \qquad H_y = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial y}e^{-j\beta z}$$

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y}e^{-j\beta z} \qquad E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial x}e^{-j\beta z}$$

$$\bar{E}\times\bar{H}^* = (E_xH_y^* - E_yH_x^*)\hat{z} - E_xH_z^*\hat{y} + E_yH_z^*\hat{x}$$

$$= \frac{\omega\mu\beta}{k_c^4}\left[\left(\frac{\partial h_z}{\partial y}\right)^2 + \left(\frac{\partial h_z}{\partial x}\right)^2\right]\hat{z} + \frac{j\omega\mu}{k_c^2}\left(\frac{\partial h_z}{\partial y}\hat{y} + \frac{\partial h_z}{\partial x}\hat{x}\right)h_z$$

So if h_z is real (or a real function times a complex constant), there is real power flow only in the z-direction.

(3.24) Write the incident, reflected, and transmitted TE_{10} fields as follows:

$$E_y^i = E_0 \sin\frac{\pi x}{a}e^{-j\beta_a z} \qquad E_y^r = E_0\Gamma \sin\frac{\pi x}{a}e^{j\beta_a z}$$

$$H_x^i = \frac{-E_0}{Z_a}\sin\frac{\pi x}{a}e^{-j\beta_a z} \qquad H_x^r = \frac{E_0\Gamma}{Z_a}\sin\frac{\pi x}{a}e^{j\beta_a z}$$

$$E_y^t = E_0 T \sin\frac{\pi x}{a}e^{-j\beta_d z}$$

$$H_x^t = \frac{-E_0 T}{Z_d}\sin\frac{\pi x}{a}e^{-j\beta_d z}$$

where $\beta_a = \sqrt{k_0^2 - (\pi/a)^2}$ $\qquad Z_a = k_0\eta_0/\beta_a$

$\beta_d = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$ $\qquad Z_d = k_0\eta_0/\beta_d$

Match fields at $z=0$ to obtain:

$$1 + \Gamma = T \qquad (E_y \text{ continuous})$$

$$\frac{1}{Z_a}(-1+\Gamma) = \frac{-T}{Z_d} \qquad (H_x \text{ continuous})$$

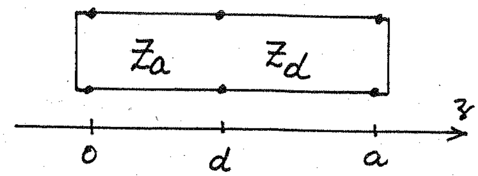
Solving for Γ gives,

$$\Gamma = \frac{Z_d - Z_a}{Z_d + Z_a} \checkmark$$

which agrees with the transmission line theory result if Z_{TE} is used as Z_0 in each region. ✓

(3.25)

$$Z_{TM} = \eta \beta / k = \eta_o \beta / k_o \sqrt{\epsilon_r}$$



for $0 < x < d$, $Z_a = \eta_o k_{xa} / k_o$

for $d < x < a$, $Z_d = \eta_o k_{xd} / \sqrt{\epsilon_r} k_o$

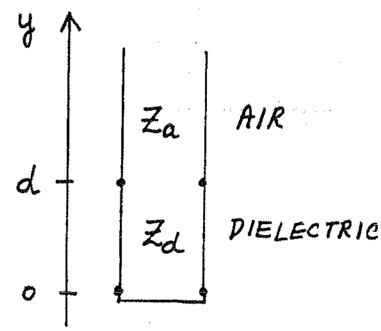
$$\beta = \sqrt{\epsilon_r k_o^2 - k_{xd}^2 - (n\pi/b)^2} = \sqrt{k_o^2 - k_{xa}^2 - (n\pi/b)^2}$$

Applying (3.215):

$$Z_a \tan \beta_a d + Z_d \tan \beta_d (a-b) = 0$$

The m-th root of this equation applies to the TM_{mn} mode.

(3.26)



$$Z_a = k_{ya} \eta_o / k_o = -jh \eta_o / k_o$$

$$Z_d = k_{yd} \eta / k = k_{yd} \eta_o / k_o$$

$$\beta = \sqrt{\mu_r k_o^2 - k_{yd}^2} = \sqrt{k_o^2 - k_{ya}^2} = \sqrt{k_o^2 + h^2}$$

Applying (3.215):

$$Z_a + j Z_d \tan k_{yd} d = 0$$

$$h = k_{yd} \tan k_{yd} = 0$$

This agrees with the solution to Problem 3.17, with $k_c = k_{yd}$. ✓

(3.27) For X-band guide, $a = 2.286$ cm.

$$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = \frac{2\pi(9500)\sqrt{2.08}}{300} = 287.\ m^{-1} \checkmark$$

$$\beta = \sqrt{k^2 - (\pi/a)^2} = 252.\ m^{-1} \checkmark$$

speed of light in Teflon $= \dfrac{c}{\sqrt{\epsilon_r}} = \dfrac{3\times10^8}{\sqrt{2.08}} = \underline{2.08\times10^8}$ m/sec $\checkmark$

phase velocity $= v_p = \dfrac{\omega}{\beta} = \dfrac{2\pi(9.5\times10^9)}{252.} = \underline{2.37\times10^8}$ m/sec $\checkmark$

From (3.231),

$$\text{group velocity} = v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \left(\frac{d\beta}{dk}\frac{dk}{d\omega}\right)^{-1} = \left(\frac{k}{\beta}\sqrt{\mu\epsilon}\right)^{-1}$$

$$= \frac{\beta}{k\sqrt{\mu\epsilon}} = \frac{252(2.08\times10^8)}{287.} = 1.83\times10^8 \text{ m/sec}$$

Note that $v_g < \dfrac{c}{\sqrt{\epsilon_r}} < v_p$.

(3.28)

$$P_{MAX} = C a^2 \ln\frac{b}{a}$$

$$\frac{dP_{MAX}}{da} = 2a\ln\frac{b}{a} - \frac{a^2}{a} = 0$$

$$2\ln\frac{b}{a} - 1 = 0$$

$$2\ln x = 1$$

$$\ln x = 0.5$$

$$x = 1.65$$

$$Z_0 = \frac{377}{2\pi}\ln\frac{b}{a} = \frac{120\pi}{2\pi}\left(\frac{1}{2}\right) = \underline{30\ \Omega}$$

Chapter 4

4.1 (This problem is essentially the same as P. 3.24)

Write the fields of incident, reflected, and transmitted TE_{10} modes in each region:

$$E_y^i = E_0 \sin\frac{\pi x}{a} e^{-j\beta_a z} \qquad E_y^r = E_0 \Gamma \sin\frac{\pi x}{a} e^{j\beta_a z}$$

$$H_x^i = \frac{-E_0}{Z_a} \sin\frac{\pi x}{a} e^{-j\beta_a z} \qquad H_x^r = \frac{E_0\Gamma}{Z_0} \sin\frac{\pi x}{a} e^{j\beta_a z}$$

$$E_y^t = E_0 T \sin\frac{\pi x}{a} e^{-j\beta_d z}$$

$$H_x^t = \frac{-E_0 T}{Z_d} \sin\frac{\pi x}{a} e^{-j\beta_d z}$$

where $\beta_a = \sqrt{k_0^2 - (\pi/a)^2}$, $Z_a = k_0\eta_0/\beta_a$

$\beta_d = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$, $Z_d = k_0\eta_0/\beta_d$

Match fields at $z=0$:

$$1 + \Gamma = T$$

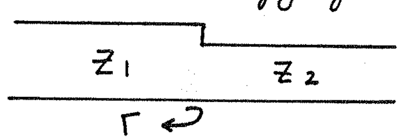
$$\frac{1}{Z_a}(-1+\Gamma) = \frac{-T}{Z_d}$$

Solving gives,

$$\Gamma = \frac{Z_d - Z_a}{Z_d + Z_a}. \quad \text{As in Example 4.2} \quad \checkmark$$

4.2 Using a transmission line analogy gives,

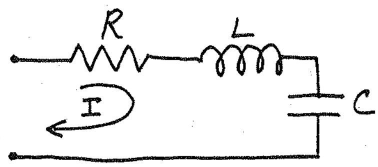
$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$



where $Z_1 = k_0\eta_0/\beta_1$, $Z_2 = k_0\eta_0/\beta_2$.

But $\beta_1 = \beta_2 = \sqrt{k_0^2 - (\pi/a)^2}$ in both regions, since only the height (b) of the guide changes. Thus, $\Gamma = 0$ from above. This is obviously not correct, as E_y should be zero for $b/2 < y < b$. Higher order TE_{1n} modes must be considered, in a mode matching procedure. This will result in a solution where $\Gamma \neq 0$. Consideration of only the dominant mode is not adequate.

4.3



$$P_\ell = \tfrac{1}{2}|I|^2 R \implies R = \frac{P_\ell}{\tfrac{1}{2}|I|^2}$$

$$W_m = \tfrac{1}{4} L |I|^2 \implies L = \frac{2W_m}{\tfrac{1}{2}|I|^2}$$

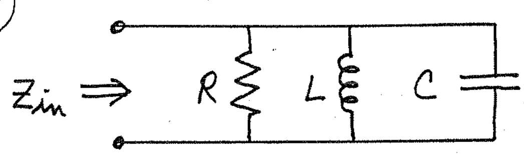
$$W_e = \tfrac{1}{4} C |V_c|^2 = \frac{1}{4\omega^2 C}|I|^2 \implies \frac{1}{\omega^2 C} = \frac{2W_e}{\tfrac{1}{2}|I|^2}$$

The input impedance is,

$$Z_{in} = R + j\left(\omega L + \frac{1}{\omega C}\right) = \frac{P_\ell + 2j\omega(W_m - W_e)}{\tfrac{1}{2}|I|^2} \quad \checkmark$$

In agreement with (4.17)

4.4



$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{1}{\frac{1}{R} + j\omega\left(C - \frac{1}{\omega^2 L}\right)}$$

$$Z(-\omega) = \frac{1}{\frac{1}{R} - j\omega\left(C - \frac{1}{\omega^2 L}\right)} = Z^*(\omega) \quad \checkmark$$

4.5
$$P_{in} = \frac{1}{2}[V]^t[I]^* = \frac{1}{2}[V]^t[Y]^*[V]^*$$
$$= \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} V_m Y_{mn}^* V_n^*$$

If lossless, $Re\{P_{in}\}=0$. Since the V_n's are independent, we first let all $V_m=0$, except for V_n. Then,

$$P_{in} = \frac{1}{2}V_n Y_{nn}^* V_n^* = \frac{1}{2}|V_n|^2 Y_{nn}^*$$
$$\therefore \; Re\{Y_{nn}^*\} = Re\{Y_{nn}\} = 0 \quad \checkmark$$

Now let all port voltages be zero except for V_m and V_n. Then,

$$P_{in} = \frac{1}{2}V_m Y_{mn}^* V_n^* + \frac{1}{2}V_n Y_{nm}^* V_m^*$$

So,
$$Re\{V_m Y_{mn}^* V_n^* + V_n Y_{nm}^* V_m^*\} = 0$$

If $Y_{mn}=Y_{nm}$ (reciprocal), then

$$Re\{Y_{mn}^*(V_m V_n^* + V_n V_m^*)\} = Re\{Y_{mn}^*[(V_m V_n^* + (V_m V_n^*)^*]\} = 0$$

Since $A+A^*$ is real, we must have $Re\{Y_{mn}\}=0$ ✓

4.6 Let $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$, and show that Z_{ij}'s can be found such that $P_{in}=0$, but not all Z_{ij}'s are pure imaginary.

$$P_{in} = \frac{1}{2}[I]^t[Z]^t[I]^* = \frac{1}{2}(I_1 Z_{11} I_1^* + I_1 Z_{21} I_2^* + I_2 Z_{12} I_1^* + I_2 Z_{22} I_2^*)$$
$$= \frac{1}{2}(Z_{11}|I_1|^2 + Z_{22}|I_2|^2 + Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*)$$

To be lossless, we must have $Re\{Z_{11}\} = Re\{Z_{22}\} = 0$.

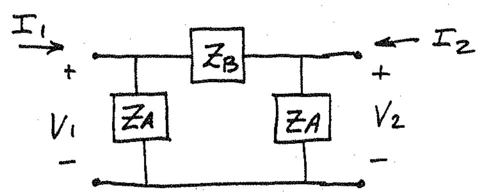
Also, $Re\{Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*\} = 0$.

This will occur if $Z_{12} = -Z_{21}^*$ (since $Re\{A-A^*\}=0$).

For example, if $Z_{12} = a+jb$, then $Z_{21} = -a+jb$.

Thus, $[Z]$ is not symmetric, and the answer is NO.

(4.7)



From (4.28),

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{V_1}{V_1\left(\dfrac{2Z_A+Z_B}{Z_A(Z_A+Z_B)}\right)} = \frac{Z_A(Z_A+Z_B)}{2Z_A+Z_B} \checkmark = Z_{22} \quad \text{(BY SYMMETRY)}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{I_1 Z_{11}\left(\dfrac{Z_A}{Z_A+Z_B}\right)}{I_1} = \frac{Z_A^2}{2Z_A+Z_B} \checkmark = Z_{12} \quad \text{(BY RECIPROCITY)}$$

From (4.29),

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{I_1}{I_1\left(\dfrac{Z_A Z_B}{Z_A+Z_B}\right)} = \frac{Z_A+Z_B}{Z_A Z_B} \checkmark = Y_{22} \quad \text{(BY SYMMETRY)}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-V_1/Z_B}{V_1} = \frac{-1}{Z_B} \checkmark = Y_{12} \quad \text{(BY RECIPROCITY)}$$

CHECK: $[Z][Y]=[U]$?

$$Z_{11}Y_{11} + Z_{12}Y_{21} = \frac{(Z_A+Z_B)^2}{Z_B(2Z_A+Z_B)} - \frac{Z_A^2}{Z_B(2Z_A+Z_B)} = \frac{2Z_AZ_B+Z_B^2}{Z_B(2Z_A+Z_B)} = 1 \checkmark$$

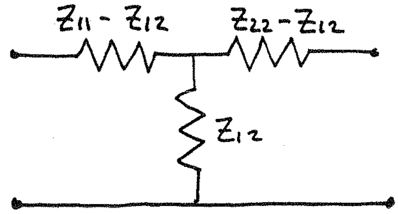
$$Z_{11}Y_{12} + Z_{12}Y_{22} = \frac{-Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} + \frac{Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} = 0 \checkmark$$

Similarly for the T-network. The results are,

$$Z_{11} = Z_{22} = \frac{Y_A+Y_B}{Y_AY_B} \checkmark \qquad Z_{12} = Z_{21} = \frac{-1}{Y_B} \checkmark$$

$$Y_{11} = Y_{22} = \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B} \checkmark \qquad Y_{12} = Y_{21} = \frac{Y_A^2}{2Y_A+Y_B} \checkmark$$

(4.8) Model the two-port as below:



Then,

$$Z_{sc}^{(1)} = Z_{11} - Z_{12} + \frac{Z_{12}(Z_{22}-Z_{12})}{Z_{22}} = Z_{11} - Z_{12}^2/Z_{22}$$

$$Z_{sc}^{(2)} = Z_{22} - Z_{12}^2/Z_{11}$$

$$Z_{oc}^{(1)} = Z_{11} \checkmark$$

$$Z_{oc}^{(2)} = Z_{22} \checkmark$$

From the first equation,

$$Z_{12}^2 = -(Z_{sc}^{(1)} - Z_{11})Z_{22} = (Z_{oc}^{(1)} - Z_{sc}^{(1)})\, Z_{oc}^{(2)}.$$

(4.9) From (4.58),

$$V_n = V_n^+ + V_n^-$$
$$Z_0 I_n = V_n^+ - V_n^-$$

Solve for V_n^+, V_n^-:

$$V_n^+ = (V_n + Z_0 I_n)/2$$
$$V_n^- = (V_n - Z_0 I_n)/2$$

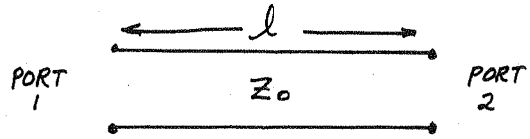
Then,

$$V_1^+ = \tfrac{1}{2}[20 + 50(.4j)] = 14.1\angle 45^\circ$$
$$V_1^- = \tfrac{1}{2}[20 - 50(.4j)] = 14.1\angle -45^\circ$$
$$V_2^+ = \tfrac{1}{2}[-4j + 50(.08)] = 2.82\angle -45^\circ$$
$$V_2^- = \tfrac{1}{2}[-4j - 50(.08)] = 2.82\angle -135^\circ$$

$$Z_{in}^{(1)} = \frac{V_1}{I_1} = \frac{20\angle 0}{0.4\angle 90} = 50\angle -90^\circ$$

$$Z_{in}^{(2)} = \frac{V_2}{I_2} = \frac{4\angle -90}{.08\angle 0} = 50\angle -90^\circ$$

4.10 a)

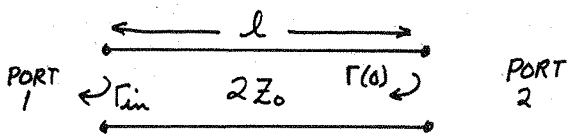


$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = 0 \qquad S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = 0$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0} = e^{-j\beta l} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = e^{-j\beta l}$$

$$[S] = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix} \quad \text{unitary} \checkmark$$

b)



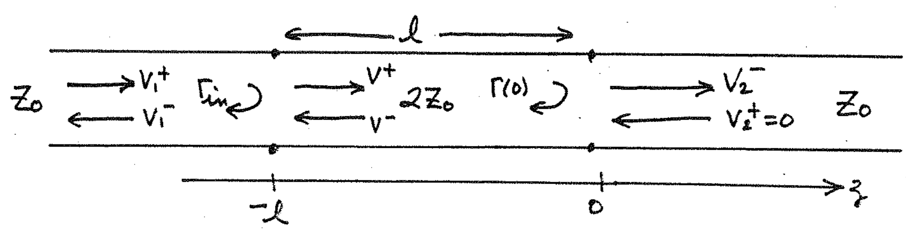
$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma_{in} = S_{22} \qquad \Gamma(0) = \frac{Z_0 - 2Z_0}{Z_0 + 2Z_0} = \frac{-1}{3}$$

$$Z_{in} = 2Z_0 \frac{1+\Gamma(0)\,e^{-2j\beta l}}{1-\Gamma(0)\,e^{-2j\beta l}} = 2Z_0 \frac{1-\frac{1}{3}e^{-2j\beta l}}{1+\frac{1}{3}e^{-2j\beta l}}$$

$$\Gamma_{in} = \frac{Z_{in}-Z_0}{Z_{in}+Z_0} = \frac{2(1-\frac{1}{3}e^{-2j\beta l})-(1+\frac{1}{3}e^{-2j\beta l})}{2(1-\frac{1}{3}e^{-2j\beta l})+(1+\frac{1}{3}e^{-2j\beta l})} = \frac{1-e^{-2j\beta l}}{3-\frac{1}{3}e^{-2j\beta l}}$$

CHECK: if $l = \lambda/2$, $\Gamma_{in} = 0$ ✓

For $S_{21} = S_{12}$, consider the following circuit:



on the 2Zo line we have,

$$V_1 = V_1^+ + V_1^- = V_1^+(1+\Gamma_{in}) = V^+e^{j\beta l} - \frac{1}{3}V^+e^{-j\beta l}$$

$$V_2 = V_2^+ + V_2^- = V_2^- = V^+(1-\frac{1}{3})$$

Thus,
$$V_2^- = \frac{2}{3}V^+ = \frac{\frac{2}{3}V_1^+(1+\Gamma_{in})}{e^{j\beta\ell} - \frac{1}{3}e^{-j\beta\ell}}$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \frac{\frac{2}{3}(1+\Gamma_{in})}{e^{j\beta\ell}-\frac{1}{3}e^{-j\beta\ell}} = \frac{8}{3}\,\frac{e^{-j\beta\ell}}{3-\frac{1}{3}e^{-2j\beta\ell}}$$

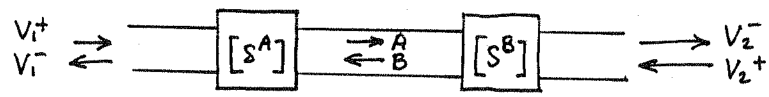
CHECK: if $\ell=\lambda$, $Z_{in}=Z_0$, $\Gamma_{in}=0$, $S_{11}=0$, $S_{21}=1$ ✓

if $\ell=\lambda/2$, $\Gamma_{in}=0$, $S_{11}=0$, $S_{21}=-1$ ✓

$$|S_{11}|^2+|S_{21}|^2 = \left|\frac{1-e^{-2j\beta\ell}}{3-\frac{1}{3}e^{-2j\beta\ell}}\right|^2 + \frac{64}{9}\,\frac{1}{|3-\frac{1}{3}e^{-2j\beta\ell}|^2}$$

$$= \frac{|1-e^{-2j\beta\ell}|^2+\frac{64}{9}}{|3-\frac{1}{3}e^{-2j\beta\ell}|^2} = \frac{1-e^{-2j\beta\ell}-e^{2j\beta\ell}+1+\frac{64}{9}}{9-e^{2j\beta\ell}-e^{-2j\beta\ell}+\frac{1}{9}} = 1 \checkmark \quad \text{(UNITARY)}$$

(4.11) Define wave amplitudes as shown:



Then,
$$\begin{bmatrix}V_1^-\\A\end{bmatrix} = [S^A]\begin{bmatrix}V_1^+\\B\end{bmatrix} \qquad \begin{bmatrix}B\\V_2^-\end{bmatrix} = [S^B]\begin{bmatrix}A\\V_2^+\end{bmatrix} \qquad \begin{bmatrix}V_1^-\\V_2^-\end{bmatrix} = [S]\begin{bmatrix}V_1^+\\V_2^+\end{bmatrix}$$

$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$. When $V_2^+=0$, we have $B = S_{11}^B A$, $V_2^- = S_{21}^B A$.

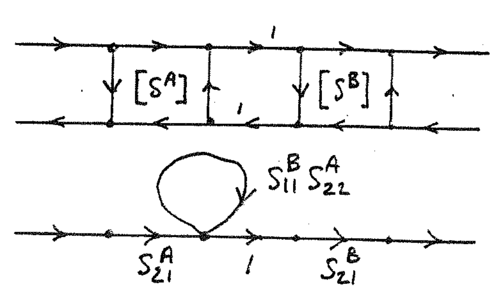
Then,
$$A = S_{21}^A V_1^+ + S_{22}^A B = S_{21}^A V_1^+ + S_{22}^A S_{11}^B A$$

$$\frac{V_2^-}{S_{21}^B} = S_{21}^A V_1^+ + S_{22}^A S_{11}^B \frac{V_2^-}{S_{21}^B}$$

$$V_2^-\left(\frac{1-S_{22}^A S_{11}^B}{S_{21}^B}\right) = S_{21}^A V_1^+$$

So,
$$S_{21} = \frac{S_{21}^A S_{21}^B}{1-S_{22}^A S_{11}^B} \checkmark$$

SIGNAL FLOWGRAPH SOLUTION:



$$\therefore S_{21} = \frac{S_{21}^A S_{21}^B}{1-S_{11}^B S_{22}^A} \checkmark$$

4.12

a) $$[S] = \begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{bmatrix}$$ $S_{12}=S_{21}$ since reciprocal

$[S]$ is unitary if lossless, so

1st Row: $|S_{11}|^2 + |S_{21}|^2 = 1$ (or 1st COL)

$|S_{21}|^2 = 1 - |S_{11}|^2$ ✓

b) $$[S] = \begin{bmatrix} S_{11} & S_{21} \\ 0 & S_{22} \end{bmatrix}$$ $S_{12} \neq S_{21}$ since nonreciprocal

1st Row: $|S_{11}|^2 + |S_{21}|^2 = 1$

1st Col.: $|S_{11}|^2 = 1$

$\therefore |S_{21}| = 0$

4.13 a matched, reciprocal, 3-port network has an $[S]$ matrix of the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

If the network is lossless, then $[S]$ must be unitary:

$|S_{12}|^2 + |S_{13}|^2 = 1$ (1) $\qquad S_{13} S_{23}^* = 0$ (4)

$|S_{12}|^2 + |S_{23}|^2 = 1$ (2) $\qquad S_{12} S_{13}^* = 0$ (5)

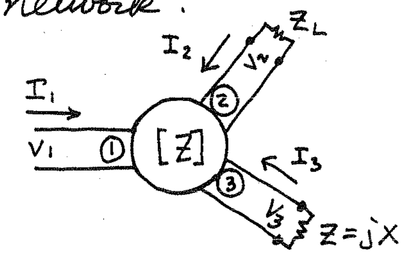
$|S_{13}|^2 + |S_{23}|^2 = 1$ (3) $\qquad S_{12} S_{23}^* = 0$ (6)

To show that a contradiction exists, assume that $S_{12}=0$, in order to satisfy (5) and (6). Then from (1), $|S_{13}|^2=1$, and from (3), $|S_{23}|=0$. But then (2) will be contradicted. Similarly, a contradiction will follow if we let $S_{13}=0$, or $S_{23}=0$.

a circulator is an example of a nonreciprocal, lossless, matched 3-port network.

(4.14) For this problem it is easiest to use the Z-matrix for a lossless reciprocal 3-port network:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} jX_{11} & jX_{12} & jX_{13} \\ jX_{12} & jX_{22} & jX_{23} \\ jX_{13} & jX_{23} & jX_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$



If we terminate port 3 in a reactance jX, then $V_3 = -jX I_3$. Then we must find jX so that $V_2 = 0$ for $V_1 \neq 0$. If $V_2 = 0$, then $I_2 = 0$:

$$V_3 = jX_{13} I_1 + jX_{33} I_3 = -jX I_3$$

$$I_3 = \frac{-X_{13} I_1}{X_{33} + X}$$

$$V_2 = jX_{12} I_1 + jX_{23} I_3 = \left(jX_{12} - \frac{jX_{23}X_{13}}{X_{33}+X}\right) I_1 = 0$$

So,

$$X_{12}X_{33} + XX_{12} - X_{13}X_{23} = 0$$

$$X = \frac{X_{13}X_{23} - X_{12}X_{33}}{X_{12}} \checkmark$$

CHECK: The input impedance at Port 1 is,

$$Z_{in}^{(1)} = \frac{V_1}{I_1} = \frac{jX_{11}I_1 + jX_{13}I_3}{I_1} = jX_{11} + jX_{13}\left(\frac{-X_{13}}{X_{33}+X}\right)$$

$$= j\left(X_{11} - \frac{X_{13}^2}{X_{33}+X}\right)$$ which is pure imaginary ✓

4.15

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

Assume the network is fed at port 1, so $V_1^+ \neq 0$. Port 2 is terminated in a matched load, so $V_2^+ = 0$. Port 3 is terminated in a reactive load, so $V_3^+ = e^{j\phi} V_3^-$. We must find $e^{j\phi}$ so that $V_1^-/V_1^+ = 0$.

$$V_3^- = S_{13} V_1^+ + S_{33} V_3^+ = e^{-j\phi} V_3^+$$

$$V_3^+ = \frac{S_{13} V_1^+}{e^{-j\phi} - S_{33}}$$

$$V_1^- = S_{11} V_1^+ + S_{13} V_3^+ = S_{11} V_1^+ + \frac{S_{13}^2 V_1^+}{e^{-j\phi} - S_{33}}$$

$$\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{13}^2}{e^{-j\phi} - S_{33}} = 0 \Rightarrow e^{-j\phi} = S_{33} - \frac{S_{13}^2}{S_{11}}. \checkmark$$

We should also verify that this quantity has unit magnitude:

$$\left| S_{33} - S_{13}^2/S_{11} \right|^2 = \frac{|S_{11}|^2 |S_{33}|^2 - |S_{13}|^4 - S_{11}^* S_{33}^* S_{13}^2 - S_{11} S_{33} S_{13}^{*2}}{|S_{11}|^2}$$

The unitary properties of [S] lead to four equations:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{11} S_{12}^* + S_{12} S_{11}^* + |S_{13}|^2 = 0$$

$$2|S_{13}|^2 + |S_{33}|^2 = 1 \qquad S_{12} S_{13}^* + S_{11} S_{13}^* + S_{13} S_{33}^* = 0$$

Eliminating S_{12} from the two equations on the right yields,

$$-2|S_{11}|^2 - \frac{S_{11} S_{13}^* S_{33}}{S_{13}} - \frac{S_{11}^* S_{13} S_{33}^*}{S_{13}^*} + |S_{13}|^2 = 0$$

or,

$$-S_{11} S_{13}^{*2} S_{33} - S_{11}^* S_{13}^2 S_{33}^* = 2|S_{11}|^2 |S_{13}|^2 - |S_{13}|^4$$

Then,

$$|S_{33} - S_{13}^2/S_{11}|^2 = \frac{|S_{11}|^2 |S_{33}|^2 + \cancel{|S_{13}|^4} + 2|S_{11}|^2 |S_{13}|^2 - \cancel{|S_{13}|^4}}{|S_{11}|^2}$$

$$= |S_{33}|^2 + 2|S_{13}|^2 = 1 \checkmark$$

(4.16)

a) To be lossless, [S] must be unitary:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = (.1)^2 + (.8)^2 + (.3)^2 = 0.74 \neq 1$$

Thus the network is <u>not lossless</u>.

b) The network <u>is reciprocal</u>, since [S] is symmetric.

c) When ports 2, 3, 4 are matched, $\Gamma = S_{11}$

So $RL = -20 \log |\Gamma| = -20 \log(.1) = \underline{20\,dB}$

d) For ports 1 & 3 terminated with Z_0,

$IL = -20 \log |S_{42}| = -20 \log(.4) = \underline{8.0\,dB}$

<u>phase delay = 60°</u>

e) For a short circuit at port 3, and matched loads at other ports, we have

$$V_2^+ = V_4^+ = 0$$

$$V_3^+ = -V_3^-$$

$$V_1^- = S_{11}V_1^+ + S_{13}V_3^+ = S_{11}V_1^+ - S_{13}V_3^-$$

$$V_3^- = S_{31}V_1^+$$

Then,

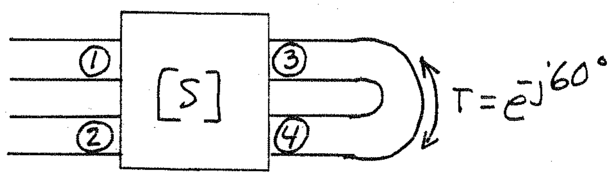
$$\Gamma^{(1)} = \frac{V_1^-}{V_1^+} = S_{11} - S_{13}S_{31} \quad \checkmark$$

$$= .1j - (.3\angle{-45°})(.3\angle{-45°}) = .1j + .09j$$

$$= .19j = \underline{.19\angle 90°} \quad \checkmark$$

(verified with Serenade)

(4.17)



$V_4^+ = T V_3^-$
$V_3^+ = T V_4^-$

Assume feed at port 1, port 2 matched. Then $V_2^+ = 0$.
Also, $S_{12} = S_{13} = S_{21} = S_{24} = S_{31} = S_{34} = S_{42} = S_{43} = 0$

DIRECT SOLUTION:

$$V_2^- = S_{23} V_3^+ = S_{23} T V_4^-$$
$$V_3^- = S_{33} V_3^+$$
$$V_4^- = S_{41} V_1^+ + S_{44} V_4^+ = S_{41} V_1^+ + S_{44} T V_3^-$$
$$= S_{41} V_1^+ + S_{44} S_{33} T V_3^+$$
$$= S_{41} V_1^+ + S_{44} S_{33} T^2 V_4^-$$

Solve for V_4^-:

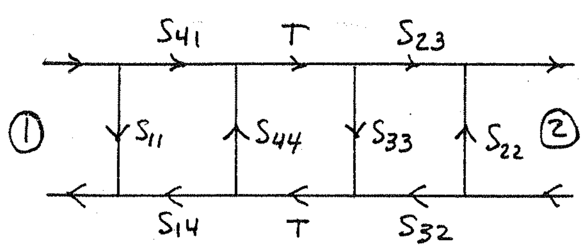
$$V_4^- = \frac{S_{41} V_1^+}{1 - S_{33} S_{44} T^2}$$

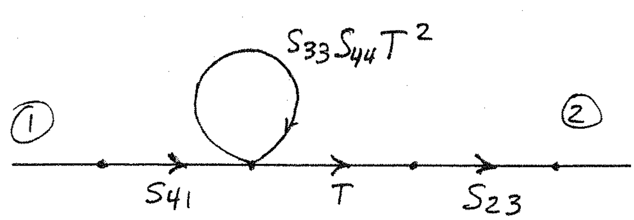
Then,

$$T_{21} = \frac{V_2^-}{V_1^+} = \frac{S_{23} S_{41} T}{1 - S_{33} S_{44} T^2} = \frac{(.7\angle{-45})(.8\angle{0})(1\angle{-60})}{1-(.7\angle{-30})(.3\angle{-30})(1\angle{-120})} = 0.463\angle{-105°}$$

$IL = -20 \log(.463) = 6.7 dB$ ✓ phase delay $= 105°$ ✓ (verified w/ Serenade)

SIGNAL FLOWGRAPH SOLUTION:

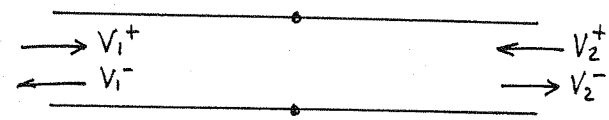




$$T_{21} = \frac{S_{41} S_{23} T}{1 - S_{33} S_{44} T^2}$$ ✓

(splitting rule and self-loop rule)

(4.18)



From (4.62),

$$S_{11} = \sqrt{\frac{Z_{01}}{Z_{01}}}\, \frac{V_1^-}{V_1^+}\bigg|_{V_2^+=0} = \frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}$$

$$S_{12} = \sqrt{\frac{Z_{02}}{Z_{01}}}\, \frac{V_1^-}{V_2^+}\bigg|_{V_1^+=0} = \frac{2Z_{01}}{Z_{01}+Z_{02}}\sqrt{\frac{Z_{02}}{Z_{01}}} = \frac{2\sqrt{Z_{01}Z_{02}}}{Z_{01}+Z_{02}}$$

$$S_{21} = \sqrt{\frac{Z_{01}}{Z_{02}}}\, \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0} = \frac{2Z_{02}}{Z_{01}+Z_{02}}\sqrt{\frac{Z_{01}}{Z_{02}}} = \frac{2\sqrt{Z_{01}Z_{02}}}{Z_{01}+Z_{02}}$$

NOTE THAT $S_{12} = S_{21}$

$$S_{22} = \sqrt{\frac{Z_{02}}{Z_{02}}}\, \frac{V_2^-}{V_2^+}\bigg|_{V_1^+=0} = \frac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}}$$

(verified with Serenade for $Z_{01} = 50\Omega$, $Z_{02} = 100\Omega$)

(4.19) Using Table 4.2 for S to Z transformation:

$(1+S_{11}) = 1.4765 \angle 28.3°$

$(1-S_{11}) = .9899 \angle -45°$

$S_{12}S_{21} = -0.36$

$(1+S_{22}) = 1.4765 \angle -28.3°$

$(1-S_{22}) = .9899 \angle 45°$

$(1-S_{11})(1-S_{22}) - S_{12}S_{21} = 1.340$

Then,

$$Z_{11} = \frac{50(1.4765\angle 28.3°)(.9899\angle 45°) - .36}{1.340} = 2.24 + j52.2\ \Omega \checkmark$$

$$Z_{12} = Z_{21} = 50\,\frac{2(.6\angle 90°)}{1.340} = j44.8\ \Omega \checkmark$$

$$Z_{22} = \frac{50(.9899\angle -45°)(1.4765\angle -28.3°) - .36}{1.340} = 2.24 - j52.2\ \Omega \checkmark$$

(verified with Serenade)

4.20 From (4.62),

$$S'_{ij} = \frac{\sqrt{Z_{0j}}}{\sqrt{Z_{0i}}} S_{ij}$$

So,

$$S'_{11} = S_{11} \checkmark$$

$$S'_{12} = \sqrt{\frac{Z_{02}}{Z_{01}}} S_{12} \checkmark$$

$$S'_{21} = \sqrt{\frac{Z_{01}}{Z_{02}}} S_{21} \checkmark$$

$$S'_{22} = S_{22} \checkmark$$

4.21 From Table 4.1 the ABCD parameters for a transmission line section are,

$A = D = \cos\beta l$, $B = jZ_0 \sin\beta l$, $C = jY_0 \sin\beta l$.

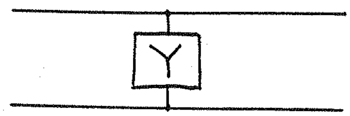
Now use Table 4.2 to convert to Z-parameters:

$$Z_{11} = \frac{A}{C} = \frac{\cos\beta l}{jY_0 \sin\beta l} = -jZ_0 \cot\beta l \checkmark$$

$$Z_{12} = Z_{21} = \frac{1}{C} = -jZ_0 \csc\beta l \checkmark$$

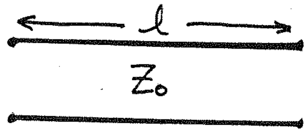
$$Z_{22} = \frac{D}{C} = \frac{\cos\beta l}{jY_0 \sin\beta l} = -jZ_0 \cot\beta l \checkmark$$

4.22



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$ ✓ $\quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = Y$ ✓

$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0$ ✓ $\quad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$ ✓



for $I_2=0$, $\quad V_1 = V^+(e^{j\beta l} + e^{-j\beta l}) = V^+ 2\cos\beta l$

$V_2 = 2V^+ = V_1/\cos\beta l$

So, $\quad A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \cos\beta l$ ✓

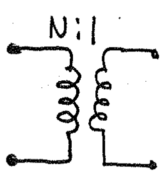
$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{V_1}{Z_{in} V_2} = \frac{\cos\beta l}{-jZ_0 \cot\beta l} = jY_0 \sin\beta l$ ✓

for $V_2=0$, $\quad V_1 = V^+(e^{j\beta l} - e^{-j\beta l}) = V^+ 2j\sin\beta l$

$I_2 = \frac{2V^+}{Z_0}$

So, $\quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = jZ_0 \sin\beta l$ ✓

$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{V_1}{Z_{in} I_2} = \frac{B}{Z_{in}} = \frac{jZ_0 \sin\beta l}{jZ_0 \tan\beta l} = \cos\beta l$ ✓



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{NV_2}{V_2} = N$ ✓

$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0$ ✓

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$ ✓

$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{1}{N}$ ✓

(4.23) NOTE: Difference in signs for Z and ABCD.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{V_2}\frac{V_2}{I_1}\right|_{I_2=0} = A/C$$

for $I_1 = 0$,

$$V_1 = AV_2 - BI_2$$

$$0 = CV_2 - DI_2 \Rightarrow \quad V_2 = DI_2/C$$

$$V_1 = \left(\frac{AD}{C} - B\right) I_2$$

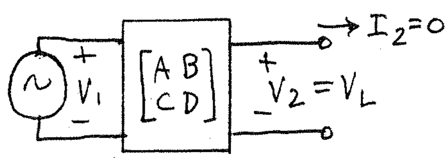
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{AD - BC}{C}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 1/C$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = D/C$$

(4.24) Using Table 4.1, the ABCD matrix of the cascade of four components (including load) is,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & j50 \\ j/50 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/25 & 1 \end{bmatrix} = \begin{bmatrix} 3j & 25j \\ j/25 & 0 \end{bmatrix}$$

3 v 

$$V_1 = AV_2 + BI_2 = AV_2 = AV_L$$

$$V_L = \frac{V_1}{A} = \frac{3}{3j} = 1\angle -90^\circ$$

(verified with Serenade)

4.25

DIRECT CALCULATION:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_1 \frac{1/Y}{Z+1/Y}} = 1+YZ$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{I_1/Y} = Y$$

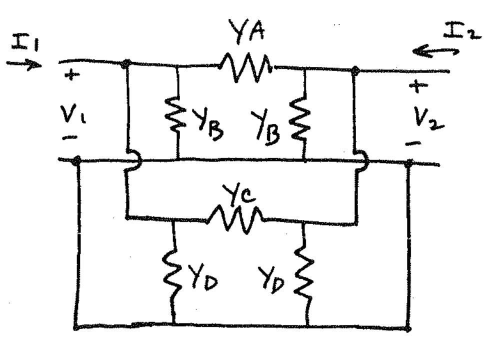
$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$$

CHECK: $AD-BC = 1+YZ-ZY = 1$ ✓

CALCULATION USING CASCADE: (From Table 4.1)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix} \checkmark$$

4.26



$$[Y_1] = \begin{bmatrix} Y_A+Y_B & -Y_A \\ -Y_A & Y_A+Y_B \end{bmatrix}$$

$$[Y_2] = \begin{bmatrix} Y_C+Y_D & -Y_C \\ -Y_C & Y_C+Y_D \end{bmatrix}$$

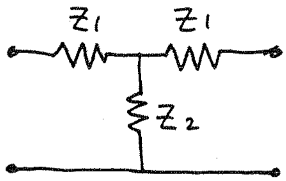
adding [Y] matrices gives:

$$[Y] = [Y_1]+[Y_2] = \begin{bmatrix} Y_A+Y_B+Y_C+Y_D & -Y_A-Y_C \\ -Y_A-Y_C & Y_A+Y_B+Y_C+Y_D \end{bmatrix}$$

By direct calculation, we obtain similar results:

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_A+Y_B+Y_C+Y_D \checkmark \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -(Y_A+Y_C) \checkmark$$

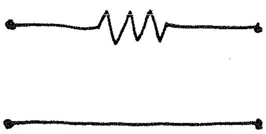
Now apply to bridged-T network (Example 5.7 of 1st edition)



$$[Z_A] = \begin{bmatrix} Z_1+Z_2 & Z_2 \\ Z_2 & Z_1+Z_2 \end{bmatrix}$$

$$[Y_A] = \frac{1}{D}\begin{bmatrix} Z_1+Z_2 & -Z_2 \\ -Z_2 & Z_1+Z_2 \end{bmatrix} \checkmark$$

$$D = (Z_1+Z_2)^2 - Z_2^2 = Z_1^2 + 2Z_1Z_2 \checkmark$$



$$[Y_B] = \begin{bmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{bmatrix} \checkmark$$

$$[Y_{TOT}] = [Y_A] + [Y_B] = \begin{bmatrix} \frac{1}{Z_3} + \frac{Z_1+Z_2}{D} & -\left(\frac{1}{Z_3} + \frac{Z_2}{D}\right) \\ -\left(\frac{1}{Z_3} + \frac{Z_2}{D}\right) & \frac{1}{Z_3} + \frac{Z_1+Z_2}{D} \end{bmatrix} \checkmark$$

4.27

$$V_1 = AV_2 - BI_2 \qquad V_n = V_n^+ + V_n^-$$

$$I_1 = CV_2 - DI_2 \qquad I_n = (V_n^+ - V_n^-)/Z_0$$

So,

$$V_1^+ + V_1^- = A(V_2^+ + V_2^-) - B(V_2^+ - V_2^-)/Z_0$$

$$V_1^+ - V_1^- = C(V_2^+ + V_2^-)Z_0 - D(V_2^+ - V_2^-)$$

For $V_2^+ = 0$,

$$V_1^+ + V_1^- = (A + B/Z_0)V_2^-$$

$$V_1^+ - V_1^- = (CZ_0 + D)V_2^-$$

eliminate V_2^+:

$$V_1^+ + V_1^- = \frac{A + B/Z_0}{CZ_0 + D}(V_1^+ - V_1^-)$$

$$V_1^-(CZ_0 + D + A + B/Z_0) = V_1^+(A + B/Z_0 - CZ_0 - D)$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \checkmark$$

eliminate V_1^-:

$$2V_1^+ = (A + B/Z_0 + CZ_0 + D)V_2^-$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \frac{2}{A + B/Z_0 + CZ_0 + D} \checkmark$$

for $V_1^+=0$ the above set reduces to,

$$V_1^- = (A - B/Z_0)V_2^+ + (A + B/Z_0)V_2^-$$
$$-V_1^- = (CZ_0 - D)V_2^+ + (CZ_0 + D)V_2^-$$

eleminate V_1^-:

$$(A - B/Z_0 + CZ_0 - D)V_2^+ + (A + B/Z_0 + CZ_0 + D)V_2^- = 0$$

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \quad ✓$$

eleminate V_2^-:

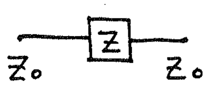
$$\frac{V_1^-}{A + B/Z_0} - \frac{A - B/Z_0}{A + B/Z_0}V_2^+ = \frac{-V_1^-}{CZ_0 + D} - \frac{CZ_0 - D}{CZ_0 + D}V_2^+$$

$$V_1^-\left(\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D}\right) = V_2^+\left(\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D}\right)$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0} = \frac{\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D}}{\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D}} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \quad ✓$$

These results agree with Table 4.2.

4.28

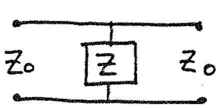


From Table 4.1, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

convert to [S] using Table 4.2:

$$S_{11} = \frac{1 + Z/Z_0 - 1}{1 + Z/Z_0 + 1} = \frac{Z}{2Z_0 + Z} \quad ; \quad S_{12} = \frac{2}{1 + Z/Z_0 + 1} = \frac{2Z_0}{2Z_0 + Z}$$

$$1 - S_{11} = \frac{2Z_0}{2Z_0 + Z} = S_{12} \quad ✓$$



From Table 4.1, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$

convert to [S]:

$$S_{11} = \frac{1 - Z_0/Z - 1}{1 + Z_0/Z + 1} = \frac{-Z_0}{2Z + Z_0} \quad ; \quad S_{12} = \frac{2}{1 + Z_0/Z + 1} = \frac{2Z}{2Z + Z_0}$$

$$1 + S_{11} = \frac{2Z}{2Z + Z_0} = S_{12} \quad ✓$$

4.29

a) Using the S-parameters, the transmission coefficient from Port 1 to Port 4 is,

$$T = \frac{V_4^-}{V_1^+} = \frac{1}{V_1^+}\left(\frac{-1}{\sqrt{2}}\right)\left(V_2^+ + jV_3^+\right) = \frac{1}{V_1^+}\left(\frac{-1}{\sqrt{2}}\right)\left(\Gamma V_2^- + j\Gamma V_3^-\right)$$

$$= \frac{1}{V_1^+}\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right)(\Gamma)(j+j)V_1^+ = j\Gamma \checkmark$$

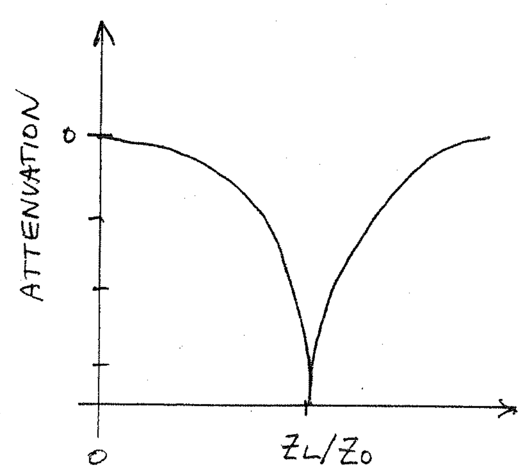
$$\text{attenuation} = |T| = |\Gamma| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

at port 1 the reflected wave is,

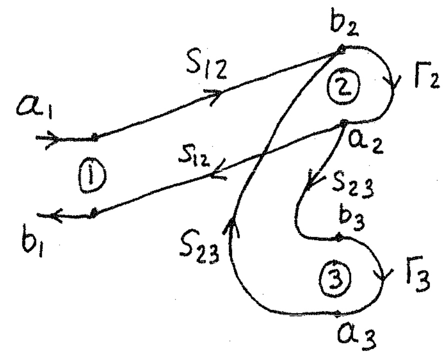
$$V_1^- = \frac{-1}{\sqrt{2}}\left(jV_2^+ + V_3^+\right) = \frac{-1}{\sqrt{2}}\Gamma\left(jV_2^- + V_3^-\right) = \frac{1}{2}\Gamma(-1+1)V_1^+ = 0 \checkmark$$

b)

Z_L/Z_0	atten. (dB)
0	0
0.172	3
1	∞
5.83	3
∞	0

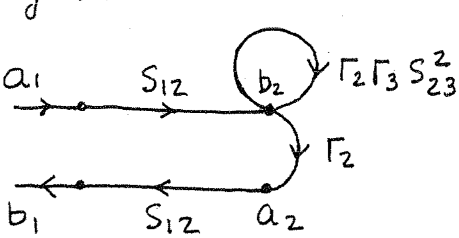


(4.30) The signal flowgraph is as follows:



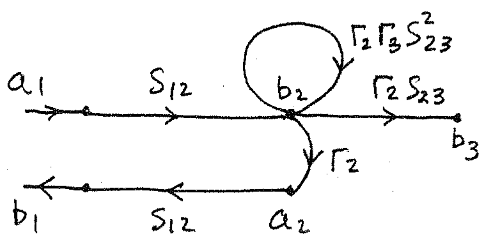
Let $\Gamma_{in} = \dfrac{b_1}{a_1}$

Using the reduction rules:



$$b_2 = a_1 \frac{S_{12}}{1-\Gamma_2\Gamma_3 S_{23}^2} \quad ✓$$

$$b_1 = a_1 \frac{S_{12}^2 \Gamma_2}{1-\Gamma_2\Gamma_3 S_{23}^2} \quad ✓$$



$$b_3 = b_2 \Gamma_2 S_{23}$$

Then,

$$\frac{P_2}{P_1} = \frac{b_2^2 - a_2^2}{a_1^2 - b_1^2} = \frac{b_2^2(1-|\Gamma_2|^2)}{a_1^2(1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2(1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2\left(1-\frac{|S_{12}^2|^2|\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)}$$

$$= \frac{|S_{12}|^2(1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2} \quad ✓$$

$$\frac{P_3}{P_1} = \frac{b_3^2 - a_3^2}{a_1^2 - b_1^2} = \frac{b_3^2(1-|\Gamma_3|^2)}{a_1^2(1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2|\Gamma_2 S_{23}|^2(1-|\Gamma_3|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2\left(1-\frac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)}$$

$$= \frac{|S_{12}|^2|S_{23}|^2|\Gamma_2|^2(1-|\Gamma_3|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2} \quad ✓$$

(verified by direct calculation using S-parameters)

(4.31)

$$Z_{oc} = -jZ_o \cot \beta\Delta \simeq \frac{-jZ_o}{\beta\Delta} = \frac{-jZ_o c}{\omega\sqrt{\epsilon_e}\,\Delta} = \frac{-j}{\omega C_f}$$

$$\therefore \ \Delta \simeq \frac{Z_o c C_f}{\sqrt{\epsilon_e}}$$ (agrees with T. Edwards, p. 123)

For $C_f = 0.075$ pF, $\epsilon_e = 1.894$, $Z_o = 50\,\Omega$,

this gives $\underline{\Delta = 0.082 \text{ cm}}$

(Using $\epsilon_r = 2.2$, $d = 0.158$ cm, $W = 0.487$ cm)

The Hammerstad & Bekkadal approximation gives

$$\Delta = 0.412d\left(\frac{\epsilon_e + .3}{\epsilon_e - .258}\right)\left(\frac{W + .262d}{W + .813d}\right) = \underline{0.075 \text{ cm}}$$

(4.32) The complex reflected power can be computed using (4.88):

$$P_r = \int_s \bar{E}^r \times \bar{H}^{r*} \cdot \hat{z}\, ds = -\int_{x=0}^{a}\int_{y=0}^{b} E_y^r H_x^{r*}\, dx\, dy$$

$$= -b\int_{x=0}^{a}\Big[\sum_n A_n \sin\frac{n\pi x}{a} e^{j\beta_n^a z}\Big]\Big[\sum_m \frac{A_m^*}{Z_m^{a*}} \sin\frac{m\pi x}{a} e^{-j\beta_m^{a*} z}\Big] dx$$

$$= \frac{-ab}{2}\sum_{n=1}^{\infty}\frac{|A_n|^2}{Z_n^{a*}} e^{j(\beta_n^a - \beta_n^{a*})z}$$

The only propagating mode is the $n=1$ (TE_{10}) mode, so β_1^a is real, and β_n^a is imaginary for $n>1$. Let $\alpha_n = j\beta_n = \sqrt{(n\pi/a)^2 - k_0^2}$ for $n>1$. Then $Z_1^a = k_0\eta_0/\beta_1^a$, and $Z_n^a = k_0\eta_0/\beta_n^a = j k_0\eta_0/\alpha_n$ for $n>1$.

Then $$P_r = \frac{-ab}{2}\left[\frac{|A_1|^2\beta_1^a}{k_0\eta_0} - j\sum_{n=2}^{\infty}\frac{|A_n|^2\alpha_n}{k_0\eta_0}e^{2\alpha_n z}\right] \quad \text{for } z<0.$$

So we see that $Im\{P_r\} > 0$, indicating an inductive load.

(4.33) This problem can be solved by setting up two equations for A_1 and A_2 using (4.97). But a general computer program had been written for Section 4.6, so this was used to obtain the results that $A_1 = 0.071\angle 50°$, and $A_2 = 0.29\angle -13°$. The computer program listing is shown on the following page. (FORTRAN program HPMODAL.FOR)

```
C MODAL ANALYSIS OF ASYMMETRIC H-PLANE STEP
       COMPLEX AM(100),QM(100,100),PM(100),ZKC,ZNA
       COMPLEX DET,XOZ0,XN,BTNA,BTKC
       DIMENSION LL(100),MM(100)
       PI=3.14159265
       Z0=377.
100    write(6,*) ' Enter lambda/a, c/a, N:'
       READ(6,*) A,C,NE
       XLAM=1.
       a=1./a
       c=c*a
       XK0=2.*PI/XLAM
C FILL ARRAYS
       BT1A=SQRT(XK0*XK0-(PI/A)**2)
       Z1A=XK0*Z0/BT1A
       DO 40 M=1,NE
       DO 10 N=1,NE
       BTNA=CSQRT((1.,0.)*XK0*XK0-(N*PI/A)**2)
       IF(AIMAG(BTNA).GT.0.) BTNA=-BTNA
       ZNA=XK0*Z0/BTNA
       QM(M,N)=(0.,0.)
       IF(M.EQ.N) QM(M,N)=A/2.
       DO 50 K=1,NE
       BTKC=CSQRT((1.,0.)*XK0*XK0-(K*PI/C)**2)
       IF(AIMAG(BTKC).GT.0.) BTKC=-BTKC
       ZKC=XK0*Z0/BTKC
       QM(M,N)=QM(M,N)+ZKC*2./C*XI(K,M,A,C)*XI(K,N,A,C)/ZNA
50     CONTINUE
10     CONTINUE
       PM(M)=(0.,0.)
       IF(M.EQ.1) PM(M)=-A/2.
       DO 60 K=1,NE
       BTKC=CSQRT((1.,0.)*XK0*XK0-(K*PI/C)**2)
       IF(AIMAG(BTKC).GT.0.) BTKC=-BTKC
       ZKC=XK0*Z0/BTKC
       PM(M)=PM(M)+ZKC*2./C*XI(K,M,A,C)*XI(K,1,A,C)/Z1A
60     CONTINUE
40     CONTINUE
C INVERT MATRIX AND COMPUTE A VECTOR (standard matrix inversion)
       CALL CMINV(QM,DET,LL,MM,100,NE)
       DO 20 M=1,NE
       AM(M)=(0.,0.)
       DO 20 N=1,NE
20     AM(M)=AM(M)+QM(M,N)*PM(N)
       write(6,*) ' A vector (mag, phase):'
       DO 30 N=1,NE
       AMM=CABS(AM(N))
       PHS=180.*BTAN2(AIMAG(AM(N)),REAL(AM(N)))/PI
30     WRITE(6,*) N,AMM,PHS
C      COMPUTE REACTANCE
       XOZ0=(0.,-1.)*(1.+AM(1))/(1.-AM(1))
       XLAMG=2.*PI/BT1A
       XN=XOZ0*XLAMG/(2.*A)
       WRITE(6,*) ' c/a=',C/A,'lambda/a=',XLAM/A,'Xn=',XN
       GOTO 100
200    CALL EXIT
       END
       FUNCTION XI(M,N,A,C)
       PI=3.14159265
       ARGM=PI*(M/C-N/A)
       ARGP=PI*(M/C+N/A)
       XIP=SIN(ARGP*C)/(2.*ARGP)
       XIM=C/2.
       IF(ABS(ARGM)*A.GT.1.E-6) XIM=SIN(ARGM*C)/(2.*ARGM)
       XI=XIM-XIP
       RETURN
       END
```

(4.34) This solution is essentially the same as the analysis in Section 4.6. Let $d = (a-c)/2$

$$E_y^i = \sin\frac{\pi x}{a}\, e^{-j\beta_1^a z} \qquad H_x^i = \frac{-1}{Z_1^a}\sin\frac{\pi x}{a}\, e^{-j\beta_1^a z}$$

$$E_y^r = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} A_n \sin\frac{n\pi x}{a}\, e^{j\beta_n^a z} \qquad H_x^r = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{A_n}{Z_n^a}\sin\frac{n\pi x}{a}\, e^{j\beta_n^a z}$$

$$E_y^t = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} B_n \sin\frac{n\pi}{c}(x-d)\, e^{-j\beta_n^c z} \qquad H_x^t = -\sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{B_n}{Z_n^c}\sin\frac{n\pi}{c}(x-d)\, e^{-j\beta_n^c z}$$

where $\beta_n^a = \sqrt{k_0^2 - (n\pi/a)^2}$, $\beta_n^c = \sqrt{k_0^2 - (n\pi/c)^2}$

The solution has the same form as (4.97):

$$\frac{a}{2}A_m + \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{k=1\\ \text{odd}}}^{\infty} \frac{2Z_k^c I_{km} I_{kn}}{c\, Z_n^a} A_n = \sum_{\substack{k=1\\ \text{odd}}}^{\infty} \frac{2Z_k^c I_{km} I_{k1}}{c\, Z_1^a} - \frac{a}{2}\delta_{m1}$$

for $m = 1, 3, 5 \ldots$,

and,
$$I_{mn} = \int_{x=d}^{d+c} \sin\frac{m\pi}{c}(x-d)\sin\frac{n\pi x}{a}\, dx$$

$$\delta_{mn} = \begin{cases} 1 & \text{for } m=n \\ 0 & \text{for } m\neq n \end{cases}$$

4.35 From (4.110) the source current is,

$$\bar{J}_s = \hat{x}\,\frac{2B^+ m\pi}{a}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} + \hat{y}\,\frac{2B^+ n\pi}{b}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

From Table 3.2, the transverse fields for ± traveling TM_{mn} modes are,

$$E_x = \frac{\mp j\beta m\pi}{k_c^2 a} C^{\pm}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$E_y = \frac{\mp j\beta n\pi}{k_c^2 b} C^{\pm}\sin\frac{m\pi x}{a}\cos\frac{m\pi y}{b} e^{\mp j\beta z}$$

$$H_x = \frac{j\omega\epsilon n\pi}{k_c^2 b} C^{\pm}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H_y = \frac{-j\omega\epsilon m\pi}{k_c^2 a} C^{\pm}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

where $C^{\pm}$ are the unknown amplitudes. At $z=0$, E_t is continuous, so $C^+ = -C^-$. Also, $\hat{z}\times(\bar{H}^+ - \bar{H}^-) = \bar{J}_s$, or $H_y^+ - H_y^- = J_{sx}$ and $-H_x^+ + H_x^- = J_{sy}$. So,

$$J_{sx}: \quad \frac{-j\omega\epsilon m\pi}{k_c^2 a}(C^+ - C^-) = 2B^+\frac{m\pi}{a} \Rightarrow C^+ = -C^- = \frac{k_c^2 B^+}{-j\omega\epsilon}$$

$$J_{sy}: \quad \frac{j\omega\epsilon n\pi}{k_c^2 b}(-C^+ + C^-) = 2B^+\frac{n\pi}{a} \Rightarrow C^+ = -C^- = \frac{k_c^2 B^+}{-j\omega\epsilon} \checkmark$$

Since these fields satisfy Maxwell's equations and the boundary conditions, they must form the unique solution.

(4.36) From (4.114), the souce current is,

$$\bar{M}_s = -\hat{x}\, 2B^+_{mn} \frac{n\pi}{b} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} + \hat{y}\, 2B^+_{mn} \frac{m\pi}{a} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

From Table 3.2, the transverse TM_{mn} fields for $\pm$ traveling waves can be written as,

$$E^{\pm}_x = \frac{-m\pi}{a} C^{\pm} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$E^{\pm}_y = \frac{-n\pi}{b} C^{\pm} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H^{\pm}_x = \frac{\pm\omega\epsilon}{\beta} \frac{n\pi}{b} C^{\pm} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H^{\pm}_y = \frac{\mp\omega\epsilon}{\beta} \frac{m\pi}{a} C^{\pm} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

where $C^{\pm}$ is the unknown amplitude. At $z=0$, $\bar{H}_t$ is continuous, so $C^+ = -C^-$. At $z=0$, $(\bar{E}^+ - \bar{E}^-)\times\hat{z} = \bar{M}_s$, so we have,

$$-E^+_x + E^-_x = M_{sy} \Rightarrow C^+ - C^- = 2B^+_{mn} \Rightarrow C^+ = -C^- = B^+_{mn}$$

$$E^+_y - E^-_y = M_{sx} \Rightarrow -C^+ + C^- = -2B^+_{mn} \Rightarrow C^+ = -C^- = B^+_{mn} \checkmark$$

Since these fields satisfy Maxwell's equations and the relevant boundary conditions, they must form the unique solution.

(4.37) Following Example 4.8:

$$\bar{J}(x,y,z) = I(y)\,\delta(x-a/2)\,\delta(z) \quad \text{for } 0<y<d.$$

$$\bar{e}_1 = \hat{y}\sin\frac{\pi x}{a}, \quad \bar{h}_1 = \frac{-\hat{x}}{Z_1}\sin\frac{\pi x}{a}, \quad Z_1 = k_0\eta_0/\beta,$$

From (4.119),
$$P_1 = \frac{ab}{Z_1}$$

From (4.118),

$$A_1^+ = \frac{-1}{P_1}\int_v \sin\frac{\pi x}{a}\, e^{j\beta_1 z}\, I(y)\,\delta(x-a/2)\,\delta(z)\,dx\,dy\,dz$$

$$= \frac{-I_0}{P_1}\int_{y=0}^{d}\frac{\sin k(d-y)}{\sin kd}\,dy \xrightarrow[(\text{let } w=d-y)]{} \frac{-I_0}{P_1\sin kd}\int_0^d \sin kw\,dw$$

$$= \frac{I_0 Z_1(\cos kd-1)}{kab\sin kd}$$

The total power flow in the TE_{10} mode is,

$$P = \frac{ab|A_1^+|^2}{2Z_1},$$

for both + and − traveling waves, since $|A_1^+| = |A_1^-|$.
Then the radiation resistance is,

$$R_{in} = \frac{2P}{I_0^2} = \frac{ab|A_1^+|^2}{I_0^2 Z_1} = \frac{Z_1}{ab}\,\frac{(1-\cos kd)^2}{k^2\sin^2 kd}.$$

$$= \frac{Z_1}{k^2ab}\,\frac{(2\sin^2\frac{kd}{2})^2}{4\sin^2\frac{kd}{2}\cos^2\frac{kd}{2}} = \frac{Z_1}{k^2ab}\tan^2\frac{kd}{2} \checkmark$$

(4.38) Following Example 4.8:

$$\bar{J}(x,y,z) = I\,\delta(z)\left[\delta(x-a/4) - \delta(x-3a/4)\right]\hat{y} \quad \text{for } 0<y<b$$

From Table 3.2,

TE$_{10}$: $\bar{e}_1 = \hat{y}\sin\frac{\pi x}{a}$ $\quad \bar{h}_1 = \frac{-\hat{x}}{Z_1}\sin\frac{\pi x}{a}$ $\quad Z_1 = k_0\eta_0/\beta_1$

TE$_{20}$: $\bar{e}_2 = \hat{y}\sin\frac{2\pi x}{a}$ $\quad \bar{h}_2 = \frac{-\hat{x}}{Z_2}\sin\frac{2\pi x}{a}$ $\quad Z_2 = k_0\eta_0/\beta_2$

$P_1 = ab/Z_1$ $\quad \beta_1 = \sqrt{k_0^2 - (\pi/a)^2}$

$P_2 = ab/Z_2$ $\quad \beta_2 = \sqrt{k_0^2 - (2\pi/a)^2}$

From (4.118):

$$A_1^+ = \frac{-1}{P_1}\int_v \bar{E}_1^- \cdot \bar{J}\,dv = -\frac{Ib}{P_1}\left(\sin\frac{\pi}{4} - \sin\frac{3\pi}{4}\right) = 0 \quad ✓$$

$$A_2^+ = \frac{-1}{P_2}\int_v \bar{E}_2^- \cdot \bar{J}\,dv = \frac{-Ib}{P_2}\left(\sin\frac{\pi}{2} - \sin\frac{3\pi}{2}\right) = \frac{-2Ib}{a}$$

Since the excitation has an odd symmetry about the center of the guide, it will only excite modes that have an electric field with an odd symmetry about $x=a/2$. This implies the TE$_{m0}$ modes, for m even, will be excited. The TE$_{10}$ mode is not excited.

(4.39) By image theory, the half-loop on the end wall can be replaced by a full loop without the endwall. For a small loop, the equivalent magnetic dipole moment is,

$$\bar{P}_m = \hat{z}\, I_o \pi r_o^2\, \delta(x)\delta(y-b/2)\delta(z)$$

$$\bar{M} = j\omega\mu_o \bar{P}_m$$

$$= \hat{z}\, j\omega\mu_o I_o \pi r_o^2\, \delta(x)\delta(y-b/2)\,\delta(z) \qquad V/m^2$$

For the TE_{10} mode,

$$\bar{e}_1 = \hat{y}\sin\frac{\pi x}{a}$$

$$\bar{h}_1 = \frac{-\hat{x}}{Z_1}\sin\frac{\pi x}{a}$$

$$h_{z1} = \frac{j\pi}{k_o\eta_o a}\cos\frac{\pi x}{a}$$

with $Z_1 = k_o\eta_o/\beta_1$, $P_1 = ab/Z_1$

From (4.122),

$$A_1^+ = \frac{1}{P_1}\int_v (-\bar{h}_1 + \hat{z}\, h_{z1})\cdot \bar{M}\, e^{j\beta_1 z}\, dv$$

$$= \frac{Z_1}{ab}\int_v h_{z1} M\, dv = \frac{-\pi^2 Z_1 I_o r_o^2}{a^2 b} = A_1^-$$

Stronger coupling can usually be obtained by coupling to the transverse field components — see Example 4.9 of the second edition.

(4.40)

FIRST SOLUTION: (all fields and currents are TE_{10})

$$E_y = B \sin\frac{\pi x}{a}\left[e^{-j\beta z} - e^{j\beta z}\right] = -2jB\sin\frac{\pi x}{a}\sin\beta z \qquad 0<z<d$$

$$H_x = \frac{B}{Z_1}\sin\frac{\pi x}{a}\left[-e^{-j\beta z} - e^{j\beta z}\right] = \frac{-2B}{Z_1}\sin\frac{\pi x}{a}\cos\beta z \qquad 0<z<d$$

This satisfies $E_y=0$ at $z=0$.

$$E_y = C\sin\frac{\pi x}{a}e^{-j\beta(z-d)} \qquad z>d$$

$$H_x = \frac{C}{Z_1}\sin\frac{\pi x}{a}e^{-j\beta(z-d)} \qquad z>d$$

at $z=d$, E_y is continuous, so

$$-2jB\sin\beta d = C$$

at $z=d$, $\hat{z}\times(\bar{H}^+ - \bar{H}^-) = \bar{J}_s$, or

$$\frac{C}{Z_1} + \frac{2B}{Z_1}\cos\beta d = \frac{2\pi A}{a}$$

Solving for B, C:

$$B = \frac{\pi Z_1 A}{a}e^{-j\beta d}, \qquad C = \frac{\pi Z_1 A}{a}\left(e^{-2j\beta d} - 1\right)$$

SECOND SOLUTION: (Using (4.105) and (4.106b)):

E_y due to J_{sy} at $z=d$:

$$E_y^{\pm} = \frac{-\pi Z_1 A}{a}\sin\frac{\pi x}{a}e^{\mp j\beta(z-d)}$$

E_y due to $-J_{sy}$ at $z=-d$:

$$E_y^{\pm} = \frac{\pi Z_1 A}{a}\sin\frac{\pi x}{a}e^{\mp j\beta(z+d)}$$

For $0<z<d$,

$$E_y = \frac{\pi Z_1 A}{a}\sin\frac{\pi x}{a}\left[e^{-j\beta(z+d)} - e^{j\beta(z-d)}\right] = \frac{-2j\pi Z_1 A}{a}e^{-j\beta d}\sin\frac{\pi x}{a}\sin\beta z \ \checkmark$$

For $z>d$,

$$E_y = \frac{\pi Z_1 A}{a}\sin\frac{\pi x}{a}\left[e^{-j\beta(z+d)} - e^{-j\beta(z-d)}\right] = \frac{-2j\pi Z_1 A}{a}\sin\beta d\sin\frac{\pi x}{a}e^{-j\beta z} \ \checkmark$$

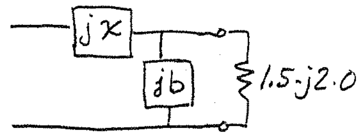
These results agree with those from the first solution.

Chapter 5

(5.1) Smith chart solutions, verified with PCAAD 5.0

a) $z_L = 1.5 - j2.0$

inside $1+jx$ circle

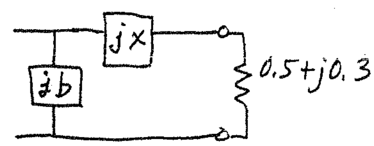


SOL'N #1 $b_1 = 0.107$ ✓ $x_1 = 1.78$ ✓

SOL'N #2 $b_2 = -0.747$ ✓ $x_2 = -1.78$ ✓

b) $z_L = 0.5 + j0.3$

outside $1+jx$ circle

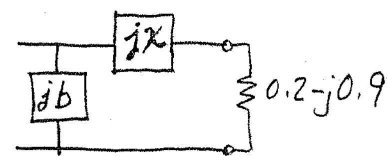


SOL'N #1 $x_1 = 0.20$ ✓ $b_1 = 1.00$ ✓

SOL'N #2 $x_2 = -0.80$ ✓ $b_2 = -1.00$ ✓

c) $z_L = 0.2 - j0.9$

outside $1+jx$ circle

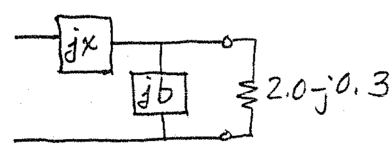


SOL'N #1 $x_1 = 1.30$ ✓ $b_1 = 2.00$ ✓

SOL'N #2 $x_2 = 0.50$ ✓ $b_2 = -2.00$ ✓

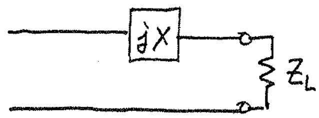
d) $z_L = 2.0 - j0.3$

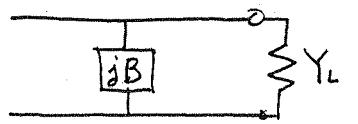
inside $1+jx$ circle



SOL'N #1 $b_1 = 0.427$ ✓ $x_1 = 1.02$ ✓

SOL'N #2 $b_2 = -0.573$ ✓ $x_2 = -1.02$ ✓

(5.2) a)  matching is possible if $Z_L = Z_0 - jX$ ✓

b)  matching is possible if $Y_L = \frac{1}{Z_0} - jB$ ✓

(5.3) Analytical Solutions:

From (5.9), $t = \dfrac{80 \pm \sqrt{100[(75-100)^2 + (80)^2]/75}}{100-75} = 3.2 \pm 3.871$

$t_1 = 7.071 \quad , \quad t_2 = -0.671$

From (5.10) the possible stub positions are,

$d_1 = \frac{\lambda}{2\pi} \tan^{-1} t_1 = 0.2276\,\lambda$ ✓

$d_2 = \frac{\lambda}{2\pi}(\pi + \tan^{-1} t_2) = 0.4059\,\lambda$ ✓

From (5.8b) the required stub susceptances are,

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0[R_L^2 + (X_L + Z_0 t)^2]}$$

$B_1 = 0.0129 \quad , \quad B_2 = -0.0129$

From (5.11a) the o.c. stub lengths are,

$\ell_1 = \frac{-\lambda}{2\pi} \tan^{-1}(B_1 Z_0) = 0.3776\,\lambda$ ✓ ($\lambda/2$ added to get $\ell_1 > 0$)

$\ell_2 = \frac{-\lambda}{2\pi} \tan^{-1}(B_2 Z_0) = 0.1224\,\lambda$ ✓

(5.4) Use B_1, B_2 from Problem 5.3 with (5.11b):

$\ell_1 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_1} = 0.1276\,\lambda$ ✓

$\ell_2 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_2} = 0.3724\,\lambda$ ✓ ($\lambda/2$ added to get $\ell_2 > 0$)

(5.5) Smith Chart Solutions (computerized Smith chart)

The normalized load impedance is $z_L = 0.6 - j0.8$

The stub positions and required reactances are,

$$d_1 = (0.5 - 0.375) + 0.1667 = 0.2917\lambda \checkmark \quad , \; x_{s1} = -1.155$$

$$d_2 = (0.5 - 0.375) + 0.3333 = 0.4583\lambda \checkmark \quad , \; x_{s2} = 1.155$$

The open-ckt stub lengths are,

$$l_1 = .3636 - .250 = 0.114\lambda \checkmark$$

$$l_2 = .25 + .1364 = 0.386\lambda \checkmark$$

(verified with PCAAD 5.0)

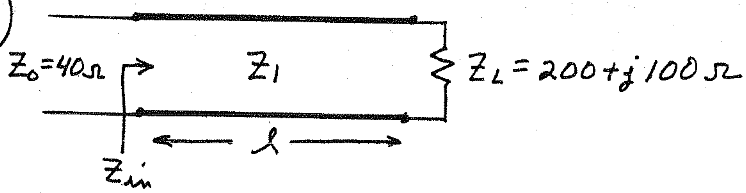
(5.6) The required stub lengths for s.c. stubs are $\lambda/4$ longer (or shorter) than the o.c. stub lengths:

$$l_1 = .114 + .25 = 0.364\lambda \checkmark$$

$$l_2 = .386 - .25 = 0.136\lambda \checkmark$$

(verified with PCAAD 5.0)

5.7



To match this load, we must find Z_1 and ℓ so that $Z_{in} = Z_0 = 40\Omega$:

$$Z_{in} = 40 = Z_1 \frac{(200 + j100) + jZ_1 t}{Z_1 + j(200 + j100)t} \quad , \text{ with } t = \tan\beta\ell.$$

$$(40Z_1 - 4000t) + j\,8000t = 200Z_1 + j(100 + Z_1 t)Z_1$$

Equating real and imaginary parts gives two equations for the two unknowns, Z_1 and t: (if they exist!)

$$\text{Re:} \quad 40Z_1 - 4000t = 200Z_1 \Rightarrow Z_1 = -25t$$

$$\text{Im:} \quad 8000t = Z_1(100 + Z_1 t)$$

$$8000t = -25t(100 - 25t^2)$$

$$t = \pm\sqrt{16.8} = \pm 4.10 \quad (\text{use } -4.10 \text{ so that } Z_1 > 0) \checkmark$$

Then, $\beta\ell = \tan^{-1}(-4.10) = -76.3° \equiv 104° \Rightarrow \underline{\ell = 0.288\lambda}$

The characteristic impedance is then,

$$Z_1 = -25(-4.10) = 102.5\,\Omega \quad \checkmark$$

(Note: Not all load impedances can be matched in this way – a good exam problem to determine which impedances can be matched using this technique!)

(5.8) From (2.91) the impedance of a terminated lossy line is,

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \quad , \quad \gamma l = \alpha l + j\beta l$$

For $Z_L = \infty$ (o.c.), the normalized input admittance is,

$$y_{in} = \tanh \gamma l = \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l}$$

The normalized input susceptance is,

$$b_{in} = \frac{\tan \beta l \,(1 - \tanh^2 \alpha l)}{1 + \tanh^2 \alpha l \tan^2 \beta l}$$

(at this point, we could find max. b_{in} by calculating b_{in} vs. l)

Since maximum susceptance for a lossless line is obtained for $\beta l = \pi/2$, we expect βl to be close to $\pi/2$ for the lossy case. So let $\beta l = \pi/2 + \Delta$, where Δ is small. Also, αl is small, so we have $\tanh \alpha l \simeq \alpha l$, and $\tan \beta l = -\cot \Delta \simeq -1/\Delta$.
Then,

$$b_{in} \simeq \frac{\frac{-1}{\Delta}(1 - \alpha^2 l^2)}{1 + \alpha^2 l^2/\Delta^2} \simeq \frac{-1}{\Delta + \alpha^2 l^2/\Delta}$$

To maximize b_{in}, we can minimize $\Delta + \alpha^2 l^2/\Delta$ with respect to l:

$$\frac{d}{dl}(\Delta + \alpha^2 l^2/\Delta) = \frac{d\Delta}{dl} + \frac{2\alpha^2 l}{\Delta} + \alpha^2 l^2 \left(\frac{-1}{\Delta^2}\right)\frac{d\Delta}{dl} = 0$$

or, since $\frac{d\Delta}{dl} = \beta$,

$$\beta + \frac{2\alpha^2 l}{\Delta} - \frac{\alpha^2 l^2}{\Delta^2}\beta = 0$$

since $\Delta = \beta l - \pi/2$, we have,

$$l^2 \beta(\alpha^2 + \beta^2) - \pi l(\alpha^2 + \beta^2) + \beta \frac{\pi^2}{4} = 0$$

Solve for l:

$$\ell = \frac{\pi(\alpha^2+\beta^2) \pm \sqrt{\pi^2(\alpha^2+\beta^2)^2 - \beta^2\pi^2(\alpha^2+\beta^2)}}{2\beta(\alpha^2+\beta^2)}$$

$$= \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta\sqrt{\alpha^2+\beta^2}} \simeq \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta^2} \qquad (\text{since } \alpha^2 << \beta^2)$$

Then,

$$\Delta = \beta\ell - \pi/2 \simeq \frac{\pi\alpha}{2\beta} \simeq \alpha\ell \qquad (\text{since } \beta \simeq \pi/2\ell)$$

The corresponding value of b_{in} is,

$$b_{in}^{MAX} = \frac{\pm 1}{\alpha\ell + \alpha\ell} = \frac{\pm 1}{2\alpha\ell} = \frac{\pm 2}{\alpha\lambda} \qquad (\text{since } \ell \simeq \lambda/4)$$

For $\alpha = 0.01$ neper/λ, $b_{in}^{MAX} = \frac{\pm 2}{.01} = \underline{\pm 200}$

(This checks with direct calculation of y_{in} vs. ℓ.)

The reactance of a short-circuited line is the dual case of the above problem, so $x_{in}^{MAX} = \pm 200$.

(5.9) Smith chart solution:

1. plot $y_L = 0.4 + j1.2$ on admittance chart
2. plot rotated $1+jb$ circle
3. add a stub susceptance of $j0.6$ or $-j1.0$ to move to rotated $1+jb$ circle
4. move $\lambda/8$ toward generator, to $1+jb$ circle
5. add a stub susceptance of $+j3.0$ or $-j1.0$ to move to center of chart.
6. the O.C. stub lengths are,

$$\ell_1 = 0.086\lambda \checkmark \quad \text{or} \quad \ell_1 = 0.375\lambda \checkmark$$

$$\ell_2 = 0.199\lambda \checkmark \quad \text{or} \quad \ell_2 = 0.375\lambda \checkmark$$

(see attached Smith chart for first solution)

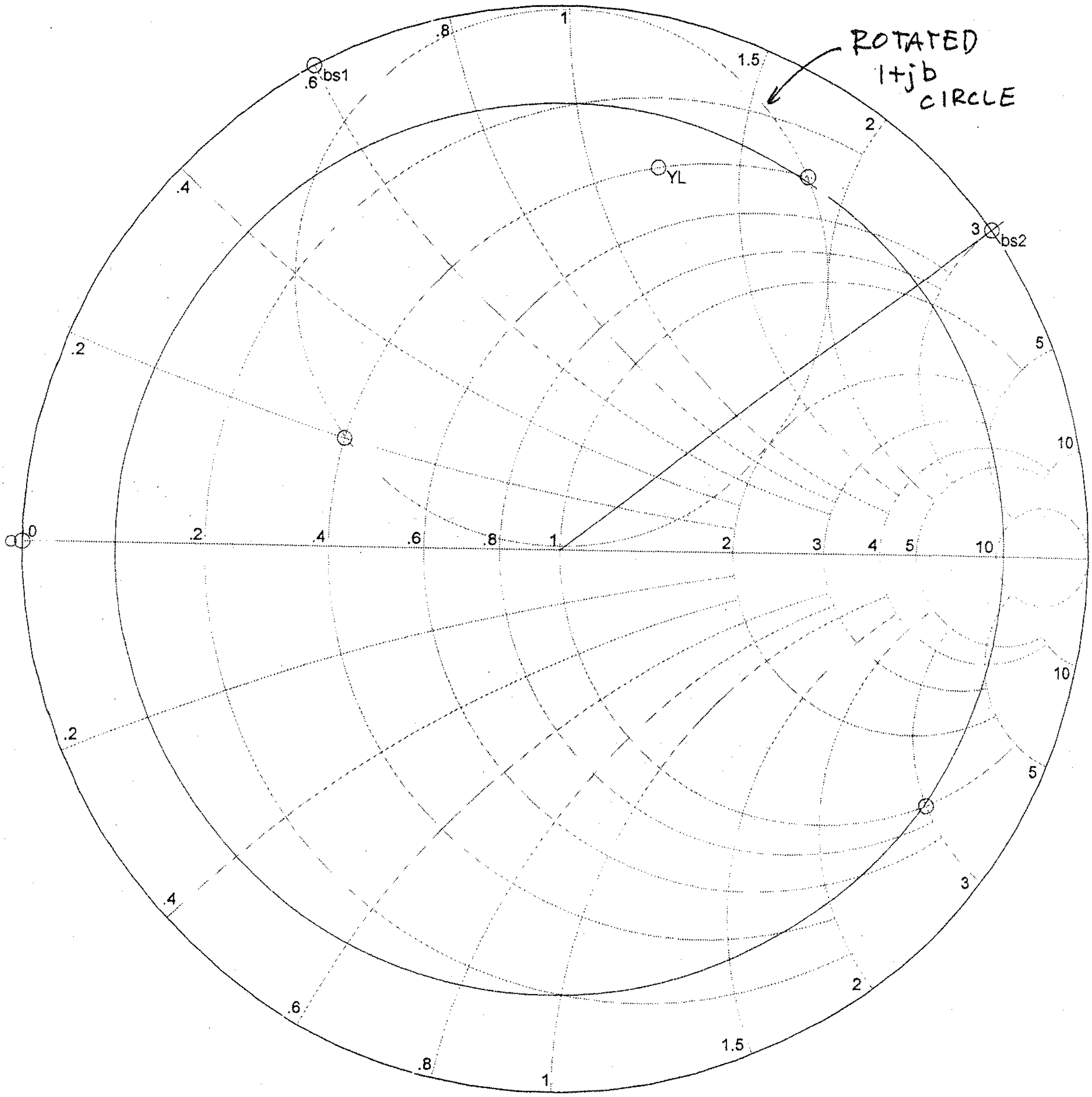
Analytic solution: $\quad t = \tan \beta d = 1$

$$b_1 = -b_L + 1 \pm \sqrt{2g_L - g_L^2} = 0.6,\ -1.0$$

$$b_2 = \frac{1}{g_L}\left[\pm\sqrt{2g_L - g_L^2} + g_L\right] = 3.0,\ -1.0$$

$$\ell_1 = \frac{\lambda}{2\pi}\tan^{-1} b_1 = 0.086\lambda \quad \text{or} \quad \ell_1 = 0.375\lambda$$

$$\ell_2 = \frac{\lambda}{2\pi}\tan^{-1} b_2 = 0.198\lambda \quad \text{or} \quad \ell_2 = 0.375\lambda$$



Smith chart for P. 5.9 (#1)

5.10 Analytic Solution: let $t = \tan\beta d = \tan 135° = -1.0$

From (5.22) the first stub susceptance is

$$b_1 = -b_L + \frac{1 \pm \sqrt{(1+t^2)g_L - g_L^2 t^2}}{t} = -3 \text{ or } -1.4 \checkmark$$

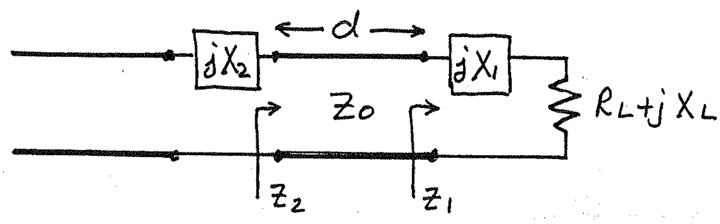
From (5.23) the second stub susceptance is

$$b_2 = \frac{\pm\sqrt{(1+t^2)g_L - g_L^2 t^2} + g_L}{g_L t} = -3 \text{ or } 1.0 \checkmark$$

The S.C. stub lengths are, from (5.24b),

$$\ell_1 = 0.051\lambda \quad \text{or } 0.0987\lambda$$
$$\ell_2 = 0.051\lambda \quad \text{or } 0.375\lambda$$

5.11



$$Z_1 = R_L + j(X_L + X_1)$$

$$Z_2 = Z_0 \frac{R_L + j(X_L + X_1 + Z_0 t)}{Z_0 + jt(R_L + jX_L + jX_1)} = Z_0 \qquad t = \tan\beta d$$

Solving for R_L:

$$R_L = Z_0 \frac{1+t^2}{2t^2}\left[1 \pm \sqrt{1 - \frac{4t^2(Z_0 - X_L t - X_1 t)^2}{Z_0(1+t^2)^2}}\right]$$

So we must have,

$$0 \le R_L \le Z_0 \frac{1+t^2}{2t^2} = \frac{Z_0}{\sin^2\beta d}$$

The first stub reactance is,

$$X_1 = -X_L + \frac{Z_0 \pm \sqrt{(1+t^2)R_L Z_0 - R_L^2 t^2}}{t}$$

The second stub reactance is,

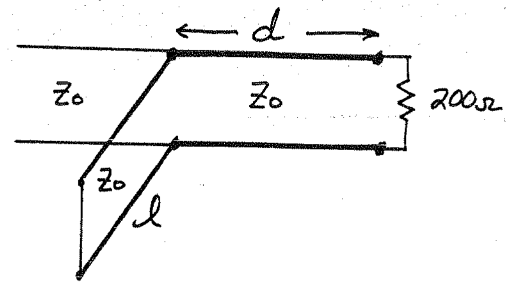
$$X_2 = \frac{\pm Z_0\sqrt{Z_0 R_L(1+t^2) - R_L^2 t^2} + R_L Z_0}{R_L t}$$

The stub lengths are given by,

$$\ell_{oc} = \frac{1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X}\right), \qquad \ell_{sc} = \frac{1}{2\pi}\tan^{-1}\left(\frac{X}{Z_0}\right)$$

5.12 Using the Smith chart ($Z_0 = 100\,\Omega$)

a) a single short-circuited shunt stub:



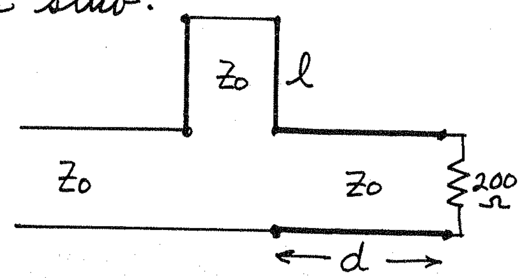
at f_0, $d_1 = 0.152\lambda$ ✓ $d_2 = 0.348\lambda$ ✓

$b_1 = -0.7$ $b_2 = +0.7$

$\ell_1 = 0.153\lambda$ ✓ $\ell_2 = 0.347$ ✓

$|\Gamma_1| = 0$ $|\Gamma_2| = 0$

b) a single short-circuited series stub:



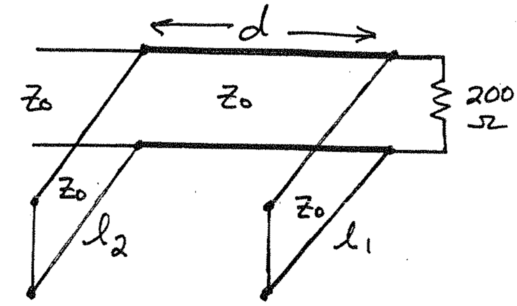
at f_0, $d_1 = 0.098\lambda$ ✓ $d_2 = 0.402\lambda$ ✓

$X_1 = 0.7$ $X_2 = -0.7$

$\ell_1 = 0.097\lambda$ ✓ $\ell_2 = 0.403\lambda$ ✓

$|\Gamma_1| = 0$ $|\Gamma_2| = 0$

c) a double short-circuited shunt stub: (let $d = \lambda/8$)



at f_0,

$b_1 = 0.14$ $b_1' = 1.85$

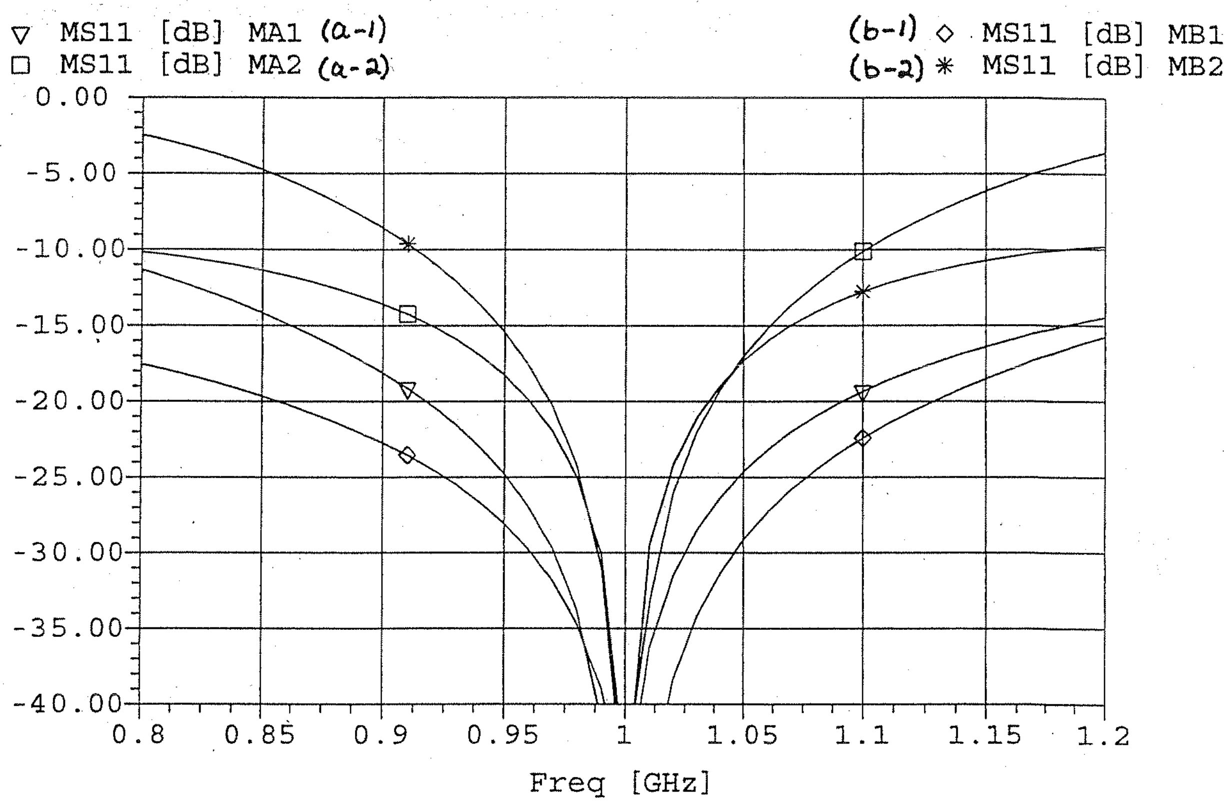
$\ell_1 = 0.272\lambda$ ✓ $\ell_1' = 0.421\lambda$ ✓

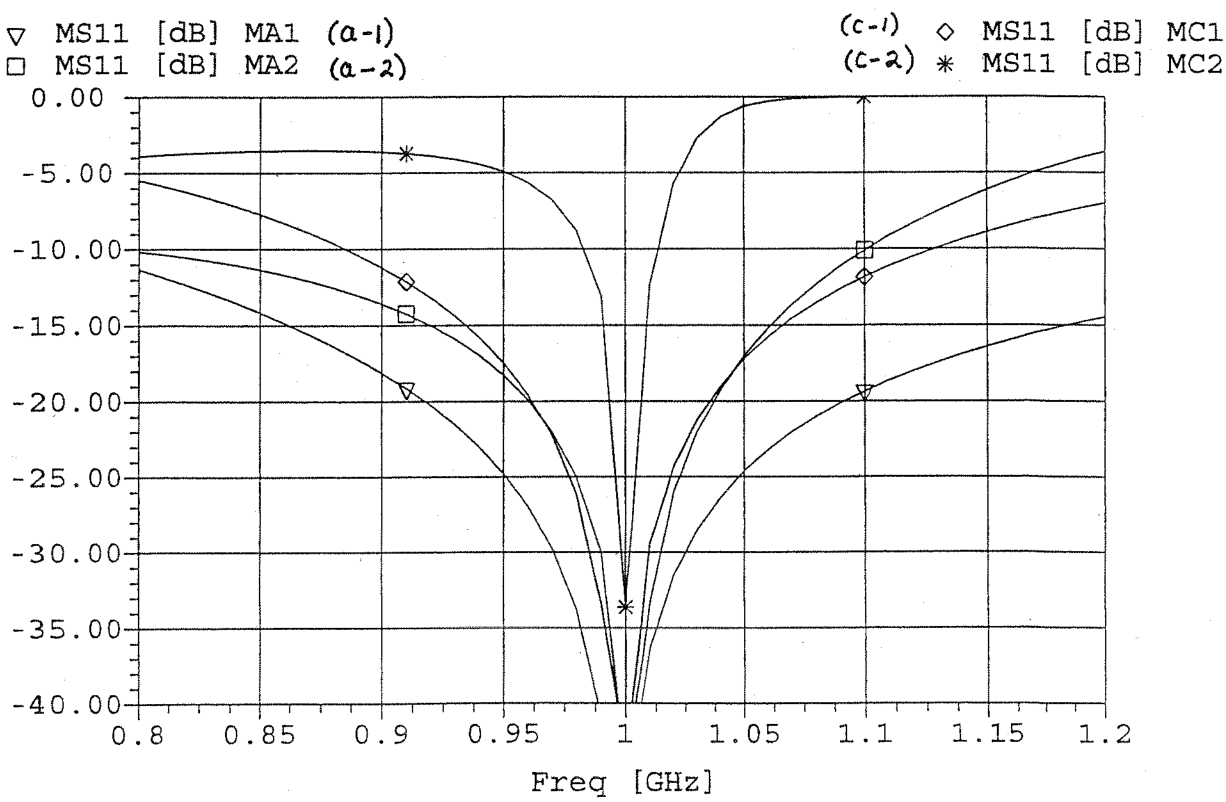
$b_2 = -0.73$ $b_2' = 2.75$

$\ell_2 = 0.15\lambda$ ✓ $\ell_2' = 0.444\lambda$ ✓

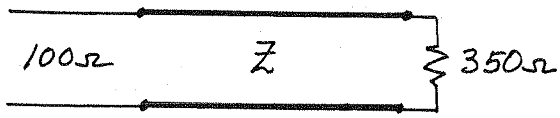
$|\Gamma| = 0$ $|\Gamma'| = 0$

Plots of return loss vs. f/f for these six solutions are shown on the following page. (only 4 curves could be plotted per graph). These results show that the tuner of solution (b-1), the series stub tuner, gives the best bandwidth. This is probably because the stub length and line length are shortest for this case, giving the smallest frequency variation.


▽ MS11 [dB] MA1 (a-1)
□ MS11 [dB] MA2 (a-2)
(b-1) ◇ MS11 [dB] MB1
(b-2) ✳ MS11 [dB] MB2
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
0.8 0.85 0.9 0.95 1 1.05 1.1 1.15 1.2
Freq [GHz]


▽ MS11 [dB] MA1 (a-1)
□ MS11 [dB] MA2 (a-2)
(c-1) ◇ MS11 [dB] MC1
(c-2) ✳ MS11 [dB] MC2
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
0.8 0.85 0.9 0.95 1 1.05 1.1 1.15 1.2
Freq [GHz]

(5.13)



$Z = \sqrt{100(350)} = 187.1\Omega$ ✓

An SWR of 2 corresponds to a reflection coefficient magnitude of,

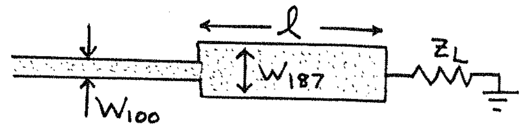
$$\Gamma_m = \frac{S-1}{S+1} = 1/3$$

Then from (5.33) the bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\ \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] = 71\%$$

MICROSTRIP LAYOUT:

$\epsilon_r = 2.2$, $d = 0.159$ cm, $f = 4$ GHz



First try $w/d < 2$:

for W_{100}, $A_{100} = 2.213$, $W_{100}/d = 0.896 < 2$ (OK), $W_{100} = 0.142$ cm

for W_{187}, $A_{187} = 4.047$, $W_{187}/d = 0.140 < 2$ (OK), $W_{187} = 0.022$ cm

From (3.195), ϵ_e for W_{187} is $\epsilon_e = 1.66$.

Then the physical length of the $\lambda/4$ transformer is,

$$l = \frac{\lambda_g}{4} = \frac{c}{4\sqrt{\epsilon_e}\, f} = 1.455 \text{ cm}$$ ✓

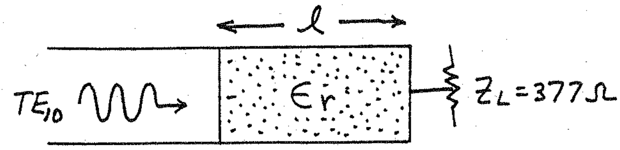
(5.14)

From (5.34) and (5.36), the partial reflection coefficients are,

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{150-100}{150+100} = 0.2\ ; \quad \Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2} = \frac{225-150}{225+150} = 0.2$$

Since the approximate expression for Γ in (5.42) is identical to the numerator for the exact expression in (5.41), the greatest error will occur when the denominator of (5.41) departs from unity to the greatest extent. This occurs for $\theta = 0$ or $180°$. Then (5.41) gives the exact Γ as 0.384, while (5.42) gives the approximate $\Gamma = 0.4$. Thus the error is about 4%.

5.15



$a = 2.286$ cm

$k_o = \frac{2\pi f}{c} = 209.4 \ m^{-1}$

In the air-filled guide,

$$\beta_a = \sqrt{k_o^2 - (\pi/a)^2} = 158.0 \ m^{-1} \ ✓$$

$$Z_a = \frac{k_o \eta_o}{\beta_a} = \frac{(209.4)(377)}{158} = 499.6 \ \Omega \ ✓$$

So the matching section impedance must be,

$$Z_m = \sqrt{Z_a Z_L} = \sqrt{(499.6)(377)} = 434.0 \ \Omega$$

$$= \frac{k_m \eta_m}{\beta_m} = \frac{k_o \eta_o}{\beta_m}$$

So the propagation constant of the matching section must be,

$$\beta_m = \frac{k_o \eta_o}{Z_m} = \frac{(209.4)(377)}{434} = 181.9 \ m^{-1}$$

$$= \sqrt{\epsilon_r k_o^2 - (\pi/a)^2}$$

Solving for ϵ_r:

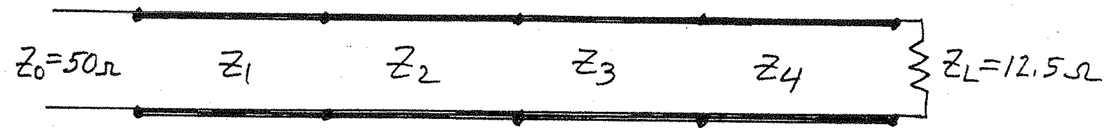
$$\epsilon_r = \frac{\beta_m^2 + (\pi/.02286)^2}{(209.4)^2} = \underline{1.185} \ ✓$$

The physical length of the matching section is,

$$\ell = \frac{\lambda_g}{4} = \frac{2\pi}{4\beta_m} = \frac{\pi}{2\beta_m} = 0.86 \text{ cm} \ ✓$$

(Note that this type of matching is not possible if $Z_L > Z_a$.)

5.16



a) Using (5.53):

$n=0$: $\ln Z_1/Z_0 = 2^{-4} C_0^4 \ln 12.5/50 \Rightarrow Z_1 = 45.85\Omega$

$n=1$: $\ln Z_2/Z_1 = 2^{-4} C_1^4 \ln 12.5/50 \Rightarrow Z_2 = 32.42\Omega$

$n=2$: $\ln Z_3/Z_2 = 2^{-4} C_2^4 \ln 12.5/50 \Rightarrow Z_3 = 19.28\Omega$

$n=3$: $\ln Z_4/Z_3 = 2^{-4} C_3^4 \ln 12.5/50 \Rightarrow Z_4 = 13.63\Omega$

Check: $n=4$: $\ln Z_5/Z_4 = 2^{-4} C_4^4 \ln 12.5/50 \Rightarrow Z_5 = 12.50\Omega = Z_L$ ✓

Can also check with data in Table 5.1, using $Z_L/Z_0 = 4$, which gives $Z_1 = 13.65\Omega$, $Z_2 = 19.30\Omega$, $Z_3 = 32.38\Omega$, $Z_4 = 45.79\Omega$
(source and load are reversed in this case)

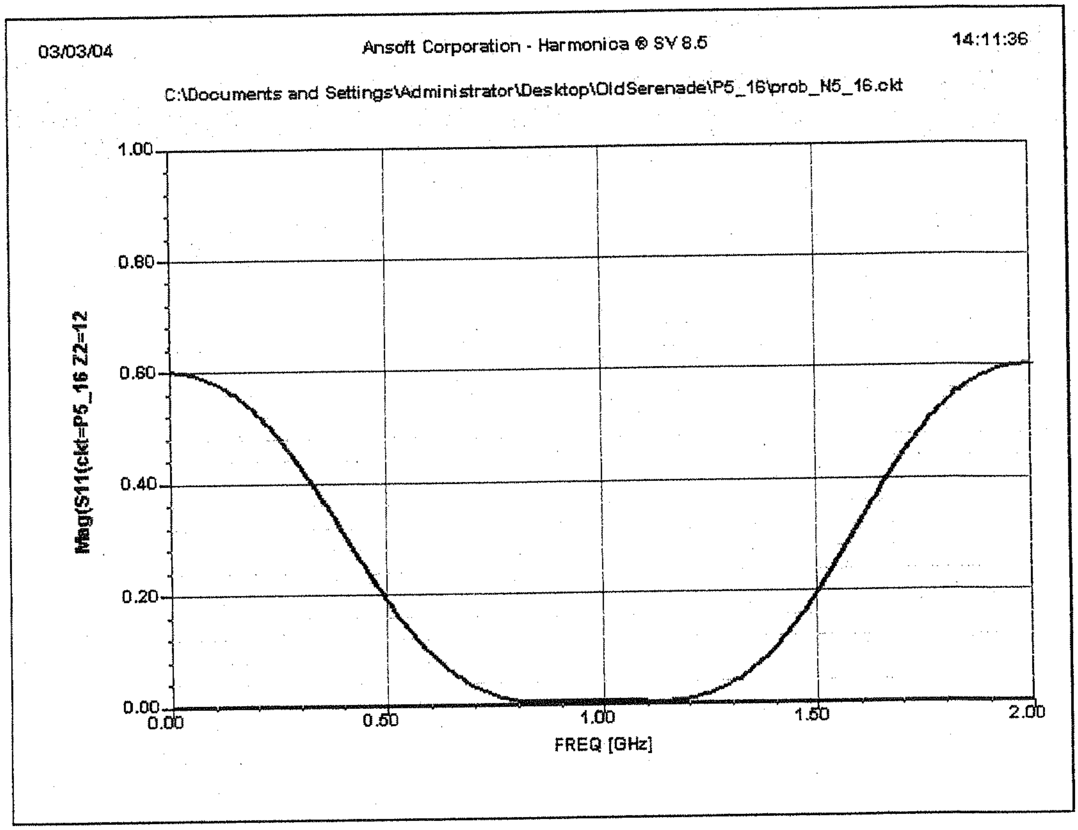
From (5.55), $A \simeq \frac{1}{2^{N+1}} \ln Z_L/Z_0 = -0.0433$

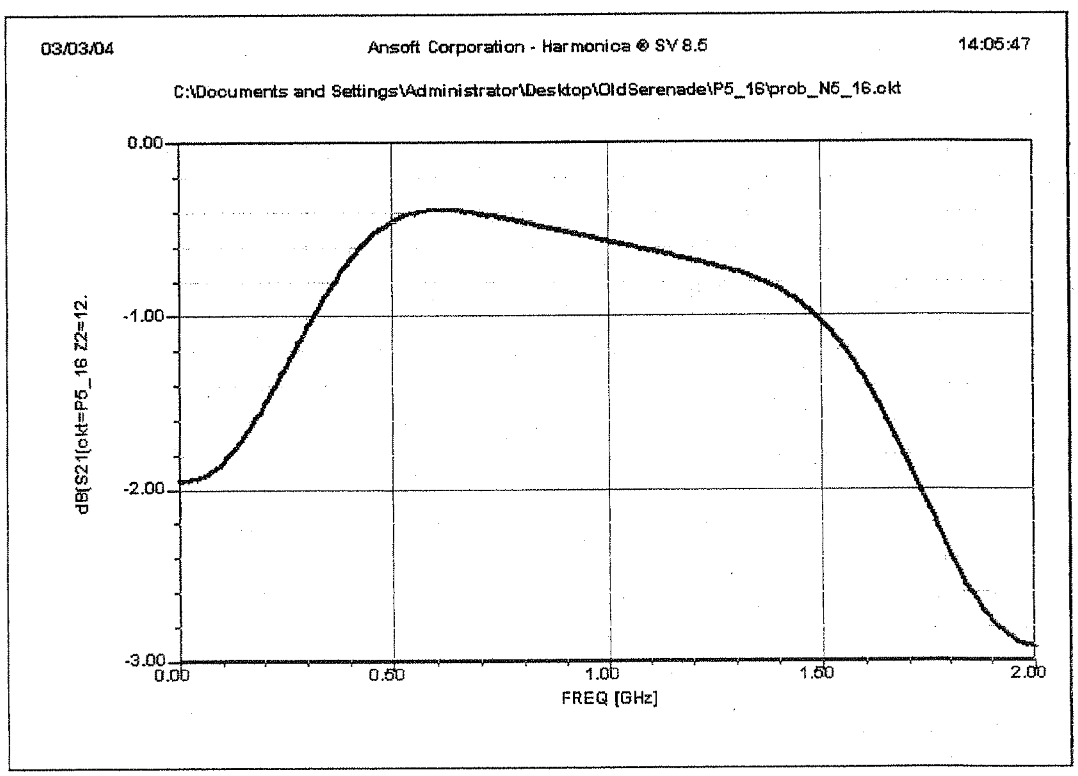
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{1}{2}\left|\frac{\Gamma_m}{A}\right|^{1/N}\right] = 69\% \quad \text{(agrees with plot)}$$

b) Microstrip line widths and lengths:

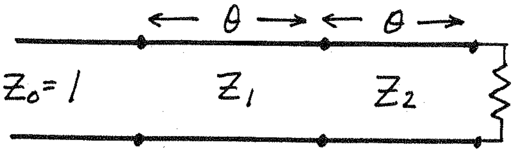
Z_C	W (cm)	ϵ_e	$\lambda_g/4$ (cm)
45.85	0.356	3.239	4.16
32.42	0.597	3.402	4.06
19.28	1.175	3.627	3.94
13.63	1.781	3.756	3.87

Results from Serenade modeling of parts a) and b) are shown on the following page. Note the good match, and the insertion loss of about 0.5dB.


03/03/04
Ansoft Corporation - Harmonica ® SV 8.5
14:11:36
C:\Documents and Settings\Administrator\Desktop\OldSerenade\P5_16\prob_N5_16.ckt
Mag(S11(ckt=P5_16 Z2=12
1.00
0.80
0.60
0.40
0.20
0.00
0.00
0.50
1.00
1.50
2.00
FREQ [GHz]


03/03/04
Ansoft Corporation - Harmonica ® SV 8.5
14:05:47
C:\Documents and Settings\Administrator\Desktop\OldSerenade\P5_16\prob_N5_16.ckt
dB(S21(ckt=P5_16 Z2=12.
0.00
-1.00
-2.00
-3.00
0.00
0.50
1.00
1.50
2.00
FREQ [GHz]

5.17



From (5.50) the desired input reflection coefficient response is (N=2):

$$\Gamma(\theta) = 2A(1+\cos 2\theta)$$

From the above circuit, we have that $\Gamma(0) = \frac{R-1}{R+1} = 0.2$, so $A = 0.2/4 = 0.05$.

Now we calculate the input reflection coefficient of the above circuit using ABCD matrices and conversion to S-parameters:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_1 \sin\theta \\ jY_1 \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_2 \sin\theta \\ jY_2 \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - Z_1 Y_2 \sin^2\theta & j(Z_1+Z_2)\cos\theta\sin\theta \\ j(Y_1+Y_2)\sin\theta\cos\theta & \cos^2\theta - Y_1 Z_2 \sin^2\theta \end{bmatrix}$$

Using Table 4.2 to convert to S-parameters gives the input reflection coefficient as,

$$\Gamma(\theta) = S_{11} + \frac{S_{12}S_{21}\Gamma_\ell}{1 - S_{22}\Gamma_\ell} = \frac{A+B-C-D}{S} + \frac{4\Gamma_\ell/S^2}{1 - \frac{-A+B-C+D}{S}\Gamma_\ell}$$

$$= \frac{(A+B-C-D)[S - \Gamma_\ell(-A+B-C+D)] + 4\Gamma_\ell}{S[S - \Gamma_\ell(-A+B-C+D)]}$$

where $S = A+B+C+D$, $\Gamma_\ell = \frac{R-1}{R+1}$.

This result can be equated to $2A(1+\cos 2\theta)$, and solved for Z_1 and Z_2, but this is a <u>very</u> lengthy procedure. Instead, we will first evaluate both expressions at $\theta = 90°$:

$\Gamma(90°) = 0$, and $\begin{bmatrix} A & B \\ C & D \end{bmatrix}\Bigg|_{\theta=90°} = \begin{bmatrix} -Z_1Y_2 & 0 \\ 0 & -Y_1Z_2 \end{bmatrix}$

So $\Gamma(\theta)$ reduces to the following equation:

$$(-Z_1Y_2 + Y_1Z_2)[-(Y_1Z_2 + Z_1Y_2) - \Gamma_\ell (Z_1Y_2 - Y_1Z_2)] + 4\Gamma_\ell = 0$$

$$(Z_1^2Y_2^2 - Y_1^2Z_2^2) + \Gamma_\ell (Z_1^2Y_2^2 + Y_1^2Z_2^2 + 2) = 0$$

$$(Z_1^4 - Z_2^4) + \Gamma_\ell (Z_1^4 + Z_2^4 + 2Z_1^2Z_2^2) = 0$$

$$(Z_1^2 - Z_2^2) + \Gamma_\ell (Z_1^2 + Z_2^2) = 0$$

$$Z_2^2 = Z_1^2 \frac{1+\Gamma_\ell}{1-\Gamma_\ell} = Z_1^2 R \Rightarrow \underline{\underline{Z_2 = Z_1\sqrt{R}}} \checkmark \quad (\text{for } Z_0 = 1)$$

Another equation is harder to find, so we will make use of the fact that the transformer will be symmetric:

$$\frac{Z_1 - 1}{Z_1 + 1} = \frac{R - Z_2}{R + Z_2} = \frac{R/Z_2 - 1}{R/Z_2 + 1}$$

Thus, $\frac{R}{Z_2} = Z_1$ or $\underline{\underline{Z_1 = R^{1/4}}}$ (for $Z_0 = 1$)

If $R = 1.5$, these results reduce to,

$$Z_1 = (1.5)^{1/4} = 1.1067 \checkmark$$

$$Z_2 = 1.1067\sqrt{1.5} = 1.3554 \checkmark$$

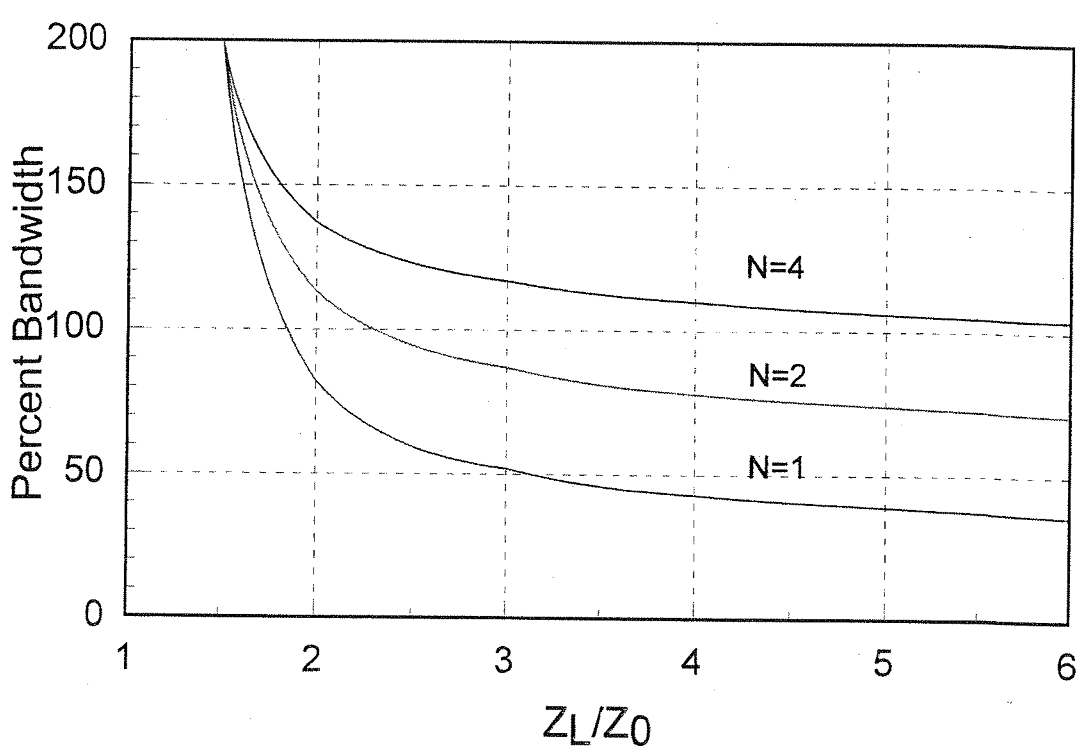
which agree with Table 5.1

(5.18) From (5.55), the fractional bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right], \text{ with } A = 2^{-N}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

Z_L/Z_0	N=1		N=2		N=4	
	A	$\Delta f/f_0$-%	A	$\Delta f/f_0$-%	A	$\Delta f/f_0$-%
1.5	0.1000	200	0.0500	200	0.0125	200
2.0	0.1667	82	0.0833	113	0.0208	137
3.0	0.2500	52	0.1250	87	0.0313	117
4.0	0.3000	43	0.1500	78	0.0375	110
5.0	0.3333	39	0.1667	74	0.0417	106
6.0	0.3571	36	0.1786	71	0.0446	104

This data is plotted in the graph below.



5.19

$$x = \sec\theta_m \cos\theta$$

$n=1$: $T_1(x) = x = \sec\theta_m \cos\theta$ (5.60a) ✓

$n=2$: $T_2(x) = 2x^2-1 = 2\sec^2\theta_m \cos^2\theta - 1$

$$= 2\sec^2\theta_m \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta\right) - 1$$

$$= \sec^2\theta_m (1+\cos 2\theta) - 1$$ (5.60b) ✓

$n=3$: $T_3(x) = 4x^3 - 3x = 4\sec^3\theta_m \cos^3\theta - 3\sec\theta_m \cos\theta$

$$= \sec^3\theta_m (3\cos\theta + \cos 3\theta) - 3\sec\theta_m \cos\theta$$ (5.60c) ✓

$n=4$: $T_4(x) = 8x^4 - 8x^2 + 1$

$$= 8\sec^4\theta_m \cos^4\theta - 8\sec^2\theta_m \cos^2\theta + 1$$

$$= \sec^4\theta_m (3 + 4\cos 2\theta + \cos 4\theta) - 4\sec^2\theta_m (1+\cos 2\theta) + 1$$

(5.60d) ✓

5.20 $S_m = 1.2 \Rightarrow |\Gamma_m| = \frac{S_m - 1}{S_m + 1} = 0.091 = A$

From (5.61),

$$\Gamma(\theta) = 2e^{-j4\theta}\left[\Gamma_0 \cos 4\theta + \Gamma_1 \cos 2\theta + \tfrac{1}{2}\Gamma_2\right]$$

$$= Ae^{-j4\theta}\, T_4(\sec\theta_m \cos\theta)$$

$$= Ae^{-j4\theta}\left[\sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m(\cos 2\theta + 1) + 1\right]$$

From (5.63),

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\left|\frac{\ln Z_L/Z_0}{2\Gamma_m}\right|\right)\right]$$

$$= \cosh\left[\frac{1}{4}\cosh^{-1}\left(\frac{1}{2(.091)}\ln\frac{60}{40}\right)\right] = 1.0655$$

so, $\theta_m = \cos^{-1}(1/1.0655) = 20°$

Equating $\cos 4\theta$ terms:

$$2\Gamma_0 = A\sec^4\theta_m \Rightarrow \Gamma_0 = 0.0586 = \Gamma_4$$

Equating $\cos 2\theta$ terms:

$$2\Gamma_1 = A(4\sec^4\theta_m - 4\sec^2\theta_m) \Rightarrow \Gamma_1 = 0.02795 = \Gamma_3$$

Equating constant terms:

$$\Gamma_2 = A(3\sec^4\theta_m - 4\sec^2\theta_m + 1) = 0.0296$$

Then the characteristic impedances are,

$$Z_1 = Z_0\frac{1+\Gamma_0}{1-\Gamma_0} = 44.98\,\Omega \checkmark$$

$$Z_2 = Z_1\frac{1+\Gamma_1}{1-\Gamma_1} = 47.57\,\Omega \checkmark$$

$$Z_3 = Z_2\frac{1+\Gamma_2}{1-\Gamma_2} = 50.47\,\Omega \checkmark$$

$$Z_4 = Z_3\frac{1+\Gamma_3}{1-\Gamma_3} = 53.37\,\Omega \checkmark$$

CHECK: $Z_5 = Z_4\frac{1+\Gamma_4}{1-\Gamma_4} = 60.01\,\Omega = Z_L \checkmark$

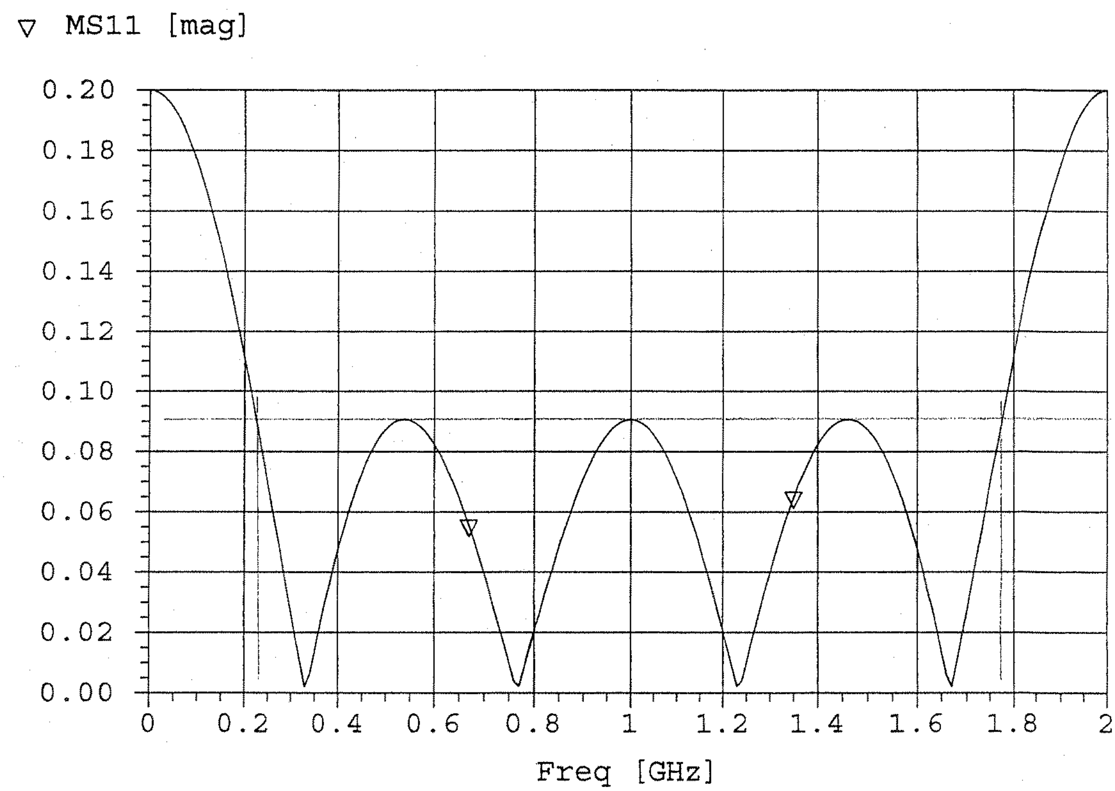
From (5.64) the bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4(20)}{180} = 156\% \checkmark$$

The calculated reflection coefficient magnitude versus frequency is shown below. The bandwidth from this graph gives a value of approximately,

$$\frac{\Delta f}{f_0} = \frac{1.78 - .23}{1} \times 100 = 155\%$$

in close agreement with the above calculation.



(5.21) From (5.61) and (5.60b),

$$\Gamma(\theta) = A e^{-2j\theta} T_2(\sec\theta_m \cos\theta) = A e^{-2j\theta}\left[\sec^2\theta_m (1+\cos 2\theta) - 1\right]$$

$$\Gamma(0) = A\, T_2(\sec\theta_m) = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \frac{R-1}{R+1} = 0.2 \quad ; \quad A = \Gamma_m = 0.05$$

As in Problem 5.17, we will evaluate $\Gamma(\theta)$ for $\theta = 90°$. Then $\Gamma(90°) = \Gamma_m$. Also, as in Problem 5.17, from symmetry we have that $Z_1 Z_2 = R$. Then,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -Z_1^2/R & 0 \\ 0 & -R/Z_1^2 \end{bmatrix} \qquad (Z_0 = 1)$$

$$\Gamma(90°) = \Gamma_m = \frac{(-Z_1^2/R + R/Z_1^2)\left[-(Z_1^2/R + R/Z_1^2) - \Gamma_\ell (Z_1^2/R - R/Z_1^2)\right] + 4\Gamma_\ell}{-(Z_1^2/R + R/Z_1^2)\left[-(Z_1^2/R + R/Z_1^2) - \Gamma_\ell (Z_1^2/R - R/Z_1^2)\right]}$$

$$= \frac{(R^2 - Z_1^4)\left[-(Z_1^4 + R^2) - \Gamma_\ell (Z_1^4 - R^2)\right] + 4\Gamma_\ell R^2 Z_1^4}{(Z_1^4 + R^2)\left[(Z_1^4 + R^2) + \Gamma_\ell (Z_1^4 - R^2)\right]}$$

$$\Gamma_m (Z_1^4 + R^2)^2 + \Gamma_m \Gamma_\ell (Z_1^4 + R^2)(Z_1^4 - R^2) = -(R^2 - Z_1^4)(Z_1^4 + R^2) + \Gamma_\ell (Z_1^4 - R^2)^2 + 4\Gamma_\ell R^2 Z_1^4$$

$$Z_1^8 (\Gamma_m - 1)(\Gamma_\ell + 1) + 2 Z_1^4 R^2 (\Gamma_m - \Gamma_\ell) - R^4 (\Gamma_m + 1)(\Gamma_\ell - 1) = 0$$

For $\Gamma_m = 0.05$, $\Gamma_\ell = 0.2$, $R = 1.5$:

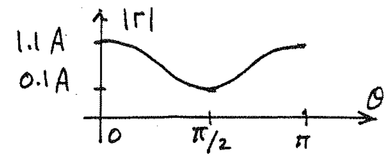
$$-1.140 Z_1^8 - 0.6750 Z_1^4 + 4.2525 = 0$$

$$Z_1^4 = \frac{0.675 \pm 4.455}{-2.280} = 1.65789 \Rightarrow Z_1 = 1.1347 Z_0 \checkmark$$

$$Z_2 = R/Z_1 = 1.3219 Z_0 \checkmark$$

These results agree with Table 5.2.

(5.22) $|\Gamma(\theta)| = A(0.1 + \cos^2\theta)\ ,\quad 0 < \theta < \pi$



From (5.46a), for $N=2$,

$$|\Gamma(\theta)| = 2(\Gamma_0 \cos 2\theta + \tfrac{1}{2}\Gamma_1) = A(0.1 + \cos^2\theta)$$

$$= A(0.6 + 0.5\cos 2\theta)$$

When $\theta = 0$, $\quad |\Gamma(0)| = 1.1A = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{1.5-1}{1.5+1} = 0.2 \Rightarrow A = 0.182$

Equating coefficients of $\cos 2\theta$:

$$2\Gamma_0 = 0.5A \quad \Rightarrow \quad \Gamma_0 = 0.0455$$

Equating constant terms:

$$\Gamma_1 = 0.6A = 0.109$$

so the characteristic impedances are,

$$Z_1 = Z_0 \frac{1+\Gamma_0}{1-\Gamma_0} = 1.095\, Z_0$$

$$Z_2 = Z_1 \frac{1+\Gamma_1}{1-\Gamma_1} = 1.245\, Z_1 = 1.363\, Z_0$$

CHECK: at $\theta = \pi/2$, the input impedance to the transformer will be,

$$Z_{in} = \frac{Z_1^2}{(Z_2^2/Z_L)} = \frac{Z_L Z_0 Z_1^2}{Z_2^2} = 0.968\, Z_0$$

So the input reflection coefficient is,

$$\Gamma_{in} = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right| = 0.016$$

which is reasonably close to $|\Gamma(\pi/2)| = 0.1A = 0.018$

(5.23)

$$\frac{d(\ln Z/Z_0)}{dz} = A \sin\frac{\pi z}{L}$$

$$\ln(Z/Z_0) = B - \frac{LA}{\pi}\cos\frac{\pi z}{L}$$

$$Z(z) = C e^{-\frac{LA}{\pi}\cos\frac{\pi z}{L}}$$

$$Z(0) = Z_0 = C e^{-LA/\pi} \quad , \quad Z(L) = Z_L = C e^{+LA/\pi}$$

Solve for C, A to get,

$$C = \sqrt{Z_0 Z_L}$$

$$A = \frac{-\pi}{2L}\ln(Z_0/Z_L) \checkmark$$

From (5.67),

$$\Gamma(\theta) = \frac{1}{2}\int_{z=0}^{L} e^{-2j\beta z}\frac{d}{dz}(\ln Z/Z_0)\,dz$$

$$= \frac{1}{2}\int_{z=0}^{L} A\sin\frac{\pi z}{L}\, e^{-2j\beta z}\,dz$$

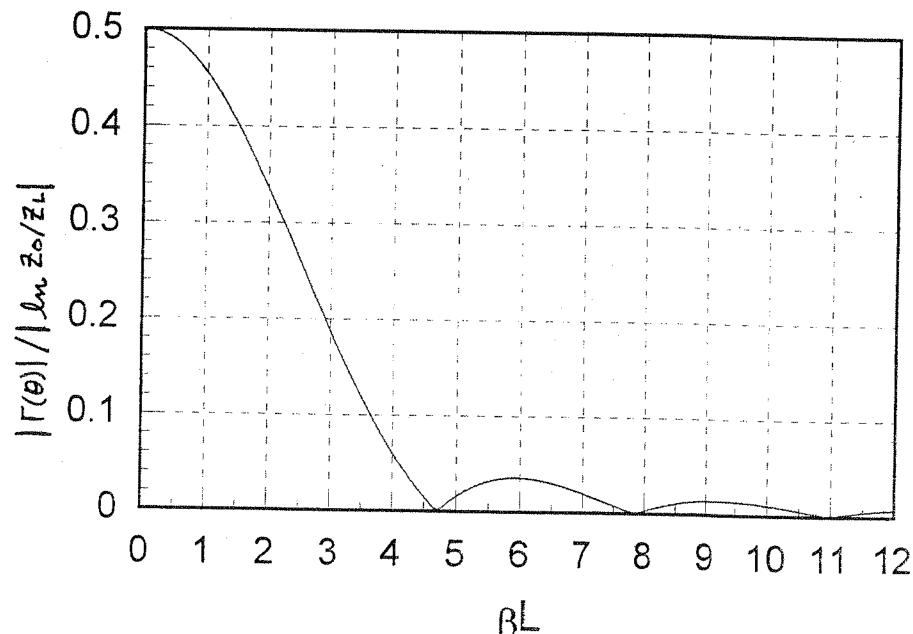
$$= \frac{A}{2}\left.\frac{e^{-2j\beta z}\left[-2j\beta\sin\frac{\pi z}{L} - \frac{\pi}{L}\cos\frac{\pi z}{L}\right]}{(\pi/L)^2 - 4\beta^2}\right|_0^L$$

$$= \frac{\pi A}{2L} e^{-j\beta L}\frac{(e^{-j\beta L} + e^{j\beta L})}{(\pi/L)^2 - 4\beta^2}$$

So,

$$|\Gamma(\theta)| = \frac{\pi^2}{2}\left|\ln\frac{Z_0}{Z_L}\right|\left|\frac{\cos\beta L}{\pi^2 - (2\beta L)^2}\right| \checkmark$$

This result is plotted as shown:



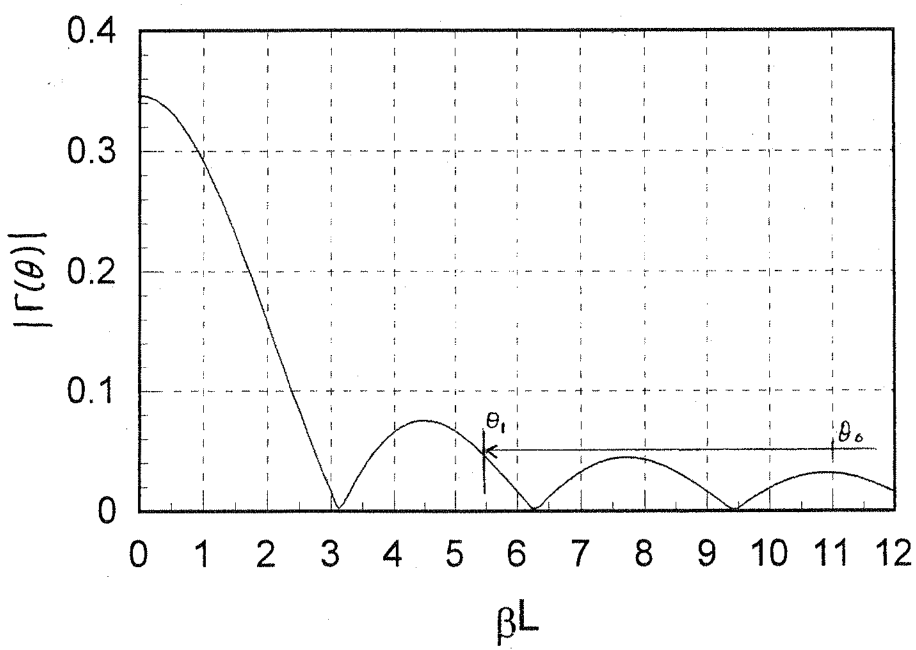
(5.24) From (5.68), $Z(z) = Z_0 e^{az}$ for $0<z<L$.

$$a = \frac{1}{L} \ln \frac{Z_L}{Z_0} = \frac{0.693}{L}$$

From (5.70),

$$|\Gamma(\theta)| = \frac{1}{2}\left|\ln \frac{Z_L}{Z_0}\right| \left|\frac{\sin\beta L}{\beta L}\right| = 0.346\left|\frac{\sin\beta L}{\beta L}\right| \checkmark$$

This result is plotted in the graph shown below:



We see that the lower frequency limit for $|\Gamma| \le 0.05$ is $\theta_1 = 5.5$. To obtain 100% bandwidth, we must have,

$$\frac{\theta_2-\theta_1}{(\theta_1+\theta_2)/2} = 1 \quad , \text{ or } \quad \theta_2 = 3\theta_1 = 16.5$$

Then at the center frequency,

$$\theta_0 = \frac{\theta_1+\theta_2}{2} = 11.0 = \beta L$$

So,

$$L = \frac{11\lambda_0}{2\pi} = 1.75\lambda_0 \checkmark$$

From (5.64), θ_m for a Chebyshev transformer with 100% bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 1 \quad \Rightarrow \quad \theta_m = \pi/4 .$$

Then from (5.63),

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right]$$

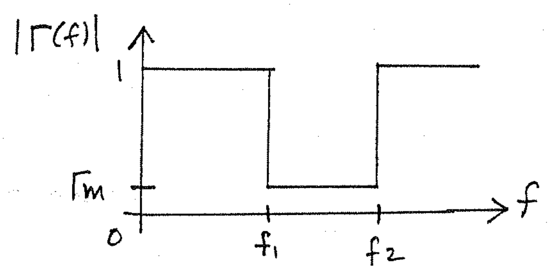
$$1.414 = \cosh\left[\frac{1}{N}(2.5846)\right] \quad \Rightarrow \quad N = 2.93 \Rightarrow \underline{N=3}$$

So $N=3$ sections would be required, for a length of $3\lambda_0/4$ at the center frequency. ✓
This is much shorter than the exponential taper matching section.

(5.25) From Figure 5.22 the Bode-Fano limit for a parallel RC load is,

$$\int_0^\infty \ln\frac{1}{|\Gamma(\omega)|}\, d\omega \leq \frac{\pi}{RC}$$

The optimum reflection coefficient magnitude response will be as shown:

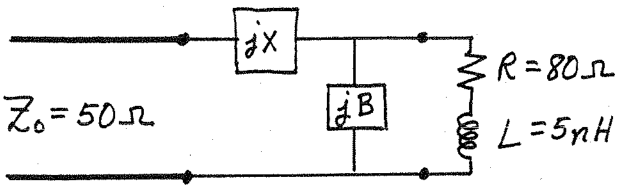


$$\text{Thus, } \ln\frac{1}{\Gamma_m} \leq \frac{\pi}{\Delta\omega RC} = \frac{\pi}{2\pi(10.6-3.1)\times10^9(75)(0.6\times10^{-12})}$$

$$\leq 1.48$$

$$\Gamma_m > 0.228 \quad \Rightarrow \quad RL < \underline{6.4\,dB}$$

(5.26) L-section matching solution:



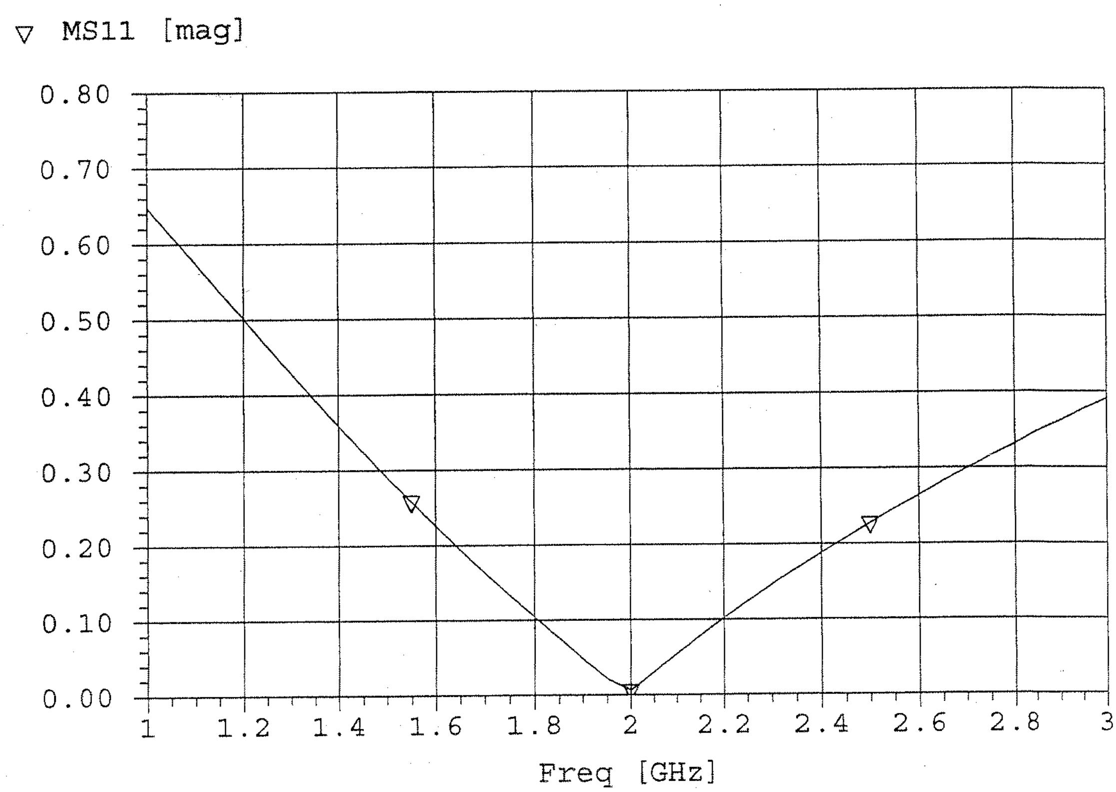
at $f = 2GHz$, $Z_L = 80 + j63\,\Omega$, $z_L = 1.6 + j1.26$ (INSIDE $1 + jx$)

a Smith chart solution gives,

$jb = -j1.8 \Rightarrow$ INDUCTOR with $L = 22.1nH$. ✓

$jx = -j1.25 \Rightarrow$ CAPACITOR with $C = 1.27pF$. ✓

The input reflection coefficient magnitude is plotted below, where it is seen that the bandwidth for $|\Gamma| < 0.1$ is 20%.



Bode-Fano limit:

From Figure 5.22d, the Bode-Fano criteria gives a bandwidth limit of

$$\Delta\omega = \frac{\pi R}{L} \frac{1}{\ln 1/\Gamma_m} = 2.18\times10^{10} = \omega_2 - \omega_1$$

$$\frac{\Delta f}{f_0} = \frac{f_2 - f_1}{f_0} = \frac{2.18\times10^{10}}{2\pi\,(2\times10^{9})} = 174\%$$

This is considerably more than the bandwidth of the L-section match.

Chapter 6

6.1 From Table 6.1 for a parallel RLC circuit:

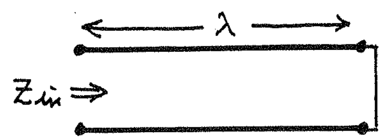
$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 355.9 \text{ MHz}$$

$$Q = \omega_o RC = 2\pi f_o RC = 17.9$$

$$Q_e = R_L/\omega_o L = \frac{R_L}{R} Q = 40.3$$

$$Q_L = \frac{1}{1/Q_e + 1/Q} = 12.4$$

6.2



$$\ell = \lambda = \frac{2\pi v_p}{\omega_o} \quad \text{for } \omega = \omega_o$$

This circuit has a series-type resonance, like the short-circuited $\lambda/2$ resonator. Thus, let

$$\beta\ell = \frac{\omega_o \ell}{v_p} + \frac{\Delta\omega \ell}{v_p} = 2\pi\left(1 + \frac{\Delta\omega}{\omega_o}\right)$$

Then from (6.24) the input impedance is,

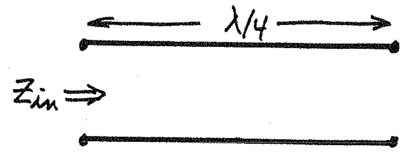
$$Z_{in} \simeq Z_o \frac{\alpha\ell + j2\pi \frac{\Delta\omega}{\omega_o}}{1 + j2\pi\frac{\Delta\omega}{\omega_o}} \simeq Z_o\left(\alpha\ell + j2\pi\frac{\Delta\omega}{\omega_o}\right) = R + 2jL\Delta\omega$$

Thus, $\quad R = Z_o \alpha\ell \quad , \quad L = \frac{\pi Z_o}{\omega_o}$.

And,

$$Q = \frac{\omega_o L}{R} = \frac{\pi Z_o}{Z_o \alpha \ell} = \frac{\pi}{\alpha\ell} = \frac{\beta}{2\alpha} \quad \left(\text{since } \ell = \lambda = \frac{2\pi}{\beta} \text{ at res.}\right)$$

6.3



$\ell = \frac{\lambda}{4} = \frac{\pi v_p}{2\omega_0}$ for $\omega = \omega_0$

This circuit has a series-type resonance, like the short-circuited λ/2 line. So let,

$$\beta\ell = \frac{\omega_0 \ell}{v_p} + \frac{\Delta\omega \ell}{v_p} = \frac{\pi}{2}\left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

Then,
$$\tan\beta\ell = \tan\frac{\pi}{2}\left(1+\frac{\Delta\omega}{\omega_0}\right) = -\cot\frac{\Delta\omega\pi}{2\omega_0} \simeq \frac{-2\omega_0}{\pi\Delta\omega}$$

The input impedance is,

$$Z_{in} = Z_0 \frac{1 + j\tan\beta\ell \tanh\alpha\ell}{\tanh\alpha\ell + j\tan\beta\ell} \simeq \frac{1 - j\frac{2\omega_0}{\pi\Delta\omega}\alpha\ell}{\alpha\ell - j\frac{2\omega_0}{\pi\Delta\omega}}$$

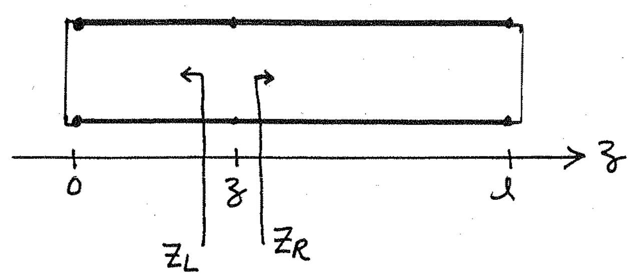
$$\simeq Z_0 \frac{\alpha\ell + j\frac{\pi\Delta\omega}{2\omega_0}}{1 + j\frac{\pi\Delta\omega}{2\omega_0}\alpha\ell} \simeq Z_0\left(\alpha\ell + j\frac{\pi\Delta\omega}{2\omega_0}\right) = R + 2jL\Delta\omega$$

$$\therefore \; R = Z_0\alpha\ell \; ✓ \quad , \quad L = \frac{\pi Z_0}{4\omega_0} \; ✓$$

Then,
$$Q = \frac{\omega_0 L}{R} = \frac{\pi}{4\alpha\ell} = \frac{\beta}{2\alpha} \; ✓$$

(since $\ell = \frac{\lambda}{4} = \frac{\pi}{2\beta}$ at resonance)

6.4

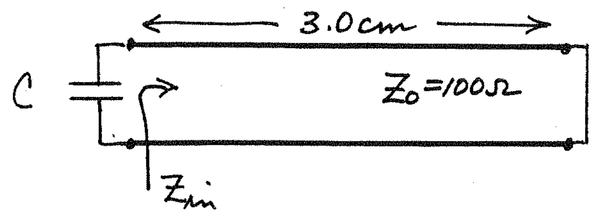


$\beta l = \pi$

$Z_L = j Z_0 \tan \beta z$

$Z_R = j Z_0 \tan \beta(l - z) = j Z_0 \tan(\pi - \beta z) = -j Z_0 \tan \beta z = Z_L^*$ ✓

6.5



$f_0 = 6$ GHz

$\beta = \frac{2\pi f}{c} = 125.7\ m^{-1}$ for an air-filled line

$\beta l = (125.7)(0.03) = 216°$ ✓

$Z_{in} = j Z_0 \tan \beta l = j(100) \tan 216° = j\,72.6\Omega = j\omega L$ ✓

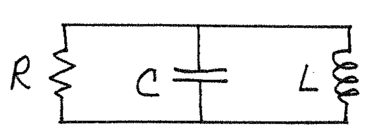
To achieve resonance we must have,

$$Z_{in} = (jX_c)^* = \frac{j}{\omega C}$$

So,

$$C = \frac{1}{\omega X_{in}} = 0.365\,pF \checkmark$$

The equivalent circuit at 6 GHz, with the shunt resistor, is as follows:



$R = 10,000\ \Omega$

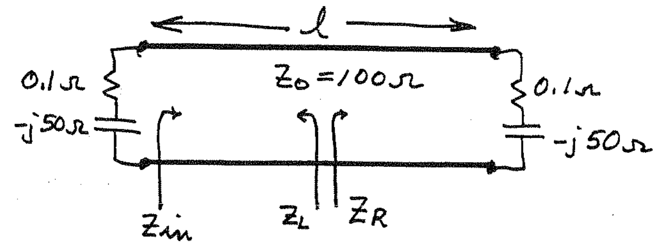
$C = 0.365\ pF$

$L = \frac{X_{in}}{\omega} = \frac{72.6}{2\pi(6 \times 10^9)} = 1.93\ nH$ ✓

So the Q is,

$$Q = \omega RC = 2\pi(6\times10^9)(10,000)(0.365\times10^{-12}) = 138. \checkmark$$

6.6



Since the resonator is symmetrical, at the midpoint of the line we must have, $Z_L = Z_R^* = Z_R$, or $Im\{Z_R\} = 0$:

Let $t = \tan \beta\ell/2$ and $Z_L = R_L + jX_L$. ($R_L = 0.1$, $X_L = -50.$)

$$Z_R = Z_0 \frac{Z_L + jZ_0 t}{Z_0 + jZ_L t} = Z_0 \frac{R_L + j(X_L + Z_0 t)}{(Z_0 - X_L t) + jR_L t}$$

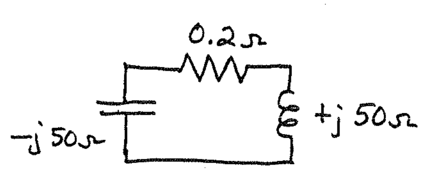
$$= Z_0 \frac{R_L(Z_0 - X_L t) + R_L t(X_L + Z_0 t) + j(X_L + Z_0 t)(Z_0 - X_L t) - jR_L^2 t}{(Z_0 - X_L t)^2 + (R_L t)^2}$$

$Im\{Z_R\} = 0 \Rightarrow$
$$(X_L + Z_0 t)(Z_0 - X_L t) - R_L^2 t = 0$$
$$-X_L Z_0 t^2 + (Z_0^2 - X_L^2 - R_L^2)t + Z_0 X_L = 0$$
$$5000t^2 + 7500t - 5000 = 0$$
$$t^2 + 1.5t - 1 = 0$$

$$t = \frac{-1.5 \pm \sqrt{(1.5)^2 + 4}}{2} = -0.75 \pm 1.25 = \begin{cases} 0.50 \Rightarrow \beta\ell = 53.1° \\ -2.00 \Rightarrow \beta\ell = -126.9° = 53.1° \end{cases}$$

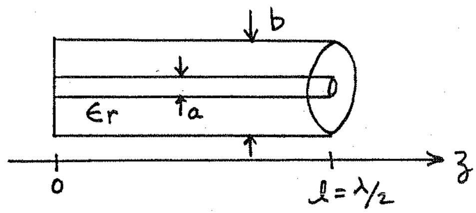
So,
$$\ell = \frac{53.1°}{360°}\lambda = 0.148\lambda \qquad \tan\beta\ell = 1.332$$

CHECK: $$Z_{in} = 100 \frac{(.1 - j50) + j133.2}{100 + j(.1 - j50)(1.332)} = 0.1 + j50\Omega \checkmark$$



$$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{50}{0.2} = 250.$$

6.7



$\beta \ell = \pi$

$\beta = \pi/\ell$

$\eta = \sqrt{\mu_0/\epsilon}$

From Section 2.2 the TEM fields of a coaxial line are,

$$\bar{E}^{\pm} = \hat{\rho}\frac{V_0}{\rho \ln b/a} e^{\mp j\beta z} \quad , \quad \bar{H}^{\pm} = \pm\hat{\phi}\frac{V_0}{\eta \ln b/a} e^{\mp j\beta z}$$

$E_\rho = 0$ at $z=0$ in the resonator, so the standing wave fields can be written as,

$$E_\rho = \frac{V_0}{\rho \ln b/a}\left[e^{-j\beta z} - e^{j\beta z}\right] = \frac{-2jV_0}{\rho \ln b/a} \sin\beta z$$

$$H_\phi = \frac{V_0}{\eta \ln b/a}\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2V_0}{\eta \ln b/a} \cos\beta z$$

From (1.84) and (1.86) the time-average stored electric and magnetic energies are,

$$W_e = \frac{\epsilon}{4}\int_v |E|^2 dv = \frac{\epsilon}{4}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell}\left(\frac{2V_0}{\rho \ln b/a}\right)^2 \sin^2\frac{\pi z}{\ell}\,\rho\, dz\, d\phi\, d\rho$$

$$= \frac{\pi\epsilon V_0^2}{\ln b/a}$$

$$W_m = \frac{\mu_0}{4}\int_v |\bar{H}|^2 dv = \frac{\mu_0}{4}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell}\left(\frac{2V_0}{\eta \ln b/a}\right)^2 \cos^2\frac{\pi z}{\ell}\,\rho\, dz\, d\phi\, d\rho$$

$$= \frac{\pi\mu_0 V_0^2}{\eta^2 \ln b/a} = \frac{\pi\epsilon V_0^2}{\ln b/a} = W_e \checkmark$$

(6.8)

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R + j(\omega L - \frac{1}{\omega C})} = \frac{1}{\frac{R}{Z_0^2} + j\omega\left(\frac{L}{Z_0^2} - \frac{1}{\omega^2 C Z_0^2}\right)}$$

The input impedance of a parallel RLC circuit is,

$$Z_{in} = \frac{1}{\frac{1}{R'} + \frac{1}{j\omega L'} + j\omega C'} = \frac{1}{\frac{1}{R'} + j\omega\left(C' - \frac{1}{\omega^2 L'}\right)}$$

Thus the original circuit acts as a parallel $R'L'C'$ resonator with $R' = Z_0^2/R$, $C' = L/Z_0^2$, $L' = C Z_0^2$.
(This is the basis for using $\lambda/4$ lines as impedance and admittance inverters.)

(6.9) From (6.40), $\sigma_{BRASS} = 2.56E7$

$$f_{101} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = 4.802 \text{ GHz} \checkmark$$

$$f_{102} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{2\pi}{d}\right)^2} = 7.075 \text{ GHz} \checkmark$$

at 4.802 GHz, $R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0272\ \Omega$, $k = 100.57\ m^{-1}$

at 7.075 GHz, $R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0330\ \Omega$, $k = 148.18\ m^{-1}$

From (6.46), $(2\ell^2 a^3 b + 2bd^3 + \ell^2 a^3 d + ad^3) = (1000 + \ell^2 576)\ cm^4$

Then,

$$Q_{101} = \frac{k^3 a^3 d^3 b \eta_0}{2\pi^2 R_s} \frac{1}{(1576)(10^{-8})} = 7,251 \checkmark$$

$$Q_{102} = \frac{k^3 a^3 d^3 b \eta_0}{2\pi^2 R_s} \frac{1}{(3304)(10^{-8})} = 9,119 \checkmark$$

(verified with FORTRAN program RECCAVITY.FOR)

(6.10) From Table 3.2, the magnetic fields of the TM_{11} waveguide mode are,

$$H_x^{\pm} = \frac{B^{\pm}}{b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} e^{\mp j\beta z}$$

$$H_y^{\pm} = \frac{B^{\pm}}{a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} e^{\mp j\beta z}$$

To have current maxima at $z=0,d$ the cavity fields must be,

$$H_x = \frac{A}{b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} \cos\frac{\pi z}{d}$$

$$H_y = \frac{A}{a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} \cos\frac{\pi z}{d}$$

The stored magnetic energy is,

$$W_m = \frac{\mu_0}{4}\int_v |\bar{H}|^2 dv = \frac{\mu_0}{4} A^2 \frac{a}{2}\frac{b}{2}\frac{d}{2}\left(\frac{1}{b^2}+\frac{1}{a^2}\right) = \frac{abd\mu_0 A^2}{32}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

The power lost in the walls is,

$$P_\ell = \frac{R_s}{2}\int_s |\bar{H}_t|^2 ds = R_s\left\{\int_{x=0}^{a}\int_{z=0}^{d} |H_x(y=0)|^2 dz\,dx + \int_{y=0}^{b}\int_{z=0}^{d} |H_y(x=0)|^2 dy\,dz + \right.$$

$$\left. + \int_{x=0}^{a}\int_{y=0}^{b}\left[|H_x(z=0)|^2 + |H_y(z=0)|^2\right] dx\,dy\right\}$$

$$= \frac{A^2 R_s}{4}\,\frac{a^3 d + b^3 d + a^3 b + a b^3}{a^2 b^2}$$

Then,

$$Q = \frac{\omega_0 (W_e + W_m)}{P_\ell} = \frac{2\omega_0 W_m}{P_\ell} = \frac{k_0 \eta_0}{4R_s}\,\frac{abd\,(a^2+b^2)}{(a^3 d + b^3 d + a^3 b + a b^3)}$$

6.11 From Section 3.3 the transverse fields of the TE_{10} mode in the two regions can be written as,

$$E_y = \begin{cases} A \sin\frac{\pi x}{a} \sin\beta_a z & \text{for } 0<z<d-t \\ B \sin\frac{\pi x}{a} \sin\beta_d (d-z) & \text{for } d-t<z<d \end{cases}$$

$$H_x = \begin{cases} -j\frac{A}{Z_a} \sin\frac{\pi x}{a} \cos\beta_a z & \text{for } 0<z<d-t \\ -j\frac{B}{Z_d} \sin\frac{\pi x}{a} \cos\beta_d (d-z) & \text{for } d-t<z<d \end{cases}$$

where $\beta_a = \sqrt{k_o^2-(\pi/a)^2}$, $\beta_d = \sqrt{\epsilon_r k_o^2-(\pi/a)^2}$

$Z_a = k_o\eta_o/\beta_a$, $Z_d = k\eta/\beta_d = k_o\eta_o/\beta_d$

Continuity of E_y, H_x at $z=d-t$:

E_y: $$A \sin\beta_a (d-t) = B \sin\beta_d t$$

H_x: $$\frac{A}{Z_a} \cos\beta_a (d-t) = \frac{B}{Z_d} \cos\beta_d t$$

Divide to obtain:

$$Z_a \tan\beta_a (d-t) = Z_d \tan\beta_d t$$

$$\beta_d \tan\beta_a (d-t) = \beta_a \tan\beta_d t$$

This equation can be solved for k_o. β_a and β_d are functions of k_o as given above.

(6.12) TM modes: $(\nabla^2 + k^2) E_z = 0$

Let $E_z(x,y,z) = X(x)\,Y(y)\,Z(z)$.
Substitute into wave equation and divide by XYZ:

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} + k^2 = 0$$

By the separation of variables argument,

$$\frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2 \quad \Rightarrow \quad X(x) = A\cos k_x x + B\sin k_x x$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2 \quad \Rightarrow \quad Y(y) = C\cos k_y y + D\sin k_y y$$

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = -k_z^2 \quad \Rightarrow \quad Z(z) = E\cos k_z z + F\sin k_z z$$

with $k^2 = k_x^2 + k_y^2 + k_z^2$.

Now, $E_z = 0$ for $x = 0, a$ and $y = 0, b$. Therefore, $A = C = 0$ and $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$. To enforce the remaining boundary conditions, we need E_x or E_y:
From Maxwell's equations,

$$E_x = \frac{1}{k^2 - k_z^2}\frac{\partial^2 E_z}{\partial x \partial z} = \frac{1}{k^2 - k_z^2}(Bk_x \cos k_x x)(D\sin k_y y) \cdot$$
$$\cdot (-k_z E \sin k_z z + k_z F \cos k_z z)$$

For $E_x = 0$ at $z = 0, d$ we must have $F = 0$, and $k_z = \frac{\ell\pi}{d}$.
Thus,

$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2,$$

which determines the resonant frequencies. The solution for TE modes is similar.

(6.13) From Table 3.5 the fields of the TM_{nm0} mode are ($\beta=0$):

$$E_z = A \sin n\phi \, J_n(k_c\rho)$$

$$H_\rho = \frac{j\omega\epsilon n}{k_c^2\rho} A \cos n\phi \, J_n(k_c\rho)$$

$$H_\phi = \frac{-j\omega\epsilon}{k_c} A \sin n\phi \, J_n'(k_c\rho) \quad , \qquad k_c = p_{nm}/a = k$$

The stored electric energy is,

$$W_e = \frac{\epsilon}{4}\int_v |\bar{E}|^2 dv = \frac{A^2\epsilon}{4}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\int_{z=0}^{d} \sin^2 n\phi \, J_n^2(k_c\rho)\,\rho\, d\rho\, d\phi\, dz$$

$$= \frac{A^2\epsilon}{4}\pi d \frac{a^2}{2} J_n'^2(p_{nm}) = \frac{A^2 a^2 \pi d \epsilon}{8} J_n'^2(p_{nm}) \quad \text{(using C.14)}$$

The power loss due to finite conductivity is,

$$P_\ell = \frac{R_s}{2}\int_s |\bar{H}_t|^2 ds$$

$$= \frac{R_s}{2}\left\{\int_{\phi=0}^{2\pi}\int_{z=0}^{d} |H_\phi(\rho=a)|^2 a\, d\phi\, dz + 2\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\left[|H_\rho|^2 + |H_\phi|^2\right]\rho\, d\rho\, d\phi\right\}$$

$$= \frac{A^2 R_s}{2}\left\{\frac{\pi a d}{\eta^2} J_n'^2(p_{nm}) + \frac{2\pi}{\eta^2}\frac{p_{nm}^2}{2k_c^2} J_n'^2(p_{nm})\right\}$$

$$= \frac{A^2 R_s \pi}{2\eta^2}(ad + a^2) J_n'^2(p_{nm})$$

Then, $$Q_c = \frac{2\omega W_e}{P_\ell} = \frac{\omega a^2 \pi d \epsilon (2\eta^2)}{4 R_s \pi a (d+a)} = \frac{a d k \eta}{2R_s(d+a)} \quad ✓$$

The power lost in the dielectric is,

$$P_d = \frac{\omega\epsilon''}{2}\int_v |\bar{E}|^2 dv = \frac{\omega\epsilon}{2}\tan\delta \int_v |\bar{E}|^2 dv = \frac{2kW_e}{\eta\epsilon}\tan\delta$$

So, $$Q_d = \frac{2\omega W_e}{P_\ell} = \frac{1}{\tan\delta} \quad ✓ \qquad \text{(as in (6.48))}$$

(6.14) From Figure 6.10, maximum Q for the TE_{111} mode occurs for $2a/d \simeq 1.7$. From (6.53a) the resonant frequency is,

$$f_{111} = \frac{c}{2\pi\sqrt{\epsilon_r}}\sqrt{\left(\frac{p'_{11}}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = \frac{3\times10^8}{2\pi\sqrt{1.5}}\sqrt{\left(\frac{1.841}{a}\right)^2 + \left(\frac{1.7\pi}{2a}\right)^2}$$

$$= \frac{1.264\times10^8}{a} = 6\times10^9 \text{ Hz} \Rightarrow \underline{a = 2.107 \text{ cm}}$$

$$d = \frac{2a}{1.7} = \underline{2.479 \text{ cm}}$$

$\sigma_{AU} = 4.1\times10^7$ S/m , $R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.024\ \Omega$

$k = 2\pi f\sqrt{\epsilon_r}/c = 153.9 \text{ m}^{-1}$

$\beta = \frac{\pi}{d} = 126.7 \text{ m}^{-1}$

From (6.57) the unloaded Q is, (due to conductor losses)

$$Q_c = \frac{(ka)^3 \eta a d \left[1-\left(\frac{1}{p'_{11}}\right)^2\right]}{4(p'_{11})^2 R_s \left\{\frac{ad}{2}\left[1+\left(\frac{\beta a}{p'^2_{11}}\right)^2\right] + \left(\frac{\beta a^2}{p'_{11}}\right)^2\left(1-\frac{1}{p'^2_{11}}\right)\right\}}$$

$= 10{,}985$ ✓

The unloaded Q due to dielectric loss is

$$Q_d = \frac{1}{\tan\delta} = 2{,}000 \ ✓$$

Then the total Q is,

$$Q = \frac{1}{\frac{1}{Q_d} + \frac{1}{Q_c}} = 1{,}692 \ ✓$$

(results checked with FORTRAN program CIRCAVITY.FOR)

6.15 Choose coordinate system so that $b<a<d$.
Then the dominant resonant mode is the TE_{101} mode:

$$f_{101} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{d}\right)^2} = 5.2 \text{ GHz}$$

or,

$$\frac{1}{a^2}+\frac{1}{d^2} = \left(\frac{2f_{101}}{c}\right)^2 = (34.7)^2$$

The next two higher modes must be either the TM_{110}, TE_{102}, or TE_{011} modes:

$$\left(\frac{2f_{110}}{c}\right)^2 = \frac{1}{a^2}+\frac{1}{b^2} = (34.7)^2 + \frac{1}{b^2}-\frac{1}{d^2}$$

$$\left(\frac{2f_{102}}{c}\right)^2 = \frac{1}{a^2}+\frac{4}{d^2} = (34.7)^2+\frac{3}{d^2}$$

$$\left(\frac{2f_{011}}{c}\right)^2 = \frac{1}{b^2}+\frac{1}{d^2}$$

Since $d>a$, $f_{011} < f_{110}$

Try $f_{011} = 6.5$ GHz ; $f_{110} = 7.2$ GHz

Then we have,

$$\frac{1}{b^2}-\frac{1}{d^2} = 1100.$$

$$\frac{1}{b^2}+\frac{1}{d^2} = 1878.$$

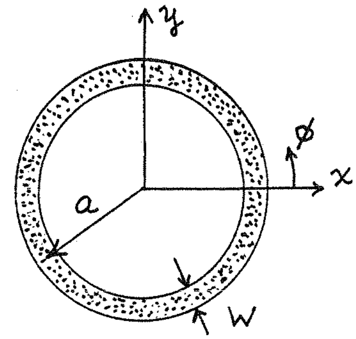
Solving gives,

$b = 2.60$ cm ✓
$d = 5.00$ cm ✓
$a = 3.53$ cm ✓

CHECK: $b<a<d$ OK ✓

$f_{102} = 7.35 \text{ GHz} > f_{110} = 7.2 \text{ GHz}$ ✓

(6.16)



$e^{\pm j\beta a\phi} = e^{\pm jn\phi}$, $n=1,2,3\ldots$

FOR PERIODICITY

So, $\beta a = \dfrac{2\pi a}{\lambda_g} = \dfrac{2\pi a\sqrt{\epsilon_e}\, f}{c} = n$

$$f = \frac{nc}{2\pi a\sqrt{\epsilon_e}} \; ; \; n=1,2,3\ldots$$

(The ring circumference is $2\pi a = n\lambda_g$)

The above result assumes $a \gg W$, so that curvature effects can be neglected. This type of resonator is most often coupled using a gap feed to a microstripline.

(6.17) For TM_{nm0} modes we have $H_z=0$ and $\frac{\partial}{\partial z}=0$. The wave equation for E_z is,

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k^2\right)E_z = 0 \qquad \text{(from 3.134)} \quad (\beta=k)$$

The general solution is,

$$E_z = (A_n \cos n\phi + B_n \sin n\phi)\, J_n(k\rho) \qquad \text{(finite at } \rho=0)$$

Since the choice of $\sin n\phi$ or $\cos n\phi$ (or any combination) depends only on the choice of the $\phi=0$ reference, we can let $B_n=0$. Then,

$$E_z = A_n \cos n\phi \; J_n(k\rho)$$

We can find H_ϕ from (3.110d):

$$H_\phi = \frac{-j\omega\epsilon}{k^2}\frac{\partial E_z}{\partial\rho} = \frac{-j\omega\epsilon}{k} A_n \cos n\phi \; J_n'(k\rho)$$

For $H_\phi=0$ at $\rho=a$ we require $J_n'(ka)=0$, or $ka = p'_{nm}$.
So the resonant frequency is,

$$f_{nm0} = \frac{ck}{2\pi\sqrt{\epsilon_r}} = \frac{c\,p'_{nm}}{2\pi a\sqrt{\epsilon_r}}$$

and,

$$f_{110} = \frac{c\,p'_{11}}{2\pi a\sqrt{\epsilon_r}} = \frac{1.841c}{2\pi a\sqrt{\epsilon_r}} \quad \checkmark$$

This solution neglects the effect of fringing fields.

(6.18) From (6.70), $\tan \beta L/2 = \alpha/\beta$,

with
$$\alpha = \sqrt{\left(\frac{2.405}{a}\right)^2 - k_o^2}$$
$$\beta = \sqrt{\epsilon_r k_o^2 - (2.405/a)^2}$$

The value of k_o at resonance must lie between $k_o = \frac{2.405}{a} = 602$, and $k_o = \frac{2.405}{a\sqrt{\epsilon_r}} = 100$.

We carry out a trial-and-error numerical search as follows:

	k_o	α	β	$\tan \beta L/2 - \alpha/\beta$
	110	592	275	−1.8
	120	590	399	−1.02
	150	583	672	.008
	145	584	631	−.12
$\longrightarrow$	149	583	664	−.0018

Thus, the resonant frequency is,

$$f_o = \frac{c k_o}{2\pi} = 7.11 \text{ GHz} \checkmark$$

(measured value is 7.8 GHz)

(6.19) Following the analysis of Section 6.5, for $TE_{01\delta}$ mode:

$$H_z = H_0 J_0(k_c\rho) e^{\pm j\beta z}$$

$$E_\phi = \frac{j\omega\mu_0 H_0}{k_c} J_0'(k_c\rho) e^{\pm j\beta z} = A J_0'(k_c\rho) e^{\pm j\beta z}$$

$$H_\rho = \frac{\mp j\beta H_0}{k_c} J_0'(k_c\rho) e^{\pm j\beta z} = \frac{\mp A}{Z_{TE}} J_0'(k_c\rho) e^{\pm j\beta z}$$

for $|z| < L/2$, $\beta = \sqrt{\epsilon_r k_0^2 - k_c^2} = \sqrt{\epsilon_r k_0^2 - (p_{01}/a)^2}$; $Z_{TE} = \frac{\omega\mu_0}{\beta} = Z_d$

for $|z| > L/2$, $j\beta = \alpha = \sqrt{k_c^2 - k_0^2} = \sqrt{(p_{01}/a)^2 - k_0^2}$; $Z_{TE} = \frac{j\omega\mu_0}{\alpha} = Z_a$

So the standing wave fields can be written as,

$$E_\phi = \begin{cases} A J_0'(k_c\rho)\left[e^{-j\beta z} - e^{j\beta z}\right] = -2jA J_0'(k_c\rho)\sin\beta z & \text{for } |z|<L/2 \\ B J_0'(k_c\rho) e^{-\alpha z} & \text{for } z>L/2 \end{cases}$$

$$H_\rho = \begin{cases} \frac{A}{Z_d} J_0'(k_c\rho)\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2A}{Z_d} J_0'(k_c\rho)\cos\beta z & \text{for } |z|<L/2 \\ \frac{B}{Z_a} J_0'(k_c\rho) e^{-\alpha z} & \text{for } z>L/2 \end{cases}$$

Continuity of E_ϕ and H_ρ at $z = L/2$ gives:

E_ϕ: $\quad -2jA\sin\beta L/2 = Be^{-\alpha L/2}$

H_ρ: $\quad \frac{2A}{Z_d}\cos\beta L/2 = \frac{B}{Z_a} e^{-\alpha L/2}$

dividing gives:

$$-jZ_d \tan\beta L/2 = Z_a$$

$$\frac{-j}{\beta}\tan\beta L/2 = j/\alpha$$

$$\tan\beta L/2 + \beta/\alpha = 0 \quad ✓$$

(6.20) assume $a>b$

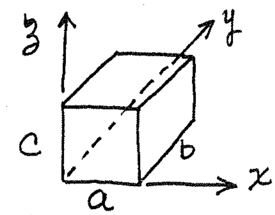
Because of the magnetic wall boundary conditions on the sidewalls, a rectangular dielectric waveguide along the z-axis would support TE modes with an H_z field of the form,

$$H_z = H_0 \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$

so the lowest order TE mode would be the TE_{11} mode. But $H_z \equiv 0$ for TM modes, so the lowest order TM mode would have,

$$H_x = H_0 \sin\frac{\pi x}{a}, \quad H_y = 0 \qquad (\text{if } a>b)$$

So the dominant mode of this resonator must be the $TM_{10\delta}$ mode. Thus we can write,



$$E_y = E_0 \sin\frac{\pi x}{a} e^{\pm j\beta z}$$

$$H_x = \frac{\pm E_0}{Z_{TM}} \sin\frac{\pi x}{a} e^{\pm j\beta z},$$

where,

$$\beta = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2} \qquad \text{for } |z| < c/2$$

$$j\beta = \alpha = \sqrt{(\pi/a)^2 - k_0^2} \qquad \text{for } z > c/2,$$

and $Z_{TM} = Z_d = \beta\eta/k = \beta\eta_0/\epsilon_r k_0 \qquad \text{for } |z| < c/2,$

$Z_{TM} = Z_a = -j\alpha\eta_0/k_0 \qquad \text{for } z > c/2$

Then the standing wave fields can be written as,

$$E_y = \begin{cases} A\sin\frac{\pi x}{a}\left[e^{-j\beta z} + e^{j\beta z}\right] = 2A\sin\frac{\pi x}{a}\cos\beta z & \text{for } |z| < c/2 \\ B\sin\frac{\pi x}{a} e^{-\alpha z} & \text{for } z > c/2 \end{cases}$$

$$H_x = \begin{cases} \frac{A}{Z_d}\sin\frac{\pi x}{a}\left[-e^{-j\beta z} + e^{j\beta z}\right] = \frac{2jA}{Z_d}\sin\frac{\pi x}{a}\sin\beta z & \text{for } |z| < c/2 \\ \frac{-B}{Z_a}\sin\frac{\pi x}{a} e^{-\alpha z} & \text{for } z > c/2 \end{cases}$$

Continuity of E_y, H_x at $z = c/2$:

$$2A\cos\beta c/2 = B e^{-\alpha c/2}$$

$$\frac{2jA}{Z_d}\sin\beta c/2 = \frac{-B}{Z_a} e^{-\alpha c/2}$$

divide to get:

$$\alpha\epsilon_r \tan\beta c/2 + \beta = 0$$

(6.21)

a) $d = \ell\lambda_o/2 = \frac{\ell}{2}\frac{c}{f_o} \Rightarrow f_o = \frac{\ell c}{2d}$ ✓

b) $E_x = E_o \sin k_o z$

$H_y = \frac{jE_o}{\eta_o} \cos k_o z$

$$W_e = \frac{\epsilon_o}{4}\int_{z=0}^{d} |E_x|^2 dz = \frac{\epsilon_o|E_o|^2}{4}\int_{z=0}^{d} \sin^2 \frac{\ell\pi z}{d} dz = \frac{\epsilon_o|E_o|^2 d}{8}$$

$$W_m = \frac{\mu_o}{4}\int_{z=0}^{d} |H_y|^2 dz = \frac{\mu_o|E_o|^2}{4\eta_o^2}\int_{z=0}^{d} \cos^2 \frac{\ell\pi z}{d} dz = \frac{\mu_o|E_o|^2 d}{8\eta_o^2} = \frac{\epsilon_o|E_o|^2 d}{8}$$

Thus $W_e = W_m$ at resonance ✓

$$P_c = 2\left(\frac{R_s}{2}\right)|H_y(z=0)|^2 = \frac{R_s|E_o|^2}{\eta_o^2}, \quad R_s = \sqrt{\frac{\omega\mu_o}{2\sigma}}$$

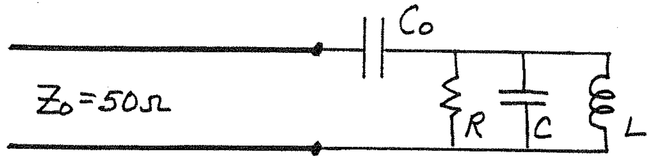
$$Q_c = \omega(W_e + W_m)/P_c = \frac{\omega\epsilon_o d\eta_o^2}{4R_s} = \frac{c\pi\ell\epsilon_o\eta_o^2}{4R_s} = \frac{\pi\ell\eta_o}{4R_s}$$

c) $f_o = \frac{(25)(3\times10^8)}{2(.04)} = \underline{93.8 \text{ GHz}}$

$R_s = 0.08\,\Omega$

$Q_c = \frac{\pi(25)(377)}{4(.08)} = \underline{92,500}$

6.22



$R = 1000\,\Omega$
$L = 1.26$ nH
$C = 0.804$ pF

The simplest way to solve this problem is graphically, with a Smith chart. The admittance of the resonator at frequencies near resonance is,

$$Y_R = \frac{1}{R} + j\frac{2Q\Delta\omega}{R\omega_0},$$

where $\omega_0 = \frac{1}{\sqrt{LC}} = 3.142 \times 10^{10}$ RPS ; $f_0 = \frac{\omega_0}{2\pi} = 5.00$ GHz

$$Q = \frac{R}{\omega_0 L} = 25.3$$

Normalized to Z_0, we have $y_R = Z_0 Y_R = 0.05 + j2.53\frac{\Delta\omega}{\omega_0}$. We can plot y_R on a Smith chart, versus $\Delta\omega/\omega_0$. For $\Delta\omega = 0$, $y_R = 0.05$. For $\Delta\omega = \pm 0.1\omega_0$, $y_R = 0.05 \pm j0.253$.
Next, convert this locus to z_R, an impedance locus. Then we see that a series capacitive reactance of $-jX_{C_0} = -j4.2$ will yield an input impedance of $z_{in} = 1$. This corresponds to a resonator admittance $y_R = 0.05 - j0.22$. So the resonant frequency will be,

$$\Delta\omega = \frac{-0.22\,\omega_0}{2.53} = -0.0869\,\omega_0$$

$$\omega_r = \omega_0 + \Delta\omega = (1 - .0869)\omega_0 = 0.913\,\omega_0$$

so,
$$f_r = \frac{\omega_r}{2\pi} = 4.566 \text{ GHz} \quad \text{(note lowering from } f_0)$$

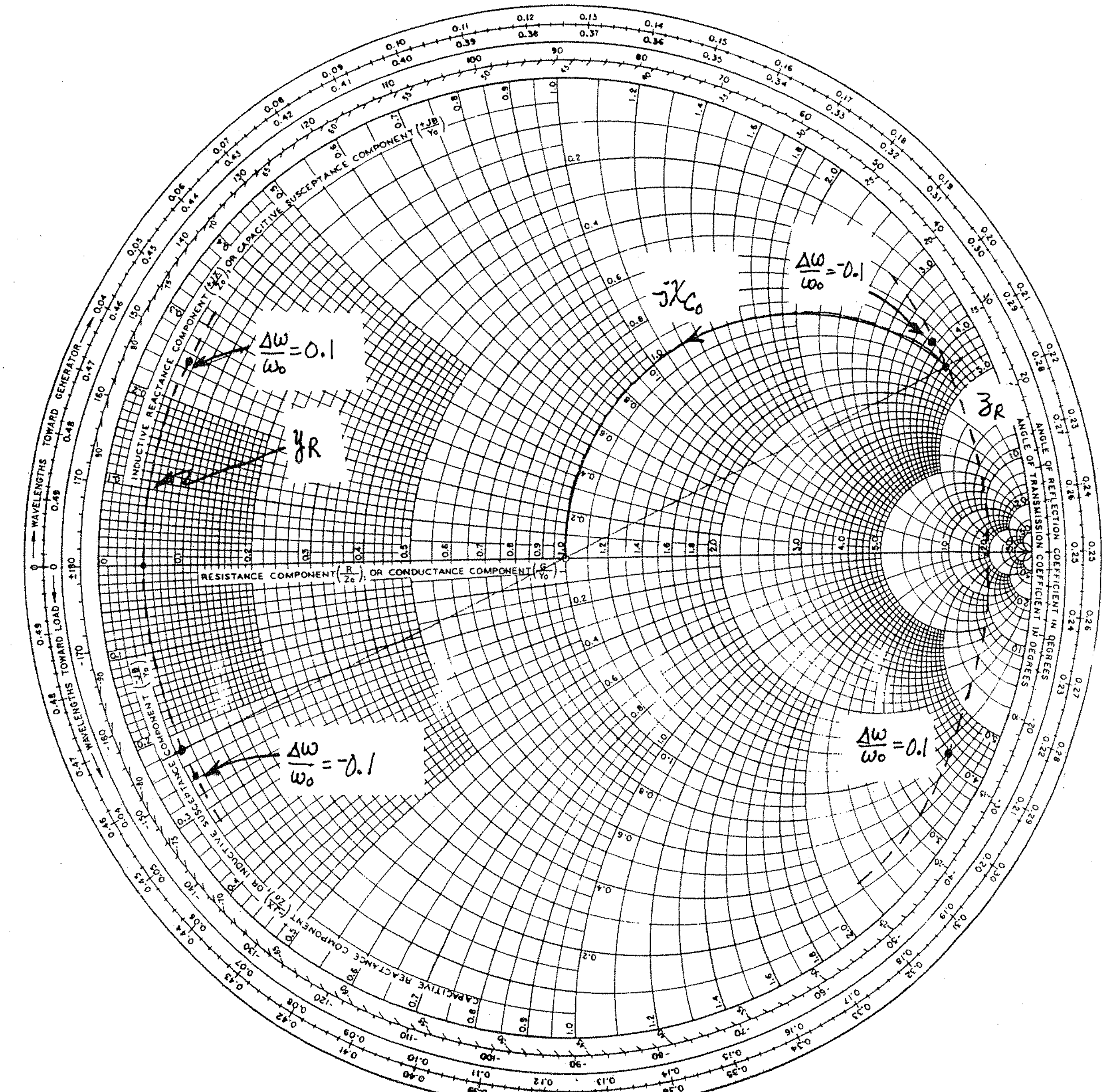
The coupling capacitor value is,

$$C_0 = \frac{1}{4.2 Z_0 \omega_r} = 0.166 \text{ pF}.$$

CHECK: at 4.566 GHz, $Y_R = (1 - j4.39) \times 10^{-3}$ S

$$Z_R = 49.2 + j216.5\,\Omega \simeq 50 + jX_{C_0}$$

$$\frac{1}{j\omega C_0} = -j210.$$



$\frac{\Delta\omega}{\omega_0} = -0.1$
$-jX_{C_0}$
$\frac{\Delta\omega}{\omega_0} = 0.1$
y_R
z_R
$\frac{\Delta\omega}{\omega_0} = -0.1$
$\frac{\Delta\omega}{\omega_0} = 0.1$
RESISTANCE COMPONENT $\left(\frac{R}{Z_0}\right)$, OR CONDUCTANCE COMPONENT $\left(\frac{G}{Y_0}\right)$
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES

(6.23) Assume TE_{101} mode, as in Section 6.6.

at 9 GHz, $k_0 = 188.\ m^{-1}$; $\beta_0 = 140.5\ m^{-1}$; $l = \frac{\lambda_g}{2} = \frac{\pi}{\beta_0} = 2.24$ cm.

$\frac{\omega_0}{2\pi} = f_0 = 9$ GHz is the resonant frequency of the closed cavity, and does not include the effect of the coupling aperture. For a high-Q cavity, the actual resonant frequency, ω_1, will be close to ω_0. So we can approximately compute x_L using ω_0. From (6.89),

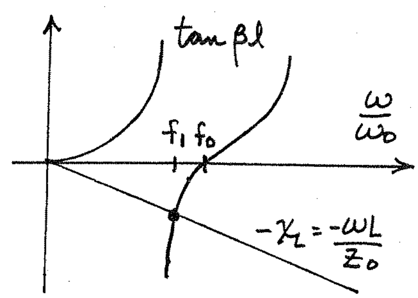
$$x_L = \sqrt{\frac{\pi k_0 \omega_1}{2Q\beta^2 C}} = 0.016 = \frac{\omega L}{Z_0} \Rightarrow \frac{L}{Z_0} = 2.83\times10^{-13}$$

Then solve (6.85) for ω:

$$\tan\beta l + x_L = 0$$

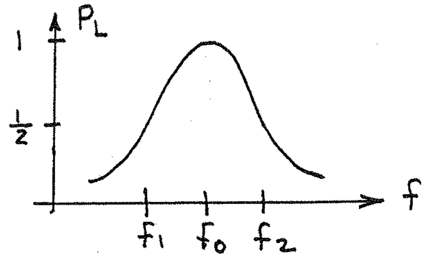
Numerical trial-and-error:

f	β	x_L	$\tan\beta l + x_L$
9	140.	.0160	.01
8.9	137.7	.0158	−.04
8.97	139.65	.0159	.0025



Thus, $f_1 = 8.97$ GHz

6.24



assuming $Q >> 1$, $f_0 = \frac{f_1 + f_2}{2} = 8.2325$ GHz

$$BW = \frac{f_2 - f_1}{f_0} = 0.3\%$$

$$Q_L = \frac{1}{BW} = 329 >> 1 \checkmark \quad \text{(loaded Q)}$$

at resonance,

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{r-1}{r+1} = 0.33 \Rightarrow r = \frac{1+\Gamma}{1-\Gamma} = 1.985$$

From (6.83),

$$Q = g Q_e = \frac{g}{\frac{1}{Q_L} - \frac{1}{Q}}$$

Solve for Q:

$$\frac{Q}{Q_L} - 1 = g$$

$$Q = Q_L(g+1)$$

If we have a series resonator, $g = \frac{Z_0}{R} = \frac{1}{r} = 0.504$ (undercoupled)

$$Q = (1+g)Q_L = 495.$$

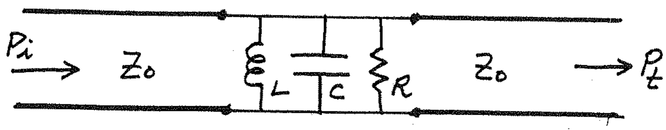
$$Q_e = \frac{Q}{g} = 982.$$

If we have a parallel resonator; $g = \frac{R}{Z_0} = r = 1.985$ (overcoupled)

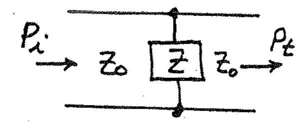
$$Q = (1+g)Q_L = 982.$$

$$Q_e = \frac{Q}{g} = 495.$$

6.25



(Q, ω_0, g)



$$\frac{P_t}{P_i} = |S_{21}|^2 \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1 + Z_0/Z + 1} = \frac{2}{2 + Z_0/Z}$$

So,
$$\frac{P_t}{P_i} = \frac{4}{|2 + Z_0/Z|^2}$$

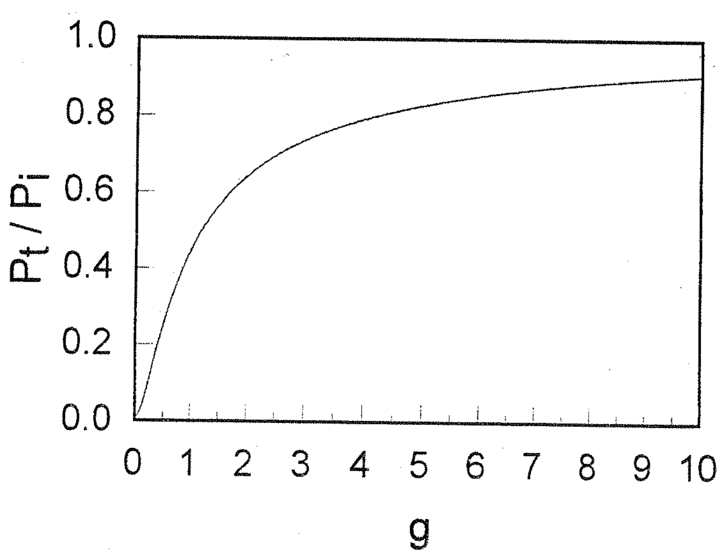
From Table 6.1, $\frac{Z_0}{Z} \simeq \frac{Z_0}{R} + j\frac{2Q\Delta\omega Z_0}{R\omega_0}$

For a parallel RLC resonator, $g = R/Z_0$, so

$$\frac{P_t}{P_i} = \frac{4}{\left(2+\frac{1}{g}\right)^2 + \left(\frac{2Q\Delta\omega}{g\omega_0}\right)^2} = \frac{4g^2}{(1+2g)^2 + \left(\frac{2Q\Delta\omega}{\omega_0}\right)^2}$$

at resonance $\Delta\omega = 0$, so this reduces to,

$$\frac{P_t}{P_i} = \frac{4g^2}{(1+2g)^2} = \left(\frac{2g}{1+2g}\right)^2$$



(6.26) The unperturbed TE_{101} cavity fields are,

$$E_y = A \sin\frac{\pi x}{a} \sin\frac{\pi z}{d}$$

$$H_x = \frac{-jA}{Z} \sin\frac{\pi x}{a} \cos\frac{\pi z}{d} \qquad ; \; Z = k\eta/\beta$$

$$H_z = \frac{j\pi A}{k\eta a} \cos\frac{\pi x}{a} \sin\frac{\pi z}{d}$$

Then the numerator in (6.95) is,

$$\int_{v_0} \left(\Delta\epsilon |\bar{E}_0|^2 + \Delta\mu |\bar{H}_0|^2\right) dv = (\mu_r - 1)\mu_0 \int_{x=0}^{a} \int_{y=0}^{b} \int_{z=0}^{t} \left(|H_x|^2 + |H_z|^2\right) dz\,dy\,dx$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \int_{z=0}^{t} \left(\frac{1}{Z^2}\cos^2\frac{\pi z}{d} + \frac{\pi^2}{k^2\eta^2 a^2} \sin^2\frac{\pi z}{d}\right) dz$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \left[\frac{1}{Z^2}\left(\frac{z}{2} + \frac{\sin\frac{2\pi z}{d}}{4\pi/d}\right)\Bigg|_0^t + \frac{\pi^2}{k^2\eta^2 a^2}\left(\frac{z}{2} - \frac{\sin\frac{2\pi z}{d}}{4\pi/d}\right)\Bigg|_0^t\right]$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \left[\frac{t}{2\eta^2} + \frac{\beta^2 - \pi^2/a^2}{k^2\eta^2}\frac{d}{4\pi}\sin\frac{2\pi t}{d}\right]$$

The denominator in (6.95) is $\frac{abd\epsilon_0 A^2}{2}$, so

$$\frac{\omega - \omega_0}{\omega_0} = \frac{-(\mu_r - 1)ab\,\eta^2[\bullet]}{abd}$$

$$= \frac{-(\mu_r - 1)}{d}\left(\frac{t}{2} + \frac{\beta^2 - \pi^2/a^2}{k^2}\frac{d}{4\pi}\sin\frac{2\pi t}{d}\right)$$

For $t \ll d$ this simplifies to,

$$\frac{\omega - \omega_0}{\omega_0} \simeq -(\mu_r - 1)\left(\frac{t}{d}\right)\left(\frac{\beta^2}{k^2}\right)$$

(6.27) Following Example 6.8:

at $x = a/2,\ z = 0$: $\quad E_y = 0$

$$H_x = \frac{-jA}{Z}, \quad Z = k_o\eta_o/\beta$$

$$H_y = 0$$

Then,

$$\int_{\Delta v} (\mu|\bar{H}_o|^2 - \epsilon|\bar{E}_o|^2)\,dv = \mu_o \frac{A^2}{Z^2}\Delta V \quad ; \quad \Delta V = \pi l r_o^2$$

$$\int_{\Delta v} (\mu|\bar{H}_o|^2 + \epsilon|\bar{E}_o|^2)\,dv = \frac{V_o\epsilon_o A^2}{2}$$

So (6.102) reduces to,

$$\frac{\omega - \omega_o}{\omega_o} = \frac{2\mu_o \Delta V}{Z^2 \epsilon_o V_o} = \frac{2\eta_o^2 \Delta V \beta^2}{k_o^2 \eta_o^2 V_o} = \frac{2\beta^2}{k_o^2}\frac{\Delta V}{V_o}$$

(an increase in resonant frequency)

Chapter 7

(7.1) This is a special case of a lossless reciprocal 3-port network; it was shown in general that such a network could not be matched at all ports (using the [S] matrix). Alternatively, we can argue as follows: If the input to each port is to be matched to its respective characteristic impedance, we must have,

$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{1}{Z_3} \qquad \text{(port 1)}$$

$$\frac{1}{Z_2} = \frac{1}{Z_1} + \frac{1}{Z_3} \qquad \text{(port 2)}$$

$$\frac{1}{Z_3} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad \text{(port 3)}$$

It is not possible to satisfy these three equations simultaneously:

$$\det\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = -4 \neq 0$$

(7.2)

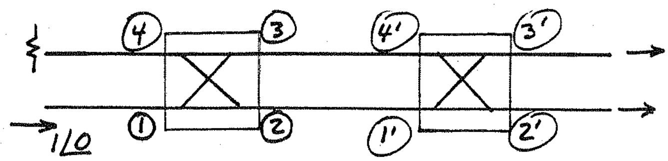
$$RL = -20 \log |\Gamma| = -20 \log |S_{11}| = -20 \log (0.05) = 26.\ \text{dB} \checkmark$$

$$C = 10 \log \frac{P_1}{P_3} = -20 \log |S_{13}| = -20 \log (0.1) = 20.\ \text{dB} \checkmark$$

$$D = 10 \log \frac{P_3}{P_4} = 20 \log \left|\frac{S_{13}}{S_{14}}\right| = 20 \log \left(\frac{.1}{.05}\right) = 6.0\ \text{dB} \checkmark$$

$$I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| = -20 \log (0.05) = 26.\ \text{dB} \checkmark$$

(7.3)



$C = 8.34\,dB \Rightarrow \beta = |S_{13}| = 0.383$

$\alpha = \sqrt{1-\beta^2} = 0.924$

If $V_1^+ = 1\angle 0°$, then from (7.17),

$$V_3^- = j\beta V_1^+ = 0.383\angle 90°$$

$$V_2^- = \alpha V_1^+ = 0.924\angle 0°$$

Then the outputs of the second coupler are,

$$V_3'^- = j\beta V_1'^+ + \alpha V_4'^+ = j\beta V_2^- + \alpha V_3^-$$

$$= (0.383)(0.924)\angle 90° + (0.924)(0.383)\angle 90° = 0.707\angle 90° \checkmark$$

$$V_2'^- = \alpha V_1'^+ + j\beta V_4'^+ = \alpha V_2^- + j\beta V_3^-$$

$$= (0.924)(0.924)\angle 0 - (0.383)(0.383)\angle 0 = 0.707\angle 0° \checkmark$$

Thus the outputs are identical to those for a single 3dB hybrid.

(7.4)

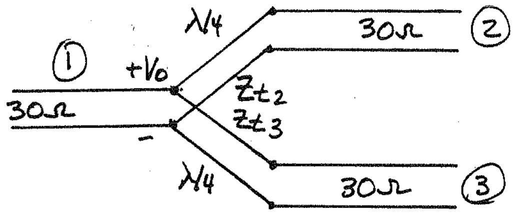
$$P_1 = 2W = 10 \log \frac{2W}{0.001W} = 33.0\ dBm \checkmark$$

$$IL = 0.7\,dB = -10 \log \frac{P_2}{P_1} \Rightarrow P_2 = P_1 - IL = 32.3\,dBm = 1.70W$$

$$C = 20\,dB = -10 \log \frac{P_3}{P_1} \Rightarrow P_3 = P_1 - C = 13.0\,dBm = 0.02W \checkmark$$

$$D = 35\,dB = 10 \log \frac{P_3}{P_4} \Rightarrow P_4 = P_3 - D = -12.0\,dBm = 0.063\,mW$$

7.5



$$\frac{P_2}{P_3} = 3$$

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{3}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{3}{4Z_0}\right)$$

$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3} = \frac{1}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{1}{4Z_0}\right)$$

So, $Z_2 = 4Z_0/3 = 40\Omega$ ✓

$Z_3 = 4Z_0 = 120\Omega$ ✓

The λ/4 matching transformers have impedances,

$Z_{t2} = \sqrt{30(40)} = 34.6\ \Omega$ ✓

$Z_{t3} = \sqrt{30(120)} = 60.0\ \Omega$ ✓

Then the S-parameters are, (phase ref. at 30Ω ports)

$$S_{11} = \frac{30-30}{30+30} = 0$$

$$S_{22} = \frac{30\|120 - 40}{30\|120 + 40} = \frac{24-40}{24+40} = -0.25 \checkmark$$

$$S_{33} = \frac{30\|40 - 120}{30\|40 + 120} = \frac{17.1-120.}{17.1+120.} = -0.75 \checkmark$$

$$S_{21} = S_{12} = \sqrt{P_2/P_1}\, e^{j\theta} = \sqrt{3/4}\,\underline{/-90^\circ} = 0.866\,\underline{/-90^\circ} \checkmark$$

$$S_{31} = S_{13} = \sqrt{P_3/P_1}\, e^{j\theta} = \sqrt{1/4}\,\underline{/-90^\circ} = 0.50\,\underline{/-90^\circ} \checkmark$$

Since the network is lossless, we should have,

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1$$

so, $S_{23} = S_{32} = \sqrt{1-(.25)^2-(.866)^2}\, e^{-2j\theta} = 0.433\,\underline{/-180^\circ}$ ✓

7.6

T-NETWORK: From Table 4.1 the ABCD parameters are

$A = 1 + R_1/R_2$ $\qquad B = 2R_1 + R_1^2/R_2$

$C = 1/R_2$ $\qquad D = 1 + R_1/R_2$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = 0 \Rightarrow \cancel{1} + \cancel{\frac{R_1}{R_2}} + \frac{2R_1}{Z_0} + \frac{R_1^2}{Z_0 R_2} - \frac{Z_0}{R_2} \cancel{-1} - \cancel{\frac{R_1}{R_2}} = 0$$

$$R_1^2 + 2R_1R_2 - Z_0^2 = 0$$

$$R_2 = \frac{Z_0^2 - R_1^2}{2R_1}$$

$$S_{12} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \alpha \Rightarrow 2 + \frac{2R_1}{R_2} + \frac{2R_1}{Z_0} + \frac{R_1^2}{Z_0R_2} + \frac{Z_0}{R_2} = \frac{2}{\alpha}$$

$$2Z_0R_2 + 2Z_0R_1 + \underbrace{2R_1R_2 + R_1^2 + Z_0^2}_{=Z_0^2} = \frac{2}{\alpha} Z_0 R_2$$

$$R_2 + R_1 + Z_0 = R_2/\alpha$$

$$\left(\frac{1}{\alpha} - 1\right)(Z_0 - R_1) = 2R_1$$

$$Z_0\left(\frac{1}{\alpha} - 1\right) = R_1\left(1 + \frac{1}{\alpha}\right)$$

$$R_1 = Z_0 \frac{1-\alpha}{1+\alpha} \checkmark$$

$$R_2 = \frac{2\alpha}{1-\alpha^2} Z_0 \checkmark$$

For $Z_0 = 50\Omega$:

α (dB)	α	$R_1(\Omega)$	$R_2(\Omega)$
3	.708	8.6	141.9
10	.316	26.0	35.1
20	.100	40.9	10.1

Π - NETWORK: From Table 4.1 the ABCD parameters are,

$$A = 1 + R_2/R_1 \qquad B = R_2$$

$$C = \frac{2}{R_1} + \frac{R_2}{R_1^2} \qquad D = 1 + R_2/R_1$$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = 0 \Rightarrow \frac{R_2}{Z_0} - \frac{2Z_0}{R_1} - \frac{R_2 Z_0}{R_1^2} = 0$$

$$R_2 R_1^2 - 2Z_0^2 R_1 - R_2 Z_0^2 = 0$$

$$R_2 = \frac{2Z_0^2 R_1}{R_1^2 - Z_0^2}$$

$$S_{12} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \alpha \Rightarrow 2 + \frac{2R_2}{R_1} + \frac{R_2}{Z_0} + \underbrace{\frac{2Z_0}{R_1} + \frac{Z_0 R_2}{R_1^2}}_{= R_2/Z_0} = \frac{2}{\alpha}$$

$$1 + \frac{R_2}{R_1} + \frac{R_2}{Z_0} = \frac{1}{\alpha}$$

$$R_1 Z_0 + R_2(Z_0 + R_1) = \frac{1}{\alpha} Z_0 R_1$$

$$2Z_0^{\cancel{2}}\cancel{R_1} = \cancel{R_1}\cancel{Z_0}\left(\frac{1}{\alpha} - 1\right)(R_1 - Z_0)$$

$$\frac{2Z_0\alpha}{1-\alpha} = R_1 - Z_0$$

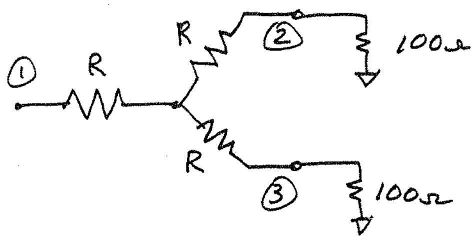
$$R_1 = Z_0\left(1 + \frac{2\alpha}{1-\alpha}\right) = \frac{1+\alpha}{1-\alpha} Z_0 \checkmark$$

$$R_2 = \frac{1-\alpha^2}{2\alpha} Z_0 \checkmark$$

For $Z_0 = 50\,\Omega$:

α (dB)	α	R_1 (Ω)	R_2 (Ω)
3	.708	292.5	17.6
10	.316	96.2	71.2
20	.100	61.1	247.5

7.7



DESIGN:

$Z_0 = 100\,\Omega$

$R = 33.3\,\Omega$ ✓

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

CASE a) ports 2 & 3 matched to $100\,\Omega$ ($V_2^+ = V_3^+ = 0$);

If $V_1^+ = 1$,

$$V_3^- = \tfrac{1}{2}[V_1^+ + V_2^+] = \tfrac{1}{2}$$

$$V_3 = V_3^+ + V_3^- = \tfrac{1}{2}$$

$$P_3 = V_3^2/Z_0 = 0.25/Z_0$$

CASE b) port 3 matched, $\Gamma = 0.3$ at port 2 ($V_3^+ = 0$):

If $V_1^+ = 1$,

$$V_2^- = \tfrac{1}{2}[V_1^+ + V_3^+] = \tfrac{1}{2}$$

$$V_2^+ = \Gamma V_2^- = (0.3)(\tfrac{1}{2}) = 0.15$$

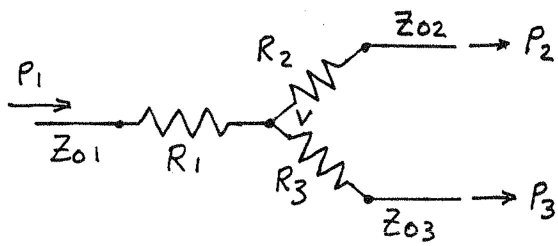
$$V_3^- = \tfrac{1}{2}[V_1^+ + V_2^+] = \tfrac{1}{2}(1.15) = 0.575\text{ v}$$

$$V_3 = V_3^+ + V_3^- = 0.575\text{ v}$$

$$P_3 = V_3^2/Z_0 = 0.331/Z_0$$

$$\frac{P_3\,(\text{PORT 2 MISMATCHED})}{P_3\,(\text{PORT 2 MATCHED})}\,(dB) = 10\log\left(\frac{0.331}{0.25}\right) = \underline{1.2\,dB}$$

7.8



$$\alpha = \frac{P_2}{P_3}$$

let $Z_1 = R_1 + Z_{01}$
$Z_2 = R_2 + Z_{02}$
$Z_3 = R_3 + Z_{03}$

For $\alpha = \frac{P_2}{P_3}$ we must have,

$$\frac{Z_{02} Z_3^2}{Z_2^2 Z_{03}} = \alpha \qquad \ldots (1)$$

For all three ports to be matched we must have,

$$Z_{01} = R_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \qquad \ldots (2)$$

$$Z_{02} = R_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} \qquad \ldots (3)$$

$$Z_{03} = R_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad \ldots (4)$$

We assume that Z_{01} and α are given; then we have 5 unknowns ($R_1, R_2, R_3, Z_{02}, Z_{03}$), and 4 equations. So we need one more equation: we will choose the condition that,

$$Z_{02} Z_{03} = Z_{01}^2 \qquad \ldots (5)$$

which will ensure that when $\lambda/4$ transformers are used to match Z_{02} and Z_{03} to a final output impedance of Z_{01}, phase tracking will be maintained for ports 2 and 3. (See Problem 7.10). Now use (1) and (5) to eliminate Z_3 and Z_{03} from (2), (3), (4). Then (2) reduces to,

$$R_1 = \frac{Z_{01}\left[Z_{02} + \sqrt{\alpha}\,(Z_{01} - Z_2)\right]}{Z_{02} + \sqrt{\alpha}\, Z_{01}}$$

$$Z_1 = \frac{Z_{01}\left[2 Z_{02} + \sqrt{\alpha}\,(2 Z_{01} - Z_2)\right]}{Z_{02} + \sqrt{\alpha}\, Z_{01}}$$

and (3) and (4) then reduce to,

$$Z_2\left[(Z_{02}+\sqrt{\alpha}\,Z_{01})^2-\alpha Z_{01}Z_{02}\right] = 2Z_{02}^2\,(Z_{02}+\sqrt{\alpha}\,Z_{01})$$

and,

$$Z_2\left[(Z_{02}+\sqrt{\alpha}\,Z_{01})^2-Z_{01}Z_{02}\right] = 2Z_{01}^2\,(Z_{02}+\sqrt{\alpha}\,Z_{01})$$

Then we obtain a quartic equation for Z_{02}:

$$Z_{02}^4 + Z_{01}(2\sqrt{\alpha}-1)Z_{02}^3 + Z_{01}^2(\alpha-1)Z_{02}^2 + Z_{01}^3(\alpha-2\sqrt{\alpha})Z_{02} - \alpha Z_{01}^4 = 0$$

after finding Z_{02} (either numerically or using the formula for a quartic equation), the above equations can be used to find Z_{03}, R_2, R_3, and R_1.

EXAMPLE: let $Z_{01}=1$, $\alpha=2$;

Then, $\quad Z_{02}^4 + 1.828\,Z_{02}^3 + Z_{02}^2 - 0.828\,Z_{02} - 2 = 0$

The solution for Z_{02} was computed numerically using an HP-15C calculator to be,

$$Z_{02} = 0.8935$$

Then,

$Z_2 = 1.041$; $\quad R_2 = 0.1478$

$Z_3 = 1.6477$; $\quad R_3 = 0.5285$; $\quad Z_{03} = 1.1192$

$R_1 = 0.3621$

CHECK: $\quad Z_{in} = R_1 + Z_2 \| Z_3 = 1.000$ ✓

NOTE: There are other choices for the relation between Z_{02} and Z_{03}, or other pairs of variables. One possibility that may give a simpler solution would be to let $Z_1 = \alpha^{1/3} Z_2$.

(7.9) From (7.37), $K^2 = P_3/P_2 = 1/3 \Rightarrow K = 0.577$

$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}} = 131.7\ \Omega$$

$$Z_{02} = K^2 Z_{03} = 43.9\ \Omega$$

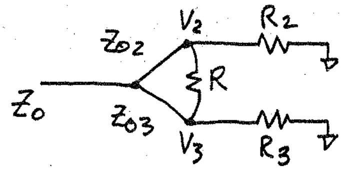
$$R = Z_0(K + 1/K) = 115.5\ \Omega$$

The output impedances are,

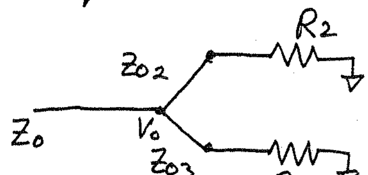
$$R_2 = Z_0 K = 28.9\ \Omega$$

$$R_3 = Z_0/K = 86.7\ \Omega$$

(7.10)



assuming the output ports are matched, no power should be dissipated in resistor R (for lossless power division). Therefore $V_2 = V_3$, and the resistor R can be removed:



Input matching requires that,

$$\frac{1}{Z_0} = \frac{R_2}{Z_{02}^2} + \frac{R_3}{Z_{03}^2}$$

Power division requires that $P_3 = K^2 P_2$:

$$P_2 = \tfrac{1}{2}|V_0|^2 R_2/Z_{02}^2$$

$$P_3 = \tfrac{1}{2}|V_0|^2 R_3/Z_{03}^2 = K^2 P_2$$

Thus,

$$K^2 \frac{R_2}{Z_{02}^2} = \frac{R_3}{Z_{03}^2}$$

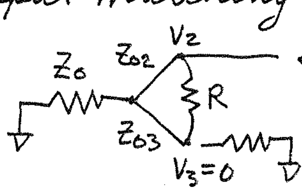
and,

$$Z_{02}^2 = Z_0 R_2 (1+K^2) \qquad \ldots (1)$$

$$Z_{03}^2 = Z_0 R_3 (1 + 1/K^2) \qquad \ldots (2)$$

For output matching,

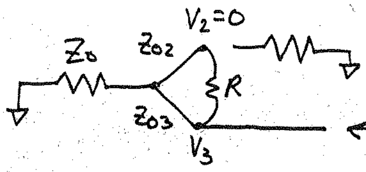
PORT 2:



$$R_2 = R \,\|\, \frac{Z_{02}^2}{Z_0} = \frac{R R_2 (1+K^2)}{R + R_2(1+K^2)}$$

$$R_2(1+K^2) = RK^2 \qquad \ldots (3)$$

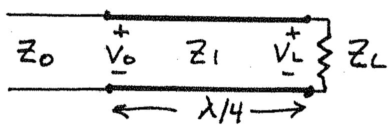
PORT 3:



$$\leftarrow R = R \,\|\, \frac{Z_{o3}^2}{Z_o} = \frac{R R_3(1+1/K^2)}{R + R_3(1+1/K^2)}$$

$$R_3(1+K^2) = R \qquad \ldots (4)$$

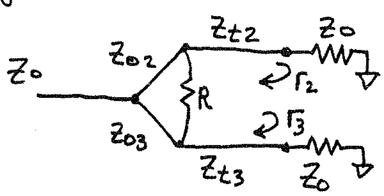
From (3) and (4) we have that $R_2 = K^2 R_3$. We need one more condition to obtain the design equations. This condition is that the output impedances R_2, R_3 be chosen so that output matching transformers have the same transfer phase at both ports:



It is easy to show that,

$$\frac{V_L}{V_o} = -j\,\frac{1+\Gamma}{1-\Gamma} \; ; \quad \Gamma = \frac{Z_L - Z_1}{Z_L + Z_1}$$

So the phase of V_L/V_o does not change when Γ is replaced by $-\Gamma$, or when Z_L and Z_1 are interchanged (Z_L, Z_1 real).



$$\Gamma_2 = \frac{Z_o/Z_{t2} - 1}{Z_o/Z_{t2} + 1}$$

$$\Gamma_3 = \frac{Z_o/Z_{t3} - 1}{Z_o/Z_{t3} + 1}$$

So we choose $Z_{t2} Z_{t3} = Z_o^2 = \sqrt{Z_o R_2}\,\sqrt{Z_o R_3} = K Z_o R_3$, which leads to the design equations,

$$R_2 = K Z_o \quad ✓ \qquad Z_{o2} = Z_o\sqrt{K(1+K^2)} \quad ✓$$

$$R_3 = Z_o/K \quad ✓ \qquad Z_{o3} = Z_o\sqrt{\frac{1+K^2}{K^3}} \quad ✓$$

$$R = Z_o(K + 1/K) \quad ✓$$

7.11 Setting $A_{10}^- = 0$ from (7.40b):

$$\left(\epsilon_0 \alpha_e + \frac{\mu_0 \alpha_m}{Z_{10}^2}\right)\sin^2\frac{\pi s}{a} - \frac{\mu_0 \pi^2 \alpha_m}{\beta^2 a^2 Z_{10}^2}\cos^2\frac{\pi s}{a} = 0$$

For a round aperture, $\alpha_e = 2r_0^3/3$; $\alpha_m = 4r_0^3/3$:

$$\left(\epsilon_0 + \frac{2\mu_0}{Z_{10}^2}\right)\sin^2\frac{\pi s}{a} - \frac{2\mu_0 \pi^2}{\beta^2 a^2 Z_{10}^2}\cos^2\frac{\pi s}{a} = 0$$

Since $Z_{10} = k_0\eta_0/\beta$, this simplifies as follows:

$$(k_0^2 + 2\beta^2)\sin^2\frac{\pi s}{a} - \frac{2\pi^2}{a^2}\cos^2\frac{\pi s}{a} = 0$$

$$\left(3k_0^2 - \frac{2\pi^2}{a^2}\right)\sin^2\frac{\pi s}{a} - \frac{2\pi^2}{a^2}\left(1 - \sin^2\frac{\pi s}{a}\right) = 0$$

$$3k_0^2\sin^2\frac{\pi s}{a} = \frac{2\pi^2}{a^2}$$

$$\frac{6a^2}{\lambda_0^2}\sin^2\frac{\pi s}{a} = 1, \text{ or } \sin\frac{\pi s}{a} = \frac{\lambda_0}{\sqrt{6}\,a} < 1 \text{ for } a > \lambda_0/2 \checkmark$$

7.12 at $f = 11$ GHz, $k_0 = 230.4\ m^{-1}$; $\beta = 116.4\ m^{-1}$; $Z_{10} = k_0\eta_0/\beta = 746.2\ \Omega$

$P_{10} = ab/Z_{10} = 1.67 \times 10^{-7}$ W

From (7.41) the position of the coupling aperture is,

$$\sin\frac{\pi s}{a} = \pi\sqrt{\frac{2}{4\pi^2 - k_0^2 a^2}} = 0.867 \Rightarrow s = 0.334a = \underline{0.528\text{ cm}}$$

Then, $\sin^2\frac{\pi s}{a} = 0.752$, $\cos^2\frac{\pi s}{a} = 0.248$

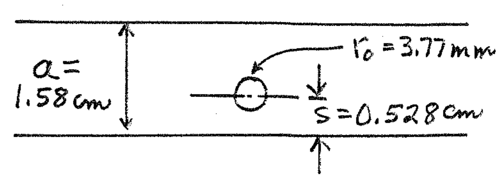
For $C = 20$dB, $\left|\frac{A_{10}^-}{A}\right| = 0.1$. From (7.40b) we have,

$$0.1 = \frac{\omega}{P_{10}}\left[\frac{2}{3}\epsilon_0\sin^2\frac{\pi s}{a} + \frac{4\mu_0}{3Z_{10}^2}\left(\sin^2\frac{\pi s}{a} - \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right]r_0^3$$

$$0.1P_{10} = \left[\frac{2}{3}\frac{k_0}{\eta_0}\sin^2\frac{\pi s}{a} + \frac{4k_0\eta_0}{3Z_{10}^2}\left(\sin^2\frac{\pi s}{a} - \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right]r_0^3$$

$$1.67 \times 10^{-8} = 3.12 \times 10^{-1} r_0^3$$

$$r_0 = \underline{3.77\text{ mm}}$$



(7.13) at $f = 17$ GHz, $k_0 = 356.\ m^{-1}$, $\beta = 295.3\ m^{-1}$,
$Z_{10} = k_0\eta_0/\beta = 454\ \Omega$, $P_{10} = ab/Z_{10} = 2.75\times10^{-7}$ W

From (7.44) the necessary skew angle is, for $s = a/2$,

$$\cos\theta = k_0^2/2\beta^2 = 0.728 \implies \underline{\theta = 43°}\ ✓$$

From (7.45) the aperture radius is,

$$\left|\frac{A_{10}^-}{A}\right| = 0.0316 = \frac{4k_0^2 r_0^3}{3ab\beta} = 4.58\times10^6 r_0^3 \implies \underline{r_0 = 1.90\ mm}\ ✓$$

(7.14) $N = 4$ (5-holes)
$k_0 = 366.5\ m^{-1}$, $\beta = 307.9\ m^{-1}$, $Z_{10} = k_0\eta_0/\beta = 448.8\ \Omega$
$P_{10} = ab/Z_{10} = 2.78\times10^{-7}\ \Omega$

From (7.40a), with $s = a/2$,

$$|K_f| = \frac{2k_0}{3\eta_0 P_{10}}\left|1 - 2\beta^2/k_0^2\right| = 9.59\times10^5$$

From (7.55),

$$C = 20dB = -20\log|K_f| - 20\log k - 20\log\sum_{n=0}^{N} C_n^N$$

For $N=4$, $\sum_{n=0}^{N} C_n^N = 1+4+6+4+1 = 16$, so

$$20 = -119.6 - 20\log k - 24.1$$
$$k = 6.53\times10^{-9}$$

From (7.54) the aperture radii are,

$$r_0 = k^{1/3} = 1.87\ mm = r_4$$
$$r_1 = (4k)^{1/3} = 2.97\ mm = r_3$$
$$r_2 = (6k)^{1/3} = 3.97\ mm$$

The spacing between the apertures is $\lambda g/4 = 5.1\ mm$.
(Smaller apertures would result if $s = a/4$ were used.)

(7.15) $N=4$ (5 hole)

$k_o = 366.5\ m^{-1}$, $\beta = 307.9\ m^{-1}$, $Z_{10} = 448.8\ \Omega$

$P_{10} = 2.78\times10^{-7}\ W$

From (7.40a,b) with $s=a/2$,

$$|K_f| = \frac{2k_o}{3\eta_o P_{10}}\left|1-\frac{2\beta^2}{k_o^2}\right| = 9.59\times10^5$$

From (7.59),

$$30\,dB = D_{min} = 20\log T_N(\sec\theta_m)$$

$$T_4(\sec\theta_m) = \cosh\left[4\cosh^{-1}(\sec\theta_m)\right] = 31.6$$

$$\sec\theta_m = 1.587$$

From (7.57),

$$C = 20\,dB = -20\log|K_f| - 20\log k - 30$$

$$k = 3.30\times10^{-9}$$

From (7.56) and (6.60d):

$$2\left[r_o^3\cos4\theta + r_1^3\cos2\theta + r_2^3\right] = k\left[\sec^4\theta_m(\cos4\theta + 4\cos2\theta + 3) - 4\sec^2\theta_m(\cos2\theta+1)+1\right]$$

Then,

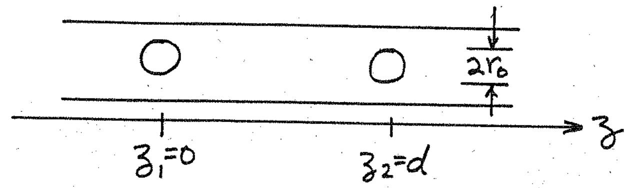
$$2r_o^3 = k\sec^4\theta_m \Rightarrow r_o = r_4 = 2.19\ mm$$

$$2r_1^3 = k\left[4\sec^4\theta_m - 4\sec^2\theta_m\right] \Rightarrow r_1 = r_3 = 2.93\ mm$$

$$2r_2^3 = k\left[3\sec^4\theta_m - 4\sec^2\theta_m + 1\right] \Rightarrow r_2 = 2.54\ mm.$$

The spacing between the apertures is $\lambda_g/4 = 5.1\ mm$.

7.16



The incident TE_{10} fields are, for $x=0$, $y=b/2$, $z=z_n$:

$$E_y = A \sin \frac{\pi x}{a} e^{-j\beta z} = 0$$

$$H_x = \frac{-A}{Z_{10}} \sin \frac{\pi x}{a} e^{-j\beta z} = 0$$

$$H_z = \frac{j\pi A}{\beta a Z_{10}} \cos \frac{\pi x}{a} e^{-j\beta z} = \frac{j\pi A}{\beta a Z_{10}} e^{-j\beta z_n}$$

$P_{10} = ab/Z_{10}$; $Z_{10} = k_0\eta_0/\beta$.

From (4.124)-(4.125) the equivalent polarization currents are: $(\hat{n}=\hat{x})$

$$\bar{P}_e = 0$$

$$\bar{P}_m = -\alpha_m H_z \delta(x)\delta(y-b/2)\delta(z-z_n) e^{-j\beta z_n}$$

Then the amplitudes of the forward and reverse coupled waves from a single aperture are,

$$A_{10}^+ = \frac{1}{P_{10}}\int_v \bar{H}_{10}^- \cdot j\omega\mu_0 \bar{P}_m \, dv = \frac{-j\omega\mu_0\alpha_m}{P_{10}}\left(\frac{j\pi A}{\beta a Z_{10}}\right) e^{j\beta z_n} H_z^- = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0}$$

$$A_{10}^- = \frac{1}{P_{10}}\int_v \bar{H}_{10}^+ \cdot j\omega\mu_0 \bar{P}_m \, dv = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0} e^{-2j\beta z_n}$$

Then the total forward and backward wave amplitudes from two apertures at $z_1=0$ and $z_2=d$ are,

$$A_{10}^+ = \frac{j\pi^2\alpha_m A(2)}{P_{10}a^2 k_0\eta_0} \qquad A_{10}^- = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0}(1+e^{-2j\beta d})$$

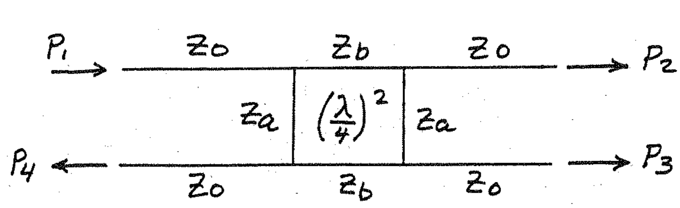
For $A_{10}^-=0$, we must have $(1+e^{-2j\beta d})=0$, or $\underline{d=\lambda_g/4}$.

Then the coupling factor is,

$$C = 20\log\left|\frac{A}{A_{10}^+}\right| = 20\log\left|\frac{P_{10}a^2k_0\eta_0}{2\pi^2\alpha_m}\right|$$

$$= 20\log\left|\frac{3a^3b\beta}{8\pi^2r_0^3}\right| \text{ dB}$$

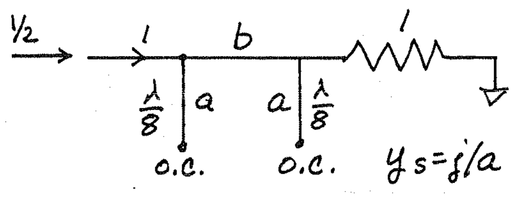
(7.17)



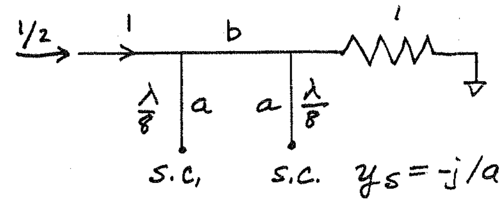
$\alpha = P_2/P_3$

let $a = \frac{Z_a}{Z_o}$, $b = \frac{Z_b}{Z_o}$

Following the analysis of Section 7.5, the even and odd circuits are: (in normalized form)



EVEN MODE



ODD MODE

The ABCD matrices are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix}\begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix} = \begin{bmatrix} -b/a & jb \\ j/b - jb/a^2 & -b/a \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix}\begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix} = \begin{bmatrix} b/a & jb \\ j/b - jb/a^2 & b/a \end{bmatrix}$$

$$\Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{j(b - 1/b + b/a^2)}{-2b/a + j(b + 1/b - b/a^2)}$$

$$\Gamma_o = \frac{A+B-C-D}{A+B+C+D} = \frac{j(b - 1/b + b/a^2)}{2b/a + j(b + 1/b - b/a^2)}$$

The reflection at port 1 is then,

$$B_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{j}{2}\left(b - \frac{1}{b} + \frac{b}{a^2}\right)\frac{-2j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)}{\left(\frac{2b}{a}\right)^2 + \left(b + \frac{1}{b} - \frac{b}{a^2}\right)^2} = 0$$

Thus, $b - \frac{1}{b} + \frac{b}{a^2} = 0$ (can't have $b + \frac{1}{b} - \frac{b}{a^2} = 0$, or else $B_2 = B_3 = 0$)

So, $a = \frac{b}{\sqrt{1-b^2}}$ Then $\frac{b}{a^2} = \frac{1}{b} - b$

Then,

$$T_e = \frac{2}{A+B+C+D} = \frac{2}{-2\frac{b}{a} + j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)} = \frac{1}{-\frac{b}{a} + jb}$$

$$T_o = \frac{2}{A+B+C+D} = \frac{2}{\frac{2b}{a} + j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)} = \frac{1}{\frac{b}{a} + jb}$$

So the output wave amplitudes at ports 2 and 3 are,

$$B_2 = \tfrac{1}{2}(T_e + T_o) = \tfrac{1}{2}\left[\frac{1}{-b/a + jb} + \frac{1}{b/a + jb}\right] = \frac{-j}{b(1+1/a^2)}$$

$$B_3 = \tfrac{1}{2}(T_e - T_o) = \tfrac{1}{2}\left[\frac{1}{-b/a + jb} - \frac{1}{b/a + jb}\right] = \frac{-1/a}{b(1+1/a^2)}$$

This shows a 90° phase shift between ports 2 and 3.

For $P_2/P_3 = \alpha$,

$$P_2 = \alpha P_3$$

$$|B_2|^2 = \alpha |B_3|^2$$

$$1 = \frac{\alpha}{a^2} \Rightarrow a = \sqrt{\alpha} \quad \text{or} \quad Z_a = \sqrt{\alpha}\, Z_0 \;\checkmark$$

Then, $$b = \frac{a}{\sqrt{1+a^2}} = \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}} = \sqrt{\frac{\alpha}{1+\alpha}}, \quad \text{or} \quad Z_b = \sqrt{\frac{\alpha}{1+\alpha}}\, Z_0 \;\checkmark$$

CHECK: When $\alpha = 1$, $Z_a = Z_0$ ✓, $Z_b = Z_0/\sqrt{2}$ ✓

at the isolated port,

$$B_4 = \tfrac{1}{2}(\Gamma_e - \Gamma_o) = \frac{j}{2}\left(b - \frac{1}{b} + \frac{b}{a^2}\right)(\cdot) = 0$$

So there is isolation.

EXAMPLE: $\alpha = 3$ (6dB), $Z_0 = 50\Omega$

Then $Z_a = 87\Omega$ ✓

$Z_b = 43\Omega$ ✓

7.18 $b = 0.32$ cm, $\epsilon_r = 2.2$, $Z_{oe} = 70\,\Omega$, $Z_{oo} = 40\,\Omega$

Thus, $\sqrt{\epsilon_r}\, Z_{oe} = 104\,\Omega$; $\sqrt{\epsilon_r}\, Z_{oo} = 59\,\Omega$

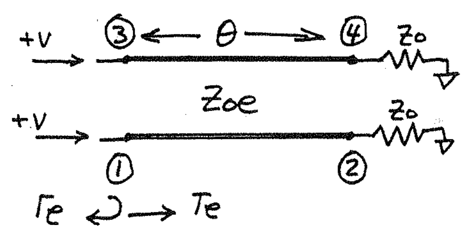
From Figure 7.29,

$$S/b = 0.075 \implies S = 0.24 \text{ mm} \checkmark$$
$$W/b = 0.67 \implies W = 2.1 \text{ mm} \checkmark$$

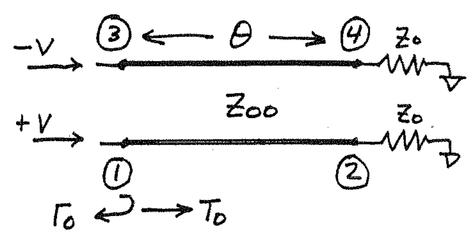
7.19 $\epsilon_r = 4.2$, $d = 0.158$ cm, $W = 0.300$ cm, $S = 0.1173$ cm

From Serenade, $Z_{oe} = 58.7\,\Omega$
$Z_{oo} = 42.6\,\Omega$

7.20



EVEN MODE



ODD MODE

For $\theta = \pi/2$,

$$\Gamma_e = \frac{Z_{oe}^2/Z_o - Z_o}{Z_{oe}^2/Z_o + Z_o} = \frac{Z_{oe}^2 - Z_o^2}{Z_{oe}^2 + Z_o^2} \qquad \Gamma_o = \frac{Z_{oo}^2/Z_o - Z_o}{Z_{oo}^2/Z_o + Z_o} = \frac{Z_{oo}^2 - Z_o^2}{Z_{oo}^2 + Z_o^2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} \cos\theta & jZ_{oe}\sin\theta \\ \frac{j}{Z_{oe}}\sin\theta & \cos\theta \end{bmatrix} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} \cos\theta & jZ_{oo}\sin\theta \\ \frac{j}{Z_{oo}}\sin\theta & \cos\theta \end{bmatrix}$$

$$T_e = S_{21} = \frac{2}{2\cos\theta + j\left(\frac{Z_{oe}}{Z_o} + \frac{Z_o}{Z_{oe}}\right)\sin\theta} \qquad T_o = S_{21} = \frac{2}{2\cos\theta + j\left(\frac{Z_{oo}}{Z_o} + \frac{Z_o}{Z_{oo}}\right)\sin\theta}$$

For a unit amplitude wave incident at port 1, the output wave amplitudes are,

$$B_1 = \tfrac{1}{2}(\Gamma_e + \Gamma_o) \qquad B_2 = \tfrac{1}{2}(T_e + T_o)$$

$$B_3 = \tfrac{1}{2}(\Gamma_e - \Gamma_o) \qquad B_4 = \tfrac{1}{2}(T_e - T_o)$$

So the reflection at port 1 is,

$$B_1 = \frac{1}{2}\left[\frac{Z_{oe}^2 - Z_o^2}{Z_{oe}^2 + Z_o^2} + \frac{Z_{oo}^2 - Z_o^2}{Z_{oo}^2 + Z_o^2}\right] = \frac{Z_{oo}^2 Z_{oe}^2 - Z_o^4}{(Z_o^2 + Z_{oe}^2)(Z_o^2 + Z_{oo}^2)} = 0$$

Thus, $\underline{Z_o = \sqrt{Z_{oe} Z_{oo}}}$ ✓ (so all ports are matched ✓)

Then, $\left(\frac{Z_{oe}}{Z_o} + \frac{Z_o}{Z_{oe}}\right) = \left(\frac{Z_o}{Z_{oo}} + \frac{Z_{oo}}{Z_o}\right)$

So, $T_e = T_o$, and $\underline{B_4 \equiv 0}$ ✓

The output waves at ports 2 and 3 are,

$$B_2 = \frac{2}{2\cos\theta + j\left(\frac{Z_{oe}}{Z_o} + \frac{Z_o}{Z_{oe}}\right)\sin\theta} = \frac{1}{\cos\theta + j\,\frac{Z_{oe} + Z_{oo}}{2Z_o}\sin\theta}$$

$$B_3 = \frac{1}{2}\left[\frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} - \frac{Z_{oo} - Z_{oe}}{Z_{oo} + Z_{oe}}\right] = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}$$

Let $C = B_3 = \dfrac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}$

Then $\sqrt{1 - C^2} = \dfrac{2Z_o}{Z_{oe} + Z_{oo}}$, and so,

$$B_2 = \frac{\sqrt{1 - C^2}}{\sqrt{1 - C^2}\cos\theta + j\sin\theta}$$

For $\theta = \pi/2$, the midband responses are,

$B_3 = C$ ✓

$B_2 = -j\sqrt{1 - C^2}$ ✓

which agree with (7.85) – (7.86).

7.21 $C = 10^{-19.1/20} = 0.1109$; $f = 8\text{GHz}$; $Z_0 = 60\Omega$

From (7.87),

$$Z_{oe} = Z_0\sqrt{\frac{1+C}{1-C}} = 67.1\Omega \qquad Z_{oo} = Z_0\sqrt{\frac{1-C}{1+C}} = 53.7\Omega$$

For a stripline with $\epsilon_r = 2.2$, $b = 0.32\text{cm}$,

$$\sqrt{\epsilon_r}\,Z_{oe} = 99.5\Omega, \qquad \sqrt{\epsilon_r}\,Z_{oo} = 79.7\Omega$$

From Figure 7.29,

$$S/b = 0.36 \implies S = 1.15\text{ mm}$$
$$W/b = 0.60 \implies W = 1.92\text{ mm}$$

The line lengths are,

$$\ell = \frac{\lambda_g}{4} = \frac{c}{4\sqrt{\epsilon_r}\,f} = 6.32\text{ mm}$$

7.22 $C = 5dB = 10^{-5/20} = 0.562$; $f_o = 8\text{GHz}$; $Z_0 = 60\Omega$

From (7.87),

$$Z_{oe} = Z_0\sqrt{\frac{1+C}{1-C}} = 113.3\Omega \qquad Z_{oo} = Z_0\sqrt{\frac{1-C}{1+C}} = 31.8\Omega$$

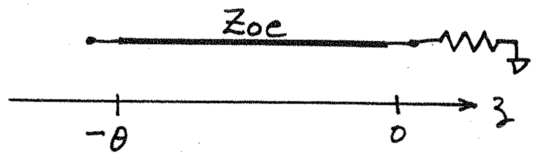
Then, $\sqrt{\epsilon_r}\,Z_{oe} = 168.1\Omega$ $\qquad \sqrt{\epsilon_r}\,Z_{oo} = 47.2\Omega$ $\qquad (\epsilon_r = 2.2)$

From Figure 7.29,

$$S/b \cong 0.009 \implies S = 0.029\text{ mm } (!)$$
$$W/b \cong 0.34 \implies W = 1.09\text{ mm}$$

This design is probably not practical due to the extremely close spacing of the lines.

(7.23) For V_{1e} or V_{1o} at port 1, we first find V_{2e} or V_{2o}:



$$\Gamma_e = \frac{Z_0 - Z_{0e}}{Z_0 + Z_{0e}}$$

$$1+\Gamma_e = \frac{2Z_0}{Z_0 + Z_{0e}}$$

$$V(z) = V^+\left(e^{-j\beta z} + \Gamma_e e^{j\beta z}\right)$$

$$V_{1e} = V(-\theta) = V^+\left(e^{j\theta} + \Gamma_e e^{-j\theta}\right)$$

$$V_{2e} = V(0) = V^+(1+\Gamma_e)$$

So,

$$V_{2e} = \frac{V_{1e}(1+\Gamma_e)}{e^{j\theta} + \Gamma_e e^{-j\theta}} = \frac{2Z_0 V_{1e}}{(Z_0+Z_{0e})e^{j\theta} + (Z_0 - Z_{0e})e^{-j\theta}}$$

$$= \frac{Z_0 V_{1e}}{Z_0\cos\theta + jZ_{0e}\sin\theta} = \frac{Z_0 V_{1e}\sec\theta}{Z_0 + jZ_{0e}\tan\theta}$$

Similarly,

$$V_{2o} = \frac{Z_0 V_{1o}\sec\theta}{Z_0 + jZ_{0o}\tan\theta}$$

Then using (7.74) and the results following (7.79) gives,

$$V_4 = V_{2e} - V_{2o} = \frac{Z_0 V\sec\theta}{2Z_0 + j(Z_{0e}+Z_{0o})\tan\theta}\left[\frac{Z_0 + jZ_{0e}\tan\theta}{Z_0 + jZ_{0e}\tan\theta} - \frac{Z_0 + jZ_{0o}\tan\theta}{Z_0 + jZ_{0o}\tan\theta}\right]$$

$$V_2 = V_{2e} + V_{2o} = \frac{2Z_0 V\sec\theta}{2Z_0 + j(Z_{0e}+Z_{0o})\tan\theta} = \frac{\frac{2Z_0}{Z_{0e}+Z_{0o}}V}{\frac{2Z_0}{Z_{0e}+Z_{0o}}\cos\theta + j\sin\theta}$$

$$= \frac{V\sqrt{1-c^2}}{\sqrt{1-c^2}\cos\theta + j\sin\theta} \checkmark \qquad \text{since } \sqrt{1-c^2} = \frac{2Z_0}{Z_{0e}+Z_{0o}}$$

(7.24) $N=3,\ C=20\,dB$, maximally flat, $Z_o=50\Omega$

(a) From (7.90) and Example

$$C=\left|\frac{V_3}{V_1}\right|=2\sin\theta\left[C_1\cos 2\theta+\tfrac{1}{2}C_2\right] \qquad ;\ C_3=C_1$$

$$=C_1\sin 3\theta+(C_2-C_1)\sin\theta \qquad \text{for } \theta=\pi/2,\ C=C_o=C_2-2C_1$$

$$\frac{dC}{d\theta}=\left[3C_1\cos 3\theta+(C_2-C_1)\cos\theta\right]\Big|_{\pi/2}\equiv 0$$

$$\frac{d^2C}{d\theta^2}=\left[-9C_1\sin 3\theta-(C_2-C_1)\sin\theta\right]\Big|_{\pi/2}=10C_1-C_2=0$$

at midband $(\theta=\pi/2)$, $C_o=10^{-20/20}=0.1=C_2-2C_1$

$$\therefore\ C_1=C_3=0.1/8=0.0125$$

$$C_2=10C_1=0.125$$

Using (7.87) gives Z_{oe}, Z_{oo}:

$$Z_{oe}^{(1)}=Z_{oe}^{(3)}=Z_o\sqrt{\frac{1+C_1}{1-C_1}}=50.63\Omega$$

$$Z_{oo}^{(1)}=Z_{oo}^{(3)}=Z_o\sqrt{\frac{1-C_1}{1+C_1}}=49.38\Omega$$

$$Z_{oe}^{(2)}=Z_o\sqrt{\frac{1+C_2}{1-C_2}}=56.69\Omega$$

$$Z_{oo}^{(2)}=Z_o\sqrt{\frac{1-C_2}{1+C_2}}=44.10\Omega$$

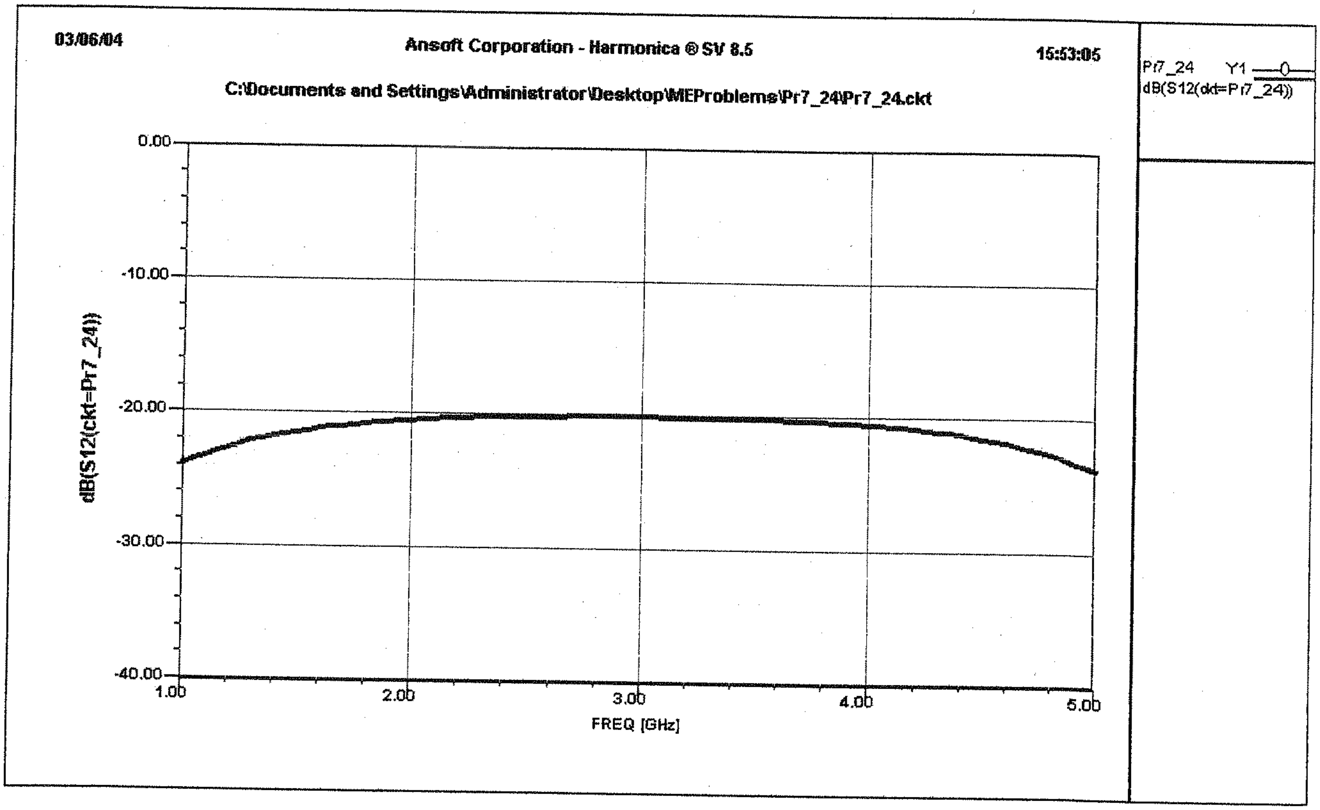
(b) $\epsilon_r=4.2,\ d=0.158\,cm,\ \tan\delta=0.02$, Cu, $0.5\,mil=t$.

From Serenade,

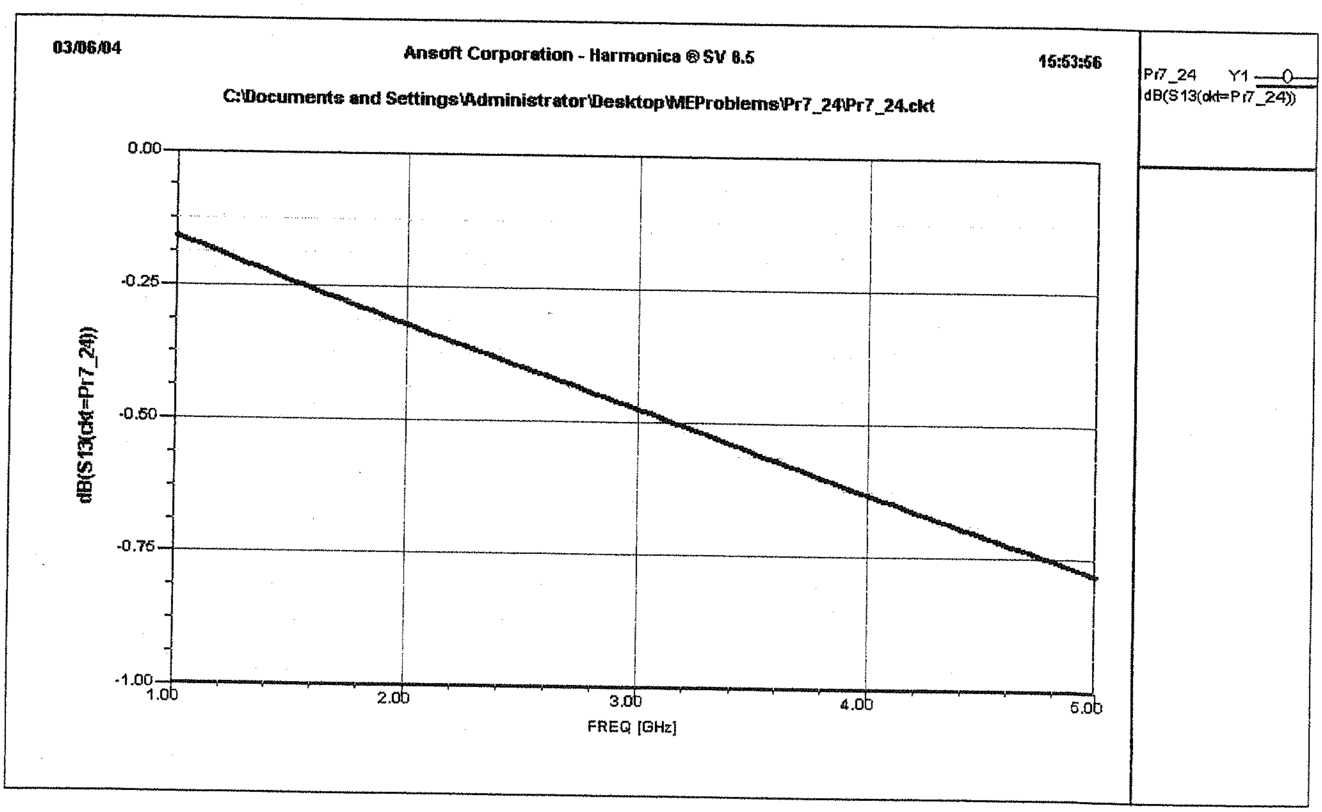
$Z_{oe}=50.63,\ Z_{oo}=49.38 \Rightarrow W=3.125\,mm,\ S=9.941\,mm,\ l=1.38\,cm$

$Z_{oe}=56.69,\ Z_{oo}=44.10 \Rightarrow W=3.054\,mm,\ S=1.601\,mm,\ l=1.39\,cm$

The resulting coupling and insertion loss are plotted on the following page.



03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
15:53:05
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_24\Pr7_24.ckt
Pr7_24 Y1
dB(S12(ckt=Pr7_24))
0.00
-10.00
-20.00
-30.00
-40.00
dB(S12(ckt=Pr7_24))
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]



03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
15:53:56
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_24\Pr7_24.ckt
Pr7_24 Y1
dB(S13(ckt=Pr7_24))
0.00
-0.25
-0.50
-0.75
-1.00
dB(S13(ckt=Pr7_24))
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

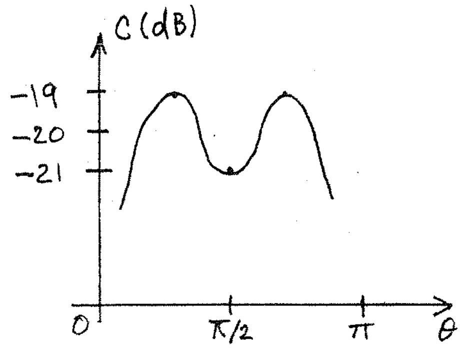
(7.25) $C = 20\,dB$, equal-ripple (1dB), $N=3$, $Z_0 = 50\Omega$

(a) From (7.90) and Example 7.8:

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[C_1\cos 2\theta + \tfrac{1}{2}C_2\right]$$

$$= C_1 \sin 3\theta + (C_2 - C_1)\sin\theta$$

We cannot equate this to a Chebyshev polynomial, since C is a polynomial in $\sin\theta$. The desired response is as shown:



So at $\theta = \pi/2$, $C_0 = -21dB = 0.08913$
$= C_2 - 2C_1$

$C_{MAX} = -19dB = 0.1122$

We can use trial-and-error to find C_1:

C_1	C_{MAX}
.05	.134
.06	.147
.03	.106
$\boxed{.035}$	$\boxed{.1125}$

Thus $C_1 = 0.035$, $C_2 = 0.1591$

Then,

$Z_{0e}^{(1)} = Z_{0e}^{(3)} = 51.78\,\Omega$

$Z_{00}^{(1)} = Z_{00}^{(3)} = 48.28\,\Omega$

$Z_{0e}^{(2)} = 58.70\,\Omega$

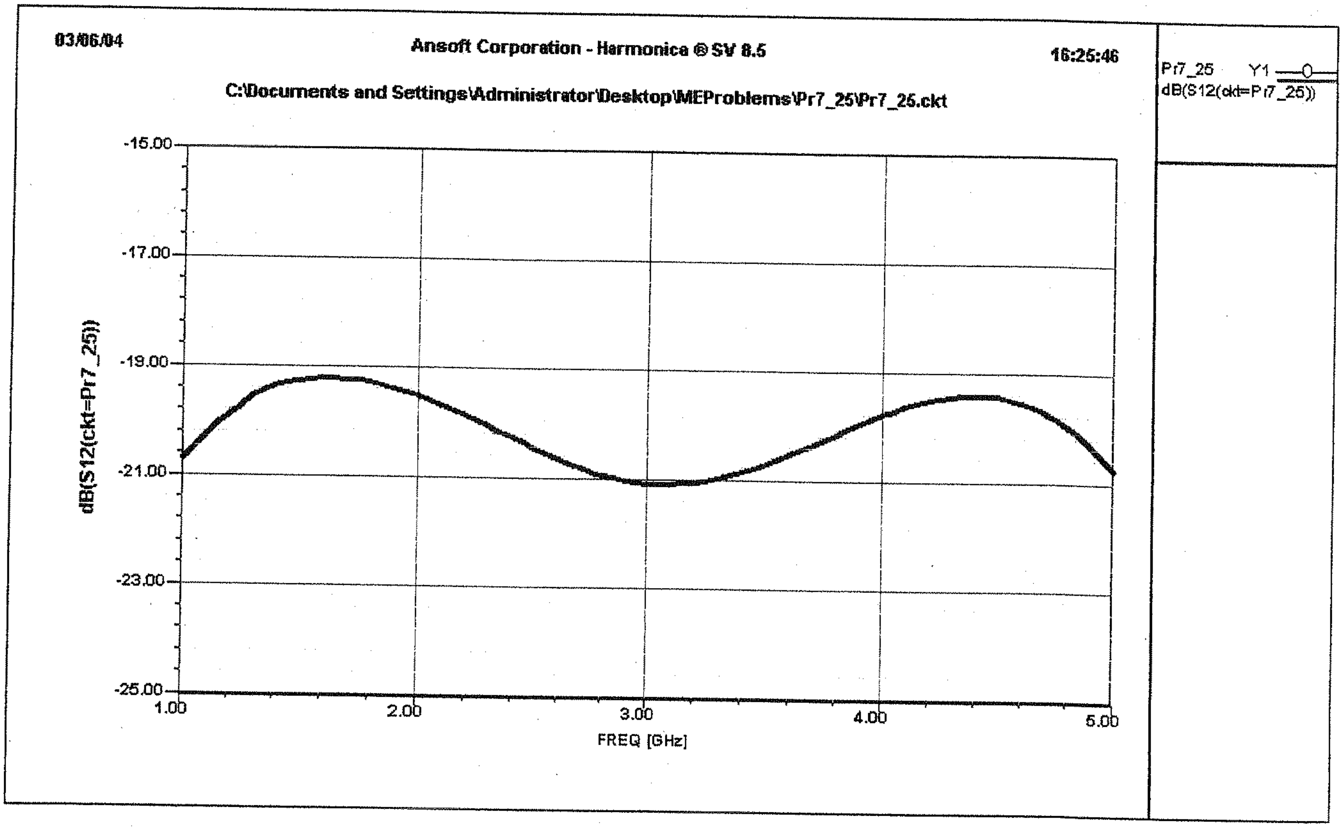
$Z_{00}^{(2)} = 42.59\,\Omega$

(b) From Serenade,

$Z_{0e} = 51.78$, $Z_{00} = 48.28 \Rightarrow W = 3.121\,mm$, $S = 4.842\,mm$, $l = 1.38\,cm$

$Z_{0e} = 58.70$, $Z_{00} = 42.59 \Rightarrow W = 3.002\,mm$, $S = 1.173\,mm$, $l = 1.39\,cm$

The coupling and insertion loss are plotted on the following page.


03/06/04
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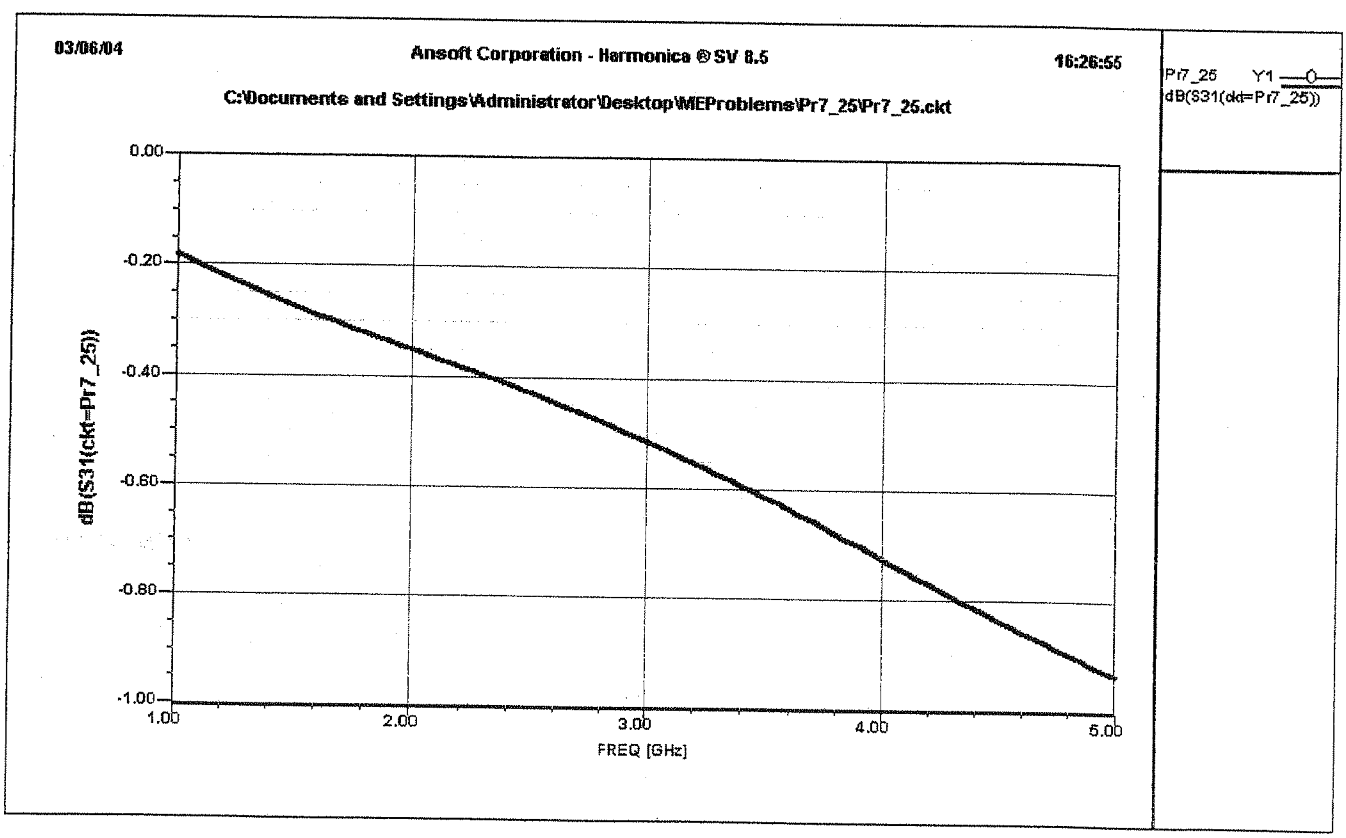
16:25:46
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_25\Pr7_25.ckt
Pr7_25 Y1
dB(S12(ckt=Pr7_25))
dB(S12(ckt=Pr7_25))
-15.00
-17.00
-19.00
-21.00
-23.00
-25.00
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]


03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
16:26:55
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_25\Pr7_25.ckt
Pr7_25 Y1
dB(S31(ckt=Pr7_25))
dB(S31(ckt=Pr7_25))
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

7.26 From (7.98) and (7.99) we can show that,

$$Z_{e4} = Z_0\sqrt{\frac{1+C}{1-C}}$$

Equating this to (7.97a) gives:

$$Z_0\sqrt{\frac{1+C}{1-C}} = \frac{Z_{oo}+Z_{oe}}{3Z_{oo}+Z_{oe}} = \frac{1+\frac{Z_{oe}}{Z_{oo}}}{3+\frac{Z_{oe}}{Z_{oo}}}\, Z_{oe}$$

Now solve (7.99) for Z_{oe} in terms of Z_{oo}:

$$3C\,(Z_{oe}^2+Z_{oo}^2) + 2C\, Z_{oe} Z_{oo} = 3(Z_{oe}^2 - Z_{oo}^2)$$

$$3(C-1)\,Z_{oe}^2 + 2C\, Z_{oe} Z_{oo} + 3(C+1)\, Z_{oo}^2 = 0$$

$$Z_{oe} = \frac{-2C Z_{oo} \pm \sqrt{4C^2 Z_{oo}^2 - 36(C^2-1) Z_{oo}^2}}{6(C-1)} = \frac{-C-\sqrt{9-8C^2}}{3(C-1)}\, Z_{oo}$$

(choose negative root since Z_{oe} and Z_{oo} are positive, and $c<1$)
Substituting for Z_{oe}/Z_{oo} in the above expression gives,

$$Z_0\sqrt{\frac{1+C}{1-C}} = \frac{2C-3-\sqrt{9-8C^2}}{8C-9-\sqrt{9-8C^2}}\, Z_{oe}$$

or

$$Z_{oe} = Z_0\sqrt{\frac{1+C}{1-C}}\; \frac{\left[8C-9-\sqrt{9-8C^2}\right]\left[2C-3+\sqrt{9-8C^2}\right]}{(2C-3)^2-(9-8C^2)}$$

$$= Z_0\sqrt{\frac{1+C}{1-C}}\; \frac{(24C^2-42C+18)+6(C-1)\sqrt{9-8C^2}}{12C(C-1)}$$

$$= Z_0\sqrt{\frac{1+C}{1-C}}\; \frac{4C-3+\sqrt{9-8C^2}}{2C} \quad ✓$$

To find Z_{oo}, simply replace C by $-C$ (because of symmetry of (7.98) and (7.99):

$$Z_{oo} = Z_0\sqrt{\frac{1-C}{1+C}}\; \frac{4C+3-\sqrt{9-8C^2}}{2C} \quad ✓$$

(7.27) $f = 5\,GHz$, $\epsilon_r = 10$, $d = 1\,mm$

$C = 10^{-3/20} = 0.708$

assume $Z_0 = 50\,\Omega$ (not stated in problem!)

From (7.100) we have,

$$Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{\frac{1-C}{1+C}}} Z_0 = 176.4\,\Omega$$

$$Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{\frac{1+C}{1-C}}} Z_0 = 52.5\,\Omega$$

(These results are very approximate; SuperCompact gives $Z_{0e} = 121\,\Omega$ and $Z_{0o} = 21\,\Omega$ for this design)

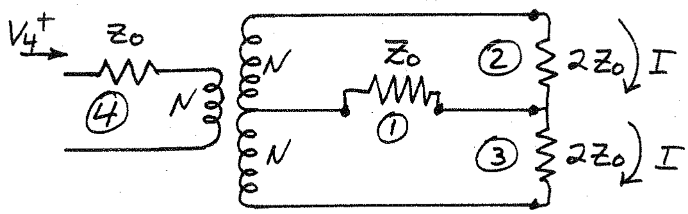
From Figure 7.30,

$$S/d \cong 0.075 \implies S = 0.075\,mm$$

$$W/d \cong 0.07 \implies W = 0.07\,mm$$

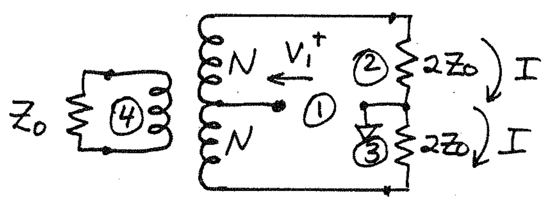
(SuperCompact gives 0.071 mm and 0.075 mm for S and W, respectively, starting with the values calculated above for Z_{0e} and Z_{0o}. These are reasonably close to our values for S and W, even though Z_{0e}, Z_{0o} are not close to the SuperCompact values.)

(7.28) First consider an incident wave at port 4, with the other ports matched:



There is no voltage drop across the resistor at port 1, so $S_{41} = S_{14} = 0$. The load impedance across the secondary is $4Z_0$, so the input impedance at port 4 is $Z_{in} = (N/2N)^2(4Z_0) = Z_0$, so $S_{44} = 0$. The voltages at ports 2 and 3 have the same magnitude, but opposite signs (relative to the center terminal). Power conservation then gives $S_{24} = S_{42} = 1/\sqrt{2}$; $S_{34} = S_{43} = -1/\sqrt{2}$.

Now consider an incident wave at port 1, with matched loads at the other ports:



This excites the transformer in an "odd mode", so $V_4 = 0$. (consistent with $S_{41} = 0$). Ports 2 and 3 are now equally excited, so $S_{21} = S_{12} = S_{31} = S_{13} = 1/\sqrt{2}$. The input impedance at port 1 is Z_0, so $S_{11} = 0$.

Finally, the unitary properties of the S-matrix for a lossless network lead to $S_{22} = S_{33} = 0$. Thus, the S-matrix is similar in form to (7.101).

(7.29) From (7.101) the [S] matrix of a 180° (3dB) hybrid is,

$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

For V_1 at port 1 and V_4 at port 4, the output voltages are (note: hybrid is matched, so $V_1 = V_1^+$, $V_4 = V_4^+$)

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ 0 \\ 0 \\ V_4 \end{bmatrix}$$

$$= \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 \\ V_1 - V_4 \\ V_1 + V_4 \\ 0 \end{bmatrix} \begin{matrix} ✓ \\ ✓ \\ ✓ \\ ✓ \end{matrix} \quad \begin{matrix} \\ \text{(difference)} \\ \text{(sum)} \\ \\ \end{matrix}$$

(7.30) $\alpha = \beta = \sqrt{2}/2$ for $C = 3dB$

From (7.115a), $\beta = \frac{2\sqrt{k}}{k+1} \Longrightarrow k = 0.1716$ ✓

From (7.115b), $\alpha = \frac{1-k}{1+k} \Longrightarrow k = 0.1716$ ✓

Then,

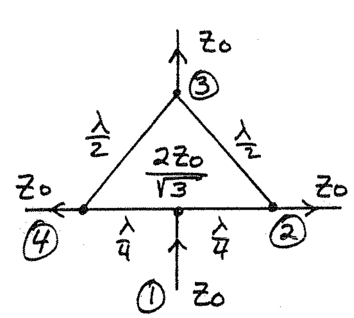
$$Z_{oe}(0) = Z_{oo}(0) = Z_o = 50\,\Omega$$

$$Z_{oe}(L) = Z_o/k = 291\,\Omega$$

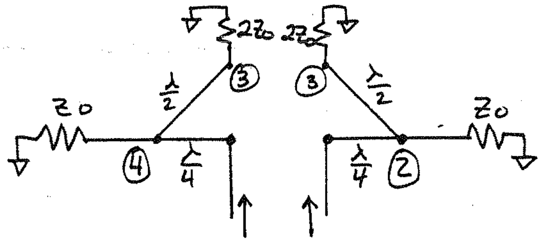
$$Z_{oo}(L) = k Z_o = 8.6\,\Omega$$

a Klopfenstein taper can be used for these taper variations.

(7.31)



First, let $V_1^+ = 1v$ at port 1, with matched loads at other ports. Then we can bisect the network as follows:



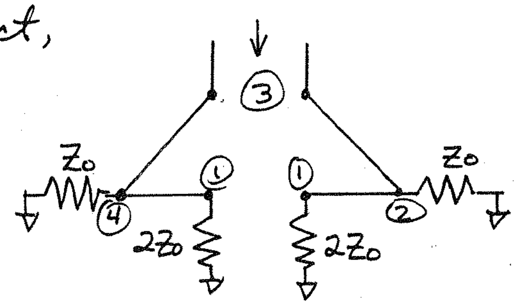
The input impedance of one of these halves is,

$\left(\frac{2Z_0}{\sqrt{3}}\right)^2 \frac{(Z_0 + 2Z_0)}{2Z_0^2} = 2Z_0$, so $Z_{in}^{①} = Z_0$, and $S_{11} = 0$. ✓

Because of the $\lambda/2$ line, the voltage magnitude at port 3 is equal to the voltage magnitude at port 4. By power conservation, $P_2 = P_4 = P_3 = P_{in}/3$. Thus,

$S_{41} = S_{21} = 1/\sqrt{3}\,\underline{/-90°}$, $S_{31} = 1/\sqrt{3}\,\underline{/-270°}$. $(|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1)$

Next, let $V_3^+ = 1v$ at port 3, with other ports matched, and bisect,



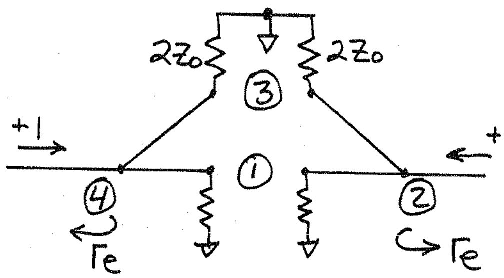
The input impedance of one of these halves is

$Z_0 \| \left(\frac{2Z_0}{\sqrt{3}}\right)^2 \frac{1}{2Z_0} = Z_0 \| \frac{2Z_0}{3} = \frac{2Z_0}{5}$, so $Z_{in}^{③} = Z_0/5$, and $S_{33} = -2/3$.

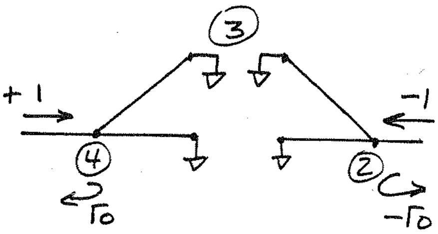
So the power delivered to each half is $\frac{1}{2} P_{inc}(1 - |S_{33}|^2) = 5/18\, P_{inc}$. Of this, 2/5 goes to port 4 and 3/5 goes to port 1.

So, $S_{43} = S_{23} = 1/3\,\underline{/-180°}$. The total power to port 1 is then $1/3\, P_{inc}$, so $S_{13} = 1/\sqrt{3}\,\underline{/-270°}$. (Then $|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{43}|^2 = \frac{1}{3} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} = 1$)

Now drive ports 2 and 4 with even and odd excitations:



EVEN MODE



ODD MODE

$Z_{in}^{e} = 2Z_0 \| \frac{2Z_0}{3} = Z_0/2$

$\Gamma_e = \frac{1/2 - 1}{1/2 + 1} = -1/3$

$Z_{in}^{o} = 0$

$\Gamma_o = -1$

Then, $S_{22} = S_{44} = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{1}{2}(-1/3 - 1) = -2/3$

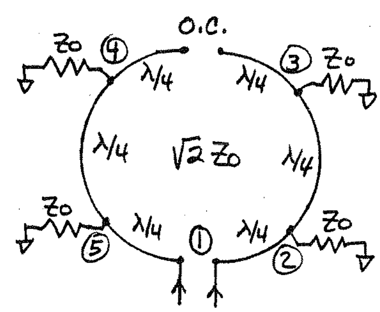
$S_{24} = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{1}{2}(-1/3 + 1) = 1/3$

So the complete S-matrix is,

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{3}\angle -90 & 1/\sqrt{3}\angle -270 & 1/\sqrt{3}\angle -90 \\ 1/\sqrt{3}\angle -90 & -2/3 & -1/3 & 1/3 \\ 1/\sqrt{3}\angle -270 & -1/3 & -2/3 & -1/3 \\ 1/\sqrt{3}\angle -90 & 1/3 & -1/3 & -2/3 \end{bmatrix} \checkmark$$

This checks with an analysis using SuperCompact.

(7.32) Let $V_1^+ = 1\angle 0$



Bisecting the network places an effective short circuit at ports 3 and 4, due to the λ/4 O.C. stubs. Thus $S_{41} = S_{31} = 0$. Then there is an effective open circuit in parallel with the Z_0 loads at ports 2 and 5. So the input impedance of one of the halves is $(\sqrt{2}Z_0)^2/Z_0 = 2Z_0$. The total input impedance at port 1 is then Z_0, so $S_{11} = 0$, and the input power divides evenly to ports 2 and 5. Thus, $V_1^- = 0$; $V_2^- = V_5^- = 0.707\angle -90°$; $V_3^- = V_4^- = 0$.

CHECK: $|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 0 + \frac{1}{2} + 0 + \frac{1}{2} = 1$ ✓

(7.33)

$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \quad \text{for hybrid}$$

a) let $b = \lambda/4$ at $f_o \Rightarrow \beta b = \pi/2$

Assume V_o^+ incident at T-junction. Then,

$$V_1^+ = V_o^+ e^{-j\beta(b-a)} = e^{-j\pi/2} e^{j\beta a}$$

$$V_4^+ = V_o^+ e^{-j\beta a}$$

$$V_2^- = \frac{-V_o^+}{\sqrt{2}}(jV_1^+ + V_4^+) = \frac{-V_o^+}{\sqrt{2}}(e^{j\beta a} + e^{-j\beta a}) = -\sqrt{2}\, V_o^+ \cos\beta a$$

$$P_2 = \tfrac{1}{2}|V_2^-|^2 = |V_o^+|^2 \cos^2\beta a$$

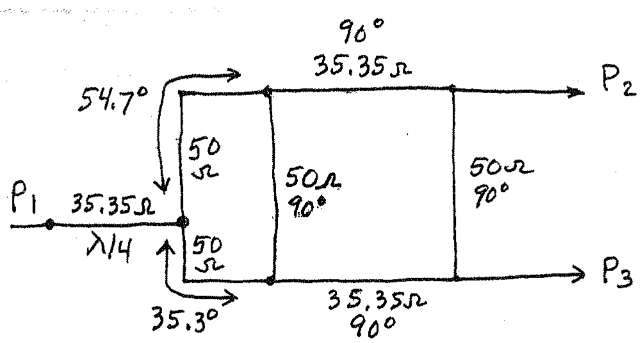
$$V_3^- = \frac{-V_o^+}{\sqrt{2}}(V_1^+ + jV_4^+) = \frac{-V_o^+}{\sqrt{2}}(-je^{j\beta a} + je^{-j\beta a}) = -\sqrt{2}\, V_o^+ \sin\beta a$$

$$P_3 = \tfrac{1}{2}|V_3^-|^2 = |V_o^+|^2 \sin^2\beta a$$

So $\frac{P_3}{P_2} = \tan^2\beta a = \tan^2\frac{\pi a}{2b}$ ✓ (since $\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{4b} = \frac{\pi}{2b}$)

b) For $\frac{P_3}{P_2} = 0.5$, $a = 0.098\lambda = 35.3°$; $(b-a) = 0.152\lambda = 54.7°$

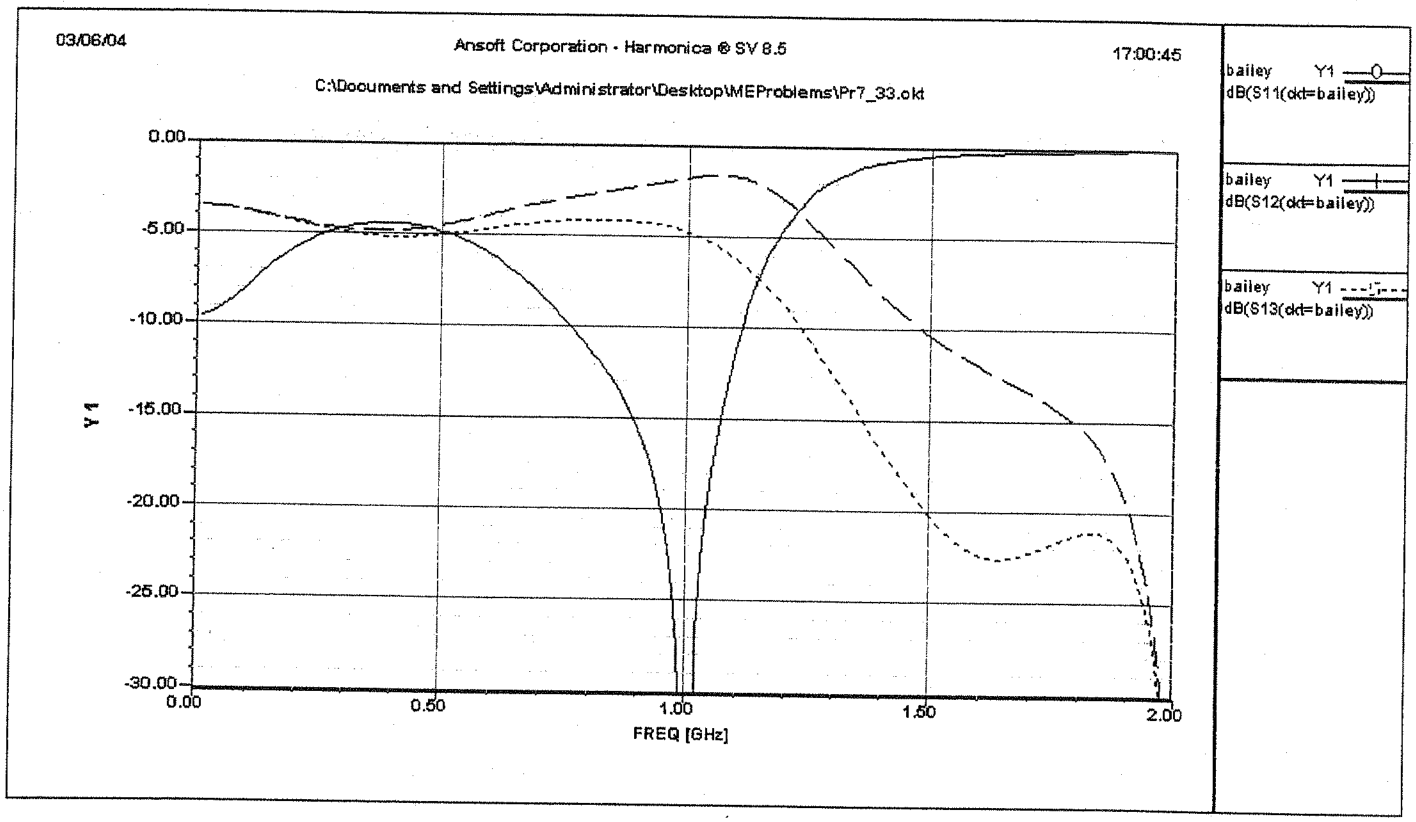
CIRCUIT:



The S-parameters are plotted on the following page, for a center frequency of 1 GHz. Note that the power output ratios,

$$\frac{P_2}{P_1} = \frac{2}{3} = -1.76\,dB \quad \text{and} \quad \frac{P_3}{P_1} = \frac{1}{3} = -4.77\,dB$$

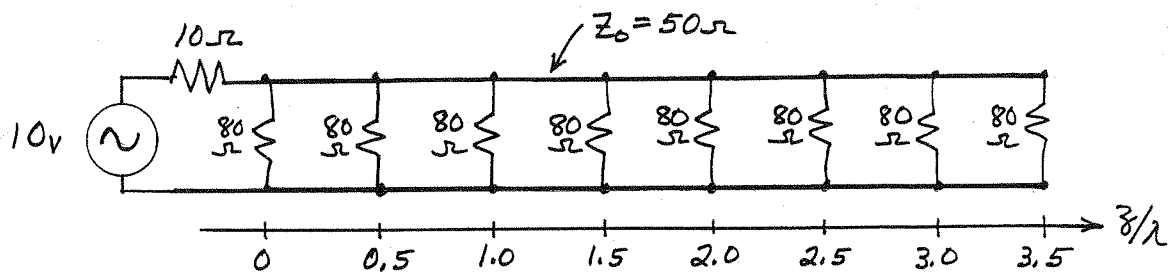
are verified.



03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
17:00:45
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_33.ckt
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
Y1
0.00
0.50
1.00
1.50
2.00
FREQ [GHz]
bailey Y1
dB(S11(ckt=bailey))
bailey Y1
dB(S12(ckt=bailey))
bailey Y1
dB(S13(ckt=bailey))

Chapter 8

8.1



The input impedance is $80\Omega/8 = 10\Omega$, so $V(0) = 5v$. In between the resistors, we must use the transmission line equations. For $0 < z < \lambda/2$, we have,

$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x}) \quad , \text{ where } x = z - \lambda/2$$

$$V(z=0) = V(x = -\lambda/2) = V^+(-1-\Gamma) = 5v$$

Thus,

$$V(x) = \frac{-5}{1+\Gamma}(e^{-j\beta x} + \Gamma e^{j\beta x}) \quad , \quad \Gamma = \frac{80/7 - 50}{80/7 + 50} = -0.628$$

$$|V(z)| = \frac{5}{1+\Gamma}\left|1 + \Gamma e^{-2j\beta z}\right| \quad \text{for } 0 < z < \lambda/2$$

So $|V(\lambda/2)| = +5v \quad (V(\lambda/2) = -5v)$. The peak occurs for $z = \lambda/4$:

$$|V(z=\lambda/4)| = \frac{5}{1+\Gamma}(1-\Gamma) = 5\,\frac{Z_0}{Z_L} = \frac{5(50)}{80/7} = 21.8v$$

Intermediate values can also be calculated in the same manner. For $\lambda/2 < z < \lambda$ we repeat the above procedure, but with $Z_L = 80/6$. Thus we have,

$|V(z = 0.75\lambda)| = 18.8v$ ✓

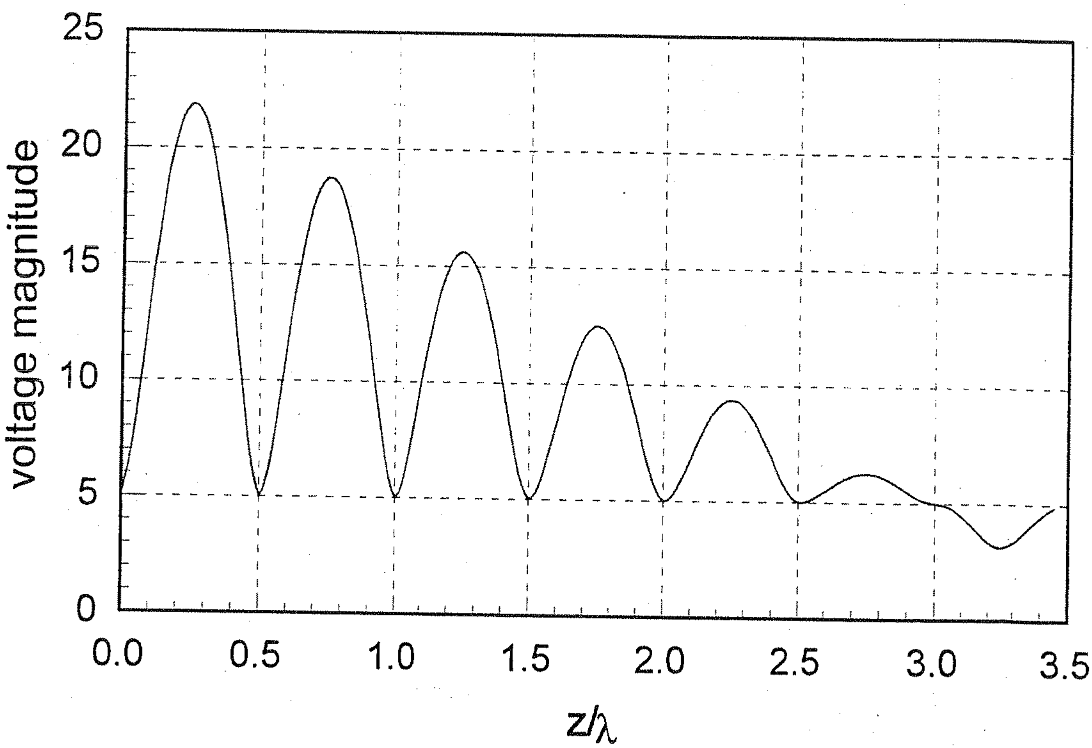
$|V(z = 1.75\lambda)| = 12.5v$ ✓

$|V(z = 2.25\lambda)| = 9.37v$ ✓

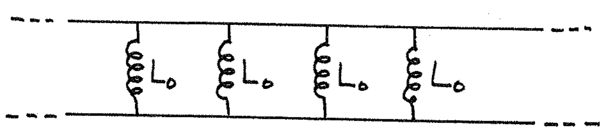
$|V(z = 2.75\lambda)| = 6.25v$ ✓

$|V(z = 3.25\lambda)| = 3.12v$ ✓

$|V(z)|$ is plotted on the following page.



(8.2)



$Z_o = 100\Omega$; $d = 1.0\,cm$
$k = k_o$; $L_o = 3\,nH$

let $\theta = k_o d$, $\quad b = \dfrac{-Z_o}{\omega L_o} = \dfrac{-Z_o}{c k_o L_o} = \dfrac{-111}{k_o}$

From (8.9) a passband occurs when,

$$|\cos\beta d| = \left|\cos\theta - \frac{b}{2}\sin\theta\right| \leq 1 \,,$$

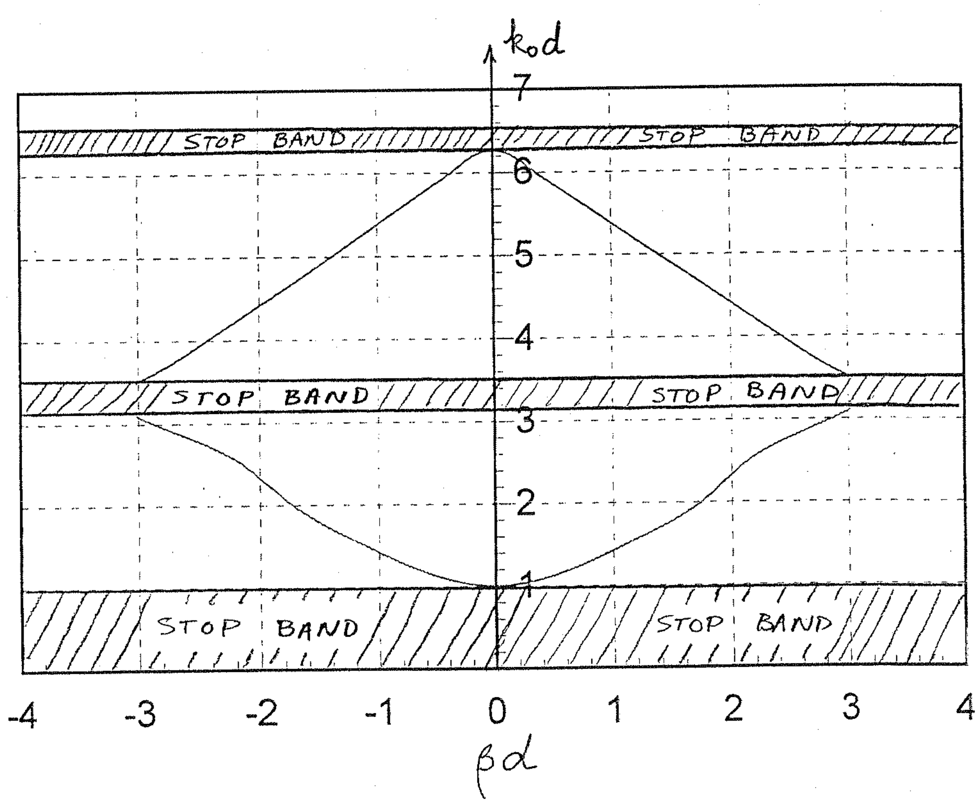
and a stopband occurs when,

$$\cosh\alpha d = \left|\cos\theta - \frac{b}{2}\sin\theta\right| \geq 1$$

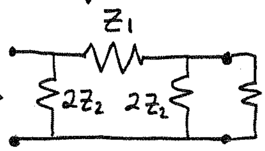
so we can compute the following data table:

	k_o	θ	$\cos\theta - b/2 \sin\theta$	βd (rad)
STOP	10	5.7°	1.55	—
	30	17.2°	1.50	—
	100	57.3°	1.007	—
PASS	110	63.0°	0.903	0.444
	150	85.9°	0.441	1.11
	200	114.6°	−0.164	1.74
	250	143.2°	−0.535	2.14
	300	171.9°	−0.964	2.87
	310	177.6°	−0.992	3.02
STOP	320	183.3°	−1.02	—
	340	194.5°	−1.009	—
PASS	350	200.5°	−0.992	3.02
	360	206.3°	−0.965	2.87
	400	229.2°	−0.758	2.43
	450	257.8°	−0.332	1.91
	500	286.5°	0.177	1.39
	550	315.1°	0.637	0.88
	600	343.8°	0.934	0.36
	625	358.1°	0.996	0.08

These passbands and stopbands are plotted below:



(8.3) $Z_{i\pi}$ can easily be derived from $Z_i = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{B}{C}}$.
Verify as follows:

$Z_{in} \Rightarrow$ 

$$Z_i = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1 + Z_1/4Z_2}} = \frac{Z_2\sqrt{Z_1}}{\sqrt{Z_2 + Z_1/4}}$$

let $$Z = 2Z_2 \| Z_i = \frac{\dfrac{2Z_2^2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}}}{2Z_2 + \dfrac{Z_2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}}} = \frac{2Z_2\sqrt{Z_1}}{2\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}}$$

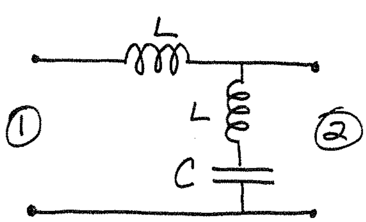
$$Z_{in} = 2Z_2 \| (Z_1 + Z) = \frac{\dfrac{4Z_1Z_2\sqrt{Z_2+Z_1/4} + 2Z_2\sqrt{Z_1}(2Z_2+Z_1)}{2\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}}}{2Z_2 + \dfrac{2Z_1\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}(2Z_2+Z_1)}{2\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}}}$$

$$= \frac{4Z_1Z_2\sqrt{Z_2+Z_1/4} + Z_2\sqrt{Z_1}(2Z_2+Z_1)(2)}{4Z_2\sqrt{Z_2+Z_1/4} + 2Z_2\sqrt{Z_1} + 2Z_1\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}(2Z_2+Z_1)}$$

$$= \frac{Z_2\left[\sqrt{Z_1}(2Z_2+Z_1) + 2Z_1\sqrt{Z_2+Z_1/4}\right]}{2\sqrt{Z_1}(Z_2+Z_1/4) + Z_1\sqrt{Z_2+Z_1/4} + 2Z_2\sqrt{Z_2+Z_1/4}}$$

$$= \frac{Z_2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}} = Z_i \quad \checkmark$$

(8.4)



$$\frac{1}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega C}{1-\omega^2 LC}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\omega C}{1-\omega^2 LC} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1-2\omega^2 LC}{1-\omega^2 LC} & j\omega L \\ \frac{j\omega C}{1-\omega^2 LC} & 1 \end{bmatrix} \quad \checkmark$$

From (8.27),

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{j\omega L(1-2\omega^2 LC)}{j\omega C}} = \sqrt{\frac{L}{C}(1-2\omega^2 LC)} \quad \checkmark$$

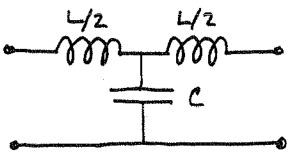
$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \frac{1}{1-\omega^2 LC}\sqrt{\frac{j\omega L}{j\omega C(1-2\omega^2 LC)}} = \frac{1}{1-\omega^2 LC}\sqrt{\frac{L}{C(1-2\omega^2 LC)}}$$

From (8.31), $\cosh\gamma = \sqrt{AD} = \sqrt{\dfrac{1-2\omega^2 LC}{1-\omega^2 LC}}$.

(8.5) $R_0 = 50\,\Omega$, $f_c = 50$ MHz, $f_\infty = 52$ MHz, LOW-PASS

Design equations from Table 8.2:

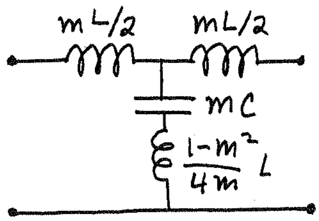
CONSTANT - k SECTION:



$L = 2R_0/\omega_c = 3.18 \times 10^{-7}$ H ; $L/2 = 159$ nH

$C = 2/\omega_c R_0 = 127.$ pF

m-DERIVED SECTION:



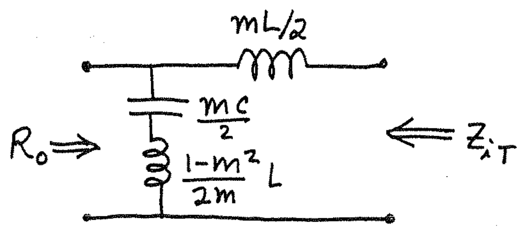
$m = \sqrt{1-(f_c/f_\infty)^2} = 0.275$

$\dfrac{mL}{2} = 43.7$ nH

$mC = 34.9$ pF

$\dfrac{1-m^2}{4m}L = 267.$ nH

MATCHING SECTIONS: $(m = 0.6)$

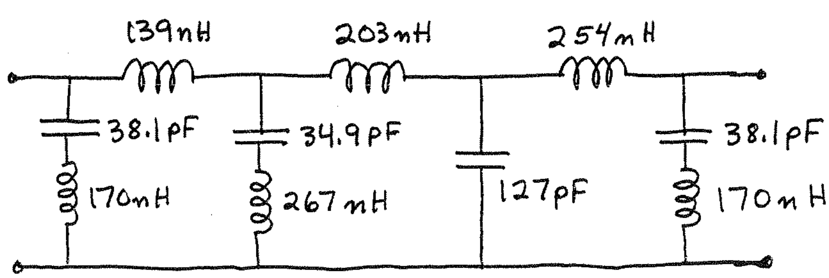


$\dfrac{mL}{2} = 95.4$ nH

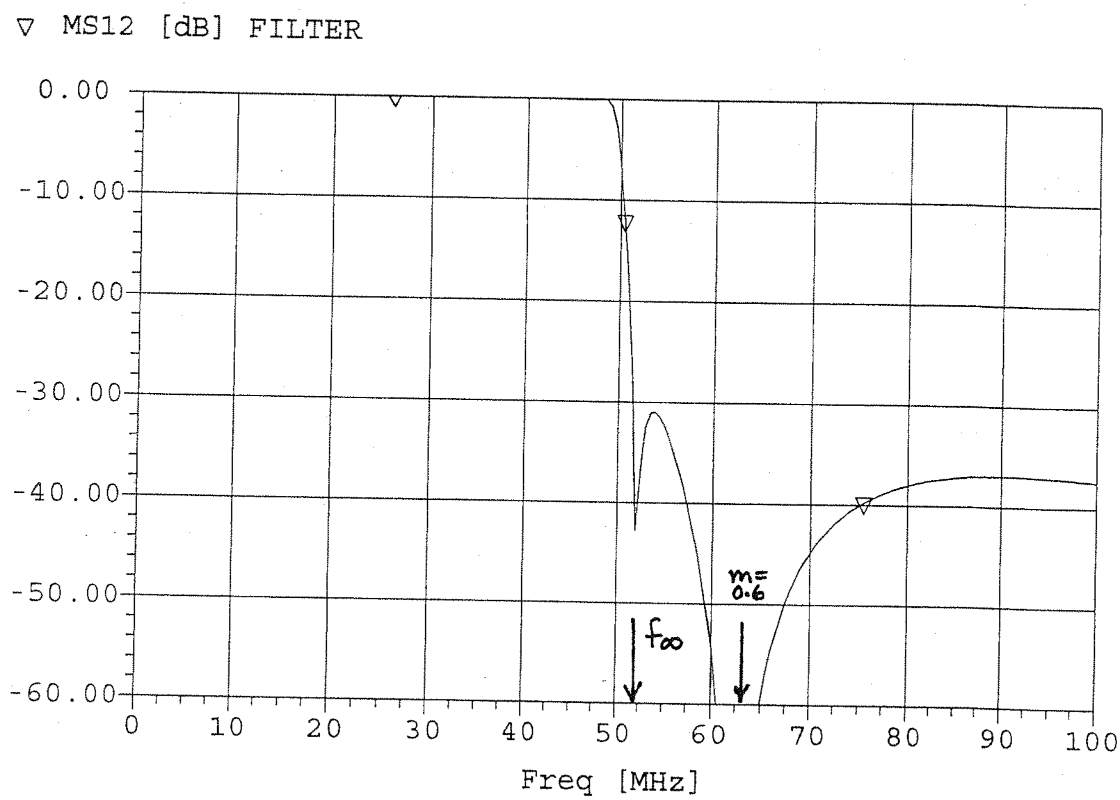
$\dfrac{mC}{2} = 38.1$ pF

$\dfrac{1-m^2}{2m}L = 170.$ nH

COMPLETE FILTER:



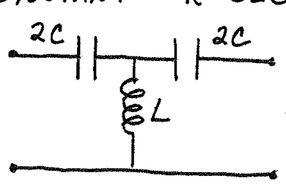
The calculated response is shown on the following page.



(8.6) $R_o = 75\Omega$, $f_c = 50$ MHz, $f_\infty = 48$ MHz, HIGH-PASS

Design equations are given in Table 8.2

CONSTANT-k SECTION:

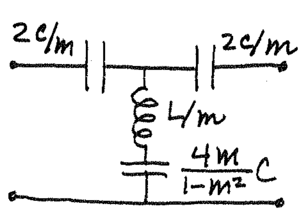


$L = R_o/2\omega_c = 119.$ nH

$C = 1/2R_o\omega_c = 21.2$ pF

$2C = 42.4$ pF

m-DERIVED SECTION:



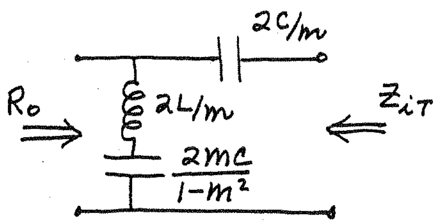
$m = \sqrt{1-(f_\infty/f_c)^2} = 0.280$

$\frac{2C}{m} = 151.$ pF

$L/m = 425.$ nH

$\frac{4mC}{1-m^2} = 25.8$ pF

MATCHING SECTION: $(m=0.6)$

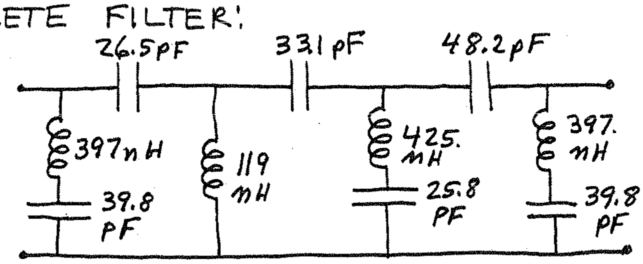


$$\frac{2C}{m} = 70.7\ pF \quad ✓$$

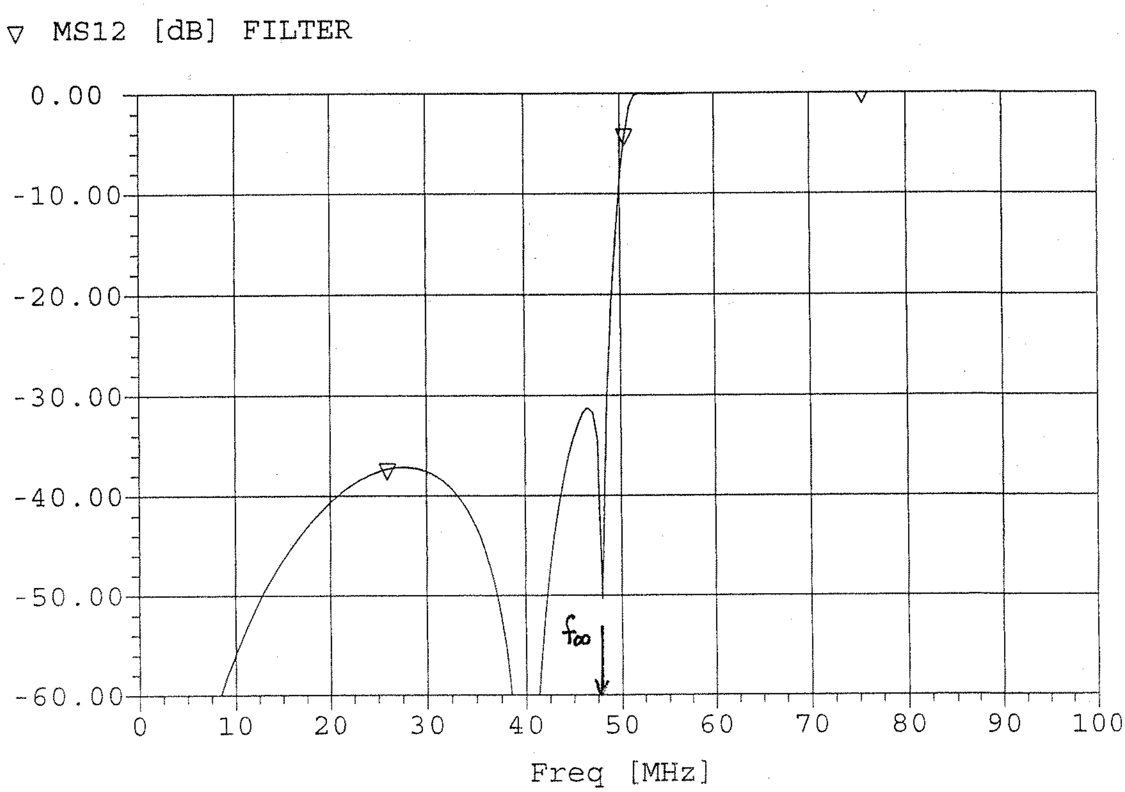
$$\frac{2L}{m} = 397.\ nH. \quad ✓$$

$$\frac{2mC}{1-m^2} = 39.8\ pF \quad ✓$$

COMPLETE FILTER:

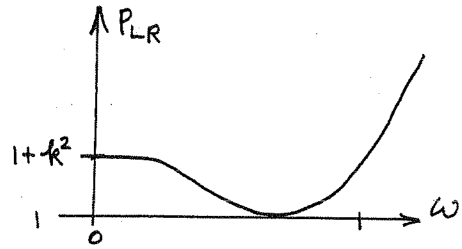


The calculated filter response is shown below.



(8.7) From (8.61),

$$P_{LR} = 1 + k^2 T_N^2(\omega) = 1 + k^2 (2\omega^2 - 1)^2 \quad \text{for } N = 2$$



$P_{LR} = 1 = 0\,dB$

$P_{LR} = 1 + k^2 = 1\,dB = 1.259$

so $k = \pm 0.509$

(We must choose $k = -0.509$, otherwise L, C are not real.)

Then from (8.63),

$$R = 1 + 2k^2 - 2k\sqrt{1+k^2} = 2.66 \checkmark$$

We also have that,

$$4k^2 = \frac{1}{4R} L^2 C^2 R^2 \implies L = \frac{-4k}{C\sqrt{R}}$$

$$-4k^2 = \frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2)$$

$$-16k^2 R = R^2 C^2 + \frac{16k^2}{C^2 R} + 8kR\sqrt{R}$$

$$R^2 C^4 + (16k^2 R + 8kR\sqrt{R})C^2 + \frac{16k^2}{R} = 0$$

$$7.08C^4 - 6.64C^2 + 1.56 = 0$$

Thus,

$$C = 0.685 \checkmark$$

$$L = 1.822 \checkmark$$

(R, L, C check with results given in Matthai, Young, and Jones.)

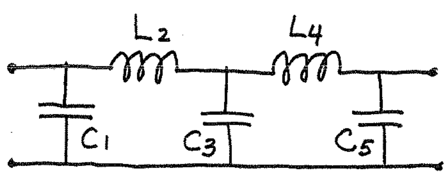
(8.8) $f_o = 3$ GHz, LOW-PASS, M.F., $Z_o = 75\,\Omega$, $\alpha = 20$ dB at 5 GHz

Following Example 8.3:

$$\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{5}{3} - 1 = 0.667,$$

so from Figure 8.26 we see that $N=5$ will give $\alpha > 20$ dB. Then from Table 8.3, the LP prototype values are,

$g_1 = 0.618$
$g_2 = 1.618$
$g_3 = 2.000$
$g_4 = 1.618$
$g_5 = 0.618$



Use (8.67) to scale cutoff frequency and impedance:

$$C_1 = \frac{g_1}{R_o \omega_c} = 0.437 \text{ pF} ✓$$

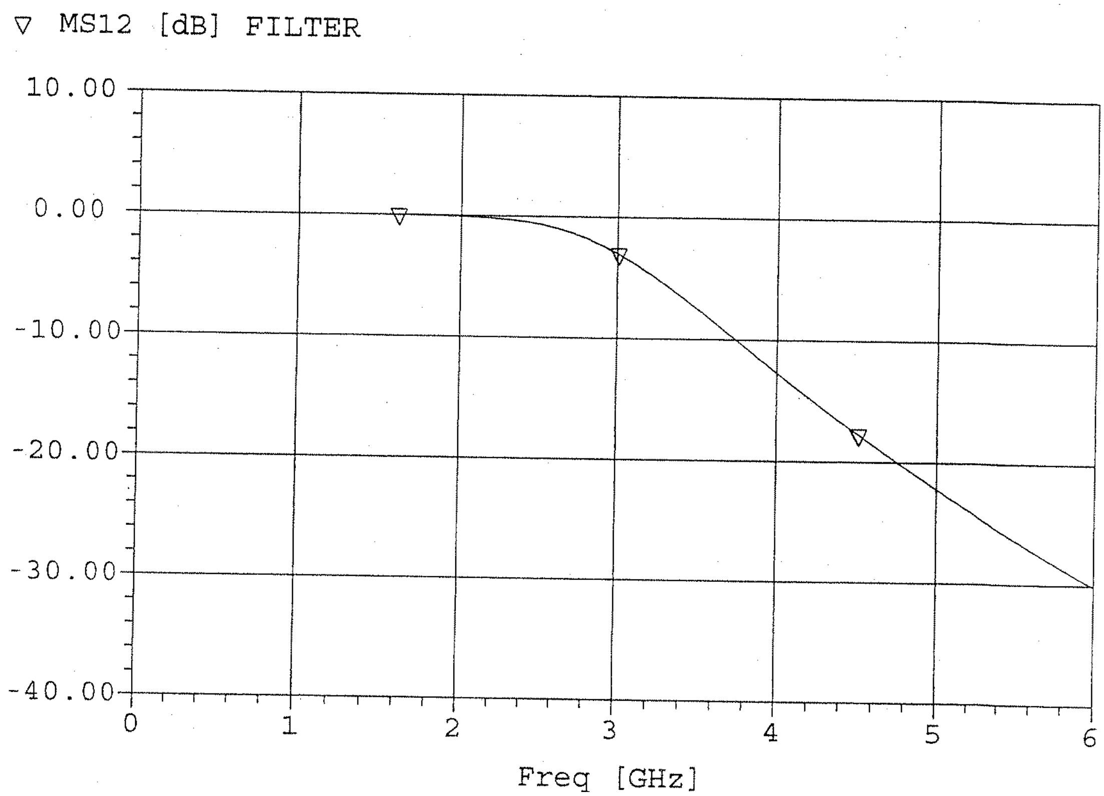
$$L_2 = \frac{R_o g_2}{\omega_c} = 6.44 \text{ nH} ✓$$

$$C_3 = \frac{g_3}{R_o \omega_c} = 1.41 \text{ pF} ✓$$

$$L_4 = \frac{R_o g_4}{\omega_c} = 6.44 \text{ nH} ✓$$

$$C_5 = \frac{g_5}{R_o \omega_c} = 0.437 \text{ pF} ✓$$

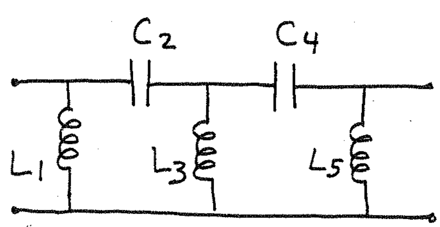
The calculated filter response is shown on the following page. Note that the insertion loss at 5 GHz is more than 20dB.



(8.9) $f_o = 1\,GHz$, HIGH PASS, 3dB E.R., $N=5$, $Z_o = 50\Omega$

at $f = 0.6\,GHz$, $\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{1}{.6} - 1 = 0.667$, so from Figure 8.27b, the attenuation for $N=5$ should be about 41 dB. From Table 8.4 the prototype values are,

$g_1 = 3.4817$
$g_2 = 0.7618$
$g_3 = 4.5381$
$g_4 = 0.7618$
$g_5 = 3.4817$



Impedance and frequency scaling using (8.70):

$$L_1 = \frac{Z_0}{\omega_c g_1} = 2.28\ \text{nH} \quad \checkmark$$

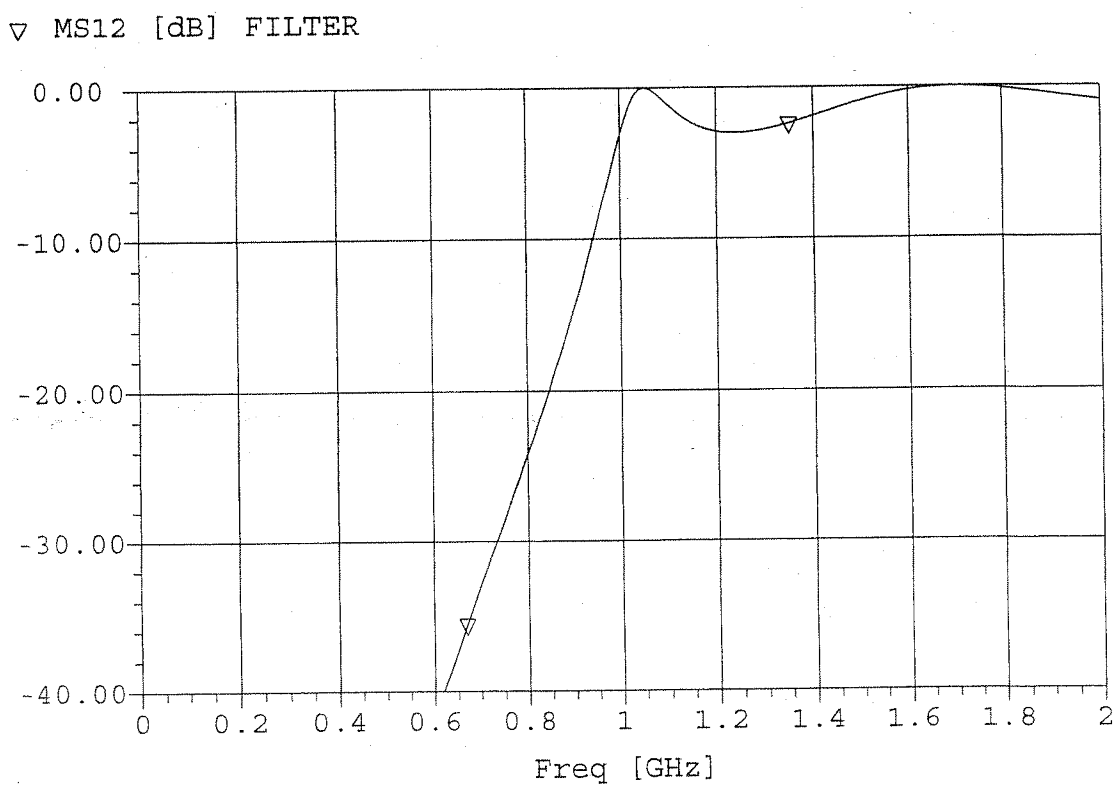
$$C_2 = \frac{1}{Z_0 \omega_c g_2} = 4.18\ \text{pF} \quad \checkmark$$

$$L_3 = \frac{Z_0}{\omega_c g_3} = 1.75\ \text{nH} \quad \checkmark$$

$$C_4 = \frac{1}{Z_0 \omega_c g_4} = 4.18\ \text{pF} \quad \checkmark$$

$$L_5 = \frac{Z_0}{\omega_c g_5} = 2.28\ \text{nH} \quad \checkmark$$

The calculated filter response is shown below. Note that the insertion loss at $f = 0.6$ GHz is just a bit more than 40dB.



(8.10) $f_o = 2GHz$, B.P., M.F.G.D., $\Delta = 0.05$, $N = 4$, $Z_0 = 50\Omega$

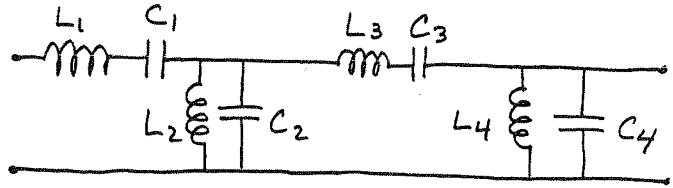
From Table 8.5 the prototype element values are,

$g_1 = 1.0598$

$g_2 = 0.5116$

$g_3 = 0.3181$

$g_4 = 0.1104$



From Table 8.6 and (8.64) the scaled element values are,

$L_1 = \frac{g_1 Z_0}{\omega_0 \Delta} = 84.3\ nH$ ✓

$L_2 = \frac{\Delta Z_0}{\omega_0 g_2} = 0.388\ nH$ ✓

$L_3 = \frac{g_3 Z_0}{\omega_0 \Delta} = 25.3\ nH$ ✓

$L_4 = \frac{\Delta Z_0}{\omega_0 g_4} = 1.80\ nH$ ✓

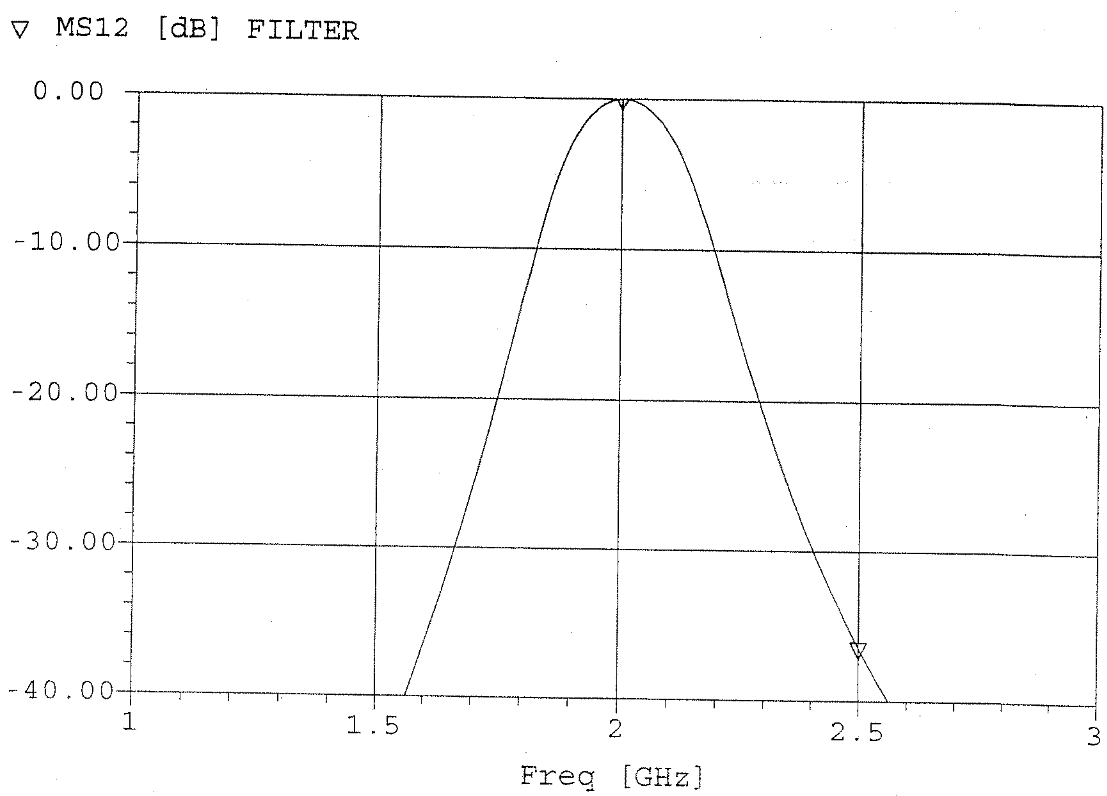
$C_1 = \frac{\Delta}{\omega_0 g_1 Z_0} = 0.075\ pF$ ✓

$C_2 = \frac{g_2}{\omega_0 \Delta Z_0} = 16.3\ pF$ ✓

$C_3 = \frac{\Delta}{\omega_0 g_3 Z_0} = 0.25\ pF$ ✓

$C_4 = \frac{g_4}{\omega_0 \Delta Z_0} = 3.51\ pF$ ✓

The calculated filter response is shown below.



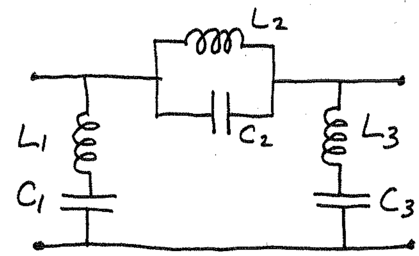
8.11 $\quad f_0 = 3\text{GHz}$, $Z_0 = 75\Omega$, $N=3$, B.S., 0.5dB E.R.

First use (8.75) to transform 3.1 GHz to a L.P. prototype response frequency:

$$\omega \leftarrow \Delta\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1} = 0.1\left(\frac{3.1}{3} - \frac{3}{3.1}\right)^{-1} = 1.52$$

So, $\left|\frac{\omega}{\omega_c}\right| - 1 = 0.52$, and Figure 8.27a gives an attenuation of 11 dB for $N=3$. From Table 8.4, the prototype values are,

$g_1 = 1.5963$
$g_2 = 1.0967$
$g_3 = 1.5963$



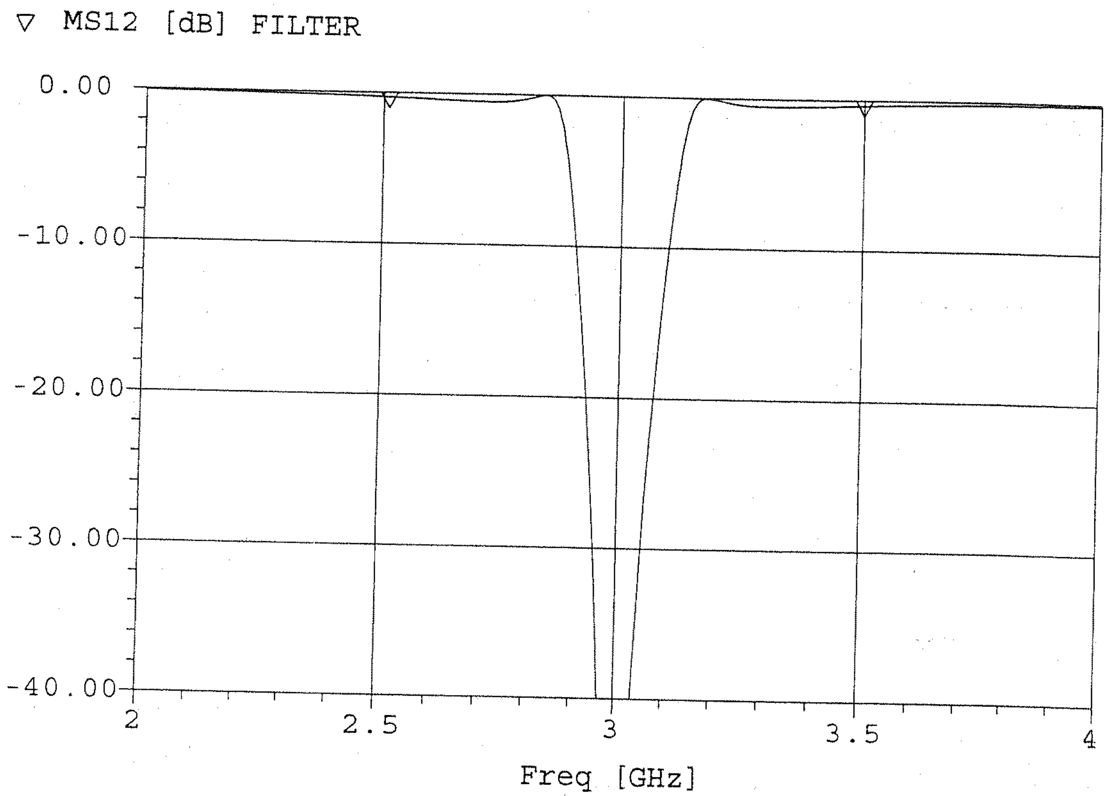
From Table 8.6 and (8.64) the scaled element values are,

$$L_1 = \frac{Z_0}{\omega_0 g_1 \Delta} = 24.9\,nH \checkmark \qquad C_1 = \frac{g_1 \Delta}{\omega_0 Z_0} = 0.113\,pF \checkmark$$

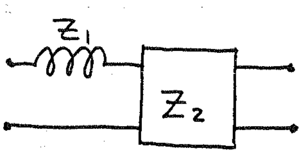
$$L_2 = \frac{g_2 \Delta Z_0}{\omega_0} = 0.436\,nH \checkmark \qquad C_2 = \frac{1}{Z_0 \omega_0 g_2 \Delta} = 6.45\,pF \checkmark$$

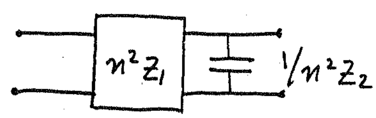
$$L_3 = \frac{Z_0}{\omega_0 g_3 \Delta} = 24.9\,nH \checkmark \qquad C_3 = \frac{g_3 \Delta}{Z_0 \omega_0} = 0.113\,pF \checkmark$$

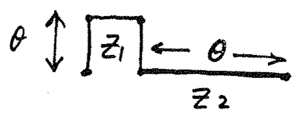
The calculated response for this filter is shown on the following page. Note that the insertion loss at 3.1 GHz is about 10 dB.

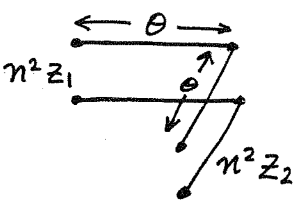


(8.12)









$Z_S = jZ_1 \tan\theta$ $\qquad$ $Y_S = \frac{j}{n^2 Z_2} \tan\theta$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & jZ_1 \tan\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_2 \sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & j(Z_1+Z_2)\sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jn^2 Z_2 \sin\theta \\ \frac{j\sin\theta}{n^2 Z_1} & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\tan\theta}{n^2 Z_2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & jn^2 Z_1 \sin\theta \\ \frac{j}{n^2}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\sin\theta & \cos\theta \end{bmatrix}$$

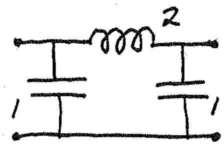
So these two matrices are equal if,

$$Z_1 + Z_2 = n^2 Z_1$$

or, $$n^2 = 1 + Z_2/Z_1 \checkmark$$

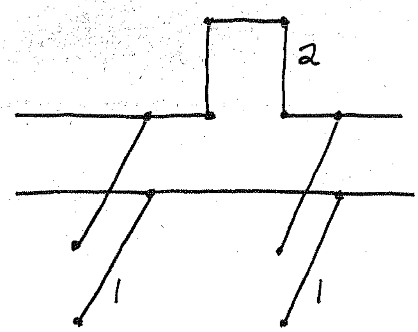
(8.13) $f_0 = 6$ GHz, $N=3$, M.F., $Z_0 = 50\Omega$

From Table 8.3 the L.P. prototype is,

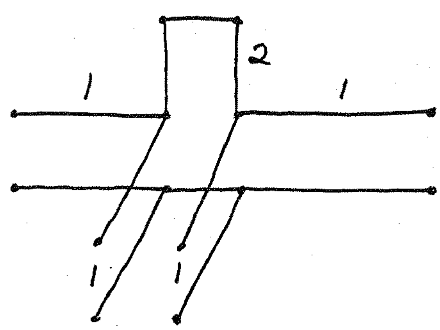


(choosing a π-circuit simplifies the problem)

Richards' transform:



add unit elements at ends:



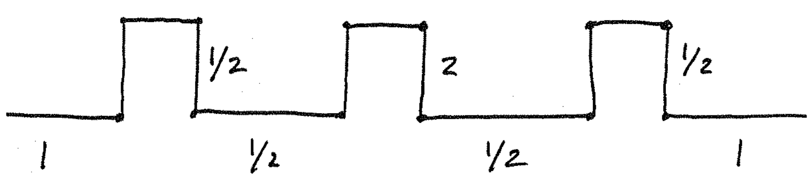
apply first Kuroda identity (twice):

$Z_1 = 1$

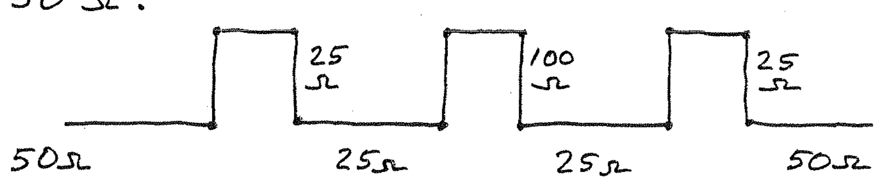
$Z_2 = 1$

$n^2 = 1 + \frac{Z_2}{Z_1}$

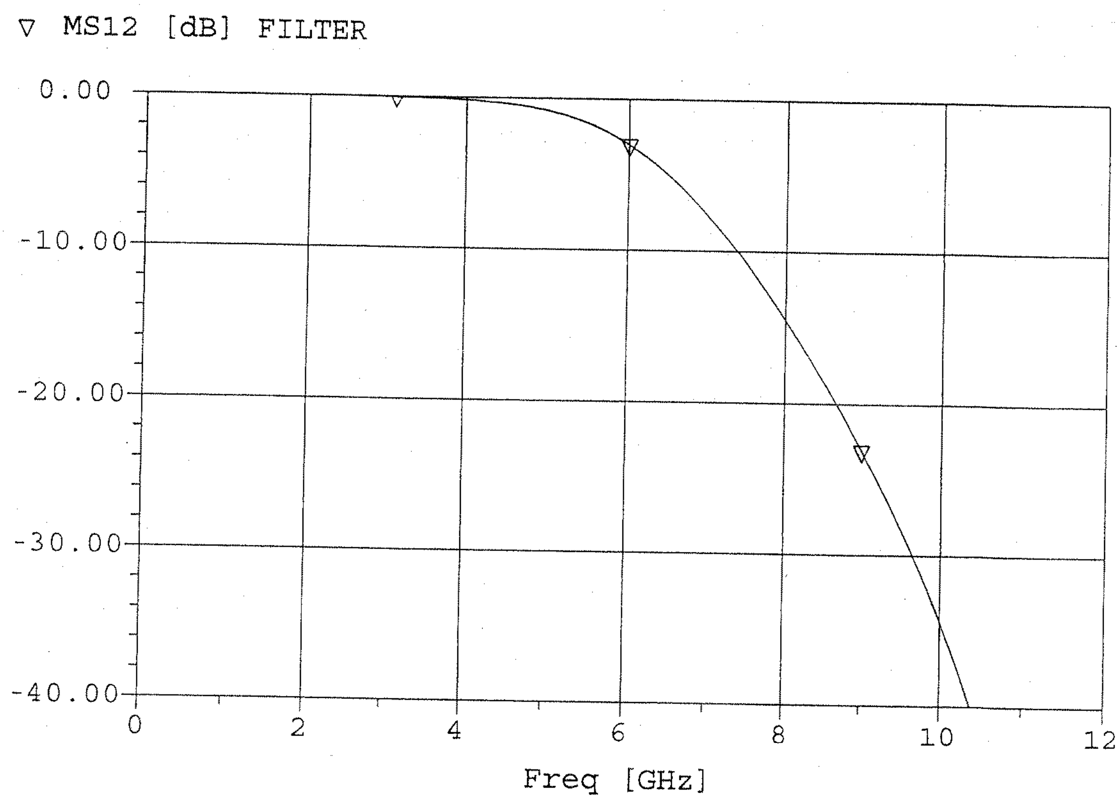
$= 2$



Scale to 50Ω:

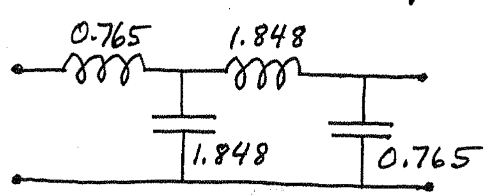


all line lengths and stub lengths are $\lambda/8$ long at 6 GHz. The calculated filter response is shown below.

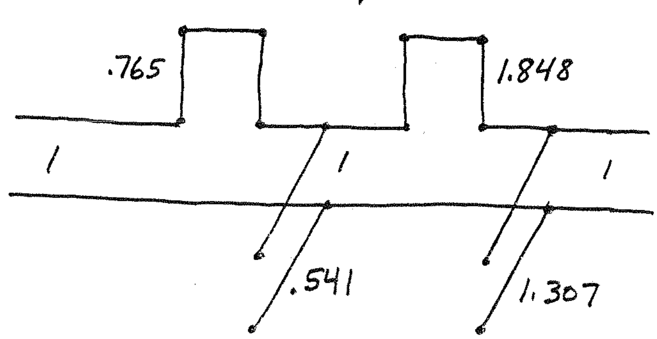


(8.14) $f_0 = 8\text{ GHz}$, $N=4$, L.P., M.F., $Z_0 = 50\,\Omega$

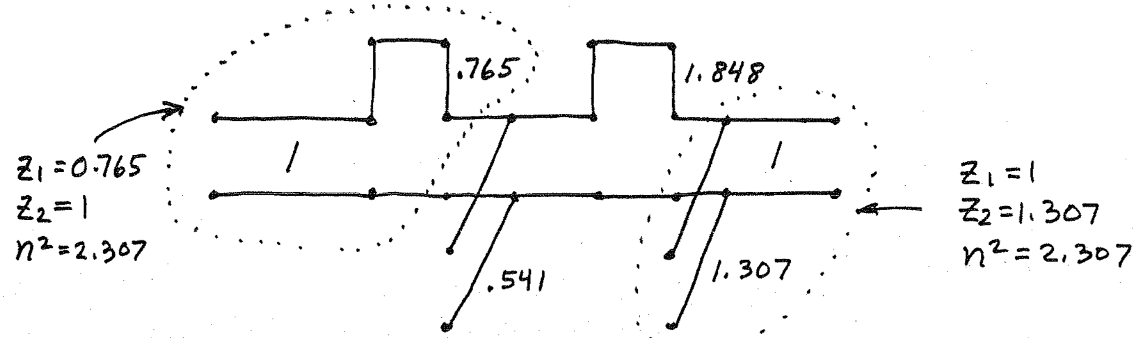
From Table 8.3 the L.P. prototype is,



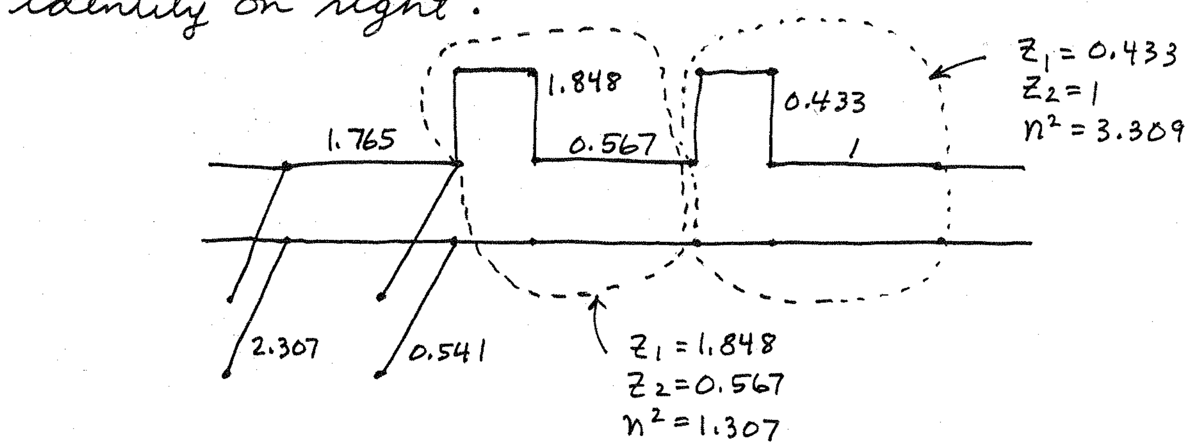
applying Richard's transform:



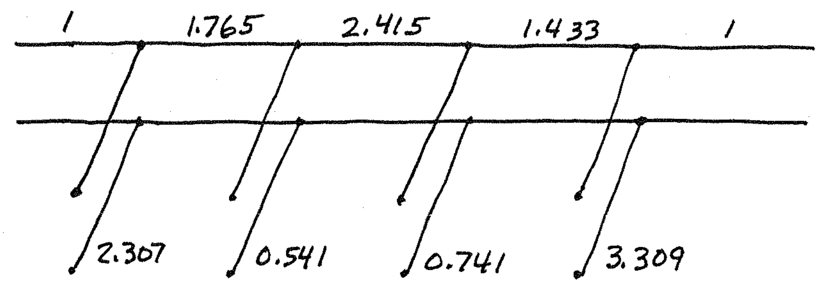
add unit elements:



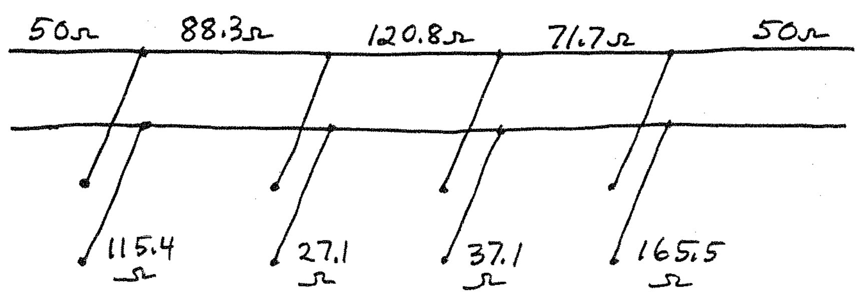
Use the second Kuroda identity on left; first Kuroda identity on right:



Now use the second Kuroda identity twice:

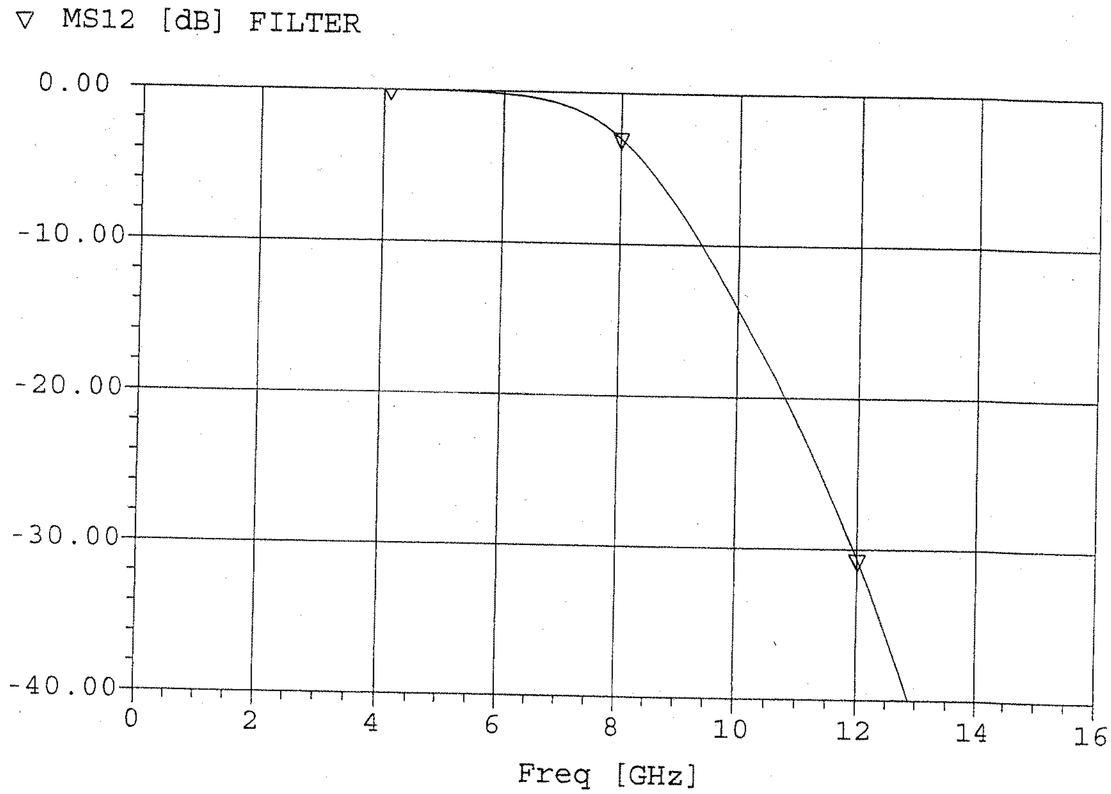


Scale to 50Ω:

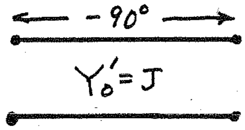


✓

all lines are $\lambda/8$ long at 8GHz. The calculated filter response is shown on the following page.

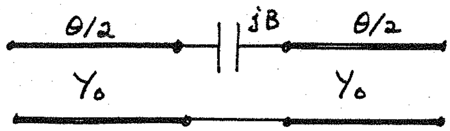

▽ MS12 [dB] FILTER
0.00
-10.00
-20.00
-30.00
-40.00
0
2
4
6
8
10
12
14
16
Freq [GHz]

(8.15) Quarter-wave line ($-\lambda/4$ long, since $\theta < 0$)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix}$$

ADMITTANCE INVERTER:



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta/2 & j/Y_0 \sin\theta/2 \\ jY_0 \sin\theta/2 & \cos\theta/2 \end{bmatrix} \begin{bmatrix} 1 & -j/B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta/2 & j/Y_0 \sin\theta/2 \\ jY_0 \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\cos\theta + \frac{Y_0}{2B}\sin\theta\right) & j\left(\frac{1}{Y_0}\sin\theta - \frac{1}{B}\cos^2\frac{\theta}{2}\right) \\ jY_0\left(\sin\theta + \frac{Y_0}{B}\sin^2\frac{\theta}{2}\right) & \left(\cos\theta + \frac{Y_0}{2B}\sin\theta\right) \end{bmatrix} \checkmark$$

Equivalence requires the following conditions:

A, D: $\cos\theta + \frac{Y}{2B}\sin\theta = 0 \Rightarrow \theta = -\tan^{-1}\left(\frac{2B}{Y_0}\right) < 0$ ✓

B: $\frac{Y_0}{J} = -\sin\theta + \frac{Y_0}{B}\cos^2\theta/2$

C: $\frac{J}{Y_0} = -\sin\theta - \frac{Y_0}{B}\sin^2\theta/2$

$\ominus$ $\frac{Y_0}{J} - \frac{J}{Y_0} = \frac{Y_0}{B}$

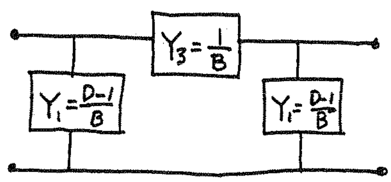
or, $B = \dfrac{Y_0}{\frac{Y_0}{J} - \frac{J}{Y_0}} = \dfrac{J}{1-(J/Y_0)^2}$ ✓

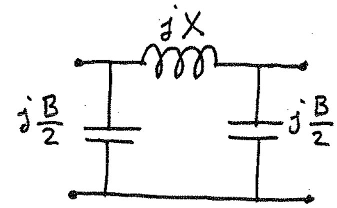
also, $\tan|\theta| = \dfrac{2B}{Y_0} = \dfrac{2(J/Y_0)}{1-(J/Y_0)^2} = \dfrac{2\tan\left|\frac{\theta}{2}\right|}{1-\tan^2\left|\frac{\theta}{2}\right|}$

so,

$\tan\left|\frac{\theta}{2}\right| = J/Y_0$ ✓

(8.16) The easiest way to do this problem is to use the π-network of Table 4.1, with the shunt and series element values given in terms of the ABCD parameters:





Then using the ABCD parameters for a transmission line gives the equivalent circuit elements as,

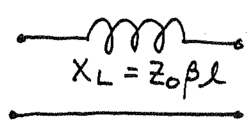
$$jX = B = jZ_0 \sin\beta l$$

$$j\frac{B}{2} = \frac{D-1}{B} = \frac{\cos\beta l - 1}{jZ_0 \sin\beta l} = \frac{j}{Z_0}\tan\beta l/2$$

For $\beta l < \pi/4$ and large Z_0, these results reduce to:

$$X \simeq Z_0 \beta l$$

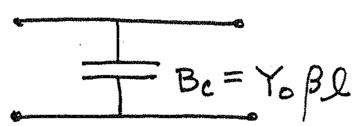
$$\frac{B}{2} \simeq 0$$



For $\beta l < \pi/4$ and small Z_0, these results reduce to:

$$X \simeq 0$$

$$\frac{B}{2} \simeq \frac{\beta l}{2 Z_0}$$



(NOTE: This problem can also be done using y-parameters, but a sign change for y_{12} will be required because of the reference direction for I_2 for the ABCD parameters.)

(8.17) $f_0 = 3\,\text{GHz}$, $N=5$, $0.5\,\text{dB}$ equal ripple, $Z_0 = 50\,\Omega$, $Z_\ell = 15\,\Omega$, $Z_h = 120\,\Omega$

a) From Table 8.4 and (8.86):

$$g_1 = 1.7058 = C_1 \Rightarrow \beta \ell_1 = g_1 Z_\ell / Z_0 = 29.3^\circ$$
$$g_2 = 1.2296 = L_2 \Rightarrow \beta \ell_2 = g_2 Z_0 / Z_h = 29.4^\circ$$
$$g_3 = 2.5408 = C_3 \Rightarrow \beta \ell_3 = g_3 Z_\ell / Z_0 = 43.7^\circ$$
$$g_4 = 1.2296 = L_4 \Rightarrow \beta \ell_4 = g_4 Z_0 / Z_h = 29.4^\circ$$
$$g_5 = 1.7058 = C_5 \Rightarrow \beta \ell_5 = g_5 Z_\ell / Z_0 = 29.3^\circ$$

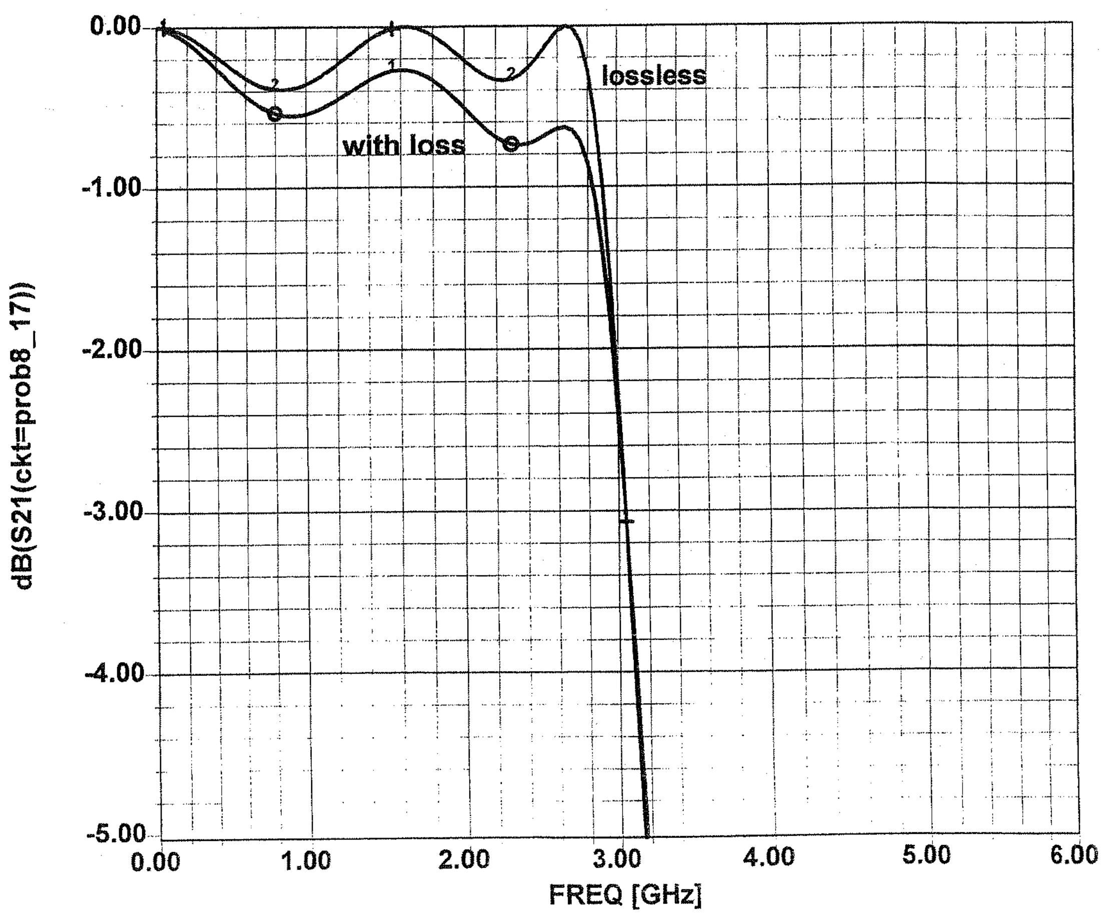
Observe that $\beta \ell_i < 45^\circ$ for all lines

b) $\epsilon_r = 4.2$, $d = 0.079\,\text{cm}$, $\tan\delta = 0.02$, Cu, $t = 0.5\,\text{mil}$

$W(15\,\Omega) = 7.98\,\text{mm}$; $\ell_1 = 0.42\,\text{cm}$; $\ell_3 = 0.63\,\text{cm}$; $\ell_5 = 0.42\,\text{cm}$

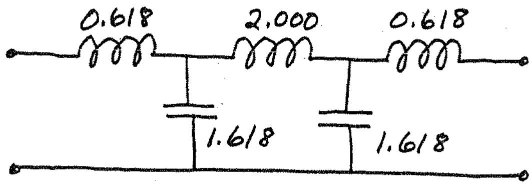
$W(120\,\Omega) = 0.213\,\text{mm}$; $\ell_2 = 0.48\,\text{cm}$; $\ell_4 = 0.48\,\text{cm}$

The response, with and without loss, is shown below.



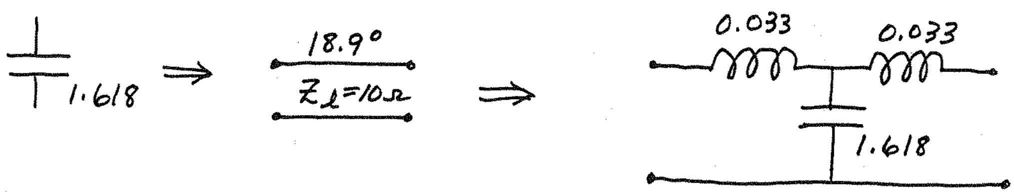
(8.18) $f_o = 2\,GHz$, L.P., M.F., $Z_o = 50\,\Omega$

From Table 8.3 the LP prototype is,



The shunt capacitors can be implemented with a length of Z_ℓ line. Using (8.83b) gives,

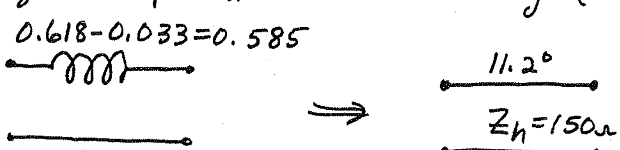
$$B = \frac{1.618}{Z_o} = \frac{\sin \beta \ell}{Z_\ell} \Rightarrow \beta \ell = 18.9°$$



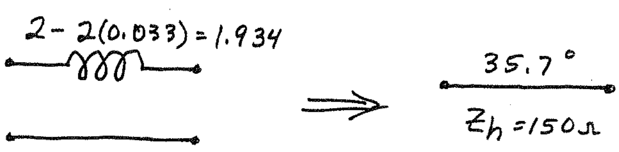
The model for this line (see Figure 8.39) shows inductors on either side, with values given by (8.83a):

$$\frac{X}{2Z_o} = \frac{Z_\ell}{Z_o} \tan \beta\ell/2 = 0.033$$

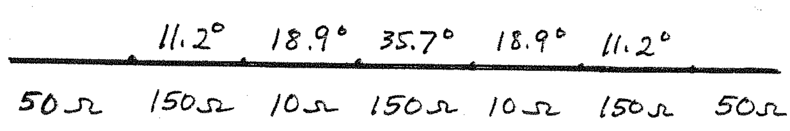
Then the end inductors of value 0.618 can be implemented as lengths of Z_h line. Using (8.83a) gives,



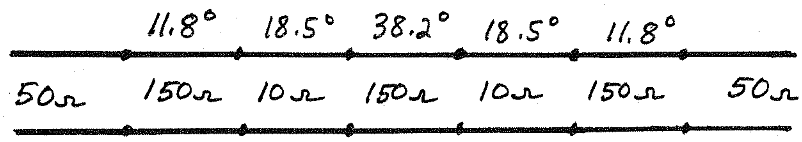
and the middle inductor of value 2.000 can be implemented as,



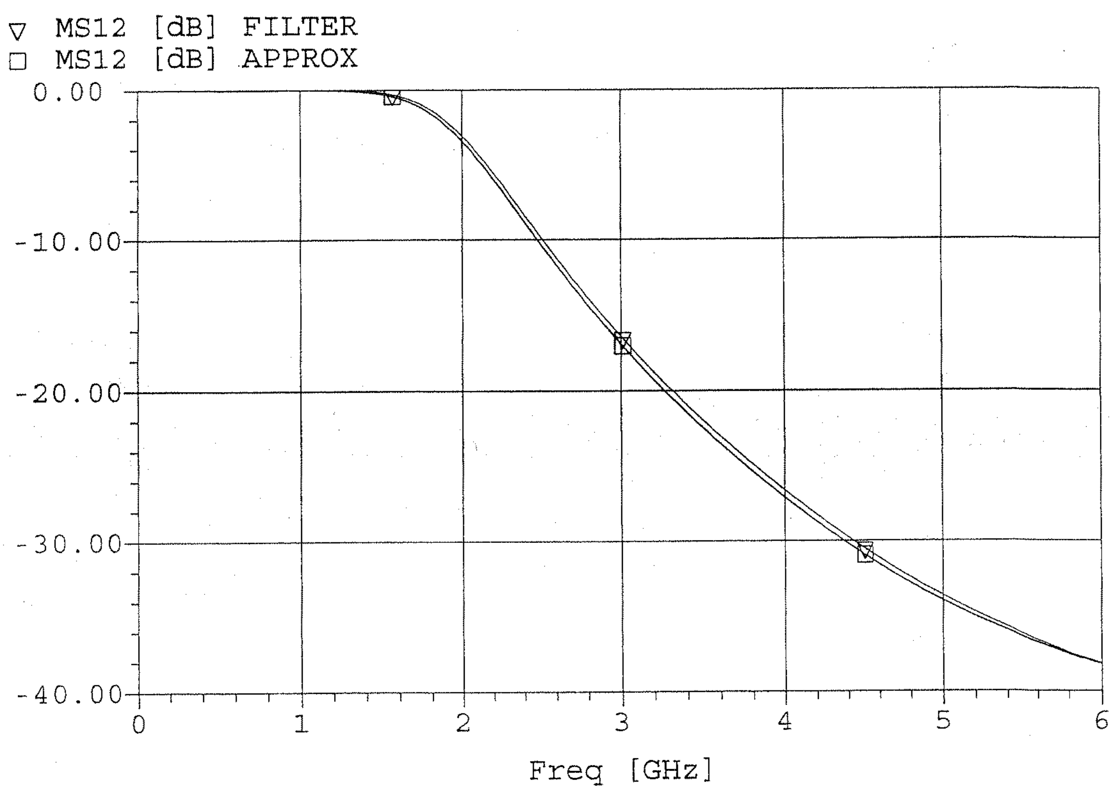
The final filter:



For comparison, the design using the approximations of (8.84) and (8.85) is,



Note that only the middle section differs very much in these two designs. The calculated filter response is shown below for both designs. Note that there is very little difference.



8.19 $f_0 = 2.45$ GHz, BW = 10%, EQUAL-RIPPLE (0.5 dB), $N=3$, $Z_0 = 50\,\Omega$

a) Use (8.71) to transform 2.1 GHz to normalized L.P. form:

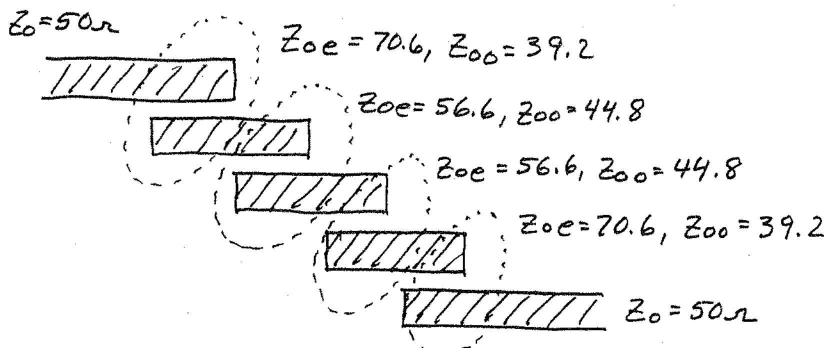
$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{0.1}\left(\frac{2.1}{2.45} - \frac{2.45}{2.1}\right) = -3.09$$

Then, $\left|\frac{\omega}{\omega_c}\right| - 1 = 2.09 \Rightarrow$ Fig 8.27 gives $\alpha \simeq 30$ dB

The L.P. prototype values are given in Table 8.4

$Z_0 J_n$ is found from (8.121), and Z_{oe}, Z_{oo} from (8.108):

n	g_n	$Z_0 J_n$	Z_{oe} (Ω)	Z_{oo} (Ω)
1	1.5963	0.3137	70.6	39.2
2	1.0967	0.1187	56.6	44.8
3	1.5963	0.1187	56.6	44.8
4	1.0000	0.3137	70.6	39.2



all lines are $\lambda/4$ long at 2.45 GHz.

b) $d = 0.158$ cm, $\epsilon_r = 4.2$, $\tan\delta = 0.01$, Cu, $t = 0.5$ mil

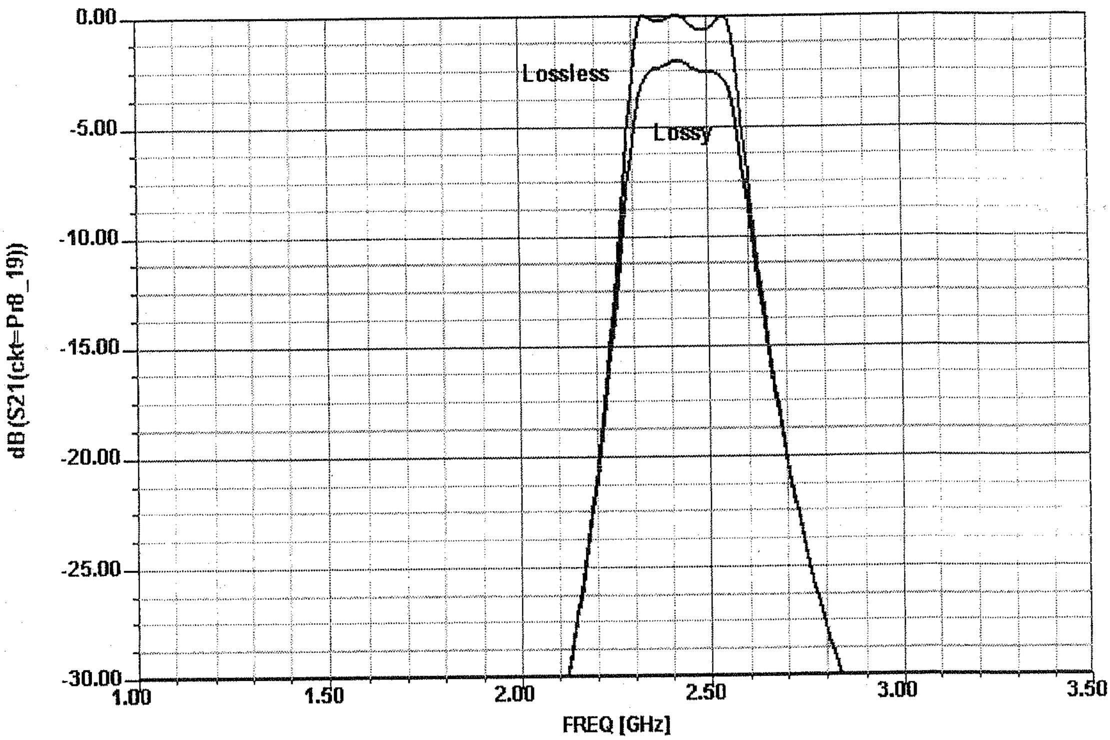
From Serenade,

$Z_{oe} = 70.6$, $Z_{oo} = 39.2 \Longrightarrow W = 2.484$ mm, $S = 0.415$ mm, $\ell = 1.74$ cm

$Z_{oe} = 56.6$, $Z_{oo} = 44.8 \Longrightarrow W = 3.026$ mm, $S = 1.723$ mm, $\ell = 1.71$ cm

The calculated response, from Serenade, is shown on the following page.

Pr8_19.ckt



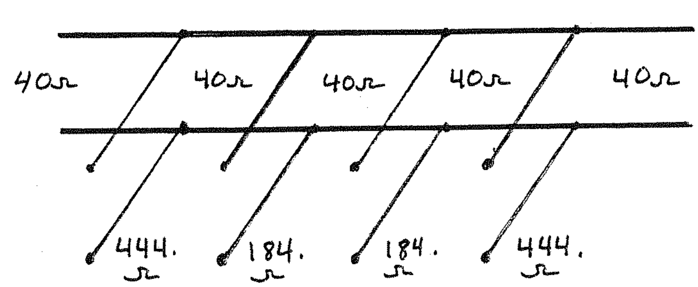
(8.20) $f_0 = 3\,GHz$, B.S., M.F., $N=4$, $\Delta = 0.15$, $Z_0 = 40\Omega$.

We find the g_n values from Table 8.3. Then the $\lambda/4$ o.c. stub characteristic impedances can be found from (8.130):

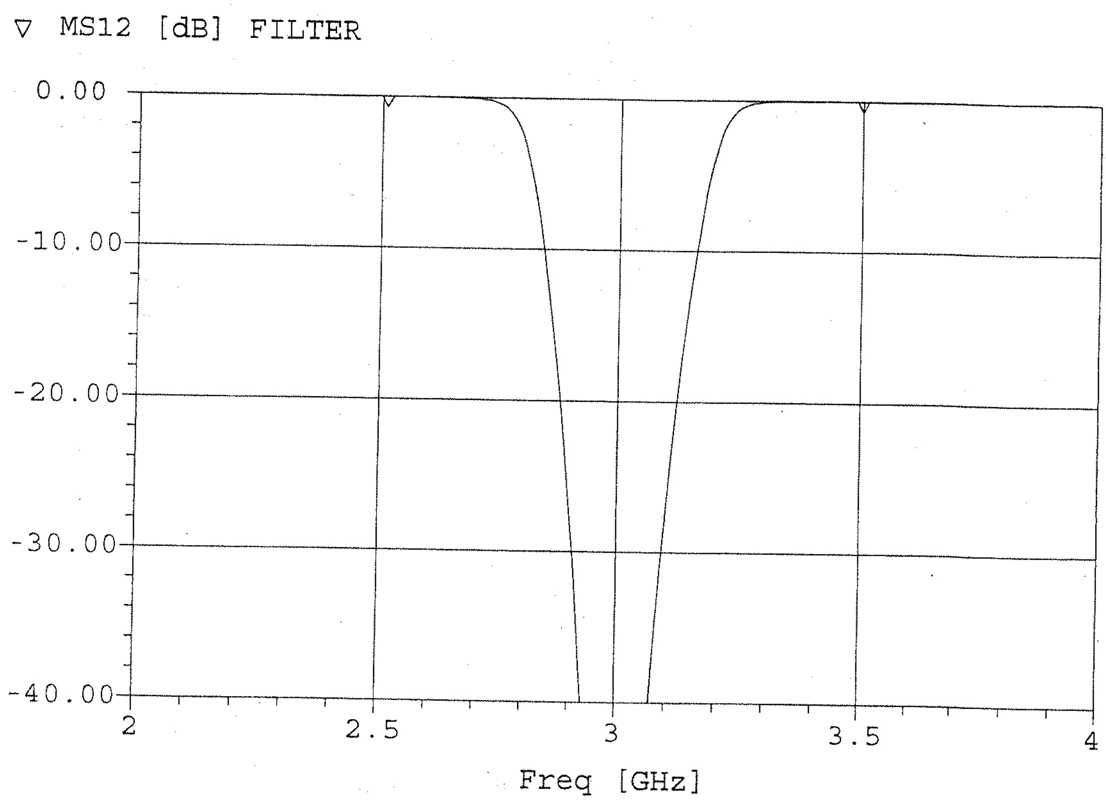
n	g_n	$Z_{0n}(\Omega)$
1	0.765	444.
2	1.848	184.
3	1.848	184.
4	0.765	444.

(Z_{01} and Z_{04} may be too high to be practical)

The final filter is shown below:



all lines and stubs are $\lambda/4$ long at 3 GHz. The calculated filter response is shown below:



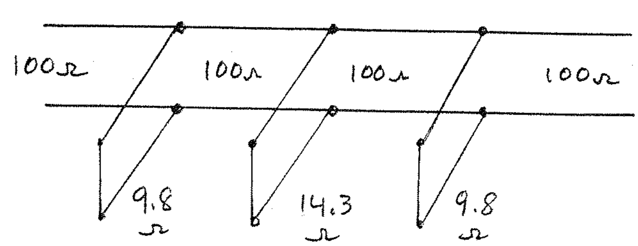
(8.21) BANDPASS, $N=3$, EQUAL RIPPLE (0.5dB), $\lambda/4$ S.C. stubs, $f_0 = 3GHz$, 20% BW

a) Find g_n from Table 8.4. Then the $\lambda/4$ short-circuited stub characteristic impedances can be found as,

n	g_n	Z_{0n} (Ω)
1	1.5963	9.8
2	1.0967	14.3
3	1.5963	9.8

$\Delta = 0.20$

$$Z_{0n} = \frac{\pi Z_0 \Delta}{4 g_n}$$

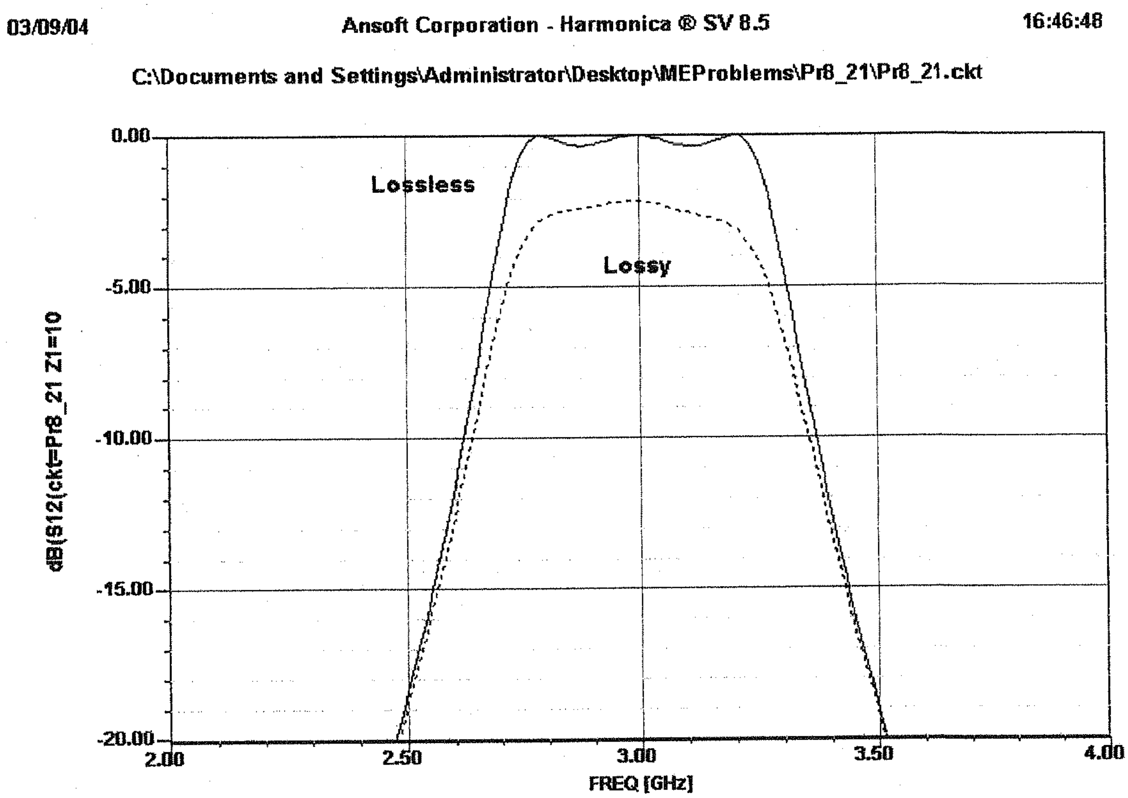


b) FR-4, $\epsilon_r = 4.2$, $d = 0.079$ cm, $\tan\delta = 0.02$, Cu, $t = 0.5$ mil

$W(9.8) = 1.307$ cm, $90° = 1.27$ cm

$W(14.3) = 0.845$ cm, $90° = 1.29$ cm

$W(100) = 0.367$ mm, $90° = 1.46$ cm



(8.22) The bandpass $\lambda/4$ resonator filter uses short-circuited stubs, which have an input admittance of,

$$Y = -jY_{on} \cot\theta \quad , \text{ where } \theta = \pi/2 \text{ for } \omega = \omega_0 .$$

Let $\omega = \omega_0 + \Delta\omega$. Then $\theta = \pi/2\,(1 + \Delta\omega/\omega_0)$, and

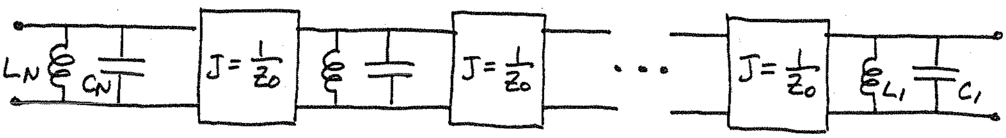
$Y = jY_{on} \tan \frac{\pi \Delta\omega}{2\omega_0} \simeq \frac{j\pi Y_{on}(\omega - \omega_0)}{2\omega_0}$. Now the admittance of a parallel LC resonator is, from Table 6.1,

$$Y \simeq 2jC_n(\omega - \omega_0).$$

So the characteristic impedance of the stub is,

$$Z_{0n} = \frac{1}{Y_{0n}} = \frac{\pi}{4\omega_0 C_n}.$$

The filter circuit can be redrawn as follows:



This is the same as the circuit in part (e) of Figure 8.45, for coupled line bandpass filters (with $Z_0 J_n = 1$). Correspondance with the lumped-element bandpass filter requires that,

$$\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{C_1'}{L_1'}}$$

$$Z_0^2 \sqrt{\frac{C_2}{L_2}} = \sqrt{\frac{L_2'}{C_2'}}$$

where C_n' and L_n' are the lumped element filter values, and $L_n C_n = L_n' C_n' = 1/\omega_0^2$. Solving for C_n gives,

$$C_1 = C_1'$$

$$C_2 = \frac{1}{\omega_0^2 Z_0^2 C_2'}.$$

Using Table 8.6 to transform back to LP prototype values gives,

$$C_1 = C_1' = \frac{g_1}{\Delta \omega_0 Z_0}$$

$$C_2 = \frac{1}{\omega_0^2 Z_0^2}\left(\frac{\omega_0 g_2 Z_0}{\Delta}\right) = \frac{g_2}{\Delta \omega_0 Z_0}$$

So the characteristic impedances are,

$$Z_{0n} = \frac{\pi}{4\omega_0}\left(\frac{\Delta \omega_0 Z_0}{g_1}\right) = \frac{\pi \Delta Z_0}{4 g_n} \checkmark$$

which agrees with (8.131).

(8.23) $f_o = 4\text{ GHz}$, $\Delta = 0.12$, B.P., M.F., $Z_o = 50\Omega$

First transform 3.6 GHz to L.P. prototype form:

$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) = \frac{1}{.12}\left(\frac{3.6}{4} - \frac{4}{3.6}\right) = -1.76$$

Then,

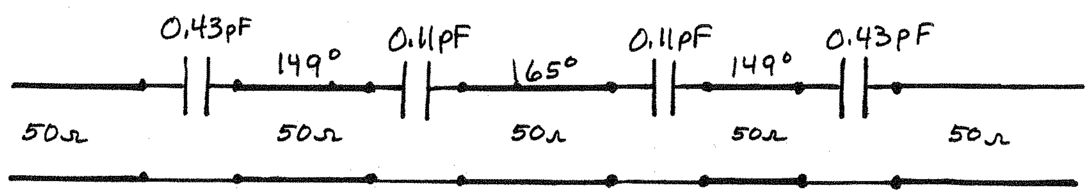
$$\left|\frac{\omega}{\omega_c}\right| - 1 = 0.76$$

So from Figure 8.26 we see that $N=3$ is required to achieve $\alpha > 12\text{ dB}$ at 3.6 GHz. (or, analytically, $P_{LR} = 1 + (\omega/\omega_c)^{2N} = 14.9\text{ dB}$ for $N=3$)

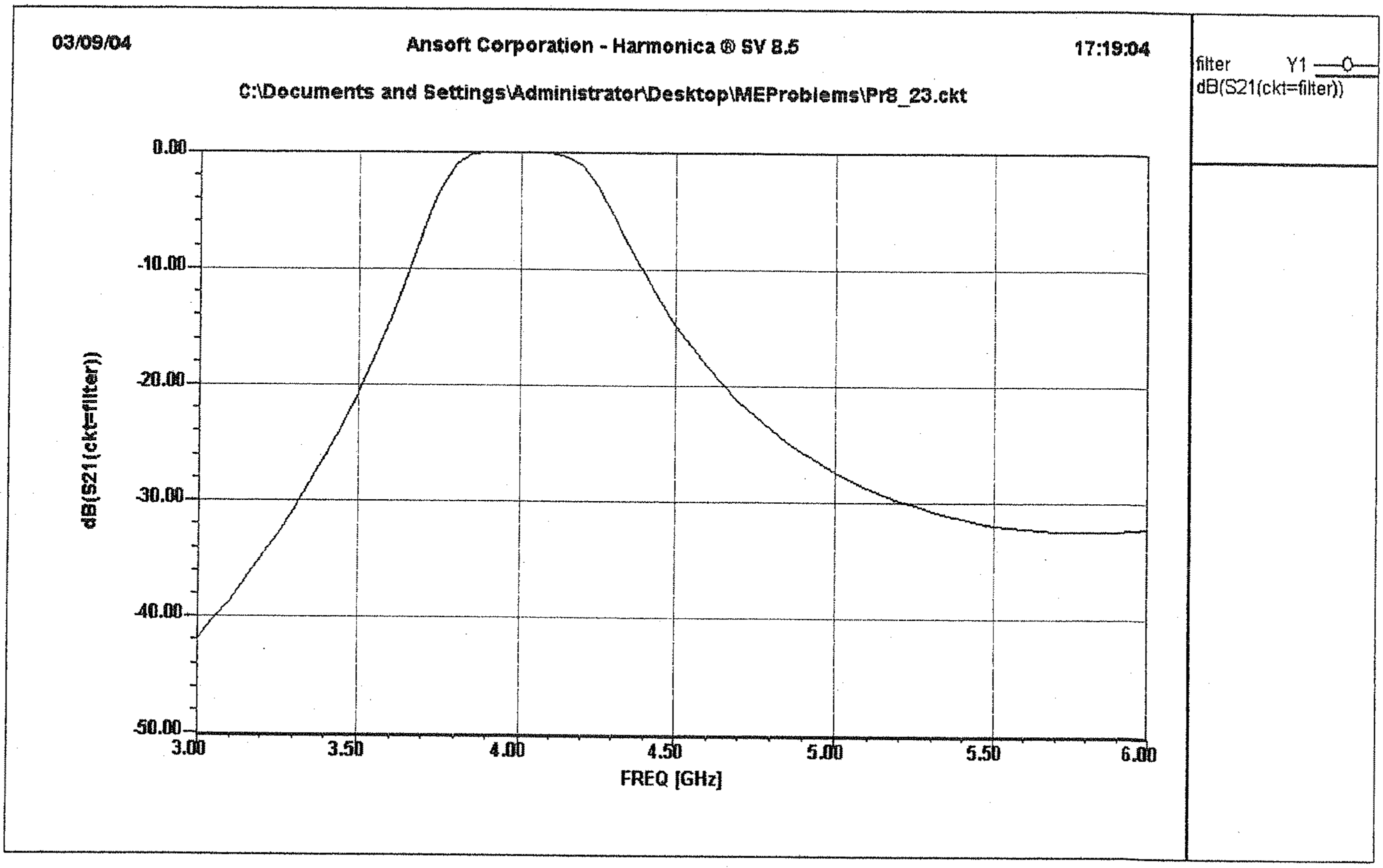
The prototype values are given in Table 8.3. Then $Z_o J_n$ can be found from (8.121). Then, $Z_o B_n = \dfrac{J_n Z_o}{1-(J_n Z_o)^2}$, and $\theta_n = \pi - \frac{1}{2}\left[\tan^{-1}(2 Z_o B_n) + \tan^{-1}(2 Z_o B_{n+1})\right]$. Also, $C_n = B_n/\omega_o$.

n	g_n	Z_oJ_n	Z_oB_n	C_n (pF)	θ_n @ 4GHz
1	1.000	0.434	0.535	0.43	149°
2	2.000	0.133	0.135	0.11	165°
3	1.000	0.133	0.135	0.11	149°
4	1.000	0.434	0.535	0.43	—

The filter circuit is shown below:



The calculated filter response is shown on the following page.



(8.24) $f_o = 836.5\,MHz$, $\Delta = 0.03$, BANDPASS, 1.0dB EQUAL RIPPLE, $Z_o = 50\Omega$

Use 0.5dB ripple design to allow for approximation errors. Use (8.136)–(8.137) to find inverter constants and coupling capacitor values:

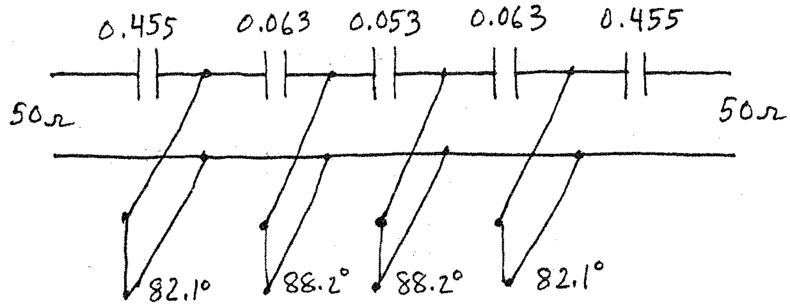
n	g_n	$Z_o J_{n-1,n}$	$C_{n-1,n}$ (PF)
1	1.6703	0.1188	0.455
2	1.1926	0.0167	0.063
3	2.3661	0.0140	0.053
4	0.8419	0.0167	0.063
5	1.9841	0.1188	0.455

Then use (8.138) and (8.141) to find the resonator lengths:

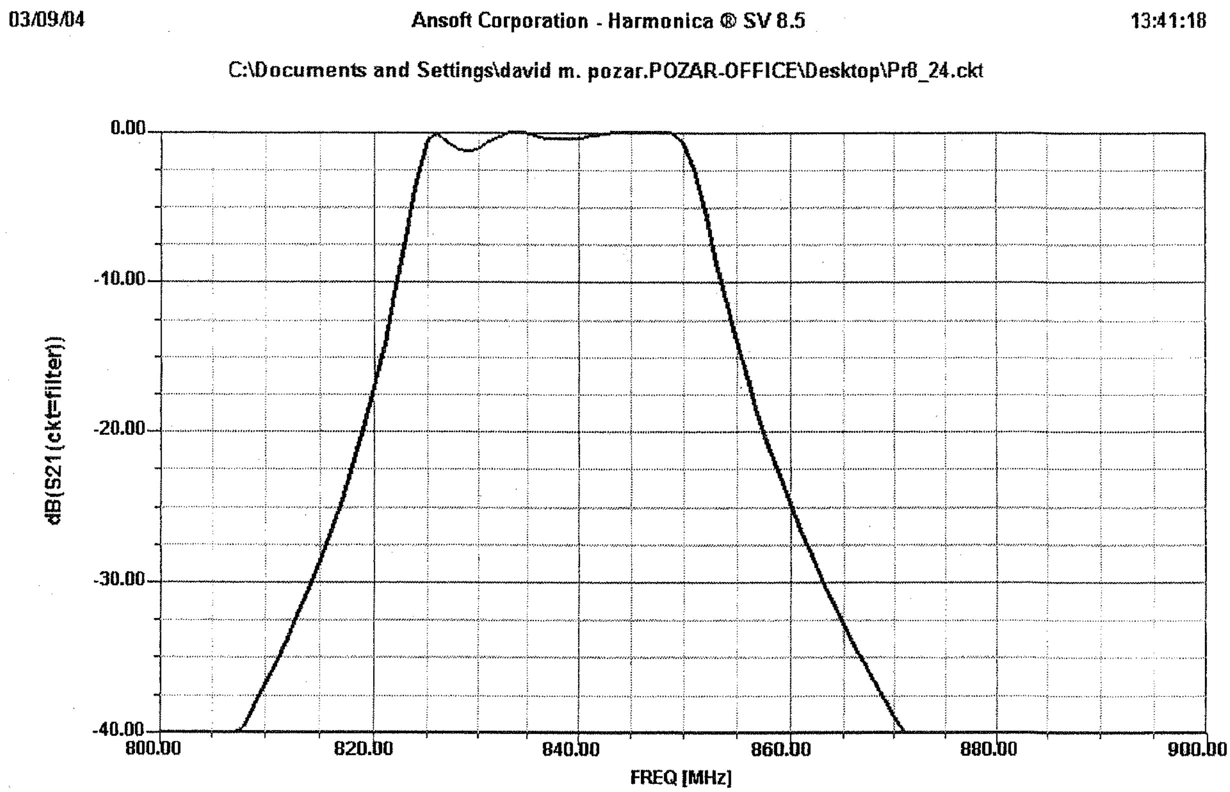
$C_n' = C_n + \Delta C_n$

n	ΔC_n (PF)	ℓ_n (λ)	ℓ°
1	−0.518	0.228	82.1
2	−0.116	0.245	88.2
3	−0.116	0.245	88.2
4	−0.518	0.228	82.1

Final filter circuit:



The resulting response is shown below.

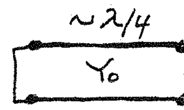


(8.25) From Matthai, Young & Jones, P. 482, we have the general results that

$$J_1 = \sqrt{\frac{G_A b_1 \Delta}{g_0 g_1 \omega_1'}} \quad , \quad J_n = \frac{\Delta}{\omega_1'}\sqrt{\frac{b_n b_{n+1}}{g_n g_{n+1}}} \quad , \quad J_N = \sqrt{\frac{G_B b_N \Delta}{\omega_1' g_N g_{N+1}}}$$

with $G_A = G_B = 1/Z_0$, $\omega_1' = 1$, $g_0 = 1$, and where

$$b_j = \frac{\omega_0}{2} \frac{dB_j}{d\omega}\bigg|_{\omega=\omega_0} = \frac{\pi}{4Z_0}$$ is the admittance slope parameter for the S.C. stub resonator:



$$\leftarrow Y = -jY_0 \cot\theta \simeq \frac{j\pi(\omega-\omega_0)}{2Z_0\omega_0}$$

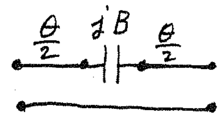
Then,

$$Z_0 J_1 = \sqrt{\frac{\pi\Delta}{4g_1}} \quad \checkmark$$

$$Z_0 J_n = \frac{\pi\Delta}{4\sqrt{g_n g_{n+1}}} \quad \checkmark$$

$$Z_0 J_N = \sqrt{\frac{\pi\Delta}{4 g_N g_{N+1}}} \quad \checkmark$$

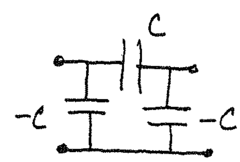
The end inverters are modeled as



(Fig. 8.38c)

for which $C_{i,i+1} = \frac{B}{\omega} = \frac{J_{i,i+1}}{\omega_0\sqrt{1-(Z_0 J_{i,i+1})^2}}$ ✓ for C_{01}, J_{01} and $C_{N,N+1}, J_{N,N+1}$.

The middle inverters are modeled as



(Fig. 8.38d)

for which $C_{n,n+1} = \frac{J_{n,n+1}}{\omega}$ ✓

Chapter 9

9.1 $\quad f = 10\,GHz, \quad 4\pi M_s = 1780\,G$

From (9.24)–(9.25),

$$[\mu] = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

where,

$$\mu = \mu_0\left(1 + \frac{\omega_0\omega_m}{\omega_0^2 - \omega^2}\right), \quad \kappa = \mu_0 \frac{\omega\omega_m}{\omega_0^2 - \omega^2}$$

a) $H_0 = M_s = 0 \Longrightarrow \omega_0 = \omega_m = 0 \Rightarrow \mu = \mu_0, \; \kappa = 0$

so,

$$[\mu] = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_0 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \checkmark$$

b) $H_0 = 1000\,Oe$

Then, $\quad f_0 = 2.8 \frac{MHz}{Oe}(1000\,Oe) = 2.8\,GHz$

$$f_m = 2.8 \frac{MHz}{Oe}(1780G)\left(\frac{1\,Oe}{1G}\right) = 4.98\,GHz$$

so,

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.849\,\mu_0$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = -0.540\,\mu_0$$

$$[\mu] = \begin{bmatrix} 0.849 & -j0.540 & 0 \\ j0.540 & 0.849 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mu_0 \checkmark$$

9.2

$$B_x = \mu H_x + j\kappa H_y$$
$$B_y = -j\kappa H_x + \mu H_y$$
$$B_z = \mu_0 H_z$$

Then,
$$B^+ = \tfrac{1}{2}(B_x + jB_y) = \tfrac{1}{2}\left[\mu H_x + j\kappa H_y + \kappa H_x + j\mu H_y\right]$$
$$= \tfrac{1}{2}(\mu+\kappa)(H_x + jH_y) = (\mu+\kappa)H^+$$
$$B^- = \tfrac{1}{2}(B_x - jB_y) = \tfrac{1}{2}\left[\mu H_x + j\kappa H_y - \kappa H_x - j\mu H_y\right]$$
$$= \tfrac{1}{2}(\mu-\kappa)(H_x - jH_y) = (\mu-\kappa)H^-$$

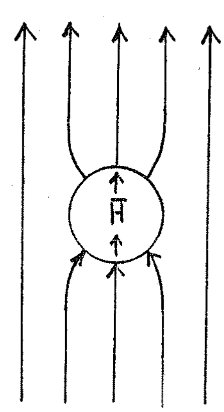
Thus,
$$\begin{bmatrix} B^+ \\ B^- \\ B_z \end{bmatrix} = \begin{bmatrix} (\mu+\kappa) & 0 & 0 \\ 0 & (\mu-\kappa) & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \begin{bmatrix} H^+ \\ H^- \\ H_z \end{bmatrix} \checkmark$$

9.3

$4\pi M_s = 1780$ G, $H_e = 1200$ Oe

From (9.41) the internal field is,
$$\bar{H} = \bar{H}_e - N\bar{M}$$

From Table 9.1 the demagnetization factors for a sphere are $N_x = N_y = N_z = 1/3$, so
$$H_z = H_{ze} - N_z M_z = 1200 - \tfrac{1}{3}(1780)$$
$$= 607 \text{ Oe}.$$



9.4 $\quad 4\pi M_s = 600\,G$

From Table 9.1 the demagnetization factors for a thin rod are $N_x = N_y = \frac{1}{2}$, $N_z = 0$. Then (9.46) gives the gyromagnetic resonance frequency as,

$$\omega_r = \mu_0 \gamma \sqrt{(H_a + \tfrac{1}{2}M_s)(H_a + \tfrac{1}{2}M_s)}$$

$$= \mu_0 \gamma (H_a + \tfrac{1}{2}M_s)$$

$$= \mu_0 \gamma (1300\,\text{Oe}) \left(\frac{1\,\text{A/m}}{4\pi \times 10^{-3}\,\text{Oe}}\right)$$

$$f_r = \frac{\omega_r}{2\pi} = \frac{\mu_0 \gamma}{2\pi} \left(\frac{1\,\text{A/m}}{4\pi \times 10^{-3}\,\text{Oe}}\right)(1300\,\text{Oe})$$

$$= 2.8\,\frac{\text{MHz}}{\text{Oe}}(1300\,\text{Oe}) = 3.64\,\text{GHz}$$

9.5 $\quad 4\pi M_s = 1200\,G$, $\epsilon_r = 10$, $H_0 = 500\,\text{Oe}$, $f = 8\,\text{GHz}$

(FARADAY ROTATION)

$$f_0 = (2.8\,\tfrac{\text{MHz}}{\text{Oe}})(500\,\text{Oe}) = 1.4\,\text{GHz}$$

$$f_m = (2.8\,\tfrac{\text{MHz}}{\text{Oe}})(1200\,G) = 3.36\,\text{GHz}$$

$$k_0 = 167.6\,\text{m}^{-1}$$

Then from (9.25),

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.924\,\mu_0$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = -0.433\,\mu_0$$

From (9.52) the propagation constants of the CP waves are,

RHCP: $\beta_+ = \omega\sqrt{\epsilon(\mu+\kappa)} = k_0\sqrt{\epsilon_r}\sqrt{0.924-0.433} = 371.4\,\text{m}^{-1}$

LHCP: $\beta_- = \omega\sqrt{\epsilon(\mu-\kappa)} = k_0\sqrt{\epsilon_r}\sqrt{0.924+0.433} = 617.4\,\text{m}^{-1}$

Then, $\Delta\beta = \beta_+ - \beta_- = -246.0\,\text{m}^{-1}$

From (9.57) the polarization rotation of an LP wave is,

$$\phi = -(\beta_+ - \beta_-)z/2$$

so,

$$z = \frac{2\phi}{\beta_- - \beta_+} = \frac{2(\pi/2)}{246\,\text{rad/m}} = 12.8\,\text{mm}$$

(9.6) $4\pi M_s = 1780$ G, $\epsilon_r = 13$, $H_0 = 2000$ Oe, $f = 5$ GHz

$$f_0 = (2.8 \text{ MHz/Oe})(2000 \text{ Oe}) = 5.60 \text{ GHz}$$
$$f_m = (2.8 \text{ MHz/Oe})(1780 \text{ Oe}) = 4.98 \text{ GHz}$$
$$k_0 = 2\pi f/c = 104.7 \text{ m}^{-1}$$

From (9.25),
$$\mu = \mu_0 \left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 5.385 \mu_0$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = 3.915 \mu_0$$

The $\hat{x}$-polarized wave has $\bar{H} = \hat{y} H_y$, and is the extraordinary wave. From (9.64)–(9.65),

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu} = 2.539 \mu_0$$

$$\beta_e = \omega\sqrt{\epsilon \mu_e} = k_0 \sqrt{\epsilon_r \mu_e/\mu_0} = 601.5 \text{ m}^{-1}$$

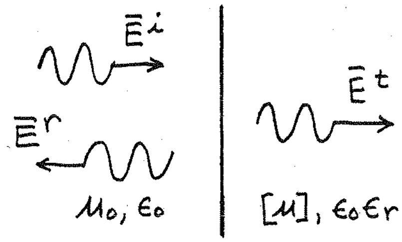
The $\hat{y}$-polarized wave has $\bar{H} = \hat{x} H_x$, and is the ordinary wave. Thus,

$$\beta_o = \sqrt{\epsilon_r}\, k_0 = 377.5 \text{ m}^{-1}$$

So the distance required for a differential phase shift of 180° is,

$$L = \frac{\pi}{\beta_e - \beta_o} = \underline{1.403 \text{ cm}}$$

9.7



The incident, reflected, and transmitted fields for a RHCP wave can be written as,

$$\bar{E}^i = E_0(\hat{x} - j\hat{y})e^{-j\beta_0 z} \qquad \bar{H}^i = \frac{E_0}{\eta_0}(\hat{y} + j\hat{x})e^{-j\beta_0 z}$$

$$\bar{E}^r = \Gamma^+ E_0(\hat{x} - j\hat{y})e^{j\beta_0 z} \qquad \bar{H}^r = \frac{-E_0}{\eta_0}\Gamma^+(\hat{y} + j\hat{x})e^{j\beta_0 z}$$

$$\bar{E}^t = T^+ E_0(\hat{x} - j\hat{y})e^{-j\beta_+ z} \qquad \bar{H}^t = \frac{E_0}{\eta_0}T^+(\hat{y} + j\hat{x})e^{-j\beta_+ z}$$

where, $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$, $\beta_+ = \omega\sqrt{\epsilon(\mu+\kappa)}$, $\eta_+ = \frac{1}{Y_+} = \sqrt{\frac{\mu+\kappa}{\epsilon}}$

matching fields at $z=0$ gives (for both $\hat{x}$ and $\hat{y}$ components)

$$1 + \Gamma^+ = T^+$$

$$\eta_+(1 - \Gamma^+) = \eta_0 T^+$$

Solving gives,

$$\Gamma^+ = \frac{\eta_+ - \eta_0}{\eta_+ + \eta_0} \quad , \quad T^+ = \frac{2\eta_+}{\eta_+ + \eta_0} \quad \checkmark$$

Similarly, for a LHCP wave we obtain,

$$\Gamma^- = \frac{\eta_- - \eta_0}{\eta_- + \eta_0} \quad , \quad T^- = \frac{2\eta_-}{\eta_- + \eta_0} \quad \checkmark$$

where, $\eta_- = \frac{1}{Y_-} = \sqrt{\frac{\mu-\kappa}{\epsilon}}$.

9.8 $4\pi M_s = 1200\,G$, $\bar{H}_0 = \hat{x} H_0$, $f = 4\,GHz$, $\bar{E} = \hat{x} E_0$

$f_m = (2.8\,MHz/Oe)(1200\,G) = 3.36\,GHz$

This is a case of birefringence. From (9.64)–(9.65),

$$\beta_e = \omega\sqrt{\mu_e \epsilon}$$

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu}$$

The wave will be cutoff when $\mu_e \leq 0$:

$$\frac{\mu^2 - \kappa^2}{\mu} < 0$$

$$\frac{\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right)^2 - \left(\frac{f f_m}{f_0^2 - f^2}\right)^2}{1 + \frac{f_0 f_m}{f_0^2 - f^2}} < 0$$

$$\frac{(f_0^2 - f^2)^2 + 2 f_0 f_m (f_0^2 - f^2) + f_m^2 (f_0^2 - f^2)}{(f_0^2 - f^2)\left[(f_0^2 - f^2) + f_0 f_m\right]} < 0$$

$$\frac{(f_0^2 - f^2) + 2 f_0 f_m + f_m^2}{f_0^2 - f^2 + f_0 f_m} < 0$$

If $f_0^2 - f^2 + f_0 f_m > 0$, then $(f_0 + f_m)^2 - f^2 < 0$

$$f_0 + f_m < f$$

$$f_0 < f - f_m = 4.0 - 3.36 = 0.64\,GHz \Rightarrow H_0 = 229\,Oe \checkmark$$

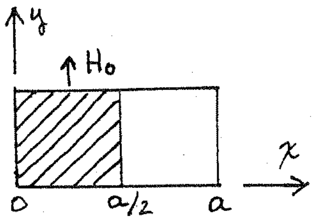
If $f_0^2 - f^2 + f_0 f_m < 0$, then $(f_0 + f_m)^2 - f^2 > 0$

$$f_0 = \frac{-f_m \pm \sqrt{f_m^2 + 4f^2}}{2} = -1.68 \pm 4.34 = 2.66\,GHz$$

$$H_0 = 950\,Oe \checkmark$$

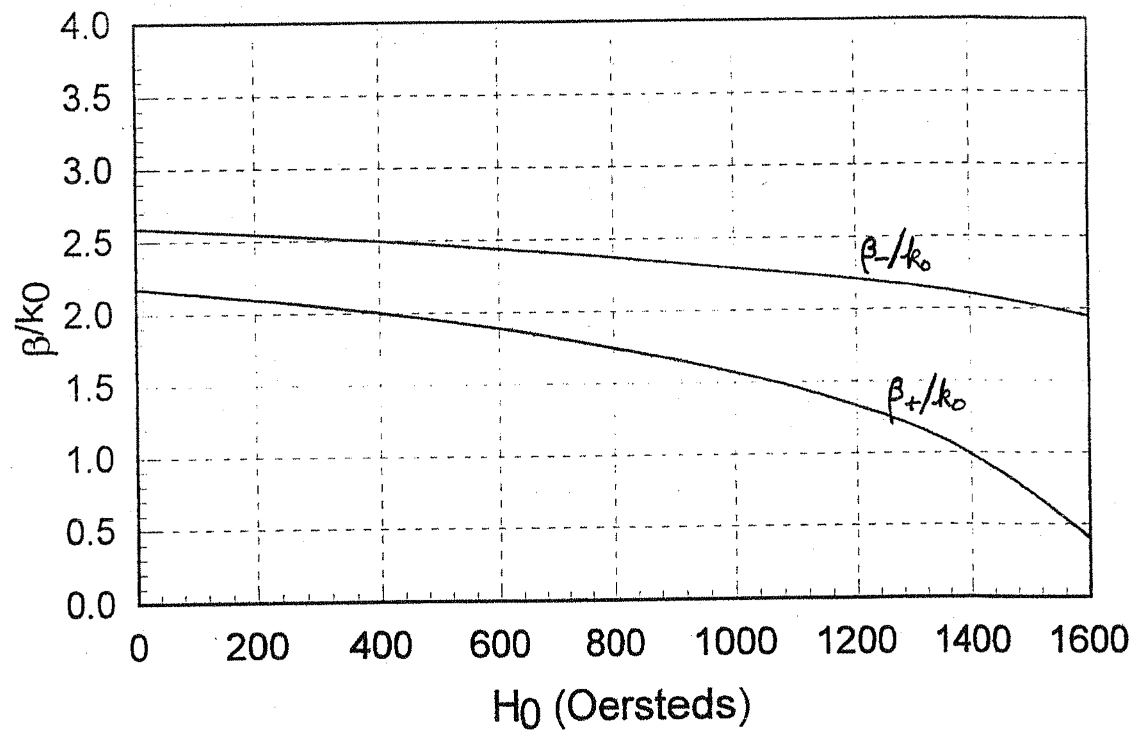
So the cutoff range is between 229 Oe and 950 Oe.

(9.9)

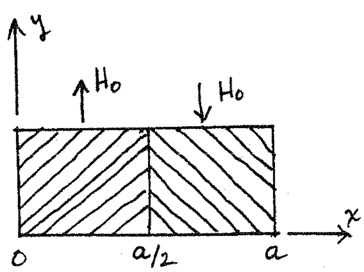


$a = 1.0$ cm $\quad (c=0,\ t=a/2)$
$f = 10$ GHz
$4\pi M_s = 1700$ G
$\epsilon_r = 13$

The propagation constants are found by solving (9.79) numerically. This was done using the FORTRAN program FLW1S.FOR, with the results shown below:

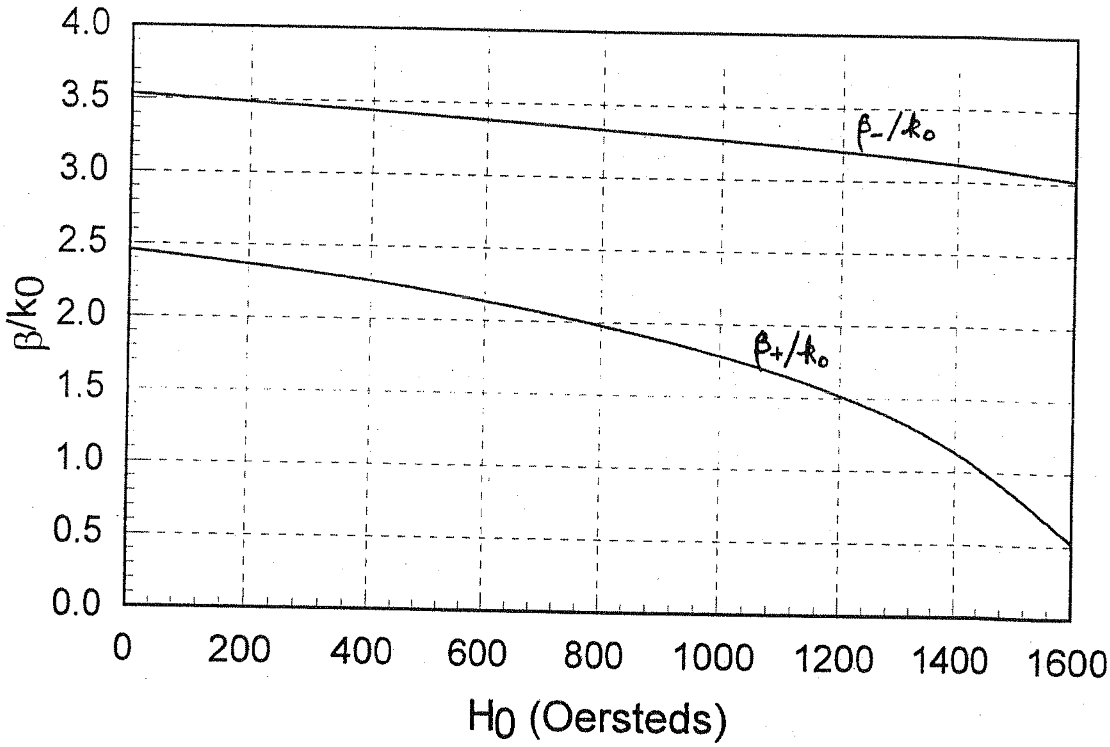


(9.10)

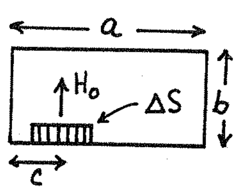


$a = 1.0$ cm $\quad (c=0,\ t=a/2)$
$f = 10$ GHz
$4\pi M_s = 1700$ G
$\epsilon_r = 13$

The propagation constants are found by solving (9.84) numerically. This was done using the FORTRAN program FLW2S.FOR, with the results shown below.



9.11



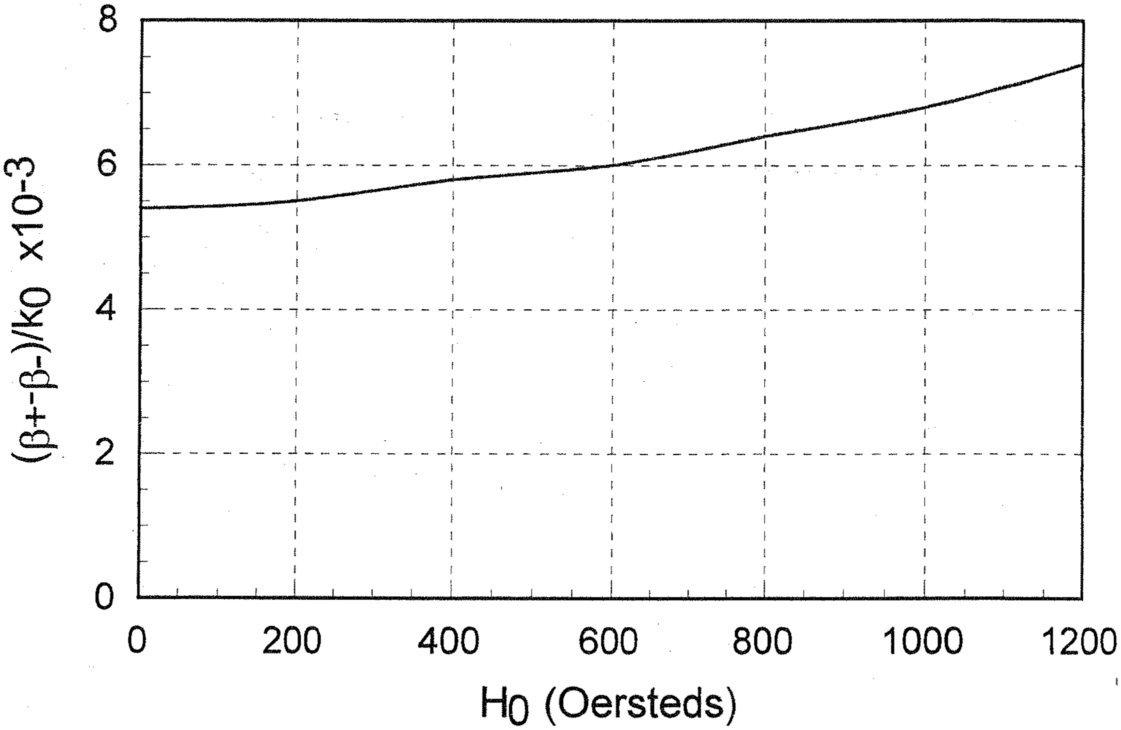
$f = 10\,GHz$, $a = 2.286$ cm, $b = 1.016$ cm
$4\pi M_s = 1700G$, $c = a/4$, $\Delta S = 2\,mm^2$
$k_o = 209.4\,m^{-1}$, $S = 232.3\,mm^2$, $f_m = 4.76\,GHz$ ✓

From (9.80) the differential phase shift is,

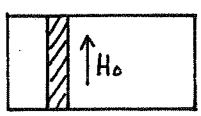
$$(\beta_+ - \beta_-)/k_o = -2\,\frac{k_c}{k_o}\,\frac{\kappa}{\mu}\,\frac{\Delta S}{S}\sin 2k_c c = -0.0113\ \kappa/\mu$$

H_o (Oe)	f_o (GHz)	κ/μ_o	μ/μ_o	$(\beta_+-\beta_-)/k_o$
0	0	−0.476	1.000	0.0054 ✓
200	0.56	−0.477	0.973	0.0055
400	1.12	−0.482	0.946	0.0058
600	1.68	−0.489	0.918	0.0060
800	2.24	−0.501	0.891	0.0064
1000	2.80	−0.516	0.855	0.0068
1200	3.36	−0.537	0.820	0.0074 ✓

This data is plotted on the following page



(9.12)



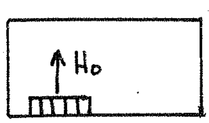
$f = 8\ GHz$, $4\pi M_s = 1500\ G$, $f_m = 4.2\ GHz$

Gyromagnetic resonance for this geometry is given approximately by (9.87): $(N_x = 1,\ N_y = N_z = 0)$

$$f = \sqrt{f_o(f_o + f_m)}$$

Solve for f_o: $\quad f_o^2 + 4.2 f_o - 64 = 0 \Rightarrow f_o = -2.1 \pm 8.27 = 6.17\ GHz$

Thus, $\quad H_o = \dfrac{6170\ MHz}{2.8\ MHz/Oe} = 2204\ Oe.$ ✓

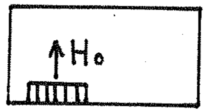


Gyromagnetic resonance for this geometry is given by the condition,

$f_o = f = 8\ GHz \qquad (N_x = N_z = 0,\ N_y = 1)$

Thus, $\quad H_o = \dfrac{8000\ MHz}{2.8\ MHz/Oe} = 2857.\ Oe$ ✓

9.13



$f = 10\,GHz$, $4\pi M_s = 1700\,G$, $\Delta H = 200\,Oe$

$\frac{\Delta S}{S} = 0.01$

Gyromagnetic resonance is given by the condition,

$$f_0 = f = 10\,GHz \quad (\text{since } N_x = N_z = 0,\ N_y = 1)$$

$$H_0 = \frac{10{,}000\,MHz}{2.8\,MHz/Oe} = 3571.\,Oe$$

The position of the ferrite slab is given by (9.86), since the RF magnetic fields, h_x and h_z, have demagnetization factors of zero. Thus,

$$\tan k_c x = \pm \frac{k_c}{\beta_0}$$

$k_0 = 209.4\ m^{-1}$ ✓

$k_c = \pi/a = 137.4\ m^{-1}$.

$\beta_0 = \sqrt{k_0^2 - k_c^2} = 158.\ m^{-1}$

$$S = x = \frac{1}{k_c}\tan^{-1}\frac{k_c}{\beta_0} = \underline{0.521\,cm}$$

The perturbation result of (9.81) must be used to find the attenuation constants, since this geometry cannot be analyzed exactly:

$$\alpha_\pm = \frac{\Delta S}{S\beta_0}\left(\beta_0^2 \chi''_{xx}\sin^2 k_c x + k_c^2 \chi''_{zz}\cos^2 k_c x \mp \chi''_{xy} k_c \beta_0 \sin 2k_c x\right)$$

where the susceptibilities are given by (9.39):

$f_0 = f = 10\,GHz$

$f_m = 1700(2.8) = 4.76\,GHz$

$$\alpha = \frac{\Delta H}{2\omega\mu_0\gamma} = \frac{(200)(2.8\,MHz)}{2(10{,}000\,MHz)} = 0.028$$

$$\chi''_{xx} = \frac{\alpha f f_m\left[f_0^2 + f^2(1+\alpha^2)\right]}{\left[f_0^2 - f^2(1+\alpha^2)\right]^2 + 4 f_0^2 f^2 \alpha^2} = 8.50$$

$$\chi''_{zz} = \chi''_{xx} = 8.50$$

$$\chi''_{xy} = \frac{2 f_0 f_m f^2 \alpha}{\left[f_0^2 - f^2(1+\alpha^2)\right]^2 + 4 f_0^2 f^2 \alpha^2} = 8.49$$

Then, $\alpha_\pm = 6.3\times10^{-5}\,(9.139\times10^4 + 9.136\times10^4 \mp 1.825\times10^5)$

$$\alpha_+ = 0.0158 \text{ neper/m} = 0.137 \text{ dB/m}$$

$$\alpha_- = 23.0 \text{ neper/m} = 200 \text{ dB/m}$$

For 30dB reverse attenuation, the required length is,

$$L = \frac{30\text{ dB}}{200\text{ dB/m}} = 0.15 \text{ m} = 15. \text{ cm}$$

Then the forward insertion loss is,

$$IL = (0.137)(0.15) = 0.02 \text{ dB}$$

Note: the calculation of $\alpha_\pm$ is numerically sensitive.

9.14 The magnetic fields for the TE_{10} waveguide mode can be written as,

$$H_x = \frac{j\beta A}{k_c} \sin k_c x \, e^{-j\beta z}$$

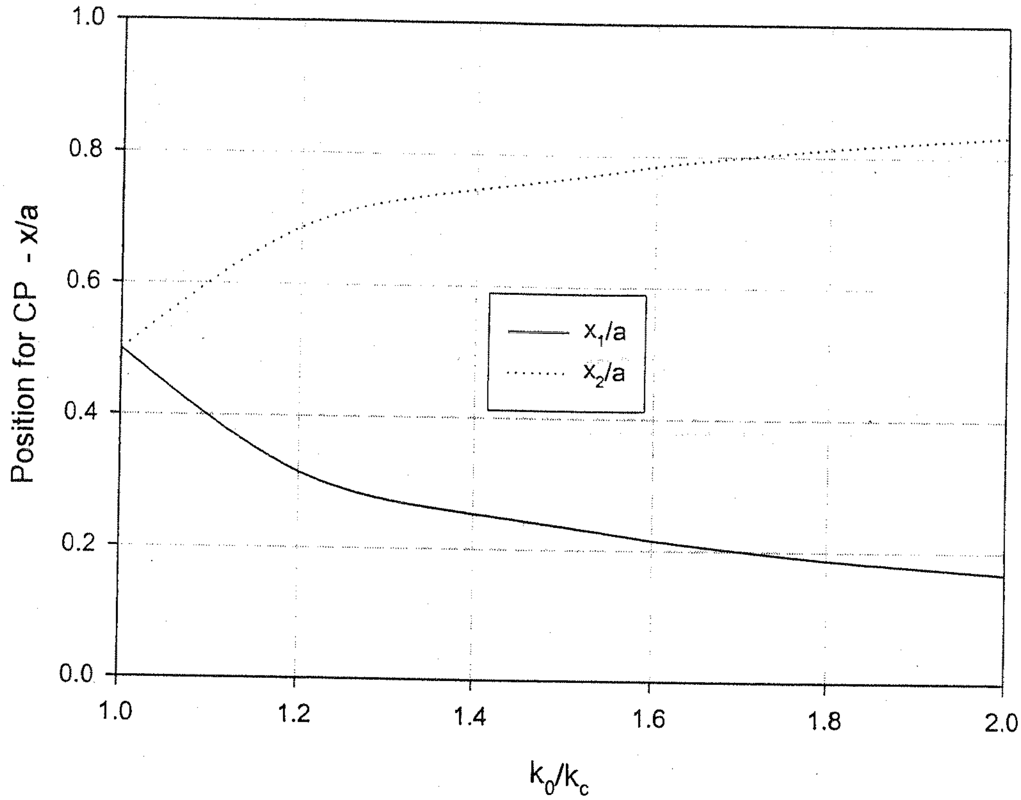
$$H_z = A \cos k_c x \, e^{-j\beta z}$$

with $k_c = \pi/a$, $\beta = \sqrt{k_0^2 - k_c^2}$. Circular polarization occurs when,

$$\frac{H_x}{H_z} = \pm j = \frac{j\beta}{k_c} \tan k_c x, \quad \text{or} \quad \tan k_c x = \pm \frac{k_c}{\beta} \qquad (9.86)$$

k_0/k_c	β/k_c	x_1/a	x_2/a
1.0	0	0.500	0.500
1.2	0.663	0.314	0.686
1.4	0.980	0.253	0.747
1.6	1.249	0.215	0.785
1.8	1.497	0.187	0.813
2.0	1.732	0.167	0.833

These results are plotted on the following page.



(9.15) $f = 5\,GHz$, $\epsilon_r = 10$, $4\pi M_r = 1200\,G$, $L = 3.65\,cm$

$$f_m = (2.8\,MHz/Oe)(1200\,Oe) = 3.36\,GHz \checkmark \qquad k_0 = 2\pi f/c = 104.7\,m^{-1}$$

In the remanent state, $H_0 = f_0 = 0$. Then,

$$\mu = \mu_0, \quad \kappa = -\frac{f_m}{f}\mu_0 = -0.672\,\mu_0, \quad \mu_e = \frac{\mu^2 - \kappa^2}{\mu} = 0.548\,\mu_0$$

In State #1, $\bar{M} = M_r\hat{x}$ and $\bar{H} = \hat{y}H_y$, so this is an extraordinary wave:

$$\beta_e = k_0\sqrt{\epsilon_r \mu_e/\mu_0} = 245.1\,m^{-1}$$

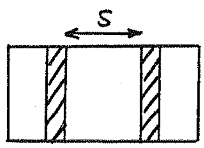
In State #2, $\bar{M} = M_r\hat{y}$ and $\bar{H} = \hat{y}H_y$, so this is an ordinary wave:

$$\beta_o = \sqrt{\epsilon_r}\,k_0 = 331.1\,m^{-1}$$

Then the differential phase shift is,

$$\Delta\phi = (\beta_2 - \beta_1)L = (\beta_o - \beta_e)L = \underline{180^\circ}$$

(9.16) $s = 2\,mm,\ f = 10\,GHz,\ 4\pi M_r = 1000\,G$



From Figure 9.17, maximum differential phase shift for $s = 2\,mm$ occurs for $t/a \cong 0.112$, so $t = 0.112a = 2.6\,mm$ ✓.
Then $(\beta_+ - \beta_-)/k_o = 0.24$, for $4\pi M_r = 1786\,G$. If we assume $(\beta_+ - \beta_-)$ is proportional to κ (and so M_r), then for $4\pi M_r = 1000\,G$ we have

$$\frac{(\beta_+ - \beta_-)}{k_o} = 0.24\left(\frac{1000}{1786}\right) = 0.134,$$

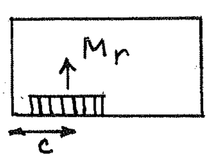
so, $(\beta_+ - \beta_-) = 0.134 k_o = 16.1°/cm$ ✓

Then the slab lengths for 180° and 90° sections are,

$$L = \frac{180°}{16.1°/cm} = 11.2\ cm \checkmark$$

$$L = \frac{90°}{16.1°/cm} = 5.6\ cm \checkmark$$

(9.17)



$a = 2.286\ cm,\ b = 1.016\ cm,\ f = 9\ GHz$
$4\pi M_r = 1200\,G,\ c = a/4,\ \Delta S = 2\ mm^2$

From (9.80) the (approximate) differential phase shift is,

$$\beta_+ - \beta_- = \frac{-2\pi}{a}\frac{\kappa}{\mu}\frac{\Delta S}{S}\sin\frac{2\pi c}{a}$$

Now for $H_o = 0$,

$$\frac{\kappa}{\mu} = -\frac{f_m}{f} = \frac{-2.8(1200)}{9000} = -0.373$$

and, $S = ab = 232.3\ mm^2$, so
$\beta_+ - \beta_- = 0.883\ rad/m = 0.506°/cm$ ✓

So the required length is,

$$L = \frac{22.5°}{0.506°/cm} = 44.5\ cm \checkmark$$

(a bit long!)

(9.18)

$f = 9$ GHz, $4\pi M_s = 1700$ G, $\Delta S = 6$ mm^2

$a = 2.286$ cm, $b = 1.016$ cm, $H_a = 4000$ Oe

From (9.41) the internal bias field is ($N_z = 1$)

$$H_0 = H_a - NM_s = 4000 - 1700 = \underline{2300 \text{ Oe}}$$

From (9.80) the (approximate) differential phase shift is,

$$\beta_+ - \beta_- = \frac{-2\pi}{a} \frac{\kappa}{\mu} \frac{\Delta S}{S} \sin \frac{2\pi c}{a}$$

Maximum phase shift will occur for $c = a/4 = \underline{0.572 \text{ cm}}$.

Then, $f_m = 2.8(1700) = 4.76$ GHz, $f_0 = 2.8(2300) = 6.44$ GHz.

So,

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.224\,\mu_0$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = -1.08\,\mu_0$$

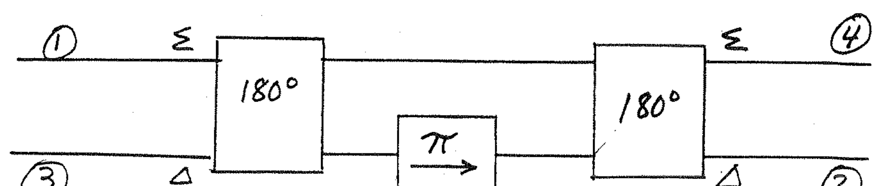
Thus,

$$\beta_+ - \beta_- = 0.342 \text{ rad/cm} = 19.6°/\text{cm}$$

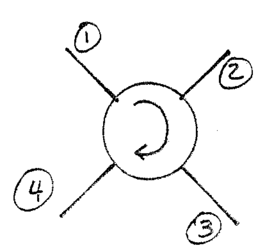
So the required length for a 180° phase shift is,

$$L = \frac{180°}{19.6°/\text{cm}} = \underline{9.2 \text{ cm}}$$

(9.19) a four-port circulator can be made using a gyrator and two 180° hybrid couplers:



a three port circulator can be obtained by shorting any one of the ports.



9.20 $RL = 10dB,\ 20dB.$

From (9.92) the scattering matrix of a mismatched circulator is,

$$[S] = \begin{bmatrix} \Gamma & \beta & \alpha \\ \alpha & \Gamma & \beta \\ \beta & \alpha & \Gamma \end{bmatrix}$$

Then $|\beta| \approx |\Gamma|$

$|\alpha| \simeq 1 - |\Gamma|^2$

For $RL = 10dB$, $I = |\beta| = 10dB$

For $RL = 20dB$, $I = |\beta| = 20dB$

10.1 From (10.6),

$$ENR = 10\log\frac{T_1-T_0}{T_0} \Rightarrow \frac{T_1-T_0}{T_0}=100,\ T_1=2.93\times10^4\ K$$

$$T_e=\frac{T_1-YT_2}{Y-1}=\frac{2.93\times10^4-(42.51)(77)}{42.51-1}=627K$$

$$F=1+\frac{T_e}{T_0}=1+\frac{627}{290}=3.16=\underline{\underline{5.0dB}}$$

10.2

$$T_e=\frac{T_1-YT_2}{Y-1}$$

$$T_e+\Delta T_e=\frac{T_1-(Y+\Delta Y)T_2}{(Y+\Delta Y)-1}$$

$$\Delta T_e=\frac{T_1-(Y+\Delta Y)T_2}{(Y+\Delta Y)-1}-\frac{T_1-YT_2}{Y-1}$$

$$=\frac{T_1-(Y+\Delta Y)T_2}{(Y-1)\left(1+\frac{\Delta Y}{Y-1}\right)}-\frac{T_1-YT_2}{Y-1}$$

$$\simeq\frac{[T_1-(Y+\Delta Y)T_2]\left[1-\frac{\Delta Y}{Y-1}\right]-(T_1-YT_2)}{Y-1}$$

$$\simeq\frac{-\frac{T_1}{Y-1}+\frac{YT_2}{Y-1}-T_2}{Y-1}\Delta Y=\frac{(T_2-T_1)}{(Y-1)^2}\Delta Y$$

$$\frac{\Delta T_e}{T_e}=\frac{(T_2-T_1)Y}{(Y-1)^2T_e}\frac{\Delta Y}{Y}=\frac{(T_1+T_e)(T_2+T_e)}{T_e(T_2-T_1)}\frac{\Delta Y}{Y}$$

minimize with respect to Te:

$$\frac{d}{dT_e}\left[\frac{\Delta T_e}{T_e}\right]=\frac{\left(\frac{T_1}{T_e}+1\right)\left(\frac{T_2}{T_e}+1\right)+T_e\left(\frac{-T_1}{T_e^2}\right)\left(\frac{T_2}{T_e}+1\right)+T_e\left(\frac{T_1}{T_e}+1\right)\left(\frac{-T_2}{T_e^2}\right)}{T_2-T_1}=0$$

Thus,

$$T_e=\sqrt{T_1T_2}\ \checkmark$$

(10.3) Find attenuation for each line.

X-band W.G.: From Chapter 3, the attenuation of copper X-band guide at 10 GHz is $\alpha_c = 0.11$ dB/m. So the total loss is $L = 0.22$ dB. ✓

RG-8/U: From appendix J, $\alpha_c = 35$ dB/100 ft, or $\alpha_c = 1.15$ dB/m. So the total loss is $L = 2.30$ dB.

Circular W.G.: From Chapter 3, the attenuation of the TE_{11} mode is,

$$\alpha_c = \frac{R_s}{ak\eta\beta}\left(k_c^2 + \frac{k^2}{p_{11}'^2 - 1}\right) = 0.0172 \text{ np/m} = 0.15 \text{ dB/m} ✓$$

So the total loss is $L = 0.30$ dB. Thus, the rectangular waveguide is the best choice.

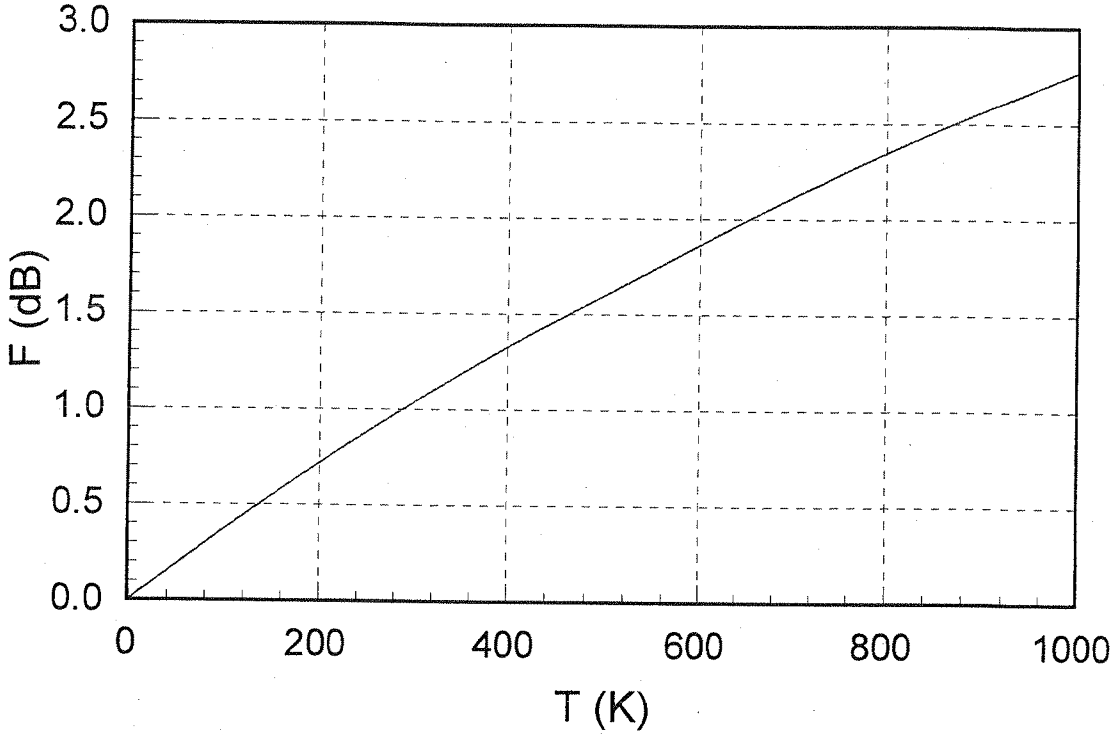
(10.4) From (10.16), the noise figure of a lossy line is,

$$F = 1 + (L-1)\frac{T}{T_0}.$$

When $T = T_0$, $F = L = 1\text{ dB} = 1.259$. Thus, $F = 1 + 0.259\left(\frac{T}{290K}\right)$

T (K)	F (dB)
0	0
250	0.88
500	1.60
750	2.23
1000	2.77

This data is plotted on the following page.



(10.5) The equivalent noise power input is,

$$P_{min} = kT_eB = (1.38\times10^{-23})(250)(10^9) = 3.5\times10^{-12}\ W$$

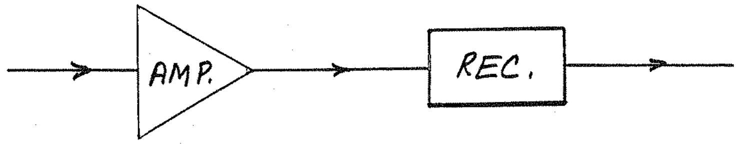
If we assume the upper limit is set by the 1dB compression point, then

$$P_{max} = -10\,dBm = 0.1\ mW = 10^{-4}\ W.$$

Thus the dynamic range is,

$$DR_\ell = 10\log\frac{P_{MAX}}{P_{MIN}} = 10\log\left(\frac{10^{-4}}{3.5\times10^{-12}}\right) = 75\,dB$$

(10.6)



$G = 12\,dB = 15.8$
$BW = 150\,MHz$
$F = 4\,dB = 2.51$

$T_e = 900\,K$

The noise figure of the receiver is, from (10.11),

$$F_2 = 1 + \frac{T_e}{T_0} = 1 + \frac{900}{290} = 4.10$$

Then the noise figure of the cascade is, from (10.21),

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1) = 2.51 + \frac{4.10 - 1}{15.8} = 2.71 = 4.3\,dB \checkmark$$

(10.7)

a) $T_e = \dfrac{P}{kB} = \dfrac{(.001)\times 10^{-9.5}}{(1.38\times 10^{-23})(75\times 10^{6})} = 305.5K \checkmark$

b) $F_\ell = 1 + (L-1)\dfrac{T}{T_0} = 1 + (1.413 - 1)\dfrac{300}{290} = 1.43$, $F_a = 1 + \dfrac{T_e}{T_0} = 1.62 = 2.1\,dB$

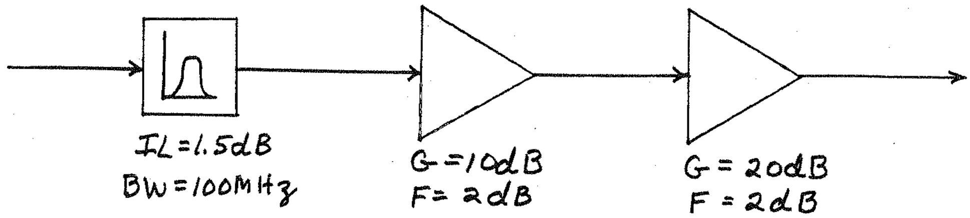
c) $F_c = F_\ell + \dfrac{F_a - 1}{G_\ell} = 1.43 + \dfrac{1.62 - 1}{1/1.413} = 2.30 = \underline{3.6\,dB} \checkmark$

$T_c = (F_c - 1)T_0 = (2.30 - 1)(290) = \underline{378K} \checkmark$

d) $N_o = k(T_c + T_i)BG = (1.38\times 10^{-23})(378 + 305.5)(75\times 10^{6})\left(\dfrac{15.8}{1.413}\right)$

$= 7.9\times 10^{-12}\,W = 7.9\times 10^{-9}\,mW = \underline{-81.0\,dBm} \checkmark$

10.8



From (10.23) the noise figure of the cascade is ($F_1 = IL = 1.5dB$)

$$F_{CAS} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.41 + (1.41)(1.58-1) + \frac{1.41}{10}(1.58-1)$$

$$= 2.31 = 3.64dB$$

If $P_{in} = -90$ dBm, then $P_{out} = -90dBm - 1.5dB + 10dB + 20dB = -61.5dBm$.
The noise power output is then,

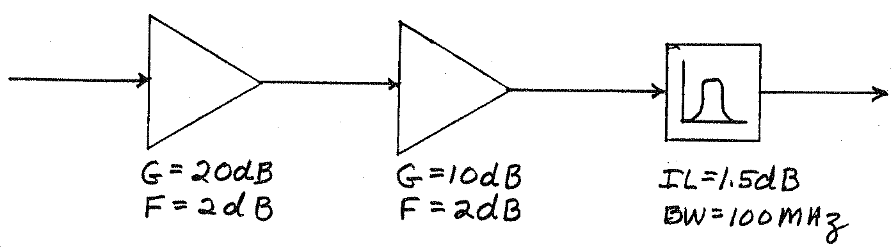
$$P_n = G_{cas} k T_{e_{cas}} B = k(F_{cas} - 1) T_0 B G_{cas}$$

$$= (1.38 \times 10^{-23})(2.31-1)(290)(10^8)(10^{28.5/10}) = 3.71 \times 10^{-10} W$$

$$= -64.3 dBm$$

Thus, $\frac{S_o}{N_o} = -61.5 + 64.3 = 2.8dB$

The best noise figure would be achieved with the arrangement shown below:



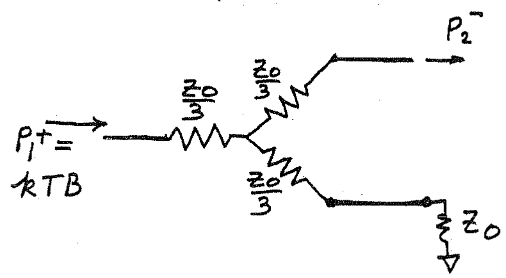
Then,

$$F_{CAS} = 1.58 + \frac{(1.58-1)}{100} + \frac{(1.41-1)}{1000} = 1.586 = 2.0dB$$

(In practice, however, the initial filter may serve to prevent overload of the amplifier, and may not be allowed to be moved.)

(10.9)

a) RESISTIVE DIVIDER



$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

When the input noise power at port 1 is kTB, and the divider is at temperature T, the system is in thermodynamic equilibrium. Thus the output noise power at port 2 must be kTB. We can also express this as due to the attenuated input noise power and noise power added by the network (ref. at input). Thus,

$$P_2^- = kTB = \frac{kTB}{4} + \frac{N_{added}}{4}$$

$$\therefore N_{added} = 3kTB$$

The equivalent noise temperature is then,

$$T_e = \frac{N_{added}}{kB} = 3T$$

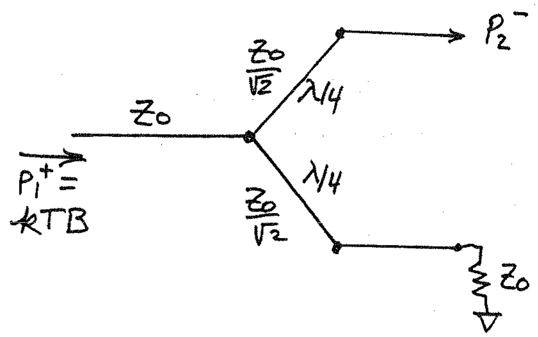
and the noise figure is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{3T}{T_0}$$

at room temperature, $T = T_0$, so $F = 4 = 6\text{dB}$.

(this result checks with that obtained using the available gain method)

b) WILKINSON DIVIDER



$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$

In this case, if the input noise power is kTB, and the system is in thermodynamic equilibrium, the net output power at port 2 is $\frac{3}{4}kTB$, because of the mismatch of the output ports (1/4 of output power is reflected). Then we have,

$$P_2^- = \frac{3}{4}kTB = \frac{kTB}{2} + \frac{N_{added}}{2} \qquad (N_{added} \text{ ref. at input})$$

$$\therefore N_{added} = \frac{1}{2}kTB$$

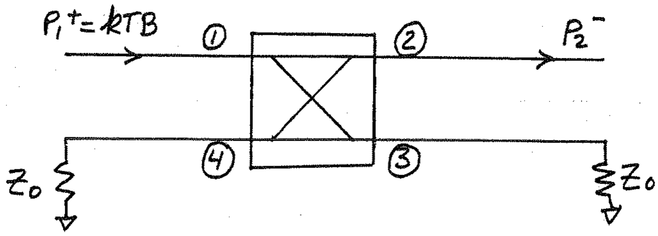
$$T_e = \frac{N_{added}}{kB} = \frac{T}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{2T_0}$$

If $T = T_0$, $F = \frac{3}{2} = 1.76\ dB$.

(Result verified with HP-MDS, calculations using available gain, and direct measurement)

c) QUADRATURE HYBRID



$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Using the same thermodynamic arguments as above, the output noise power is kTB (outputs are matched). Thus,

$$P_2^- = kTB = \frac{kTB}{2} + \frac{N_{added}}{2}$$

$$\therefore N_{added} = kTB$$

$$T_e = \frac{N_{added}}{kB} = T$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{T_0}$$

If $T = T_0$, we have $F = 2 = 3dB$

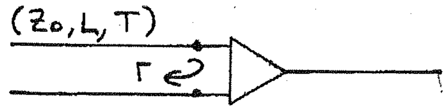
(10.10) From (10.33), $T_e = \dfrac{(L-1)(L+|\Gamma_s|^2)}{L(1-|\Gamma_s|^2)} T$

let $x^2 = |\Gamma_s|^2$; $C = (L-1)T/L$. Then $T_e = C \dfrac{L+x^2}{1-x^2}$

$$\frac{dT_e}{dx} = C \frac{(1-x^2)(2x)+(2x)(L+x^2)}{(1-x^2)^2} = \frac{2x(1+L)}{(1-x^2)^2} = 0$$

Thus $x = 0$, so $|\Gamma_s| = 0$ minimizes T_e ✓

10.11



Solution using noise temperature:

Let $N_i = kT_0B$

Then $N_o = \underbrace{\frac{kT_0BG}{L}(1-|\Gamma|^2)}_{\text{INPUT NOISE}} + \underbrace{\frac{(L-1)}{L}kTB(1-|\Gamma|^2)G}_{\text{NOISE ADDED BY LINE}} + \underbrace{kT_0(F-1)GB}_{\text{NOISE ADDED BY AMP}}$

also,
$$S_o = \frac{G(1-|\Gamma|^2)}{L} S_i$$

So,
$$F_{CAS} = \frac{S_i N_o}{S_o N_i} = \frac{L}{\cancel{G}(1-|\Gamma|^2)} \cdot \frac{\frac{\cancel{kT_0B}\cancel{G}}{L}(1-|\Gamma|^2) + \left(\frac{L-1}{L}\right)\cancel{G}\cancel{k}T\cancel{B}(1-|\Gamma|^2) + \cancel{k}T_0(F-1)\cancel{G}\cancel{B}}{\cancel{kT_0B}}$$

$$= 1 + (L-1)\frac{T}{T_0} + \frac{L(F-1)}{1-|\Gamma|^2} \checkmark$$

Solution using cascade formula:

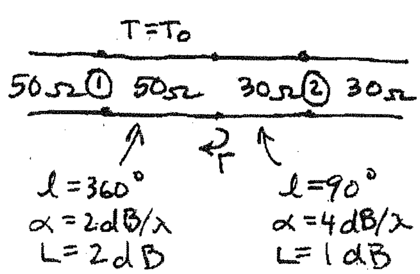
$$T_{e\,(LINE)} = (L-1)T$$
$$F_{(LINE)} = 1 + (L-1)T/T_0$$
$$G_{(LINE)} = \frac{1}{L}(1-|\Gamma|^2)$$

$$F_{CAS} = F_{(LINE)} + \frac{F_{AMP}-1}{G_{(LINE)}} = 1 + (L-1)\frac{T}{T_0} + \frac{L}{1-|\Gamma|^2}(F-1) \checkmark$$

CHECK: IF $\Gamma = 0$, $F_{CAS} = 1 + (L-1)\frac{T}{T_0} + L(F-1)$ ✓

IF $\Gamma = 0$ AND $T = T_0$, $F_{CAS} = 1 + (L-1) + L(F-1) = LF$ ✓

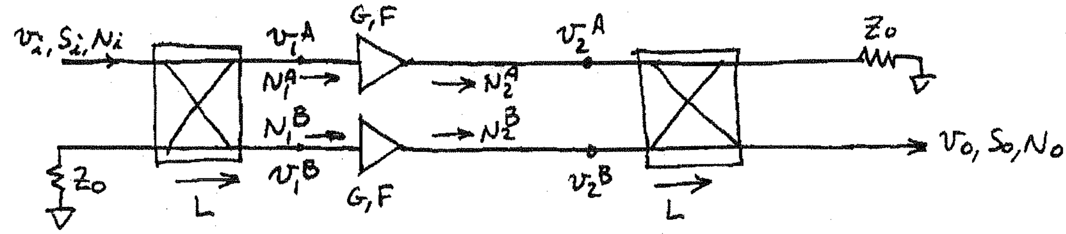
NUMERICAL CHECK:



$$\Gamma = \frac{30-50}{30+50} = \frac{-1}{4}$$

$F_{CAS} = 3.06$ dB – AGREES WITH SERENADE.

(10.12)



$$S_i = v_i^2/2$$

$$v_1^A = \frac{v_i}{\sqrt{2L}} \qquad v_1^B = -j\frac{v_i}{\sqrt{2L}}$$

$$v_2^A = \frac{v_i\sqrt{G}}{\sqrt{2L}} \qquad v_2^B = -j\frac{v_i\sqrt{G}}{\sqrt{2L}}$$

$$v_0 = -j\frac{v_2^A}{\sqrt{2L}} + \frac{v_2^B}{\sqrt{2L}} = -j\frac{v_i\sqrt{G}}{2L} - j\frac{v_i\sqrt{G}}{2L} = -j\frac{v_i\sqrt{G}}{L}$$

$$S_0 = \frac{v_0^2}{2} = \frac{v_i^2 G}{2L^2} = \frac{G S_i}{L^2} \checkmark$$

$$N_1^A = N_1^B = kT_0B$$

$$N_2^A = N_2^B = kT_0BG + kT_eBG = kT_0BG(1+F-1) = kT_0BGF$$

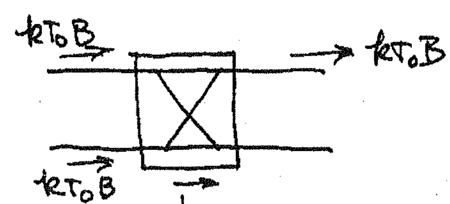
$$N_0 = \frac{N_2^A}{2L} + \frac{N_2^B}{2L} + \frac{N_{ADDED}}{2L} = \frac{kT_0BG}{L}F + \underbrace{\frac{kT_0B}{2L}(2L-2)}_{\text{SEE BELOW}}$$

$$= \frac{kT_0BG}{L}F + kT_0B\left(1-\frac{1}{L}\right)$$

$$F_{TOT} = \frac{S_iN_0}{S_0N_i} = \frac{L^2}{G}\left[\frac{GF}{L} + \left(1-\frac{1}{L}\right)\right] = LF + \frac{L}{G}(L-1) \checkmark$$

CHECK: IF $L=1$, $F_{TOT} = F$ ✓

N_{ADDED} FOR HYBRID:



$$N_0 = \frac{kT_0B}{2L} + \frac{kT_0B}{2L} + \frac{N_{ADDED}}{2L} = kT_0B$$

(REF. AT INPUT, for N_{ADDED})

$\therefore$ $N_{ADDED} = 2kT_0B(L-1)$ (REF. AT INPUT)

(10.13)

$N_i = -105\,dBm = 3.16\times10^{-14}\,W$

$F = 6dB = 4$

$G = 30dB = 10^3$

$B = 20MHz$

$T_e = (F-1)T_o = 3(290) = 870K$ ✓

$N_o = G(N_i + kT_eB)$

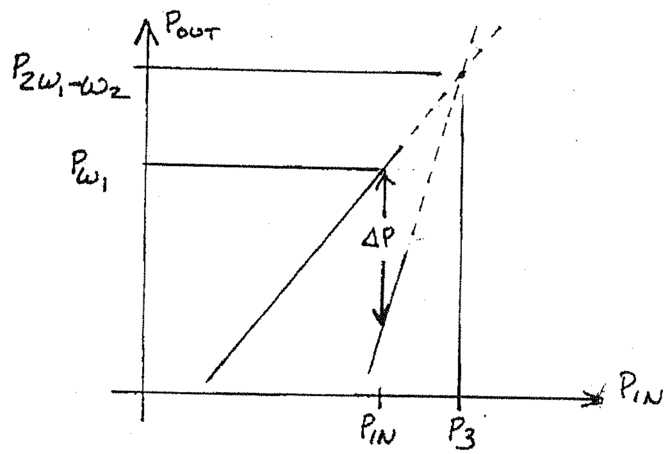
$= 10^3\left[3.16\times10^{-14} + (1.38\times10^{-23})(870)(20\times10^6)\right]$

$= 2.72\times10^{-10}\,W = -65.7\,dBm$ ✓

$DR_\ell = \frac{P_1}{N_o} = 21dBm + 65.7\,dBm = \underline{86.7dB}$ ✓

$DR_f = \frac{2}{3}(P_3 - N_o - SNR) = \frac{2}{3}(33 + 65.7 - 8) = \underline{60.5\,dB}$ ✓

(10.14)



$P_{\omega_1} = P_{in} + b_1$ (EQ. OF LINE, SLOPE = 1)

$P_{2\omega_1-\omega_2} = 3P_{in} + b_2$ (EQ. OF LINE, SLOPE = 3)

SUBTRACT:

$\Delta P = P_{\omega_1} - P_{2\omega_1-\omega_2} = -2P_{in} + b_1 - b_2$

Now, $P_3 = P_{in}$ when $\Delta P = 0$, so

$\Delta P = -2P_{in} + b_1 - b_2$

$0 = -2P_3 + b_1 - b_2$

so, $\Delta P = -2P_{in} + 2P_3$

or, $P_3 = \underbrace{P_{in} + \Delta P/2}_{\text{RELATIVE TO INPUT}} = \underbrace{P_{\omega_1} + \Delta P/2}_{\text{RELATIVE TO OUTPUT}}$ ✓

(10.15) Retaining only the terms that give rise to the third order intermodulation products:

$$v_o \sim k(v_1 \cos\omega_1 t + v_2 \cos\omega_2 t)^3$$

$$\sim k(v_1^2 v_2 \cos^2\omega_1 t \cos\omega_2 t + v_1 v_2^2 \cos\omega_1 t \cos^2\omega_2 t)$$

$$\sim k(v_1^2 v_2 \cos 2\omega_1 t \cos\omega_2 t + v_1 v_2^2 \cos\omega_1 t \cos 2\omega_2 t)$$

$$\sim \frac{k}{2}\left[v_1^2 v_2 \cos(2\omega_1 - \omega_2)t + v_1 v_2^2 \cos(2\omega_2 - \omega_1)t\right]$$

So the ratio of the powers in the two outputs is,

$$\left(\frac{v_1^2 v_2}{v_1 v_2^2}\right)^2 = \left(\frac{v_1}{v_2}\right)^2 = 6\,dB$$

Note that the individual output powers vary as P_{in}^3.

(10.16) Moving the reference for P_3' to the output of the mixer gives, $P_3' = 13dBm - 6dB = 7dB$ (ref. at output)

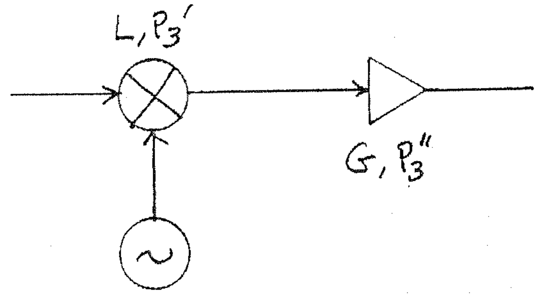
Numerical values:

$P_3'' = 22dBm = 158\,mW$		(AMP)
$P_3' = 7\,dBm = 5\,mW$		(MIXER)
$G_2 = 20dB = 100$		(AMP)

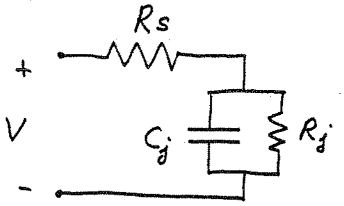
Then from (10.54),

$$P_3 = \left(\frac{1}{G_3 P_3'} + \frac{1}{P_3''}\right)^{-1} = \left[\frac{1}{(100)(5)} + \frac{1}{158}\right]^{-1} = 120\,mW$$

$$= \underline{20.8\,dBm}$$



(10.17) The AC model of the diode is shown below:



The voltage sensitivity is, from (10.63) and (10.64),

$$\beta_v = \beta_i R_j = \frac{\Delta I_{dc}}{P_{in}} R_j \quad ,$$

where P_{in} is the RF input power and ΔI_{dc} is the change in DC current through the diode. If the RF voltage across the diode is V, then the RF input power is,

$$P_{in} = \frac{|V|^2}{2} Re\{Y_d\} = \frac{|V|^2}{2} Re\left\{\frac{1}{R_s + \frac{R_j(1/j\omega C_j)}{R_j + 1/j\omega C_j}}\right\}$$

$$= \frac{|V|^2}{2} Re\left\{\frac{1/R_j + j\omega C_j}{(1+R_s/R_j) + j\omega C_j R_s}\right\} = \frac{|V|^2}{2} \frac{\frac{1}{R_j}(1+R_s/R_j) + \omega^2 C_j^2 R_s}{(1+R_s/R_j)^2 + (\omega C_j R_s)^2}$$

From (10.62) the change in DC current is,

$$\Delta I_{dc} = \frac{|v_o|^2}{4} G_d' = \frac{|v_o|^2}{4} \frac{\alpha}{R_j}$$

where v_o is the peak RF junction voltage. This is the current when the junction is short-circuited. When the packaged diode is shorted, the effect of R_s must be included:

$$\Delta I_{dc} = \frac{|v_o|^2}{4} \frac{\alpha}{R_j} \frac{R_j}{R_j + R_s} = \frac{\alpha |v_o|^2}{4(R_j + R_s)}$$

The relation between v_o^2 and $|V|^2$ is,

$$v_o = V \frac{\frac{R_j/j\omega C_j}{R_j + 1/j\omega C_j}}{R_s + \frac{R_j/j\omega C_j}{R_j + 1/j\omega C_j}} = V \frac{1}{(1+R_s/R_j) + j\omega C_j R_s}$$

So,
$$|v_o|^2 = \frac{|V|^2}{(1+R_s/R_j)^2 + (\omega C_j R_s)^2}$$

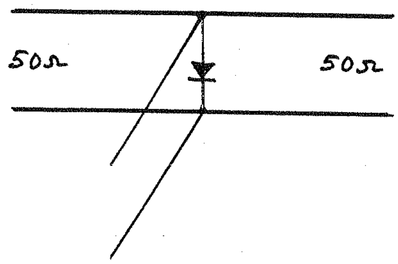
Finally,
$$\beta_v = \frac{\alpha R_j}{2(1+R_s/R_j)[(1+R_s/R_j)+(\omega C_j)^2 R_s R_j]} \text{ V/W} \quad ✓$$

at $f = 10\text{ GHz}$, $\omega C_j = 2\pi(10^{10})(0.1\times10^{-12}) = 0.0063$; $\alpha = 1/25\text{ mV}$

I_o (μA)	R_j (Ω)	β_v (V/mW)
0	2.5×10^5	33.
20	1.24×10^3	14.
50	4.99×10^2	3.

$$R_j = \frac{1}{\alpha(I_o + I_s)}$$

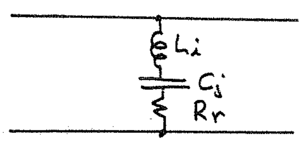
(10.18)



$\omega L_i = 7.5\,\Omega$

$1/\omega C_j = 79.6\,\Omega$

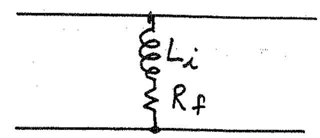
SWITCH ON: (DIODE OFF)



$Z_d = 0.5 - j72\,\Omega$

$Y_d = (0.096 + j13.9)\,mS$

SWITCH OFF: (DIODE ON)



$Z_d = 0.3 + j7.5\,\Omega$

$Y_d = (5.3 - j133.)\,mS$

To minimize the insertion loss for the ON state, let the stub susceptance by $Y_s = -j0.0139 = -j0.695/Z_0$. So the stub length should be $l = 0.403\lambda$.

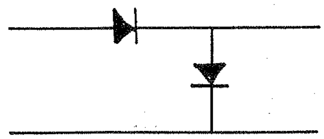
In the ON state, the shunt impedance is then, $Z = 1/0.096\times10^{-3} = 10{,}417\,\Omega$, so the insertion loss is, from (10.68b),

$$IL = -20 \log \left| \frac{2Z}{2Z+Z_0} \right| = 0.021\,dB$$

In the OFF state, the shunt impedance is $Z = 0.246 + j6.8\,\Omega$, so the insertion loss is,

$$IL = -20 \log \left| \frac{2Z}{2Z+Z_0} \right| = 11.7\,dB$$

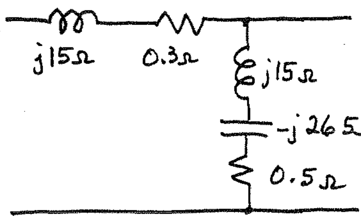
(10.19)



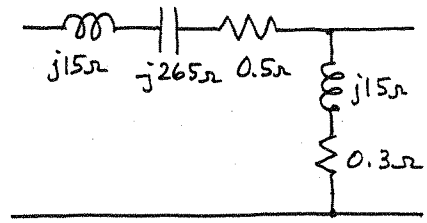
$\omega L_i = 15\,\Omega$

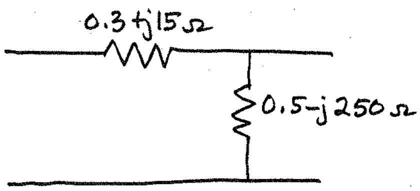
$1/\omega C_j = 265\,\Omega$

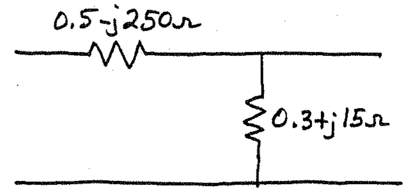
SWITCH ON:



SWITCH OFF:

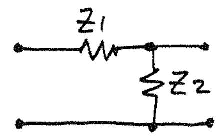






ABCD matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} = \begin{bmatrix} 1+Z_1/Z_2 & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}$$



Convert to S_{21}:

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{1 + Z_1/Z_2 + Z_1/Z_0 + Z_0/Z_2 + 1} = \frac{2}{2 + \frac{Z_1}{Z_2} + \frac{Z_1}{Z_0} + \frac{Z_0}{Z_2}}$$

ON STATE: $Z_1 = 0.3 + j15\,\Omega$, $Z_2 = 0.5 - j250\,\Omega$

$S_{21} = 0.995\underline{/-14^\circ}$

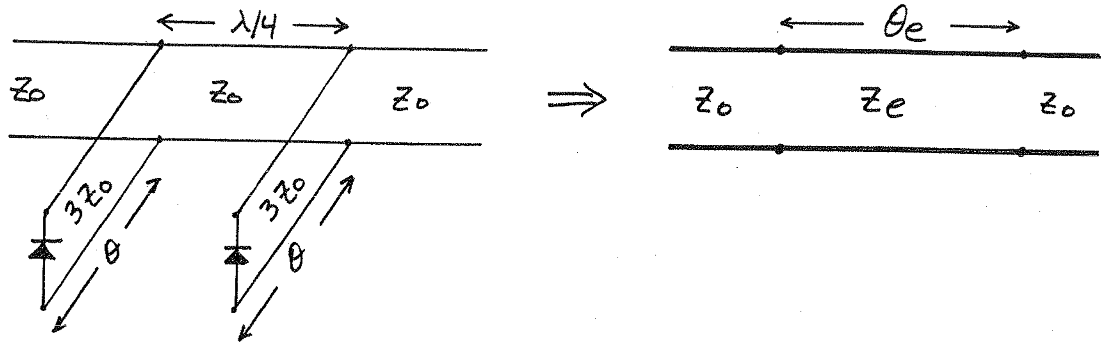
$IL = 0.044\text{ dB}$

OFF STATE: $Z_1 = 0.5 - j250\,\Omega$, $Z_2 = 0.3 + j15\,\Omega$

$S_{21} = 0.118\underline{/149^\circ}$

$IL = 18.6\text{ dB}$

10.20



From (10.74), $\cos\theta_e = -b$

$Z_e = Z_0/\sqrt{1-b^2}$

where b is the normalized stub susceptance.

For diodes ON, $b = -\frac{1}{3}\cot\theta$

$\cos\theta_e = \frac{1}{3}\cot\theta$

For diodes OFF, $b = 1/3 \tan\theta$

$\cos\theta_e = \frac{-1}{3}\tan\theta$

So $\Delta\phi = 45° = \cos^{-1}(\frac{1}{3}\cot\theta) - \cos^{-1}(\frac{-1}{3}\tan\theta)$ (ON-OFF)

Solving this equation numerically:

θ	$\Delta\phi$
110°	73°
120°	46°
130°	40°
122°	44.3°
121°	45.2°

So we choose $\theta = 121°$. (Using $\theta = 31°$ gives $\Delta\phi = -45°$)

Insertion loss for $\theta = 121°$:

Using (10.73), $b = BZ_0$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{-BZ_0 + j(1 - B^2Z_0^2) - BZ_0} = \frac{2}{-2b + j(2 - b^2)}$$

$$|S_{21}|^2 = \frac{4}{4b^2 + (2-b^2)^2}$$

DIODES ON: $b = \frac{-1}{3}\cot\theta = 0.20$

$|S_{21}|^2 = 0.9996$

$IL = 0.0017\,dB \sim 0\,dB$ ✓

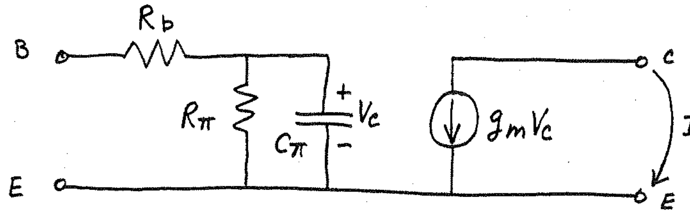
DIODES OFF: $b = \frac{1}{3}\tan\theta = -0.555$

$|S_{21}|^2 = 0.977$

$IL = 0.102\,dB$ ✓

(SuperCompact analysis gives $IL_{ON} = 0\,dB$, $\phi_{ON} = -101.5°$, $IL_{OFF} = 0.10\,dB$, $\phi_{OFF} = -56.7°$, thus $\Delta\phi = 44.8°$)

10.21 *Unilateral bipolar transistor model:*



From (10.77) the short-circuit current gain is,

$$G_i^{sc} = \left|\frac{I_c}{I_b}\right|\Bigg|_{V_{ce}=0} = \frac{g_m V_c}{I_b} = \frac{g_m I_b \left|\frac{R_\pi/j\omega C_\pi}{R_\pi + 1/j\omega C_\pi}\right|}{I_b}$$

$$= g_m \frac{R_\pi}{|1+j\omega R_\pi C_\pi|} = \frac{g_m}{\left|\frac{1}{R_\pi} + j\omega C_\pi\right|} \simeq \frac{g_m}{\omega C_\pi} \quad \text{since } R_\pi \gg 1/\omega C_\pi$$

(e.g., if $R_\pi = 110\Omega$, $C_\pi = 18pF$, $f = 1GHz$, then $1/\omega C_\pi = 9\Omega$)

10.22 $R_i = 7\Omega$, $C_{ds} = 0.12pF$, $R_{ds} = 400\Omega$, $C_{gs} = 0.3pF$, $C_{gd} = 0$, $g_m = 30mS$, $f = 5GHz$.

$$y_{11} = \frac{j\omega C_{gs}}{1+j\omega R_i C_{gs}} = \frac{j0.00942}{1+j0.06597} = 0.0094\angle 86° = 0.00062 + j0.0094 \checkmark$$

$$y_{21} = \frac{g_m}{1+j\omega R_i C_{gs}} = \frac{.03}{1+j0.06597} = 0.03\angle 4° \checkmark \qquad y_{12} = 0 \checkmark$$

$$y_{22} = \frac{1}{R_{ds}} + j\omega C_{ds} = 0.0025 + j0.00377 = 0.00452\angle 56.5° \checkmark$$

$$\Delta y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21} = (.0226\angle 24.5°)(.0228\angle 9.5°) = 0.000515\angle 34°$$

$$S_{11} = \frac{(Y_0 - Y_{11})(Y_0+Y_{22})}{\Delta Y} = \frac{Y_0 - Y_{11}}{Y_0 + Y_{11}} = \frac{.0215\angle -25.9°}{.0226\angle 24.5°} = 0.951\angle -50° \checkmark$$

$$S_{12} = 0$$

$$S_{21} = \frac{-2Y_{21}Y_0}{\Delta y} = \frac{(-.04)(.03\angle 4°)}{.000515\angle 34°} = 2.33\angle 150°$$

$$S_{22} = \frac{Y_0 - Y_{22}}{Y_0 + Y_{22}} = \frac{.0179\angle -12°}{.0228\angle 9.5°} = 0.785\angle -22° \checkmark$$

If conjugately matched, the unilateral transducer gain is,

$$G_{TU} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 148.8 = 21.7\,dB$$

The corresponding result from the circuit model, as given in (11.18), is

NOTE: THIS RESULT IS NOT GIVEN UNTIL CHAPTER 11.

$$G_{TU} = \frac{g_m^2 R_{ds}}{4\omega^2 R_i C_{gs}^2} = 21.6\,dB$$

(verified with Serenade)

Chapter 11

(11.1) The [S] matrix for a 3-dB matched attenuator is,

$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

For $Z_L = 50\Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = 0$, $\Gamma_{out} = 0$

Then from (11.12), (11.13), and (11.8) we have

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)} = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

For $Z_L = 25\Omega$: $\Gamma_L = -1/3$, $\Gamma_S = \Gamma_{out} = 0$, $\Gamma_{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = -1/6$

$$G_A = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = |S_{21}|^2(1-|\Gamma_L|^2) = 0.444 \checkmark$$

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)} = 0.457 \checkmark$$

For $Z_S = 25\Omega$, $Z_L = 50\Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = -1/3$, $\Gamma_{out} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} = -1/6$

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{(1-|\Gamma_{out}|^2)} = 0.457$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{1} = 0.444$$

$$G = |S_{21}|^2 = 0.5$$

(11.2) $S_{11} = 0.61\angle{-170°}$, $S_{12} = 0.06\angle{70°}$, $S_{21} = 2.3\angle{80°}$, $S_{22} = 0.72\angle{-25°}$

$Z_S = 25\Omega$, $Z_L = 100\Omega$, $V_S = 2V$.

a)

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.641\angle{-174°} \checkmark$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.777\angle{-25°} \checkmark$$

$$G = 12.8 = \underline{11.1\,dB} \checkmark$$

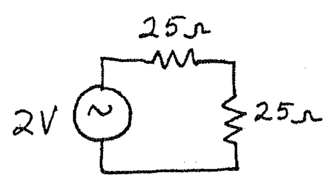
$$G_A = 18.4 = \underline{12.6\,dB} \checkmark$$

$$G_T = 10.8 = \underline{10.3\,dB} \checkmark$$

$$G_{TU} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2\,|1-S_{22}\Gamma_L|^2} = 10.4 = \underline{10.2\,dB} \checkmark$$

(verified with Serenade)

b)



$$P_{AVS} = \frac{1}{2}\left(\frac{2}{2}\right)^2\frac{1}{25} = \underline{0.02\,W}$$

$$P_L = G_T P_{AVS} = (10.8)(.02) = \underline{0.216\,W}$$

(11.3)

From (11.25) - (11.26) the centers and radii of the stability circles are:

$\Delta = S_{11} S_{22} - S_{12} S_{21} = 0.117 \angle -50°$

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 2.56 \angle 28° \checkmark$$

$$R_L = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| = 1.37 \checkmark$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = 3.77 \angle 174° \checkmark$$

$$R_L = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| = 2.53 \checkmark$$

From (11.28),

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 |S_{12} S_{21}|} = 1.35 > 1 \quad \checkmark$$

Since $K > 1$ and $|\Delta| < 1$, the transistor is unconditionally stable.

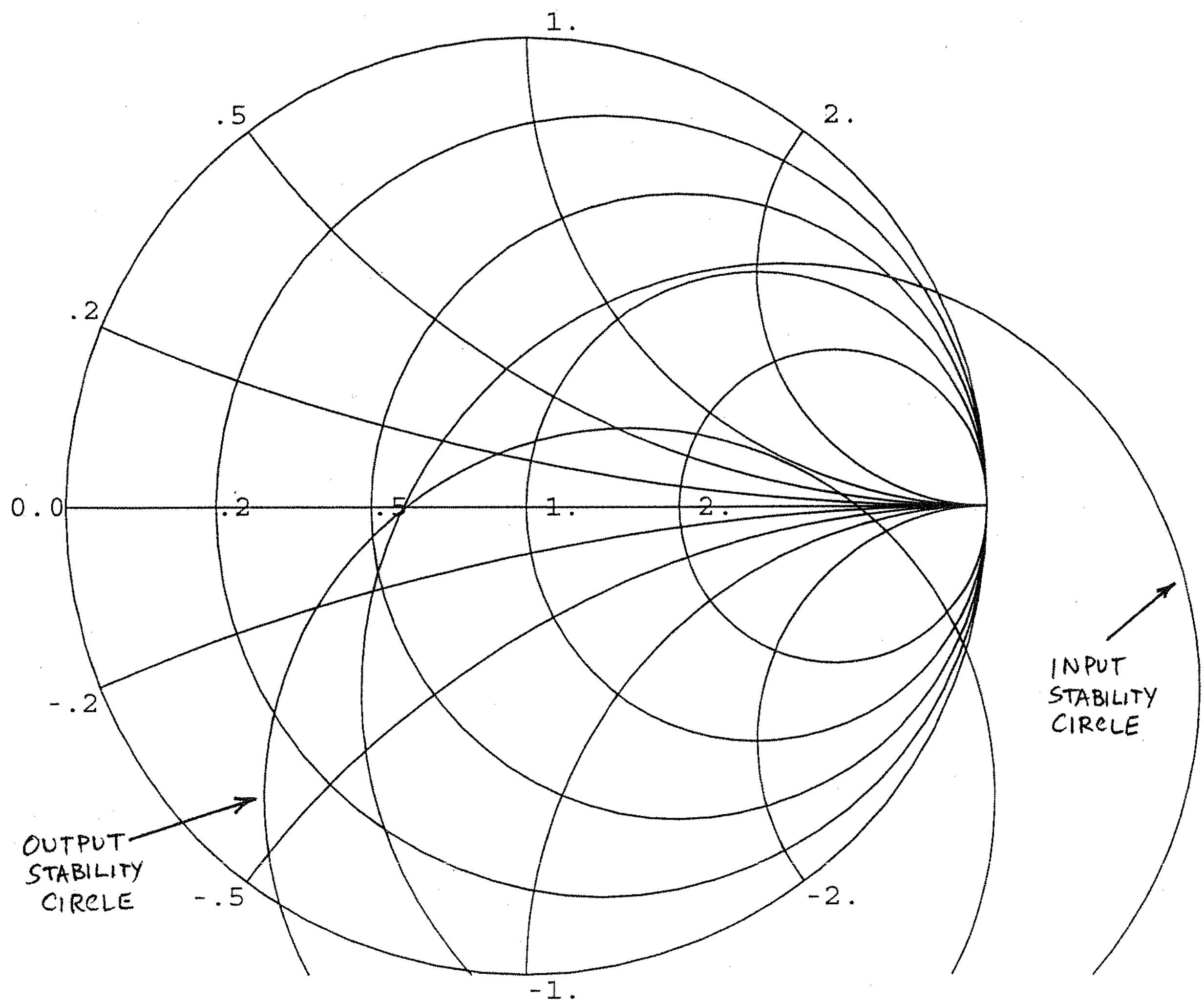
(11.4) $S_{11} = 0.8 \angle -90°$, $S_{12} = 0.3 \angle 70°$, $S_{21} = 5.1 \angle 80°$, $S_{22} = 0.62 \angle -40°$

$\Delta = 1.52 \angle -49°$

$C_L = 0.66 \angle -70°$ $\quad R_L = 0.79$

$C_S = 0.68 \angle -35°$ $\quad R_S = 0.91$ $\quad K = 0.75$ ✓

Since $K<1$ the transistor is potentially unstable. The stability circles are plotted below.



(11.5) Using (11.30) to compute μ:

DEVICE	S_{11}	S_{12}	S_{21}	S_{22}	μ	
A	$0.34\angle{-170°}$	$0.06\angle{70°}$	$4.3\angle{80°}$	$0.45\angle{-25°}$	1.193	UNC. STABLE
B	$0.75\angle{-60°}$	$0.2\angle{70°}$	$5.0\angle{90°}$	$0.5\angle{60°}$	0.283	POT. UNSTABLE
C	$0.65\angle{-140°}$	$0.04\angle{60°}$	$2.4\angle{50°}$	$0.7\angle{-65°}$	1.057	UNC. STABLE

Device A has the best stability.

(11.6) From (11.30) the μ-parameter test is,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*\Delta| + |S_{21}S_{12}|} > 1$$

If $S_{12}=0$ (unilateral) then we have,

$$\Delta = S_{11}S_{22}$$

So,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-|S_{11}|^2 S_{22}|} = \frac{1-|S_{11}|^2}{|S_{22}||1-|S_{11}|^2|} > 1$$

Since the denominator is positive and μ is positive, the numerator must also be positive, thus $|S_{11}|<1$. Then the above reduces to,

$$\mu = \frac{1}{|S_{22}|} > 1,$$

So,

$$|S_{22}| < 1.$$

11.7 From the definitions of (11.41),

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad , \quad C_1 = S_{11} - \Delta S_{22}^* .$$

Similar to the expansion used after (11.35), it can be verified by direct expansion that,

$$|C_1|^2 = |S_{11} - \Delta S_{22}^*|^2 = |S_{12}S_{21}|^2 + (1-|S_{22}|^2)(|S_{11}|^2 - |\Delta|^2)$$

So the condition that $B_1^2 - 4|C_1|^2 > 0$ implies that,

$$(1+|S_{11}|^2 - |S_{22}|^2 - |\Delta|^2)^2 > 4|S_{12}S_{21}|^2 + 4(1-|S_{22}|^2)(|S_{11}|^2 - |\Delta|^2)$$

$$1 + 2|S_{11}|^2 - 2|S_{22}|^2 - 2|\Delta|^2 + |S_{11}|^4 - 2|S_{11}|^2|S_{22}|^2 - 2|S_{11}|^2|\Delta|^2 + |S_{22}|^4$$
$$+ 2|\Delta|^2|S_{22}|^2 + |\Delta|^4 > 4|S_{12}S_{21}|^2 + 4(|S_{11}|^2 - |\Delta|^2 - |S_{11}|^2|S_{22}|^2 + |\Delta|^2|S_{22}|^2)$$

$$1 - 2|S_{11}|^2 - 2|S_{22}|^2 + 2|\Delta|^2 + |S_{11}|^4 + 2|S_{11}|^2|S_{22}|^2 - 2|S_{11}|^2|\Delta|^2 + |S_{22}|^4$$
$$- 2|\Delta|^2|S_{22}|^2 + |\Delta|^4 > 4|S_{12}S_{21}|^2$$

$$(1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2)^2 > 4|S_{12}S_{21}|^2$$

or,

$$K^2 = \frac{(1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2)^2}{4|S_{12}S_{21}|^2} > 1 \quad \checkmark$$

11.8 $S_{11} = 0.65\angle{-140°}$, $S_{21} = 2.4\angle{50°}$, $S_{12} = 0.04\angle{60°}$, $S_{22} = 0.70\angle{-65°}$

First we check stability:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.393\angle{165°}$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.26$$

Since $|\Delta| < 1$ and $K > 1$ the transistor is unconditionally stable at 5 GHz. For maximum gain, the transistor should be conjugately matched: (using 11.40)

$$\Gamma_S = \Gamma_{in}^* = 0.826\angle{147°} \checkmark$$

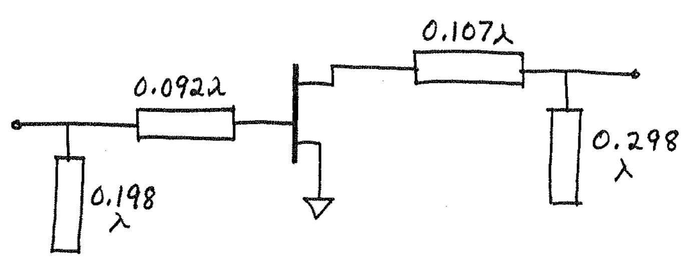
$$\Gamma_L = \Gamma_{out}^* = 0.850\angle{71°} \checkmark$$

The gains can then be calculated as,

$$G_S = 3.14, \quad G_0 = 5.76, \quad G_L = 1.64$$

So the overall transducer gain is $G_T = 29.7 = 14.7\,dB$ ✓

Matching was done on a Smith chart. The final AC amplifier circuit is shown below:



SuperCompact analysis gives $|S_{11}| = 0.05$, $|S_{22}| = 0.04$, $G = 14.7\,dB$ ✓

(11.9) $S_{11} = 0.61\angle{-170°}$, $S_{21} = 2.24\angle{32°}$, $S_{12} = 0$, $S_{22} = 0.72\angle{-83°}$

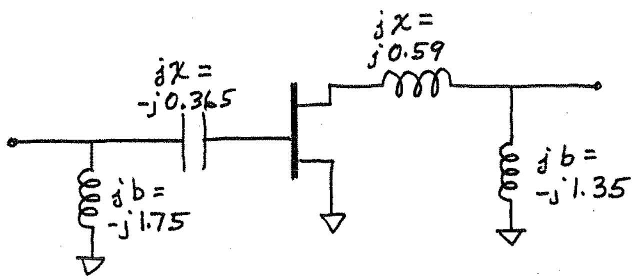
The transistor is unconditionally stable since $K = \infty$ and $|\Delta| < 1$. Since the transistor is unilateral,

$$\Gamma_S = S_{11}^* = 0.61\angle{170°} \checkmark \quad , \quad \Gamma_L = S_{22}^* = 0.72\angle{83°} \checkmark$$

and the maximum gain is, from (11.45),

$$G_{TU_{MAX}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 16.6 = 12.2\,dB$$

Matching was done with a Smith chart. The final circuit is:



The matching element values are, at 6 GHz,

$$C = \frac{-1}{\omega Z_0 x_c} = 1.45\,pF \checkmark \qquad L = \frac{Z_0 x_L}{\omega} = 0.78\,mH \checkmark$$

$$L = \frac{-Z_0}{\omega b_L} = 0.76\,mH \checkmark \qquad L = \frac{-Z_0}{\omega b_L} = 0.98\,mH \checkmark$$

SuperCompact analysis gives $|S_{11}| = 0.035$, $|S_{22}| = 0.008$, and $G = 12.2\,dB$ ✓

(11.10) $S_{11} = 0.61\angle{-170^\circ}$, $S_{21} = 2.24\angle{32^\circ}$; $S_{12} = 0$, $S_{22} = 0.72\angle{-83^\circ}$
$G = 10\,dB$, $G_S = 1\,dB$, $G_L = 2\,dB$

Since $K = \infty$ and $|\Delta| < 1$, the transistor is unconditionally stable. From (11.45), we have

$$G_{S_{MAX}} = \frac{1}{1-|S_{11}|^2} = 1.59 \checkmark, \qquad G_{L_{MAX}} = \frac{1}{1-|S_{22}|^2} = 2.08 \checkmark$$

So for $G_S = 1\,dB = 1.26$, and $G_L = 2\,dB = 1.58$, we have from (11.46),

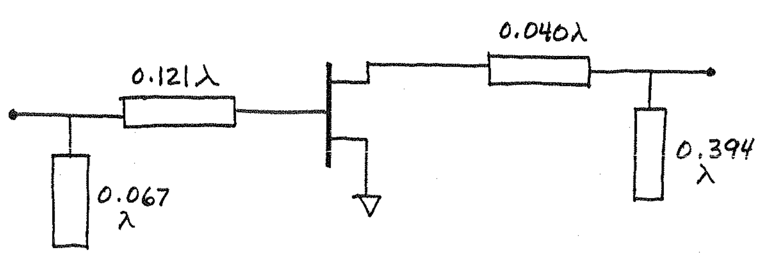
$$g_S = \frac{G_S}{G_{S_{MAX}}} = 0.792, \qquad g_L = \frac{G_L}{G_{L_{MAX}}} = 0.760$$

Then the centers and radii of the constant gain circles can be found from (11.49)-(11.50):

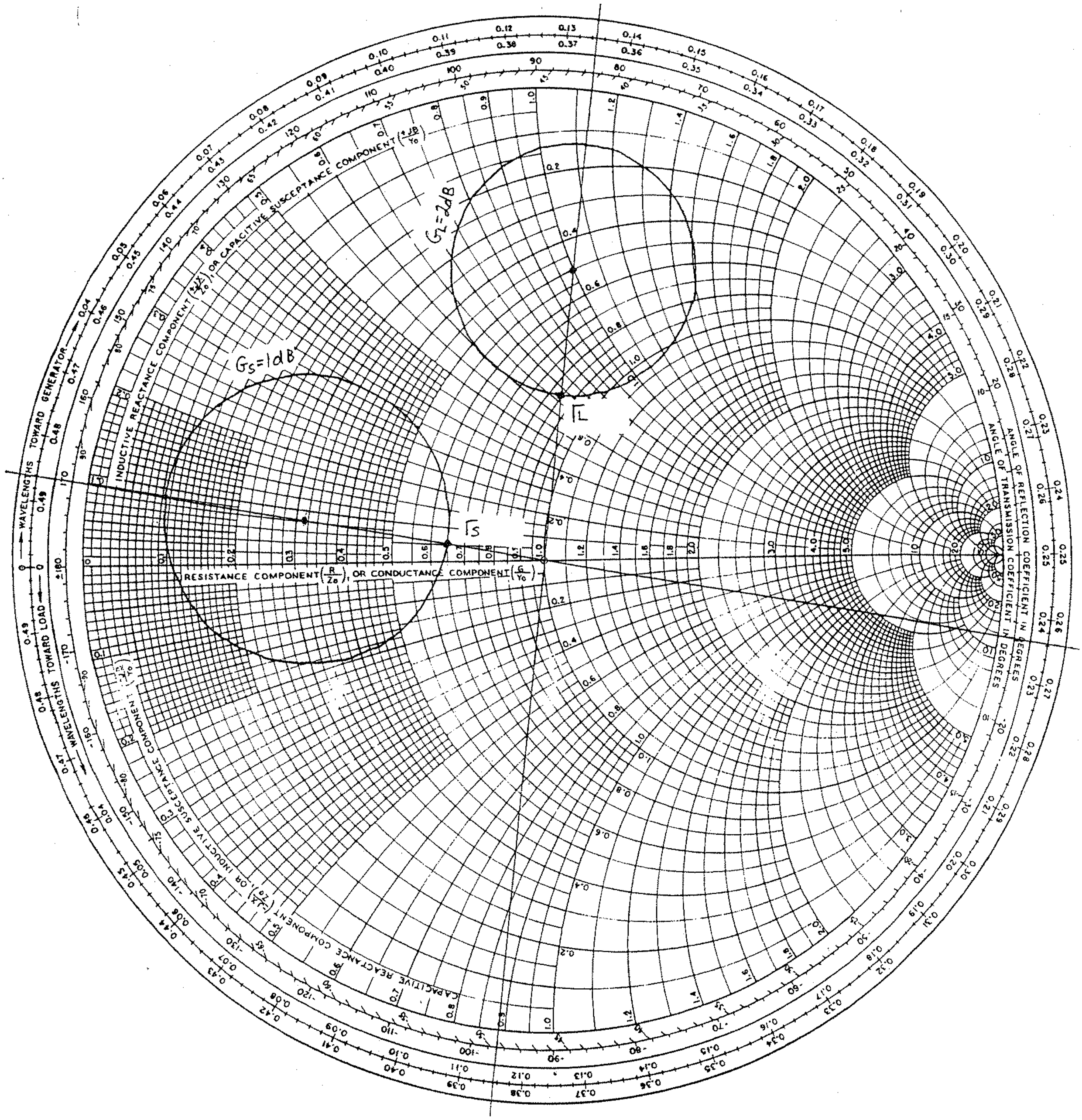
$$C_S = 0.524\angle{170^\circ} \checkmark \qquad C_L = 0.625\angle{83^\circ} \checkmark$$

$$R_S = 0.310 \checkmark \qquad R_L = 0.269 \checkmark$$

Since $G_o = 10 \log |S_{21}|^2 = 7.0\,dB$, using the $G_S = 1\,dB$ and the $G_L = 2\,dB$ gain circles will give an overall gain of $10\,dB$. We plot these circles on the Smith chart, and choose $\Gamma_S = 0.215\angle{170^\circ}$ ✓ and $\Gamma_L = 0.361\angle{83^\circ}$ ✓ to minimize the magnitude of these values. After matching, we have the following amplifier circuit:



SuperCompact analysis gives $|S_{11}| = 0.45$, $|S_{22}| = 0.48$, $G = 10.05\,dB$ ✓ (reflections at input and output serve to reduce the gain to 10 dB). Smith chart shown on following page.



(11.11) From (11.46) the unilateral figure of merit is,

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = \frac{(.06)(4.3)(.34)(.45)}{[1-(.34)^2][1-(.45)^2]} = 0.056 \checkmark$$

So from (11.45) this bounds the error in G_T/G_{TU} by,

$$0.897 = \frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} = 1.122$$

or,

$$-0.47\,dB < G_T(dB) - G_{TU}(dB) < 0.5\,dB$$

(11.12) From (11.48) and (11.47), when $G_S = 1$ we have,

$$g_s = \frac{1}{G_{S_{MAX}}} = 1 - |S_{11}|^2 \quad , \quad 1 - g_s = |S_{11}|^2$$

so (11.51) reduces to,

$$C_S = \frac{(1-|S_{11}|^2)\, S_{11}^*}{1-|S_{11}|^4} = \frac{S_{11}^*}{1+|S_{11}|^2}$$

$$R_S = \frac{|S_{11}|(1-|S_{11}|^2)}{1-|S_{11}|^4} = \frac{|S_{11}|}{1+|S_{11}|^2}$$

So the equation of the constant gain circle becomes,

$$\left| \Gamma_S - \frac{S_{11}^*}{1+|S_{11}|^2} \right| = \frac{|S_{11}|}{1+|S_{11}|^2}$$

one solution to this equation occurs for $\Gamma_S = 0$, so the circle must pass through the center of the Smith chart.

(11.13) $S_{11} = 0.7\angle{-110°}$, $S_{12} = 0.02\angle{60°}$, $S_{21} = 3.5\angle{60°}$, $S_{22} = 0.8\angle{-70°}$

$F_{MIN} = 2.5\,dB$, $\Gamma_{OPT} = 0.7\angle{120°}$, $R_N = 15\Omega$

First check stability: $K = 1.07$, $|\Delta| = 0.53$

Since $K>1$ and $|\Delta|<1$ the device is unconditionally stable. Minimum noise figure occurs for $\Gamma_S = \Gamma_{OPT} = 0.7\angle{120°}$. Then we maximize gain by conjugate matching the output. From (11.41b),

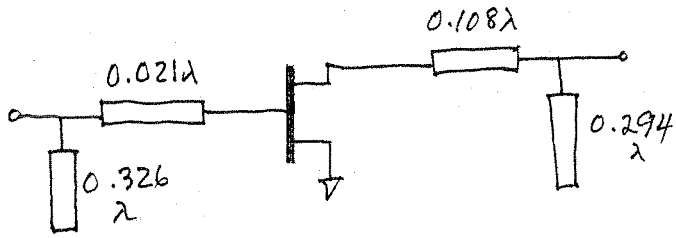
$$\Gamma_L = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_S^*}{1-S_{11}\Gamma_S} \right)^* = 0.873\angle{74°} \ ✓$$

So the noise figure will be $F = F_{min} = 2.5\,dB$, and the gain will be,

$$G_T = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}\,|S_{21}|^2\,\frac{1-|S_{22}|^2}{|1-S_{22}\Gamma_L|^2}$$

$$= (1.85)(12.25)(3.81) = 86.3 = 19.4\,dB$$

Impedance matching is done with a Smith chart; the final amplifier circuit is shown below.



Serenade analysis of this amplifier gives

$|S_{11}| = 0.33$, $|S_{22}| = 0.13$,

$G = 19.7\,dB$, $F = 2.5\,dB$ ✓.

The solution is simpler if S_{12} is set to zero, resulting in $G = 18\,dB$.

11.14 $S_{11}=0.6\angle{-60°}$, $S_{21}=2.\angle{181°}$, $S_{12}=0$, $S_{22}=0.7\angle{-60°}$

$F_{MIN}=2.0dB$, $\Gamma_{OPT}=0.62\angle{100°}$, $R_N=20\Omega$

Since $S_{12}=0$ and $|S_{11}||S_{22}|<1$, the device is unconditionally stable. The overall gain is, $G_{TU}=G_S G_o G_L$, where $G_o=|S_{21}|^2=4=6dB$. ✓ So $G_S+G_L=0dB$.

Plot noise figure circles for F=2.0, 2.05, 2.2, and 3.0 dB:

F(dB)	N	C_F	R_F
2.05	0.0134	$0.61\angle{100°}$	0.09
2.20	0.055	$0.59\angle{100°}$	0.18
3.00	0.30	$0.48\angle{100°}$	0.40
2.00	0.	$0.62\angle{100°}$	0

Now plot constant gain circles for $G_S=G_L=0dB$:

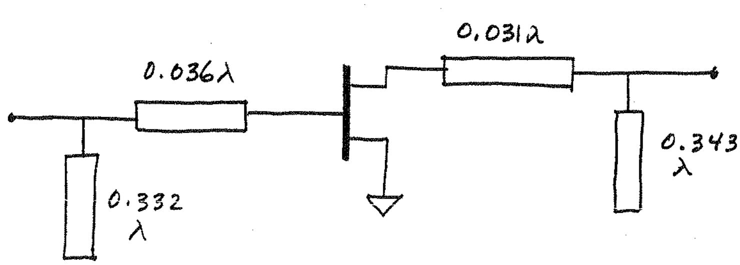
$G_{S_{MAX}}=1.56$ ✓ $G_{L_{MAX}}=1.96$ ✓

$g_s=0.641$ $g_L=0.510$

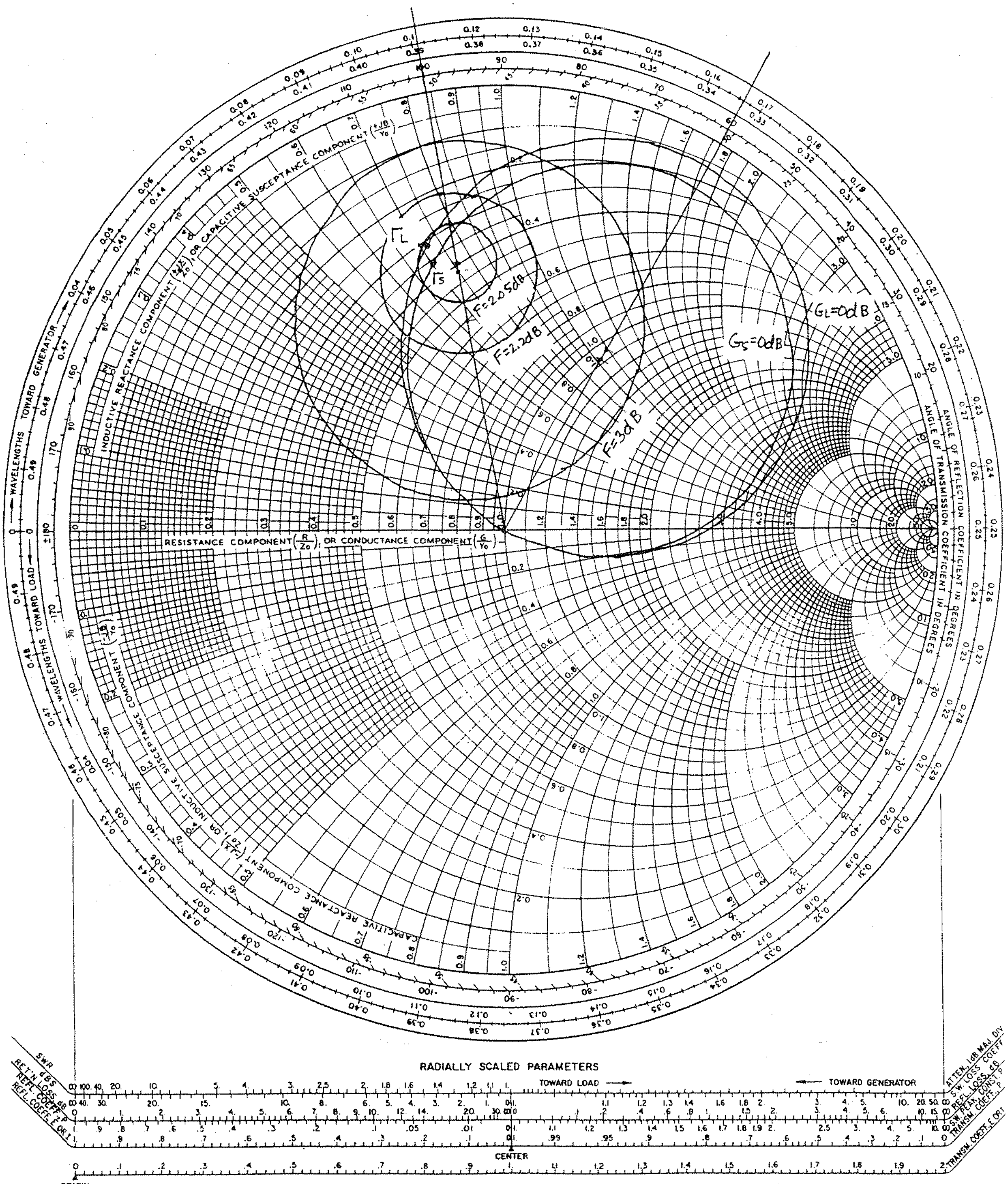
$C_S=0.44\angle{60°}$ $C_L=0.47\angle{60°}$

$R_S=0.44$ $R_L=0.47$

These two circles are close together near the F=2dB point. We choose $\Gamma_L=0.66\angle{105°}$, $\Gamma_S=0.62\angle{105°}$. Then we should obtain $F\simeq 2.04dB$. The final AC amplifier circuit is:



Super Compact analysis gives $|S_{11}|=0.62$, $|S_{22}|=0.67$, $G=6.1dB$, and $F=2.04dB$ ✓ The gain and noise circles are shown below.



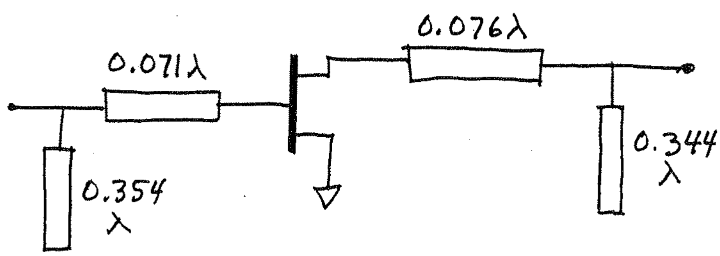
Γ_L
Γ_S
F=2.05dB
F=2.2dB
F=3dB
G_L=0dB
G_S=0dB
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN

(11.15) S-parameters and noise parameters of Problem 11.14
Plot the $F = 2.5\,dB$ constant noise figure circle:
$N = 0.141$, $C_F = 0.543\angle 100°$, $R_F = 0.286$

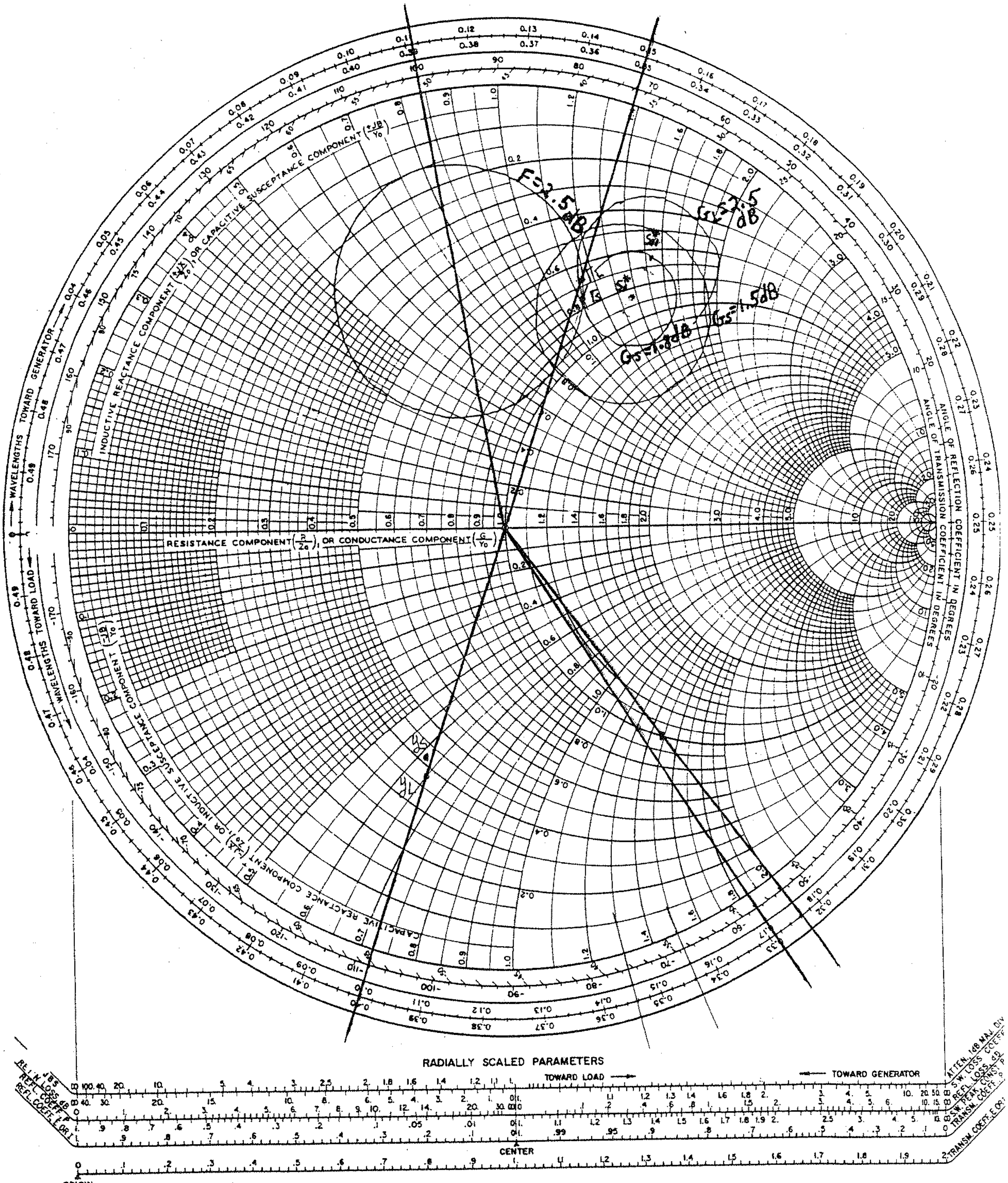
Now, $G_{S_{MAX}} = 1.56 = 1.93\,dB$, $G_{L_{MAX}} = 1.96 = 2.92\,dB$
But these points (S_{11}^*, S_{22}^*) do not lie on the $F = 2.5\,dB$ circle. We can plot some gain circles to just give intersections with the $F = 2.5\,dB$ noise circle:

$G_S = 1.5\,dB$	$g_S = 0.905$	$C_S = 0.56\angle 60°$	$R_S = 0.204$
$G_L = 2.5\,dB$	$g_L = 0.907$	$C_L = 0.67\angle 60°$	$R_L = 0.163$
$G_S = 1.7\,dB$	$g_S = 0.948$	$C_S = 0.58\angle 60°$	$R_S = 0.149$
$G_S = 1.8\,dB$	$g_S = 0.970$	$C_S = 0.59\angle 60°$	$R_S = 0.112$

The $G_S = 1.8\,dB$ and $G_L = 2.5\,dB$ circles are close to optimum (the $F = 2.5\,dB$ noise circle). Thus we have $\Gamma_S = 0.545\angle 70°$, $\Gamma_L = 0.59\angle 72°$, which will yield a gain of $G_T = 1.8 + 2.5 + 6 = 10.3\,dB$. The final AC amplifier circuit is shown below:



SuperCompact analysis of this circuit gives $|S_{11}| = 0.20$, $|S_{22}| = 0.28$, $G = 10.3\,dB$ ✓, and $F = 2.4\,dB$ ✓. The noise and gain circles are shown on the following page.

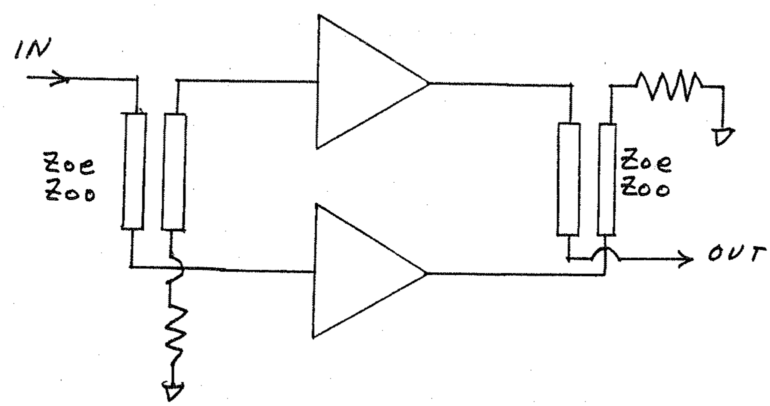

RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN
RESISTANCE COMPONENT $(\frac{R}{Z_0})$, OR CONDUCTANCE COMPONENT $(\frac{G}{Y_0})$
F=2.5dB
Gs=1.5dB
Gs=1.0dB

(11.16) FOR THE COUPLED LINE COUPLERS:

$$C = 10^{-3/20} = 0.708$$

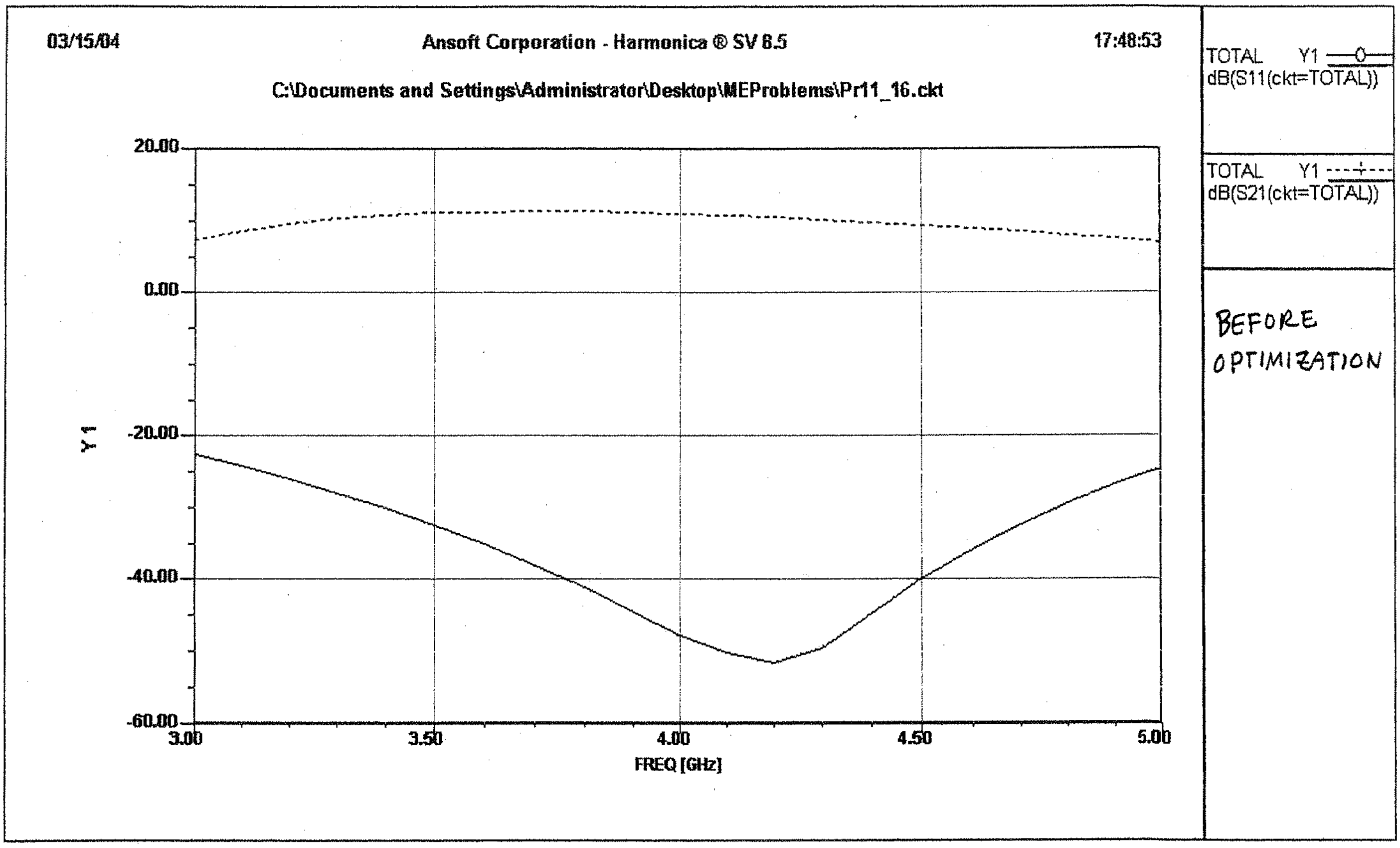
$$Z_{0e} = 50\sqrt{\frac{1+C}{1-C}} = 121\,\Omega$$

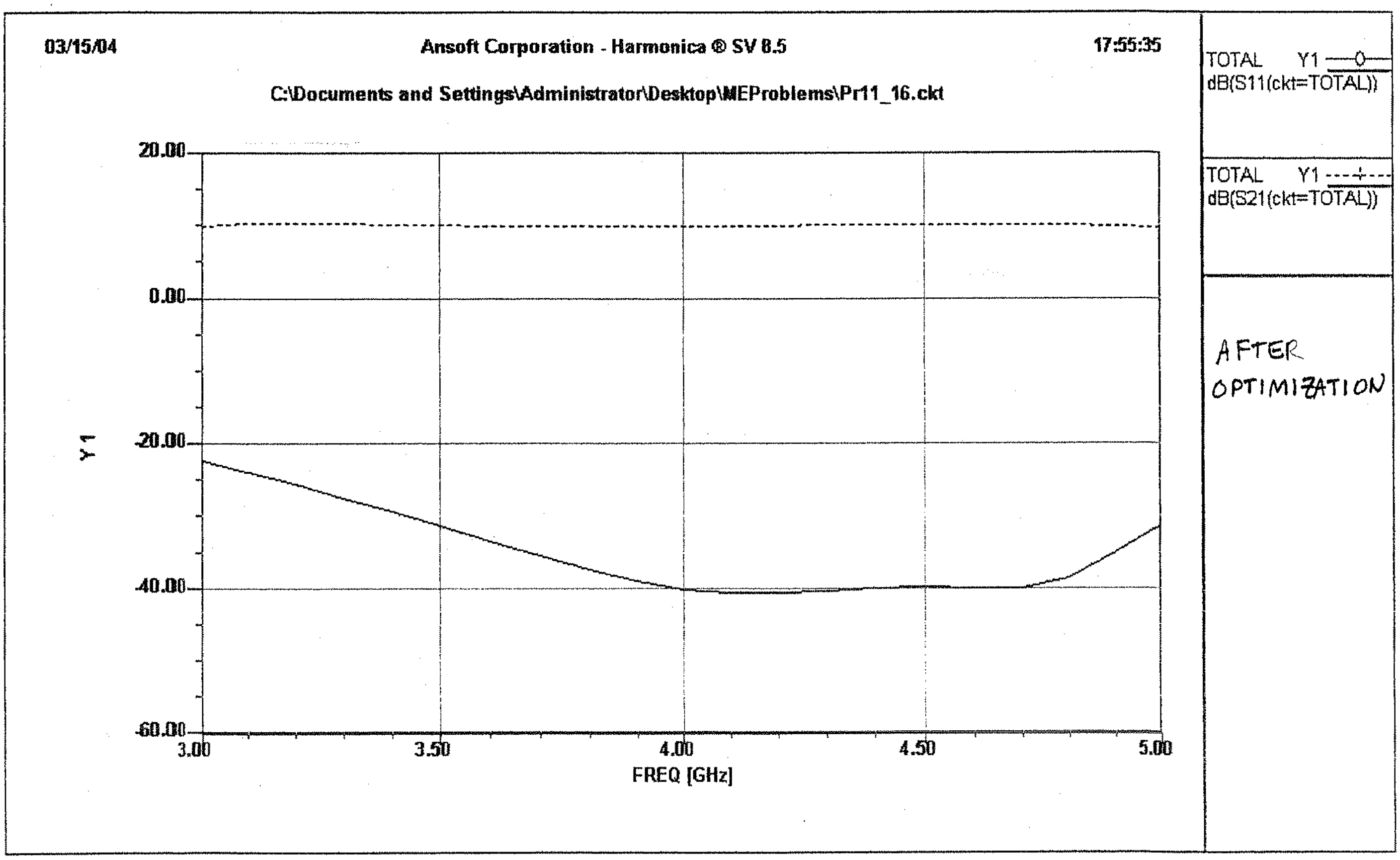
$$Z_{00} = 50\sqrt{\frac{1-C}{1+C}} = 21\,\Omega$$



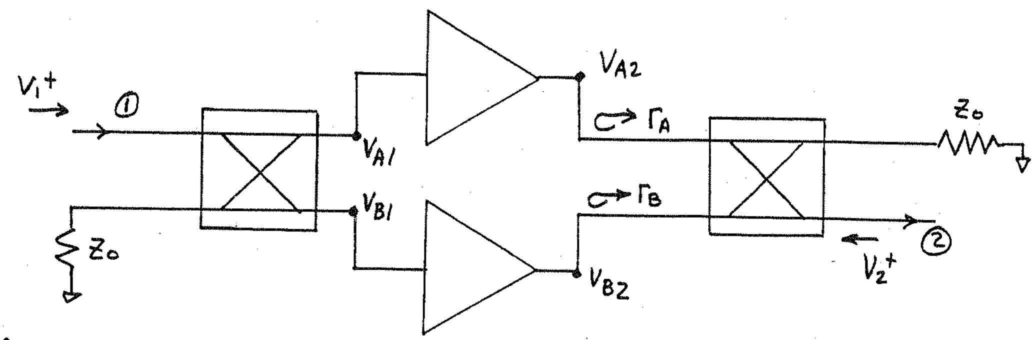
The amplifier circuit of Example 11.4 was used for both amplifiers here. As in Example 11.6, the amplifier matching networks were optimized using SuperCompact to give a flat 10dB gain response, with good input matching. Results of the optimization are given below, including the line and stub lengths before and after optimization, the SuperCompact data file, and the calculated gain and input return loss of the balanced amplifier before and after optimization. Results seem to be a bit better than those of Example 11.6.

PARAMETER	BEFORE OPT.	AFTER OPT.
INPUT SECTION STUB LENGTH	0.100λ	0.125λ
INPUT SECTION LINE LENGTH	0.179λ	0.119λ
OUTPUT SECTION LINE LENGTH	0.045λ	0.089λ
OUTPUT SECTION LINE LENGTH	0.432λ	0.458λ


03/15/04
Ansoft Corporation - Harmonica ® SV 8.5
17:48:53
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr11_16.ckt
Y1
20.00
0.00
-20.00
-40.00
-60.00
3.00
3.50
4.00
4.50
5.00
FREQ [GHz]
TOTAL Y1
dB(S11(ckt=TOTAL))
TOTAL Y1
dB(S21(ckt=TOTAL))
BEFORE OPTIMIZATION


03/15/04
Ansoft Corporation - Harmonica ® SV 8.5
17:55:35
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr11_16.ckt
Y1
20.00
0.00
-20.00
-40.00
-60.00
3.00
3.50
4.00
4.50
5.00
FREQ [GHz]
TOTAL Y1
dB(S11(ckt=TOTAL))
TOTAL Y1
dB(S21(ckt=TOTAL))
AFTER OPTIMIZATION

(11.17)



The analysis for S_{22} is identical to that for S_{11} in eqs (11.61)–(11.65), but with input V_2^+ at port 2.

Thus, if the input at port 2 is V_2^+, then the voltages incident at the amplifiers are,

$$V_{A2}^- = \frac{1}{\sqrt{2}} V_2^+$$

$$V_{B2}^- = \frac{-j}{\sqrt{2}} V_2^+$$

Then the reflected output voltage at port 2 is,

$$V_2^- = \frac{1}{\sqrt{2}} V_{A2}^+ + \frac{-j}{\sqrt{2}} V_{B2}^+ = \frac{1}{\sqrt{2}} \Gamma_A V_{A2}^- + \frac{-j}{\sqrt{2}} \Gamma_B V_{B2}^-$$

$$= \frac{1}{2} V_2^+ (\Gamma_A - \Gamma_B)$$

Thus,

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \frac{1}{2}(\Gamma_A - \Gamma_B) \checkmark$$

(11.18) From (11.77), $G = \dfrac{g_m^2 Z_d Z_g}{4} \dfrac{\left(e^{-N\alpha_g l_g} - e^{-N\alpha_d l_d}\right)^2}{\left(e^{-\alpha_g l_g} - e^{-\alpha_d l_d}\right)^2}$

differentiating with respect to N and setting to zero gives,

$$\alpha_g l_g e^{-N\alpha_g l_g} - \alpha_d l_d e^{-N\alpha_d l_d} = 0$$

$$\ln \alpha_g l_g - N\alpha_g l_g = \ln \alpha_d l_d - N\alpha_d l_d$$

$$\ln \frac{\alpha_g l_g}{\alpha_d l_d} = N(\alpha_g l_g - \alpha_d l_d)$$

$$N = \frac{\ln(\alpha_g l_g / \alpha_d l_d)}{\alpha_g l_g - \alpha_d l_d} \checkmark$$

11.19 $R_i = 5\Omega$, $R_{ds} = 200\Omega$, $C_{gs} = 0.35\,pF$, $g_m = 40\,mS$

Assume $Z_g = Z_d = 50\Omega$.

We use (11.68) and (11.71) to find $\alpha_g l_g$, $\alpha_d l_d$. Then use (11.76) to find G:

f	G(N=4)	G(N=8)	G(N=16)
2	9.9dB	14.0dB	16.5dB
4	9.8	13.7	15.9
8	9.2	12.5	13.4
12	8.4	10.7	9.5
16	7.2	8.4	4.7
18	6.5	7.0	2.2
20	5.7	5.6	-0.2

Gain at f = 18 GHz vs. N:

N	G(dB)
5	7.1
6	7.3
7	7.2
8	7.0
9	6.7

So the optimum value of N is seen to be $N_{opt} = 6$.

Using (11.77) to find N_{opt} directly:

$$\alpha_g l_g = 0.1958$$

$$\alpha_d l_d = 0.125$$

$$N_{opt} = \frac{\ln(\alpha_g l_g / \alpha_d l_d)}{\alpha_g l_g - \alpha_d l_d} = 6.33$$

(11.20) $S_{11} = 0.76\angle 169°$, $S_{12} = 3.08\angle 69°$, $S_{21} = 0.079\angle 53°$, $S_{22} = 0.36\angle -169°$

$\Gamma_{SP} = 0.797\angle -187°$, $\Gamma_{LP} = 0.491\angle 185°$, $G_P = 10\,dB$. $f = 1\,GHz$.

Check stability using small-signal S-parameters:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.452\angle -27°$$

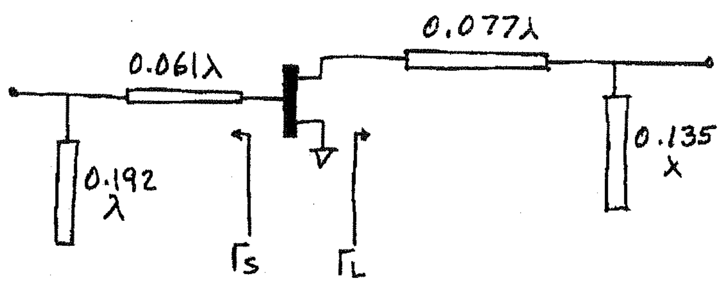
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.02$$

Since $|\Delta| < 1$ and $K > 1$, the device is unconditionally stable at this frequency.

Using the given large-signal source and load reflection coefficients gives,

$$\Gamma_S = 0.797\angle -187°, \quad \Gamma_{LP} = 0.491\angle 185°$$

Then the matching circuits can be designed, resulting in the following AC circuit:

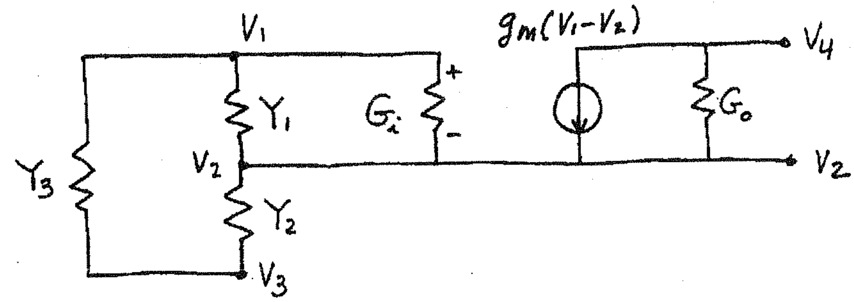


Since the gain with this Γ_S, Γ_P is 10 dB, the input power for a 1 W output is,

$$P_{in} = P_{out} - G_P = 30\,dBm - 10 = 20\,dBm = 100\,mW.$$

Chapter 12

(12.1)



Writing KCL for nodes V_1, V_2, V_3, V_4:

V_1: $(V_3-V_1)Y_3 + (V_2-V_1)Y_1 + (V_2-V_1)G_i = 0$

V_2: $(V_1-V_2)Y_1 + (V_3-V_2)Y_2 + (V_1-V_2)G_i + g_m(V_1-V_2) + (V_4-V_2)G_o = 0$

V_3: $(V_1-V_3)Y_3 + (V_2-V_3)Y_2 = 0$

V_4: $(V_2-V_4)G_o - g_m(V_1-V_2) = 0$

Rearranging:

$$
\begin{aligned}
&V_1(Y_1+Y_3+G_i) + V_2(-Y_1-G_i) + V_3(-Y_3) + V_4(0) = 0\\
&V_1(-Y_1-G_i-g_m) + V_2(Y_1+Y_2+G_i+G_o+g_m) + V_3(-Y_2) + V_4(-G_o) = 0\\
&V_1(-Y_3) + V_2(-Y_2) + V_3(Y_2+Y_3) + V_4(0) = 0\\
&V_1(g_m) + V_2(-G_o-g_m) + V_3(0) + V_4 G_o = 0
\end{aligned}
$$

which agrees with the matrix of (12.3).

12.2 From (12.4),

$$\det\begin{bmatrix}(Y_1+Y_3+G_i) & -Y_3 \\ (g_m-Y_3) & (Y_2+Y_3)\end{bmatrix}=0 \quad \text{for oscillation.}$$

For a Colpitts oscillator, let $Y_1=j\omega C_1$, $Y_2=j\omega C_2$, $Z_3=R+j\omega L_3$.

Then,

$$\det[\cdot]=\left(j\omega C_1+\frac{1}{R+j\omega L_3}+G_i\right)\left(j\omega C_2+\frac{1}{R+j\omega L_3}\right)+\left(\frac{1}{R+j\omega L_3}\right)\left(g_m-\frac{1}{R+j\omega L_3}\right)=0$$

$$\left[1+(G_i+j\omega C_1)(R+j\omega L_3)\right]\left[1+j\omega C_2(R+j\omega L_3)\right]+g_m(R+j\omega L_3)-1=0$$

$$\cancel{1}+j\omega C_2\,\cancel{(R+j\omega L_3)}+(G_i+j\omega C_1)\cancel{(R+j\omega L_3)}+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)^{\cancel{2}}+$$
$$+g_m\cancel{(R+j\omega L_3)}-\cancel{1}=0$$

$$j\omega C_2+G_i+j\omega C_1+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)+g_m=0$$

Re: $\quad G_i+g_m-\omega^2 L_3 G_i C_2-\omega^2 C_1 C_2 R=0$

Im: $\quad \omega C_2+\omega C_1+\omega C_2 G_i R-\omega^3 C_1 C_2 L_3=0$

$$C_1+C_2+C_2 G_i R-\omega^2 C_1 C_2 L_3=0$$

$$\omega=\sqrt{\frac{C_1+C_2+C_2G_iR}{C_1C_2L_3}}=\sqrt{\frac{1}{L_3}\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{G_iR}{C_1}\right)} \quad ✓$$

(12.3) $f_o = 30\,MHz$, $\beta = 40$, $R_i = 800\,\Omega$

Choose $\underline{C_1' = C_2 = 500\,pF}$. Then (12.20) gives L_3 as,

$$L_3 = \frac{1}{\omega_o^2}\left(\frac{1}{C_1'} + \frac{1}{C_2}\right) = \underline{0.11\,\mu H}$$

From (12.22), the maximum value of inductor resistance is,

$$R_{MAX} = G_i\left[\frac{1 + g_m/G_i}{\omega_o^2 C_1 C_2} - \frac{L_3}{C_1}\right]$$

Since $G_i = 1/R_i$, $g_m/G_i = \beta$, and assuming $RG_i << 1$ so that $C_1 \simeq C_1'$, we have,

$$R_{MAX} = \frac{1}{R_i}\left(\frac{1+\beta}{\omega_o^2 C_1' C_2} - \frac{L_3}{C_1'}\right) = 5.5\,\Omega$$

So the minimum inductor Q is,

$$Q_{MIN} = \frac{\omega_o L_3}{R_{MAX}} = \underline{3.8} \qquad \text{Then } C_1 \simeq 500\,pF.$$

(12.4) $f_o = 10\,MHz$, $R = 30\,\Omega$, $C = 27\,pF$, $C_o = 5.5\,pF$.

From (12.23a), $L = \dfrac{1}{\omega_s^2 C} = 9.4\,mH$ $\quad(\omega_s = 2\pi \cdot 10\,MHz)$

From (12.23b), $L = \dfrac{C_o + C}{\omega_p^2 C_o C} = 9.4\,mH$ $\quad(\omega_p = 2\pi \cdot 10\,MHz)$

Using L in (12.23a,b) gives $f_s = 9.990\,MHz$, $f_p = 10.015\,MHz$, for a percentage difference of 0.25%.

The Q is

$$Q = \frac{\omega L}{R} = 20{,}000.$$

(12.5) From (12.24), $Z_L + Z_{in} = 0$ for oscillation.

Then,
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-Z_{in} - Z_0}{-Z_{in} + Z_0} = \frac{Z_{in} + Z_0}{Z_{in} - Z_0} = \frac{1}{\Gamma_{in}}$$

(12.6) Let $Z = R + jX$, $R > 0$ (POSITIVE RESISTANCE)

Then $z = Z/Z_0 = r + jx$, $r > 0$

$$\Gamma = \frac{z-1}{z+1} = \frac{(r-1) + jx}{(r+1) + jx}$$

Now let $Z = -R + jX$, $R > 0$ (NEGATIVE RESISTANCE)

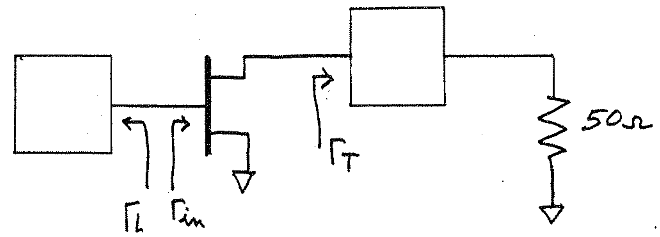
$z = -r + jx$, $r > 0$

Then,
$$\Gamma = \frac{z-1}{z+1} = \frac{-(r+1) + jx}{(-r+1) + jx} = \frac{(r+1) - jx}{(r-1) - jx}$$

So,
$$\frac{1}{\Gamma^*} = \frac{(r-1) + jx}{(r+1) + jx}$$

which has the same form as Γ for positive resistance. So we can read the resistance circles as negative, and interpret the "reflection coefficient" read from the Smith chart as $1/\Gamma^*$.

(12.7) $S_{11} = 0.9\angle{-150^\circ}$, $S_{21} = 2.6\angle{150^\circ}$, $S_{12} = 0.2\angle{-15^\circ}$, $S_{22} = 0.5\angle{-105^\circ}$



The output stability circle is,

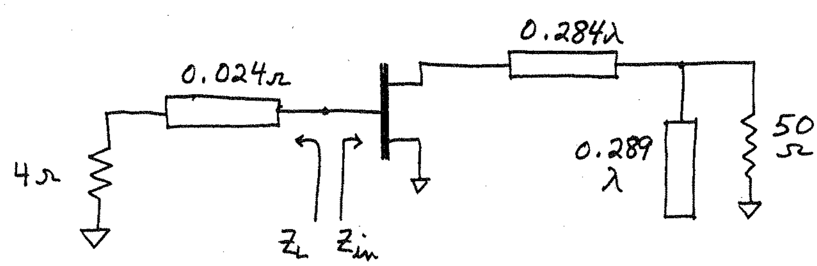
$$C_T = 8.09\angle{-15} \quad , \quad R_T = 8.28$$

From (11.3a),

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_T}{1 - S_{22}\Gamma_T}$$

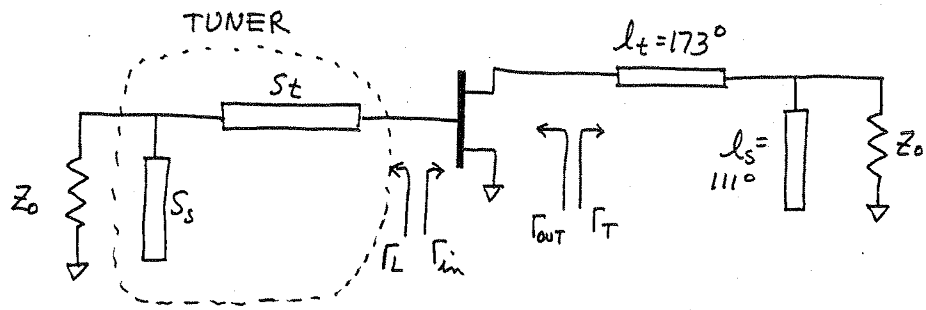
Choose Γ_T so that $|\Gamma_T| < 1$ and Γ_{in} is large. By trial-and-error, we select $\Gamma_T = 0.9\angle{130^\circ}$. Then $\Gamma_{in} = 1.61\angle{-162^\circ}$, or $Z_{in} = -12. - j\,7.5\,\Omega$. So $Z_L = -R_{in}/3 - jX_{in} = 4 + j\,7.5\,\Omega = (0.08 + j0.15)Z_0$.

Matching networks were designed on the Smith chart. The final AC circuit is shown below.



(See Fig. 12.8 for definitions of Γ_L and Γ_T)

(12.8)

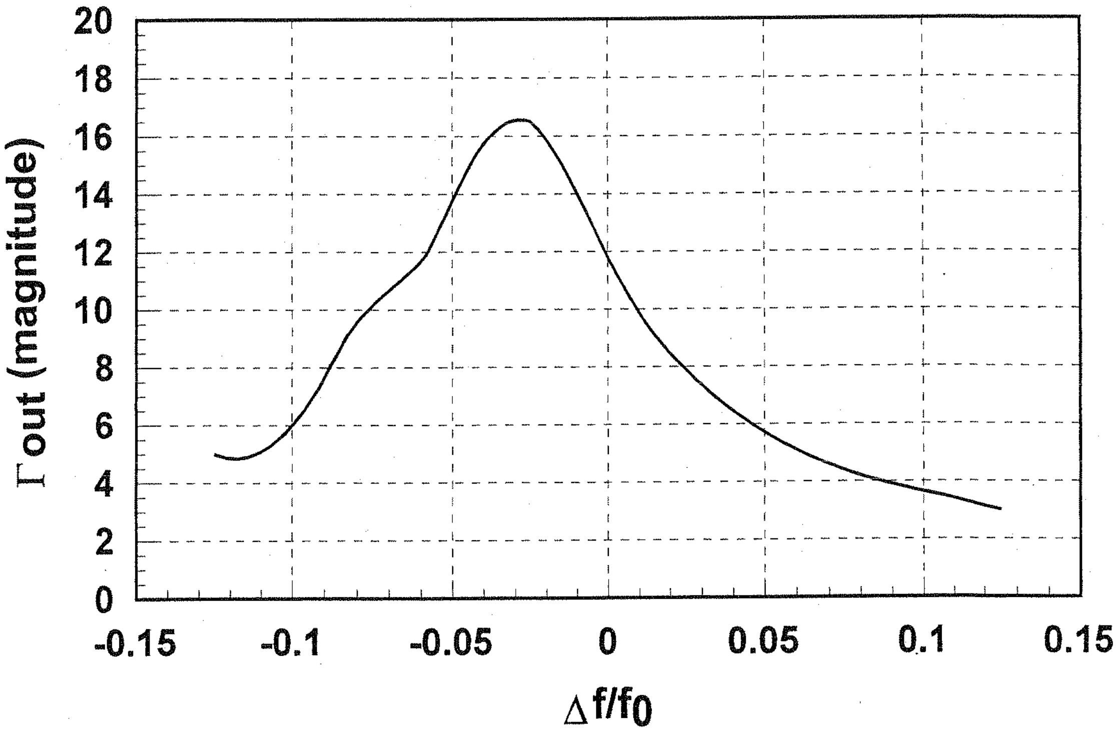


As in Example 12.4, choose $\Gamma_L = 0.6 \angle -130°$. The Γ_{out}, Z_{out}, Z_T, l_t, and l_s are unchanged. Then we have the simple matching problem of using the stub tuner to match 50Ω to Γ_L. The stub susceptance is $jb_S = +j1.56$, or a stub length of $S_s = 0.158\lambda$. The line length is $S_t = 0.18 - 0.176 = 0.004\lambda$.

We then analyze the above circuit to compute Γ_{out} versus frequency:

f (GHz)	$\Delta f / f_0$	$\lvert\Gamma_{out}\rvert$
2.10	−0.125	5.0
2.18	−0.092	7.2
2.20	−0.083	9.1
2.26	−0.058	11.9
2.30	−0.042	15.4
2.34	−0.025	16.5
2.38	−0.008	13.6
2.40	0	11.8
2.42	0.008	10.2
2.46	0.025	7.9
2.50	0.042	6.3
2.60	0.083	4.1
2.66	0.110	3.4
2.70	0.125	3.0

The maximum of $\lvert\Gamma_{out}\rvert$ does not occur at $\Delta f = 0$ because the tuner is not resonant at f_0. The "Q" is much lower than in Example 12.4. This problem shows the advantage of using a high-Q resonator for the oscillator. $\lvert\Gamma_{out}\rvert$ vs f is plotted on the following page.



(12.9) $S_{11} = 1.2\angle 150°$, $S_{12} = 0.2\angle 120°$, $S_{21} = 3.7\angle -72°$, $S_{22} = 1.3\angle -67°$

as in Example 12.4, maximize $|\Gamma_{out}|$ by choosing $S_{11}\Gamma_L \simeq 1$, since

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{11}\Gamma_L}$$

Thus let $\Gamma_L = 0.8\angle -150°$ ✓. Then $\Gamma_{out} = 15.88\angle -99.3°$, and

$$Z_{out} = Z_0 \frac{1+\Gamma_{out}}{1-\Gamma_{out}} = -7.6 + j1.9\,\Omega$$

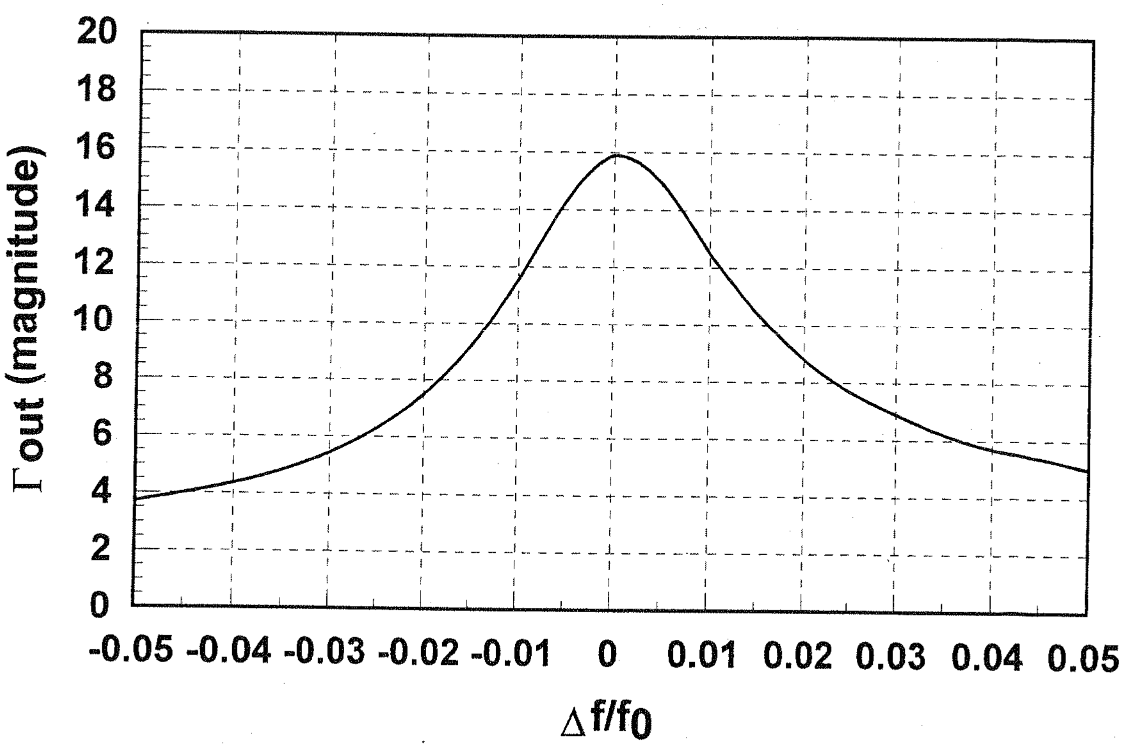
$$Z_T = \frac{-R_{out}}{3} - jX_{out} = 2.53 - j1.9\,\Omega \quad (z_T = 0.0506 - j0.038)$$

matching Z_T to the load impedance gives $l_t = 0.031\lambda$ with a required stub susceptance of $+j4$. Thus $l_s = 0.21\lambda$.

at the dielectric resonator, $\Gamma_L' = \Gamma_L e^{2j\beta l_r} = (0.8\angle -150°)e^{2j\beta l_r} =$ $= 0.8\angle 180°$. Thus $l_r = 0.4583\lambda$.

$$Z_L' = Z_0 \frac{1+\Gamma_L'}{1-\Gamma_L'} = 5.55\,\Omega = N^2 R$$

$|\Gamma_{out}|$ vs f was calculated with SuperCompact, and is plotted below:



(12.10) From (12.49),

$$S_\phi = \frac{kT_0F}{P_0}\left(\frac{K\omega_\alpha\omega_h^2}{\Delta\omega^3} + \frac{\omega_h^2}{\Delta\omega^2} + \frac{K\omega_\alpha}{\Delta\omega} + 1\right)$$

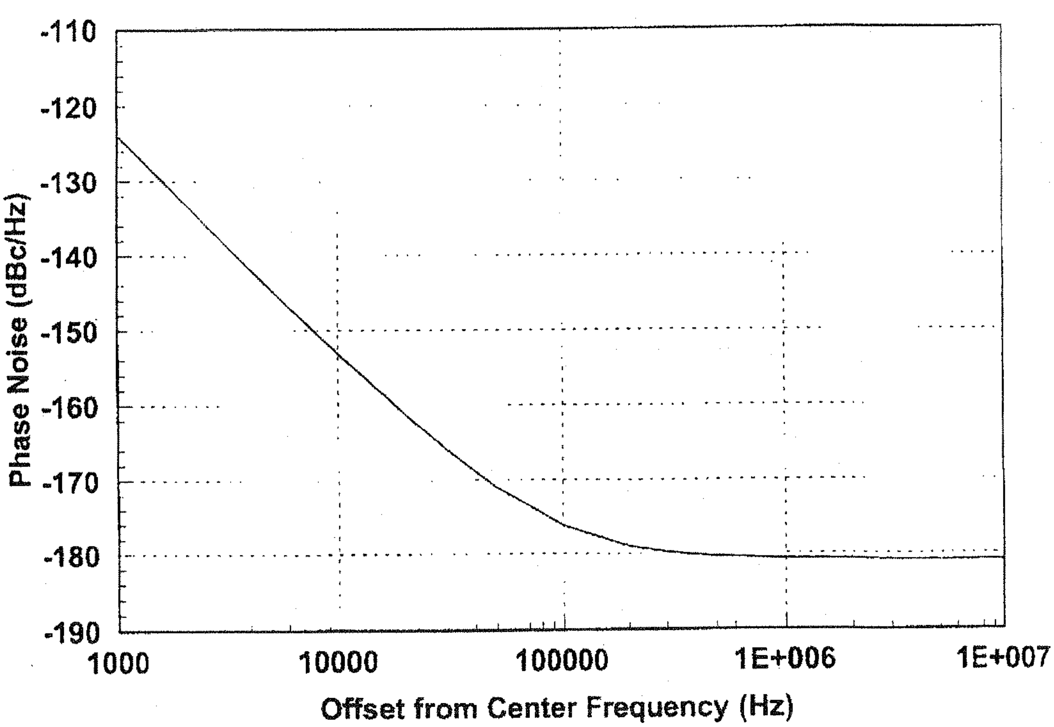
$$\mathcal{L}(f) = S_\phi/2 .$$

For $F = 6dB = 4$, $f_0 = 100 MHz$, $Q = 500$, $P_0 = 10 dBm = 10 mW$, $K = 1$,
$f_\alpha = 50 kHz$, $f_h = f_0/2Q = 100 kHz$, $\Delta f = f - f_0$

a short computer program was written to compute data for the plot shown below.

(a) $\Delta f = 1 MHz$, $S_\phi = -178 dBm$, $\mathcal{L}(1MHz) = -181 dBc/Hz$

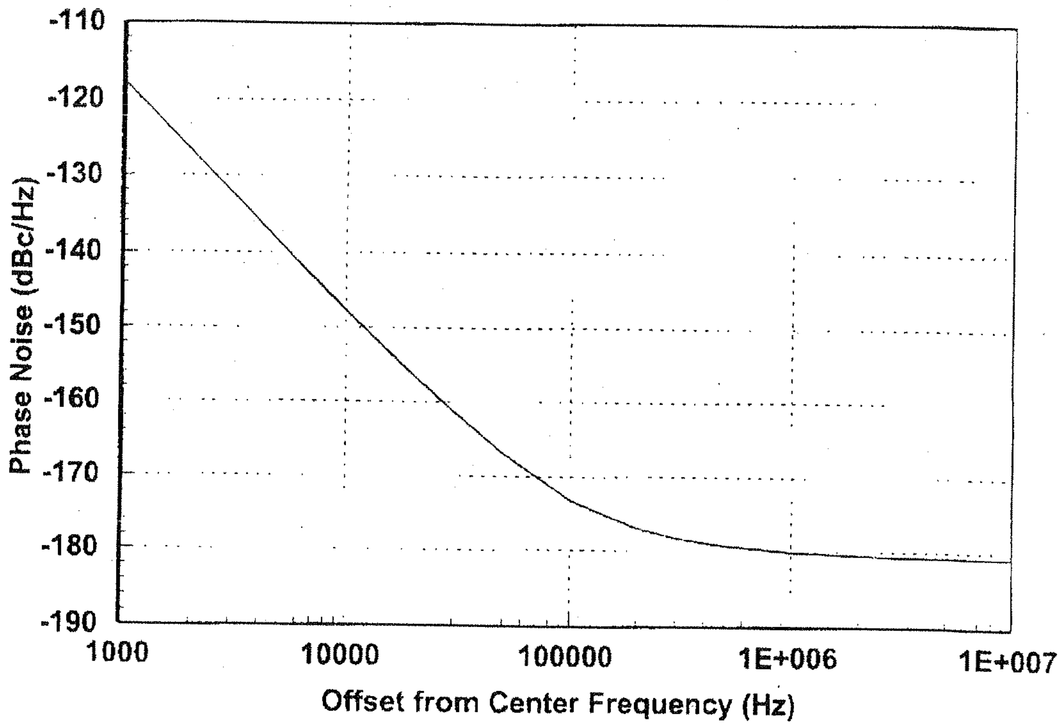
(b) $\Delta f = 10 kHz$, $S_\phi = -150 dBm$, $\mathcal{L}(10kHz) = -153 dBc/Hz$



(12.11) This calculation is similar to that of Problem 12.10, but with $f_\alpha = 200\,kHz$. Plot shown below.

(a) $\Delta f = 1\,MHz$, $S_\phi = -177\,dBm$, $\mathcal{L}(1\,MHz) = -180\,dBc/Hz$

(b) $\Delta f = 10\,kHz$, $S_\phi = -144\,dBm$, $\mathcal{L}(10\,kHz) = -147\,dBc/Hz$.



12.12

If C is the desired signal level, I is the undesired signal level, S is the desired rejection ratio, $\mathcal{L}(f)$ the phase noise, and B the filter bandwidth, then

$$S = \frac{C}{I B \mathcal{L}(f)}$$

In dB,

$$\mathcal{L}(f) = C(dBm) - I(dBm) - S(dB) - 10\log(B). \checkmark$$

12.13 $B = 12\ kHz$, $S = 80\ dB$, $C = I$.

From (12.50),

$$\mathcal{L}(30\,kHz) = C\,(dBm) - I(dBm) - S(dB) - 10\log(B)$$
$$= -80\,dB - 10\log(12\times10^3)$$
$$= \underline{-121\ dBm}$$

(12.14) Assume excitation at f_1, f_2; o.c. at all other frequencies except $f_3 = f_1 + f_2$. Then all power terms are zero except for $n = \pm 1, m = 0$; $n = 0, m = \pm 1$; and $n = m = \pm 1$. So the Manley-Rowe relations give,

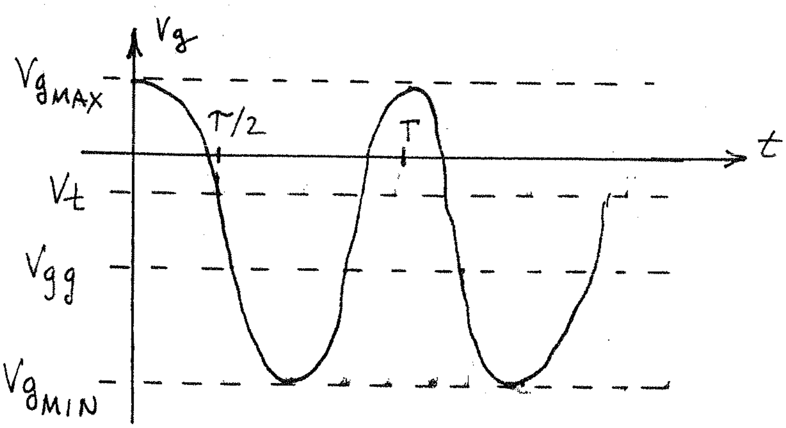
$$\frac{P_{10}}{\omega_1} + \frac{P_{11}}{\omega_1 + \omega_2} = 0$$

$$\frac{P_{01}}{\omega_2} + \frac{P_{11}}{\omega_1 + \omega_2} = 0$$

For sources at f_1, f_2, we have $P_{10} > 0$ and $P_{01} > 0$. Then $P_{11} < 0$, representing power at $f_3 = f_1 + f_2$ $(m = n = 1)$. Conversion gain is then,

$$G_C = \frac{-P_{11}}{P_{10}} = \frac{\omega_1 + \omega_2}{\omega_1} = 1 + \omega_2/\omega_1 . \quad ✓$$

(12.15)



$$V_g = V_{gg} + V_g \cos\frac{2\pi t}{T}$$

$$V_g = (V_{g_{MAX}} - V_{g_{MIN}})/2 \quad \text{(PEAK)}$$

$$V_{gg} = (V_{g_{MAX}} + V_{g_{MIN}})/2 \quad \text{(AVG)}$$

$$V_t = V_{gg} + V_g \cos\frac{\pi\tau}{T}$$ solve for $\cos\frac{\pi\tau}{T}$:

$$\cos\frac{\pi\tau}{T} = \frac{V_t - V_{gg}}{V_g} = \frac{2V_t - V_{g_{MAX}} - V_{g_{MIN}}}{V_{g_{MAX}} - V_{g_{MIN}}} \quad ✓$$

(12.16) $v_{RF}(t) = V_{RF}\left[\cos(\omega_{LO}-\omega_{IF})t + \cos(\omega_{LO}+\omega_{IF})t\right]$

$v_{LO}(t) = V_{LO}\cos\omega_{LO}t$

after mixing and LPF:

$$v_{OUT}(t) = \frac{KV_{RF}V_{LO}}{2}\left[\cos\omega_{IF}t + \cos\omega_{IF}t\right] = KV_{RF}V_{LO}\cos\omega_{IF}t$$

(both sidebands convert to same IF)

(12.17) $i(t) = e^{3v(t)} - 1$, $v(t) = 0.1\cos\omega_1 t + 0.1\cos\omega_2 t$

$$i(t) = i\Big|_{v=0} + \frac{di}{dv}\Big|_{v=0} v + \frac{d^2 i}{dv^2}\Big|_{v=0}\frac{v^2}{2} + \frac{d^3 i}{dv^3}\Big|_{v=0}\frac{v^3}{6} + \cdots$$

$$i\big|_{v=0} = 0 \; ; \; \frac{di}{dv}\Big|_{v=0} = 3 \; , \; \frac{d^2 i}{dv^2}\Big|_{v=0} = 9 \; ; \; \frac{d^3 i}{dv^3}\Big|_{v=0} = 27.$$

So,

$$i(t) = 3v + 4.5v^2 + 4.5v^3 + \cdots$$

$$v^2 = .01\left[\cos^2\omega_1 t + 2\cos\omega_1 t\cos\omega_2 t + \cos^2\omega_2 t\right]$$

$$= .01\left[1 + \tfrac{1}{2}\cos^2\omega_1 t + \tfrac{1}{2}\cos 2\omega_2 t + \cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\right]$$

$$v^3 = .001\left[\cos^3\omega_1 t + 3\cos^2\omega_1 t\cos\omega_2 t + 3\cos\omega_1 t\cos^2\omega_2 t + \cos^3\omega_2 t\right]$$

$$= .001\Big[\tfrac{1}{4}\cos 3\omega_1 t + \tfrac{3}{4}\cos\omega_1 t + \tfrac{3}{2}\cos\omega_2 t + \tfrac{3}{4}\cos(2\omega_1-\omega_2)t$$
$$+ \tfrac{3}{4}\cos(2\omega_1+\omega_2)t + \tfrac{3}{2}\cos\omega_1 t + \tfrac{3}{4}\cos(\omega_1-2\omega_2)t$$
$$+ \tfrac{3}{4}\cos(\omega_1+2\omega_2)t + \tfrac{3}{4}\cos\omega_2 t + \tfrac{1}{4}\cos 3\omega_2 t\Big]$$

ω	I	
0	(4.5)(.01)	=0.045 ✓
ω_1, ω_2	$(3)(.1)+(4.5)(.001)(\frac{3}{2}+\frac{3}{4})$	=0.3101 ✓
$2\omega_1, 2\omega_2$	$(4.5)(.01)(\frac{1}{2})$	=0.0225 ✓
$3\omega_1, 3\omega_2$	(4.5)(.001)(1/4)	=0.00113 ✓
$\omega_1+\omega_2$	(4.5)(.01)(1)	=0.045 ✓
$\omega_1-\omega_2$	(4.5)(.01)(1)	=0.045 ✓
$2\omega_1-\omega_2$	(4.5)(.001)(3/4)	=0.003375 ✓
$2\omega_1+\omega_2$	(4.5)(.001)(3/4)	=0.003375 ✓
$\omega_1-2\omega_2$	(4.5)(.001)(3/4)	=0.003375 ✓
$\omega_1+2\omega_2$	(4.5)(.001)(3/4)	=0.003375 ✓

(12.18) $f_{RF} = 900\ MHz$

$f_{IF} = 80\ MHz$

Two possible LO frequencies are,

$$f_{LO} = f_{RF} \pm f_{IF} = 980\ MHz,\ 820\ MHz \checkmark$$

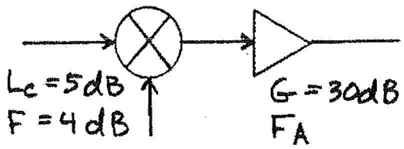
The image frequency for $f_{LO} = 980\ MHz$ is,

$$f_{IM} = f_{LO} + f_{IF} = 980 + 80 = 1060\ MHz \checkmark$$

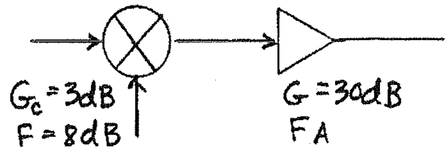
The image frequency for $f_{LO} = 820\ MHz$ is,

$$f_{IM} = f_{LO} - f_{IF} = 820 - 80 = 740\ MHz. \checkmark$$

12.19



CASE #1



CASE #2

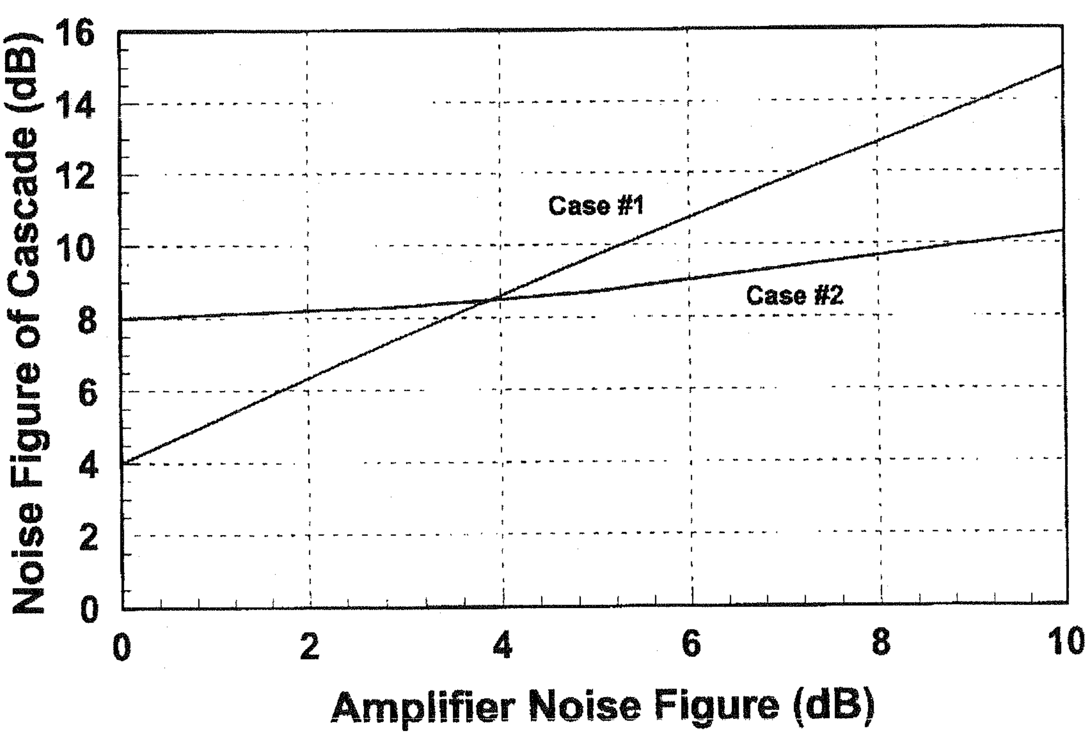
$$F_c = 2.51 + \frac{F_A - 1}{1/3.16}$$

$$F_c = 6.31 + \frac{F_A - 1}{2.0}$$

F_A (dB)	F_c(#1) dB	F_c(#2) dB
0	4.0	8.0
3	7.5	8.3
5	9.7	8.7
10	14.9	10.3

3 dB = 2.0
4 dB = 2.51
5 dB = 3.16
8 dB = 6.31

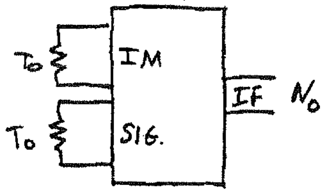
RESULTS ARE PLOTTED BELOW:



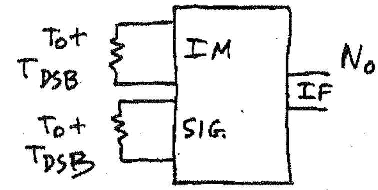
12.20

DSB

LOSSY



LOSSLESS



$$N_o = N_{ADDED} + \frac{kT_oB}{L} + \frac{kT_oB}{L}$$

(B in SSB)

$$N_o = \frac{2kB}{L}(T_o + T_{DSB})$$

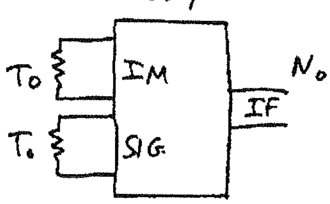
$$\therefore \quad N_{ADDED} = \frac{2kB}{L} T_{DSB}$$

$$F_{DSB} = \frac{S_i N_o}{S_o N_i} = \frac{N_o L}{2kT_o B} = 1 + \frac{T_{DSB}}{T_o}$$

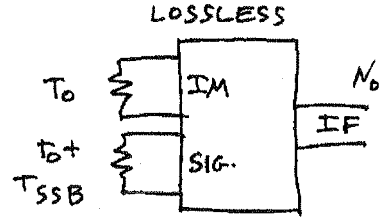
(INPUT NOISE – $N_i = 2kT_oB$)

SSB

LOSSY



LOSSLESS



$$N_o = N_{ADDED} + \frac{2kT_oB}{L}$$

$$N_o = \frac{2kBT_o}{L} + \frac{kBT_{SSB}}{L}$$

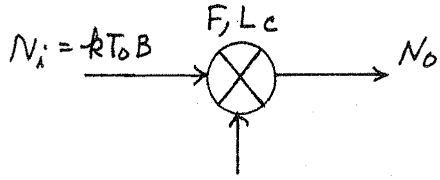
$$\therefore \quad T_{SSB} = \frac{L\,N_{ADDED}}{kB}$$

$$\underline{T_{SSB} = 2\,T_{DSB}} \quad ✓$$

$$F_{SSB} = \frac{S_i N_o}{S_o N_i} = \frac{N_o L}{kT_oB} = 2 + \frac{T_{SSB}}{T_o} = 2\left(1 + \frac{T_{DSB}}{T_o}\right) = 2\,F_{DSB} \quad ✓$$

(INPUT NOISE – $N_i = kT_oB$)

(12.21)



$$N_0 = \underbrace{\frac{kT_0B}{L_c}}_{\text{INPUT NOISE}} + \underbrace{\frac{kT_0B(F-1)}{L_c}}_{\text{MIXER NOISE}} = \frac{kT_0BF}{L_c} \checkmark$$

(12.22) $v_1 = V_0 \cos\omega t$; $v_2 = V_0 \cos(\omega t + \theta)$

as in (12.112)-(12.113), the diode currents in a mixer using a quadrature hybrid will be,

$$i_1 = k v_0^2 [\cos(\omega t - \pi/2) + \cos(\omega t + \theta - \pi)]^2$$
$$= k v_0^2 [\sin\omega t - \cos(\omega t + \theta)]^2$$

$$i_2 = -k v_0^2 [\cos(\omega t - \pi) + \cos(\omega t + \theta - \pi/2)]^2$$
$$= -k v_0^2 [-\cos\omega t + \sin(\omega t + \theta)]^2$$

Low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \tfrac{1}{2}\sin\theta)$$
$$i_2 = -k v_0^2 (1 - 1/2 \sin\theta)$$

so the output is $i_1 + i_2 = k v_0^2 \sin\theta$ ✓

If a mixer with a 180° hybrid is used, the diode currents become,

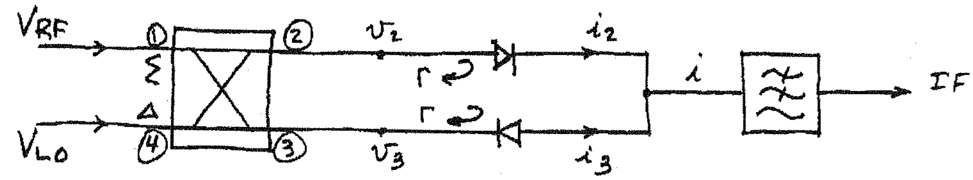
$$i_1 = k v_0^2 [\cos\omega t + \cos(\omega t + \theta)]^2$$
$$i_2 = -k v_0^2 [\cos\omega t - \cos(\omega t + \theta)]^2$$

Then low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \tfrac{1}{2}\cos\theta)$$
$$i_2 = -k v_0^2 (1 - \tfrac{1}{2}\cos\theta)$$

so the output is $i_1 + i_2 = k v_0^2 \cos\theta$ ✓

(12.23)



$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

let $v_{RF}(t) = V_{RF}\cos\omega_{RF}t = v_1(t)$

$v_{LO}(t) = V_{LO}\cos\omega_{LO}t = v_4(t)$

Then the diode voltages are,

$$v_2(t) = \frac{1}{\sqrt{2}}V_{RF}\cos(\omega_{RF}t - 90°) + \frac{1}{\sqrt{2}}V_{LO}\cos(\omega_{LO}t + 90°)$$

$$v_3(t) = \frac{1}{\sqrt{2}}V_{RF}\cos(\omega_{RF}t - 90°) + \frac{1}{\sqrt{2}}V_{LO}\cos(\omega_{LO}t - 90°)$$

Assume $i_2 = k v_2^2$, $i_3 = -k v_2^2$. $\omega_{IF} = \omega_{RF} - \omega_{LO}$.

Then, after LP filtering, the diode currents are,

$$i_2 = \frac{k}{4}V_{RF}V_{LO}\cos(\omega_{RF}t - 90° - \omega_{LO}t - 90°) = \frac{-k}{4}V_{RF}V_{LO}\cos\omega_{IF}t.$$

$$i_3 = \frac{-k}{4}V_{RF}V_{LO}\cos(\omega_{RF}t - 90° - \omega_{LO}t + 90°) = \frac{-k}{4}V_{RF}V_{LO}\cos\omega_{IF}t.$$

So the IF output current is $i(t) = -\frac{k}{2}V_{RF}V_{LO}\cos\omega_{IF}t$ ✓

AT RF INPUT:

$$V_2^+ = \Gamma V_2^- = \frac{-j}{\sqrt{2}}\Gamma V_{RF}^+ \; ; \; V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}}\Gamma V_{RF}^+$$

$$V_{RF}^{\Sigma} = V_1^- = V_2^+(-j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{-\Gamma V_{RF}^+} \checkmark$$

$$V_{RF}^{\Delta} = V_4^- = V_2^+(j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{0} \checkmark$$

AT LO INPUT:

$$V_2^+ = \Gamma V_2^- = \frac{j}{\sqrt{2}}\Gamma V_{LO}^+ \; ; \; V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}}\Gamma V_{LO}^+$$

$$V_{LO}^{\Sigma} = V_1^- = V_2^+(-j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{0} \checkmark$$

$$V_{LO}^{\Delta} = V_4^- = V_2^+(j/\sqrt{2}) + V_3^+(-j/\sqrt{2}) = \underline{-\Gamma V_{LO}^+} \checkmark$$

Assume now that

$$v_{LO}(t) = V_{LO}^{(2)}\cos 2\omega_{LO}t.$$

Then after LPF,

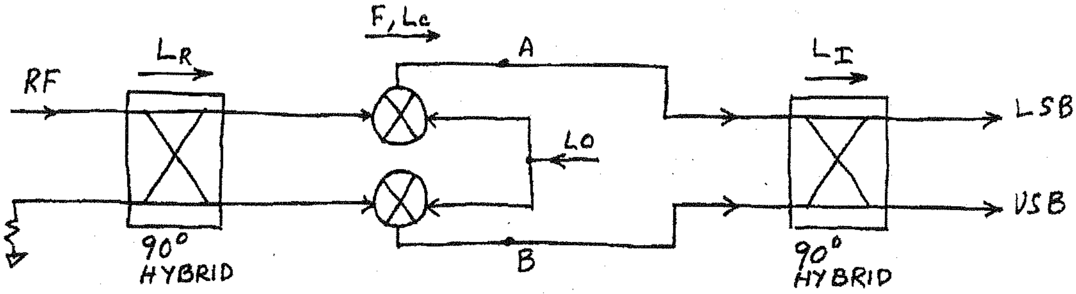
$$v_2^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t + 2\omega_{LO} t + 90°) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{LO} t + 90°)$$

$$v_3^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t + 2\omega_{LO} t + 90°) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{LO} t + 90°)$$

Then forming

$$i(t) = k(v_2^2 - v_3^2)\Big|_{LPF} = 0 \text{ for } \omega_{RF} \pm 2\omega_{LO} \text{ frequencies.}$$

12.24



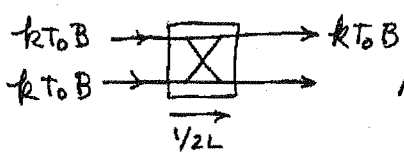
The noise power due to the RF hybrid and the mixer, ref. to IF output of mixer, is

$$N_A = N_B = \frac{kB}{L_c}\left[T_0 + (F-1)\,T_0\right] = \frac{kBT_0F}{L_c},$$

since the noise power output of the matched hybrid is kT_0B. The total noise power output is (at either LSB or USB),

$$N_o = \frac{N_A}{2L_I} + \frac{N_B}{2L_I} + \frac{N_{added}}{2L_I} = \frac{kBT_0F}{L_IL_c} + \frac{N_{added}}{2L_I}$$

N_{added} is the output noise power of the IF hybrid when not terminated at second input port:



$$N_o = \frac{kT_0B}{2L} + \frac{kT_0B}{2L} + \frac{N_{added}}{2L} = kT_0B$$

Thus $N_{added} = 2kT_0B(L-1)$

So, $\quad N_o = \dfrac{kBT_0F}{L_IL_c} + kT_0B\left(1 - \dfrac{1}{L_I}\right) \quad ; \quad S_o = \dfrac{4S_i}{L_c}\,\dfrac{1}{4L_IL_R} = \dfrac{S_i}{L_cL_IL_R}$

$N_i = kT_0B$

and then,

$$F_{TOT} = \frac{S_iN_o}{S_oN_i} = \frac{L_cL_IL_R}{kT_0B}\left[\frac{kBFT_0}{L_IL_c} + kT_0B\left(1 - \frac{1}{L_I}\right)\right] = \underline{FL_R + L_cL_RL_I - L_cL_R} \checkmark$$

CHECK: if $L_R = L_I = 1$, $F_{TOT} = F + 2L_c - 2L_c = F$ ✓ (mixer noise only)

CHECK: if $F = L_c$ (passive mixer loss only), $F_{TOT} = L_cL_IL_R$ ✓

(The cascade noise figure formula can be used to obtain the same result if we set $F_R = L_R$, $F_I = L_I$.)

Chapter 13

(13.1) $F_\theta(\theta, \phi) = A \sin\theta \sin\phi$

MAIN BEAM occurs at $\theta = 90°$; $\phi = 90°$ or $270°$ ✓

3 dB points in $\theta = 90°$ plane:

$\sin\phi = 0.707 \Longrightarrow \phi = 45°$ or $135°$

$HPBW_\theta = 135 - 45 = 90°$ ✓

3 dB points in $\phi = 90°$ plane:

$\sin\theta = 0.707 \Longrightarrow \theta = 45°$ or $135°$

$HPBW_\phi = 135 - 45 = 90°$ ✓

$$D = \frac{4\pi F_{MAX}^2}{\iint F^2(\theta,\phi)\sin\theta \, d\theta \, d\phi} = \frac{4\pi}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin^3\theta \sin^2\phi \, d\theta \, d\phi}$$

$$= \frac{4\pi}{\frac{4}{3}\cdot\pi} = 3 = \underline{\underline{4.8 \text{ dB}}}$$ (verified with PCAAD)

(13.2) $F_\theta(\theta, \phi) = \begin{cases} A\sin\theta & \text{for } 0 \le \theta \le \pi/2 \\ 0 & \text{otherwise} \end{cases}$

$$D = \frac{4\pi F_{MAX}^2}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} F_\theta^2(\theta,\phi)\sin\theta \, d\theta \, d\phi} = \frac{4\pi}{2\pi\int_{\theta=0}^{\pi/2}\sin^3\theta \, d\theta} = \frac{4\pi}{2\pi\left(\frac{2}{3}\right)}$$

$$= 3 = \underline{\underline{4.8 \text{ dB}}}$$ (verified with PCAAD)

(13.3) $f = 12.4\text{ GHz}$, $Diam = 18'' = 0.457\text{ m}$, $\eta_{ap} = 65\%$

$$\lambda = \frac{c}{f} = 0.0242\text{ m} \checkmark$$

$$A = \pi R^2 = \pi\left(\frac{Diam}{2}\right)^2 = 0.164\text{ m}^2$$

From (13.13),

$$D = \eta_{ap}\frac{4\pi A}{\lambda^2} = (0.65)\frac{4\pi(0.164)}{(0.0242)^2} = 2287 = \underline{\underline{33.6\text{dB}}} \checkmark$$

(13.4) $f = 38\text{GHz}$, $G = 39.0\text{dB}$, $Diam = 12.0''$, $\eta_{rad} = 90\%$

a) From (13.11), $D = \dfrac{G}{\eta_{rad}} = \dfrac{10^{39/10}}{0.9} = 8{,}826.$

From (13.13), $D = \dfrac{4\pi A}{\lambda^2}\eta_{ap}$

$$\eta_{ap} = \frac{\lambda^2 D}{4\pi A} = \left(\frac{c}{\pi f\, Diam}\right)^2 D = \underline{\underline{60\%}}$$

b) From (13.9), $D = \dfrac{32{,}400}{\theta_3^2}$

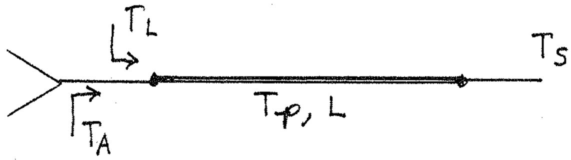
$$\theta_3 = \sqrt{\frac{32{,}400}{8826}} = \underline{\underline{1.9^\circ}}$$

(13.5) From (13.18), $T_A = \eta_{rad} T_b + (1-\eta_{rad})T_p$

$$= (T_b - T_p)\eta_{rad} + T_p$$

Thus, $\eta_{rad} = \dfrac{T_A - T_p}{T_b - T_p} = \dfrac{105-290}{5-290} = \underline{65\%}$

(13.6)



$$T_A = \eta_{rad} T_b + (1-\eta_{rad}) T_p \quad \text{(at antenna)} \quad (13.18)$$

$$T_L = (L-1) T_p \quad \text{(at antenna)} \quad (10.15)$$

$$T_S = \frac{1}{L}(T_A + T_L) \quad \text{(at output)}$$

$$= \frac{1}{L}\left[\eta_{rad} T_b + (1-\eta_{rad}) T_p\right] + \frac{(L-1)}{L} T_p \quad (13.20) \checkmark$$

(13.7) $T = \eta_{rad} T_b + (1-\eta_{rad}) T_p + T_R$

$T_b = 50K$
$T_p = 290K$
$F = 1.1dB = 1.29$
$L = 2.5dB = 1.78$
$G = 33.5dB = 2240$

The noise temperature of the receiver is,

$$T_R = (F-1)T_o = (1.29-1)(290) = 84K$$

The efficiency of the array is,

$$\eta_{rad} = \frac{1}{L} = \frac{1}{1.78} = 0.56$$

Thus,

$$T = (0.56)(50) + (1-0.56)(290) + 84 = \underline{240K}$$

Then, $\frac{G}{T}(dB) = 10 \log \frac{2240}{240} = 9.7 \underline{dB/K}$

This value is well below the desired minimum of 12 dB/K.

(13.8) Solving (13.23) for G gives,

$$G = \frac{4\pi S R^2}{P_t} = \frac{4\pi(7.5\times10^{-3})(300)^2}{85} = 100 = \underline{\underline{20\,dB}}$$

(13.9)

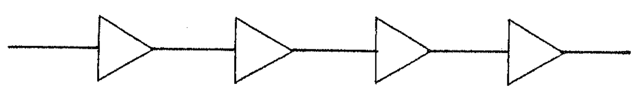
1) RADIO LINK: $f = 28\,GHz \Rightarrow \lambda = 0.0107\,m$; $G_t = G_r = 25\,dB = 316$.

Let $P_r = 1W$. Then,

$$P_t = \frac{(4\pi R)^2}{P_r G_t G_r \lambda^2} = \frac{(4\pi)^2(5000)^2}{(1)(316)^2(0.0107)^2} = 3.45\times10^8\,W$$

$$\text{ATTENUATION} = 10\log\frac{P_r}{P_t} = 10\log\frac{1}{3.45\times10^8} = \underline{-85.4\,dB}$$ ✓

2) WIRED LINK: $\alpha = 0.05\,dB/m = 0.0057$ neper/m ; $4\times30\,dB$ REPEATERS.



$$\text{ATTENUATION OF LINE} = 10\log e^{-2\alpha R}$$
$$= 10\log e^{-2(0.0057)(5000)}$$
$$= -250\,dB$$

TOTAL LOSS $= -250 + 4(30) = \underline{\underline{-130\,dB}}$ ✓

The radio link has much less link loss than the wired link, and will thus require less transmit power.

(13.10) $f = 882\ MHz \Rightarrow \lambda = 0.34\ m$; $F = 8\ dB = 6.31$; $P_t G_t = 20$
$SNR = 18\ dB = 63.1$; $G = 1\ dBi = 1.26$; $B = 30\ kHz$

$$T_{sys} = T_A + T_R = T_A + (F-1)T_o$$
$$= 400 + (6.31-1)(290) = 1940\ K$$

$$N_o = k T_{sys} B = (1.38\times10^{-23})(1940)(3\times10^4)$$
$$= 8.03\times10^{-16}\ W \quad \text{(at receiver input)}$$

$$S_o = \left(\frac{S_o}{N_o}\right) N_o = (63.1)(8.03\times10^{-16}) = 5.07\times10^{-14}\ W$$
$$= -103\ dBm.$$

$$R = \sqrt{\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 S_o}} = \sqrt{\frac{(20)(1.26)(.34)^2}{(4\pi)^2(5.07\times10^{-14})}} = \underline{6.03\times10^5\ m}$$
$$= 603\ km.$$

(This is unrealistically large due to the assumption of free-space.)

(13.11) Carrier power at receiver:

$$C = S_i G_A G/L \qquad (S_i \text{ ref. to antenna w/ 0 dBi})$$

at input to amplifier:

$$T_e = T_A + (F-1)T_0 + (L-1)T_0/G$$

The noise power at input to receiver:

$$N = kT_e G/L = \frac{C}{(C/N)}$$

$L = 25\,dB = 316.2$
$S_i = 1\times10^{-16}$
$G_A = 5\,dB = 3.16$
$\frac{C}{N} = 32\,dB = 1580.$
$G = 10\,dB = 10$

So,

$$T_e = \frac{CL}{k(\frac{C}{N})G} = \frac{S_i G_A}{k(\frac{C}{N})}$$

$$F = 1 + \frac{T_e}{T_0} - \frac{T_A}{T_0} - \frac{(L-1)}{G} = 1 + \frac{S_i G_A}{kT_0(\frac{C}{N})} - \frac{T_A}{T_0} - \frac{(L-1)}{G}$$

$$= 1 + \frac{(1\times10^{-16})(3.16)}{(1.38\times10^{-23})(290)(1.58\times10^{3})} - \frac{300}{290} - \frac{(316.2-1)}{10}$$

$$= 18.4 = \underline{12.6\,dB}$$

(13.12) $N_o = kT_bB = S_o$ for $S_o/N_o = 0\,dB$

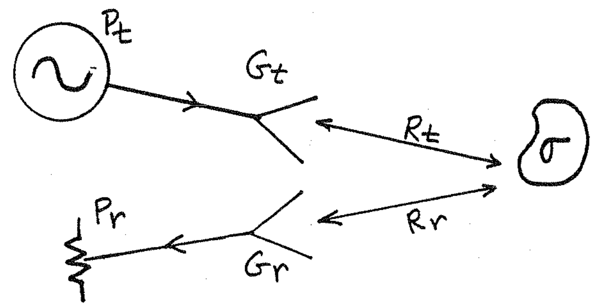
$$kT_bB = P_r = \frac{P_t G^2 \lambda^2}{(4\pi R)^2}$$

$$R = \sqrt{\frac{P_t G^2 \lambda^2}{16\pi^2 k T_b B}} = \sqrt{\frac{(1000)(2.51)^2(4.48)^2}{16\pi^2(1.38\times10^{-23})(4)(4\times10^6)}} = \underline{1.9\times10^9\,m}$$

If $SNR = 30\,dB = 1000$, $R = 6.0\times10^7\,m$

$R_{VENUS} = 4.2\times10^{10}\,m$, so the signal will not even reach the nearest planet.

13.13



From (13.23) the power density incident on the target is,

$$S = \frac{P_t G_t}{4\pi R_t^2}$$

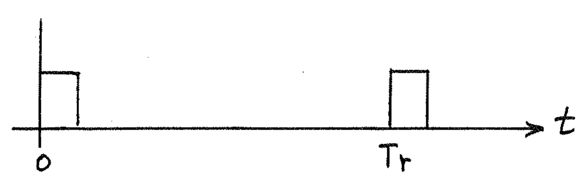
The scattered power density at the receiver is, from (13.37),

$$S_r = \frac{P_t G_t \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^2 R_t^2 R_r^2},$$

where $\sigma(\theta_t, \phi_t; \theta_r, \phi_r)$ is the radar cross-section of the target seen at θ_r, ϕ_r with an incident wave at θ_t, ϕ_t. Then the received power can be found using (13.14):

$$P_r = P_t \frac{G_t G_r \lambda^2 \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^3 R_t^2 R_r^2} \quad ✓$$

13.14



When a pulse is transmitted at $t=0$, the return pulse must come back before the next pulse in transmitted at $t = T_r$, to avoid an ambiguity in range. The round-trip time for a pulse return is,

$$T = 2R/c,$$

so the maximum unambiguous range is,

$$R_{MAX} = \frac{cT_r}{2} = \frac{c}{2f_r} \quad ✓$$

(13.15) From (13.40) the doppler frequencies are,

$$f_{d(MIN)} = \frac{2 v_{MIN} f_o}{c} = \frac{2(1\,\frac{m}{sec})(12\,GHz)}{3\times10^8\,m/sec} = 80\ Hz$$

$$f_{d(MAX)} = \frac{2 v_{MAX} f_o}{c} = \frac{2(20\,m/sec)(12\,GHz)}{3\times10^8\,m/sec} = 1.6\ kHz$$

so the necessary passband is 80–1600 Hz.

(13.16) From (13.38) the received power is,

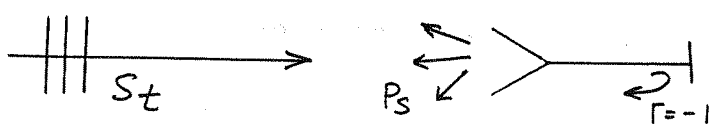
$$P_r = P_t \frac{G^2\lambda^2\sigma}{(4\pi)^3 R^4} = 1000\,\frac{(1000)^2(0.15)^2(20)}{(4\pi)^3(10^4)^4} = 2.27\times10^{-11}\ W$$

$$= -76\ dBm$$

The transmitter power is $10\log(10^4)(10^3) = 70\ dBm$. So the isolation between receiver and transmitter must be,

$$I = 70\ dBm - (-76\ dBm) + 10\ dB = 156\ dB$$

(13.17)



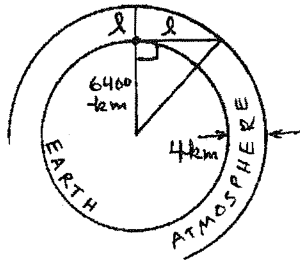
assume an incident plane wave with power density S_t. Then the received power of the antenna is, from (13.14) and (13.15):

$$P_\ell = S_t A_e = S_t \frac{\lambda^2 G}{4\pi}$$

Because of the short-circuit termination, all of this power is re-transmitted (assuming a lossless antenna), giving a radiated power in the main beam direction of, $P_s = G P_\ell$. Then the RCS can be found from (13.36):

$$\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2}{4\pi} \checkmark$$

13.18



LOOKING TOWARD ZENITH, $\ell = 4000$ m. $= 4$ km.

LOOKING TOWARD HORIZON,

$$\ell = \sqrt{(6404)^2 - (6400)^2} = 226 \text{ km}.$$

$\alpha = 0.005$ dB/km

T= 4K   L, T_o $\Rightarrow T_e = \frac{4}{L} + (L-1)T_o$

AT ZENITH: $L = (0.005 \text{ dB/km})(4 \text{ km}) = 0.02 \text{dB} = 1.0046$

$T_e = 5.3K$

AT HORIZON: $L = (0.005 \text{dB/km})(226 \text{km}) = 1.13 \text{dB} = 1.297$

$T_e = 89K$

13.19 $f = 28 \text{GHz} \Rightarrow \lambda_o = 0.0107 m$, $G = 32dB = 1585$, $P_t = 5W$

a) $R = \sqrt{\frac{GP_t}{4\pi S}} = \sqrt{\frac{(1585)(5)}{4\pi(.01 W/cm^2)}} = 251 \text{ cm} = 2.51 m$

b) $G = 32 - 10 = 22 \text{ dB} = 158.5$

$R = \sqrt{\frac{(158.5)(5)}{4\pi \ (.01 W/cm^2)}} = 79.5 \text{ cm} = 0.8 m$

c) $d = \sqrt{\frac{\lambda^2 D}{\pi^2}} = 0.175 m$ (use $D = \frac{G}{\eta_{ap}} = 2641.$)

$R_{ff} = \frac{2d^2}{\lambda} = 5.7m$

(neither distance is in the far-field of the antenna)

(13.20)

$$S = 1300 \text{ W/m}^2 = \frac{1}{2}|E||\bar{H}| = \frac{1}{2\eta_0}|E|^2 = \frac{\eta_0}{2}|\bar{H}|^2$$

$$|E| = \sqrt{2\eta_0 S} = \sqrt{2(377)(1300)} = 990 \text{ V/m}$$

$$|\bar{H}| = \sqrt{\frac{2S}{\eta_0}} = \sqrt{\frac{2(1300)}{377}} = 2.6 \text{ A/m}$$

check: $\frac{1}{2}|E||\bar{H}| = \frac{1}{2}(990)(2.6) = 1300 \text{ W/m}^2$ ✓