Session 4: MPI Parallel-Distributed Computation on Kubernetes

Sunday, August 5, 2018 · 2:30PM - 3:30PM

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Topics

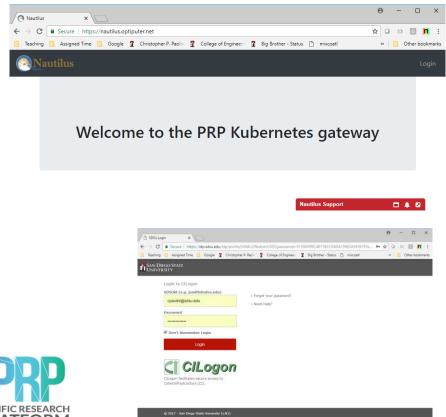
- Configuring and testing an environment for using OpenMPI on K8s
- Benchmarking PRP K8s with LINPACK
- Example PRP science driver: subsurface CO₂ and waste-water injection simulation

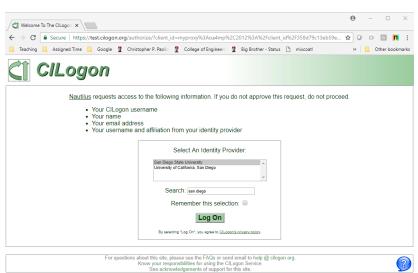




PRP Kubernetes Gateway: kubectl Client Config

Login to https://nautilus.optiputer.net/ to obtain the latest kubectl client config bundle



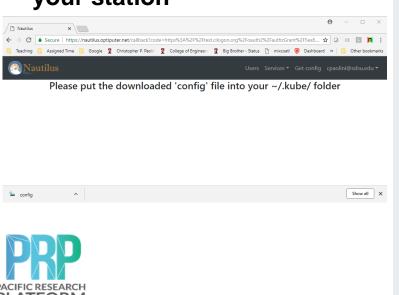


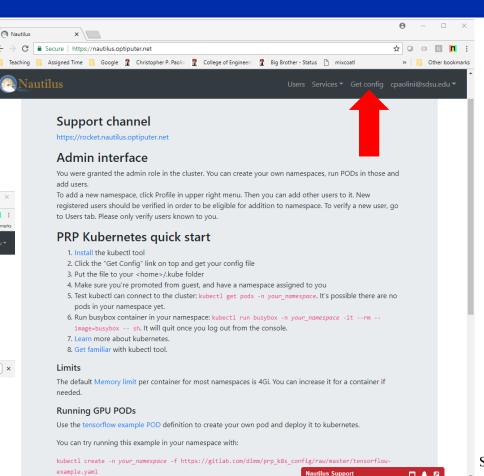




PRP Kubernetes Gateway: kubectl Client Config cont.

Download kubectl client config bundle via URL https://nautilus.optiputer.net/authConfig and move file to \$HOME/.kube/config on your station







Using OpenMPI on K8s: Create Pods

Check the kubectl configuration

```
[paolini@ps-40g ~]$ kubectl -n sdsu cluster-info
Kubernetes master is running at https://67.58.53.147:6443
```

Create cluster of pods using a YAML configuration

[paolini@fiona k8s]\$ kubectl create -f subflow.yaml

Verify all pods are in the Running state

[paolini@fiona k8s]\$ kubectl get pods -o wide -n sdsu

	_					
NAME	READY	STATUS	RESTARTS	AGE	IP	NODE
subflow-74c57d67d4-bqnc2	1/1	Running	0	3d	10.244.1.151	k8s-nvme-01.sdsc.optiputer.net
subflow-74c57d67d4-gd8sn	1/1	Running	0	3d	10.244.16.45	fiona.cac.washington.edu
subflow-74c57d67d4-jjx15	1/1	Running	0	3d	10.244.15.182	dtn-main.ucr.edu
subflow-74c57d67d4-j172q	1/1	Running	0	3d	10.244.11.71	siderea.ucsc.edu
subflow-74c57d67d4-19n8t	1/1	Running	0	3d	10.244.24.46	dtn2-daejeon.kreonet.net
subflow-74c57d67d4-1j8xw	1/1	Running	0	3d	10.244.10.96	fiona-dtn-1.ucsc.edu
subflow-74c57d67d4-q8mbj	1/1	Running	0	3d	10.244.19.134	fiona.nwsc.ucar.edu
subflow-74c57d67d4-x5j6r	1/1	Running	0	3d	10.244.12.100	k8s-epyc-01.sdsc.optiputer.net



Using OpenMPI on K8s: Create Pods cont.

[paolini@fiona k8s] \$ kubectl get pod -n sdsu -o=custom-columns=NODE:.spec.nodeName

NODE

k8s-nvme-01.sdsc.optiputer.net

fiona.cac.washington.edu

dtn-main.ucr.edu

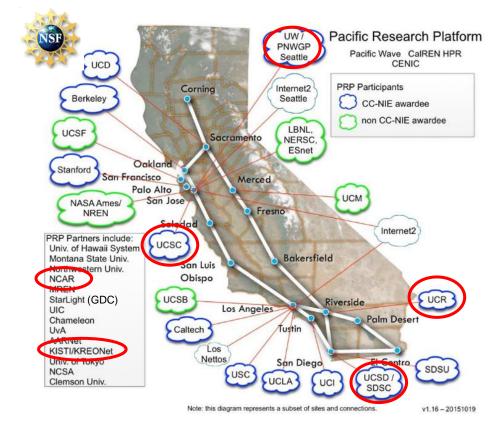
siderea.ucsc.edu

dtn2-daejeon.kreonet.net

fiona-dtn-1.ucsc.edu

fiona.nwsc.ucar.edu

k8s-epyc-01.sdsc.optiputer.net







Using OpenMPI on K8s: YAML configuration

```
apiVersion: apps/v1beta1
kind: Deployment
metadata:
  name: subflow
  namespace: sdsu
spec:
  replicas: 8
  template:
    metadata:
      labels:
        k8s-app: subflow
      containers:
      - name: subflow
        image: phusion/baseimage:0.9.19
        imagePullPolicy: IfNotPresent
        args: ["sleep", "infinity"]
        resources:
          limits:
            memory: "48Gi"
          requests:
            memory: "32Gi"
        volumeMounts:
        - name: nfs
          mountPath: /nfs
      volumes:
      - name: nfs
          server: 10.109.158.238
          path: "/"
```

Request each container to have 32GiB of allocatable memory, with an upper limit of 48GiB

Mount existing NFS (Network File System) share (nfs://10.109.158.238/) on mount-point /nfs in pod.

Persistent location of git source clone and build for k8s pods





Using OpenMPI on K8s: Initialize Cluster

```
[paolini@fiona k8s]$ ./initcluster.sh
[paolini@fiona k8s]$ cat initcluster.sh
                #!/bin/bash
                hosts=""
                n=0
                for host in `kubectl get pod -n sdsu -o=custom-columns=IP:status.podIP,NAME:.metadata.name | grep subflow
                    hosts="$hosts $host"
                    if [ $n -eq 1 ];
                    then
                        cores=`kubectl exec -it $host -n sdsu -- bash -c "grep -c ^processor /proc/cpuinfo
                        cores=`tr -dc '[[:print:]]' <<< " $cores"`
                        hosts+=$cores
                        n=0
                    else
                        n=1
                    fi
                done
                pods=`kubectl get pod -n sdsu -o=custom-columns=NAME:.metadata.name|sed -e '/NAME/d'|egrep '^subflow-
                for pod in $pods
                do
                    kubectl exec -it $pod -n sdsu -- bash -c "/nfs/subflow/k8s/setup.sh $hosts"
                done
                for pod in $pods
                    kubectl exec -it $pod -n sdsu -- bash -c "/nfs/subflow/k8s/updatekeys.sh"
                done
```

Build string array of 3tuples (IP Address, hostname, processor count) needed for MPI hostfile

Invoke setup.sh on each pod with string array as augment list

setup.sh starts ssh server on each pod

Gather ssh public keys of all pods for genenerating /etc/ssh/ssh known hosts (requires sshd be running on all pods)



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```
[paolini@fiona k8s]$ cat setup.sh
#!/bin/bash
apt-get --quiet update;
apt-get --quiet -y install screen emacs git python mlocate openmpi-bin openmpi-doc libopenmpi-dev make
valgrind g++ m4 gfortran liblapacke-dev libnetcdf-dev iputils-ping openssh-server cmake mesa-utils-extra
if [ ! -f /etc/hosts.DIST ]
then
    cp /etc/hosts /etc/hosts.DIST
fi
    id subflow >/dev/null 2>&1
then
    groupadd subflow; useradd -g subflow -d /home/subflow -m -c 'Subflow Execution' -s /bin/bash subflow
fi
if [ ! -f /root/.ssh/id rsa.pub ]
then
    echo -e 'y\n'|/usr/bin/ssh-keygen -f /root/.ssh/id rsa -t rsa -N ''
```

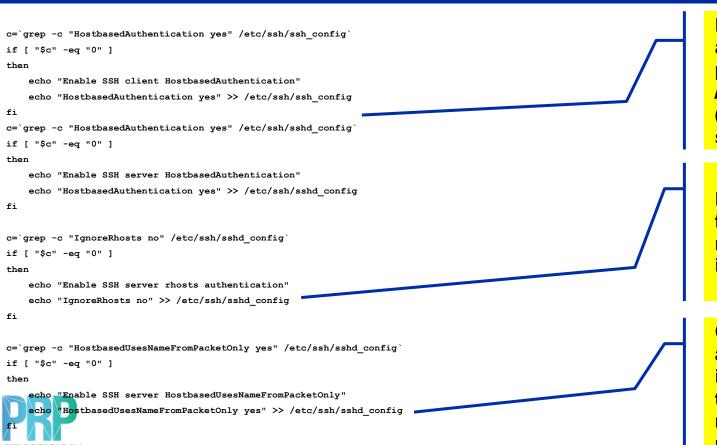
Install required packages for running MPI programs and building target

Backup original /etc/hosts file

Create fictitious user account if you don't want to run MPI processes as root

Generate protocol
version 2 ssh
authentication keys for
passwordless, public
key authentication
between pods

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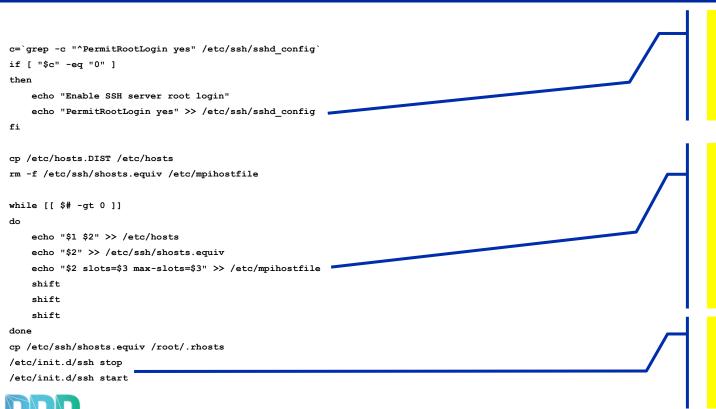


Permit user
authentication between
pods listed in
/etc/ssh/shosts.equiv
(enable in client and
server config files)

Enable authentication for root user, based on names of (trusted) pods in /root/.rhosts

Configure sshd to accept the hostname information provided in the connection itself, rather than use DNS resolution

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Permit root
authentication between
pods to allow MPI
processes to run as root

Rebuild /etc/hosts, /etc/ssh/shosts.equiv, and /etc/mpihostfile using string array argument (IP Address, hostname, processor count)

Restart ssh server on pod to enable updated configuration



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```
[paolini@fiona k8s]$ cat updatekeys.sh
#!/bin/bash
rm -f /etc/ssh/ssh known hosts
                                                            Each host gathers the
                                                            public ssh host keys of
for host in `cat /etc/ssh/shosts.equiv`
                                                            all hosts
do
    ssh-keyscan -t rsa $host >> /etc/ssh/ssh known hosts
    cp /etc/ssh/ssh known hosts /root/.ssh/known hosts
done
```





Using OpenMPI on K8s: Using a Custom Image

```
apiVersion: apps/v1beta1
kind: Deployment
metadata:
  name: subflow
  namespace: sdsu
spec:
  replicas: 8
  template:
    metadata:
      labels:
        k8s-app: subflow
    spec:
      containers:
      - name: subflow
        image: dimm0/subflow:latest
        imagePullPolicy: IfNotPresent
        args: ["sleep", "infinity"]
        resources:
          limits:
            memory: "48Gi"
          requests:
            memory: "32Gi"
        volumeMounts:
        - name: nfs
          mountPath: /nfs
      volumes:
      - name: nfs
          server: 10.109.158.238
          path: "/"
```

```
[paolini@fiona image]$ cat Dockerfile
FROM phusion/baseimage: 0.9.19
MAINTAINER Dmitry Mishin <dmishin@sdsc.edu>
RUN apt-get -y update && \
    apt-get -v install screen emacs git python mlocate openmpi-bin
openmpi-doc libopenmpi-dev make valgrind g++ m4 gfortran liblapacke-dev
libnetcdf-dev iputils-ping openssh-server cmake mesa-utils-extra
[root@fiona image] # docker pull dimm0/subflow
Using default tag: latest
latest: Pulling from dimm0/subflow
f069f1d21059: Pull complete
ecbeec5633cf: Pull complete
ea6f18256d63: Pull complete
54bde7b02897: Pull complete
a3ed95caeb02: Pull complete
ce9e695a6234: Pull complete
346026b9659b: Pull complete
d8a2ef8e4be5: Pull complete
Digest:
sha256:eb39c3151cd0ed776f6188949bc7f93a64db2eb04ab4391591c2670e0c821806
Status: Downloaded newer image for dimm0/subflow:latest
```





Using OpenMPI on K8s: Using a Custom Container



[root@fiona image] # docker images

	[rooteriona image]# docker images				
	REPOSITORY	TAG	IMAGE ID	CREATED	SIZE
	rocketchat/rocket.chat	latest	e98d34f3e7ab	16 hours ago	1.38GB
•	dimm0/subflow	latest	6aab3414d3be	44 hours ago	1.17GB
	us.gcr.io/prp-k8s/nautilus-portal	latest	e5f31849537f	7 days ago	38.7MB
	us.gcr.io/prp-k8s/nautilus-portal	<none></none>	1c868229707b	7 days ago	38.7MB
	us.gcr.io/prp-k8s/perfsonar_testpoint_h	latest	2c3315ea9b6d	10 days ago	818MB
	us.gcr.io/prp-k8s/perfsonar_testpoint_h	<none></none>	3f220a2c3190	10 days ago	818MB
	rocketchat/rocket.chat	<none></none>	587a250b077b	12 days ago	1.37GB
	us.gcr.io/prp-k8s/meshconfig	latest	ee2c46848fde	2 weeks ago	34.2MB
	sameersbn/postgresql	latest	3c0142eb3992	2 weeks ago	204MB
	robcurrie/tensorflow-gpu	latest	90592bb75ee1	4 weeks ago	4.28GB
	nvidia/k8s-device-plugin	1.10	a7c090961376	5 weeks ago	63.1MB
	gcr.io/google_containers/kube-proxy-amd64	v1.11.0	1d3d7afd77d1	5 weeks ago	97.8MB
	us.gcr.io/prp-k8s/prp-tuner	latest	62dbae4fe2e1	2 months ago	52.2MB
	gcr.io/runconduit/proxy	v0.4.1	fd47f6c1933d	3 months ago	111MB
	<pre>gcr.io/runconduit/proxy-init</pre>	v0.4.1	60eb863cee15	3 months ago	105MB
	quay.io/calico/node	v3.1.1	d94b64ac210d	3 months ago	248MB
	quay.io/calico/cni	v3.1.1	482f47df27e2	3 months ago	68.8MB
	us.gcr.io/prp-k8s/perfsonar_testpoint_h	<none></none>	2fafb9b60974	3 months ago	784MB
	us.gcr.io/prp-k8s/perfsonar_testpoint	latest	fec1df3352a3	3 months ago	784MB
	quay.io/coreos/kube-rbac-proxy	v0.3.0	543e2018dcac	4 months ago	40.2MB
	rook/rook	v0.7.1	ee1353c748c0	4 months ago	435MB
	traefik	1.5.3	8c72b944d569	5 months ago	49.6MB
	k8s.gcr.io/pause	3.1	da86e6ba6ca1	7 months ago	742kB
	quay.io/prometheus/node-exporter	v0.15.2	ff5ecdcfc4a2	8 months ago	22.8MB





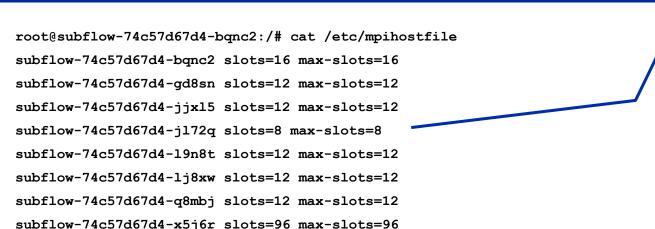
Using OpenMPI on K8s: Using a Custom Container

```
[paolini@fiona k8s]$ kubectl create -f comet.vaml
deployment.apps "subflow-comet" created
[paolini@fiona k8s]$ ./initcluster.sh sdsu-comet
Generating public/private rsa key pair.
Your identification has been saved in /root/.ssh/id rsa.
Your public key has been saved in /root/.ssh/id rsa.pub.
The key fingerprint is:
SHA256:cOvHueOgYOI/x0zWRSvEwnvT6lgnNEmcFTNMWZ6pPdM root@subflow-comet-6788dc75d5-778bz
The key's randomart image is:
+---[RSA 2048]----+
    *=0*Booo
     B.B=.=..
   . +o.B.=
    00=.0 * 0
    ++ B S E
+----[SHA256]----+
Enable SSH client HostbasedAuthentication
Enable SSH server HostbasedAuthentication
Enable SSH server rhosts authentication
Enable SSH server HostbasedUsesNameFromPacketOnly
Enable SSH server root login
 * Stopping OpenBSD Secure Shell server sshd
 * Starting OpenBSD Secure Shell server sshd
# subflow-comet-6788dc75d5-778bz:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-p8z2q:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-pgw78:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-q6wvc:22 SSH-2.0-OpenSSH_7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-778bz:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-p8z2g:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-pqw78:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-g6wyc:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-778bz:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-p8z2g:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-pgw78:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-q6wvc:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-778bz:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-p8z2g:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
# subflow-comet-6788dc75d5-pgw78:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4
```

subflow-comet-6788dc75d5-q6wvc:22 SSH-2.0-OpenSSH 7.2p2 Ubuntu-4ubuntu2.4

[OK]





Number of *slots* is the number of processors available to the pod





Using OpenMPI on K8s: MPI Ring Test

```
[paolini@fiona k8s]$ more RUN
#!/bin/bash
KUBE NAMESPACE=sdsu
export KUBE NAMESPACE
MPI CLUSTER NAME=subflow
export MPI CLUSTER NAME
case "$1" in
    ring)
        kubectl -n $KUBE NAMESPACE exec -it $MPI CLUSTER NAME-74c57d67d4-bqnc2 -- mpirun --allow-run-as-root \
            --hostfile /etc/mpihostfile \
                                                                                  Direct Open MPI to use TCP-
            --mca btl tcp,self \ -
                                                                                 based communications over
            -n 8 -npernode 1 --bind-to core
                                                                                  IP interfaces. Modular
            /nfs/subflow/k8s/ring
                                                                                 Component Architecture
        ;;
                                                                                 (MCA) parameter: byte
                                                                                 transfer layer (BTL)
                                                                                  Bind each MPI process to a
                                                                                  core.
                                                                                                      UNIVERSITY
```

Using OpenMPI on K8s: MPI Ring Test cont.

```
[paolini@fiona k8s]$ ./RUN ring
process 0 of 8 (on subflow-74c57d67d4-bqnc2)
process 2 of 8 (on subflow-74c57d67d4-jjxl5)
process 1 of 8 (on subflow-74c57d67d4-gd8sn)
process 6 of 8 (on subflow-74c57d67d4-q8mbj)
process 5 of 8 (on subflow-74c57d67d4-lj8xw)
process 3 of 8 (on subflow-74c57d67d4-jl72q)
process 7 of 8 (on subflow-74c57d67d4-x5j6r)
process 4 of 8 (on subflow-74c57d67d4-l9n8t)
token= 8192.0000001, matches TIMES_AROUND*nprocs (things look ok).
doing long send...
...passed long send test.
[paolini@fiona k8s]$
```

Pass a (float) token around the "ring" of 8 processes 2¹⁰ times. Each process increments the token by +1 when the token is received from its "left" process neighbor, prior to sending the token to its "right" neighbor.





Benchmarking PRP K8s with LINPACK

- HPL LINPACK Benchmark: commonly used performance metric to evaluate a cluster's performance
- Project URL: https://www.top500.org/project/linpack/
- FAQs:

http://www.netlib.org/utk/people/JackDongarra/faq-linpack.html http://www.netlib.org/benchmark/hpl/faqs.html

- Download URL: http://www.netlib.org/benchmark/hpl/
- Repeatedly solves a linear system of N equations using LU decomposition with partial row pivoting, giving a performance metric R.
- We want to identify R_{max} for a particular N (with respect to a chosen number of processes and other parameters).





Benchmarking PRP K8s with LINPACK cont.

- Solve a linear system Ax = b of order N
- Compute LU factorization of A with row partial pivoting (operations stored in P) of the n x (n+1) = N coefficient matrix [A b]
- [A b] = [[L,U] y]
- $PA=LU \rightarrow LUx=Pb$
- Solve Ly=Pb for y, using forward substitution, then solve Ux=y for x using backward substitution
- O(2/3n3) FLOP
- $n \times (n+1)$ coefficient matrix logically partitioned into $NB \times NB$ blocks, that are cyclically assigned onto a $P \times Q$ process grid.





Benchmarking PRP K8s with LINPACK: Block LU

• Partition A into blocks (sub-matrices) of size NBxNB and distribute to each process:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Partitioning yields the following block equations for partitioned LU factorization:

$$\begin{bmatrix} A_{11} = L_{11}U_{11} & A_{12} = L_{11}U_{12} & A_{13} = L_{11}U_{13} \\ A_{21} = L_{21}U_{11} & A_{22} = L_{21}U_{12} + L_{22}U_{22} & A_{23} = L_{21}U_{13} + L_{22}U_{23} \\ A_{31} = L_{31}U_{11} & A_{32} = L_{31}U_{12} + L_{32}U_{22} & A_{33} = L_{31}U_{13} + L_{32}U_{23} + L_{33}U_{33} \end{bmatrix}$$

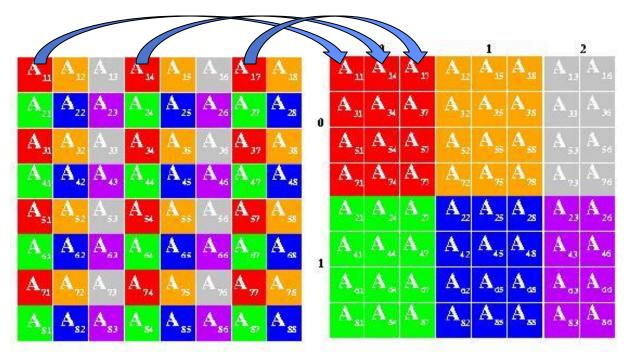
 Block size, NB, will affect performance. We can experiment and see how changing the block size yields different performance metrics R (Gflops) on the PRP K8s cluster.





Benchmarking PRP K8s with LINPACK: Processor Grid (PxQ)

• Example: $P \times Q = 2 \times 3$; each color corresponds to an MPI process assignment.





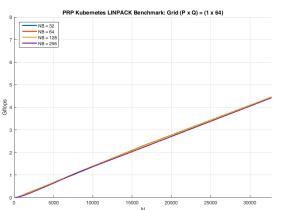


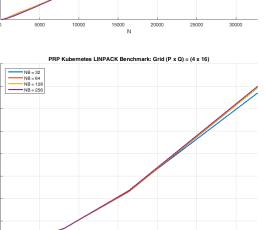
Benchmarking PRP K8s with LINPACK: Launching HPL

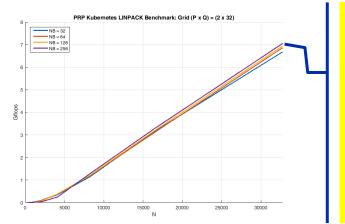
```
[paolini@fiona k8s]$ more RUN
#!/bin/bash
KUBE NAMESPACE=sdsu
export KUBE NAMESPACE
MPI CLUSTER NAME=subflow
export MPI CLUSTER NAME
case "$1" in
    hpl)
        kubectl -n $KUBE NAMESPACE exec -it $MPI CLUSTER NAME-74c57d67d4-bqnc2 -- mpirun --allow-run-as-root \
            --hostfile /etc/mpihostfile \
                                                                                  Max is 8 nodes with 8
            --mca btl tcp,self \
                                                                                  processes per node (64)
            --display-map \
                                                                                  HPL application is xhpl
            -n 64 -npernode 8 --bind-to core:overload-allowed
            sh -c 'cd /nfs/hpl-2.2/bin/k8s; ./xhpl > xhpl.out 2 \times 1'
        ;;
```

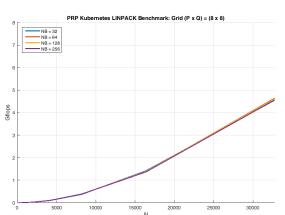
San Diego State University

Benchmarking PRP K8s with LINPACK: Results









- Blocksize = 256
- (PxQ)=(2x32)
- Panel fact.: Crout
- Recursive Panel fact.: Crout
- 7.056 Gflops



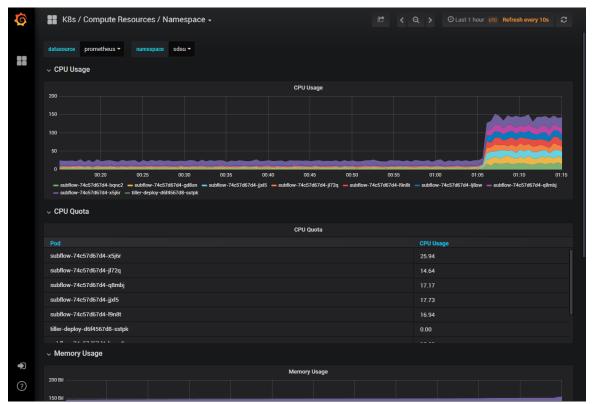
15000

25000



Benchmarking PRP K8s with LINPACK: Grafana Monitoring

- https://grafana.nautilus.optiputer.net/
- https://grafana.nautilus.optiputer.net/d/4r_GftHmz/k8s-compute-resources-namespace?refresh=10s&orgld=1&var-datasource=prometheus&var-namespace=sdsu







Benchmarking PRP K8s with LINPACK: HPL.dat

```
root@subflow-74c57d67d4-bqnc2:/nfs/hpl-2.2/bin/k8s# cat HPL.dat
HPLinpack benchmark input file
Innovative Computing Laboratory, University of Tennessee
HPL.out
             output file name (if any)
             device out (6=stdout,7=stderr,file)
            # of problems sizes (N)
128 256 512 1024 2048 4096 8192 16384 32768 Ns
             # of NBs
32 64 128 256
             PMAP process mapping (0=Row-,1=Column-major)
             # of process grids (P x Q)
64 32 16 8
             Qs
16.0
             threshold
             # of panel fact
         PFACTs (0=left, 1=Crout, 2=Right)
             # of recursive stopping criterium
2 4
             NBMINs (>= 1)
             # of panels in recursion
             NDIVs
             # of recursive panel fact.
         RFACTs (0=left, 1=Crout, 2=Right)
             # of broadcast
             BCASTs (0=1rg,1=1rM,2=2rg,3=2rM,4=Lng,5=LnM)
             # of lookahead depth
             DEPTHs (>=0)
             SWAP (0=bin-exch,1=long,2=mix)
             swapping threshold
             L1 in (0=transposed,1=no-transposed) form
             U in (0=transposed,1=no-transposed) form
             Equilibration (0=no,1=yes)
```

memory alignment in double (> 0)

11 values of *N*, from 2⁷ to 2¹⁵ such that N is NB aligned

Test with 4 block sizes NB.

"good" block sizes are almost
always in the [32 .. 256]
interval. -- Jack Dongarra

Test with 4 processor grid configurations with Q ≥ P and PQ = 64.

Crout's algorithm typically performs better than left- and right- looking panel methods



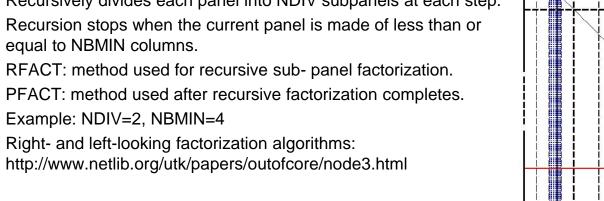


Benchmarking PRP K8s with LINPACK: Panel Factorization

```
PFACTs (0=left, 1=Crout, 2=Right)
# of recursive stopping criterium
NBMINs (>= 1)
# of panels in recursion
NDTVs
# of recursive panel fact.
RFACTs (0=left, 1=Crout, 2=Right)
```

- Crout matrix decomposition: returns lower triangular matrix and a unit upper triangular matrix (Prescott Durand Crout).
- Recursively divides each panel into NDIV subpanels at each step.
- equal to NBMIN columns.

- Right- and left-looking factorization algorithms:







Benchmarking PRP K8s with LINPACK: Right Looking

• Suppose we have factored the blocks in the first row and column of *A*:

$$A = LU = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{13} & U_{14} & U_{15} \end{bmatrix}$$
 Factored blocks

• Since the remaining green block submatrix is independent of the yellow completed region, the green

region can be factored in isolation:
$$\begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix} \Rightarrow \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = P \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix} = P \begin{bmatrix} L_{11}U_{11} & L_{11}U_{12} \\ L_{21}U_{11} & L_{21}U_{12} + L_{22}U_{22} \end{bmatrix}$$

• Factoring the first column of the block submatrix yields L_{11} , L_{21} , and U_{11} . We can solve for U_{12} .

$$A_{12} = L_{11}U_{12} \rightarrow U_{12} = L_{11}^{-1}A_{12}$$

• Finally, we update block A_{22} and repeat this process:

Remaining block submatrix to factor

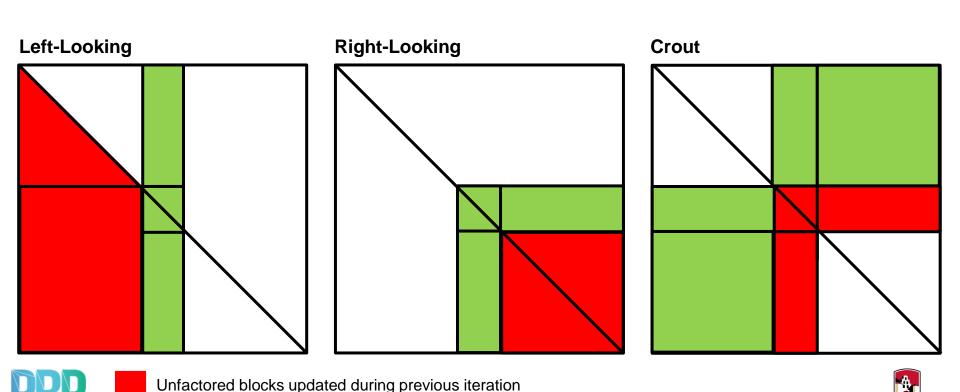
$$A_{22} = L_{21}U_{12} + L_{22}U_{22} \Rightarrow L_{22}U_{22} = \underbrace{A_{22} - L_{21}U_{12}}_{12}$$

Perform LU factorization to solve for L_{22} and U_{22}





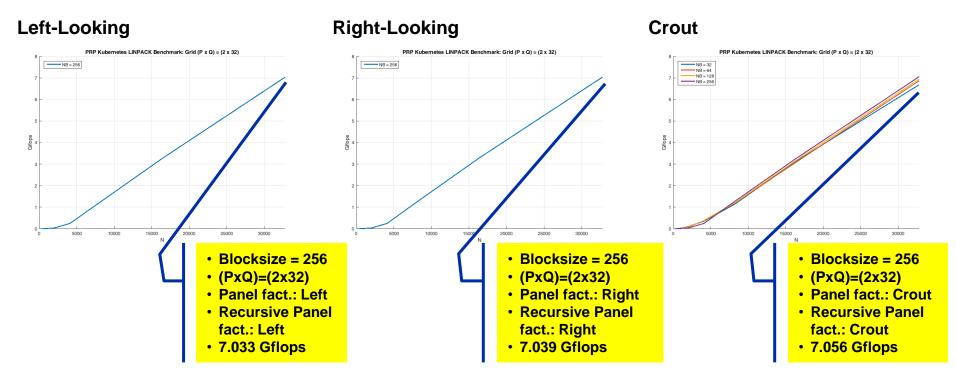
Benchmarking PRP K8s with LINPACK: Memory Access



San Diego State University

Blocks currently being factored

Benchmarking PRP K8s with LINPACK: Comparison







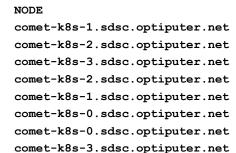
Benchmarking PRP K8s with LINPACK: SDSC Comet

- Comet: eXtreme Science and Engineering Discovery Environment (XSEDE) cluster
- 1,944 nodes, 24 cores/node → 46,656 cores (Haswell)
- FDR InfiniBand Interconnect (MPI latency 1.03-1.97 μs)
- K8s on Comet configured by Dmitry Mishin <dmishin@ucsd.edu>



[paolini@fiona k8s]\$ kubectl create -f comet.yaml
[paolini@fiona k8s]\$ kubectl get pods -o wide -n sdsu-comet

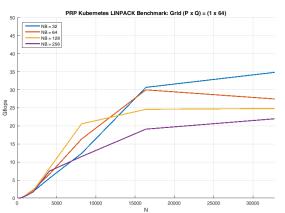
[pastinicizona nob] + nabecot get pour o wide in basa comes						
NAME	READY	STATUS	RESTARTS	AGE	IP	
subflow-comet-7c8b94f6b6-2hpr6	1/1	Running	0	1h	10.244.66.4	
subflow-comet-7c8b94f6b6-b42tb	1/1	Running	0	1h	10.244.68.4	
subflow-comet-7c8b94f6b6-gnzgr	1/1	Running	0	1h	10.244.72.4	
subflow-comet-7c8b94f6b6-htxxd	1/1	Running	0	1h	10.244.68.3	
subflow-comet-7c8b94f6b6-rwc48	1/1	Running	0	1h	10.244.66.3	
subflow-comet-7c8b94f6b6-th6qd	1/1	Running	0	1h	10.244.67.5	
subflow-comet-7c8b94f6b6-v7cpn	1/1	Running	0	1h	10.244.67.4	
subflow-comet-7c8b94f6b6-vfwkh	1/1	Running	0	1h	10.244.72.3	







Benchmarking PRP K8s with LINPACK: Comet Results



PRP Kubernetes LINPACK Benchmark; Grid (P x Q) = (4 x 16)

15000

25000

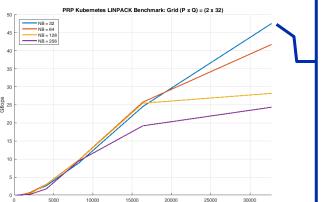
NB = 32

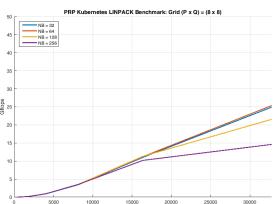
- NB = 64

- NB = 128

- NB = 256









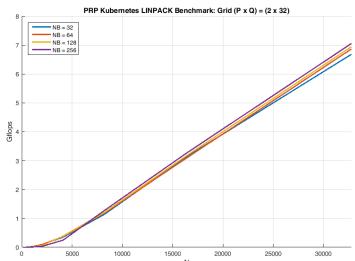
- (PxQ)=(2x32)
- Panel fact.: Crout
- Recursive Panel fact.: Crout
- 47.54 Gflops





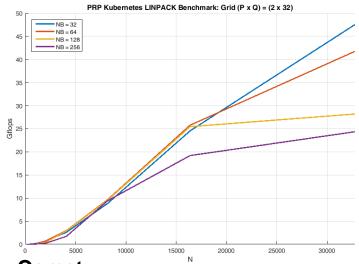
Benchmarking PRP K8s with LINPACK: Pros and Cons

- Slower numerical performance; however:
- Unlimited walltime (so far)
- No formal approval required (so far)
- Required limited administrator intervention



PRP Network

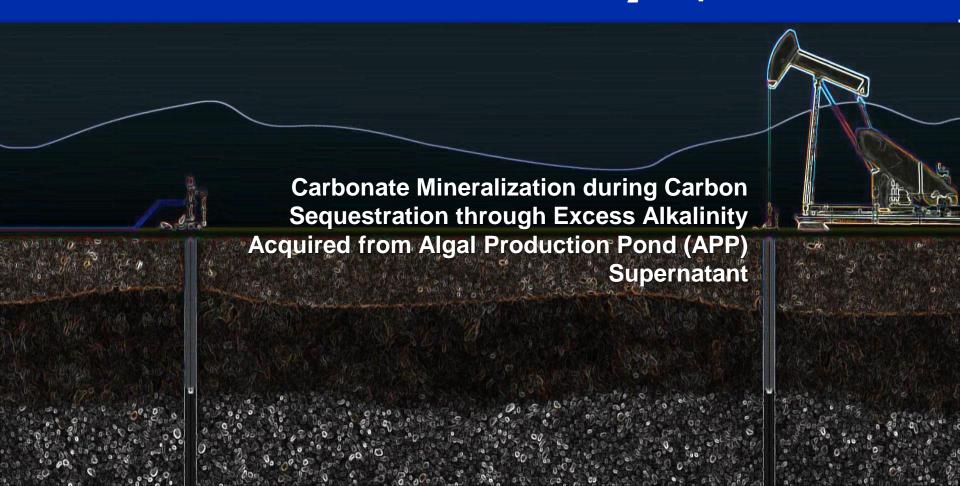
- Superior numerical performance; however:
- Maximum walltime for Comet jobs is 2 days
- Requires XSEDE allocation via approved proposal
- Idle VMs waste allocation SUs
- Required administrator intervention (Dmitry Mishin)





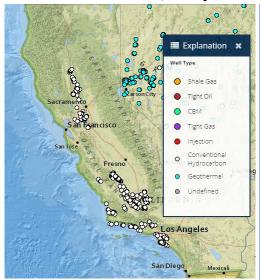


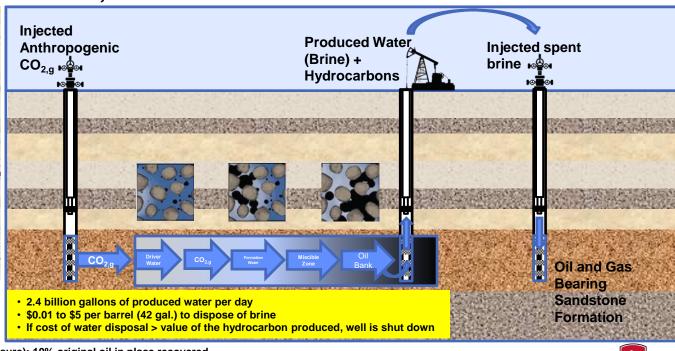
PRP Science Driver: Numerical CO₂ Sequestration



Synergistic CCS, EOR, and Biofuel Generation

 Exploring Synergies in an Enhanced Waste Management System that Produces Biofuel, Recovers Oil, Sequesters Carbon, and Treats Wastewater





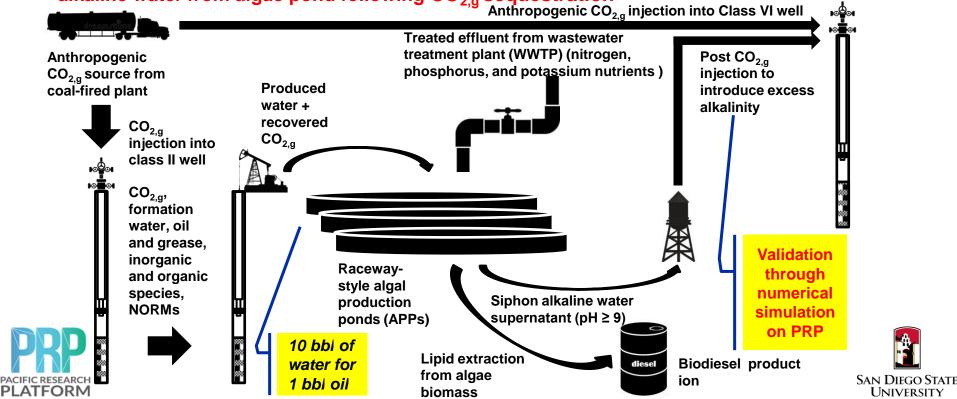


- Primary recovery (natural pressure): 10% original oil in place recovered
- Secondary recovery (water injection): 20% 40% original oil recovered
- Tertiary EOR (CO₂ injection): 30% to 60%
- US: 114 CO₂ injection projects, 2 billion cubic feet of CO₂ produce 280,000 barrels of oil per day
- · Water management: majority (55%) of well operating expenses



Synergistic CCS, EOR, and Biofuel Generation

Novel idea: proposed pilot project to (1) reuse produced water as medium for algal growth to eliminate cost of disposal and (2) increase rate of carbonate mineralization through injection of alkaline water from algae pond following CO_{2,g} sequestration
 Anthropogenic CO_{2,g} injection into Class VI well



"Locking" Captured CO_{2,q} in Place

- Ideal storage mechanism: transform atmospheric CO_{2,g} into subsurface solid phase carbonate mineral (e.g. CaCO₃, MgCO₃, CaMg(CO₃)₂)
- Neutralization of carbonic acid by alkaline earth metals (Ca²⁺,Mg²⁺)
- Solid phase storage: lower risk from upward gas migration and release into atmosphere



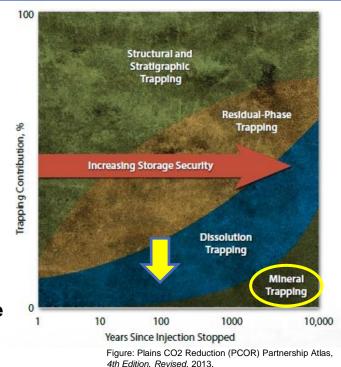
- Chemical reaction among dissolved CO_{2,g} and mineral solutes in formation water
- "Mineral Trapping": typically occurs 100 to 10,000 years after injection
- How can we achieve rapid mineralization to lower risk?





How CO₂ is Sequestered in Rock

- Structural and Stratigraphic Trapping:
 CO_{2(g)} rises through porous rock until it reaches impermeable cap rock (e.g. shale)
- Residual-Phase Trapping
 ScCO₂ stored as residual droplets in (sandstone) rock pore space
- Dissolution Trapping
 CO₂ rich water sinks to the bottom of a formation because ρ_{H20(I)+CO2(q)+H2CO3(aq)}>ρ_{H20(I)}
- (Ideally) Mineral Trapping
 Reaction between dissolved CO₂ in formation water with surrounding solutes to create carbonate minerals (e.g. CaCO₃, MgCO₃, CaMg(CO₃)₂)



Question: how might we **stimulate** a reservoir, post CO₂ injection, to induce carbonate mineral formation **sooner**?





Problem: Calcite-CO₂ Equilibrium

CO₂ solvation

$$CO_{2,g} \rightleftharpoons CO_{2,aq}$$

$$CO_{2,aq} + H_2O \rightleftharpoons H_2CO_{3,aq}$$

$$H_2CO_{3,aq} \rightleftharpoons HCO_3^- + H^+$$

$$HCO_3^- \rightleftharpoons CO_3^{2-} + H^+$$

$$H_2O \rightleftharpoons H^+ + OH^-$$

$$(1)$$

$$CaCO_{3,s} + H_2CO_{3,aq} \Longrightarrow Ca^{+2} + 2HCO_3^-$$
 (2)

Le Chatelier's principle

- Increase [CO_{2,q}] → (1,2) shifts to right → CaCO_{3,s} dissolves
- Decease $[CO_{2,q}] \rightarrow (1,2)$ shifts to left $\rightarrow CaCO_{3.s}$ precipitates
- Problem: how to store CO_{2.a} and (rapidly) precipitate calcite?





Carbonate Alkalinity

Carbonate alkalinity A_c

$$A_C = [HCO_3^-] + 2[CO_3^{2-}] + [OH^-] - [H^+]$$

Alkalinity A_C increases through carbonate dissolution:

$$CaCO_{3s} \rightleftharpoons Ca^{+2} + CO_{3s}^{-1}$$

• Carbonate precipitation thus reduces A_C and increases P_{CO2} by converting bicarbonate to CO_2 :

$$CaCO_{3,s} + H_2CO_{3,aq} \rightleftharpoons Ca^{+2} + 2HCO_3^{-1}$$
 $CO_{2,aq} + H_2O \rightleftharpoons H_2CO_{3,aq}$
 $CO_{2,g} \rightleftharpoons CO_{2,aq}$







Excess Alkalinity and Hardness

$$A_C = [HCO_3^-] + 2[CO_3^{2-}] + [OH^-] - [H^+]$$

Idea 1: introduce external source of alkalinity as OH⁻

$$CaCO_{3,s} + H_2O \Longrightarrow Ca^{+2} + HCO_3^- + OH^-$$

• Idea 2: introduce external source of hardness as Ca²⁺ and Mg²⁺

 Can we show, through modeling and numerical simulation on a PRP cluster, increased carbon sequestration as carbonate mineralization from underground injection of excess alkalinity and hardness?





Excess Alkalinity Source

 Reduction of carbonic acid to formaldehyde and hydroxide production from photosynthesis

$$H_2CO_3 \rightleftharpoons CH_2O + O_2$$
 $HCO_3^- + H_2O \rightleftharpoons CH_2O + O_2 + OH^ CO_3^{2-} + 2H_2O \rightleftharpoons CH_2O + O_2 + 2OH^-$



High rate algal pond (HRAP) CO₂ addition sump (Craggs et al., 2014)



Seambiotic's commercial algae farm in China uses CO₂ captured from power plant flue gas for photosynthesis





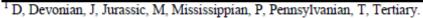
Produced Water Chemistry

 Produced waters of different geologic ages compared with average concentrations in 35% seawater (Collins, A.G. 1975, Geochemistry of Oilfield Waters, Elsevier Scientific Publishers, New York. 496 pp)

Source of hardness

Element/Ion	Seawater	Produced Water		
		Highest	Range of Mean	
		Concentration (Age ¹)	Concentrations	
Salinity	35,000		<5000 - >300,000,000	
Sodium	10,760	120,000 (J)	23,000 - 57,300	
Chloride	19,353	270,000 (P)	46,100 - 141,000	
Calcium	416	205,000 (P)	2530 - 25,800	
Magnesium	1294	26,000 (D)	530 – 4300	
Potassium	387	11,600 (D)	130 – 3100	
Sulfate	2712	8400 (T)	210 - 1170	
Bromide	87	6000 (J)	46 – 1200	
Strontium	0.008	4500 (P)	7 – 1000	
Ammonium		3300 (P)	23 – 300	
Bicarbonate	142	3600 (T)	77 – 560	
Iodide	167	1410 (P)	3 – 210	
Boron	4.45	450 (T)	8 – 40	
Carbonate		450 (M)	30 – 450	
Lithium	0.17	400 (J)	3 – 50	

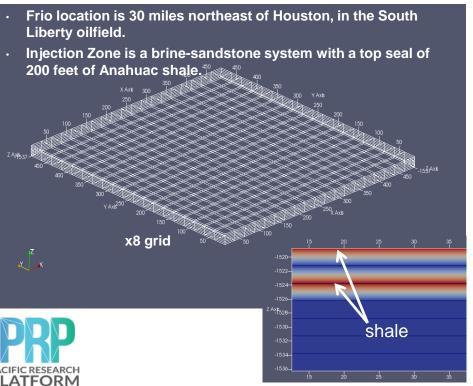






Modeled Reservoir based on Frio Formation

- 0.5 km x 0.5 km x 17.5 m volume
- Three sandstone layers separated by two shale layers
- 4 grid configurations: 20x20x{8, 16, 32, 64}





UNIVERSITY

Molar Concentration of CO_{2,aq}

Mineralogical Composition

Shale (impervious caprock, κ=7x10⁻³ mD)

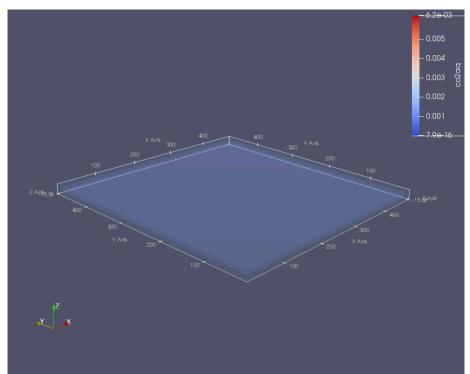
Mineral	Volume Fraction
Calcite (CaCO ₃)	0.20
Quartz (SiO ₂)	0.28
k-feldspar (KAISi ₃ O ₈)	0.01
Illite $((K,H_3O)(Al,Mg,Fe)_2(Si,Al)_4O_{10}[(OH)_2)$	0.41

Sandstone (Semi-pervious, κ = 17.2 mD)

Mineral	Volume Fraction
Quartz (SiO ₂)	0.53
k-feldspar (KAlSi ₃ O ₈)	0.11 (subarkose)

10 mM CO₂-rich Injectant

Species	Molarity
H+	2.6e-06
H2O	1
CO _{2aq} /HCO ₃ -/CO ₃ complex	0.01





Modeling Porous Media Flow

Darcy's Law (fluid flux [g/(cm² s)])

$$v_{\text{res}} = -\frac{K}{\mu} \rho_0 \left(\nabla p_{\text{res}} - \rho g \nabla z \right)$$

Fluid Conservation (continuity equation)

$$\partial_t(\rho_0\phi) + \nabla \cdot v_{\text{res}} = q_{\text{res}}$$

1				
	V	Fluid mass flux [g/(cm ² s)]	g	Acceleration due to gravity [cm/s ²]
	K	Permeability tensor of porous medium	abla z	Gravitational direction unit vector [cm]
		[1/cm]	φ	Porosity of medium (dimensionless)
	μ	Dynamic viscosity [g/(cm s)]	q	Fluid source density [g/(cm ³ s)]
	ρ	Fluid density [g/cm ³]		
	p	Fluid pressure [gfsc]		





Poroelastic Models

Strain: the symmetric gradient of displacement

$$\epsilon(u) = \frac{1}{2} \Big(\nabla u + \nabla u^T \Big)$$

Rock Stress

$$\sigma(u) = \lambda (\nabla \cdot u) I + 2G\epsilon(u)$$

Poroelastic Stress

$$\sigma_{por}(u, p) = \sigma(u) - \alpha pI$$

- λ First Lame parameter [gfsc]
- G Second Lame parameter (Shear modulus) [gfsc]
- α Biot-Willis coefficient (dimensionless)





Solute Mass Transport Model

Elemental Conservation of Mass per Unit Volume

$$\frac{\partial e_{\beta}}{\partial t} = \sum_{\alpha=1}^{N_{\alpha}} \left[\phi D_{\alpha} \nabla^{2} c_{\alpha} - \phi \nabla \cdot (c_{\alpha} u) \right] - \sum_{\gamma=1}^{M} \nu_{\beta \gamma} A_{\gamma} G_{\gamma}$$
(Park, 2014)

Elemental mass rate of change term: rate of increase of

concentration of a solute atom β in a

fluid element

Diffusive term: net rate of increase of solute activity in a fluid element due to diffusive forces Advective term:
net rate of flow of
solute activity out
of a fluid element
due to advective
forces

Source term: net rate of the increase or decrease of a mineral in a fluid element due to chemical kinetics

• Evolution of chemical elemental mass depends on mass-transfer from diffusive and advective forces as well as the precipitation and dissolution of minerals governed by kinetic reaction rates

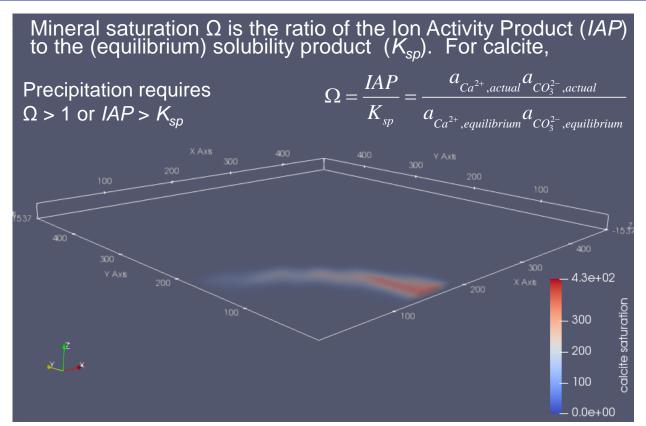
- e elemental mass in fluid [mol/cm³]
- D diffusion coefficient [cm²/s]
- c solute concentration [mol/cm³]
- α aqueous solute species index
- u fluid velocity [cm/s]

- ϕ porosity [unitless]
- β solute atom index
 - y mineral index
- G mineral reaction rate [mol/(cm² s)]
- A mineral spec. surface area [cm²/cm³]





Calcite Saturation







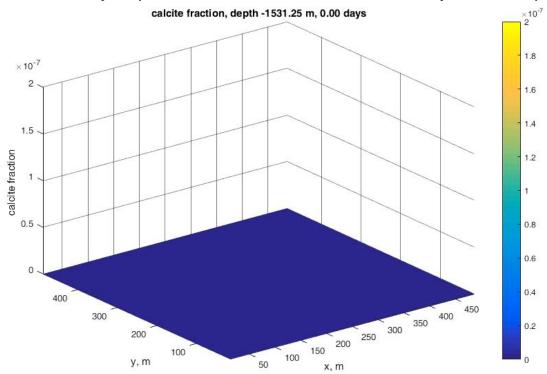
Calcite saturation after 1 day of produced water solution #1

PRP Results: 1 kg/s, 5 years Equilibrium

Calcite fraction after 1 day of produced water solution #1 and 5 years of equilibrium (2e-7)

Solute	Concentration
OH-	1e-2 M, pH = 12
Ca++	2530 ppm
Na+	23000 ppm
Mg ⁺⁺	530 ppm
CI-	46100 ppm

High alkalinity, low salinity, low hardness test water solution #1





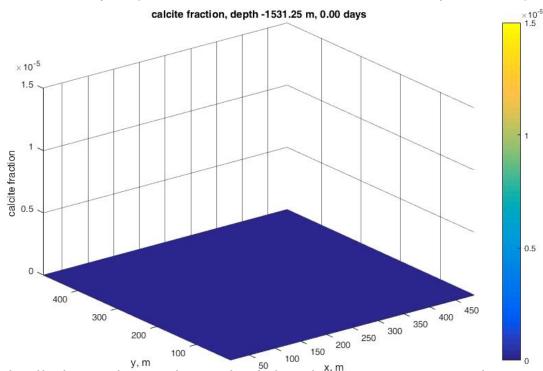


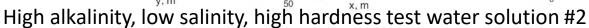
Comet Results: High Hardness, 1 kg/s, 1 year Equilibrium

Calcite fraction after 1 day of produced water solution #2 and 1 year of equilibrium (1.5e-5)

Solute	Concentration
OH-	1e-2 M, pH = 12
Ca++	25800 ppm
Na+	23000 ppm
Mg ⁺⁺	4300 ppm
Cl-	46100 ppm

High alkalinity, low salinity, high hardness test water solution #2









PRP Science Driver: Running Subflow on Comet

```
demo-comet)
        KUBE NAMESPACE=sdsu-comet
        export KUBE NAMESPACE
        HEADNODE=$MPI_CLUSTER_NAME-comet-6788dc75d5-pgw78
        export HEADNODE
        kubectl -n $KUBE NAMESPACE exec -it $HEADNODE -- mpirun --allow-run-as-root \
            --hostfile /etc/ssh/shosts.equiv \
            --display-map -n 4 -npernode 1 \
             --mca btl tcp,self \
            sh -c 'cd /nfs/subflow/exe; GRID=FrioTHMC20x20x16; export GRID; ./subflow -omp 4 -i demo.sdb -g $GRID.grid -k $GR
ID > OUTPUT/$GRID-demo.out 2>&1'
        ;;
[paolini@ps-40g k8s]$ ./RUN.sh demo-comet
Data for JOB [57040,1] offset 0
                            JOB MAP
 Data for node: subflow-comet-6788dc75d5-778bz Num slots: 24
                                                                Max slots: 0
                                                                                Num procs: 1
        Process OMPI jobid: [57040,1] App: 0 Process rank: 0
 Data for node: subflow-comet-6788dc75d5-p8z2g Num slots: 24
                                                                Max slots: 0
                                                                                Num procs: 1
        Process OMPI jobid: [57040,1] App: 0 Process rank: 1
 Data for node: subflow-comet-6788dc75d5-pgw78 Num slots: 24
                                                                Max slots: 0
                                                                                Num procs: 1
        Process OMPI jobid: [57040,1] App: 0 Process rank: 2
 Data for node: subflow-comet-6788dc75d5-q6wvc Num slots: 24
                                                                Max slots: 0
                                                                                Num procs: 1
        Process OMPI jobid: [57040,1] App: 0 Process rank: 3
```





Concluding Remarks

Simulations show it may be feasible to induce early carbonate mineral formation through alkaline produced water injection following CO₂ injection

Further Work

Intellectual merit

- What species of algae (e.g. *Dunaliella salina*, *Botryococcus braunii*, *Chlorella*, *Dunaliella tertiolecta*, *Gracilaria*, *Pleurochrysis carterae*, *Sargassum*) thrive best, if at all, in produced waters from different oil and gas sites?
- How does produced water salinity affect supernatant alkalinity?
- How do trace amounts of naturally occurring radioactive materials (NORMs) such as Radium affect algae growth?
 Radium isotopes present in produced water and barite (barium sulfate) scale are ²²⁶Ra and ²²⁸Ra.
- How does alkaline supernatant injection increase the mineralization rates of calcite, magnesite, and dolomite following CO_{2,q} injection?
- What is the optimal ratio of produced water to waste water to promote algae growth?

Broader Impacts

- Cost of produced water disposal is a significant factor in determining the profit of oil and gas production. Reducing or eliminating cost of disposal by diverting produced water to algal production ponds (APPs) for biofuel generation reduces per barrel cost, which lowers fuel cost borne by consumer.
- Transforming atmospheric CO_{2,g} to subsurface solid phase carbonate mineral (e.g. CaCO₃, MgCO₃, CaMg(CO₃)₂) through mineral trapping is a superior technology to mitigate harmful climate-change from fossil fuel combustion. Geologic mineral trapping processes typically occur at the 100yr time scale. Reducing this time reduces risk of unwanted CO_{2,g} subsurface transport and leakage back to the atmosphere.





Questions?



