Felippa's Gauss Integration Rules for Hexahedra

Line Gauss Quadrature Module

```
In[1]:= LineGaussRuleInfo[{rule_, numer_}, point_] :=
    Module[{g2 = {-1, 1}/Sqrt[3], w3 = {5/9, 8/9, 5/9}, g3 = {-Sqrt[3/5], 0, Sqrt[3/5]},
      w4 = {(1/2)-Sqrt[5/6]/6, (1/2)+Sqrt[5/6]/6, (1/2)+Sqrt[5/6]/6, (1/2)-Sqrt[5/6]/6},
      g4 = {-Sqrt[(3+2*Sqrt[6/5])/7], -Sqrt[(3-2*Sqrt[6/5])/7], Sqrt[(3-2*Sqrt[6/5])/7],
        Sqrt[(3+2*Sqrt[6/5])/7]}, g5 = {-Sqrt[5+2*Sqrt[10/7]], -Sqrt[5-2*Sqrt[10/7]],
        0, Sqrt[5-2*Sqrt[10/7]], Sqrt[5+2*Sqrt[10/7]]}/3, w5 =
       {322-13*Sqrt[70], 322+13*Sqrt[70], 512, 322+13*Sqrt[70], 322-13*Sqrt[70]}/900,
      i = point, p = rule, info = {Null, 0}}, If[p == 1, info = {0, 2}];
    If[p == 2, info = {g2[[i]], 1}];
    If[p == 3, info = {g3[[i]], w3[[i]]}];
    If[p == 4, info = {g4[[i]], w4[[i]]}];
    If[p == 5, info = {g5[[i]], w5[[i]]}];
    If[numer, Return[N[info, 20]], Return[Simplify[info]]];];
```

Hexahedra Gauss Quadrature Module

```
In[2]:= HexaGaussRuleInfo[{rule_, numer_}, point_] := Module[
    {Ξ, H, M, p1, p2, p3, p12, i, j, jj, k, m, w1, w2, w3, info = {{Null, Null, Null}, 0}},
    If[Length[rule] == 3, {p1, p2, p3} = rule, p1 = p2 = p3 = rule];
    If[Length[point] == 3, {i, j, k} = point, m = point;
     p12 = p1*p2; k = Floor[(m-1)/p12]+1; jj = m-p12*(k-1);
     j = Floor[(jj-1)/p1]+1; i = jj-p1*(j-1)];
    {Ξ, w1} = LineGaussRuleInfo[{p1, numer}, i];
    {H, w2} = LineGaussRuleInfo[{p2, numer}, j];
    {M, w3} = LineGaussRuleInfo[{p3, numer}, k];
    info = {{Ξ, H, M}, w1*w2*w3};
    If[numer, Return[N[info, 20]], Return[Simplify[info]]];];
```

Results

In[3]:= ```
ToRSTW[info_] := Module[{r, s, t, w},
    {{r, s, t}, w} = info;
    Return[{r, s, t, w}];];
```

## Rule 64

In[4]:= ```
Table[ToRSTW[HexaGaussRuleInfo[{4, True}, i]], {i, 64}]
```

Out[4]= {{-0.86113631159405257522, -0.86113631159405257522,
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