

Bezier curves

Quadratic

$$\text{In}[1]:= \text{Bezier2}[t_]:= (1-t)^2 Qa + 2 t (1-t) Qc + t^2 Qb$$

$$\text{In}[2]:= \text{Solve}[\text{Bezier2}[0] == xa \&\& \text{Bezier2}\left[\frac{1}{2}\right] == xc \&\& \text{Bezier2}[1] == xb, \{Qa, Qb, Qc\}]$$

$$\text{Out}[2]= \left\{\left\{Qa \rightarrow xa, Qb \rightarrow xb, Qc \rightarrow \frac{1}{2} (-xa - xb + 4 xc)\right\}\right\}$$

Cubic

$$\text{In}[3]:= \text{Bezier3}[t_]:= (1-t)^3 Qa + 3 t (1-t)^2 Qc + 3 t^2 (1-t) Qd + t^3 Qb$$

$$\text{In}[4]:= \text{Solve}[\text{Bezier3}[0] == xa \&\& \text{Bezier3}\left[\frac{1}{3}\right] == xc \&\&$$

$$\text{Bezier3}\left[\frac{2}{3}\right] == xd \&\& \text{Bezier3}[1] == xb, \{Qa, Qb, Qc, Qd\}]$$

$$\text{Out}[4]= \left\{\left\{Qa \rightarrow xa, Qb \rightarrow xb, Qc \rightarrow \frac{1}{6} (-5 xa + 2 xb + 18 xc - 9 xd), Qd \rightarrow \frac{1}{6} (2 xa - 5 xb - 9 xc + 18 xd)\right\}\right\}$$

Convert “t” to “ξ”

$$\text{In}[5]:= \xi a = -1 \quad (* \quad t = 0 \quad *)$$

$$\text{Out}[5]= -1$$

$$\text{In}[6]:= \xi b = 1 \quad (* \quad t = 1 \quad *)$$

$$\text{Out}[6]= 1$$

$$\text{In}[7]:= L = 2 \quad (* \text{ length of Lin on natural space } *)$$

$$\text{Out}[7]= 2$$

$$\text{In}[8]:= \xi c = \xi a + \frac{L}{3} \quad (* \quad t = 1/3 \quad *)$$

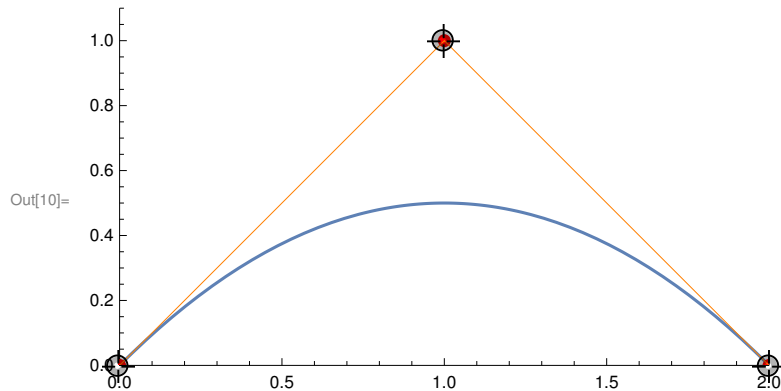
$$\text{Out}[8]= -\frac{1}{3}$$

$$\text{In}[9]:= \xi d = \xi b - \frac{L}{3} \quad (* \quad t = 2/3 \quad *)$$

$$\text{Out}[9]= \frac{1}{3}$$

Plots

```
In[10]:= DynamicModule[{p = {{0, 0}, {1, 1}, {2, 0}}}, LocatorPane[
  Dynamic[p], Dynamic@
    ParametricPlot[
      {Bezier2[t] /. {Qa → p[[1]][1], Qc → p[[2]][1], Qb → p[[3]][1]},
        Bezier2[t] /. {Qa → p[[1]][2], Qc → p[[2]][2], Qb → p[[3]][2]}}, {t, 0, 1},
      Epilog → {Red, PointSize[0.02], Point[p], Orange, Line[p]},
      PlotRange → {{0, 2}, {0, 1.1}}
    ]]]
```



```
In[11]:= DynamicModule[{p = {{0, 0}, {1/2, 1}, {3/2, 1}, {2, 0}}}, LocatorPane[
  Dynamic[p], Dynamic@
    ParametricPlot[
      {Bezier3[t] /. {Qa → p[[1]][1], Qc → p[[2]][1], Qd → p[[3]][1], Qb → p[[4]][1]},
        Bezier3[t] /. {Qa → p[[1]][2], Qc → p[[2]][2], Qd → p[[3]][2], Qb → p[[4]][2]}}, {t, 0, 1},
      Epilog → {Red, PointSize[0.02], Point[p], Orange, Line[p]},
      PlotRange → {{0, 2}, {0, 1.1}}
    ]]]
```

