Felippa's Gauss Integration Rules for Quadrilaterals

Line Gauss Quadrature Module

```
In[1]:= LineGaussRuleInfo[{rule_, numer_}, point_] :=
        Module[{g2 = {-1, 1}/Sqrt[3], w3 = {5/9, 8/9, 5/9}, g3 = {-Sqrt[3/5], 0, Sqrt[3/5]},
          w4 = {(1/2)-Sqrt[5/6]/6, (1/2)+Sqrt[5/6]/6, (1/2)+Sqrt[5/6]/6, (1/2)-Sqrt[5/6]/6},
          g4 = {-Sqrt[(3+2*Sqrt[6/5])/7], -Sqrt[(3-2*Sqrt[6/5])/7], Sqrt[(3-2*Sqrt[6/5])/7],
            Sqrt[(3+2*Sqrt[6/5])/7]}, g5 = {-Sqrt[5+2*Sqrt[10/7]], -Sqrt[5-2*Sqrt[10/7]],
            0, Sqrt[5-2*Sqrt[10/7]], Sqrt[5+2*Sqrt[10/7]]}/3, w5 =
           {322-13*Sqrt[70], 322+13*Sqrt[70], 512, 322+13*Sqrt[70], 322-13*Sqrt[70]}/900,
          i = point, p = rule, info = {{Null, Null}, 0}}, If[p == 1, info = {0, 2}];
        If[p == 2, info = {g2[[i]], 1}];
        If[p == 3, info = {g3[[i]], w3[[i]]}];
        If[p == 4, info = {g4[[i]], w4[[i]]}];
        If[p == 5, info = {g5[[i]], w5[[i]]}];
        If[numer, Return[N[info, 20]], Return[Simplify[info]]];];
```

Quadrilateral Gauss Quadrature Module

```
In[2]:= QuadGaussRuleInfo[{rule_, numer_}, point_] :=
        Module[{ξ, η, p1, p2, i, j, w1, w2, m, info = {{Null, Null}, 0}},
          If[Length[rule] == 2, {p1, p2} = rule, p1 = p2 = rule];
          If[p1 < 0, Return[QuadNonProductGaussRuleInfo[{-p1, numer}, point]]];
          If[Length[point] == 2, {i, j} = point, m = point;
            j = Floor[(m-1)/p1]+1; i = m-p1*(j-1)];
          {ξ, w1} = LineGaussRuleInfo[{p1, numer}, i];
          {η, w2} = LineGaussRuleInfo[{p2, numer}, j];
          info = {{ξ, η}, w1*w2};
          If[numer, Return[N[info, 20]], Return[Simplify[info]]];];
```

Results

```
In[3]:=  ToRSTW[info_] := Module[{r, s, t, w},
             {{r, s}, w} = info;
             Return[{r, s, 0, w}];];
```

Rule 1

```
In[4]:=  Table[ToRSTW[QuadGaussRuleInfo[{1, False}, i]], {i, 1}]
```

Out[4]= {{0, 0, 0, 4}}

Rule 4

```
In[5]:=  Table[ToRSTW[QuadGaussRuleInfo[{2, True}, i]], {i, 4}]
```

Out[5]= {{-0.57735026918962576451, -0.57735026918962576451, 0, 1.0000000000000000000},
  {0.57735026918962576451, -0.57735026918962576451, 0, 1.0000000000000000000},
  {-0.57735026918962576451, 0.57735026918962576451, 0, 1.0000000000000000000},
  {0.57735026918962576451, 0.57735026918962576451, 0, 1.0000000000000000000}}

Rule 9

```
In[6]:=  Table[ToRSTW[QuadGaussRuleInfo[{3, True}, i]], {i, 9}]
```

Out[6]= {{-0.77459666924148337704, -0.77459666924148337704, 0, 0.30864197530864197531},
  {0, -0.77459666924148337704, 0, 0.49382716049382716049},
  {0.77459666924148337704, -0.77459666924148337704, 0, 0.30864197530864197531},
  {-0.77459666924148337704, 0, 0, 0.49382716049382716049},
  {0, 0, 0, 0.79012345679012345568},
  {0.77459666924148337704, 0, 0, 0.49382716049382716049},
  {-0.77459666924148337704, 0.77459666924148337704, 0, 0.30864197530864197531},
  {0, 0.77459666924148337704, 0, 0.49382716049382716049},
  {0.77459666924148337704, 0.77459666924148337704, 0, 0.30864197530864197531}}

Rule 16

In[7]:= **Table[ToRSTW[QuadGaussRuleInfo[{4, True}, i]], {i, 16}]**

Out[7]= {{-0.86113631159405257522, -0.86113631159405257522, 0, 0.12100299328560200552},
{-0.33998104358485626480, -0.86113631159405257522, 0, 0.22685185185185185185},
{0.33998104358485626480, -0.86113631159405257522, 0, 0.22685185185185185185},
{0.86113631159405257522, -0.86113631159405257522, 0, 0.12100299328560200552},
{-0.86113631159405257522, -0.33998104358485626480, 0, 0.22685185185185185185},
{-0.33998104358485626480, -0.33998104358485626480, 0, 0.42529330301069429078},
{0.33998104358485626480, -0.33998104358485626480, 0, 0.42529330301069429078},
{0.86113631159405257522, -0.33998104358485626480, 0, 0.22685185185185185185},
{-0.86113631159405257522, 0.33998104358485626480, 0, 0.22685185185185185185},
{-0.33998104358485626480, 0.33998104358485626480, 0, 0.42529330301069429078},
{0.33998104358485626480, 0.33998104358485626480, 0, 0.42529330301069429078},
{0.86113631159405257522, 0.33998104358485626480, 0, 0.22685185185185185185},
{-0.86113631159405257522, 0.86113631159405257522, 0, 0.12100299328560200552},
{-0.33998104358485626480, 0.86113631159405257522, 0, 0.22685185185185185185},
{0.33998104358485626480, 0.86113631159405257522, 0, 0.22685185185185185185},
{0.86113631159405257522, 0.86113631159405257522, 0, 0.12100299328560200552}}