

Shape functions of Hex20

Coordinates of points in the reference space

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In[1]:= x = {{(*1*){-1, -1, -1}, (*2*){1, -1, -1}, (*3*){1, 1, -1}, (*4*){-1, 1, -1},
  (*5*){-1, -1, 1}, (*6*){1, -1, 1}, (*7*){1, 1, 1}, (*8*){-1, 1, 1}, (*9*)
  {0, -1, -1}, (*10*){1, 0, -1}, (*11*){0, 1, -1}, (*12*){-1, 0, -1},
  (*13*){0, -1, 1}, (*14*){1, 0, 1}, (*15*){0, 1, 1}, (*16*){-1, 0, 1},
  (*17*){-1, -1, 0}, (*18*){1, -1, 0}, (*19*){1, 1, 0}, (*20*){-1, 1, 0}};
```

Shape functions

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In[2]:= group1 = {9, 11, 13, 15};
group2 = {10, 12, 14, 16};
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In[4]:= ShapeFunc[n_] :=
  If[n ≤ 8,  $\frac{1}{8} (1 + r x[n, 1]) (1 + s x[n, 2]) (1 + t x[n, 3]) (r x[n, 1] + s x[n, 2] + t x[n, 3] - 2)$ ,
  If[MemberQ[group1, n],  $\frac{1}{4} (1 - r^2) (1 + s x[n, 2]) (1 + t x[n, 3])$ ,
  If[MemberQ[group2, n],  $\frac{1}{4} (1 + r x[n, 1]) (1 - s^2) (1 + t x[n, 3])$ ,
   $\frac{1}{4} (1 + r x[n, 1]) (1 + s x[n, 2]) (1 - t^2)$ ]]]
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In[5]:= AllShapeFunc = Table[ShapeFunc[m], {m, 20}];
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In[6]:= AllShapeFunc // MatrixForm
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Out[6]//MatrixForm=
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$$\begin{pmatrix} \frac{1}{8} (1-r) (1-s) (1-t) (-2-r-s-t) \\ \frac{1}{8} (1+r) (1-s) (1-t) (-2+r-s-t) \\ \frac{1}{8} (1+r) (1+s) (1-t) (-2+r+s-t) \\ \frac{1}{8} (1-r) (1+s) (1-t) (-2-r+s-t) \\ \frac{1}{8} (1-r) (1-s) (1+t) (-2-r-s+t) \\ \frac{1}{8} (1+r) (1-s) (1+t) (-2+r-s+t) \\ \frac{1}{8} (1+r) (1+s) (1+t) (-2+r+s+t) \\ \frac{1}{8} (1-r) (1+s) (1+t) (-2-r+s+t) \\ \frac{1}{4} (1-r^2) (1-s) (1-t) \\ \frac{1}{4} (1+r) (1-s^2) (1-t) \\ \frac{1}{4} (1-r^2) (1+s) (1-t) \\ \frac{1}{4} (1-r) (1-s^2) (1-t) \\ \frac{1}{4} (1-r^2) (1-s) (1+t) \\ \frac{1}{4} (1+r) (1-s^2) (1+t) \\ \frac{1}{4} (1-r^2) (1+s) (1+t) \\ \frac{1}{4} (1-r) (1-s^2) (1+t) \\ \frac{1}{4} (1-r) (1-s) (1-t^2) \\ \frac{1}{4} (1+r) (1-s) (1-t^2) \\ \frac{1}{4} (1+r) (1+s) (1-t^2) \\ \frac{1}{4} (1-r) (1+s) (1-t^2) \end{pmatrix}$$

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In[7]:= substitution = {
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  (1-r) → rm, (1-s) → sm, (1-t) → tm,
  (1+r) → rp, (1+s) → sp, (1+t) → tp,
  (1-r^2) → rr, (1-s^2) → ss, (1-t^2) → tt
};
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In[8]:= interp = AllShapeFunc /. substitution;
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Print[CForm[interp]]
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```
List((rm*sm*(-2-r-s-t)*tm)/8.,(rp*sm*(-2+r-s-t)*tm)/8.,(rp*sp*(-2+r+s-
-t)*tm)/8.,(rm*sp*(-2-r+s-t)*tm)/8.,(rm*sm*(-2-r-s+t)*tp)/8.,(rp*sm*(-2
+r-s+t)*tp)/8.,(rp*sp*(-2+r+s+t)*tp)/8.,(rm*sp*(-2-r+s+
t)*tp)/8.,(rr*sm*tm)/4.,(rp*ss*tm)/4.,(rr*sp*tm)/4.,(rm*ss*tm)/4.,(rr*sm*tp)/4.,(rp*ss*tp)/4.,(rr*sp*
tp)/4.,(rm*ss*tp)/4.,(rm*sm*tt)/4.,(rp*sm*tt)/4.,(rp*sp*tt)/4.,(rm*sp*tt)/4.)
```

Derivative of shape functions

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In[10]:= AllDerivShapeWrtR = D[AllShapeFunc, r];
AllDerivShapeWrtS = D[AllShapeFunc, s];
AllDerivShapeWrtT = D[AllShapeFunc, t];
```

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In[13]:= AllDerivShapeWrtR // MatrixForm
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Out[13]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{8}(1-r)(1-s)(1-t) - \frac{1}{8}(1-s)(1-t)(-2-r-s-t) \\ \frac{1}{8}(1+r)(1-s)(1-t) + \frac{1}{8}(1-s)(1-t)(-2+r-s-t) \\ \frac{1}{8}(1+r)(1+s)(1-t) + \frac{1}{8}(1+s)(1-t)(-2+r+s-t) \\ -\frac{1}{8}(1-r)(1+s)(1-t) - \frac{1}{8}(1+s)(1-t)(-2-r+s-t) \\ -\frac{1}{8}(1-r)(1-s)(1+t) - \frac{1}{8}(1-s)(1+t)(-2-r-s+t) \\ \frac{1}{8}(1+r)(1-s)(1+t) + \frac{1}{8}(1-s)(1+t)(-2+r-s+t) \\ \frac{1}{8}(1+r)(1+s)(1+t) + \frac{1}{8}(1+s)(1+t)(-2+r+s+t) \\ -\frac{1}{8}(1-r)(1+s)(1+t) - \frac{1}{8}(1+s)(1+t)(-2-r+s+t) \\ -\frac{1}{2}r(1-s)(1-t) \\ \frac{1}{4}(1-s^2)(1-t) \\ -\frac{1}{2}r(1+s)(1-t) \\ -\frac{1}{4}(1-s^2)(1-t) \\ -\frac{1}{2}r(1-s)(1+t) \\ \frac{1}{4}(1-s^2)(1+t) \\ -\frac{1}{2}r(1+s)(1+t) \\ -\frac{1}{4}(1-s^2)(1+t) \\ -\frac{1}{4}(1-s)(1-t^2) \\ \frac{1}{4}(1-s)(1-t^2) \\ \frac{1}{4}(1+s)(1-t^2) \\ -\frac{1}{4}(1+s)(1-t^2) \end{pmatrix}$$

```
In[14]:= AllDerivShapeWrtS // MatrixForm
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Out[14]//MatrixForm=
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$$\begin{pmatrix} -\frac{1}{8}(1-r)(1-s)(1-t) - \frac{1}{8}(1-r)(1-t)(-2-r-s-t) \\ -\frac{1}{8}(1+r)(1-s)(1-t) - \frac{1}{8}(1+r)(1-t)(-2+r-s-t) \\ \frac{1}{8}(1+r)(1+s)(1-t) + \frac{1}{8}(1+r)(1-t)(-2+r+s-t) \\ \frac{1}{8}(1-r)(1+s)(1-t) + \frac{1}{8}(1-r)(1-t)(-2-r+s-t) \\ -\frac{1}{8}(1-r)(1-s)(1+t) - \frac{1}{8}(1-r)(1+t)(-2-r-s+t) \\ -\frac{1}{8}(1+r)(1-s)(1+t) - \frac{1}{8}(1+r)(1+t)(-2+r-s+t) \\ \frac{1}{8}(1+r)(1+s)(1+t) + \frac{1}{8}(1+r)(1+t)(-2+r+s+t) \\ \frac{1}{8}(1-r)(1+s)(1+t) + \frac{1}{8}(1-r)(1+t)(-2-r+s+t) \\ -\frac{1}{4}(1-r^2)(1-t) \\ -\frac{1}{2}(1+r)s(1-t) \\ \frac{1}{4}(1-r^2)(1-t) \\ -\frac{1}{2}(1-r)s(1-t) \\ -\frac{1}{4}(1-r^2)(1+t) \\ -\frac{1}{2}(1+r)s(1+t) \\ \frac{1}{4}(1-r^2)(1+t) \\ -\frac{1}{2}(1-r)s(1+t) \\ -\frac{1}{4}(1-r)(1-t^2) \\ -\frac{1}{4}(1+r)(1-t^2) \\ \frac{1}{4}(1+r)(1-t^2) \\ \frac{1}{4}(1-r)(1-t^2) \end{pmatrix}$$

```
In[15]:= AllDerivShapeWrtT // MatrixForm
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Out[15]//MatrixForm=
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$$\begin{pmatrix} -\frac{1}{8}(1-r)(1-s)(1-t) - \frac{1}{8}(1-r)(1-s)(-2-r-s-t) \\ -\frac{1}{8}(1+r)(1-s)(1-t) - \frac{1}{8}(1+r)(1-s)(-2+r-s-t) \\ -\frac{1}{8}(1+r)(1+s)(1-t) - \frac{1}{8}(1+r)(1+s)(-2+r+s-t) \\ -\frac{1}{8}(1-r)(1+s)(1-t) - \frac{1}{8}(1-r)(1+s)(-2-r+s-t) \\ \frac{1}{8}(1-r)(1-s)(1+t) + \frac{1}{8}(1-r)(1-s)(-2-r-s+t) \\ \frac{1}{8}(1+r)(1-s)(1+t) + \frac{1}{8}(1+r)(1-s)(-2+r-s+t) \\ \frac{1}{8}(1+r)(1+s)(1+t) + \frac{1}{8}(1+r)(1+s)(-2+r+s+t) \\ \frac{1}{8}(1-r)(1+s)(1+t) + \frac{1}{8}(1-r)(1+s)(-2-r+s+t) \\ -\frac{1}{4}(1-r^2)(1-s) \\ -\frac{1}{4}(1+r)(1-s^2) \\ -\frac{1}{4}(1-r^2)(1+s) \\ -\frac{1}{4}(1-r)(1-s^2) \\ \frac{1}{4}(1-r^2)(1-s) \\ \frac{1}{4}(1+r)(1-s^2) \\ \frac{1}{4}(1-r^2)(1+s) \\ \frac{1}{4}(1-r)(1-s^2) \\ -\frac{1}{2}(1-r)(1-s)t \\ -\frac{1}{2}(1+r)(1-s)t \\ -\frac{1}{2}(1+r)(1+s)t \\ -\frac{1}{2}(1-r)(1+s)t \end{pmatrix}$$

```
In[16]:= derivR = AllDerivShapeWrtR /. substitution;
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Print[CForm[derivR]]
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```
List(-0.125*(rm*sm*tm) - (sm*(-2 - r - s - t)*tm)/8., (rp*sm*tm)/8. + (sm*(-2 + r - s - t)*tm)/8., (rp*sp*tm)/8. + (sp*(-2 + r + s - t)*tm)/8., -0.125*(rm*sp*tm) - (sp*(-2 - r + s - t)*tm)/8., -0.125*(rm*sm*tp) - (sm*(-2 - r - s + t)*tp)/8., (rp*sm*tp)/8. + (sm*(-2 + r - s + t)*tp)/8., (rp*sp*tp)/8. + (sp*(-2 + r + s + t)*tp)/8., -0.125*(rm*sp*tp) - (sp*(-2 - r + s + t)*tp)/8., -0.5*(r*sm*tm), (ss*tm)/4., -0.5*(r*sp*tm), -0.25*(ss*tm), -0.5*(r*sm*tp), (ss*tp)/4., -0.5*(r*sp*tp), -0.25*(ss*tp), -0.25*(sm*tt), (sm*tt)/4., (sp*tt)/4., -0.25*(sp*tt))
```

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In[18]:= derivS = AllDerivShapeWrtS /. substitution;
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Print[CForm[derivS]]
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```
List(-0.125*(rm*sm*tm) - (rm*(-2 - r - s - t)*tm)/8., -0.125*(rp*sm*tm) - (rp*(-2 + r - s - t)*tm)/8., (rp*sp*tm)/8. + (rp*(-2 + r + s - t)*tm)/8., (rm*sp*tm)/8. + (rm*(-2 - r + s - t)*tm)/8., -0.125*(rm*sm*tp) - (rm*(-2 - r - s + t)*tp)/8., -0.125*(rp*sm*tp) - (rp*(-2 + r - s + t)*tp)/8., (rp*sp*tp)/8. + (rp*(-2 + r + s + t)*tp)/8., (rm*sp*tp)/8. + (rm*(-2 - r + s + t)*tp)/8., -0.25*(rr*tm), -0.5*(rp*s*tm), (rr*tm)/4., -0.5*(rm*s*tm), -0.25*(rr*tp), -0.5*(rp*s*tp), (rr*tp)/4., -0.5*(rm*s*tp), -0.25*(rm*tt), -0.25*(rp*tt), (rp*tt)/4., (rm*tt)/4.)
```

```

In[20]:= derivT = AllDerivShapeWrtT /. substitution;
Print[CForm[derivT]]

List(-0.125*(rm*sm*(-2 - r - s - t)) - (rm*sm*tm)/8., -0.125*(rp*sm*(-2 + r - s - t)) -
(rp*sm*tm)/8., -0.125*(rp*sp*(-2 + r + s - t)) - (rp*sp*tm)/8., -0.125*(rm*sp*(-2 - r + s -
t)) - (rm*sp*tm)/8., (rm*sm*(-2 - r - s + t))/8. + (rm*sm*tp)/8., (rp*sm*(-2 + r - s + t))/8.
+ (rp*sm*tp)/8., (rp*sp*(-2 + r + s + t))/8. + (rp*sp*tp)/8., (rm*sp*(-2 - r + s + t))/8. +
(rm*sp*tp)/8., -0.25*(rr*sm), -0.25*(rp*ss), -0.25*(rr*sp), -0.25*(rm*ss), (rr*sm)/4., (rp*ss)/4., (rr*sp)/
4., (rm*ss)/4., -0.5*(rm*sm*t), -0.5*(rp*sm*t), -0.5*(rp*sp*t), -0.5*(rm*sp*t))

```