

Encryption Database and Biometrics Library and Software Suite

(EnDaBi)

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Documentation

(English Version)

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documentation.

Introduction

The caveman, the life choice he made to settle down from hunting and gathering provided him with more security inside his cave. but it limited his freedom to roam vast lands.

The same aspect can be found in modern society. If a company decreases its security measures on its entrances, the employees can get in and out faster (not having to pull so many cards out, remember so many passwords and such), but it loses some of its security points. and vice versa.

The EnDaBi Project tackles that problem, and tries to find balance between security and mobility.

By utilizing state of the art schemes in Encryption, Database and Biometrics technologies and embedding them all into one efficient and coherent library or suite.

About The Project

The premise of the project is simple. For starters, we're focusing on Encryption. We spent a fair amount of time searching for material that serve that particular purpose.

We found those published by Professor Christof Paar to be most suitable for implementing and most easy to understand.

The Current Project Steps :

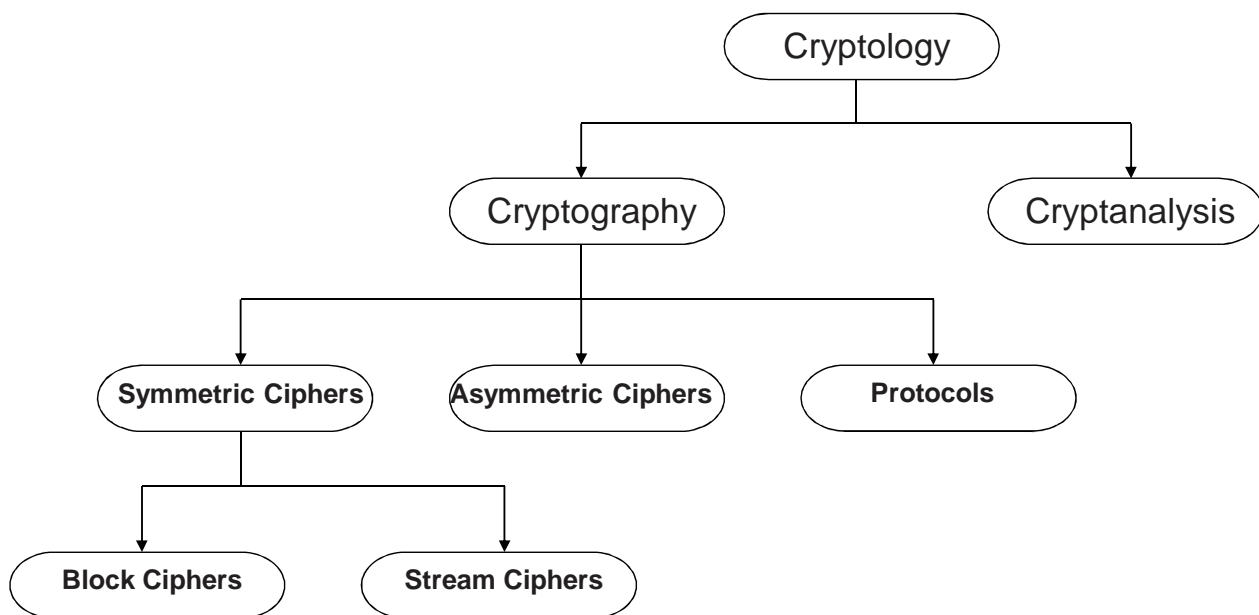
- 1) Choosing the RSA algorithm for implementation. for the sake of mobility and security (it being an Asymmetric Cryptographic Scheme).
- 2) Trying to make the Implementation as Simple and Usable as possible.
- 3) Trying to Enhance Speed and Efficiency.
- 4) Porting the Library to as many languages and platforms as possible.

Cryptography

Cryptography is a science that deals with Rendering Information that is available for everyone to see, and make it such that only a few can understand.

Cryptography can be divided into a handful of Classifications.

■ Classification of the Field of Cryptology



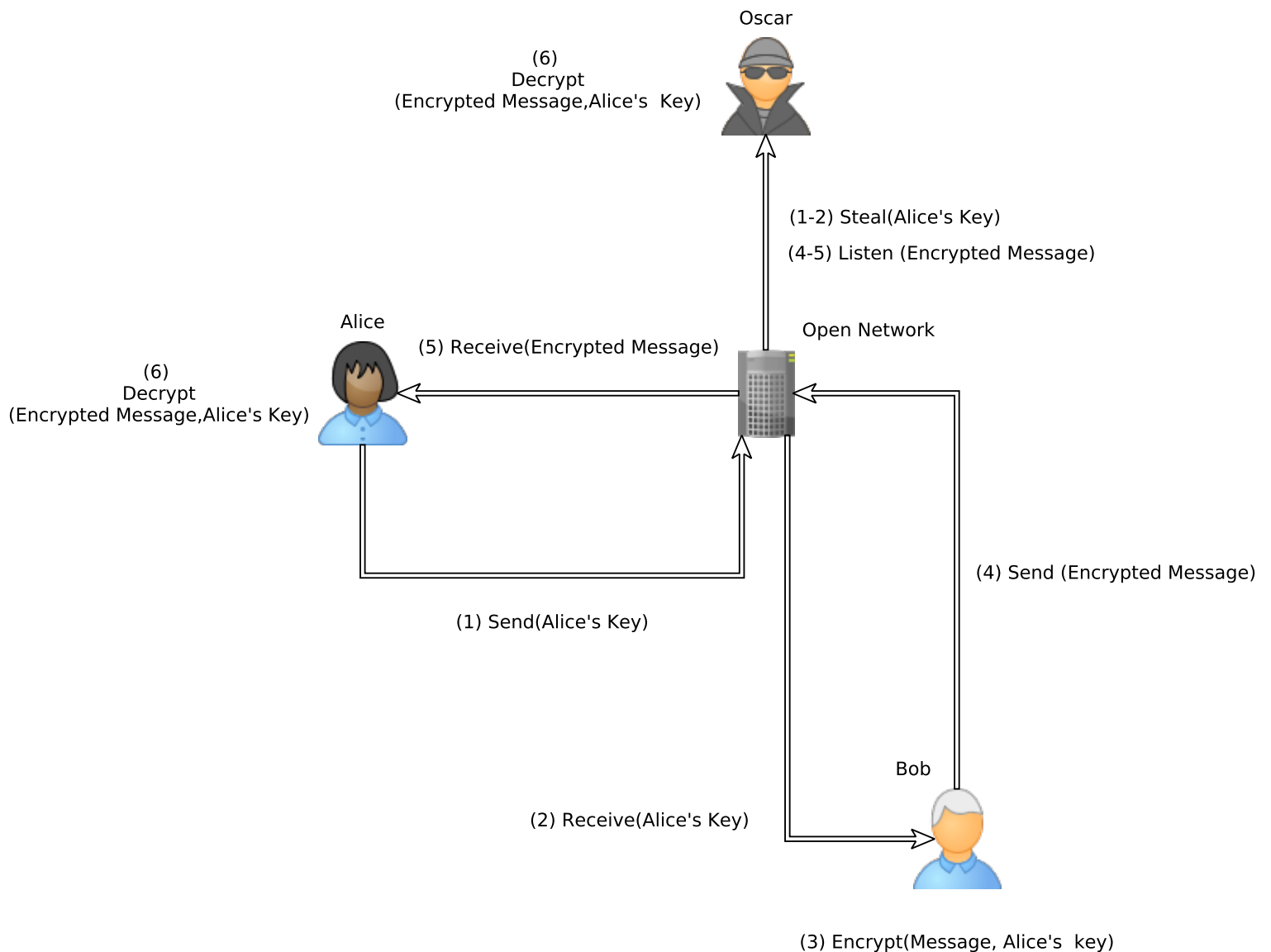
Symmetric Cryptography

Symmetric Cryptography is a classification in which a **Shared Key** is exchanged prior to Encryption.

The **Key** is used for both **Encryption and Decryption**.

The **Key** must be Exchanged over a **secure channel** or else if captured it will be used by a **Malicious Third Party** to decrypt the Encrypted message.

Symmetric Cryptography



Asymmetric Cryptography

This Encryption Classification Includes a **Pair of Keys (Public and Private)**.

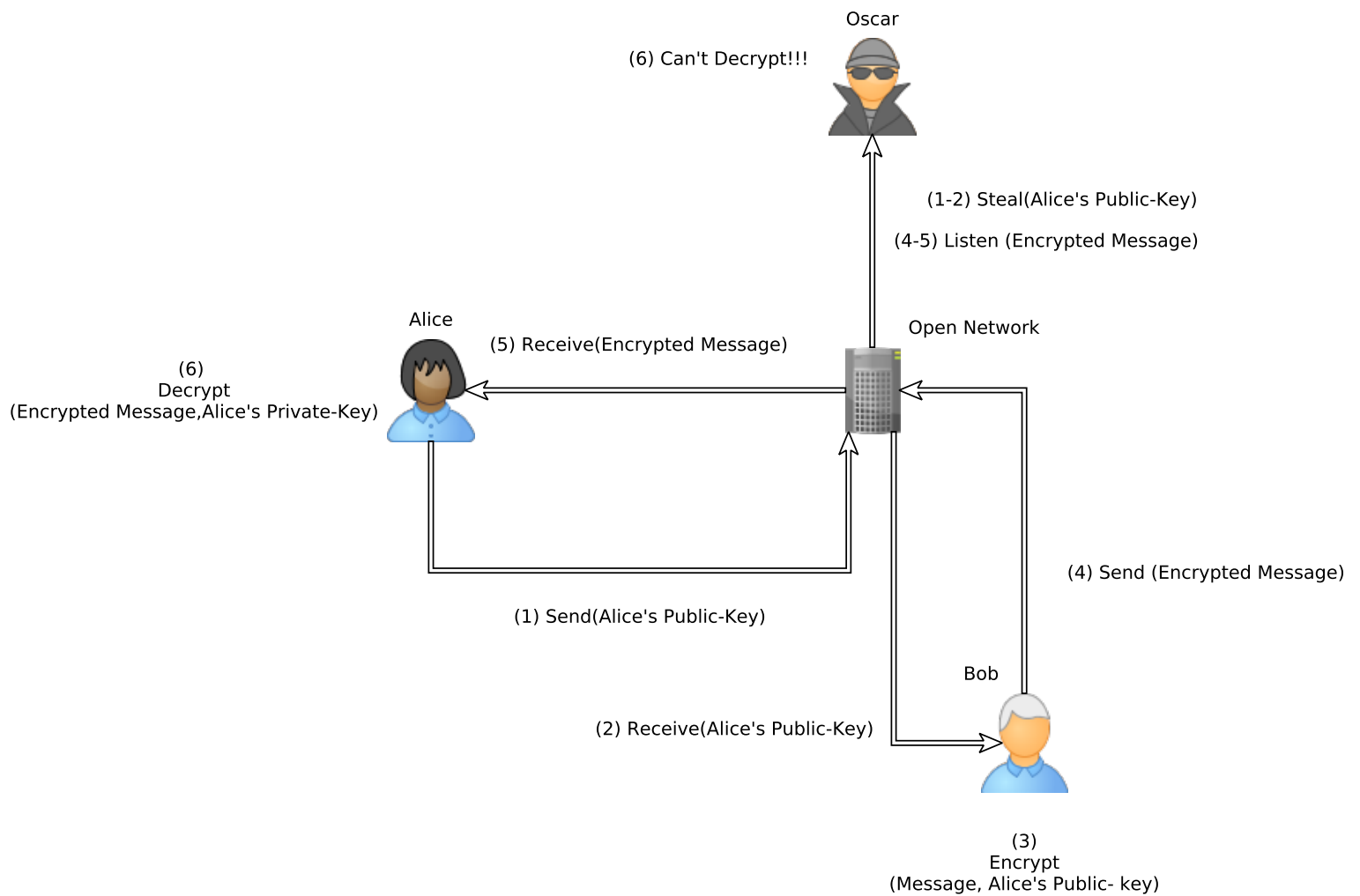
The Public Key is used for **Encryption** solely.

The Private Key is used for **Decryption** solely.

Only the **Public Key** is Exchanged.

If a Malicious Third Party Captures the Public Key, it is of no use to them **(it can't Decrypt)**.

ASymmetric Cryptography



Mathematical Background

The RSA Algorithm utilizes a number of Well known mathematical Methods and Formulae to achieve a Successful and Secure Encryption/Decryption.

Those are :

- 1) The Euclidean Algorithm.
- 2) The Extended Euclidean Algorithm.
- 3) Euler's Phi Function.
- 4) Fermat's Little Theorem.
- 5) Euler's Theorem.
- 6) Binary Exponentiation
(Square-and-Multiply).
- 7) Primality Testing.

The Eucledian Agorithm

This Algorithm Computes the Greatest Common Divisor of r_0 and r_1 . $\gcd(r_0, r_1)$.

It does so by following these simple steps :

- 1) Test if ($r_1 == 0$), if that's the case, then the final answer is the current r_0 .
- 2) Make $Temp = r_1$
- 3) Make $r_1 = r_0 \bmod r_1$
- 4) Make $r_0 = Temp$
- 5) Repeat Recursively.

The Extended Euclidean Algorithm

Suppose $\gcd(r_0, r_1) = 1$.

The theory states that you can write that as the following :

$$\gcd(r_0, r_1) = s * r_0 + t * r_1$$

Just like the Euclidean Algorithm, we go on calculating the \gcd recursively, making

$$r_i = r_{i-2} \bmod r_{i-1}$$

$$q_{i-1} = (r_{i-2} - r_i) / r_{i-1}$$

$$t_i = t_{i-2} - q_{i-1} * t_{i-1}$$

until we reach :

$$\gcd(r_0, r_1) \equiv 1$$

at that point $t = t_{i-1}$

now we submit the equation to the modulo operation :

$$\gcd(r_0, r_1) \equiv 1 \equiv s * r_0 + t * r_1$$

$$1 \bmod r_0 \equiv (s * r_0 + t * r_1) \bmod r_0$$

$$1 \bmod r_0 \equiv t * r_1 \bmod r_0$$

$$\text{and since : } 1 \bmod r_0 \equiv r_1^{-1} * r_1 \bmod r_0$$

$$\text{then : } r_1^{-1} \equiv t$$

And That is **One** way to calculate the
Modular Inverse

■ Euler's Phi Function 1/2

- *New problem, important for public-key systems, e.g., RSA:*

Given the set of the m integers $\{0, 1, 2, \dots, m-1\}$,

How many numbers in the set are **relatively prime to m** ?

- Answer: **Euler's Phi function $\Phi(m)$**

- **Example** for the sets $\{0,1,2,3,4,5\}$ ($m=6$), and $\{0,1,2,3,4\}$ ($m=5$)

$$\gcd(0,6) = 6$$

$$\gcd(1,6) = 1 \quad \leftarrow$$

$$\gcd(2,6) = 2$$

$$\gcd(3,6) = 3$$

$$\gcd(4,6) = 2$$

$$\gcd(5,6) = 1 \quad \leftarrow$$

$$\gcd(0,5) = 5$$

$$\gcd(1,5) = 1 \quad \leftarrow$$

$$\gcd(2,5) = 1 \quad \leftarrow$$

$$\gcd(3,5) = 1 \quad \leftarrow$$

$$\gcd(4,5) = 1 \quad \leftarrow$$

→ 1 and 5 relatively prime to $m=6$,
hence **$\Phi(6) = 2$**

→ **$\Phi(5) = 4$**

- Testing one gcd per number in the set is **extremely slow for large m** .

■ Euler's Phi Function 2/2

- If canonical factorization of m known: $m = p_1^{e_1} \cdot p_2^{e_2} \cdot \dots \cdot p_n^{e_n}$
(where p_i primes and e_i positive integers)
- then calculate Phi according to the relation
$$\Phi(m) = \prod_{i=1}^n (p_i^{e_i} - p_i^{e_i-1})$$
- Phi especially easy for $e_i = 1$, e.g., $m = p \cdot q \rightarrow \Phi(m) = (p-1) \cdot (q-1)$
- **Example** $m = 899 = 29 \cdot 31$:
 $\Phi(899) = (29-1) \cdot (31-1) = 28 \cdot 30 = \mathbf{840}$
- **Note:** Finding $\Phi(m)$ is computationally easy **if factorization of m is known**
(otherwise the calculation of $\Phi(m)$ becomes computationally infeasible for large numbers)

■ Fermat's Little Theorem

- Given a **prime** p and an **integer** a : $a^p \equiv a \pmod{p}$
- Can be rewritten as $a^{p-1} \equiv 1 \pmod{p}$
- **Use: Find modular inverse**, if p is prime. Rewrite to $a \cdot a^{p-2} \equiv 1 \pmod{p}$
- Comparing with definition of the modular inverse $a \cdot a^{-1} \equiv 1 \pmod{m}$
 $\rightarrow a^{-1} \equiv a^{p-2} \pmod{p}$ is the modular inverse modulo a prime p

Example: $a = 2, p = 7$

$$a^{p-2} = 2^5 = 32 \equiv 4 \pmod{7}$$

$$\text{verify: } 2 \cdot 4 \equiv 1 \pmod{7} \quad \checkmark$$

- Fermat's Little Theorem works only **modulo a prime** p

■ Euler's Theorem

- Generalization of Fermat's little theorem to **any integer modulus**
- Given two **relatively prime integers** a and m : $a^{\Phi(m)} \equiv 1 \pmod{m}$
- **Example:** $m=12$, $a=5$

1. Calculate Euler's Phi Function

$$\Phi(12) = \Phi(2^2 \cdot 3) = (2^2 - 2^1)(3^1 - 3^0) = (4 - 2)(3 - 1) = 4$$

2. Verify Euler's Theorem

$$5^{\Phi(12)} = 5^4 = 25^2 = 625 \equiv 1 \pmod{12}$$

- Fermat's little theorem = special case of Euler's Theorem
- for a prime p : $\Phi(p) = (p^1 - p^0) = p - 1$
→ Fermat: $a^{\Phi(p)} = a^{p-1} \equiv 1 \pmod{p}$

■ Square-and-Multiply

- **Basic principle:** Scan exponent bits from left to right and square/multiply operand accordingly

Algorithm: Square-and-Multiply for $x^H \bmod n$

Input: Exponent H , base element x , Modulus n

Output: $y = x^H \bmod n$

1. Determine binary representation $H = (h_t, h_{t-1}, \dots, h_0)_2$
2. **FOR** $i = t-1$ **TO** 0
3. $y = y^2 \bmod n$
4. **IF** $h_i = 1$ **THEN**
5. $y = y * x \bmod n$
6. **RETURN** y

- Rule: Square in every iteration (Step 3) and multiply current result by x if the exponent bit $h_i = 1$ (Step 5)
- Modulo reduction after each step keeps the operand y small

■ Example: Square-and-Multiply

- Computes x^{26} without modulo reduction
- Binary representation of exponent: $26 = (1, 1, 0, 1, 0)_2 = (h_4, h_3, h_2, h_1, h_0)_2$

Step		Binary exponent	Op	Comment
1	$x = x^1$	$(1)_2$		Initial setting, h_4 processed
1a	$(x^1)^2 = x^2$	$(10)_2$	SQ	Processing h_3
1b	$x^2 * x = x^3$	$(11)_2$	MUL	$h_3 = 1$
2a	$(x^3)^2 = x^6$	$(110)_2$	SQ	Processing h_2
2b	-	$(110)_2$	-	$h_0 = 0$
3a	$(x^6)^2 = x^{12}$	$(1100)_2$	SQ	Processing h_1
3b	$x^{12} * x = x^{13}$	$(1101)_2$	MUL	$h_1 = 1$
4a	$(x^{13})^2 = x^{26}$	$(11010)_2$	SQ	Processing h_0
4b	-	$(11010)_2$	-	$h_0 = 0$

- Observe how the exponent evolves into $x^{26} = x^{11010}$

Primality Testing

There are many ways to determine whether integer P is prime or not.

One way is we make sure P is prime, By factorization.

The other is depending on **Equivalence Rules** that only apply to primes. and Testing whether these Rules apply to P or not.

If They do, then P is **probably** Prime.

If They don't, then P is **definitely** Composite.

Using Sieves for Factorization

Sieves are a great way to Shrink the amount of time needed to test the primality of **Many or More** test cases.

A Sieve is a Boolean Array, that uses the Index to signal the number we're storing, and uses the boolean value to signal Primality (1 being Prime, 0 being Composite)

The Array is initialized with 1. as in all numbers are initially considered Primes.

Then we start going through numbers (indexes).

for each index that has a value of 1, do the following:

go through all the indexes that are multiples of that prime index, and switch their values to 0.

Sieve Tricks

Consider the Following :

- 1) For a given Prime index, all multiples of that index up until $index^2$ are switched to composites (value to 0), by the **Previous Prime** index **Passes**.

Therefor : only switch the multiples of a Prime index that are above $index^2$.

- 2) All Prime indexes needed to determine the primality of an index are below $\sqrt[2]{index}$.

Therefor : keep going through prime indexes until index = $\sqrt[2]{Arraylimit}$.

Prime Rules

Consider a Prime number P , it can be written

$$\text{as : } P = (r_1 * r_2 * \dots * r_i) + 1$$

where : $r_1 \dots r_i$ is a factor of $P - 1$.

Take a random number a where :

$$a > 1 \text{ and } a < P \Rightarrow \gcd(P, a) = 1 .$$

now find the result of this equation :

$$a^{P-1} = ? \bmod P .$$

$$a^{(r_1 * r_2 * \dots * r_i)} = ? \bmod P .$$

$$((a^{r_1})^{r_2})^{r_i} = ? \bmod P$$

with $answer = (a^{r_i} \bmod P) \in [1 \dots P - 1]$

We've got $P-1$ tries, and each try produces a different *answer* (since the modulus operation is cyclic and $\gcd(P, a) = 1$) until we reach

$$a^{r_i} \equiv 1 \pmod{P}$$

at that point $1^{r_i} \equiv 1 \pmod{P}$

The above proves **Fermat's Little Theorem**.

It makes it that if a given number P passes the above Equivalence Rule, then it's a **Probable Prime**.

But also if we consider a^P , by a few steps of mathematical manipulation we derive that :

$$a^P \equiv a \pmod{P}$$

if we multiply both sides by a^{-1} :

$$a^P * a^{-1} \equiv a * a^{-1} \pmod{P}$$

$$1 \equiv a * a^{-1} \pmod{P} \Rightarrow a^{P-1} \equiv a^{-1}$$

and that is a **Second** way to calculate the

Modular Inverse .

■ The RSA Cryptosystem

- Martin Hellman and Whitfield Diffie published their landmark public-key paper in 1976
- Ronald Rivest, Adi Shamir and Leonard Adleman proposed the asymmetric RSA cryptosystem in 1977
- Until now, RSA is the most widely used asymmetric cryptosystem although elliptic curve cryptography (ECC) becomes increasingly popular
- RSA is mainly used for two applications
 - Transport of (i.e., symmetric) keys (cf. Chptr 13 of *Understanding Cryptography*)
 - Digital signatures (cf. Chptr 10 of *Understanding Cryptography*)

■ Encryption and Decryption

- RSA operations are done over the integer ring Z_n (i.e., arithmetic modulo n), where $n = p * q$, with p, q being large primes
- Encryption and decryption are simply exponentiations in the ring

Definition

Given the public key $(n, e) = k_{pub}$ and the private key $d = k_{pr}$ we write

$$y = e_{k_{pub}}(x) \equiv x^e \pmod{n}$$

$$x = d_{k_{pr}}(y) \equiv y^d \pmod{n}$$

where $x, y \in Z_n$.

We call $e_{k_{pub}}()$ the encryption and $d_{k_{pr}}()$ the decryption operation.

- In practice x, y, n and d are very long integer numbers (≥ 1024 bits)
- The security of the scheme relies on the fact that it is hard to derive the „private exponent“ d given the public-key (n, e)

■ Key Generation

- Like all asymmetric schemes, RSA has set-up phase during which the private and public keys are computed

Algorithm: RSA Key Generation

Output: public key: $k_{pub} = (n, e)$ and private key $k_{pr} = d$

1. Choose two large primes p, q
2. Compute $n = p * q$
3. Compute $\Phi(n) = (p-1) * (q-1)$
4. Select the public exponent $e \in \{1, 2, \dots, \Phi(n)-1\}$ such that $\gcd(e, \Phi(n)) = 1$
5. Compute the private key d such that $d * e \equiv 1 \mod \Phi(n)$
6. **RETURN** $k_{pub} = (n, e), k_{pr} = d$

Remarks:

- Choosing two large, distinct primes p, q (in Step 1) is non-trivial
- $\gcd(e, \Phi(n)) = 1$ ensures that e has an inverse and, thus, that there is always a private key d

Why the RSA is Secure!

- 1) To calculate the private Key Efficiently you need to use the Extended Euclidean Algorithm on the e and $\phi(n)$.
- 2) to Calculate $\phi(n)$ you need to have p and q .
- 3) The only way to get p and q from a non-secure channel is by getting n (which is public).
- 4) Factoring n proves Impractical for (> 1024) bit Integers ,by today's standard computing power.

RSA's Shortcomings

1) When choosing a relatively small Exponent e as a Public-Key, if $message^e < modulus$, then by deduction :

$$message^e \equiv message^e \bmod modulus.$$

And the message can be regenerated using

$$message = \sqrt[e]{message^e}$$

Therefor : Avoid Relatively Small numbers when choosing a Public-Key.

2) Attackers can Encrypt a Predicted Plain text with the Public-Key, and Compare it to the Original Encrypted text.

Solution : **Padding** the Text Message prior to Encryption with Random Values.

EnDaBi Implementations

These Implementations include :

- . The EnDaBi RSA Core Library.
- . a Console Demo showcasing the Library's main functions.
- . a GUI Demo showcasing the Library's main functions.
- . an Example Segmented Sieve Program
- . a Small java program that utilizes built-in primality testing.
- . a make file used to compile the demos.

Toolkits, Libraries and Programming Languages We Used

C++ : Strongly typed, Fast and efficient, library extended Programming-Language.

D : Strongly typed, Fast and efficient Programming-Language with Syntax Similar to that of C and Java.

Java : Strongly typed, byte-code compiled programming language that devotes to binary portability (compile once, run everywhere).

FLTK : Fast Light Tool Kit ("FLTK", pronounced "fulltick") is a C++ graphical user interface toolkit for the X Window System, MacOS, and Microsoft Windows that supports OpenGL.

ENDABI RSA DEMO GUI is based in part on the work of the FLTK project
<http://www.fltk.org>.

LaTeX : is a high-quality typesetting system; it includes features designed for the production of technical and scientific documentation. LaTeX is the de-facto standard for the communication and publication of scientific documents.

InfInt : Arbitrary-Precision Integer Arithmetic Library. licenced under LGPL 2.1. Copyright (C) 2013 Sercan Tutar

code.google.com/p/infint/

Software we Used

Ubuntu 14.04 LTS : Free and Open Source Linux-Based Operating System.

GCC : GNU project C and C++ compiler.

GDC : A GCC-based compiler for the D language.

Javac : Java programming language compiler.

TeXstudio : is a LaTeX editor with a graphical user interface.

Code::Blocks : The open-source, cross-platform IDE.

SciTE : a programmers text editor

Vim : is a text editor that is upwards compatible to Vi. It can be used to edit all kinds of plain text. It is especially useful for editing programs.

Eclipse : extensible tool platform and Java IDE.

nano : is a small, free and friendly editor which aims to replace Pico, the default editor included in the non-free Pine package.

make : GNU make utility to maintain groups of programs

Git : is a fast, scalable, distributed revision control system with an unusually rich command set that provides both high-level operations and full access to internals.

yEd : is a powerful desktop application that can be used to quickly and effectively generate high-quality diagrams.

How To Use Our Software

1) This Installation process was tested on
Ubuntu 14.04 LTS.

2) Install Prerequisites

In Terminal, Type :

```
sudo apt-get install gcc g++ gdc openjdk-7-jdk  
vim libfltk1.3-dev fltk1.3 fltk1.3-doc  
build-essential make git
```

3) Pull EnDaBi Sources from Github

In Terminal, Type :

```
git clone https://github.com/EnDaBi/EnDaBi.git
```


4) Navigate to the EnDaBi Folder

In Terminal, Type :

```
cd EnDaBi
```

5) Compile the Sources

In Terminal, Type :

```
make
```

6) Run Demos

In Terminal, Type :

```
./ENDABI_RSA_DEMO_GUI
```

or

```
./ENDABI_RSA_DEMO
```

What's Next

Regarding the RSA Core :

- 1) Implementing the Chinese Remainder Theorem for Faster Private Key Exponentiation (Decryption).
- 2) Implementing Our Own BigInteger Library.
- 3) Implementing our Own Padding System.
- 4) Implementing Our own Primality Testing Classes.

Regarding the Project :

- 1) Adding more Encryption Schemes.
- 2) Developing Database Classes.
- 3) Developing Biometrics Classes.

References

- [1] Paar, C., and Pelzl, J.
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A Textbook for Students and Practitioners.
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www.crypto-textbook.com
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