COMS30017
COMPUTATIONAL NEUROSCIENCE

LECTURE: LEAKY INTEGRATE-AND-FIRE MODEL OF NEURON

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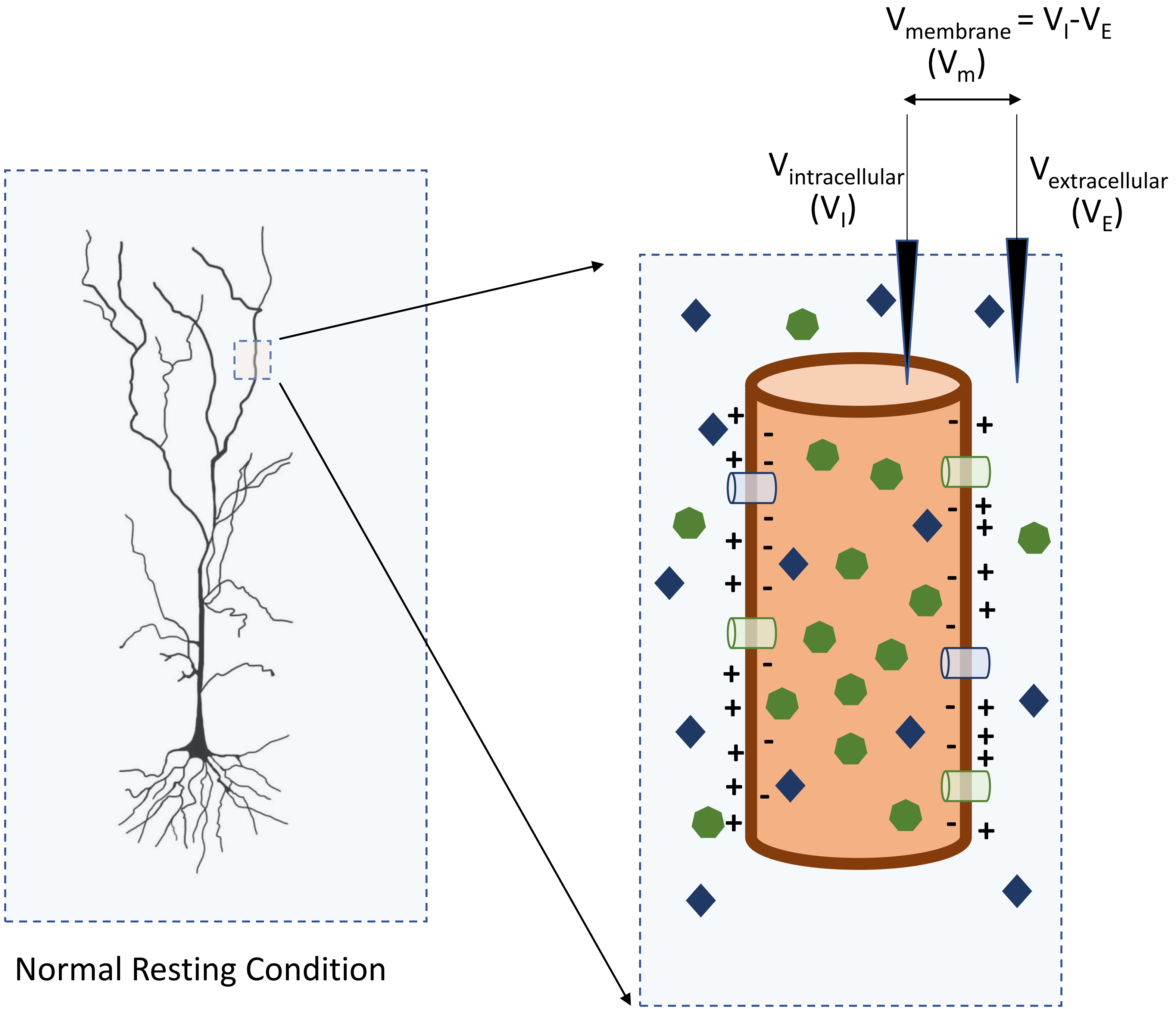
Intended learning outcomes

- Understand the systematic construction of leaky Integrate-and-Fire (LIF) model of the neuron from electrophysiology of neuron
- The working details of the LIF model
- Frequency-current (F-I) response of the LIF model
- Low-pass filtering by the LIF model



Normal Resting Condition

- More Na^+ outside than inside. Na^+ has a strong urge to go in
- More K^+ inside than outside. K^+ has a strong urge to go out
- Inside electric potential is lesser than outside potential, $\mathbf{V_m}$ is –ve
- Membrane as such does not allow the ion flow. It acts as a Capacitor, $\mathbf{C_m}$
- Only ion channels (Na^+ channels and K^+ channels) would allow flow of ions in an ion-specific manner.
- Channels act as passages for current flow and, thus, have a certain resistance **R** to the current flow.
- Both the channels' conductances are influenced by V_m: **Voltage-gated channels!!**

Voltage-dependent behaviours of Na^+ and K^+ channels: Nonlinear switching

- At normal resting condition with V_m -60 to -70mV, more K^+ channels are open than Na+ channels: **Resting membrane potential V_{rest}**

- External supply of positive current $\mathbf{I_{ext}}$ to the inside of neuron makes Vm to reach a threshold potential Vth -50 to -55mV: **Reaching the threshold potential**

- A sudden nonlinear switching in situation happens with many open Na^+ channels at threshold. Strong flow of Na^+ happen towards inside, making inside more positive than outside: **Generation of Action potential**.

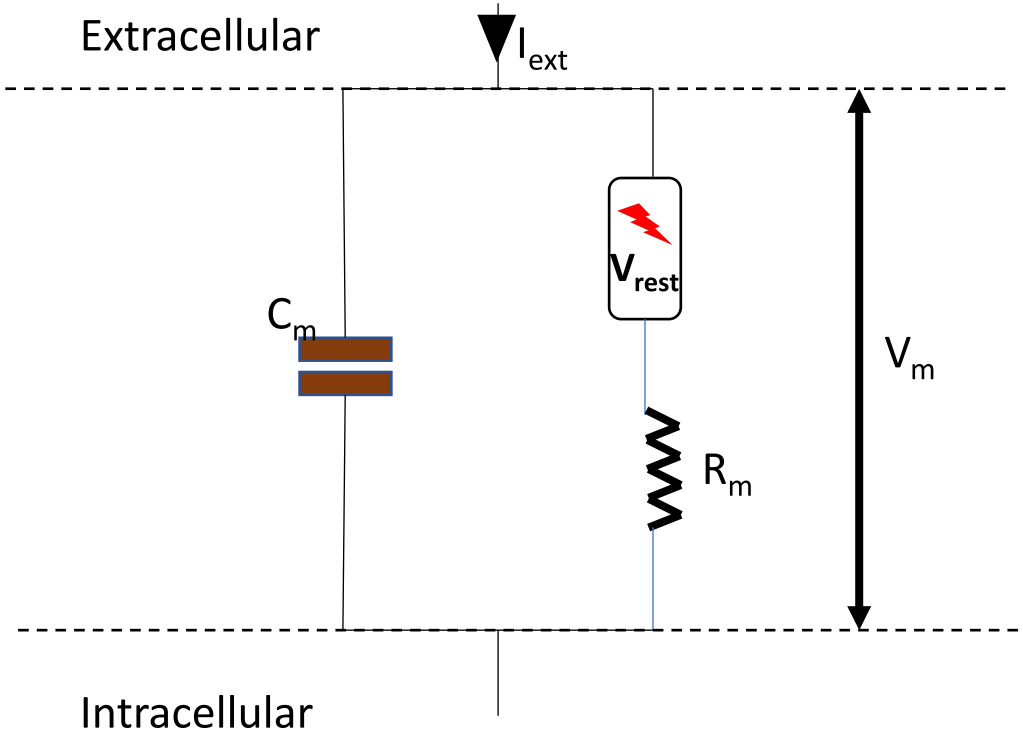
- After a while (1 ms), Na^+ channels start closing again, plugging the inward Na^+ flow. Open K^+ channels boost loss of +ve ions from inside towards outside, bringing back the V_m to the resting condition: **Reset of the membrane potential**.

- The event of action potential generation involves nonlinear switching dynamics in the channels' conductance.

- The famous **Hodgkin-Huxley Model** deals with the entire dynamics using a complex set of **nonlinear differential equations**

- Everything at least upto the V_{th} follows a very simple linear dynamics: **Linear Subthreshold Dynamics**

Electrical RC circuit model of the Subthreshold Dynamics



$$C_m \frac{dV_m}{dt} = \frac{1}{R_m}(V_{rest} - V_m) + I_{ext}$$

$$\Rightarrow C_m R_m \frac{dV_m}{dt} = (V_{rest} - V_m) + R_m I_{ext}$$

$$\Rightarrow \tau_m \frac{dV_m}{dt} = (V_{rest} - V_m) + R_m I_{ext}$$

with $\tau_m = C_m R_m$, the membrane time constant

$$\frac{dV_m}{dt} = \underbrace{\frac{1}{\tau_m}(V_{rest} - V_m)}_{\text{Leak}} + \underbrace{\frac{R_m}{\tau_m} I_{ext}}_{\text{Excitation}}$$

Leak: making V_m always to relax back to V_{rest}

Excitation: making V_m to rise from V_{rest}

Some typical magnitudes and units to remember

V_m: in millivolts (mV)

V_{th}: -55mV

V_{rest}: -70mV

I_{ext}: in pA (pico-Ampere)

C_m : 100pF (pico-Farad)

R_m : 100MΩ (Mega-Ohm)

τ_m :10ms

Can you check whether $\tau_m = R_m C_m$ turns out to be 10ms? Particularly, the unit of ms?

Leaky Integrate-and-Fire (LIF) Model (1907)

Louis Lapique

- The leaky integrate-and-fire neuron model has two key components:

1. An equation describing the voltage dynamics.

$$\tau_m \frac{dV_m}{dt} = (V_{rest} - V_m) + R_m I_{ext}$$

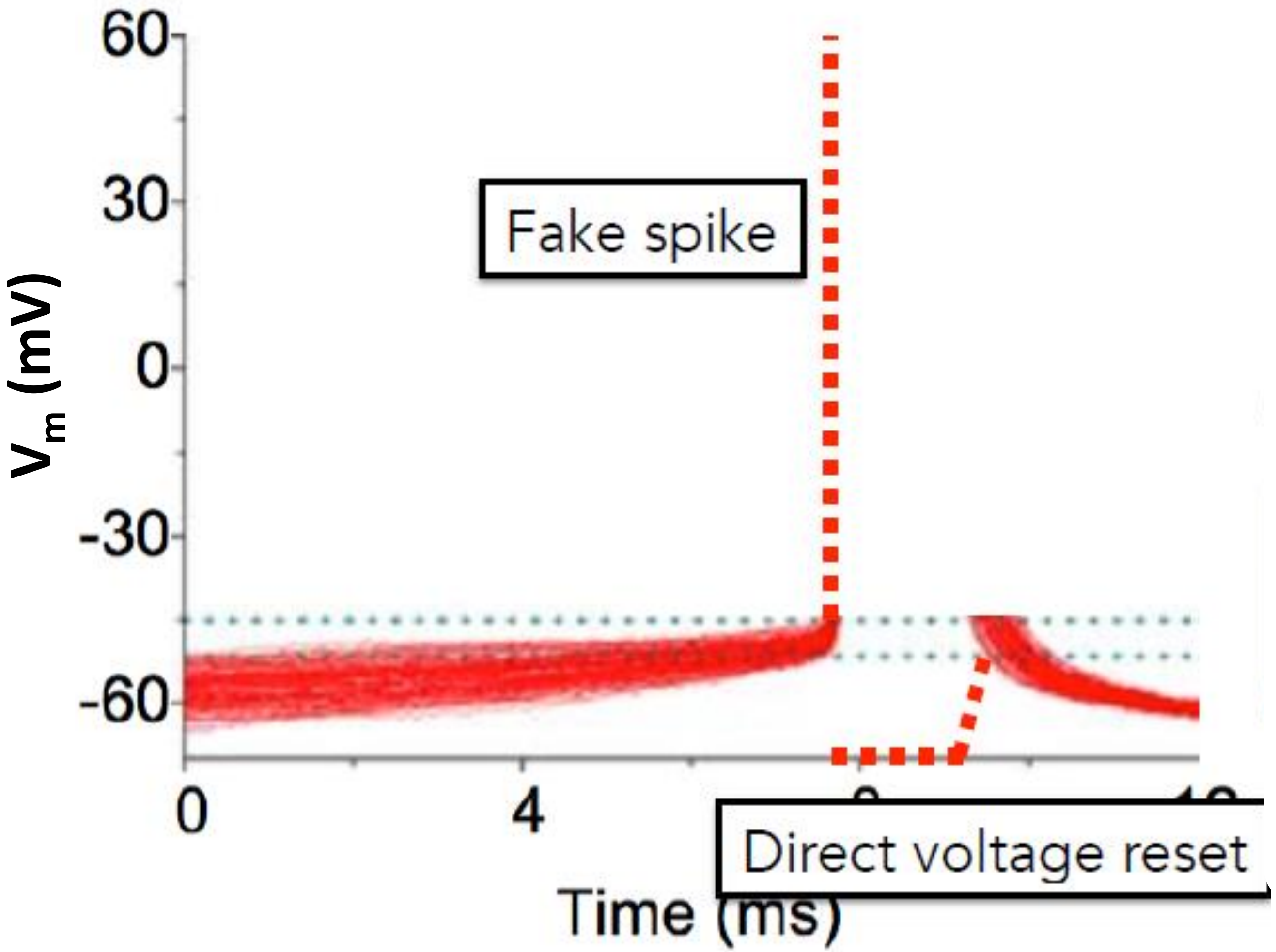
2. A voltage-reset mechanism, mimicking a spike.

If $V_m \geq V_{th}$: $V_m \rightarrow V_{rest}$, and a spike is realized in an ad hoc manner

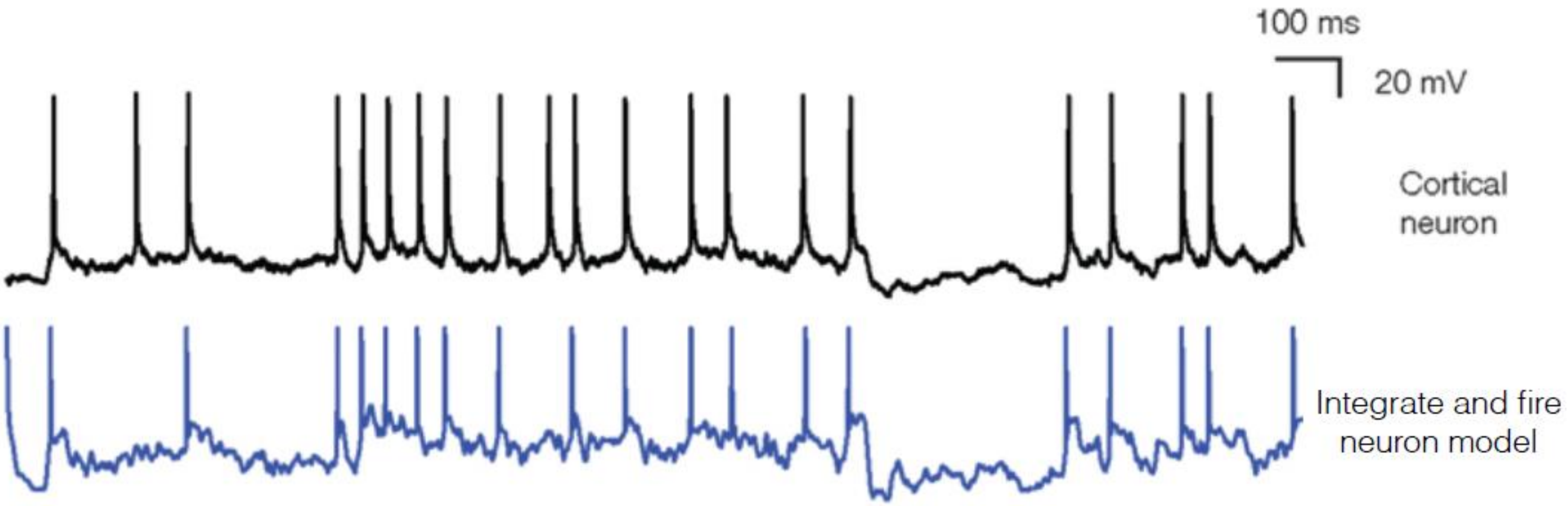
- The name is a bit misleading, the LIF model doesn’t actually generate any spikes.

- The LIF is heavily used in computational neuroscience because of its simplicity and analytical tractability.

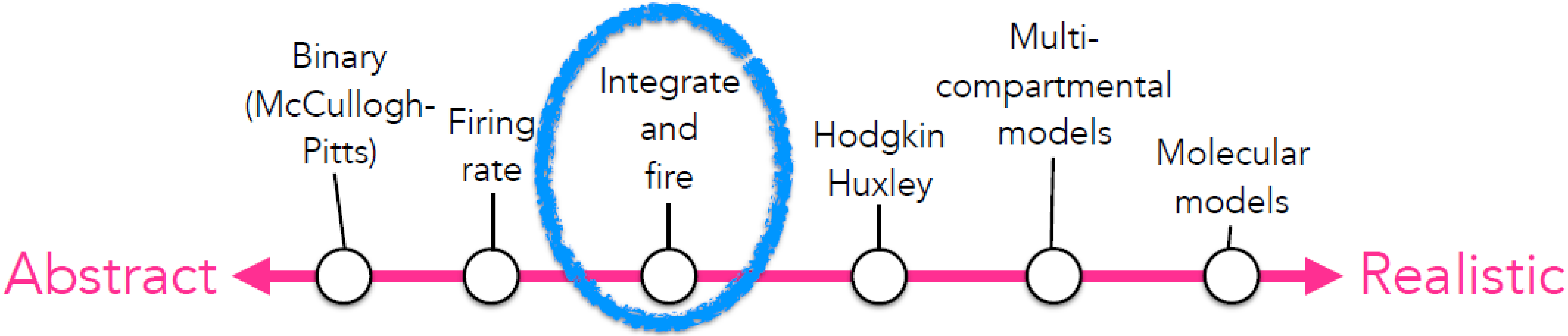
Leaky Integrate-and-Fire (LIF) Model



Success of Leaky Integrate-and-Fire (LIF) Model



Rossant et al., *Frontiers in Neurosci* (2011)



Abstract models		Realistic models
Simple	vs	Detailed
Hard to relate to biology	vs	Contains stuff you could measure
Few parameters	vs	Lots of parameters
Fast simulation	vs	Slow simulation
Mathematical analysis	vs	Intractable
Generic	vs	Specific

Remember the differential equation lecture for analytical solution !!!

$$\tau_m \frac{dV_m}{dt} = (V_{rest} - V_m) + R_m I_{ext}, \qquad V_m(t=0) = V_{rest}$$

Where I_{ext} is a constant input

Analytical Solution ⇒ $$V_m(t) = V_{rest} + R_m I_{ext}\left(1 - e^{-\frac{t}{\tau_m}}\right)$$

Asymptotic Steady State
$\boldsymbol{t \to \infty}$ ⇒ $$V_m(t) = V_{rest} + R_m I_{ext}$$

Time to reach the steady state is governed by $\boldsymbol{\tau_m}$: Larger is the time constant, slower will be the act of reaching to steady state

Remember the numerical method lecture for solving ODEs !!!

$$\tau_m \frac{dV_m}{dt} = (V_{rest} - V_m) + R_m I_{ext}, \qquad V_m(t=0) = V_{rest}$$

Euler Method:

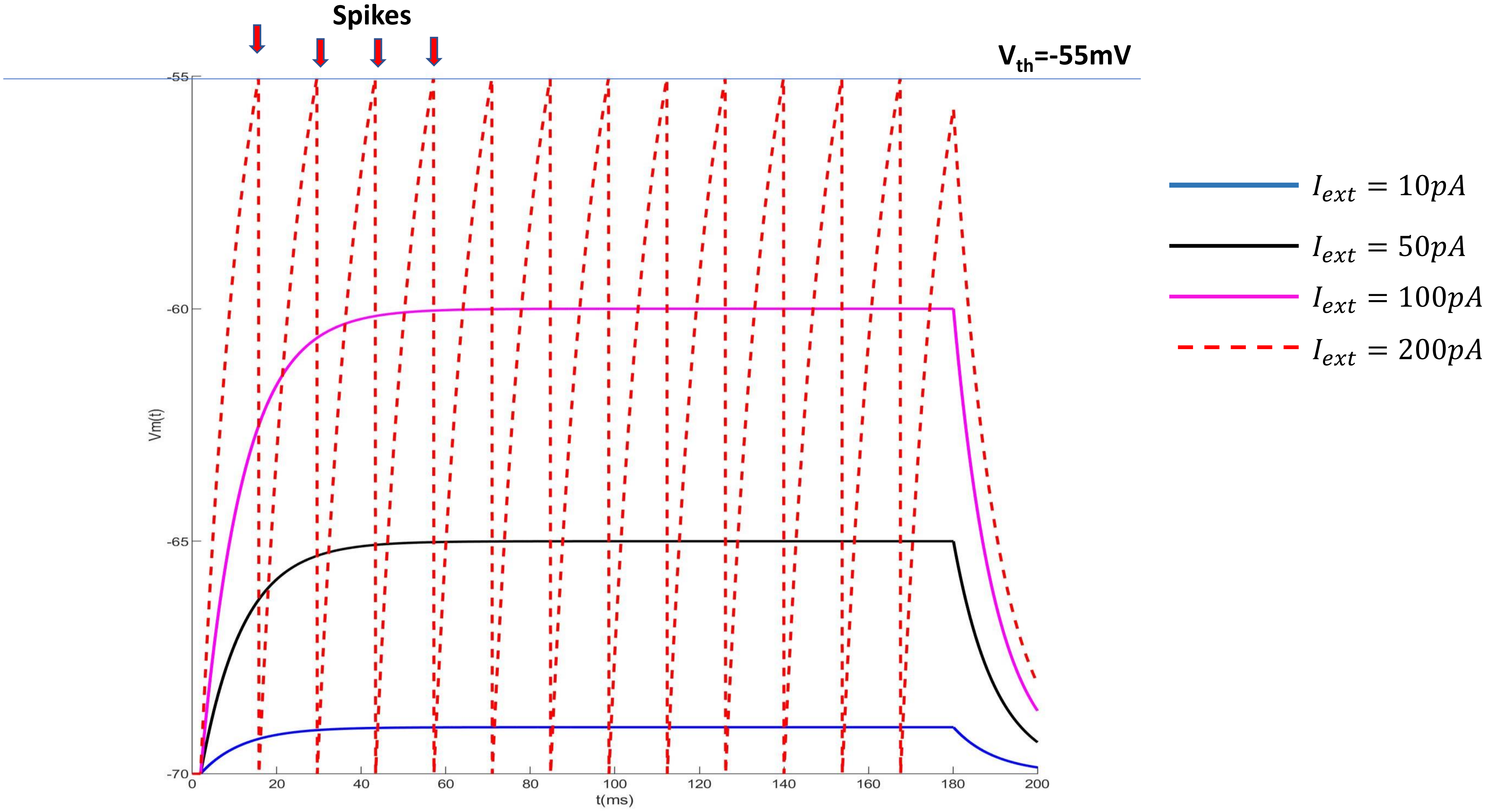
$$V_m(t_0 + \Delta t) = V_m(t_0) + \Delta t.\left(\tau_m\left(\left(V_{rest} - V_m(t_0)\right) + R_m I_{ext}\right)\right)$$

$$\Downarrow$$

$$\Downarrow$$

$$\Downarrow$$

$$V_m(t_0 + n\Delta t) = V_m(t_0 + (n-1)\Delta t) + \Delta t.\left(\tau_m\left(\left(V_{rest} - V_m(t_0 + (n-1)\Delta t)\right) + R_m I_{ext}\right)\right)$$



Spikes
V_{th}=-55mV
-55
-60
-65
-70
Vm(t)
0
20
40
60
80
100
120
140
160
180
200
t(ms)
$I_{ext} = 10pA$
$I_{ext} = 50pA$
$I_{ext} = 100pA$
$I_{ext} = 200pA$