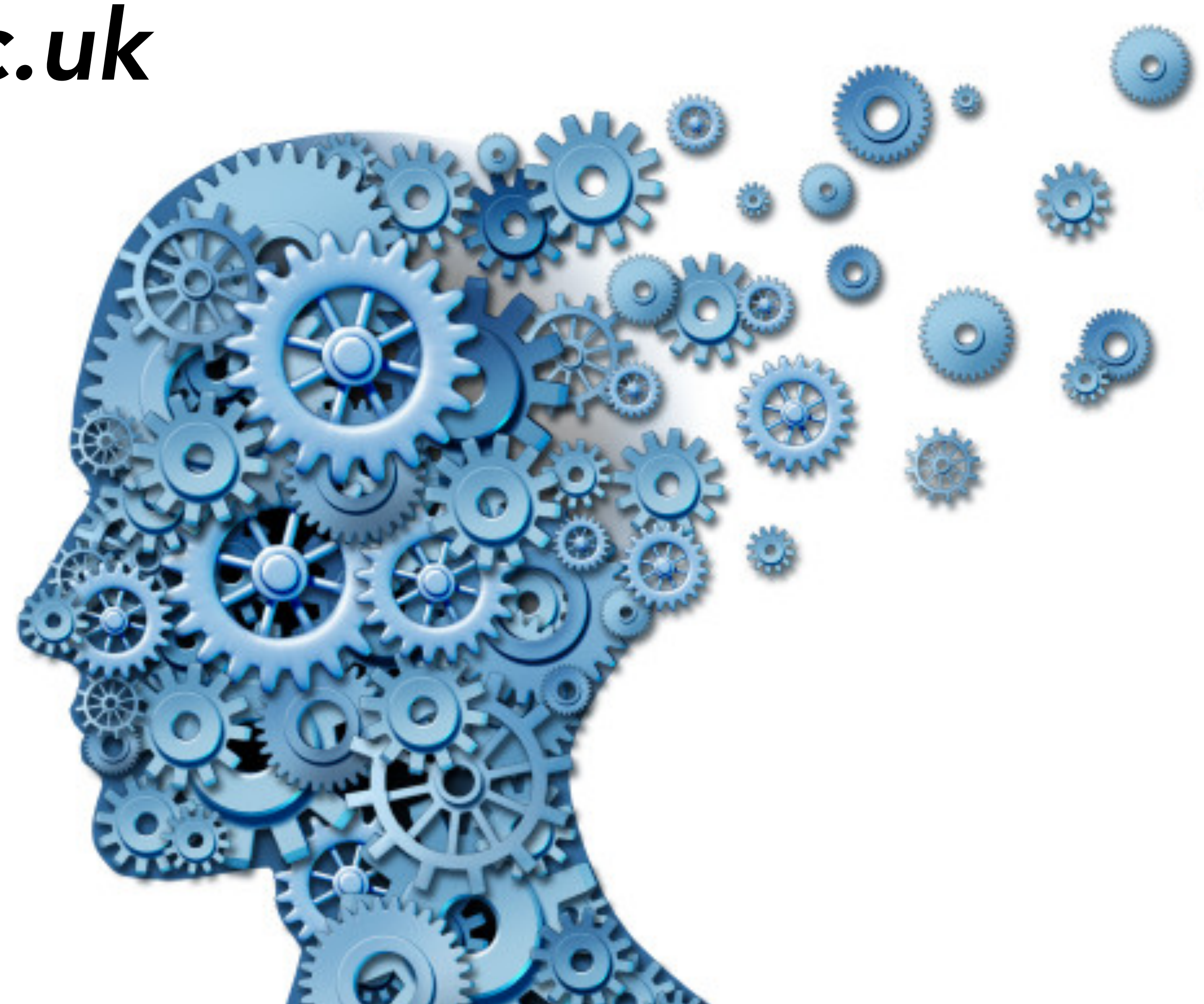
COMS30017
Computational Neuroscience

Week 5 / Video 4 / Hopfield networks (discrete attractors)

Dr. Laurence Aitchison

laurence.aitchison@bristol.ac.uk

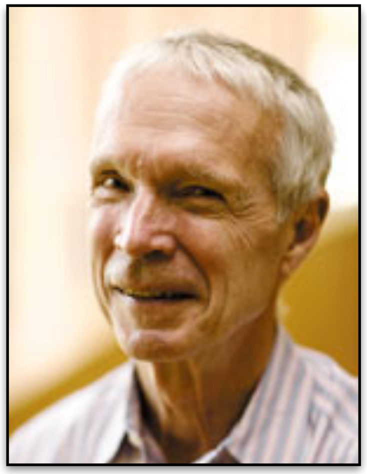


slides adapted from Dr. Cian O'Donnell

Intended Learning Outcomes

- Hopfield network evolution and energy function
- Hebbian plasticity and the Hopfield network

Hopfield networks



John Hopfield

- A Hopfield network is a recurrent network of MP neurons.
- Proposed by John Hopfield in 1982.
- The network state evolves dynamically, typically toward some "attractor" state.
- A simple synaptic plasticity rule can imprint attractors in the network weights.
- Forms a basic model of associative memory recall.
- Incredibly influential model in the history of computational neuroscience (attracted a generation of physicists to the field).

Hopfield networks

- evolves as: $$x_i(t+1) = \begin{cases} +1 & \text{if } 0 < \sum_j w_{ij} x_j(t) - \theta \\ -1 & \text{otherwise} \end{cases}$$
- There are two common flavours: synchronous or asynchronous updates.
- Usually the weights are symmetric: $w_{ij} = w_{ji}$ and the connectivity is all-to-all.
- The network dynamics evolve to minimise an "energy": $$E = -\frac{1}{2} \sum_{ij} w_{ij} x_i x_j + \sum_i \theta_i x_i$$

Hopfield network dynamics

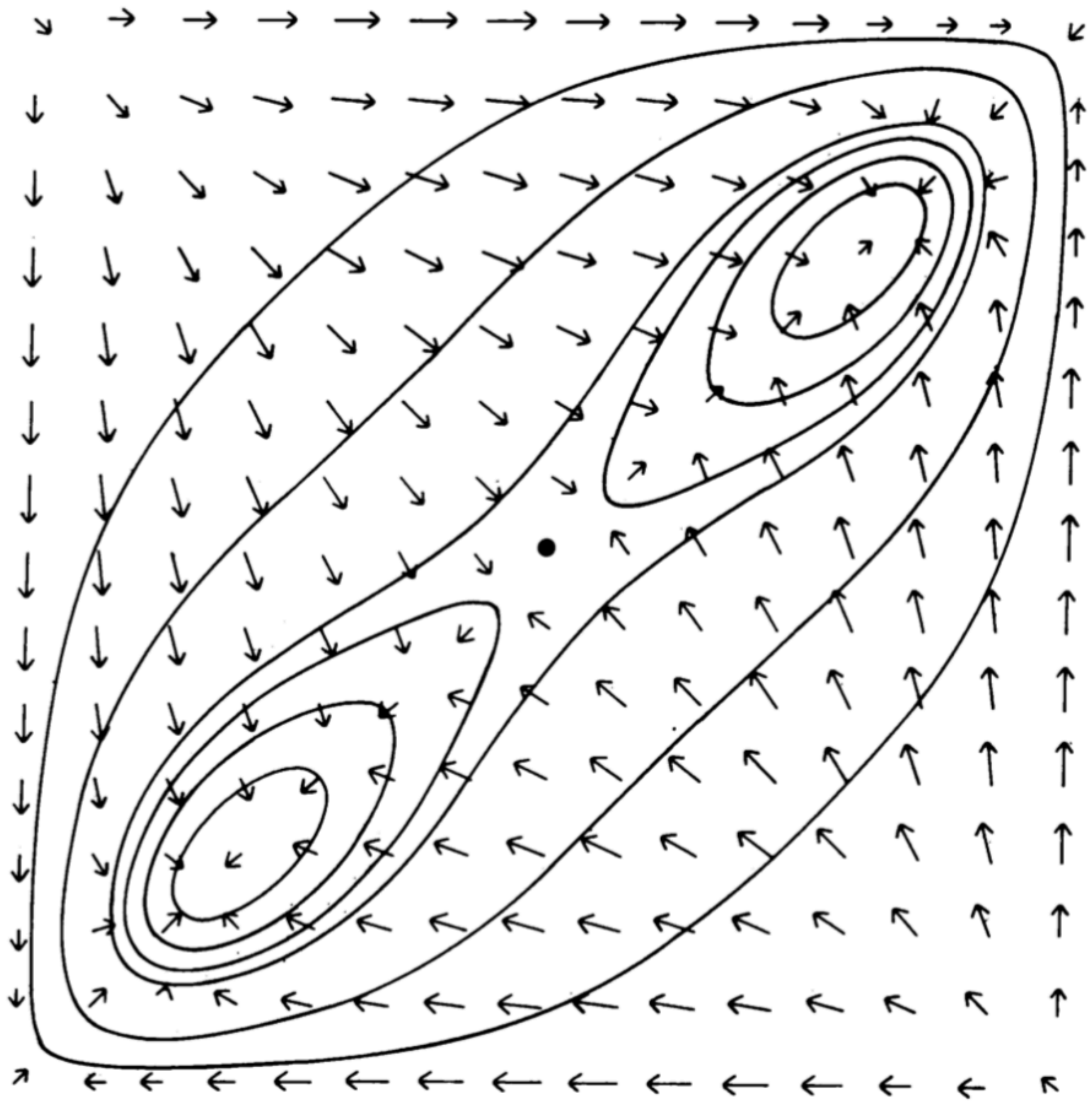


FIG. 3. An energy contour map for a two-neuron, two-stable-state system. The ordinate and abscissa are the outputs of the two neurons. Stable states are located near the lower left and upper right corners, and unstable extrema at the other two corners. The arrows show the motion of the state from Eq. 5. This motion is not in general perpendicular to the energy contours. The system parameters are $T_{12} = T_{21} = 1$, $\lambda = 1.4$, and $g(u) = (2/\pi)\tan^{-1}(\pi\lambda u/2)$. Energy contours are 0.449, 0.156, 0.017, −0.003, −0.023, and −0.041.

Hopfield, *PNAS* 1984

- Each of local minima in the energy landscape is known as an "attractor".
- The network can do pattern completion: retrieving the full pattern from a partial cue.
- The capacity of the network, or maximum number of attractors, is $\sim 0.14N$.

Learning attractors in Hopfield networks

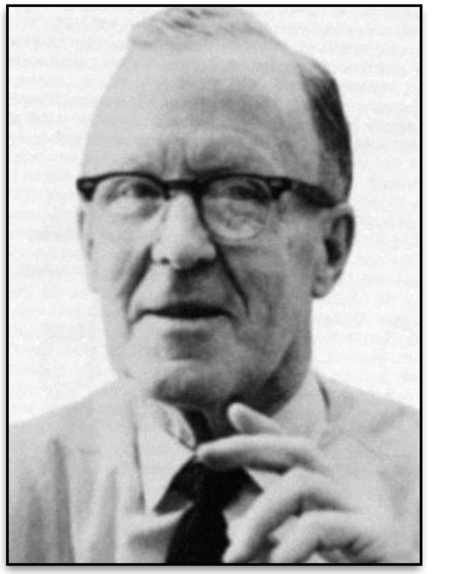
We can imprint desired attractor states into the synaptic weights by using a learning rule:

$$w_{ij} = \frac{1}{P} \sum_{a=1}^{P} x_i^a x_j^a$$

P is the total number of attractors to store.

a indexes the attractor activity pattern in the sum.

Hebbian plasticity

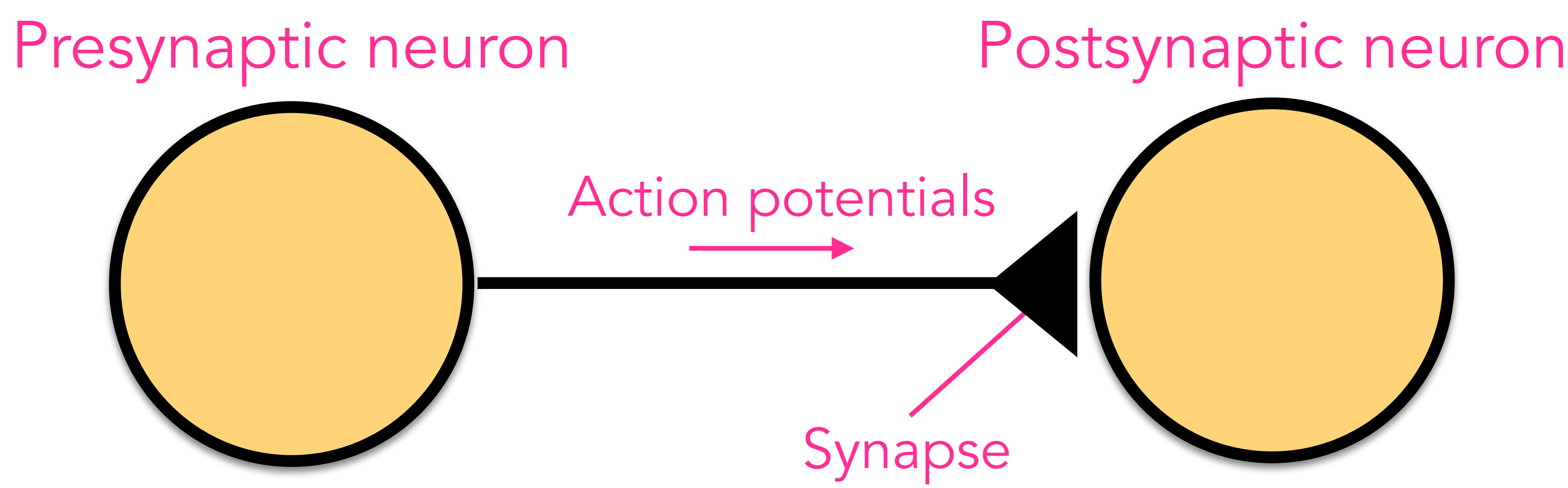


Donald Hebb

"When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased."
— Donald Hebb (1949)

a.k.a. "neurons that fire together wire together."

Hebbian plasticity



- Many synapses in the brain strengthen if their pre- and post-synaptic neurons are simultaneously active for a sufficient period of time.
- This strengthening process is called “long-term potentiation”.
- There are other stimulation patterns that can weaken synapses (the corresponding process is called “long-term depression”).
- It is a candidate brain mechanism for associative learning.
- It is also embodied in the Hopfield network learning rule.

Limitations of the Hopfield network

- Binary neurons (no spikes)
- Symmetric connectivity
- No excitatory and inhibitory cells (a single cell's output can be both)
- Discrete temporal dynamics

References

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- Hebb, D.O. (1949) The Organization of Behavior. Wiley, New York.

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