

## Solutions to questions from week 2

### **Introduction to Differential Equations pt. 1**

1. Dimensionality = 1  
Order = 2
2. Non-homogeneous.
3. Nonlinear.
4.  $\frac{dx}{dt} = \frac{m}{\rho}x + \frac{2}{\rho}t$  .
5. The integrated solution  $x(t)$  is still undetermined by constant. To find it, we need to define an initial value  $t_0$  to solve the equation. Thus the equation is solved in the interval  $[t_0, t]$ .

### **Introduction to Differential Equations pt. 2**

1.  $x_{ss} = 2$  .
2. It is the behaviour to which the function converges near a limit.
3. The solution is  $x(t) = 4 - 2e^{\frac{-t}{\tau}}$ . The higher the value of  $t$ , the lower the value of  $e^{\frac{-t}{\tau}}$ , going towards zero. Therefore,  $x(t \rightarrow \infty) = 4$ .
4. Linear differential equations with a time-varying input tend to follow the input. However, if the input is changed at much faster rate than the differential equation, it will only be able to follow the 'slower' components of the input. Higher frequency components come and go too fast, before the differential equation can change. As a result the input is said to be 'averaged out' through a low-pass filter, as the low-frequency components exert more influence over the solution.

### **Numerical Methods for Differential Equations pt. 1**

1. Nonlinear.
2. A Taylor series is an infinite expansion.  
If terms of fifth and higher orders are ignored, the truncation error is of order 5.
3. -False  
-True  
-False  
-False  
-False

## Numerical Methods for Differential Equations pt. 2

1. *smaller*
2. The solution can be calculated faster. Considering a maximal ‘tolerance’ for the error: smaller timesteps are used only in intervals where the estimated error is large, otherwise larger timesteps can be used.

## Leaky Integrate-and-Fire Model of Neuron pt. 1

1. *more*
2. Between  $-60$  to  $-70\text{mV}$ .
3.  $V_m = V_i - V_e = -70\text{mV}$
4. Typical  $V_{th}$  is between  $-50$  and  $-55\text{mV}$ .  
LIF follows a linear subthreshold dynamics.
5. pF (picoFarad)
6. When the membrane voltage ( $V_m$ ) reaches the threshold value ( $V_{th}$ ), it is reset to the resting value ( $V_{rest}$ ) and a spike is registered to have been generated. The LIF model itself doesn’t generate any action potential dynamics.

## Leaky Integrate-and-Fire Model of Neuron pt. 2

1. It is the minimal external current necessary to induce activity (action potentials) in the neuron.
2. The time constant.
3. Its dynamics will follow the input more and more closely.
4. The Impedance, which is equal to the ratio between the voltage amplitude and the current amplitude.
5. Defined by the membrane resistance  $R_m$ .
6. - The introduction of a refractory period;  
-A mechanism that implements spike frequency adaptation;  
-A non-fixed value for the spiking threshold;