Increasing Calculation Capacity By Number Theoretic Extension

1. Multidimensional numbers

"..to divide divine."

Tomas Ukkonen, 2015

We extend our real number system $\mathbb{R}$ by having dimensional basis. Instead of pure scalar α, we define dimensional numbers $\alpha\, r^d$ with dimension d which can be any real number (including negative numbers) but which we restrict here to be integer number.

Motivation

This approach can be motivated by trying to measure dimensionality of Koch's snowflake fractal (with real numbered dimension).

If we look at one side of the fractal we can see that it is self-repeating, one "measure stick length" of fractal can be used four times to measure one side of the snowflake, but equal result can be got by "stretching fractal measure stick" to be 3 times larger (longer). This leads into equation

$$4\, r^d = (3\, r)^d$$

And solving this for $d = \log(4) / \log(3) = 1.2619$ leads into real numbered dimension higher than one. It is then no wonder that length of snowflake (or fractal's) curve is infinite. The dimension is larger than one and we are measuring with "1d measure stick" $\alpha\, r^{1.2619} / r^1 = \alpha\, r^{0.2619}$ and our measurement in dimension 1 is infinite ($r^\beta = \infty$ when $\beta \neq 0$).

Another example: integration through sum.

Another idea: black holes in physics are generated when mass increases to "infinity". In our theory this means that dimension of the system increases (like 3d fractal). In higher dimensions $(7+)$ surface area of a unit hypersphere becomes smaller and smaller meaning that more and more volume can fit to the same surface area (most of the volume is near edges and volume of center is insignificantly small). This then means that things are more and more separated from each other (all states are different). On the other extreme, in a perfect vacuum (quantum physics), mass goes to zero. This could then be intepreted as "subzero" values meaning that dimension of the objects become smaller than 1. This means that surface area of sub 1-dimensional "fractal" unit sphere (proof?) becomes smaller and smaller with same volume and more importantly things are more and more concentrated to the same point meaning that things and states are inseparable.

Mathematics (discrete computer implementation)

We can define our multidimensional numbers (integer dimensions) as a vector sum $a = \sum_{i=0}^{D-1} \alpha_i r^i$. Here we can see this is not only "polynomial" but also vector space because dimensions r^i are perpendicual to each other, there is no real number α which can be used get into another dimension (only way to get into another dimension through division by zero or by process which leads to infinity: $1 / \alpha, \alpha \rightarrow 0$ or $\alpha, \alpha \rightarrow \infty$). The way to get from higher dimensions into lower dimensions is through division by infinity: $1 / \alpha, \alpha \rightarrow \infty$. Additionally, the number of dimensions is restricted to a **prime number** P in order to create closed system where calculation of inverse is always possible. We now define addition and multiplication as follows:

$$a + b = \sum_{i=0}^{P} (\alpha_i + \beta_i) r^i$$

$$a\, b = \sum_{i,j} \alpha_i\, \beta_j\, r^{(i+j) \bmod P}$$

By resticting exponents to a modular arithmetic we can create a number system which is well defined and always works. This also means that "division by zero" is always mathematically well defined and all numbers (including "zero") has always somekind of inverse although by restricting the number of dimensions to P means that higher dimensions can "overflow" back to lower dimensions and results into unintuitive results.

Required: proof that number system defined this way works. (requires Fermat's theorem and discrete mathematics). I did this earlier but lost documents. Partial mini proof: multiplication of $r^i\,r^j = r^{(i+j) \bmod P}$ leads to number system which have always inverse and is well defined (normal modular arithmetic). Therefore multiplication of r^d:s are well defined. The right name for this kind of numbers are ***polynomial rings*** which are well-studied in discrete mathematics.

Algorithm for division

Next we want algorithm to calculate a^{-1} in order to divide a/b (even by zero). We notice that the circular structure used in multiplication is similar to discrete finite Fourier transform which also uses circular buffers/circular convolution. Therefore somekind of transform approach could maybe make sense.

TODO: implement code which creates working multidimensional numbers

`dinrhiw2/math/modular.h` has integer modular integer code which can be used to implement superdimensional numbers. Modify `dinrhiw2/math/superrresolution.h` to work with modular integer math of polynomial exponents. Implement circular convolution and solve multiplication and inverse using Fourier transform and inverse Fourier transform (KissFFT has BSD style license).

Write testcase to check multidimensional number code work always (real and complex number coefficients) and then use it as datatype for `nnetwork<>`.

2. Extending Neural Network Capacity

After showing multidimensional numbers fulfills number theoretic properties of **field/ring(?)**. We can use it as our neural networks numbers and test the improved computing capacities of multidimensional neural network.

- how to calculate derive and calculate gradient of multidimensional numbers(?). [modular arithmetic loses its capacity to compare largeness of numbers so in general we cannot say functions are continuous in a typical sense. We can define gradient in each dimension d but what are requirements for general differentiability(?). In complex analysis this is already a problem.

- initially implement random search and compare its results to real number version. Multiplications are now very non-linear ("cryptographic" non-linearities) so learning functions should be complicated and gradient descent may not work very well(??).