$\boxed{5.14}$ NMOS, $t_{ox} = 9\,nm$, $\mu_n = 500\ \frac{cm^2}{V\cdot s}$, $V_t = 0.7\,V$, $W/L = 10$

FIND DRAIN CURRENT FOR THE FOLLOWING CASES:

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}\ F/m}{9 \times 10^{-9}\ m} = 0.003833\ F/m^2 = 3.833 \times 10^{-7}\ F/cm^2$$

a) $v_{GS} = 5\,V,\ v_{DS} = 1\,V$

$v_{GS} - V_t = 5 - 0.7 = 4.3\,V$

$v_{DS} < v_{GS} - V_t$

$1\,V < 4.3\,V \Rightarrow$ TRIODE RGN.

$$i_D = \mu_n C_{ox} \frac{W}{L}\left[(v_{GS} - V_t)\, v_{DS} - \tfrac{1}{2} v_{DS}^2\right]$$

$$i_D = \left(500\ \frac{cm^2}{V\cdot s}\right)\left(3.833\times10^{-7}\ \frac{F}{cm^2}\right)(10)\left[(5-0.7)\,1 - \tfrac{1}{2}(1)^2\right]$$

$$i_D = \left(0.00191667\ \frac{F}{V\cdot s}\right)(4.3 - 0.5)\ V^2 = 0.0072833\ A = \boxed{7.283\ mA}$$

b) $v_{GS} = 2\,V,\ v_{DS} = 1.3\,V$

$v_{GS} - V_t = 2 - 0.7 = 1.3\,V$

$v_{DS} = v_{GS} - V_t$

$1.3\,V = 1.3\,V \Rightarrow$ SATURATION RGN.

$$i_D = \tfrac{1}{2} k_n' \left(\frac{W}{L}\right)(v_{GS} - V_t)^2, \quad k_n' = \mu_n C_{ox} = 191.67 \times 10^{-6}\ \frac{F}{V\cdot s} = 191.67 \times 10^{-6}\ \frac{A}{V^2}$$

$$i_D = \tfrac{1}{2}\left(191.67\times10^{-6}\ \frac{A}{V^2}\right)(10)(2-0.7)^2 = 1.62\times10^{-3}\ A = \boxed{1.62\ mA}$$

c) $v_{GS} = 5\,V,\ v_{DS} = 0.2\,V$

$v_{GS} - V_t = 5 - 0.7 = 4.3\,V$

$v_{DS} = 0.2\,V < 4.3\,V \Rightarrow$ TRIODE RGN.

$$i_D = k_n'\left(\frac{W}{L}\right)\left[(v_{GS} - V_t)\, v_{DS} - \tfrac{1}{2} v_{DS}^2\right] = \left(191.67\times10^{-6}\ \frac{A}{V^2}\right)(10)\left[(5-0.7)(0.2) - \tfrac{1}{2}(0.2)^2\right]$$

$$\boxed{i_D = 1.61\ mA}$$

(d) $v_{GS} = v_{DS} = 5\,V$

$v_{GS} - V_t = 5 - 0.7 = 4.3\,V$

$v_{DS} > v_{DS} - V_t \Rightarrow 5\,V \geq 4.3\,V \Rightarrow$ SATURATION RGN.

$$i_D = \tfrac{1}{2} k_n'\left(\frac{W}{L}\right)(v_{GS} - V_t)^2 = \tfrac{1}{2}\left(191.67\times10^{-6}\ A/V^2\right)(10)(5 - 0.7\,V)^2 = \boxed{17.7\ mA}$$

$\boxed{5.17}$ NMOS, $V_t = 1\,V$

TRIODE OPERATION AND $v_{DS} \equiv$ SMALL.

@ $V_{GS} = 1.5\,V$ $r_{DS} = 1\,k\Omega$

$$I_D = k_n'\left(\frac{W}{L}\right)\left[(V_{GS} - V_t)\,v_{DS} - \frac{1}{2}v_{DS}^2\right], \quad v_{DS} \equiv \text{SMALL} \Rightarrow I_D = k_n'\left(\frac{W}{L}\right)(V_{GS} - V_t)\,v_{DS}$$

$$\therefore \quad r_{DS} = \frac{v_{DS}}{I_D} = \frac{1}{k_n'\left(\frac{W}{L}\right)(V_{GS} - V_t)}$$

@ $V_{GS} = 1.5\,V$, $r_{DS} = 1\,k\Omega$,

$$1\,k\Omega = \frac{1}{k_n'\left(\frac{W}{L}\right)(1.5 - 1)} \Rightarrow k_n'\left(\frac{W}{L}\right) = \frac{1}{(1\,k\Omega)(0.5\,V)} = 2\,\frac{mA}{V^2}$$

IF r_{DS} IS TO BE $200\,\Omega$ $(0.2\,k\Omega)$, WHAT SHOULD V_{GS} BE?

$$0.2\,k\Omega = \frac{1}{(2\,mA/V^2)(V_{GS} - 1)} \Rightarrow V_{GS} - 1 = \frac{1}{(2\,\frac{mA}{V^2})(0.2\,k\Omega)} = 2.5\,V$$

$$\boxed{V_{GS} = 3.5\,V}$$

IF DEVICE HAS WIDTH OF $2W$, WHAT ARE CORRESPONDING VALUES OF r_{DS}?

$$k_n'\left(\frac{2W}{L}\right) = 2\left(2\,\frac{mA}{V^2}\right) = 4\,\frac{mA}{V^2}$$

FOR $V_{GS} = 1.5\,V \Rightarrow r_{DS} = \dfrac{1}{\left(4\,\frac{mA}{V^2}\right)(1.5 - 1\,V)} = \boxed{0.5\,k\Omega = 500\,\Omega}$

FOR $V_{GS} = 3.5\,V \Rightarrow r_{DS} = \dfrac{1}{(4\,mA/V^2)(3.5 - 1\,V)} = \boxed{0.1\,k\Omega = 100\,\Omega}$

$\boxed{5.18}$ $V_t = 0.5\,V$, $k_n'\left(\frac{W}{L}\right) = 0.1\ \frac{mA}{V^2}$ } OPERATED IN SATURATION @ $i_D = 12.5\,\mu A$

FIND v_{GS} AND MINIMUM REQUIRED v_{DS} ($= v_{DS_{min}}$).
REPEAT FOR $i_D = 50\,\mu A$.

IN SATURATION, $i_D = \frac{1}{2} k_n'\left(\frac{W}{L}\right)\left(v_{GS} - V_t\right)^2$, $k_n'\left(\frac{W}{L}\right) = 0.1\ \frac{mA}{V^2} = 100\ \frac{\mu A}{V^2}$

$$12.5\,\mu A = \frac{1}{2}\left(100\ \frac{\mu A}{V^2}\right)\left(v_{GS} - 0.5\right)^2$$

$$\left(v_{GS} - 0.5\right)^2 = \frac{12.5\,\mu A}{\frac{1}{2}\left(100\ \frac{\mu A}{V^2}\right)} = 0.25\ V^2$$

$$v_{GS} - 0.5 = \pm 0.5\,V \implies v_{GS} = 1.0\,V,\ 0\,V$$

$$\therefore \boxed{v_{GS} = 1.0\,V} \quad \left(v_{GS} \geq V_t\right)$$

$$v_{DS_{min}} = v_{GS} - V_t = 1.0 - 0.5 = \boxed{0.5\,V = v_{DS_{min}}}$$

$$i_D = 50\,\mu A = \frac{1}{2}\left(100\ \frac{\mu A}{V^2}\right)\left(v_{GS} - 0.5\right)^2 = \left(v_{GS} - 0.5\right)^2 = \frac{50\,\mu A}{\frac{1}{2}\left(100\ \frac{\mu A}{V^2}\right)} = 1.0\,V^2$$

$$v_{GS} - 0.5 = \pm 1.0\,V \implies v_{GS} = 1.5\,V,\ -0.5\,V$$

$$\therefore \boxed{v_{GS} = 1.5\,V} \quad \left(v_{GS} \geq V_t\right)$$

$$v_{DS_{min}} = v_{GS} - V_t = 1.5 - 0.5\,V = \boxed{1.0\,V = v_{DS_{min}}}$$

$\boxed{5.19}$ NMOS: $I_D = 0.4\,mA$ @ $V_{GS} = V_{DS} = 2V$ ①

$I_D = 0.1\,mA$ @ $V_{GS} = V_{DS} = 1.5V$ ②

FIND k_n & V_t FOR THIS DEVICE

$\hookrightarrow \left[k_n = k_n' \left(\frac{W}{L}\right) = \mu_n C_{ox} \left(\frac{W}{L}\right) \right]$

NOTE: WHEN $V_{DS} = V_{GS} \implies$ <u>SATURATION REGION OPERATION</u>

① $0.4\,mA = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (2V - V_t)^2$

① $0.4 = \frac{1}{2} k_n (2 - V_t)^2 \implies k_n (2 - V_t)^2 = 0.8$, $k_n = \frac{0.8}{(2 - V_t)^2}$

② $0.1 = \frac{1}{2} k_n (1.5 - V_t)^2 \implies k_n (1.5 - V_t)^2 = 0.2$

$$\frac{0.8}{(2 - V_t)^2} (1.5 - V_t)^2 = 0.2$$

$$\frac{0.8}{0.2} (1.5 - V_t)^2 = (2 - V_t)^2$$

$$4 (1.5 - V_t)^2 = (2 - V_t)^2$$

$$2 (1.5 - V_t) = \pm (2 - V_t)$$

$$3 - 2V_t = \pm (2 - V_t)$$

$$3 - 2V_t = 2 - V_t, \quad -2 + V_t$$

$3 - 2V_t = 2 - V_t$ $\qquad$ $3 - 2V_t = -2 + V_t$

$V_t = 1V$ $\qquad$ OR $\qquad$ $V_t = 1.667\,V$

FOR TRIODE <u>OR</u> SATURATION,

$V_{GS} \geq V_t$

$V_t \leq V_{GS}$ ($V_{GS} = 2V, 1.5V$ ABOVE)

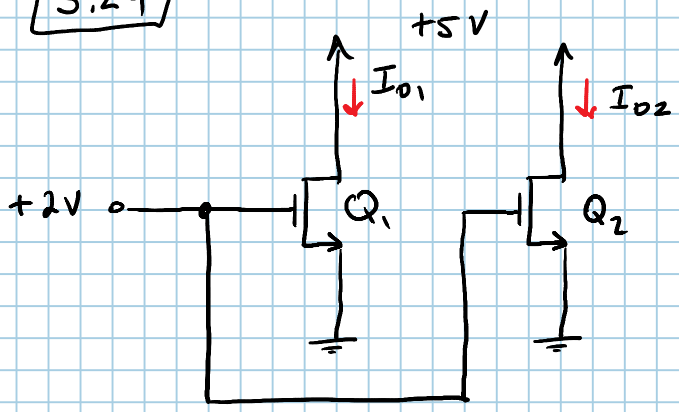
$\therefore \boxed{V_t = 1.0\,V}$

$$k_n = \frac{0.8}{(2 - 1)^2} = \boxed{0.8 \frac{mA}{V^2}}$$

$\boxed{5.27}$ $V_t = 1.0V$, NMOS $V_{GS} = V_G - V_S$, $V_{OV} = V_{GS} - V_t = V_{GS} - 1.0$, $V_{DS} = V_D - V_S$

CASE	V_S	V_G	V_D	V_{GS}	V_{OV}	V_{DS}	REGION
a	+1	+1	+2	0	-1	+1	C.O.
b	+1	+2.5	+2	+1.5	+0.5	+1	SAT
c	+1	+2.5	+1.5	+1.5	+0.5	+0.5	SAT
d	+1	+1.5	0	+0.5	-0.5	-1	C.O.
e	0	+2.5	+1	+2.5	+1.5	+1	TRI
f	+1	+1	+1	0	-1	0	C.O.
g	-1	0	0	+1	0	+1	SAT
h	-1.5	0	0	+1.5	+0.5	+1.5	SAT
i	-1.0	0	+1	+1	0	+2	SAT
j	+0.5	+2	+0.5	+1.5	+0.5	0	TRI

5.29



$Q_1 + Q_2$: MATCHED, EXCEPT FOR MAXIMUM MIS-MATCH IN $\frac{W}{L}$ OF 2%

$$k'_{n_1} = k'_{n_2} = k'_n$$

$$V_{t_1} = V_{t_2} = V_t$$

$$\frac{W_1}{L_1} = 1.02\,\frac{W_2}{L_2}$$

a)

$$I_{D_1} = \tfrac{1}{2}k_n'\left(\frac{W_1}{L_1}\right)(V_{GS}-V_t)^2 \qquad I_{D2} = \tfrac{1}{2}k_n'\left(\frac{W_2}{L_2}\right)(V_{GS}-V_t)^2$$

$$I_{D_1} = \tfrac{1}{2}k_n'\left(1.02\,\frac{W_2}{L_2}\right)(V_{GS}-V_t)^2 \qquad I_{D2} = \tfrac{1}{2}k_n'\left(\frac{W_2}{L_2}\right)(V_{GS}-V_t)^2$$

$$I_{D_1} = 1.02\underbrace{\left[\tfrac{1}{2}k_n'\left(\frac{W_2}{L_2}\right)(V_{GS}-V_t)^2\right]}_{I_{D2}} \qquad I_{D2} = \tfrac{1}{2}k_n'\left(\frac{W_2}{L_2}\right)(V_{GS}-V_t)^2$$

$I_{D_1} = 1.02\,I_{D2} \Rightarrow$ $\boxed{\text{A 2\% CHANGE IN } \frac{W}{L} \text{ PRODUCES A 2\% CHANGE IN } I_D.}$

b)

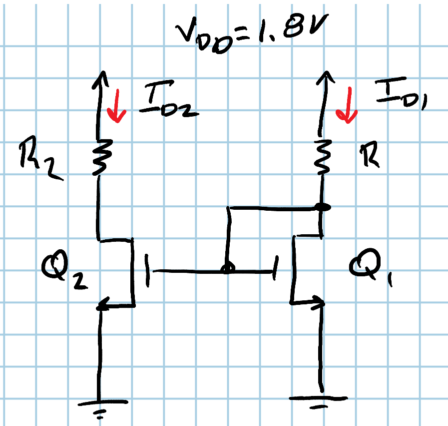
$$I_{D_1} = \tfrac{1}{2}k_n'(V_{GS}-V_t)^2 \qquad I_{D2} = \tfrac{1}{2}k_n'(V_{GS}-V_t)^2$$

$$I_{D_1} = \tfrac{1}{2}k_n'(2-1)^2 \qquad I_{D2} = \tfrac{1}{2}k_n'(2-1.01)^2$$

$$I_{D_1} = \tfrac{1}{2}k_n' \qquad I_{D2} = \tfrac{1}{2}k_n'(0.98)$$

$\therefore$ $\boxed{\text{A 10 mV MIS-MATCH IN } V_t \text{ PRODUCES A -2\% CHANGE IN } I_D}$

$\boxed{5.46}$



$V_t = 0.5V$

$k_n' = 0.4 \ mA/V^2$

$L_1 = L_2 = 0.36 \ \mu m$

$W_1 = 1.8 \ \mu m$

a) $V_{GS} = V_{DS} \Rightarrow Q_1$ IS IN SATURATION

$$I_{D1} = 90\mu A = 0.09 \ mA = \tfrac{1}{2}\left(0.4 \ mA/V^2\right)\left(\frac{1.8 \mu m}{0.36 \mu m}\right)\left(V_{GS1} - 0.5\right)^2$$

$$0.09 \ mA = \left(1 \ mA/V^2\right)\left(V_{GS} - 0.5\right)^2$$

$$\left(V_{GS} - 0.5\right) = \pm 0.3$$

$$V_{GS} = 0.5 \pm 0.3 = \underline{0.8 V}, \ 0.2 V \quad \left(V_{GS} \geq V_t\right)$$

$$V_{DS} = 0.8V \Rightarrow V_D = 0.8V$$

$$R = \frac{V_{DD} - V_D}{I_D} = \frac{1.8 - 0.8}{0.09 mA} = \boxed{11.11 \ k\Omega}$$

b) Q_2 @ EDGE OF SATURATION $\Rightarrow V_{DS2} = V_{GS2} - V_t$

$$V_{GS2} = V_{GS1} = 0.8 V$$

$$V_{DS2} = 0.8 - 0.5 = 0.3V \ (= V_{D2})$$

$$R_2 = \frac{V_{DD} - V_{D2}}{I_{D2}} = \frac{(1.8 - 0.3)^V}{0.9 \ mA} = \frac{1.5 \ V}{0.9 mA} = \boxed{1.67 \ k\Omega}$$

$$0.9 mA = \tfrac{1}{2}\left(0.4 \ mA/V^2\right)\left(\frac{W_2}{0.36 \mu m}\right)\left(0.8 - 0.5\right)^2 \Rightarrow \boxed{W_2 = 18 \ \mu m}$$