# Attitude and Heading Reference System for Quadrotor Based on

# MEMS Sensors

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Abstract—This paper deals with designing and implementation of Attitude and Heading Reference System (AHRS) which based on data fusion of multi-sensor. And it presents an approach of designing a low cost AHRS which used Micro Electronic Mechanical System (MEMS). In our approach, AHRS is made up of a 3-axes MEMS accelerometers, a 3-axes MEMS rate-gyros, an electronic compass and a FPGA which is used as the signal processor. Furthermore, a rotation quaternion and a Kalman Filter were the options as multi-sensor data fusion and integration. Finally, an experiment was introduced to analyze the attitude data carried on quadrotor. The simulation test shows when the carrier is in dynamic environment, the attitudes error are within 2°. The results show that the designed AHRS provided stabilized attitude and heading angles in dynamic conditions which improved the quadrotor's attitude and heading control.

Keywords: AHRS; quaternion; Kalman; quadrotor

### I. INTRODUCTION

With the rapid development of micro electrical technology and advanced control technology, a small Unmanned Aerial Vehicle (UAV) got fast development. The attitude and heading system based on the MEMS technology, which include three-axis MEMS accelerometers, three-axis MEMS rate-gyros and electronic

compass, accomplishing a collective of rate angular, acceleration and magnetic field intensity of body tri-axes respectively. A quadrotor aircraft can take off and land in limited spaces, even hover over a target and fly through narrow space and stay in low-speed or high-speed motion object easily<sup>[1-4]</sup>. With these advantages, the quadrotor has received a strong of attention in the last decade. Moreover the increasing interest in quadrotor has made people to find out a way to design a low-cost lightweight and low-power consumption attitude and heading reference system.

## II. AHRS DESIGN PHILOSOPHY

The attitude of quadrotor is defined using the earth-surface inertial reference frame and aircraft-body coordinate frame. Consider the earth-surface inertial reference frame as  $(X_0,Y_0,Z_0)$  and the aircraft-body coordinate frame as  $(X_1,Y_1,Z_1)$ . So the relationship between the earth-surface inertial reference frame and aircraft-body coordinate frame as below<sup>[5]</sup>,

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = R_E^B \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{1}$$

Where R<sup>B</sup><sub>E</sub> is the rotation matrix from the earth-surface inertial reference frame to the aircraft-body coordinate frame.

$$R_{E}^{B} = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \cos\theta\sin\phi \\ \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$
(2)

Where,  $\theta$  is the pitch angle;  $\Phi$  is the roll angle;  $\psi$  is the yaw angle.



# ATTITUDE CONTROL DESIGN

Accelerometers: Consider the gravity acceleration in two coordinate frame: the earth-surface inertial reference frame  $A_{Eg}=(0,0,g)^T$ ,  $g = 9.81 \text{ m/sec}^2$  denotes the gravitational constant and the aircraft-body coordinate frame  $A_{Bg}=(g_{bx}, g_{by}, g_{bz})^{T}$ . From the equation (2),  $\theta$  and  $\Phi$  can be written as <sup>[5]</sup>:

$$\theta = -\arcsin(g_{bx}/g)$$

$$\phi = \arctan(g_{by}/g_{bz})$$
(3)

But under the condition of  $\theta = \pi/2$  or  $g_{bz} = 0$ ,  $\Phi$ can't be solved out.

2) Magnetometers: The magnetic field vector expressed in earth-surface inertial reference frame is supposed to be  $B_E = (B_{Ex}, B_{Ey}, B_{Ez})^T$ , and in body frame the vector is  $B_B = (B_{Bx}, B_{By}, B_{Bz})^T$ . As the pitch angle  $\theta$  and the roll angle  $\Phi$  to be known, so we have

$$B_{Ey} = B_{By} \bullet \cos \theta + B_{Bx} \bullet \sin \theta \bullet \sin \phi$$

$$-B_{Bz} \bullet \cos \phi \bullet \sin \theta$$

$$B_{Ex} = B_{Bx} \bullet \cos \phi + B_{Bz} \bullet \sin \phi$$

$$\psi = \arctan(B_{Ex}/B_{Ey})$$
(4)

But  $\psi$  don't have any answer when  $B_{Ey} = 0$ .

3) State equation and observer equation based on quaternion:

From the earth-surface inertial reference frame to the body frame, the quaternion  $\mathbf{q}$  is defined as follows.

$$q = (q_0, q_1, q_2, q_3) = q_0 + q_1 \vec{i} + q_2 \vec{j} + q_3 \vec{k}$$
 (5)

To deal with the attitude's measurement, a method is to design a differential equation which the quaternion **q** satisfies <sup>[7-9]</sup>:

$$\stackrel{\bullet}{q} = \Omega_{B}q \qquad (6)$$

And the  $\Omega_B$  is

$$\Omega_{B} = \begin{bmatrix} 0 & -w_{Bx}/2 & -w_{By}/2 & -w_{By}/2 \\ w_{Bx}/2 & 0 & w_{Bz}/2 & -w_{By}/2 \\ w_{By}/2 & -w_{Bz}/2 & 0 & w_{Bx}/2 \\ w_{Bz}/2 & w_{By}/2 & -w_{Bx}/2 & 0 \end{bmatrix}$$
(7) (3) Calculate the Kalman filter gain 
$$K_{k} = P_{k|k-1}H_{k}^{T}(H_{k}P_{k|k-1}H_{k}^{T} + C_{v})^{-1}$$
(12) (12)

Where, w<sub>Bx</sub> is angular velocity in X axis of body frame; w<sub>By</sub> is angular velocity in Y axis of body frame; w<sub>Bz</sub> is angular velocity in Z axis of body frame.

From equation (6), we design an observational equation as follows,

$$q_{k} = (I + \Omega_{B(k-1)})q_{k-1} + W_{k-1}$$

$$Z_{k} = H_{k}q_{k} + V_{k}$$
(8)

Where,  $W_{k-1}$  satisfy  $E(W_iW_i)=C_w\delta_{ii}$ ;  $V_{k-1}$  satisfy  $E(V_iV_i)=C_v\delta_{ii}$ ;  $H_k$  satisfy  $H_k=I_{3\times 3}$ .

Thus, the quaternion  $\mathbf{q}$  can be written in terms of attitude angular:

$$q = \begin{bmatrix} C(\frac{\phi}{2})C(\frac{\theta}{2})C(\frac{\psi}{2}) + S(\frac{\phi}{2})S(\frac{\theta}{2})S(\frac{\psi}{2}) \\ S(\frac{\phi}{2})C(\frac{\theta}{2})C(\frac{\psi}{2}) - C(\frac{\phi}{2})S(\frac{\theta}{2})S(\frac{\psi}{2}) \\ C(\frac{\phi}{2})S(\frac{\theta}{2})C(\frac{\psi}{2}) + S(\frac{\phi}{2})C(\frac{\theta}{2})S(\frac{\psi}{2}) \\ C(\frac{\phi}{2})C(\frac{\theta}{2})S(\frac{\psi}{2}) - S(\frac{\phi}{2})S(\frac{\theta}{2})C(\frac{\psi}{2}) \end{bmatrix}^{(9)}$$

Where C is represent for **cos(.)** and S is represent for sin(.).

- 4) Kalman filter based on attitude quaternion [10]. Assumption: The mean squared error of system noise and observation noise were a constant. The Kalman filter is defined as follow:
- 1 The first step of Kalman filter aimed at predicting current value through the last value, it follows

$$\hat{q}_{k|k-1} = (I + \Omega_{B(k)})\hat{q}_{k-1}$$
 (10)

Where, q was initial value when k=1.

② A step to prediction mean square error

$$P_{k|k-1} = (I + \Omega_{B(k)}) P_{k-1} (I + \Omega_{B(k)})^T + C_w$$
(11)

③ Calculate the Kalman filter gain

$$K_{k} = P_{k|k-1} H_{k}^{T} (H_{k} P_{k|k-1} H_{k}^{T} + C_{v})^{-1}$$
(12)

4 Update the state equation

$$\hat{q}_{k} = (I + \Omega_{B(k)})\hat{q}_{k-1} + K_{k}[Z_{k} - H_{k}(I + \Omega_{B(k)})\hat{q}_{k-1}]$$
(13)

Where, the observation value Zk used the last filter value  $\hat{q}_{k-1}$ 

⑤ Update the mean square error

$$P_{k} = (I - K_{k} H_{k}) P_{k|k-1} \tag{14}$$

5) Calculate the attitude angle based on quaternion

The quaternion  $\mathbf{q}$  is a vector denoted by  $(\mathbf{q}_0, \mathbf{q})$ , where  $q_0 \in \mathbb{R}$ ,  $q \in \mathbb{R}^3$  are the scalar part and the vector of the unit quaternion respectively. According to the quaternion and Euler rotation matrix [7-8], we get

$$\tan \phi = \frac{2(q_0 q_1 + q_2 q_3)}{{q_0}^2 - {q_1}^2 - {q_2}^2 + {q_3}^2}$$

$$\sin \theta = 2(q_0 q_2 - q_1 q_3)$$

$$\tan \psi = \frac{2(q_1 q_2 + q_0 q_3)}{{q_0}^2 + {q_1}^2 - {q_2}^2 - {q_3}^2}$$
(15)
$$q_0^2 + {q_1}^2 + {q_2}^2 + {q_3}^2 = 1$$
So from the equation (15), we have

So from the equation (15), we have,

$$\phi_{(calc)} = \arctan(-D/C)$$
 $\theta = \arcsin(E)$ 
 $\psi_{(calc)} = \arctan(B/A)$ 
(16)

Where A is represent for  $(q_0)^2 + (q_1)^2 - (q_2)^2 - (q_3)^2$ ;

B is represent for  $2(q_1q_2+q_0q_3)$ ;

C is represent for  $(q_0)^2 - (q_1)^2 - (q_2)^2 + (q_3)^2$ ;

D is represent for  $2(q_0q_1+q_2q_3)$ ;

E is represent for  $2(q_0q_2-q_1q_3)$ .

But there were some special situation describe as table 1 and table 2.

A	В	Ψ(finally)
→0	>0	90°
→0	<0	-90°
>0	>0	Ψ(calc)
>0	<0	Ψ(calc)
<0	>0	Ψ(calc)+180°
<0	<0	Ψ(calc)-180°

Table 1. The true value table of the yaw angle  $\psi$ 

Φ(calc)	С	Φ(finally)
>0	>0	Φ(calc)
<0		
>0	<0	$\Phi$ (calc)-180°
<0	<0	Φ(calc)+180°

Table 2. The true value table of the roll angle  $\Phi$ 

### HARDWARE DESIGN

AHRS is used to collect the attitude of quadrotor with multi-sensors<sup>[11]</sup>. In order to improve its effectiveness, an embedded system was designed with good Real-time, reliability and low-power consumption, etc.

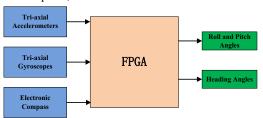


Fig.1, Block diagram of AHRS

The paper consider the FPGA as the main processor for making full use of system resources, improving the process performance, reducing the system response time. The sensor suite is based on a sensor board equipped with a tri-axis accelerometer (ADXL345), a tri-axis gyro (L3G4200D) and a tri-axis magnetometer (HMC5883L). Compares with the traditional unmanned aerial vehicle system, it has the high integration, and while having very strong data-handling capacity. All sensors outputs are digital which use the Serial Peripherals Interface(SPI) and I2C bus system as underlying physical communication layer.

#### V. EXPERIMENTAL RESULTS

To verify the results, some experiments were performed as on the ground as in the flight test. Data were collected to evaluate the accuracy and performance of the AHRS designed system. The attitudes of AHRS were processed for data fusion strategy through MATLAB running on PC. The data of attitude  $(\Phi, \Theta, \Psi)$  before data fusion compared with the attitude ( $\Phi_f$ ,  $\Theta_f$ ,  $\Psi_f$ ) after data fusion ,which describe in Figure 2 to

## Figure 4 as below;

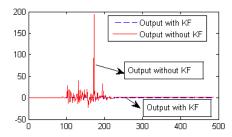


Fig.2 the Roll Angle  $\Phi$  and  $\Phi_f$  output of AHRS

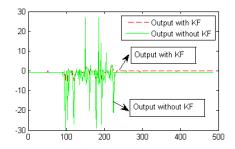


Fig .3, the Pitch Angle  $\Theta$  and  $\Theta_f$  output of AHRS

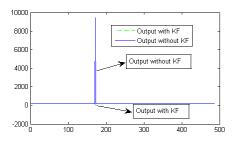


Fig. 4 the Yaw Angle  $\Psi$  and  $\Psi_f$  output of AHRS

# VI. CONCLUSIONS AND FUTURE WORK

As the figure see, the AHRS has reduce the effect of the integral drifting which brings by gyro and the effect of vibrating. So the designed AHRS provided stabilized attitude and heading angles in dynamic conditions which improved the quadrotor's attitude and heading control. Furthermore the AHRS has been used in quadrotor's flight control. Besides the FPGA provides sufficient interface for user to develop such as GPS etc and the AHRS also can be used in other robots.

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