

## POSITION ESTIMATION FOR AUTONOMOUS FLIGHT OF UNMANNED HELICOPTER BASED ON LOW-COST DUAL GPS

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### Abstract:

A novel technique to estimate the position of an unmanned helicopter using dual GPS units is presented in this paper. Our approach is to use recursive least square (RLS) method to estimate the noise characteristics with AR(2) model, and adopt Kalman filter to estimate the helicopter position under AR(2) noise model. The uniqueness of our attempt lies in using inexpensive commercial GPS units. Simulation results of the estimation system show its effect.

### Keywords:

Kalman filter; GPS; Recursive least square; AR model

### 1 Introduction

Position estimation plays a fundamental role in autonomous robots. Nowadays, the estimation technology has been widely used in all kinds of robots including aerial vehicles (for auto-landing and auto-takeoff especially).

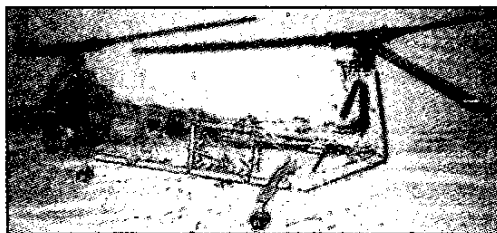


Fig.1 THUH-1 unmanned tandem helicopter

Unmanned helicopter appeared in the mid 90's of the last century and became the most prevalent research platform. At Tsinghua University, much work has been done on unmanned helicopter for a period of time. Fig.1 shows the THUH-1 unmanned tandem helicopter of the system. As part of a large context, our goal is to establish an autonomous helicopter that can be used as a platform for research and application.

Recently our goal is to develop a position estimation method that is inexpensive and can accurately estimate the unmanned helicopter position in a manner of auto-landing

and auto-takeoff. Main sensors include commercial GPS, INS for aero-model, azimuth compass and other general sensors.

GPS is a method for vehicles to determine their absolute position. A compact GPS unit is a convenient positioning system that directly gives the absolute longitude, latitude and height coordinates of the object. However, the position information provided by commercial grade GPS unit is subjected to much disturbance, data refresh rate, and intentional degradation process called Selective Availability (SA) [2]. Though SA has become the history, the position error is still as large as 15m in vertical direction and 10m in horizontal direction [2]. And the error that introduced by communication thorough troposphere and ionosphere also affects the GPS signal.

In order to minimize the effort of these degradations, differential GPS that uses dual GPS units is introduced. By receiving signals from the same satellites, it is able to decrease the effort of SA and calculate the relative coordinates between two units. We just use this kind of thought to design our system, but our system is relatively less inexpensive because we do not demand the GPS unit gives the pseudorange.

Kalman filter [3] is a well-known technique for state and parameter estimation. It is a recursive estimation procedure using sequential measurement data sets. The key problem of Kalman filter is the model of noise. In common, the Kalman filter is previously investigated under the assumption that the dynamic driving noise and measurement noise are white Gaussian noise and uncorrelated. Nevertheless, the noise of the real world is not ideal. Sometimes the ARMA model is regarded as a simplified alternative to the complicated nature [4].

There are a variety of sensors and estimation architectures that can be used for helicopter position estimation. Jun et. al. [1] estimate the state of helicopter using sensor model and Kalman filter. The raw information comes from gyroscopes, accelerometers and a GPS unit. The solution to the position estimation problem proposed

by Conway et. al.<sup>[7]</sup> comes from using a Carrier Phase Differential GPS (CD-GPS). They use 4 antennas located on the helicopter and an on-station antenna for attitude and position estimation. Another method is developed by Rock et. al.<sup>[8]</sup>, they presented a solution which combined CD-GPS and a stereo vision system.

Bosse et. al.<sup>[9]</sup>, deal with state estimation of helicopter using hybrid sensors. An inertial measurement unit provides high bandwidth motion estimations while a differential GPS unit provides periodic updates of absolute positions. At the same time, a sonar altimeter provides the altitude information and a compass is utilized to prevent long term drifts in heading estimation.

In this paper, the position estimation problem of unmanned helicopter is focused on when the helicopter hovers near the spot of the ground control station using dual inexpensive commercial GPS units and Kalman filter technique. One of the two units is installed in the ground control station and its position is not changed while the other is placed on the body of helicopter. In this case, we can safely suppose that the on-station GPS unit and the on-board unit receive the signals from the same satellites and the output data are interfered by the same kind of noise. Therefore, the parameters, which are estimated by using the on-station GPS data, can be utilized to assist the position estimation based on the data of on-board GPS unit by using Kalman filter.

## 2 Noise Models and Characteristics

The key difficulty in position estimate using GPS is the low frequency noise component, which is also referred as bias or drift that violates the white noise assumption required for standard Kalman filtering. The satellite signals are interfered in the process of communication by atmospheric refraction and even the vapor and dust in the air. So the white Gaussian noise model did not meet the requirement of modeling the noise of GPS data. Under this condition, we use AR(2) model as the model of the noise of GPS data. AR(n) model can be seen as an approximation of ARMA model<sup>[6]</sup>. Considering the practical conditions, we use AR(2), but not higher order model for its more error risk, to approximate the GPS position noise.

Denoting  $\xi(k)$  as the noise of time  $k$ , it satisfies the following formulation.

$$\xi(k+1) = a(k)\xi(k) + b(k)\xi(k-1) + e(k) \quad (1)$$

In (1),  $e(k)$  is a white Gaussian noise process of mean zero and variance  $Q_e$ .

## 3 System Model

System model includes two parts: on-board model (combined by the helicopter moving model and its carrying GPS measure model), and the on-station model (composed by the ground control station moving model and its GPS measure model).

As we all known, the model of helicopter is a complicated nonlinear model. In this paper, only position estimation problem of helicopter is focused on, so that there is no necessity to consider the dynamics of the helicopter body. What is concerned is the motion and position of the body. If the sample time is short enough, it can be supposed that the velocities in all directions are almost constant in a very short period of time. In this context, uniform velocity model is used in three dimensions. Two reasons exist here: one is that the helicopter has no ability to change the velocity dramatically through any direction in a very short time; the other is that the higher orders the model is, the more error will occur with very small and unavoidable modeling error.

On-board model is described as equation (2), where the subscript  $b$  denotes this model. Describe  $X_b(k)$  as the system state vector containing velocity  $v_b(k)$  and position  $r_b(k)$  in all three dimensions,  $w_b(k)$  as the modeling noise, a zero-mean white Gaussian process with variance  $Q_w$ .  $Y_b(k)$  is the output of the system, i.e. the output of the GPS receiver;  $\xi_b(k)$  is the measure noise (the noise of GPS output) that we described previously in this paper.

$$X_b(k) = [r_b(k) \quad v_b(k)]^T \quad (2a)$$

$$X_b(k+1) = A_b X_b(k) + w_b(k) \quad (2b)$$

$$Y_b(k) = C_b X_b(k) + \xi_b(k) \quad (2c)$$

For the hypothesis of uniform velocity model, system matrix  $A_b$  and output matrix  $C_b$  are:

$$A_b = \begin{bmatrix} I_3 & T \\ 0 & I_3 \end{bmatrix} \quad C_b = [I_3 \quad 0] \quad (3)$$

In (3),  $T$  is the sample interval.

For the same reason, the uniform velocity hypothesis is used for on-station model. The system descriptions are following and the definition of variables is the same with

on-board model while the subscript  $s$  denotes the corresponding information. The state of GPS on-station (velocity and position) is zero for it is immovably. If we take the point of station as origin, the output of GPS on-station subtracting the coordinate of origin is the noise.

$$X_s(k) = [r_s(k) \quad v_s(k)]^T \quad (4a)$$

$$X_s(k+1) = A_s X_s(k) + w_s(k) \quad (4b)$$

$$Y_s(k) = C_s X_s(k) + \xi_s(k) \quad (4c)$$

Considering position estimation of helicopter when it hovers near the station, it can be supposed that the GPS on-board and the one on-station receive the signals from the same satellites. So the noise of the two GPS units has the same characteristics. As a result, it can be safely supposed that  $\xi_b(k) = \xi_s(k)$ . Thus the data of GPS unit on-station can be used to estimate the noise.

#### 4 Architectures of Estimation

As the model described in (2), it is clear that it does not meet the required condition of standard Kalman filter since the measure noise is not white. Consequently, the first step is to identify the noise and change the model in order to satisfy Kalman filter.

There are many successful approaches to identify the AR(2) noise model. Among them, the recursive least square (RLS) method [5] may be the most effective and practical. This method will be used in our noise estimation approach. The GPS unit on-station is fixed, so the output of GPS on station is the noise. The noise model is described in (1) and the best estimation value  $\hat{a}(k)$  and  $\hat{b}(k)$  to approximate  $\xi_s(k)$  will be calculated later. The estimation model is  $\hat{\xi}_s(k)$  in (5).

$$\hat{\xi}_s(k) = \hat{a}(k)\hat{\xi}_s(k-1) + \hat{b}(k)\hat{\xi}_s(k-2) + e(k) \quad (5)$$

In the practice,  $\hat{a}(k)$  and  $\hat{b}(k)$  may be considered time-variable, the recursive least square method with forgetting factor is reasonable [5], which will decrease the weight of old information. Take  $\mu$  as the forgetting factor, the procedure of RLS are following equations.

$$\begin{bmatrix} \hat{a}^T(k) \\ \hat{b}^T(k) \end{bmatrix} = \begin{bmatrix} \hat{a}^T(k-1) \\ \hat{b}^T(k-1) \end{bmatrix} + K(k) \left( y_s^T(y+1) - \begin{bmatrix} y_s^T(y) & y_s^T(y-1) \end{bmatrix} \begin{bmatrix} \hat{a}^T(k-1) \\ \hat{b}^T(k-1) \end{bmatrix} \right)$$

$$K(k) = P(k-1) \begin{bmatrix} y_s(k) \\ y_s(k-1) \end{bmatrix} \left( \begin{bmatrix} y_s^T(k) & y_s^T(k-1) \end{bmatrix} P(k-1) \begin{bmatrix} y_s(k) \\ y_s(k-1) \end{bmatrix} + \mu \right)^{-1} \quad (6a)$$

$$P(k) = P(k-1) \begin{bmatrix} y_s(k) \\ y_s(k-1) \end{bmatrix} \left( \begin{bmatrix} y_s^T(k) & y_s^T(k-1) \end{bmatrix} P(k-1) \begin{bmatrix} y_s(k) \\ y_s(k-1) \end{bmatrix} + \mu \right)^{-1} \quad (6b)$$

$$P(k) = \frac{1}{\mu} (I - K(k) \begin{bmatrix} y_s^T(k) & y_s^T(k-1) \end{bmatrix}) P(k-1) \quad (6c)$$

In order to keep  $P(k)$  symmetrical, we rewrite (6c) to (6c').

$$P(k) = \frac{1}{\mu} P(k-1) - K(k) K^T(k) \left( \frac{1}{\mu} \begin{bmatrix} y_s^T(k) & y_s^T(k-1) \end{bmatrix} P(k-1) \begin{bmatrix} y_s(k) \\ y_s(k-1) \end{bmatrix} + 1 \right)^{-1} \quad (6c')$$

In practical application, one vector that contains all the parameters is identified in the recursive procedure.

In this procedure, it is needed to estimate the variance of  $\hat{e}(k)$  in (5). The estimation value  $\hat{Q}_e$  can be given by (7).

$$\hat{Q}_e(k) = \left( y_s^T(k+1) - \begin{bmatrix} y_s^T(k) & y_s^T(k-1) \end{bmatrix} \begin{bmatrix} \hat{a}^T(k) \\ \hat{b}^T(k) \end{bmatrix} \right)^T \left( y_s^T(k+1) - \begin{bmatrix} y_s^T(k) & y_s^T(k-1) \end{bmatrix} \begin{bmatrix} \hat{a}^T(k) \\ \hat{b}^T(k) \end{bmatrix} \right) + 1 \quad (7)$$

To improve the system equation of GPS on-board meeting the demand of Kalman filter, the estimated noise model is used to on-board model. The output equation can be adapted as following by generate a new output variable  $Y'_b(k)$ .

$$\begin{aligned} Y'_b(k) &= Y_b(k) - \hat{a}(k)Y_b(k-1) - \hat{b}(k)Y_b(k-2) \\ &= (C_b - \hat{a}(k)C_b A_b^{-1} - \hat{b}(k)C_b A_b^{-2}) X_b(k) \\ &\quad + (\xi_b(k) - \hat{a}(k)\xi_b(k-1) - \hat{b}(k)\xi_b(k-2)) \\ &\quad + ((\hat{a}(k)C_b A_b^{-1} + \hat{b}(k)C_b A_b^{-2}) w_b(k-1) - \hat{b}(k)C_b A_b^{-1} w_b(k-2)) \\ &= C'_b(k) X_b(k) + \xi'_b(k) \end{aligned} \quad (8)$$

The  $\xi'_b(k)$  in equation is a zero-mean white Gaussian noise and satisfies Kalman filter, its variance is:

$$Q_{\xi} = \hat{Q}_e + [\hat{a}(k)C_b A_b^{-1} + \hat{b}(k)C_b A_b^{-2}] P_w [\hat{a}(k)C_b A_b^{-1} + \hat{b}(k)C_b A_b^{-2}]^T + [\hat{b}(k)C_b A_b^{-1}] P_w [\hat{b}(k)C_b A_b^{-1}]^T \quad (9)$$

According the noise characteristics, the Kalman filter equations are sited as following.

$$\hat{X}_b(k+1|k+1) = A_b \hat{X}_b(k|k) + K(k+1|k+1)(Y'_b(k+1) - C'_b(k+1)A_b \hat{X}_b(k|k)) \quad (10a)$$

$$K(k+1|k+1) = P(k+1|k)C_b'^T(k+1)(C_b'(k+1)P(k+1|k)C_b'^T(k+1) + Q_{\xi})^{-1} \quad (10b)$$

$$P(k+1|k) = A_b P(k|k)A_b^T + Q_w \quad (10c)$$

$$P(k+1|k+1) = (I - K(k+1|k+1)C_b'(k+1))P(k+1|k) \quad (10d)$$

In the recursive procedure, the synchronization problem must be concerned. The commercial GPS units used gives out position information once every second after power-up while the sample rate of control system is much faster than it. So in the application, once the information of estimation is updated, the innovation must be used in the coming control cycle.

## 5 Simulation Results

As mentioned above, we plan to use this estimation system to estimate the all three-dimension position of the helicopter. In simulation work, we take one dimension to test the validity of the technique.

Fig. 2 shows the noise used in the simulation, which stands for  $\xi(k)$ . Then we use RLS with forgetting factor to identify the noise and get the estimation of it. Taking the forgetting factor  $\mu$  as 0.95. The estimation value shows in Fig.3. The plot shows that the estimation can faithfully reflect the characteristics of the noise.

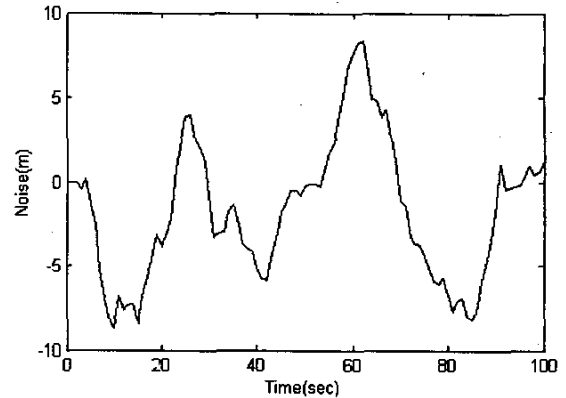


Fig.2 Noise

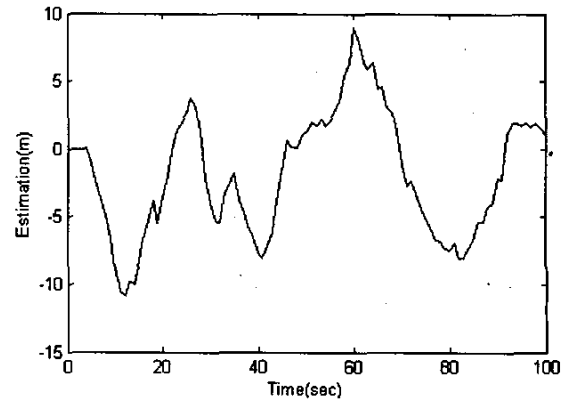


Fig.3 Estimation of noise

Kalman filter is used to estimate the position of helicopter that moves along one direction such as along the North. Fig.4 shows three error plots: the difference between GPS data and real position (GPS error), the difference between standard Kalman filter estimation position and real position (Kalman filter error), the difference between our estimation position and real position (our estimation error). The graph (Fig.4) indicates that the new method is much better than standard Kalman filter. The performance improvement can be clearly seen by using the new technique.

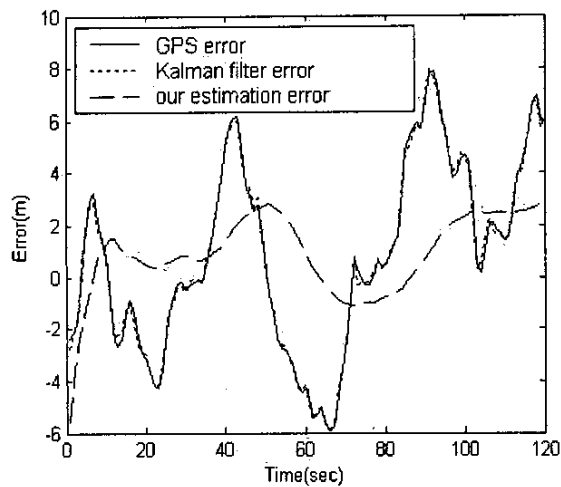


Fig.4 Error Plots

## 6 Conclusions and Future Work

In this paper, a position estimation using dual GPS units is developed for an unmanned helicopter. It provides a relative precise position estimation using inexpensive instruments. In the next step, INS (Inertial Navigation System) will be added to the system. Combining the virtue of GPS and INS, more advanced position estimation with dynamic and static performance will be developed.

It remains as future work to verify our estimation system in experiments. Our ultimate goal is to implement our system on the real unmanned tandem helicopter in our laboratory.

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