

# Sensors-based data fusion solution design for 3D motion estimation with application in Bio-logging

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**Abstract.** *The main purpose of this paper deals with the orientation estimation problem of a rigid-body motion in space. We present an algorithm for attitude estimation, expressed in quaternion representation, using low-cost sensors as 3-axis accelerometer, 3-axis magnetometer and 3-axis gyroscope. The algorithm is based on a complementary nonlinear observer coupled with a Levenberg Marquardt Algorithm (LMA). Moreover, the proposed solution exploits kinematic equation model and includes the estimation of rate gyros biases to compensate angular velocity measurements. This algorithm is developed in order to address the well-known problem of the weak dynamics of the attitude sensors (accelerometer and magnetometer). The efficiency of the proposed observer is illustrated by an attitude estimation example in presence of realistic measurements provided by low-cost sensors. Some preliminary experimental results are provided also to prove the performance of the proposed method. The developed approach will be applied in future works in Bio-logging area which interests to study the animal behavior and its energy expenditure by determining its movement patterns (3D motion or orientation).*

**Keywords.** Sensor fusion; attitude determination; nonlinear observer; Levenberg Marquardt Algorithm; Micro Electro-Mechanical System (MEMS); inertial and magnetic sensors; Bio-logging

## 1. Introduction

The rigid-body attitude estimation problem has been widely studied in literature (Walking Robots [1], Micro Air Vehicles [2], rehabilitation and biomedical engineering [3],...). The quality of this information is a fundamental requirement in many purposes as control, stabilization or patient's movements study. The efficiency of these purposes depends on the availability and reliability of attitude measurements.

Generally, these data are derived from inertial and magnetic sensors as gyroscope, magnetometer, accelerometer, inclinometer and velocity sensor. Nowadays, the progress in Micro Electro-Mechanical System (MEMS) has enabled the development of low-cost Inertial Measurement Units (IMU). The attitude determination process usually involves some errors (e.g., sensor noise, bias, misalignments, and inaccuracy in predictions). For this reason, most attitude determination methods aim to combine two or more sensor measurements in an optimal fashion and/or use state estimation techniques to increase the accuracy of the solutions.

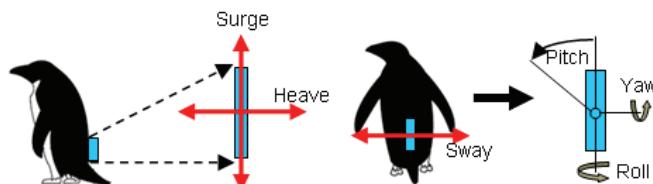
Recently, the problem of attitude and orientation tracking has been treated in another scientific area: the Bio-logging. This field is at the intersection of animal behavior and bioengineering and aims at obtaining new information on the natural world and providing new insights into the hidden lives of animal's species [4]. Generally, Bio-logging involves a free-ranging animal-attached device that records aspects of the animal's biology [5] (3D movement, behavior and physiology) and its environment. In this paper, the goal is to present another way to estimate body motions (body attitude or orientation) for marine animals: the case of the king penguin. This animal's specie is studied for the energy expenditure evaluation in extreme environment when direct observation is impossible since king penguin spends most of its live underwater. Recently, few researches started to focus on spatio-temporal movements of marine animals [6], [7]. The main idea in these works is to use only 3-axis accelerometer and 3-axis magnetometer to track the animal's posture estimates. Unfortunately, the authors consider only static and quasi-static situations of free-ranging animals with the assumption that accelerometer's readings give only earth's gravity measurements in body coordinates. Note that in dynamic situations, where motion accelerations are introduced, the assumption is not valid, and the performance of the attitude estimation is degraded. In [8], a low-pass filter is used to extract the gravity's components (the lowest frequencies of the natural panel of animal's movements) from the acceleration readings. Based on this information, the authors can deduce the attitude. Note that, the use of this type of filter introduces, in many cases, an error about the attitude information since the gravity measurements are not accurately extracted by the filter.

To circumvent these problems found in Bio-logging, we propose in this paper the addition of 3-axis gyroscope to the sensors already used (accelerometer and magnetometer) [9]. Then, another approach in this area is proposed to estimate the 3D motion (orientation) and which will be used after to study free-ranging animal movement patterns. The main guideline is to use a complementary nonlinear observer coupled with a Levenberg Marquardt Algorithm (LMA) and based on measurements from 3-axis gyroscope, 3-axis magnetometer and 3-axis accelerometer. The proposed approach combines a strap-down system, based on the integral of the angular velocity, with an optimal attitude determination system that uses Earth's magnetic field and gravity vector to calculate attitude measurements. Moreover, we will show how the measurements provided by the sensors can be used in the observer design and we will also provide some stability conditions in order to guarantee an asymptotic convergence of the estimate errors. The main advantage of this contribution is that it uses only kinematic equation, and then the proposed algorithm doesn't require the knowledge of the animal motion model.

This paper is organized as follows: Section 2 presents the problem statement. Section 3 describes the rigid-body attitude parameterization, the sensor-based framework and the kinematic model. Section 4 details the structure of the complementary nonlinear attitude observer and its stability conditions. Section 5 recalls some notions of attitude determination from vector observations (LMA). The efficiency of the proposed approach is illustrated in section 6 and 7 based on some numerical simulations and preliminary experiments. Finally, section 8 summarizes the main conclusions of the paper.

## 2. Problem statement

The concept of Bio-logging refers to the use of autonomous electronic devices to monitor something related to free-ranging animal itself and then to study its behaviour, physiology and ecology. This scientific field is essential to manage and conserve endangered species and their habitats and to mitigate human impacts. There are many examples of Bio-logging researches that illustrate the usefulness of this technique. The king penguin is one of the major model of diving birds studied in Strasbourg University thanks to the Bio-logging technology [10]. Then, knowing in details its foraging activities and understanding its energetic strategy need the development of new Bio-loggers. This generation of logger contains 3-axis accelerometer, 3-axis gyroscope, 3-axis magnetometer, ambient temperature, luminosity and barometric pressure sensors. The king penguin will be equipped with this kinematical logger (see Fig. 1).



**Fig. 1.** Schematic diagram showing how electronic device (logger) is logged on king penguin

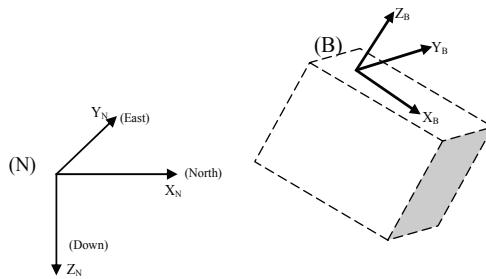
The prototype will then collect and store the sensor's data until the animal returns to a place where the tag can be recovered. After that all calculations are performed offline by extracting the measurements recorded on the memory card using a PC and without any satellite or RF transmission. Note that it is easy to locate the tag since each animal used in the tests is marked. Before deploying this new logger, the goal in this paper is to be able to convert this complex set of raw data (acceleration, angular rate and earth's magnetic field) in relevant information as 3D motion (attitude) and how it is possible to improve body motions estimation results in free-ranging animal. Thanks to the knowledge of the attitude in quaternion form, this information is exploited to calculate the Dynamic Body Acceleration (DBA) and then it is possible, even when measured without a high accuracy, to reconstitute the daily diary of the animal model and its energy expenditure [11]. The algorithm that will exploit the

measurements from 3-axis accelerometer, 3-axis magnetometer and 3-axis gyroscope are the main concerns of this work.

### 3. Attitude and kinematics

#### 3.1. Rigid-body attitude description

Generally, in the navigation field, the attitude estimation problem requires the transformation of measured and computed quantities between various frames. The attitude of a rigid-body is based on measurements from sensors attached to this latter. Indeed, inertial sensors (accelerometer, gyroscope,...) are attached to the body-platform and provide inertial measurements expressed relative to the instrument axes. In most systems, the instrument axes are nominally aligned with the body-platform axes. Since the measurements are performed in the body frame we describe in Fig. 2 the orientation of the body-fixed frame  $B(X_B, Y_B, Z_B)$  with respect to the Earth-fixed frame  $N(X_N, Y_N, Z_N)$  which is tangent on the Earth's surface (Local Tangent Plane, LTP). This local coordinate is particularly useful to express the attitude of a moving rigid-body on the surface of the earth. The  $X_N$ -axis points to true north. The  $Z_N$ -axis points toward the interior of the Earth, perpendicular to the reference ellipsoid. The  $Y_N$ -axis completes the right-handed coordinate system, pointing east (NED: North, East, Down) [12].



**Fig. 2.** Definition of the reference frames

There is one major question to be considered when designing an algorithm for rigid-body rotations that is what representation to use? Rigid-body kinematics has a somewhat delicate structure as the rotations are most naturally described as elements of the manifold  $SO(3)$  [2]. It is common practice to consider some parameterization of  $SO(3)$  such as various versions of Euler angles or quaternion. In this study we will use quaternion representation. Thus, the problem of singularity found, for example, in Euler angles is avoided using the quaternion form. A unit quaternion is a hyper complex number. It is composed of a unit vector  $\vec{u}$ , and a rotation angle  $\phi$  around this axis. It can be defined by [13]:

$$q = \begin{bmatrix} \cos \frac{\phi}{2} & \vec{u} \sin \frac{\phi}{2} \end{bmatrix}^T = q_0 + q_1 i + q_2 j + q_3 k = \begin{bmatrix} q_0 \\ \tilde{q} \end{bmatrix} \in \mathcal{Q} \quad (1)$$

with  $q_0$ ,  $q_1$ ,  $q_2$ , and  $q_3$  are real numbers and  $i$ ,  $j$ , and  $k$  are the components of the vector  $\vec{u}$ .  $\mathcal{Q}$  can be defined as:

$$\mathcal{Q} = \left\{ q \mid q^T q = 1, q_0 \in \mathbb{R}^{1 \times 1}, \tilde{q} = [q_1 \quad q_2 \quad q_3]^T \in \mathbb{R}^{3 \times 1} \right\} \quad (2)$$

Let us define some quaternion's properties that will be used later in the proposed approach. The multiplication of two quaternions  $q_a = [q_{a0} \quad \tilde{q}_a^T]^T$  and  $q_b = [q_{b0} \quad \tilde{q}_b^T]^T$  is expressed as:

$$q_a \otimes q_b = \begin{bmatrix} q_{a0} & -\tilde{q}_a^T \\ \tilde{q}_a & I_{3 \times 3} q_{a0} + [\tilde{q}_a^\times] \end{bmatrix} \begin{bmatrix} q_{b0} \\ \tilde{q}_b \end{bmatrix} \quad (3)$$

where

$$[\tilde{q}_a^\times] = \begin{bmatrix} q_{a1} \\ q_{a2} \\ q_{a3} \end{bmatrix}^\times = \begin{bmatrix} 0 & -q_{a3} & q_{a2} \\ q_{a3} & 0 & -q_{a1} \\ -q_{a2} & q_{a1} & 0 \end{bmatrix} \quad (4)$$

The unit quaternion satisfies the following constraint:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (5)$$

Using quaternion representation, there are two means to express a vector rotation in space. Let  $x_q$  and  $y_q$  be two quaternions associated with the vectors  $x \in \mathbb{R}^{3 \times 1}$  and  $y \in \mathbb{R}^{3 \times 1}$  expressed in  $N$  and  $B$ , respectively:

$$\begin{cases} x_q = [0 \quad x^T]^T \\ y_q = [0 \quad y^T]^T \end{cases} \quad (6)$$

1)  $x_q$  and  $y_q$  are bounded by the following relation:

$$x_q = q \otimes y_q \otimes \bar{q} \quad (7)$$

where  $\bar{q}$  represents the complementary quaternion that can be expressed as:

$$\bar{q} = [q_0 \quad -\tilde{q}^T]^T \quad (8)$$

2) The operation using equation (7) is equivalent to a multiplication by a rotation matrix:

$$x = M_N^B(q)y \quad (9)$$

where the rotation matrix  $M_N^B(q)$  is expressed in terms of quaternion by the following formula [14]:

$$M_N^B(q) = (q_0^2 - \tilde{q}^T \tilde{q}) I_{3 \times 3} + 2(\tilde{q} \tilde{q}^T - q_0 [\tilde{q}^\times]) \quad (10)$$

where  $[\tilde{q}^\times]$  is given by (4) with replacing  $\tilde{q}_a$  by  $\tilde{q}$ .

Finally, one obtains:

$$M_N^B(q) = \begin{bmatrix} 2(q_0^2 + q_1^2) - 1 & 2(q_1 q_2 + q_0 q_3) & 2(q_1 q_3 - q_0 q_2) \\ 2(q_1 q_2 - q_0 q_3) & 2(q_0^2 + q_2^2) - 1 & 2(q_0 q_1 + q_2 q_3) \\ 2(q_0 q_2 + q_1 q_3) & 2(q_2 q_3 - q_0 q_1) & 2(q_0^2 + q_3^2) - 1 \end{bmatrix} \quad (11)$$

### 3.2. Sensor-based framework

The sensors configuration consists of 3-axis gyroscope, 3-axis accelerometer and 3-axis magnetometer containing MEMS technologies. A detailed study of these sensors is given in [15]. Their outputs are expressed in the frame  $B$ , respectively, by:

$$\omega_G = [\omega_{Gx} \quad \omega_{Gy} \quad \omega_{Gz}]^T = \omega + b + \delta_G \quad (12)$$

$$f = [f_x \quad f_y \quad f_z]^T = M_N^B(q)(g + a) + \delta_f \quad (13)$$

$$h = [h_x \quad h_y \quad h_z]^T = M_N^B(q)m + \delta_h \quad (14)$$

where  $\omega = [\omega_x \quad \omega_y \quad \omega_z]^T \in \mathbb{R}^{3 \times 1}$  represents the real angular velocity vector measured in  $B$ .  $g = [0 \quad 0 \quad 9.81]^T \in \mathbb{R}^{3 \times 1}$  and  $a = [a_x \quad a_y \quad a_z]^T \in \mathbb{R}^{3 \times 1}$  represent the gravity vector and the body acceleration, respectively.  $a$  and  $g$  are expressed in  $N$ .  $m = [m_x \quad 0 \quad m_z]^T = [\|m\|\cos(\theta) \quad 0 \quad \|m\|\sin(\theta)]^T \in \mathbb{R}^{3 \times 1}$  represents the magnetic field measured in  $N$ . The theoretical model of the magnetic field nearest to reality considers a magnetic field vector with an inclination angle  $\theta = 60^\circ$  and a norm vector  $\|m\| = 0.5$  Gauss [16].  $b = [b_x \quad b_y \quad b_z]^T \in \mathbb{R}^{3 \times 1}$  is a function varying slowly in time

and representing an unknown gyro-bias modeled by a Gauss-Markov process [17]. It is described by:

$$\dot{b} = -T^{-1}b \quad (15)$$

where  $T = \tau I_{3 \times 3}$  is a diagonal matrix of the time constant  $\tau$ .

$\delta_G \in \Re^{3 \times 1}$ ,  $\delta_f \in \Re^{3 \times 1}$  and  $\delta_h \in \Re^{3 \times 1}$  are assumed uncorrelated white Gaussian measurements noises with null mean and covariance matrix. Since a normalized unit quaternion is used, then  $g$  is also normalized to a unit vector such as:

$$g = \frac{[0 \ 0 \ 9.81]^T}{\sqrt{9.81^2}} = [0 \ 0 \ 1]^T \quad (16)$$

In the same way as for the accelerometer,  $m$  is normalized to a unit vector such as:

$$m = \frac{[0.5\cos(60^\circ) \ 0 \ 0.5\sin(60^\circ)]^T}{\sqrt{0.5^2\cos^2(60^\circ)+0.5^2\sin^2(60^\circ)}} = \left[0.5 \ 0 \ \frac{\sqrt{3}}{2}\right]^T \quad (17)$$

### 3.3. Rigid-body kinematic model

The kinematic equation, expressed using the unit quaternion, describes the relation between the variation of the rigid-body attitude in time and the angular velocity. It is given by [14]:

$$\begin{bmatrix} \dot{q}_0 \\ \dot{\tilde{q}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\tilde{q}^T \\ I_3 q_0 + [\tilde{q}^x] \end{bmatrix} \omega \quad (18)$$

where  $\omega = [\omega_x \ \omega_y \ \omega_z]^T$  represents the real angular velocity rate vector of a rigid-body measured by gyroscope in the frame  $B$ .  $[\tilde{q}^x]$  is given by (4) with replacing  $\tilde{q}_a$  by  $\tilde{q}$ .

## 4. Design of the complementary nonlinear observer approach for the rigid-body attitude estimation

In this paper, the objective is to design a rigid-body attitude estimation algorithm based on inertial and magnetic MEMS sensors (low-cost and low-power sensors). The proposed approach will be subsequently used to estimate the orientation, under several motions, of the studied animal's specie: the king penguin. Amongst various

navigation systems, strap-down system based on the integral of the angular velocity from gyroscope leads to a good attitude estimates (in dynamic situations) for short time but suffers from gyro-bias which causes the orientation estimates to drift away from the true orientation rapidly. Accelerometer and magnetometer are good sensors in attitude estimation for static and quasi-static motion but with poor dynamic performance [2].

We propose an observer approach to take advantages from accelerometers, magnetometers and gyroscopes in order to estimate the most accurate attitude. Moreover, the proposed solution includes the estimation of rate gyros biases to compensate angular velocity measurements. It is important to stress that the resulting approach structure is complementary: high bandwidth rate gyro measurements are combined with low bandwidth vector observations to provide an accurate estimates of the attitude [17]. In our knowledge, the proposed approach using this triad of sensors is the first work applied in Bio-logging area [18].

Firstly, one considers that the real angular velocity vector  $\omega$  is written as:

$$\omega = \omega_G - b - \delta_G \quad (19)$$

To achieve our goal, let us consider the following nonlinear system described in (20) and obtained from (18), (15) and (19):

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{q}} \\ \dot{b} \end{bmatrix} &= \begin{bmatrix} 1 & -\tilde{q}^T \\ \frac{1}{2} I_{3 \times 3} q_0 + [\tilde{q}]^* & -T^{-1}b \end{bmatrix} \omega \\ &= \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} \omega_G - b - \delta_G \\ 1 \\ 2 \end{bmatrix} \\ y = q' &= [q'_0 \quad q'_1 \quad q'_2 \quad q'_3]^T \end{aligned} \quad (20)$$

where  $q \in \mathbb{R}^{4 \times 1}$  is the quaternion and  $b \in \mathbb{R}^{3 \times 1}$  is the bias (system's states).

$I_{3 \times 3} \in \mathbb{R}^{3 \times 3}$  is an identity matrix.  $q' \in \mathbb{R}^{4 \times 1}$  is the system output that is determined based on optimal fusion of accelerometer and magnetometer measurements. The method that allows to calculate  $q'$  is presented in the next section.

In order to estimate the attitude, the following complementary nonlinear attitude observer is proposed:

$$\begin{cases} \dot{\hat{q}} = \frac{1}{2} \left[ -\hat{q}^T \\ I_{3 \times 3} \hat{q}_0 + \left[ \begin{smallmatrix} \hat{q}^\times \\ \hat{q}^\times \end{smallmatrix} \right] \right] \left[ \omega_G - \hat{b} + k_1 \tilde{q}_e \right] \\ \dot{\hat{b}} = -T^{-1} \hat{b} - k_2 \tilde{q}_e \end{cases} \quad (21)$$

$\hat{q}$  and  $\hat{b}$  represent the estimated states. The observer gains  $k_1, k_2 \in \mathbb{R}^{1 \times 1}$  are positive constants.  $\left[ \begin{smallmatrix} \hat{q}^\times \\ \hat{q}^\times \end{smallmatrix} \right]$  is given by (4) with replacing  $\tilde{q}_a$  by  $\hat{q}$ .  $\tilde{q}_e \in \mathbb{R}^{3 \times 1}$  represents the vector part of the error quaternion  $q_e$ .

$q_e$  is obtained by using the quaternion product (3) between the measured quaternion  $q'$  and the complementary estimated quaternion  $\bar{\hat{q}}$  with replacing  $q_a$  by  $\bar{\hat{q}}$  and  $q_b$  by  $q'$ :

$$q_e = \bar{\hat{q}} \otimes q' = \left[ \begin{smallmatrix} q_{e0} & \tilde{q}_e^T \end{smallmatrix} \right]^T \quad (22)$$

Based on [2], [19], a detailed mathematical analysis of the observer convergence and the global stability are derived. Suppose that  $\delta_G = 0$  and  $q' \approx q$ .

**Theorem 1:** Consider the kinematic equation (18) for a time-varying  $q(t)$  and with measurements given by  $q'$  and  $\omega_G$ . Let  $(\hat{q}(t), \hat{b}(t))$  denote the solution of (21). Define error variables  $q_e = \bar{\hat{q}} \otimes q'$  and  $b_e = b - \hat{b}$ . Then, the error  $\left[ \begin{smallmatrix} q_e^T(t) & b_e^T(t) \end{smallmatrix} \right]$  is globally asymptotically stable to  $\left[ \begin{smallmatrix} \pm 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{smallmatrix} \right]$ . For almost all initial conditions  $(q_e(t_0), b_e(t_0))$ , the trajectory  $(\hat{q}(t), \hat{b}(t))$  converges to the trajectory  $(q(t), b)$ .

**Proof:** Let us consider the two error equations given by:

$$q_e = \left[ \begin{smallmatrix} q_{e0} & q_{e1} & q_{e2} & q_{e3} \end{smallmatrix} \right]^T = \bar{\hat{q}} \otimes q' \quad (23)$$

$$b_e = \left[ \begin{smallmatrix} b_{ex} & b_{ey} & b_{ez} \end{smallmatrix} \right]^T = b - \hat{b} \quad (24)$$

Suppose that  $q' \approx q$  and using the definition of quaternion product  $\otimes$  given in (3), equation (23) can be written as:

$$q_e = \left[ \begin{smallmatrix} q_0 \hat{q}_0 + q_1 \hat{q}_1 + q_2 \hat{q}_2 + q_3 \hat{q}_3 \\ -q_0 \hat{q}_1 + q_1 \hat{q}_0 + q_2 \hat{q}_3 - q_3 \hat{q}_2 \\ -q_0 \hat{q}_2 - q_1 \hat{q}_3 + q_2 \hat{q}_0 + q_3 \hat{q}_1 \\ -q_0 \hat{q}_3 + q_1 \hat{q}_2 - q_2 \hat{q}_1 + q_3 \hat{q}_0 \end{smallmatrix} \right] \quad (25)$$

Now, differentiating (24) and (25), then one obtains:

$$\begin{cases} \dot{\tilde{q}}_e = \begin{bmatrix} \dot{q}_0\hat{q}_0 + q_0\dot{\hat{q}}_0 + \dot{q}_1\hat{q}_1 + q_1\dot{\hat{q}}_1 + \dot{q}_2\hat{q}_2 + q_2\dot{\hat{q}}_2 + \dot{q}_3\hat{q}_3 + q_3\dot{\hat{q}}_3 \\ -\dot{q}_0\hat{q}_1 - q_0\dot{\hat{q}}_1 + \dot{q}_1\hat{q}_0 + q_1\dot{\hat{q}}_0 + \dot{q}_2\hat{q}_3 + q_2\dot{\hat{q}}_3 - \dot{q}_3\hat{q}_2 - q_3\dot{\hat{q}}_2 \\ -\dot{q}_0\hat{q}_2 - q_0\dot{\hat{q}}_2 - \dot{q}_1\hat{q}_3 - q_1\dot{\hat{q}}_3 + \dot{q}_2\hat{q}_0 + q_2\dot{\hat{q}}_0 + \dot{q}_3\hat{q}_1 + q_3\dot{\hat{q}}_1 \\ -\dot{q}_0\hat{q}_3 - q_0\dot{\hat{q}}_3 + \dot{q}_1\hat{q}_2 + q_1\dot{\hat{q}}_2 - \dot{q}_2\hat{q}_1 - q_2\dot{\hat{q}}_1 + \dot{q}_3\hat{q}_0 + q_3\dot{\hat{q}}_0 \end{bmatrix} \\ \dot{\tilde{b}}_e = \dot{b} - \dot{\hat{b}} \end{cases} \quad (26)$$

Substitute  $\dot{q} = [\dot{q}_0 \quad \dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3]^T$ ,  $\dot{b} = [\dot{b}_x \quad \dot{b}_y \quad \dot{b}_z]^T$ ,  $\dot{\hat{q}} = [\dot{\hat{q}}_0 \quad \dot{\hat{q}}_1 \quad \dot{\hat{q}}_2 \quad \dot{\hat{q}}_3]^T$  and  $\dot{\hat{b}} = [\dot{\hat{b}}_x \quad \dot{\hat{b}}_y \quad \dot{\hat{b}}_z]^T$  given in (20) and (21).

Finally, the observation error dynamic's is written as follows:

$$\begin{cases} \dot{\tilde{q}}_e = \frac{1}{2} \begin{bmatrix} 0 & [b_e + k_1\tilde{q}_e]^T \\ -[b_e + k_1\tilde{q}_e] & [2\omega^2] + [[b_e + k_1\tilde{q}_e]]^T \end{bmatrix} \begin{bmatrix} q_{e0} \\ \tilde{q}_e \end{bmatrix} \\ \dot{\tilde{b}}_e = -T^{-1}b_e + k_2\tilde{q}_e \end{cases} \quad (27)$$

It is easily verified that:

$$[q_e^T \quad b_e^T] = [+1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad (28)$$

and

$$[q_e^T \quad b_e^T] = [-1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad (29)$$

are the equilibrium states of the error dynamics (27).

Let us consider a domain  $D$  as:

$$D = \left\{ q_e \in \mathbb{R}^{4 \times 1} / -1 \leq q_e(i) \leq 1, \{i=1..4\} \text{ and } b_e \in \mathbb{R}^{3 \times 1} / -\infty < b_e(i) < +\infty, \{i=1..3\} \right\} \quad (30)$$

Let us define two candidate Lyapunov functions  $V_1$  and  $V_2$ . These functions are continuous, positive definite, bounded and belong to the class  $C^2$  as:

$$V_1 = \frac{1}{2} b_e^T b_e + k_2 \left( (1 - q_{e0})^2 + \tilde{q}_e^T \tilde{q}_e \right), \text{ if } q_{e0} \geq 0 \quad (31)$$

$$V_2 = \frac{1}{2} b_e^T b_e + k_2 \left( (1 + q_{e0})^2 + \tilde{q}_e^T \tilde{q}_e \right), \text{ if } q_{e0} < 0 \quad (32)$$

In our case, the motion can change randomly, and then  $q_{e0}$  can take positive or negative value according to the value of  $\phi$ . If we choose the case  $q_{e0} \geq 0$ , then we consider the Lyapunov function in (31). Equation (31) can be written also as:

$$V = \frac{1}{2} b_e^T b_e + k_2 (2(1 - q_{e0})) \quad (33)$$

When differentiating (33) and using (27), one obtains:

$$\dot{V} = -2k_2 \dot{q}_{e0} + b_e^T \dot{b}_e \quad (34)$$

Finally, one obtains:

$$\dot{V} = -k_2 k_1 \tilde{q}_e^T \tilde{q}_e - b_e^T T^{-1} b_e \quad (35)$$

Since  $0 \leq q_{e0} \leq 1$ , then:

$$\tilde{q}_e^T \tilde{q}_e = 1 - q_{e0}^2 \geq 0 \quad (36)$$

When the gains  $k_1, k_2$  are positive constants, we can write:

$$\dot{V} \leq 0 \quad (37)$$

It is clear that  $\dot{V}$  is negative semi-definite and for equilibrium states (28) and (29), the condition  $V(q_e, b_e) = \dot{V}(q_e, b_e) = 0$  is satisfied. Thus,  $(\tilde{q}_e, b_e) \rightarrow 0$  and consequently  $q_{e0} \rightarrow \pm 1$  (the norm of  $q_e$  is always equal 1 [13]).

In the same way, for the case  $q_{e0} < 0$ , the associated Lyapunov function (32) leads to the same result given by (35).

Let  $\Upsilon = \{(q_e, b_e) \in D / \dot{V}(q_e, b_e) = 0\}$ . Therefore, the only solution that can stay identically in  $\Upsilon$  is the trivial solutions in (28) and (29). Now, applying Krasovskii-LaSalle's principle [20], one can conclude that the equilibrium states (28) and (29) are globally asymptotically stable, which ends the proof. ■

## 5. Optimal determination of $q'$ from vector observations

In this section, one presents an optimal method to determine the attitude  $q'$  that is used as measurements for the nonlinear observer in (21). The problem of optimal attitude determination algorithm using two sensor's measurements (vector observations) is known as the *Wahba's problem* [21]. In this paper, we consider the earth's magnetic field  $m = [m_x \ m_y \ m_z]^T$  defined in (17) and the gravity vector  $g = [g_x \ g_y \ g_z]^T$  expressed in (16) as vector observations. These quantities are locally constant in the Earth-fixed frame  $N$ .

To solve this problem, let us define firstly the modelling error:

$$\delta(\hat{q}) = (y - \hat{y}) \quad (38)$$

where  $y$  represents the theoretical values of gravity and earth's magnetic fields:

$$\mathbf{y} = \begin{bmatrix} g_x & g_y & g_z & m_x & m_y & m_z \end{bmatrix}^T \quad (39)$$

$\hat{\mathbf{y}}$  depicts the estimated values of  $\mathbf{g}$  and  $\mathbf{m}$ :

$$\hat{\mathbf{y}} = \begin{bmatrix} \hat{g}_x & \hat{g}_y & \hat{g}_z & \hat{m}_x & \hat{m}_y & \hat{m}_z \end{bmatrix}^T \quad (40)$$

Note that these estimated values are obtained from the followings two equations:

$$\hat{\mathbf{g}}_q = \begin{bmatrix} 0 & \hat{g}_x & \hat{g}_y & \hat{g}_z \end{bmatrix}^T = \hat{\mathbf{q}} \otimes \mathbf{f}_q \otimes \bar{\hat{\mathbf{q}}} \quad (41)$$

$$\hat{\mathbf{m}}_q = \begin{bmatrix} 0 & \hat{m}_x & \hat{m}_y & \hat{m}_z \end{bmatrix}^T = \hat{\mathbf{q}} \otimes \mathbf{h}_q \otimes \bar{\hat{\mathbf{q}}} \quad (42)$$

where  $\mathbf{f}_q$  represents the quaternion form associated to the acceleration vector as

$\mathbf{f}_q = \begin{bmatrix} 0 & f_x & f_y & f_z \end{bmatrix}^T$  and  $\mathbf{h}_q$  the quaternion form associated to the earth's magnetic field vector as  $\mathbf{h}_q = \begin{bmatrix} 0 & h_x & h_y & h_z \end{bmatrix}^T$ .  $\hat{\mathbf{q}}$  depicts the estimated attitude form the nonlinear observer in (21).

To minimize the error  $\delta(\hat{\mathbf{q}})$ , an iterative method is used to locate the minimum of the scalar squared error criterion function:

$$J(\hat{\mathbf{q}}) = \delta(\hat{\mathbf{q}})^T \delta(\hat{\mathbf{q}}) \quad (43)$$

Several approaches have proved to be effective as standard technique for nonlinear least-squares problems. In this paper, Levenberg Marquardt Algorithm (LMA) is used to minimise the nonlinear function  $J(\hat{\mathbf{q}})$ . It outperforms the Gauss-Newton Algorithm (GNA) and the method of gradient descent. The LMA is more robust than GNA in the region of local minimum [22]. Then the unique minimum can be written in the following form [23]:

$$\zeta(\hat{\mathbf{q}}) = c \left[ \mathbf{H}^T \mathbf{H} + \lambda \mathbf{I}_{3 \times 3} \right]^{-1} \mathbf{H}^T \delta(\hat{\mathbf{q}}) \quad (44)$$

where  $\lambda$  is a constant that adjusts the convergence rate of the algorithm and guarantees that the inverted term will be non-singular.  $c$  is a smoothing parameter chosen between 0 and 1 [6].

$\mathbf{H} \in \Re^{6 \times 3}$  is the Jacobian matrix defined as [24]:

$$\mathbf{H} = -2 \begin{bmatrix} [\hat{\mathbf{g}}^\times] & [\hat{\mathbf{m}}^\times] \end{bmatrix}^T \quad (45)$$

where  $[\hat{\mathbf{g}}^\times]$  and  $[\hat{\mathbf{m}}^\times]$  are defined in (4) with replacing  $\tilde{\mathbf{q}}_a$  by  $\hat{\mathbf{g}}$  and  $\hat{\mathbf{h}}$ .

Then, from (45), one obtains the following matrix:

$$H = -2 \begin{bmatrix} 0 & -\hat{g}_z & \hat{g}_y & 0 & -\hat{m}_z & \hat{m}_y \\ \hat{g}_z & 0 & -\hat{g}_x & \hat{m}_z & 0 & -\hat{m}_x \\ -\hat{g}_y & \hat{g}_x & 0 & -\hat{m}_y & \hat{m}_x & 0 \end{bmatrix}^T \quad (46)$$

Note that  $\zeta(\hat{q}) \in \Re^{3 \times 1}$  represents the vector part of the error quaternion of the LMA. Then one takes  $\zeta(\hat{q}) = \tilde{q}_{error}$ . Finally, to obtain the quaternion  $q'$ , that is used after in (22), one uses the following equation:

$$q'(k+1) = q'(k) \otimes [1 \ \tilde{q}_{error}]^T \quad (47)$$

For  $k = [1..n]$  ( $n$  is the final integration step), one chooses to use always the condition  $q'(k) = \hat{q}(k)$  to obtain faster convergence of the LMA. Note that the scalar part of the error quaternion  $[1 \ \tilde{q}_{error}]^T$  is very close to unit since the incremental quaternion corresponds to a small angle rotation and then the only dynamics of the system will be contained in the vector part of the perturbation quaternion. Note that this algorithm is limited to the lowest frequencies of the natural panel of movements [2] (the static or quasi-static situations) and the values of  $q'$  are strongly disturbed in dynamic situations.  $q'$  is smoothed after by the proposed observer in (21).

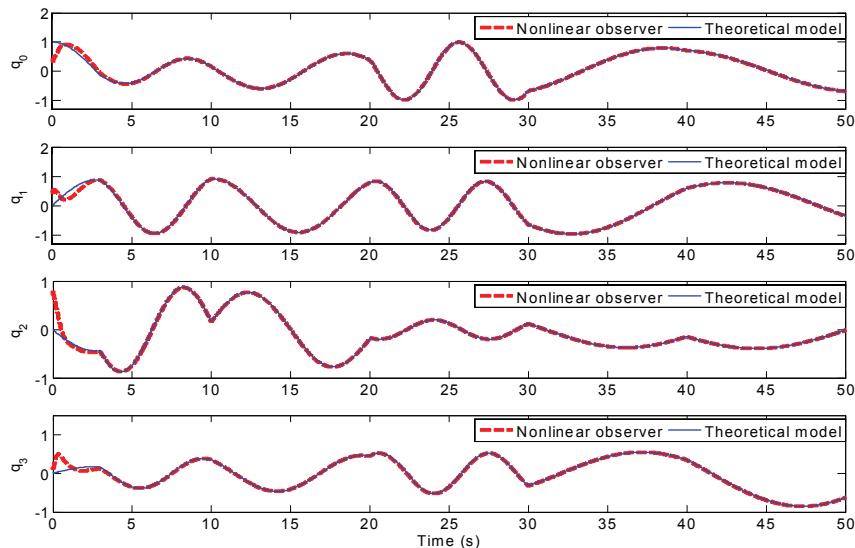
## 6. Simulation results

This section aims at illustrating the performance and accuracy of the complementary nonlinear observer designed in (21). Some numerical simulations were carried out under Matlab to estimate the rigid-body attitude based on the measurable inertial and magnetic measurements. To achieve these simulations, one starts by generating a theoretical example of attitude variation that was the subject of angular velocity data over 100 sec. The kinematic differential equation in (18) is used to obtain the continuous time motion in quaternion representation based on the considered angular velocity measurements. The obtained motion (quaternion term) is used in these simulations as reference and which will be estimated after by the proposed observer. To represent the sensor imperfections, an additive random zero-mean white Gaussian noise was considered for all measurements (see Table 1).

The sampling and calculation rate was chosen as 100Hz for all measurements (0.01sec). The angular velocity measurements are also assumed to be corrupted by a theoretical bias. This latter is generated based on equation (15). Notice that the bias is very common and undesirable characteristic of low-cost rate gyros. The observer gains  $k_1$  and  $k_2$  that guarantee convergent estimates, are set according to the considered sensor noise levels and sampling rate as:  $k_1 = 25$  and  $k_2 = 40$ . The constant  $c$  used in the LMA is set to  $c = 1/3$ . In these simulations, one chooses to initialize the theoretical states of the model (the quaternion  $q$  and bias  $b$ ) and those

of the observer with different random values. These conditions are summarized in Table 2. Notice that this choice allows illustrating the convergence of the observer although it was initialized far from the actual states.

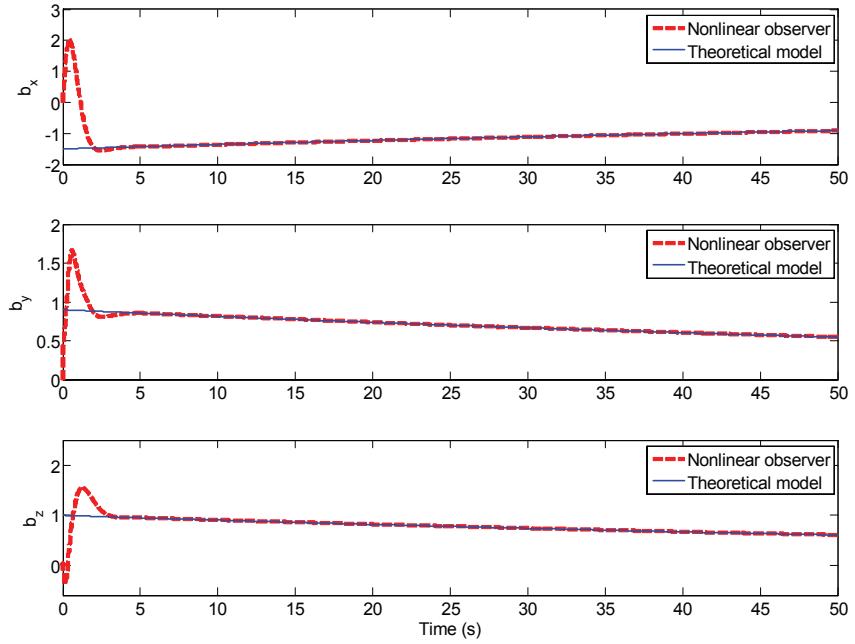
The time history of the seven states to be estimated (quaternion and bias components) and observer estimates are shown in Figs. 3 and 4. Despite the nonlinear observer and the theoretical model of the quaternion and bias were initialized with different initial conditions, one can note that the estimated quaternion and bias converge rapidly towards its theoretical values. Notice that one has tried to change the initial conditions and the same performance was obtained. It is clear that the domain of attraction seems to be large enough since the states are initialized far from their true values and the observer converges quite fast. Notice that the nonlinear observer copes well with the rate gyro bias. Moreover the noises of the accelerometer and magnetometer are rejected. In order to evaluate the overall attitude estimation performance, one chooses to plot the time history evolution of the estimation errors on the quaternion and bias. Figs. 5 and 6 depict the convergence of these errors towards zero during the motion. These obtained results show the effectiveness of the complementary nonlinear observer to the attitude estimation during the considered rigid-body motion.



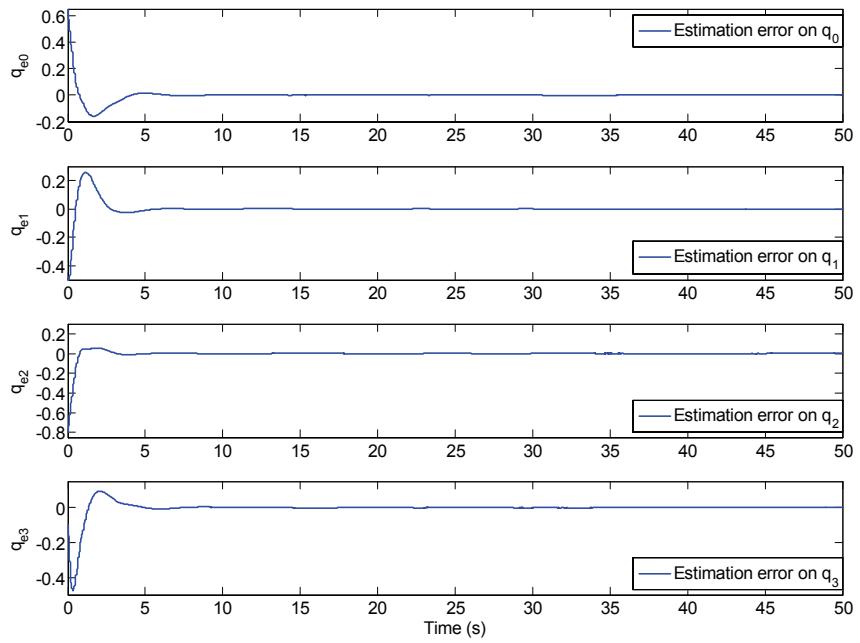
**Fig. 3.** Theoretical quaternion  $q$  and its estimated values  $\hat{q}$

**Table 1.** Characteristics of the various noises for the sensors' measurements

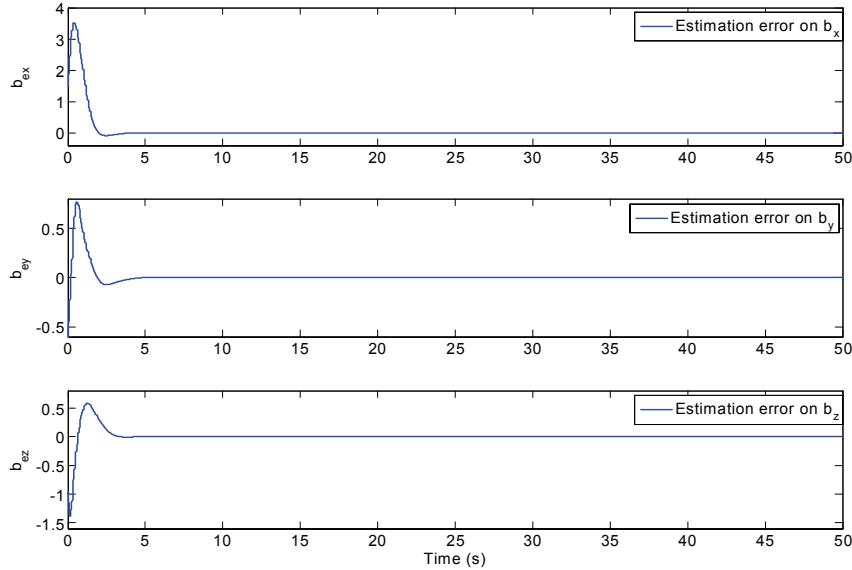
Sensors	Parameters	Standard deviations	Units
Accelerometer	$\delta_f$	0.01	$m/s^2$
Magnetometer	$\delta_h$	0.03	Gauss
Gyroscope	$\delta_G$	0.01	$rad/s$



**Fig. 4.** Theoretical bias  $b$  and its estimation  $\hat{b}$



**Fig. 5.** Quaternion estimation errors



**Fig. 6.** Gyro-bias estimation errors

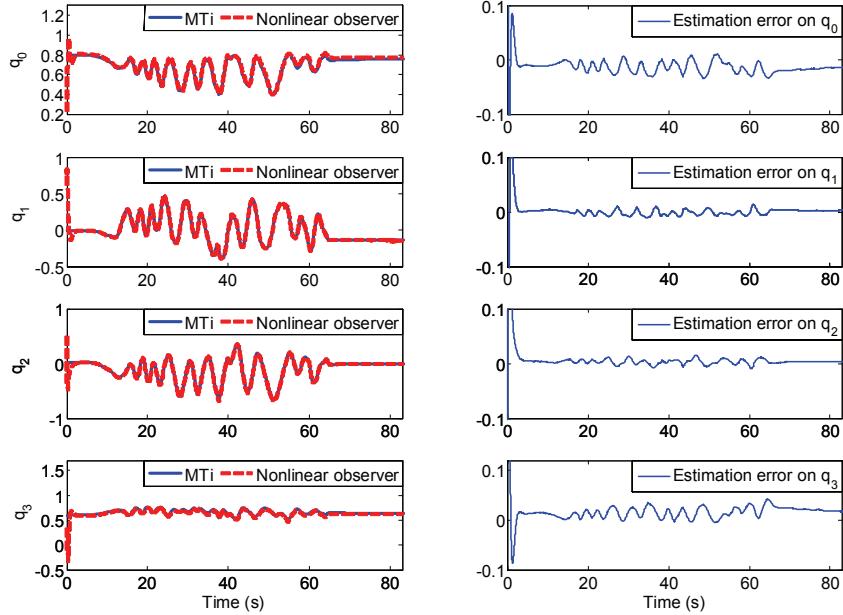
**Table 2.** Initial conditions

	Quaternion	Bias
Theoretical model	$q(t_0) = [1 \ 0 \ 0 \ 0]^T$	$b(t_0) = [-1.5 \ 0.9 \ 1]^T$
Nonlinear observer	$\hat{q}(t_0) = [0.3 \ 0.5 \ 0.8 \ 0.1]^T$	$\hat{b}(t_0) = [0 \ 0 \ 0]^T$

## 7. Experimental results

In order to evaluate the efficiency of the proposed approach in real word applications, an experimental setup was developed resorting to an inertial and magnetic sensor module. In this study, an Inertial Measurement Units (IMU) was employed: the MTi from Xsens Technologies [25], which outputs data at a rate of 100Hz. This MEMS device is a miniature, light weight, 3D digital output sensor (it outputs 3D acceleration (from accelerometer), 3D angular rate (from gyroscope), and 3D magnetic field data (from magnetometer)) with built-in bias, sensitivity, and temperature compensation. In the set of experiments, the calibrated data from MTi are used as inputs to the proposed nonlinear observer. In addition, this device is designed to track the body 3-D attitude output in Euler angles, quaternion or rotation matrix representations. The attitude from MTi is computed using an internal algorithm based on Xsens Kalman Filter (XKF) [25]. Note that MTi serves as tools for evaluation of the efficiency of the proposed attitude estimation algorithm.

In this paper, we choose to start our preliminary experiments by a simple motion in space with the MTi to validate the proposed approach. Those on king penguin will be made when the developed algorithm is implemented on the designed embedded prototype. The experiments were realized as follow: the MTi is posed on the ground without any motion for few moments. After that we take the MTi and we make it undergo a motion in all directions. Finally we return back to the static position on the ground. During this experiment, the MTi records inertial and magnetic measurements and the 3D-orientation as a quaternion, and transmits these readings to a PC via USB port. After that, the calibrated data are used to generate the estimated attitude using the proposed nonlinear observer in (21). Figure 7 illustrates the calculated attitude, in quaternion term, by the MTi and the estimated one obtained by the nonlinear observer. This figure contains also the corresponding estimation error on each quaternion component. Notice that the convergence rate is very fast and the mismatch is always small. This figure illustrates the efficiency of the proposed approach to estimate the attitude of the MTi during the motion.



**Fig. 7.** Experimental and estimated attitude – the corresponding estimation errors

## 8. Conclusion and future work

An approach based on complementary nonlinear observer algorithm has been proposed to estimate the attitude (orientation) in order to improve the quality of measures obtained by using low-cost sensors. The goal of this work, dedicated to Bio-logging area, is to produce an animal posture tracking over wide motion range. The algorithm was developed based on the rigid-body kinematic equation. A quaternion-

based process model is built to avoid the problem of singularities as in Euler angle representations. The observer exploits the complementary aspect of measurements given by the attitude sensors (3-axis accelerometer and 3-axis magnetometer) and 3-axis gyroscope. The proposed approach combines a strap-down system, based on the integral of the angular velocity, with a Levenberg Marquardt Algorithm (LMA) that uses Earth's magnetic field and gravity vector to calculate attitude measurements. Moreover, this algorithm includes the estimation of rate gyros biases to compensate angular velocity measurements. The efficiency of the proposed approach has been shown from a set of numerical simulations and some experimental results. The first prototype, dedicated to this animal application, will be logged in future works on some animals (dog and horse) for preliminary tests.

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