Robust Complementary Filter Design and Its Application to GPS/INS Vertical Channel Design

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Abstract – In this paper, a new approach is proposed for the design of a robust complementary filter. The design of a complementary filter is a method that clearly solves the problem of merging information from sensors whose frequency regions are complementary. For designing a robust complementary filter, a generalized, unbiased complementary filter structure is introduced, and H_∞ filtering technique is applied for adding robustness to the filter. In addition, the weighted H_∞ norm is used in order to suitably consider characteristics of sensor errors. To show a good performance of the proposed filter, its application for the design of Inertial Navigation System (INS) vertical channel damping loop using Global Positioning System (GPS) as its reference is presented. GPS has advantage of offering information of navigation, which are insensitive to time and place. However, because GPS altitude error has the non-stationary statistics, conventional design methods demanding the complete description of measurement errors are not usually appropriate. The proposed filter provides good performance even when error statistics are unknown, and the method is compatible to design a robust INS vertical channel damping loop. Simulation results show better performance of the proposed method over the existing vertical channel loop design schemes.

Keywords: Robust complementary filter, H_∞ filter, GPS/INS vertical damping loop, GPS error characteristic.

1. Introduction

Complementary filter is one of the most effective ways to properly merge information of sensors whose information locates at complementary frequency regions. In the linear time-invariant complementary filter design case, identity operator is decomposed into a stable low pass transfer function (TF) and a high pass TF. Each of them is assigned to the sensor with proper frequency region. In other words, the information of a sensor is used at low frequency only, while that of the other sensor is used at the complementary region. In case that one sensor provides biases or drifts, the complementary filter can eliminates these errors by referencing the other sensor information at the low frequency region. Also, high frequency disturbance possibly included in the sensor information referred at the low frequency region naturally is illuminated.

In this paper an approach to give robustness to complementary filter is proposed. Our scheme is composed of two steps. First, an unbiased filter structure is introduced to represent generalized complementary filters. And then H_∞ filtering technique is applied to design a robust complementary filter. The gains of unbiased filter are replaced by the gains of H_∞ filter in steady state. And the H_∞ filter is designed to optimize the H_∞ cost function which properly considers related sensor errors. Since the resultant filter does not require any stochastic quantity or assumptions of the sensor errors, it is robust against even non-stationary sensor errors, and weights of sensor errors can be given by designers.

As an application for demonstrating our results, we have designed a complementary filter for GPS/INS vertical channel damping. Nowadays, GPS has received much interest as a cheap but practical position-fix reference sensor. It is known that GPS provides relatively uniform navigation information with low sensitivity to time and place. However as shown in the Fig. 1 and Fig. 2, because of the non-stationary error properties of GPS height measurements, the conventional methods based on stochastic optimization may not be appropriate for GPS/INS vertical channel damping loop design. [1][2][3]

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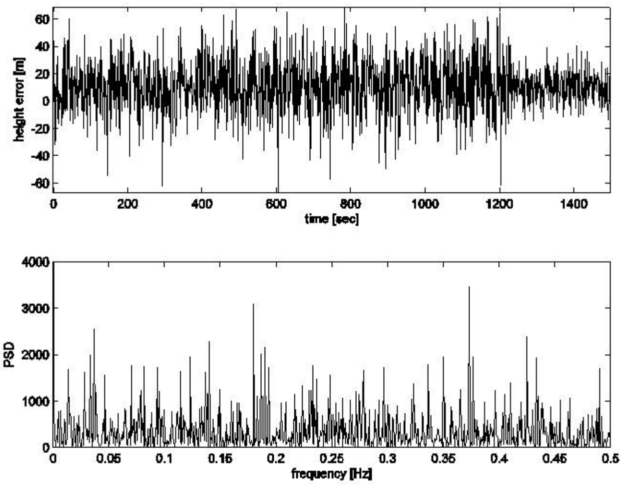

Fig. 1 Power Spectral Density of GPS errors

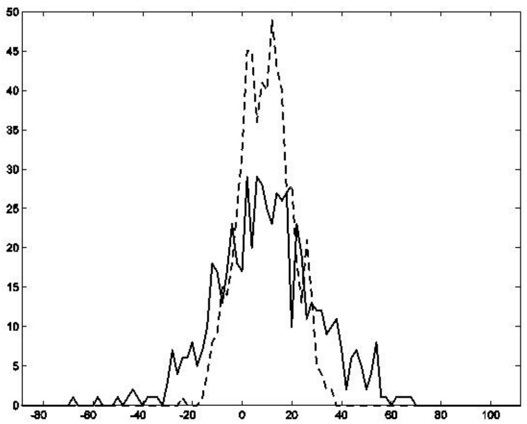

Fig. 2 Probability Density Function example of GPS errors in the different satellite uses

However, the proposed complementary filter design scheme in the paper provides an efficient INS vertical damping loop with robustness, which effectively copes with the non-stationary GPS errors. Simulations are performed in order to compare the performances of the proposed filter and conventional methods. The results demonstrate that the proposed filter works better even in extreme cases with highly non-stationary GPS errors, while the conventional method shows drastic performance degradation in those cases.

In section 2, robust complementary filter design method is introduced. The proposed method is applied with the application, INS 3rd order vertical channel damping loop in section 3. The simulation results are shown in section 4, and in the end, conclusion is in Section 5.

2. Robust complementary filter design

As shown in Fig. 3, we consider n+1 order complementary filter structure to remove biases included in the input.

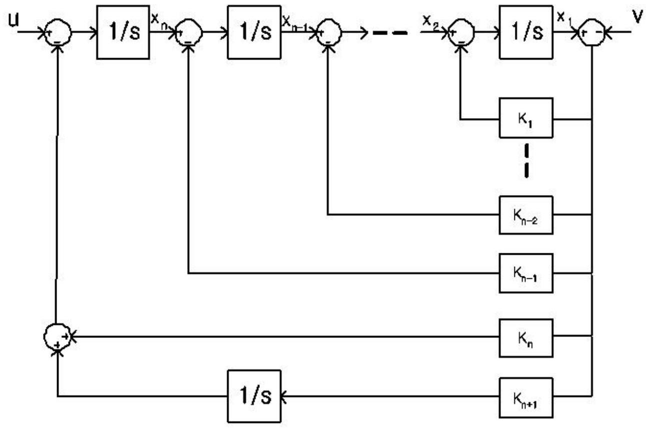

Fig. 3 Complementary filter with bias estimation

From Fig. 3, it can be seen that the state-space equation can be written as:

$$\dot{x}(t)=\begin{bmatrix}-K' & I_{n\times n}\\ -K_{n+1} & O_{1\times n}\end{bmatrix}x(t)+\begin{bmatrix}O_{n-1\times 1}\\ 1\\ 0\end{bmatrix}u(t)+\begin{bmatrix}K'\\ K_{n+1}\end{bmatrix}v(t) \qquad (1)$$

$$\hat{y}(t)=\begin{bmatrix}1 & 0 & 0\end{bmatrix}x(t), \quad K'=\begin{bmatrix}K_1\\ K_2\\ \vdots\\ K_n\end{bmatrix}$$

To show that the equation (1) is regarded as a structure of unbiased filter, the following stochastic linear system is introduced:

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)-v(t)\\ z(t)=Lx(t)\end{cases} \qquad (2)$$

$$A=\begin{bmatrix}O_{n\times 1} & I_{n\times n}\\ 0 & O_{1\times n}\end{bmatrix},\quad B=\begin{bmatrix}O_{n-1\times 1}\\ 1\\ 0\end{bmatrix},\quad C=\begin{bmatrix}1 & O_{1\times n}\end{bmatrix}$$

where $x(t)$ is the state, $y(t)$ is the output, $z(t)$ is the performance output, and $u(t)$ and $v(t)$ are the process and measurement noises respectively.

The next, a stable linear filter for the system (2) is considered as follows:

$$\begin{cases}\dot{x}(t)=Hx(t)+Ky(t)\\ \hat{z}(t)=L\hat{x}(t)\end{cases} \qquad (3)$$

Then, defining the error as $x_e(t)=x(t)-\hat{x}(t)$, the error dynamic equation can be written as

$$\dot{x}_e(t)=Hx_e(t)+\begin{bmatrix}B & K\end{bmatrix}\begin{bmatrix}u(t)\\ v(t)\end{bmatrix}+(A-H-KC)x(t)\cdot \qquad (4)$$

It is noted that if the filter is unbiased, it satisfies the following condition

$$H=A-KC\,. \qquad (5)$$

From the condition (5), the error dynamics (4) is written as follows:

$$\dot{x}_e(t) = (A - KC)x_e(t) + \begin{bmatrix} B & K \end{bmatrix}\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \quad (6)$$

It can be seen that the error system (6) is equivalent to the complementary filter structure (1).

If H_∞ filtering technique is used to obtain the gains of the filter, that problem can be solved by the suboptimal H_∞ filter which bounds the H_∞ norm (7) under the prescribed noise attenuation level γ^2. [4]

$$\sup_{u,v \in L_2} \frac{\int (\hat{z} - Lx)^T (\hat{z} - Lx) dt}{\int u^T u dt + \int v^T v dt} < \gamma^2 \quad (7)$$

If one error variance is comparatively much larger than another, the above constraint cannot suitably consider the effect of both errors. It may results in a degradation of the filter performance. For solving that problem, the weighted H_∞ norm (8) with proper weighing matrices Q and R is introduced.

$$\sup_{u,v \in L_2} \frac{\int (\hat{z} - Lx)^T (\hat{z} - Lx) dt}{\int u^T Q^{-1} u dt + \int v^T R^{-1} v dt} < \gamma^2 \quad (8)$$

The gains of unbiased H_∞ filter to satisfy the above constraint are calculated as follows:

$$K = P_\infty C^T R^{-1}, \quad (9)$$

where error covariance matrix P_∞ satisfies the algebraic Riccati equation

$$AP_\infty + P_\infty A + BQB^T - P_\infty C^T R^{-1} C P_\infty + \frac{1}{\gamma^2} P_\infty L^T L P_\infty = 0 \cdot (10)$$

3. The application: GPS/INS 3rd order vertical damping loop design

3.1 3rd order Baro/INS vertical damping loop and design methods

Pure INS generates velocity and position outputs by integrating measured accelerations from inertial measurement unit. In order to obtain the vertical component of INS, the gravity correction is needed. However, the used gravity model is different from real gravity at the position of measurement, and the difference can cause inherent divergence of the vertical component of INS. Accordingly, a stabilizing loop is required for the vertical channel. Moreover, this loop has the structure of complementary filter. Generally a 3rd order damping loop of INS vertical channel with Barometer altitude is as shown in Fig. 4. [4]

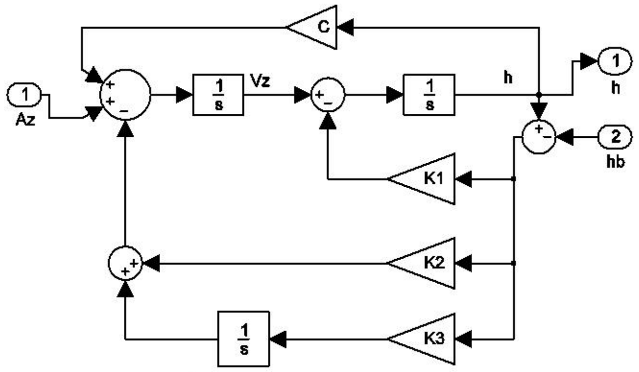

Fig. 4 3rd order Baro/INS vertical damping loop

System dynamics can be written directly from Fig.4 as follows:

$$\begin{cases} \dot{h} = v_z - k_1(h - h_b) \\ \dot{v}_z = A_z + ch - k_2(h - h_b) - b_a \\ \dot{b}_a = k_3(h - h_b) \end{cases} \quad (11)$$

One of the loop design method is pole assignment relying on the designer's experience. [1] In this method, the system characteristic equation (12) yields three poles at the complex frequency $s = -1/\tau$.

$$s^3 + k_1 s^2 + (k_2 - c)s + k_3 = 0, \quad c = 2g_o / r_o \quad (12)$$

Then the filter gains are

$$k_1 = \frac{3}{\tau},\ k_2 = \frac{3}{\tau^2} + 2\frac{g_o}{r_o},\ k_3 = \frac{1}{\tau^3} \quad . \quad (13)$$

The stochastic optimization with error modeling is to find optimal gains minimizing mean-squared velocity errors based on stochastic optimal control theory. The optimal gains are represented by the spectral densities $Q_{a1}, Q_{a2}, Q_{b1}, Q_{b2}$ of errors, assuming white noise and random walk. [2] When the reference error noise densities are comparatively large, filter gains and poles are as follows:

$$k_1 = -(p_1 + p_2 + p_3), \quad (14)$$
$$k_2 = c + p_1 p_2 + p_2 p_3 + p_3 p_1,\ k_3 = -p_1 p_2 p_3$$
$$p_1 = -\sqrt{Q_{b2}/Q_{b1}},\ p_2 = p_3 = -\sqrt{c}$$

In the stochastic optimization method, acceleration and external reference errors are modeled as white noise with known spectral densities. However, GPS is sometimes difficult to obtain the appropriate error statistics, and thus those conventional methods cannot guarantee suitable performance when GPS is used as a reference.

3.2 GPS/INS vertical damping loop with the robust complementary filter design

In order to stabilize INS vertical channel using GPS altitude, we bring in the structure of the 3rd order Baro/INS vertical channel which stabilizes the complementary filter.

Let w_a and w_g be the Disturbance included in INS and GPS respectively. From the error system model of Fig. 5, error dynamics area written as follows:

$$\begin{bmatrix} \delta\dot{h} \\ \delta\dot{v}_z \\ \delta\dot{a} \end{bmatrix} = \begin{bmatrix} -K_1 & 1 & 0 \\ -K_2+c & 0 & 1 \\ -K_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta h \\ \delta v_z \\ \delta a \end{bmatrix} + \begin{bmatrix} 0 & -K_1 \\ 1 & -K_2 \\ 0 & -K_3 \end{bmatrix} \begin{bmatrix} w_a \\ w_g \end{bmatrix} \tag{15}$$

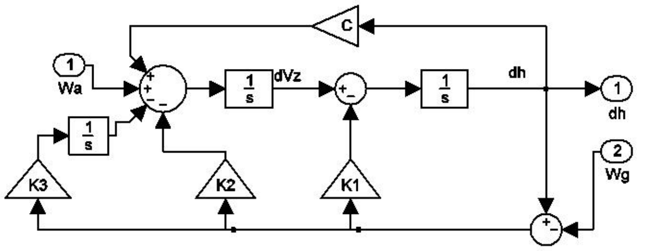

Fig. 5 Error model of GPS/INS vertical channel damping loop

Error dynamics (15) are the same as that of the structure of the given system (2) with the relevant matrices as shown below.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ c & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T \tag{16}$$

Therefore, the weighted H_∞ norm constraint to obtain the gain of the GPS/INS vertical channel damping loop is

$$\sup_{w_a, w_g \in L_2} \frac{\int e^T e dt}{\int w_a^T Q^{-1} w_a dt + \int w_g^T R^{-1} w_g dt} < \gamma^2 \tag{17}$$

where $e = \hat{z} - z$.

As mentioned above, the weighted norm can sufficiently consider the effect of disturbances w_a and w_g. It it noted that usually w_g is much larger than w_a.

4. Simulation results

Simulation is performed by implementing GPS simulator. For ensuring good performance of the proposed method, the proposed filter is compared with conventional methods: pole assignment and stochastic optimization method.

In the pole assignment method, τ set to 100. In the stochastic optimization method, Q_{b1} set to 100, and Q_{b2} set to 1. Design parameters of the proposed filter are as follows:

$$Q = 10^{-4}, \quad R = 1, \quad L = [2 \quad 0.5 \quad 0], \quad \gamma = 2.11$$

The gains of three filters are in table Ⅰ.

Table I Gains of the designed filters

	Pole assignment	Stochastic optimization	The proposed method
K_1	3.00×10^{-2}	1.04×10^{-1}	9.61×10^{-1}
K_2	3.03×10^{-4}	3.57×10^{-4}	3.83×10^{-2}
K_3	1.00×10^{-6}	3.07×10^{-7}	2.06×10^{-16}

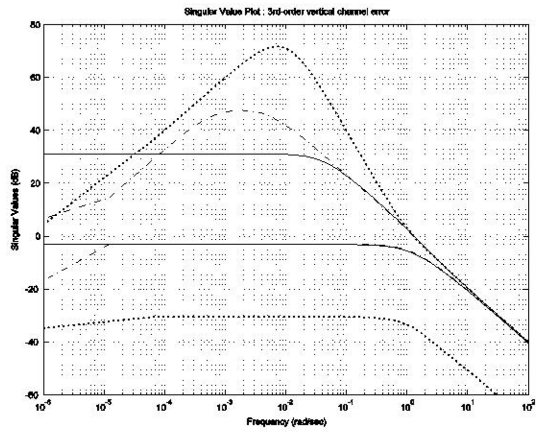

Fig. 6 Singular value plots of designed filters (dotted: pole-assignment, dashed: stochastic optimization, solid: proposed)

The proposed filter has comparatively low gains in whole frequency and moreover the energy gains of the proposed filter are lower than those of other filters in a part of frequency region.

Simulations in time domain are performed in the four cases as magnitudes of INS accelerometer short correlation time error and GPS error. Spectral density of random walk error is $1.25\times10^{-8} m^2 s^{-5}$ in all cases, and those of other errors are summarized in table Ⅱ.

Table II Simulation cases

Case	Accelerometer error	GPS error
Ⅰ	$6.00\times10^{-3} m^2 s^{-3}$	$100m^2$ (dotted in fig.2)
Ⅱ	$6.00\times10^{-3} m^2 s^{-3}$	$400m^2$ (solid in fig.2)
Ⅲ	$3.00\times10^{-1} m^2 s^{-3}$	$100m^2$ (dotted in fig.2)
Ⅳ	$3.00\times10^{-1} m^2 s^{-3}$	$400m^2$ (solid in fig.2)

The following figures (Fig. 7 through Fig. 10) represent time responses of three filters in four cases. The gains of filters are the same as those in table Ⅰ, but stochastic optimization filter in case Ⅲ and Ⅳ have the gains of $K_1 = 1.53\times10^{-1}$, $K_2 = 5.37\times10^{-3}$ and $K_3 = 1.12\times10^{-6}$ because INS accelerometer error is relatively large.

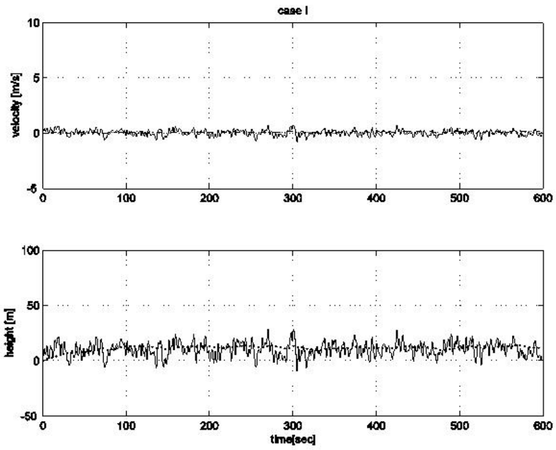

Fig. 7 Time response of case Ⅰ
(dotted: pole-assignment, dashed: stochastic optimization, solid: proposed)

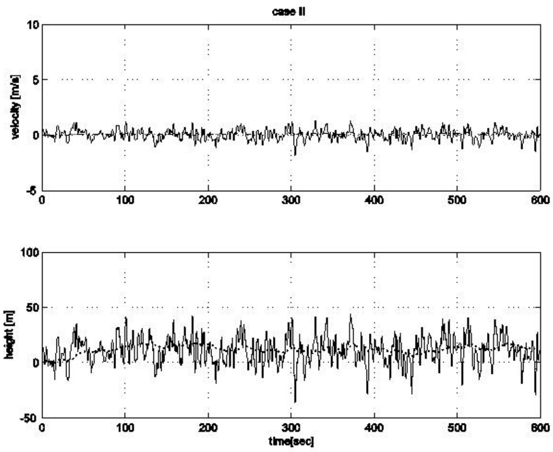

Fig. 8 Time response of case Ⅱ
(dotted: pole-assignment, dashed: stochastic optimization, solid: proposed)

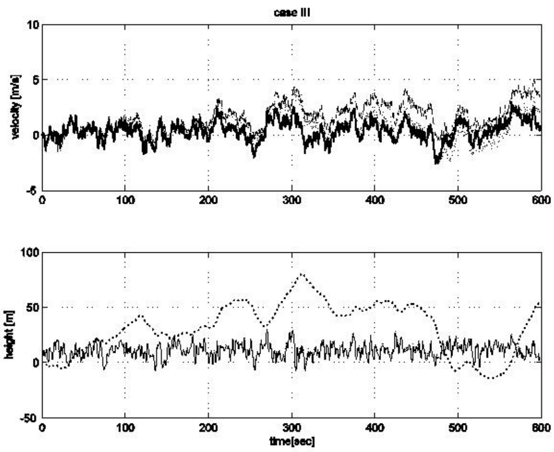

Fig. 9 Time response of case Ⅲ
(dotted: pole-assignment, dashed: stochastic optimization, solid: proposed)

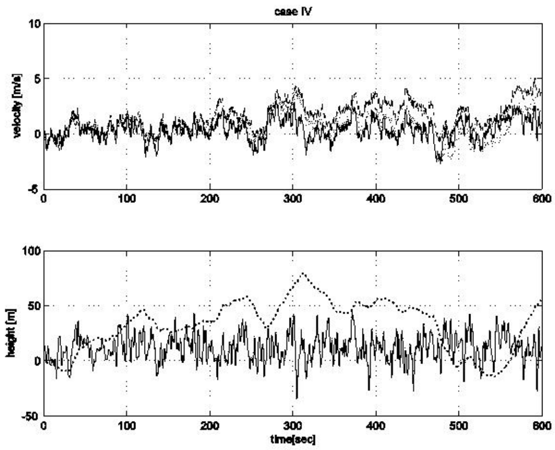

Fig. 10 Time response of case Ⅳ
(dotted: pole-assignment, dashed: stochastic optimization, solid: proposed)

In nominal case (case Ⅰ), conventional methods provide good performance with negligible bias errors. However, in worse cases with severe errors, they show drastic performance degradation, while the proposed filter works well even in those cases as shown.

5. Conclusions

In this paper, an approach to design a robust complementary filter which efficiently merges information of two sensors was proposed. The n+1 order robust complementary filter design method using H_∞ filtering technique was introduced, and it was applied to INS vertical channel stabilizing filter. The existing schemes designing INS vertical channel damping loop have used barometer altitude as the external reference. Nowadays, GPS is adopted as the reference in the INS compensation loop, because it is the cheap but efficient navigation system. In this paper, a systematic methodology to design the robust damping loop for GPS/INS vertical channel was introduced and simulated. Performance of the proposed filter was compared with those of conventional INS vertical channel damping loop design method, pole assignment and stochastic optimization method. Simulation results proved that the proposed filter has more reliable performance even though input errors were large and had non-stationary statistics.

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