A Novel Method for Modeling and Simulation of Brushless DC Motor with Kalman Filter*

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Abstract. Based on the mathematical model of the Brushless DC motor (BLDCM), a novel method of modeling and simulation for the speed control system of BLDCM with Kalman Filter is presented. In MATLAB/Simulink, the isolated functional blocks combine with the S-functions; and the model of BLDCM is established. Based on the discrete-state-space model, the recurrence formula for Kalman Filter is deduced, its block is established also. The speed feedback of BLDCM is filtered by Kalman Filter, and the PI speed controller is used. The reasonability and validity are testified by the coincidence of the simulation results and theoretical analysis. The results of experiment verified that the state space formula is correct; the correction matrix for Kalman Filter and the relevant coefficient are reasonable; and the effect of the filter is excellent. The novel method in this paper is also suitable for verifying the reasonability of other control algorithms, it offers a new thinking for designing and debugging actual motors.

Keywords: Brushless DC Motor, MATLAB/Simulink, Kalman Filter, Modeling, Simulation.

1 Introduction

The brushless DC motor, with such advantages as small volume, light weight, high efficiency, small rotational inertia and high control accuracy and so on, is widely applied in servo control, numerical control system and robot and other fields[1]. With the continuous expansion of application filed, it is required that the control system has a simple design, rational control algorithm and short development cycle, so establishment of the simulation model of brushless DC motor may effectively save the design time of control system. In this paper, a new method for establishment of brushless DC motor system simulation model based on MATLAB/Simulink is proposed on the basis of analysis of brushless DC motor mathematical model. The method, based on the motor discrete state space mathematical model, generalizes and

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deduces the Kalman Filter recursion formula and establishes the filter simulation module and the motor revolving speed is subjected to Kalman Filter for filtering and the system adopts the speed-regulation mode of PI revolving speed regulation.

2 Brushless DC Motor Mathematical Model

2.1 Model of Continuous State

The brushless DC motor operates under the state of two-phase conducting and three-phase six state, and suppose that the motor magnetic circuit is not saturated in the operation process and magnetic hysteresis loss are not calculated and the magnetic steel has a high impedance and the rotor inductive current is neglected, then the voltage balance equation of three phase winding may be expressed as[2]:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} r_a & 0 & 0 \\ 0 & r_b & 0 \\ 0 & 0 & r_c \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_a & L_{ab} & L_{ac} \\ L_{ba} & L_b & L_{bc} \\ L_{ca} & L_{cb} & L_c \end{bmatrix} \times D \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{1}$$

Where: e is the stator winding electromotive force (V): L_a~L_c is the self-inductance (H) of each phase winding; L_{ab}~L_{cb} is mutual inductance (H) between each two phase windings; and differential operator $D = d/dt$.

If the three phase windings are completely symmetric, the back electromotive force waveform has a trapezoidal wave with the flattened width of 120° electrical angle, and then it may be regarded that the three phase winding self-inductance and mutual inductance between windings are constant, then:

$$i_a + i_b + i_c = 0 \tag{2}$$

So:

$$M \cdot i_b + M \cdot i_c = -M \cdot i_a \tag{3}$$

Formula (1) is simplified as:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \cdot D \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{4}$$

The motor electromagnetic torque equation is:

$$T_e = \frac{e_a \cdot i_a + e_b \cdot i_b + e_c \cdot i_c}{\omega} \tag{5}$$

The conversion between ω and rotor electric angle speed ω_e: $\omega_e = n_p \cdot \omega$, where: n_p is motor rotor role pair number; and the motor mechanical movement equation is:

$$T_e - T_L = J \cdot \frac{d\omega}{dt} \tag{6}$$

According to the state space theory, formula (4) may be rewritten as the space equation:

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \\ \boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u} \end{cases} \tag{7}$$

Take the state vector $\boldsymbol{x} = [i_a \quad i_b \quad i_c]^T$, the coefficient matrix may be expressed as:

$$A = \begin{bmatrix} -r/(L-M) & 0 & 0 \\ 0 & -r/(L-M) & 0 \\ 0 & 0 & -r/(L-M) \end{bmatrix},$$

$$B = \begin{bmatrix} 1/(L-M) & 0 & 0 \\ 0 & 1/(L-M) & 0 \\ 0 & 0 & 1/(L-M) \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = 0;$$

The system input vector $\boldsymbol{u} = [u_a - e_a \quad u_b - e_b \quad u_c - e_c]^T$, solve the equation above and then it may obtain the motor winding current i_a, i_b and i_c.

2.2 System Transfer Function

Because the motor has only two-phase conduction at one moment, it may deduced that: $u = 2 \cdot r \cdot i + 2 \cdot (L-M) \cdot \frac{di}{dt} + 2 \cdot e$, $T_e = \frac{2 \cdot e \cdot i}{\omega}$; the winding back electromotive force waveform is the trapezoidal wave of the 120° flattened width, and when each phase winding is conduct, its back electromotive force is always in the flattened area, so $e = K_e \cdot \omega$, where K_e is the motor electromotive force coefficient, and the calculation equation above is subjected to the Laplace transform, to obtain:

$$\begin{cases} U(s) = 2 \cdot r \cdot I(s) + 2 \cdot (L-M) \cdot I(s) \cdot s + 2 \cdot E(s) \\ T_e(s) = 2 \cdot K_e \cdot I(s) \\ T_e(s) - T_L(s) = J \cdot \omega(s) \cdot s \\ E(s) = K_e \cdot \omega(s) \end{cases} \tag{8}$$

At the same time, $\omega(s) = \frac{\pi}{30} \cdot N(s)$ based on $\omega = 2 \cdot \pi \cdot n/60$; and if the load torque T_L is regarded as a kind of interference and the no load condition is considered, the motor transfer function may be obtained from equation (8):

$$\frac{N(s)}{U(s)} = \frac{\dfrac{30 \cdot K_e}{J \cdot (L-M) \cdot \pi}}{s^2 + \dfrac{r}{L-M} \cdot s + \dfrac{2 \cdot K_e^2}{J \cdot (L-M)}} = \frac{a}{s^2 + b \cdot s + c} \tag{9}$$

2.3 Model of Discrete State

In order to facilitate programming, the motor continuous transfer function is discretized. Supposed that d_1 and d_2 are two unequal roots of $s^2 + b \cdot s + c = 0$ and zero-order hold is considered and equation (10) may be obtained according to conversion of Z for equation (9) according to reference [3]:

$$\frac{N(s)}{U(s)} = Z[\frac{1-e^{-Ts}}{s} \frac{a}{s^2 + b \cdot s + c}] = \frac{A \cdot z + B}{z^2 + F \cdot z + D} \tag{10}$$

T is the system sampling cycle, in this paper $T = 0.1$ ms.

In order to obtain system state space equation, equation (10) is subjected to Z inverse transformation, to obtain:

$$n(k+2) + F \cdot n(k+1) + D \cdot n(k) = A \cdot u(k+1) + B \cdot u(k) \tag{11}$$

Take the state function: $x_1(k) = n(k)$, $x_2(k) = x_1(k+1) - A \cdot u(k)$, then it may obtain the discrete state space equation of brushless DC motor:

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{G} \cdot \boldsymbol{x}(k) + \boldsymbol{H} \cdot u(k) \\ n(k) = \boldsymbol{C} \cdot \boldsymbol{x}(k) \end{cases} \tag{12}$$

Where: $u(k)$ is the motor winding terminal voltage under the control of PWM; and $n(k)$ is the motor revolving speed; $\boldsymbol{G} = \begin{bmatrix} 0 & 1 \\ -D & -F \end{bmatrix}$; $\boldsymbol{H} = \begin{bmatrix} A \\ B - A \cdot F \end{bmatrix}$; $\boldsymbol{C} = [1 \quad 0]$; $\boldsymbol{G}$, $\boldsymbol{H}$ and $\boldsymbol{C}$ are constant matrix as for the system indicated in the paper.

3 Kalman Filter

Kalman filtering method is a kind of linear minimum variance estimation and belongs to a kind of optimal estimation algorithm and it is quite easy for its discrete algorithm to realize digitization.

The system state equation is:

$$\boldsymbol{x}(k) = \Phi(k-1) \cdot \boldsymbol{x}(k-1) + w(k-1) \tag{13}$$

The system measurement equation is:

$$\boldsymbol{y}(k) = H(k) \cdot \boldsymbol{x}(k) + v(k) \tag{14}$$

Where: Φ is transfer matrix from state $\boldsymbol{x}(k\text{-}1)$ to state $\boldsymbol{x}(k)$; $\boldsymbol{y}(k)$ is the observation vector at moment $t(k)$; H is the measurement matrix; $w(k)$ and $v(k)$ are the system noise and measurement noise respectively, and their statistical properties are:

$$E[w(k), w(i)^T] = \begin{cases} \boldsymbol{Q}(k) & i = k \\ 0 & i \neq k \end{cases} \tag{15}$$

$$E[v(k), v(i)^T] = \begin{cases} \boldsymbol{R}(k) & i = k \\ 0 & i \neq k \end{cases} \tag{16}$$

$$E[w(k), v(i)^T] = 0 \tag{17}$$

Where: $\boldsymbol{Q}(k)$ is the symmetric positive semi-definite matrix and $\boldsymbol{R}(k)$ is the symmetric positive definite matrix.

The Kalman filtering process is actually a recursive process, including two steps: prediction and update.

Step 1: prediction. Under the condition that the optimum estimated value $\hat{\boldsymbol{x}}(k)$ at $t(k)$ moment is known, the system state prediction value $\tilde{\boldsymbol{x}}(k+1)$ and error covariance matrix, $\tilde{\boldsymbol{P}}(k+1)$ at the next moment of $t(k+1)$ moment, are predicted.

$$\tilde{\boldsymbol{x}}(k+1) = \Phi(k)\hat{\boldsymbol{x}}(k) \tag{18}$$

$$\tilde{\boldsymbol{P}}(k+1) = \Phi(k)\boldsymbol{P}(k)\Phi(k)^T + \boldsymbol{Q}(k) \tag{19}$$

The Kalman gain matrix $\boldsymbol{K}(k+1)$ is obtained on its basis:

$$\begin{aligned} \boldsymbol{K}(k+1) &= \tilde{\boldsymbol{P}}(k+1)H(k+1)^T \\ &\times [H(k+1)\tilde{\boldsymbol{P}}(k+1)H(k+1)^T + \boldsymbol{R}(k+1)]^{-1} \end{aligned} \tag{20}$$

Step 2: update. The estimated value is modified according to the principle of observation error and minimum variance, so as to obtain the optimum estimated value $\hat{\boldsymbol{x}}(k+1)$ of state variable at moment $t(k+1)$ and obtain the optimum estimation variance matrix $\boldsymbol{P}(k+1)$ at the same time.

$$\hat{\boldsymbol{x}}(k+1) = \tilde{\boldsymbol{x}}(k+1) + \boldsymbol{K}(k+1)[\boldsymbol{y}(k+1) - H(k+1)\tilde{\boldsymbol{x}}(k+1)] \tag{21}$$

$$\boldsymbol{P}(k+1) = \tilde{\boldsymbol{P}}(k+1) - \boldsymbol{K}(k+1)H(k+1)\tilde{\boldsymbol{P}}(k+1) \tag{22}$$

The Kalman Filter recursion formula applicable to the system is.

$$
\begin{cases}
\hat{\boldsymbol{x}}(k+1) = \boldsymbol{G}\cdot\hat{\boldsymbol{x}}(k) + \boldsymbol{H}\cdot\boldsymbol{u}(k) + \boldsymbol{K}(k+1) \\
\qquad\qquad \times[n(k+1) - \boldsymbol{C}\cdot\boldsymbol{G}\cdot\hat{\boldsymbol{x}}(k) - \boldsymbol{C}\cdot\boldsymbol{H}\cdot\boldsymbol{u}(k)] \\
\tilde{\boldsymbol{P}}(k+1) = \boldsymbol{G}\cdot\boldsymbol{P}(k)\cdot\boldsymbol{G}^{\mathrm{T}} + \boldsymbol{H}\cdot\boldsymbol{Q}\cdot\boldsymbol{H}^{\mathrm{T}} \\
\boldsymbol{K}(k+1) = \tilde{\boldsymbol{P}}(k+1)\cdot\boldsymbol{C}^{\mathrm{T}}\cdot[\boldsymbol{C}\cdot\tilde{\boldsymbol{P}}(k+1)\cdot\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{R}]^{-1} \\
\boldsymbol{P}(k+1) = \tilde{\boldsymbol{P}}(k+1) - \boldsymbol{K}(k+1)\cdot\boldsymbol{C}\cdot\tilde{\boldsymbol{P}}(k+1)
\end{cases}
\tag{23}
$$

Formula（23） shows that $\tilde{\boldsymbol{P}}(k)$ and $\boldsymbol{K}(k)$ and $\boldsymbol{P}(k)$ have no relationship with state and may be subjected to off-line calculation；under the condition that the initial values of $\hat{\boldsymbol{x}}(0)$ and $\boldsymbol{P}(0)$ are known, the matrix $\boldsymbol{Q}$ and $\boldsymbol{R}$ shall be selected appropriately and they are subjected to many times of iterative computation to obtain the correction matrix $\boldsymbol{K}(k)$ steady state value of Kalman Filter; and then the optimum state estimated value $\hat{\boldsymbol{x}}(k)$ at moment k may be recursively calculated based on measured values $\boldsymbol{y}(k)$ and $\boldsymbol{K}(k)$ at moment k.

4 Model of Brushless DC Motor

Under the environment of MATLAB/Simulink, the thinking of using the module modeling is used to realize the graphical presentation of motor state space and Kalman recursion formula, and then substitute the coefficient matrix into various modules to establish the simulation model of brushless DC motor speed control system and the speed control strategy adopts the revolving speed closed loop PI control.

Fig. 1 is motor simulation mode control block diagram, mainly including: motor body module, revolving speed PI regulation module, PWM generator module, voltage inverter module and Kalman Filter module and so on, and the main modules of model are shown as in Fig. 2.

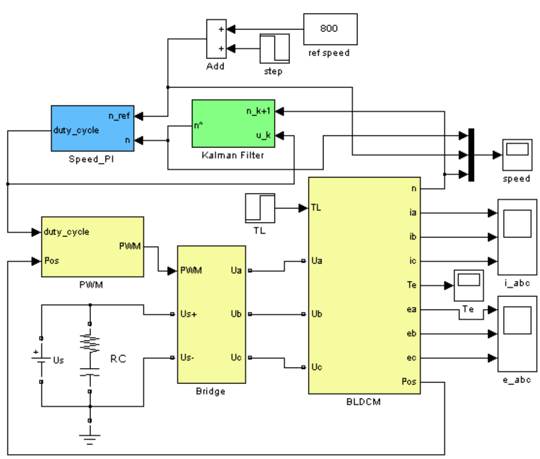
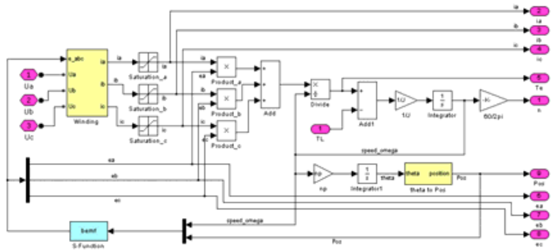
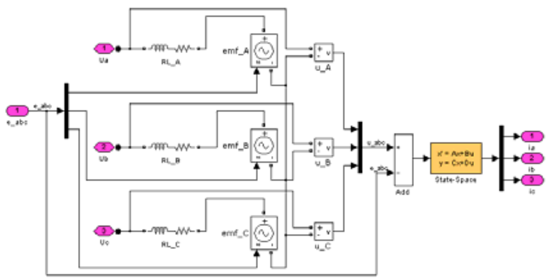


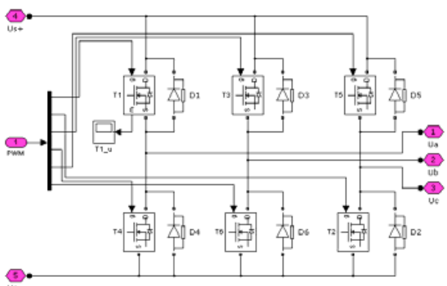
Fig. 1. Brushless DC motor simulation model



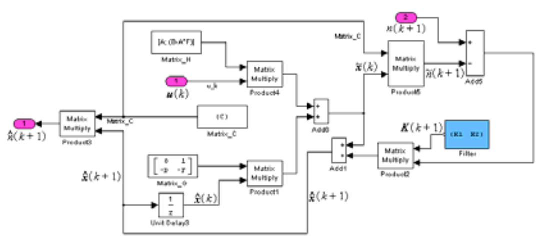
(*a*) motor block



(*b*) winding module



(*c*) inverter module



(*d*) Kalman Filter module

Fig. 2. Main function modules of motor

In the motor body module of Fig. 2(*a*), Pos is rotor electrical angle signal and the model adopts S function to realize the calculation equation. Firstly, the module shall obtain the motor phase current ia and ib and ic and obtain the electromagnetic torque Te after calculation; electromagnetic torque Te may be obtained after calculation; the motor electric angle speed ωe may be obtained after calculation of Te and then the rotor electrical angle theta may be obtained after integral; and the theta is finally converted into rotor position Pos.

Fig. 2(*b*) is the motor winding module and the paper adopts the segment linear method[4][5] to obtain winding back electromotive force and supply power and one operation cycle of motor is divided into 6 phases; each $\pi/3$ is a reversing phase and each operation phase of each phase may be indicated by a line segment; and the operation states of all the phases at one moment may be determined according to the rotor position and revolving speed signal and the back electromotive force waveform may be obtained through linear equation.

As for the inverter circuit in Fig. 2(*c*), 6 IGBTs compose three phase full bridge inverter circuit; (*d*) is Kalman filter module, where K1 and K2 included in $\boldsymbol{K}(k+1)$ is Kalman Filter correction matrix steady state value obtained through offline recursive calculation, and after calculation the finial value is K1=0.0348.

5 Simulation Analysis

The brushless DC motor parameters relevant to the paper are: P_N = 700W, T_e = 1.5N·m, I_N = 10A, n = 800rpm, winding resistance r = 2.1Ω, L = 20mH, M = 10mH, J = 7×10^{-4}kg·m^2, K_e = 0.1V/(rad/s), number of pole pairs np = 8, system power supply is 72V DC.

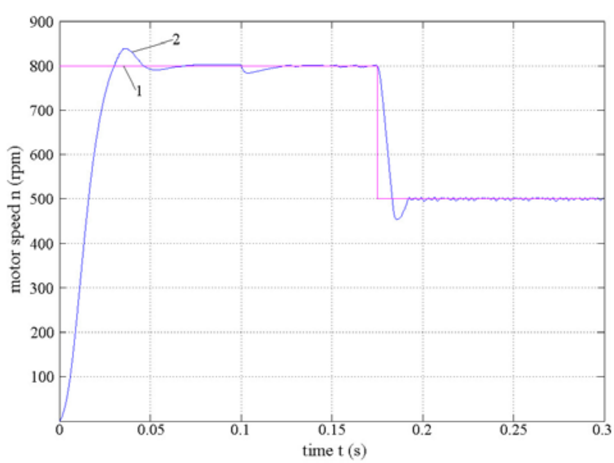


Fig. 3. Response curve of motor speed
1--target revolving speed; 2--motor revolving speed

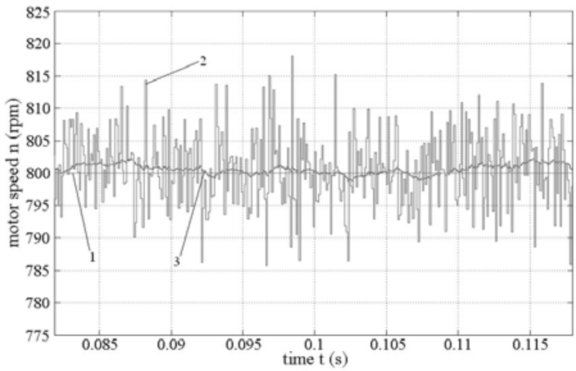


Fig. 4. Filtering effect of Kalman Filter
1--target speed; 2—motor speed without filter;
3—motor speed with filter

In order to verify the motor model static and dynamic performance, the system performs the no load startup and after it enters into the steady stat, the sudden load of T_L = 1N·m shall be increased at t = 0.1s and the target revolving speed shall be reduced to 500rpm at t = 0.175s. Fig. 3 shows that the model has a good servo performance and the steady operation is stable. In order to verify the filtering effect, the random interference is added in the system with the sampling time interval of T = 0.1 ms and diagram 4 shows the comparison of Kalman Filter filtering effect. The diagram shows that the revolving speed after filtering becomes more stable, which proves that the system state equation is correct and the steady state value obtained after recurrence is ideal; and the filtering effect is obvious.

6 Conclusion

In this paper, the brushless DC motor discrete state space mathematical model is established and in the Simulink environment the S function is combined to establish the motor simulation model and the Kalman Filter is added in the speed closed loop. The simulation result verifies the correctness and reliability of motor discrete state space model; the rationality of Kalman Filter recursion formula is proved; moreover, the model in this paper provides some simulation modules with a good versatility and portability. Therefore, the model provides a effective method and tool for analysis and design of the brushless DC motor control system and offers a new thinking for design and debugging for the motor control system.

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