MODULE *MatchingPennies*

This is a model for the Matching Pennies game. An explanation of the game can be found here: https://*www.gametheory.net*/dictionary/Games/*MatchingPennies.html*

EXTENDS *Naturals*

Matcher: The player who wins when the pennies match

Mismatcher: The player who wins when the pennies don't match

Winner: The player who won, either "Matcher" or "*Mismatcher*

GameOver: A boolean flag to prevent the model from exploring states after a winner has been decided

VARIABLES *Matcher*, *Mismatcher*, *Winner*, *GameOver*

$vars \triangleq \langle Matcher,\ Mismatcher,\ Winner,\ GameOver \rangle$

$$
\begin{aligned}
Init \triangleq\ & \land\ Matcher = \text{“”} \\
& \land\ Mismatcher\ = \text{“”} \\
& \land\ Winner\ = \text{“”} \\
& \land\ GameOver = \text{FALSE}
\end{aligned}
$$

The *Next* state is guarded by the *GameOver* boolean; the model only advances to the next state if the game is NOT over. The rest of the predicate enumerates the states each player can take, and the states that winner can take based on the next states of the players. The next state of *GameOver* is always TRUE, since this game only has one round

$$
\begin{aligned}
Next \triangleq\ & \land\ GameOver = \text{FALSE} \\
& \land\ \lor\ Matcher' = \text{“Heads”} \\
& \quad\ \lor\ Matcher' = \text{“Tails”} \\
& \land\ \lor\ Mismatcher' = \text{“Heads”} \\
& \quad\ \lor\ Mismatcher' = \text{“Tails”} \\
& \land\ Winner' = \text{IF}\ Matcher' = Mismatcher' \\
& \qquad\qquad\ \text{THEN}\ \text{“Matcher”} \\
& \qquad\qquad\ \text{ELSE}\ \text{“Mismatcher”} \\
& \land\ GameOver' = \text{TRUE}
\end{aligned}
$$

Weak Fairness is included to prevent infinite stutter steps. Although it's overkill to specify these temporal properties for such a small system (visual inspection is sufficient), it's a good exercise. Here we are saying that eventually *GameOver* will be TRUE and from that point on it will always be TRUE. Winner will eventually reach either the state "Matcher" or "Mismatcher"

Note that this model will deadlock, so deadlock detection should be turned off

$$
\begin{aligned}
Spec \triangleq\ & Init \land \Box[Next]_{vars} \land \mathrm{WF}_{vars}(Next) \\
Properties \triangleq\ & \land\ \Diamond\Box(GameOver = \text{TRUE}) \\
& \land\ \Diamond(Winner = \text{“Matcher”} \lor Winner = \text{“Mismatcher”})
\end{aligned}
$$