In[2]:=
```
dot[v_, w_] := v[[1]] * w[[1]] + v[[2]] * w[[2]] + v[[3]] * w[[3]] - v[[4]] * w[[4]]
```

In[3]:=
```
length[v_] := Sqrt[dot[v, v]]
```

In[4]:=
```
normalize[v_] := v / length[v]
```

In[5]:=
```
dist[p_, q_] := ArcCosh[-dot[p, q]]
```

In[6]:=
```
exp[p_, v_] := Cosh[length[v]] * p + (Sinh[length[v]] / length[v]) * v
```

In[7]:=
```
exp0[v_] := exp[{0, 0, 0, 1}, v]
```

In[8]:=
```
expinv[p_, q_] := dist[p, q] / Sinh[dist[p, q]] * (q + dot[p, q] * p)
```

In[9]:=
```
angle[v_, w_] := ArcCos[dot[normalize[v], normalize[w]]]
```

In[10]:=
```
angle[a_, b_, c_] := angle[expinv[b, a], expinv[b, c]]
```

In[11]:=
```
exp0[{x, y, z, 0}]
```

Out[11]=
$$\left\{\frac{x \sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}}, \frac{y \sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}}, \frac{z \sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}}, \cosh\left[\sqrt{x^2+y^2+z^2}\right]\right\}$$

In[12]:=
```
Simplify[Grad[exp0[{x, y, z, 0}], {x, y, z, w}]] // MatrixForm
```

Out[12]//MatrixForm=
$$\begin{pmatrix} \frac{x^2 \sqrt{x^2+y^2+z^2}\ \cosh\left[\sqrt{x^2+y^2+z^2}\right]+\left(y^2+z^2\right)\sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\left(x^2+y^2+z^2\right)^{3/2}} & x\,y\left(\frac{\cosh\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2}-\frac{\sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\left(x^2+y^2+z^2\right)^{3/2}}\right) & \\ x\,y\left(\frac{\cosh\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2}-\frac{\sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\left(x^2+y^2+z^2\right)^{3/2}}\right) & \frac{y^2 \sqrt{x^2+y^2+z^2}\ \cosh\left[\sqrt{x^2+y^2+z^2}\right]+\left(x^2+z^2\right)\sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\left(x^2+y^2+z^2\right)^{3/2}} & \\ x\,z\left(\frac{\cosh\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2}-\frac{\sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\left(x^2+y^2+z^2\right)^{3/2}}\right) & y\,z\left(\frac{\cosh\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2}-\frac{\sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\left(x^2+y^2+z^2\right)^{3/2}}\right) & \frac{z^2 \sqrt{x} \\ \frac{x \sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}} & \frac{y \sinh\left[\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}} & \end{pmatrix}$$

In[13]:=
```
ReplaceAll[Simplify[Grad[exp0[{x, y, z, 0}], {x, y, z, w}]],
  {Sqrt[x^2 + y^2 + z^2] → r, x^2 + y^2 + z^2 → r^2}] // MatrixForm
```

Out[13]//MatrixForm=
$$\begin{pmatrix} \frac{r\,x^2\cosh[r]+\left(y^2+z^2\right)\sinh[r]}{\left(r^2\right)^{3/2}} & x\,y\left(\frac{\cosh[r]}{r^2}-\frac{\sinh[r]}{\left(r^2\right)^{3/2}}\right) & x\,z\left(\frac{\cosh[r]}{r^2}-\frac{\sinh[r]}{\left(r^2\right)^{3/2}}\right) & 0 \\ x\,y\left(\frac{\cosh[r]}{r^2}-\frac{\sinh[r]}{\left(r^2\right)^{3/2}}\right) & \frac{r\,y^2\cosh[r]+\left(x^2+z^2\right)\sinh[r]}{\left(r^2\right)^{3/2}} & y\,z\left(\frac{\cosh[r]}{r^2}-\frac{\sinh[r]}{\left(r^2\right)^{3/2}}\right) & 0 \\ x\,z\left(\frac{\cosh[r]}{r^2}-\frac{\sinh[r]}{\left(r^2\right)^{3/2}}\right) & y\,z\left(\frac{\cosh[r]}{r^2}-\frac{\sinh[r]}{\left(r^2\right)^{3/2}}\right) & \frac{r\,z^2\cosh[r]+\left(x^2+y^2\right)\sinh[r]}{\left(r^2\right)^{3/2}} & 0 \\ \frac{x\sinh[r]}{\sqrt{r^2}} & \frac{y\sinh[r]}{\sqrt{r^2}} & \frac{z\sinh[r]}{\sqrt{r^2}} & 0 \end{pmatrix}$$