In[1]:= `dot[v_, w_] := v[[1]] * w[[1]] + v[[2]] * w[[2]] + v[[3]] * w[[3]] - v[[4]] * w[[4]]`

In[2]:= `length[v_] := Sqrt[dot[v, v]]`

In[3]:= `normalize[v_] := v / length[v]`

In[4]:= `dist[p_, q_] := ArcCosh[-dot[p, q]]`

In[5]:= `exp[p_, v_] := Cosh[length[v]] * p + (Sinh[length[v]] / length[v]) * v`

In[6]:= `exp0[v_] := exp[{0, 0, 0, 1}, v]`

In[7]:= `expinv[p_, q_] := dist[p, q] / Sinh[dist[p, q]] * (q + dot[p, q] * p)`

In[8]:= `angle[v_, w_] := ArcCos[dot[normalize[v], normalize[w]]]`

In[9]:= `angle[a_, b_, c_] := angle[expinv[b, a], expinv[b, c]]`

In[14]:= `exp0[{x, y, z, 0}]`

Out[14]= $\left\{ \dfrac{x \, \text{Sinh}\left[\sqrt{x^2 + y^2 + z^2}\right]}{\sqrt{x^2 + y^2 + z^2}}, \dfrac{y \, \text{Sinh}\left[\sqrt{x^2 + y^2 + z^2}\right]}{\sqrt{x^2 + y^2 + z^2}}, \dfrac{z \, \text{Sinh}\left[\sqrt{x^2 + y^2 + z^2}\right]}{\sqrt{x^2 + y^2 + z^2}}, \text{Cosh}\left[\sqrt{x^2 + y^2 + z^2}\right] \right\}$

In[15]:= `Simplify[Grad[exp0[{x, y, z, 0}], {x, y, z, w}]] // MatrixForm`

Out[15]//MatrixForm=

$$\begin{pmatrix} \dfrac{x^2\sqrt{x^2+y^2+z^2}\,\text{Cosh}\left[\sqrt{x^2+y^2+z^2}\right]+(y^2+z^2)\,\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)^{3/2}} & x\,y\left(\dfrac{\text{Cosh}\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2} - \dfrac{\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)^{3/2}}\right) & \vdots \\ x\,y\left(\dfrac{\text{Cosh}\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2} - \dfrac{\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)^{3/2}}\right) & \dfrac{y^2\sqrt{x^2+y^2+z^2}\,\text{Cosh}\left[\sqrt{x^2+y^2+z^2}\right]+(x^2+z^2)\,\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)^{3/2}} & \vdots \\ x\,z\left(\dfrac{\text{Cosh}\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2} - \dfrac{\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)^{3/2}}\right) & y\,z\left(\dfrac{\text{Cosh}\left[\sqrt{x^2+y^2+z^2}\right]}{x^2+y^2+z^2} - \dfrac{\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{(x^2+y^2+z^2)^{3/2}}\right) & \dfrac{z^2\sqrt{x}}{} \\ \dfrac{x\,\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}} & \dfrac{y\,\text{Sinh}\left[\sqrt{x^2+y^2+z^2}\right]}{\sqrt{x^2+y^2+z^2}} & \end{pmatrix}$$

In[17]:= `ReplaceAll[Simplify[Grad[exp0[{x, y, z, 0}], {x, y, z, w}]],`
`    {Sqrt[x^2 + y^2 + z^2] → r, x^2 + y^2 + z^2 → r}] // MatrixForm`

Out[17]//MatrixForm=

$$\begin{pmatrix} \dfrac{r\,x^2\,\text{Cosh}[r]+(y^2+z^2)\,\text{Sinh}[r]}{r^{3/2}} & x\,y\left(\dfrac{\text{Cosh}[r]}{r} - \dfrac{\text{Sinh}[r]}{r^{3/2}}\right) & x\,z\left(\dfrac{\text{Cosh}[r]}{r} - \dfrac{\text{Sinh}[r]}{r^{3/2}}\right) & 0 \\ x\,y\left(\dfrac{\text{Cosh}[r]}{r} - \dfrac{\text{Sinh}[r]}{r^{3/2}}\right) & \dfrac{r\,y^2\,\text{Cosh}[r]+(x^2+z^2)\,\text{Sinh}[r]}{r^{3/2}} & y\,z\left(\dfrac{\text{Cosh}[r]}{r} - \dfrac{\text{Sinh}[r]}{r^{3/2}}\right) & 0 \\ x\,z\left(\dfrac{\text{Cosh}[r]}{r} - \dfrac{\text{Sinh}[r]}{r^{3/2}}\right) & y\,z\left(\dfrac{\text{Cosh}[r]}{r} - \dfrac{\text{Sinh}[r]}{r^{3/2}}\right) & \dfrac{r\,z^2\,\text{Cosh}[r]+(x^2+y^2)\,\text{Sinh}[r]}{r^{3/2}} & 0 \\ \dfrac{x\,\text{Sinh}[r]}{\sqrt{r}} & \dfrac{y\,\text{Sinh}[r]}{\sqrt{r}} & \dfrac{z\,\text{Sinh}[r]}{\sqrt{r}} & 0 \end{pmatrix}$$