

Compute guided elastic wave (GEW) dispersion in MATLAB.

GEWtool is an advanced system to compute and postprocess guided elastodynamic waves (GEWs) in plates and cylinders. It is simple to use yet provides full access to the computational results as well as the underlying code. You are welcome to contribute to this open-source project.

Features:

- Multi-layered plates, tubes and rods
- Super fast, guarantees to find all solutions
- General anisotropy, dissipation
- Compute real frequencies, complex wavenumbers, or ZGV points
- Choose polarization (Lamb/SH/coupled) and parity (S/A)



Code repository: <https://github.com/dakiefer/GEWtool>

Example: Lamb waves

```
mat = Material('steel');           % load from database (or create your own)
h = 1e-3;                         % thickness in m
N = 12;                           % number of nodes (dictates accuracy)
k = linspace(1e-2, 12, 100)/h;    % wavenumbers to solve for
plate = Plate(mat, h, N);         % create waveguide description
gews = plate.LambSA; tic;         % choose S+A Lamb waves (assembles matrices)
dat = computeW(gews, k, 4); toc;  % solve and save 4 modes (argument optional)
plot(dat(1).k, dat(1).w/2/pi, 'Color', '#3B518B'); hold on;           % symmetric
plot(dat(2).k, dat(2).w/2/pi, 'Color', '#5EC962'); ylim([0, 6e6]); % anti-sym
xlabel('wavenumber k in rad/m'), ylabel('frequency f in Hz')
```

output: > Elapsed time is 0.010129 seconds.

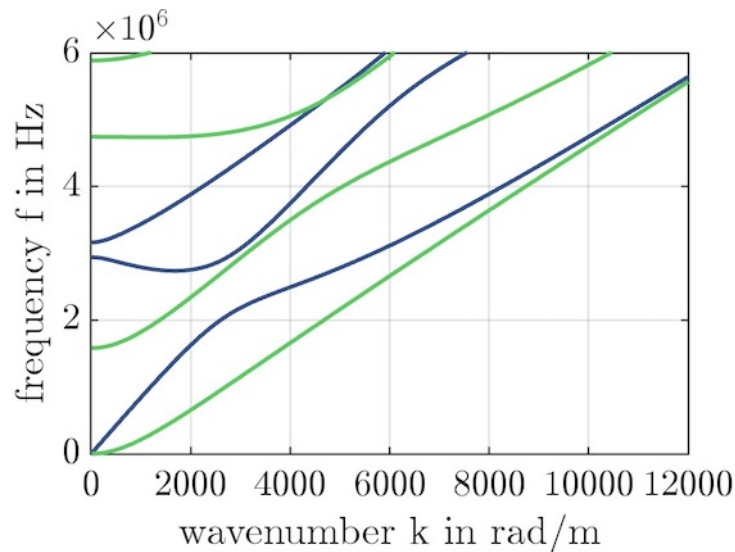


Figure 1: Lamb waves in steel

Proceed by inspecting the laser-ultrasonic excitability of the waves computed above (product of tangential and normal displacements $u_x \cdot u_y$):

```
k = linspace(1e-2, 12, 200)/h;    % use more wavenumbers
gew = plate.Lamb;                 % choose all Lamb waves
dat = computeW(gew, k, 7);        % compute
exc = excitabilityLUS(gew, dat, 'top'); % ux*uy at top surface (value of 1 at 100x median)
```

```

exc = 20*log10(exc); % in decibel
scatter(dat.k(:)/1e3, dat.w(:)/2/pi/1e6, 15, exc(:), 'filled'), ylim([0, 6]);
colormap(flipud(colormap)); cb = colorbar; caxis([-50, 0]);
xlabel('wavenumber k in rad/mm'), ylabel('frequency f in MHz')
title('laser-ultrasonic excitability in dB')

```

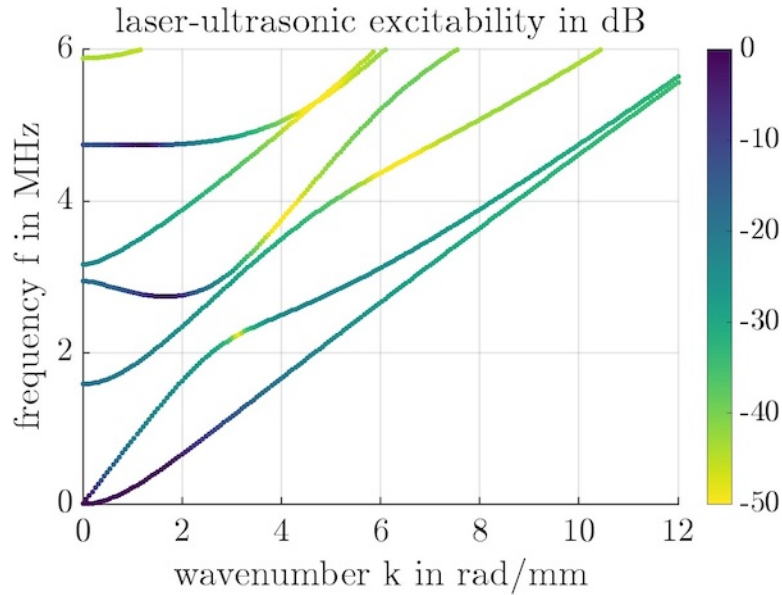


Figure 2: LUS excitability

Installation

Add `GEWtool` and the relevant subfolders to the Matlab path and save it for future sessions. To achieve this:

1. change to the `GEWtool` folder (e.g., by navigating or using `cd`)
2. execute `install`

Enjoy!

Getting started

To get started, explore the `examples` directory.

You can also display help for all functions and classes, e.g., by typing `help Plate`. The most important ones are

- Material representation: `Material`, `MaterialIsotropic`
- Waveguides: `Plate`, `Cylinder`
- Solvers: `computeW`, `computeK`, `computeZGV`

Known limitations

- Cylinders: only axial waves are supported for now.
- Leaky waves: not implemented for now.

Contact me if you have questions: daniel.kiefer@espci.fr

Mathematical and physical background

`GEWtool` implements the *Spectral Element Method (SEM)* (higher-order Finite Elements) to solve the *waveguide problem*, i.e., the boundary value problem that describes wave propagation in the structure. Such an approach is commonly qualified as ‘semi-analytical’. Contrary to classical root-finding of the characteristic

equation, this method does not miss solutions. Moreover, unlike Finite Elements, the Spectral Elements lead to small but dense matrices.

Solutions are computed with machine precision provided you have set the discretization order N sufficiently high. The higher you go in frequency-thickness, the higher N should be. As a rule of thumb: half of the obtained modes will be accurate. The figure below shows the convergence with respect to the Rayleigh-Lamb root of the S1 mode at 5.6 rad/mm in an aluminum plate (solution close to 5 MHz mm). A Spectral Collocation implementation is shown in comparison. 15 digits accuracy is attained with $N = 16$ in this case.

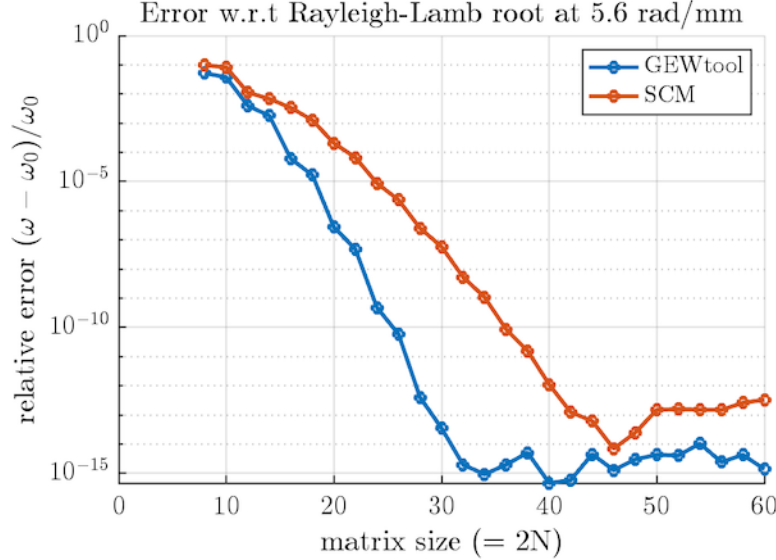



Figure 3: relative error w.r.t. Rayleigh-Lamb root

For general information on the formulation of the elastic waveguide problem refer to

D. A. Kiefer, *Elastodynamic quasi-guided waves for transit-time ultrasonic flow metering*, ser. FAU Forschungen, Reihe B, Medizin, Naturwissenschaft, Technik, vol. 42. Erlangen: FAU University Press, 2022, doi: 10.25593/978-3-96147-550-6. 

Dependencies

GEWtool depends on the functions `barylag`, `collocD`, `lglnodes` and `lgwt` created by Greg von Winkel. They are bundled together with their license files in the `resourcesAndDeps` directory. You may also find them on

Greg von Winkel, MATLAB Central File Exchange, <https://fr.mathworks.com/matlabcentral/profile/authors/869721>.

The function `computeZGVDirect` depends on the `MultiParEig` toolbox by Bor Plestenjak and Andrej Muhič:

Bor Plestenjak (2022). `MultiParEig` (<https://www.mathworks.com/matlabcentral/fileexchange/47844-multipareig>), MATLAB Central File Exchange.

The function `legendUnq` by Adam Danz is used in the example scripts:




Adam Danz (2024). `legendUnq` (<https://www.mathworks.com/matlabcentral/fileexchange/67646-legendunq>), MATLAB Central File Exchange.

Citing GEWtool

If this code is useful to you, please cite it as (always indicating the DOI):

D. A. Kiefer (2023). GEWtool. <https://doi.org/10.5281/zenodo.10114243> (<https://github.com/dakiefer/GEWtool>)

Please also cite publication(s) as relevant, e.g.,

- [1] D. A. Kiefer, S. Mezil, and C. Prada, “Beating resonance patterns and extreme power flux skewing in anisotropic elastic plates,” *Science Advances*, vol. 9, no. 51, p. eadk6846, Dec. 2023, doi: 10.1126/sciadv.adk6846. 
- [2] H. Gravenkamp, B. Plestenjak, and D. A. Kiefer, “Notes on osculations and mode tracing in semi-analytical waveguide modeling,” *Ultrasonics*, vol. 135, p. 107112, Jul. 2023, doi: 10.1016/j.ultras.2023.107112. 
- [3] D. A. Kiefer, B. Plestenjak, H. Gravenkamp, and C. Prada, “Computing zero-group-velocity points in anisotropic elastic waveguides: Globally and locally convergent methods,” *The Journal of the Acoustical Society of America*, vol. 153, no. 2, pp. 1386–1398, Feb. 2023, doi: 10.1121/10.0017252. 

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Numerical methods to compute ZGV points

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Homogenization, quaternions, rotation angles

Acknowledgments - Many inspiring discussions influenced GEWtool. D. A. Kiefer is thankful to: Hauke Gravenkamp, Pierre Chantelot, Claire Prada, Clemens Grünsteidl and Michael Ponschab

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I have several years of experience in waveguide modeling and numerical implementations thereof. In January 2022 I decided to create a new modular and versatile code from scratch. The result is GEWtool. My hope is that it be a valuable research tool and at the same time a helpful educational resource for those interested in numerical methods and elastic waves.

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