

GEWtool DOI 10.5281/zenodo.10114243

Compute guided elastic wave (GEW) dispersion in MATLAB.

GEWtool is an advanced system to compute and postprocess guided elastodynamic waves (GEWs) in plates and cylinders. It is simple to use yet provides full access to the computational results as well as the underlying code. You are welcome to contribute to this open-source project.

Features:

- multi-layered plates, tubes and rods
- anisotropy, **piezoelectricity**, dissipation
- super fast, guarantees to find all solutions
- compute real frequencies, complex wavenumbers, or ZGV points
- choose polarization (Lamb/SV/coupled) and parity (S/A)



Code repository: <https://github.com/dakiefer/GEWtool>

[!WARNING] **v2.0 introduces the following backward-incompatibilities:**

- directory structure updated: you will need to re-run `install.m` after updating to v2.0.
- postprocessing functions: now only require the `GEWdat` object returned by the solver, e.g.,
`cex = energyVelAxial(dat);` instead of `cex = energyVelAxial(gew,dat);`
- `rotateEuler()` changed from extrinsic to intrinsic rotations! This only affects you if you do several rotations in a row. Reverse the order of the rotation sequence in your old scripts to get the same results as previously. See `help rotateEuler` for more information.
- changed coordinate system
 - plate: ex-ey-ez system: ex is propagation direction, ez is normal to the plate surface (previously, this was ey)
 - cylinder: ex-ephi-er system: ex is propagation direction, er is the radial direction

[!NOTE] Coming soon:

- Leaky waves

Contact me if you have questions: daniel.kiefer@espci.fr

Example: Lamb waves

```
mat = MaterialIsotropic('steel'); % load from database (or create your own)
h = 1e-3; % thickness in m
N = 22; % number of nodes (dictates accuracy)
k = linspace(1e-2, 12, 100)/h; % wavenumbers to solve for
plate = Plate(mat, h, N); % create waveguide description
gews = plate.LambSA; tic; % choose S+A Lamb waves (assembles matrices)
dat = computeW(gews, k, 6); toc; % solve and save 6 modes (argument optional)
plot(dat); % plot dispersion curves
```

output: > Elapsed time is 0.136516 seconds.

The result can be explored interactively using the purpose-built graphical user interface GEWinspector:

```
GEWinspector(dat) % opens interactive window
```

All data is available for postprocessing and visualization in the returned object `dat`. For example, you can plot the laser-ultrasonic excitability of the waves (product of tangential and normal displacements `ux · uz`):

```
k = linspace(1e-2, 12, 200)/h; % use more wavenumbers
gew = plate.Lamb; % choose all Lamb waves this time
dat = computeW(gew, k, 7); % compute
exc = excitabilityLUS(dat, 'top'); % vx*uz at top surface
% (value of 1 at 100x median)
exc = 20*log10(exc); % in decibel
scatter(dat.k(:)/1e3, dat.w(:)/2/pi/1e6, 15, exc(:), 'filled'), ylim([0, 6]);
```

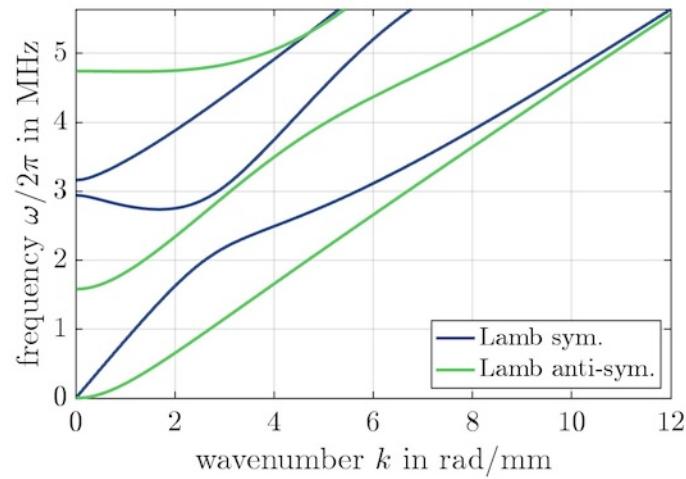


Figure 1: Lamb waves in steel

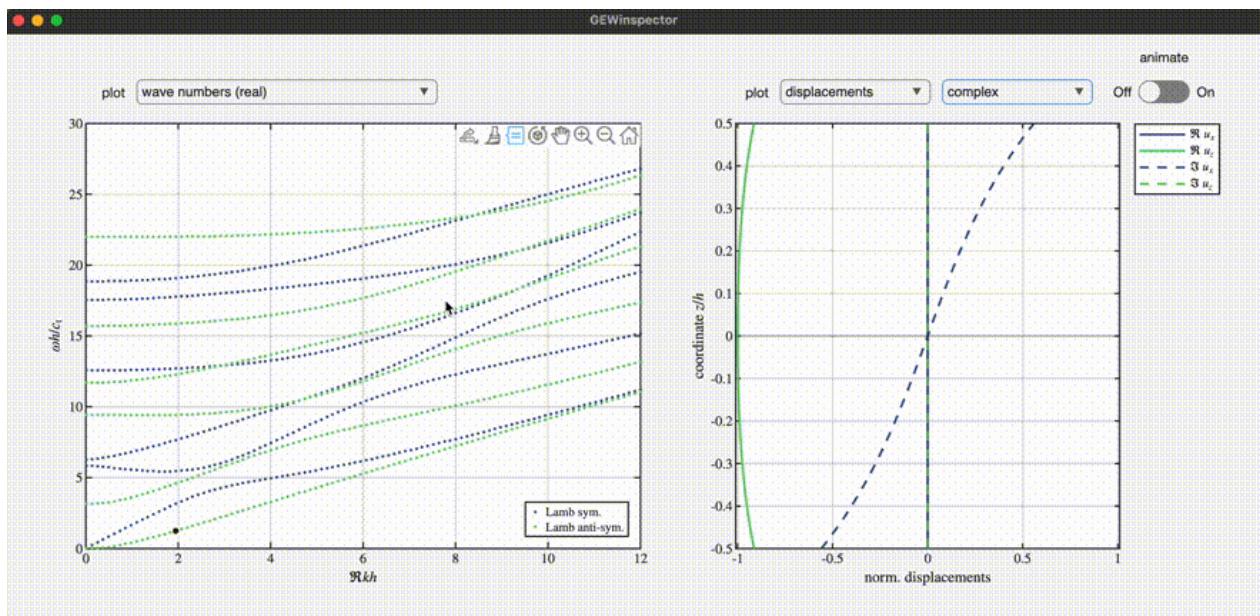


Figure 2: GEWinspector graphical user interface

```

colormap(flipud(colormap)); cb = colorbar; caxis([-50, 0]);
xlabel('wavenumber k in rad/mm'), ylabel('frequency f in MHz')
title('laser-ultrasonic excitability in dB')

```

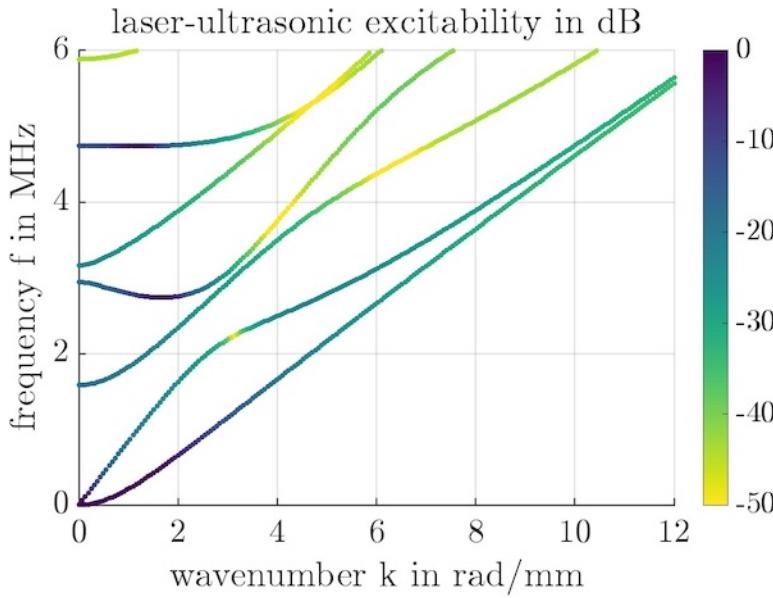


Figure 3: LUS excitability

Installation

Add `GEWtool` and the relevant subfolders to the Matlab path and save it for future sessions. To achieve this:

1. change to the `GEWtool` folder (e.g., by navigating or using `cd`)
2. execute `install`

Enjoy!

Getting started

To get started, explore the `examples` directory.

You can also display help for all functions and classes, e.g., by typing `help Plate`. The most important ones are

- Material representation: `Material`, `MaterialIsotropic`, `MaterialPiezoelectric`
- Waveguides: `Plate`, `Cylinder`, `CylinderCircumferential`
- Solvers: `computeW`, `computeK`, `computeZGV`
- Graphical user interface: `GEWinspector`

Known limitations

- Circumferential waves: post-processing is not fully supported. While group velocity is working, energy velocity cannot be computed at the moment.

Mathematical and physical background

`GEWtool` implements the *Spectral Element Method (SEM)* (higher-order Finite Elements) to solve the *waveguide problem*, i.e., the boundary value problem that describes wave propagation in the structure. Such an approach is commonly qualified as ‘semi-analytical’. Contrary to classical root-finding of the characteristic equation, this method does not miss solutions. Moreover, unlike Finite Elements, the Spectral Elements lead to small but dense matrices.

Solutions are computed with machine precision provided you have set the number of nodes N sufficiently high. The higher you go in frequency-thickness, the higher N should be. As a rule of thumb: half of the obtained modes will be accurate. The figure below shows the convergence with respect to the Rayleigh-Lamb root of the S1 mode at 5.6 rad/mm in an aluminum plate (solution close to 5 MHz mm). This root has been computed with numerical precision using Matlab's `fzero()` function. A Spectral Collocation Method (SCM) is shown in comparison as well as the classical linear Finite Elements Method (FEM) that is also implemented in GEWtool. When using SEM, GEWtool attains 10 digits accuracy with $N = 13$ (matrices of size 26x26) in this case.

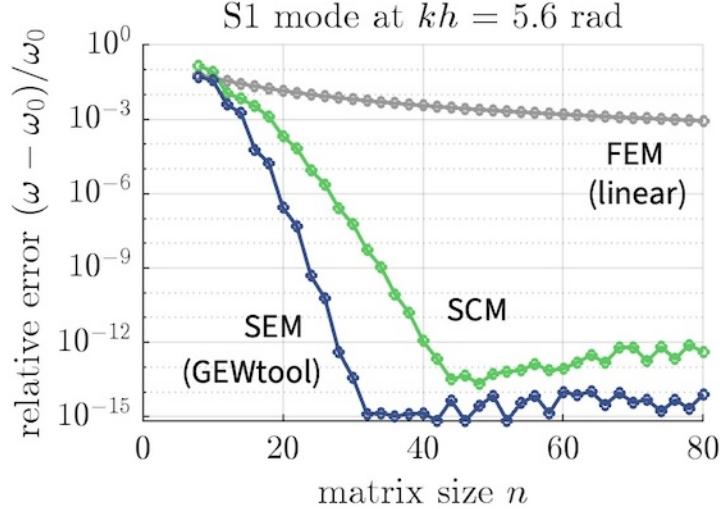


Figure 4: relative error w.r.t. Rayleigh-Lamb root

The computational time does not only depend on the matrix size but also on the algorithm chosen to solve the eigenvalue problem. A comparison between a direct algorithm (`eig`) and a subspace method (`eigs`) is depicted in the figure below. GEWtool automatically chooses the appropriate algorithm to use (heuristically with a fixed threshold). This way, the computation will be fast even when you are modeling waveguides with hundreds of layers, which results in large matrices.

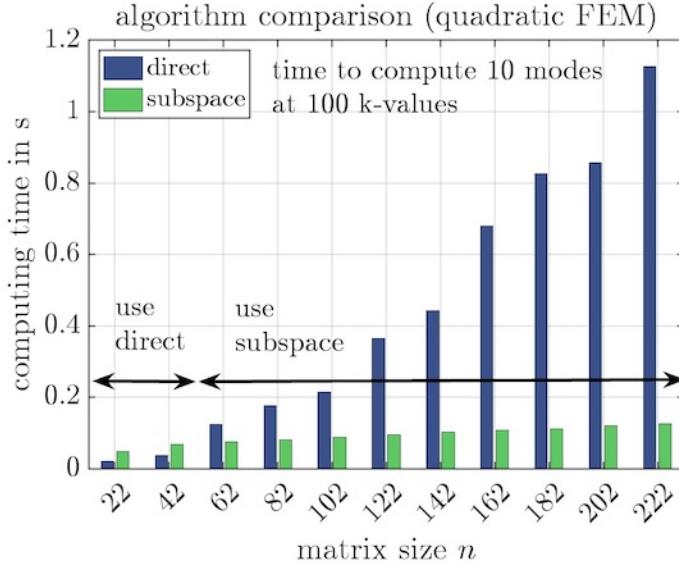


Figure 5: computing time vs algorithm

For more information on the waveguide problem and the numerical methods, refer to the literature cited in the sections below.

Dependencies

GEWtool depends on the functions `barylag`, `collocD`, `lglnodes` and `lgwt` created by Greg von Winckel. They are bundled together with their license files in the `resourcesAndDeps` directory. You may also find them on

Greg von Winckel, MATLAB Central File Exchange, <https://fr.mathworks.com/matlabcentral/profile/authors/869721>.

The function `computeZGVDirect` depends on the `MultiParEig` toolbox by Bor Plestenjak and Andrej Muhič:

Bor Plestenjak (2022). MultiParEig (<https://www.mathworks.com/matlabcentral/fileexchange/47844-multipareig>), MATLAB Central File Exchange.

The function `legendUnq` by Adam Danz is used in the example scripts:

Adam Danz (2024). legendUnq (<https://www.mathworks.com/matlabcentral/fileexchange/67646-legendunq>), MATLAB Central File Exchange.

Citing GEWtool

If this code is useful to you, please cite it as (always indicating the DOI):

D. A. Kiefer (2023). GEWtool. <https://doi.org/10.5281/zenodo.10114243> (<https://github.com/dakiefer/GEWtool>)

Please also cite the related publication(s) as relevant, e.g.,

- [1] D. A. Kiefer, S. Mezil, and C. Prada, “Beating resonance patterns and extreme power flux skewing in anisotropic elastic plates,” *Science Advances*, vol. 9, no. 51, p. eadk6846, Dec. 2023, doi: 10.1126/sciadv.adk6846. 
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- [3] D. A. Kiefer, G. Watzl, K. Burgholzer, M. Rzy, and C. Grünsteidl, “Electroelastic guided wave dispersion in piezoelectric plates: Spectral methods and laser-ultrasound experiments,” *Journal of Applied Physics*, vol. 137, no. 11, p. 114502, Mar. 2025, doi: 10.1063/5.0250494. 
- [4] H. Gravenkamp, B. Plestenjak, and D. A. Kiefer, “Notes on osculations and mode tracking in semi-analytical waveguide modeling,” *Ultrasonics*, vol. 135, p. 107112, Jul. 2023, doi: 10.1016/j.ultras.2023.107112. 
- [5] D. A. Kiefer, B. Plestenjak, H. Gravenkamp, and C. Prada, “Computing zero-group-velocity points in anisotropic elastic waveguides: Globally and locally convergent methods,” *The Journal of the Acoustical Society of America*, vol. 153, no. 2, pp. 1386–1398, Feb. 2023, doi: 10.1121/10.0017252. 

Further literature

- [1] T. J. R. Hughes, *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Mineola, New York: Dover Publications, 2012.
- [2] G. Waas, *Linear two dimensional analysis of soil dynamics problems in semi-infinite layered media*, Ph. D. thesis, University of California, Berkley, 1972.
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- [5] A. T. I. Adamou and R. V. Craster, “Spectral methods for modelling guided waves in elastic media,” *The Journal of the Acoustical Society of America*, vol. 116, no. 3, Sep. 2004, doi: 10.1121/1.1777871.
- [6] S. Finnveden, “Evaluation of modal density and group velocity by a finite element method,” *Journal of Sound and Vibration*, vol. 273, no. 1, May 2004, doi: 10.1016/j.jsv.2003.04.004.
- [7] A. Marzani, E. Viola, I. Bartoli, F. Lanza di Scalea, and P. Rizzo, “A semi-analytical finite

- element formulation for modeling stress wave propagation in axisymmetric damped waveguides," Journal of Sound and Vibration, vol. 318, no. 3, Dec. 2008, doi: 10.1016/j.jsv.2008.04.028.
- [8] H. Gravenkamp, C. Song, and J. Prager, "A numerical approach for the computation of dispersion relations for plate structures using the Scaled Boundary Finite Element Method," Journal of Sound and Vibration, vol. 331, no. 11, May 2012, doi: 10.1016/j.jsv.2012.01.029.
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- [10] D. A. Kiefer, *Elastodynamic quasi-guided waves for transit-time ultrasonic flow metering*, ser. FAU Forschungen, Reihe B, Medizin, Naturwissenschaft, Technik, vol. 42. Erlangen: FAU University Press, 2022, doi: 10.25593/978-3-96147-550-6. 

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Numerical methods to compute ZGV points

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Homogenization, quaternions, rotation angles

Clemens Grünsteidl, Research Center for Non Destructive Testing GmbH (RECENDT), Austria
Piezoelectricity

Acknowledgments - Many inspiring discussions influenced GEWtool. D. A. Kiefer is thankful to:
Hauke Gravenkamp, Claire Prada, Matt Clark, Pierre Chantelot and Michael Ponschab

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I have several years of experience in waveguide modeling and numerical implementations thereof. In January 2022 I decided to create a new modular and versatile code from scratch. The result is GEWtool. My hope is that it be a valuable research tool and at the same time a helpful educational resource for those interested in numerical methods and elastic waves.

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