

UNIVERSITY NAME

DOCTORAL THESIS

Hierarchical deterministic wallet

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in the

Research Group Name
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Declaration of Authorship

I, Daniele FORNARO, declare that this thesis titled, “Hierarchical deterministic wallet” and the work presented in it are my own. I confirm that:

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- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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“Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism.”

Dave Barry

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Abstract

Faculty Name
Department or School Name

Mathematical Engineering

Hierarchical deterministic wallet

by Daniele FORNARO

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor...

Contents

Declaration of Authorship	iii
Abstract	vii
Acknowledgements	ix
1 Elliptic Curve Geometry	1
1.1 Introduction	1
1.2 Elliptic Curve over \mathbb{F}_p	1
1.2.1 Operations	1
Symmetry	1
Point addition	2
Scalar multiplication	3
1.2.2 Group order	3
Cyclic subgroups	3
1.3 Bitcoin private-public key cryptography	4
1.3.1 Bitcoin Elliptic Curve	4
1.3.2 Bitcoin keys representation and addresses	4
Uncompressed public key	4
Compressed public key	5
WIF Private Key	5
Address	5
2 Wallet	7
2.1 What is a Bitcoin Wallet	7
2.2 Random Wallets	7
2.2.1 Pros and Cons	7
2.3 Deterministic Wallets	7
2.3.1 Pros and Cons	7
2.4 Hierarchical deterministic Wallets	7
2.4.1 Key Concept	7
Seed	7
Extended Key	7
Mnemonic and Passphrase	7
2.4.2 Pros and Cons	7
3 Seed to Master Private Key	9
3.1 Functional explanation	9
3.2 Code implementation	9

4	Child Key Derivation	11
4.1	Functional explanation	11
4.2	Normal derivation	11
4.2.1	Derive public child from public parent	11
4.2.2	Possible Risk	11
4.3	Hardened derivation	11
5	Mnemonic to Seed	13
5.1	Functional explanation	13
5.2	BIP 39 derivation	13
5.2.1	Mnemonic generation	13
5.2.2	Seed derivation	13
5.3	Electrum derivation	13
5.3.1	Mnemonic generation	13
5.3.2	Seed derivation	13
5.4	BIP39 vs Electrum derivation	13
6	How to use a HD Wallet	15
6.1	BIP 44	15
A	Frequently Asked Questions	17
A.1	How do I change the colors of links?	17
	Bibliography	19

List of Figures

1.1	Points on the Elliptic Curve $y^2 = x^3 - 7x + 10 \pmod{p}$, with $p =$ 19, 97, 127, 487	2
1.2	Elliptic Curve $y^2 = x^3 - 7x + 10 \pmod{97}$	2

List of Tables

List of Abbreviations

LAH List Abbreviations **Here**
WSF What (it) Stands **For**

Physical Constants

Speed of Light $c_0 = 2.997\,924\,58 \times 10^8 \text{ m s}^{-1}$ (exact)

List of Symbols

a	distance	m
P	power	W (J s ⁻¹)
ω	angular frequency	rad

For/Dedicated to/To my...

Chapter 1

Elliptic Curve Geometry

1.1 Introduction

Bitcoin security is based on public and private key cryptography. The main concept is that it is simple to compute the public key, knowing the private, but it is infeasible to calculate the private key, knowing the public.

In order to obtain this result a particular Elliptic Curve is used.

1.2 Elliptic Curve over \mathbb{F}_p

A point Q , which coordinates are x and y , belong to an Elliptic Curve if and only if Q satisfies the following equation:

$$y^2 = x^3 + ax + b \quad \text{over } \mathbb{F}_p \quad (1.1)$$

Where \mathbb{F}_p is the finite field defined over the set of integers modulo p and a and b are the coefficients of the curve.

We can rewrite the equation 1.1 in the following way:

$$y^2 = x^3 + ax + b \quad \text{mod } p \quad (1.2)$$

Figure 1.1 shows some examples of Elliptic Curve over \mathbb{F}_p with $a = -7$ and $b = 10$

1.2.1 Operations

A point on the Elliptic Curve has some particular properties:

- Symmetry
- Point addition
- Scalar multiplication

Symmetry

For every point in the x axis exists two points in the y axis. Suppose that a point $P(x, y)$ belongs to the Elliptic Curve, then it must satisfy the equation 1.1. So it is easy to prove that the point $Q(x, p - y)$ belongs to the curve too.

Furthermore we have $P = -Q$, from the moment that $P + Q = 0$ (see addition below).



FIGURE 1.1: Points on the Elliptic Curve $y^2 = x^3 - 7x + 10 \pmod{p}$, with $p = 19, 97, 127, 487$

Point addition

We need to change our definition of addition in order to make it works in \mathbb{F}_p . In this framework we claim that if three points are aligned over the finite field \mathbb{F}_p , then they have zero sum.

So $P + Q = R$ if and only if P, Q and $-R$ are aligned, in the sense shown in figure 1.2



FIGURE 1.2: Elliptic Curve $y^2 = x^3 - 7x + 10 \pmod{97}$

The equations for calculating point additions are the follow:
Suppose that A and B belong to the Elliptic Curve.

$$A = (x_1, y_1) \quad B = (x_2, y_2)$$

Let's defined $A + B := (x_3, y_3)$

So we have:

$$s = \begin{cases} \frac{y_2 - y_1}{x_2 - x_1}, & \text{if } x_1 \neq x_2 \\ \frac{3x_1^2 + a}{2y_1}, & \text{if } x_1 = x_2 \end{cases}$$

$$\begin{aligned} x_3 &= s^2 - x_1 - x_2 \pmod{p} \\ y_3 &= s(x_1 - x_3) - y_1 \pmod{p} \end{aligned}$$

Scalar multiplication

Once defined the addition, any multiplication can be defined as:

$$nP = \underbrace{P + P + \dots + P}_{n \text{ times}}$$

When n is a very large number can be difficult or even infeasible to compute nP in this way, but we can use the *double and add algorithm* in order to perform multiplication in $\mathcal{O}(\log n)$ steps.

1.2.2 Group order

An elliptic curve defined over a finite field is a group and so it has a finite number of points. This number is called order of the group.

If the prime order is a very large number, it is impossible to count all the point in that field, but there is an algorithm that allows to calculate the order of a group in a fast and efficient way: *Schoof's algorithm*.

Cyclic subgroups

Let's consider a generic point P , we have:

$$nP + mP = \underbrace{P + \dots + P}_{n \text{ times}} + \underbrace{P + \dots + P}_{m \text{ times}} = \underbrace{P + \dots + P}_{n+m \text{ times}} = (n+m)P$$

So multiple of P are closed under addition and this is enough to prove that the set of the multiples of P is a cyclic subgroup of the group formed by the elliptic curve.

The point P is called **generator** of the cyclic subgroup.

Remark The order of P is linked to the order of the elliptic curve by Lagrange's theorem, which states that the order of a subgroup is a divisor of the order of the parent group.

Remark If the order of the group is a prime number, all the point P generate a subgroup with the same order of the group.

1.3 Bitcoin private-public key cryptography

1.3.1 Bitcoin Elliptic Curve

Bitcoin uses a specific Elliptic Curve defined over the finite field of the natural numbers, where $a = 0$ and $b = 7$.

The equation 1.1 becomes:

$$y^2 = x^3 + 7 \pmod{p} \quad (1.3)$$

The \pmod{p} (modulo prime number) indicates that this curve is over a finite field of prime order $p = 2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$.

The *order* of this Elliptic Curve is a very large prime number, close to 2^{256} , but smaller than p .

Let's consider a particular point G , called generator, with:

$$\begin{aligned} x = & 79BE667EF9DCBBAC55A06295CE870B07029BFCDB2DCE28D959F2815B16F81798 \\ y = & 483ADA7726A3C4655DA4FBFC0E1108A8FD17B448A68554199C47D08FFB10D4B8 \end{aligned}$$

From the moment that the order of the group is a prime number, the order of any subgroup is equal to the order of the entire group. In particular the order of the subgroup generated by G is equal to *order*.

Definition A private key is a number chosen in the range between 1 and *order*.

Definition A public key W is a point in the Bitcoin EC, derived from a private key k in the following way:

$$W = k \cdot G \quad (1.4)$$

Where the multiplication between k and G is defined in the previous chapter.

This is a *one way* function, in the sense that computing the scalar multiplication, knowing the private key is simple, but make the reverse is infeasible.

Remark It is infeasible to calculate a private key knowing the public key.

1.3.2 Bitcoin keys representation and addresses

In order to make it easy to store and recognise keys, some encodings were designed.

A public key, a point in the EC, can be represented in two ways: *uncompressed* or *compressed*.

Uncompressed public key

An uncompressed public key is represented in hexadecimal digits, and it is obtained simply concatenating the x coordinate with the y coordinate and adding 04 at the beginning, for a total of 130 hexadecimal digits.

Example of an uncompressed public key:

0450863AD64A87AE8A2FE83C1AF1A8403CB53F53E486D8511DAD8A04887E5B235
22CD470243453A299FA9E77237716103ABC11A1DF38855ED6F2EE187E9C582BA6

Compressed public key

A compressed public key is obtained simply taking the x coordinate and adding 02 at the beginning if the y coordinate is even, 03 otherwise.

This is due to the *symmetry properties* of a point of the EC.

Example of a public key compressed:

0250863AD64A87AE8A2FE83C1AF1A8403CB53F53E486D8511DAD8A04887E5B2352

WIF Private Key

WIF stands for wallet import format and is the standard way used to write down a private key.

- Add a version number (80 for Bitcoin) in front of the private key, in order to recognize quickly for what cryptocurrency that private key was used.
- Add 01 at the end of the private key if you want a WIF *compressed*, none if you want a WIF *uncompressed*. The difference between these two types is that from a *compressed* private key a *compressed* public key is expected and from a *uncompressed* private key a *uncompressed* public key is expected.
- Add a checksum at the end, obtained applying the SHA256 function twice to the string previously obtained, take the first 4 bytes (8 hexadecimal digits) and put them at the end of the string.
- Compute the Base58Encode, obtaining a 52 digit string.

Example of private key WIF:

KwdMAjGmerYanjeui5SHS7JkmpZvVipYvB2LJGU1ZxJwYvP98617

Address

Among the Bitcoin transactions, one of the most used is a *Pay-to-PubkeyHash*, meaning that in the transaction you will not write directly the public key, but the hash of that public key.

The hash function used in this framework is the HASH160 function, applied to the *compressed* public key. This is an irreversible procedure, so you cannot obtain the public key from the public key hash.

In order to obtain a valid Bitcoin address, it is needed to encode the *PubkeyHash* in base58, adding first the version in front, the checksum at the end and then encode everything with Base58Encode, obtaining a 34 digit string.

Example of an Address:

1BvBMSEYstWetqTFn5Au4m4GFg7xJaNVN2

Chapter 2

Wallet

2.1 What is a Bitcoin Wallet

A Bitcoin wallet is a structure used to store keys.

There exist two different type of wallets:

- Nondeterministic Wallets
- Deterministic Wallets

Remark *Bitcoin wallets contains keys, not coins.*

2.2 Random Wallets

This is the simplest

2.2.1 Pros and Cons

2.3 Deterministic Wallets

2.3.1 Pros and Cons

2.4 Hierarchical deterministic Wallets

2.4.1 Key Concept

Seed

Extended Key

Mnemonic and Passphrase

2.4.2 Pros and Cons

Chapter 3

Seed to Master Private Key

3.1 Functional explanation

3.2 Code implementation

Chapter 4

Child Key Derivation

4.1 Functional explanation

4.2 Normal derivation

4.2.1 Derive public child from public parent

4.2.2 Possible Risk

4.3 Hardened derivation

Chapter 5

Mnemonic to Seed

5.1 Functional explanation

5.2 BIP 39 derivation

5.2.1 Mnemonic generation

5.2.2 Seed derivation

5.3 Electrum derivation

5.3.1 Mnemonic generation

5.3.2 Seed derivation

5.4 BIP39 vs Electrum derivation

Chapter 6

How to use a HD Wallet

6.1 BIP 44

Appendix A

Frequently Asked Questions

A.1 How do I change the colors of links?

The color of links can be changed to your liking using:

```
\hypersetup{urlcolor=red}, or  
\hypersetup{citecolor=green}, or  
\hypersetup{allcolor=blue}.
```

If you want to completely hide the links, you can use:

```
\hypersetup{allcolors=.}, or even better:  
\hypersetup{hidelinks}.
```

If you want to have obvious links in the PDF but not the printed text, use:

```
\hypersetup{colorlinks=false}.
```


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