Investigations into New Lattice Orderings

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Context-theoretic semantics (Clarke, 2007) defines a lattice ordering on the vector space formed from contexts of words. The lattice meet and join are defined by taking component-wise minimums and maximums with respect to the basis defined by contexts: there is a basis vector (dimension) for each context a word can occur in.

However, this ordering is only one that is possible on the vector space. Here we consider two others and evaluate their plausibility for defining entailment between vector representations of meaning.

1 Motivation

In (Clarke, 2012), a context theory is defined as follows:

Definition 1 (Context Theory) *A context theory is a tuple $\langle A, \mathcal{A}, \xi, V, \psi \rangle$, where A is a set (the alphabet), $\mathcal{A}$ is a unital algebra over the real numbers, ξ is a function from A to $\mathcal{A}$, V is an abstract Lebesgue space and ψ is an injective linear map from $\mathcal{A}$ to V.*

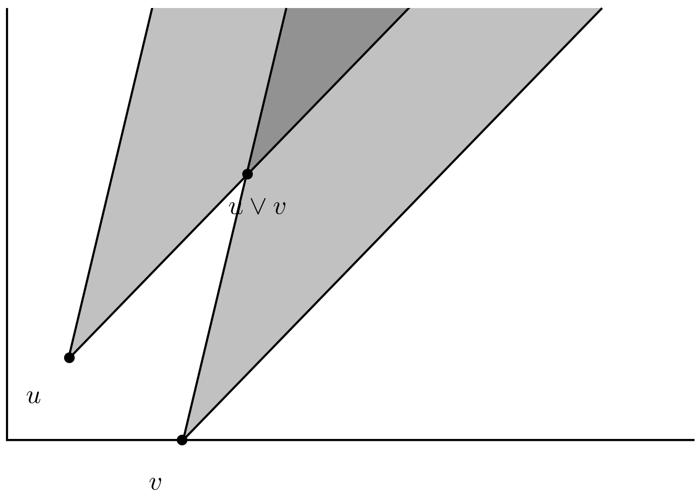
The definition is quite complex because the space V in which the ordering is defined is a bigger space than the space $\mathcal{A}$ used to define composition. If we were able to define an ordering on $\mathcal{A}$ instead, then this would significantly simplify the definition, to $\langle A, \mathcal{A}, \xi \rangle$.

2 Background

A **partially ordered vector space** is a vector space V together with a partial ordering $\leq$ that satisfies:

- If $u \leq v$ then $u + w \leq v + w$
- If $u \leq v$ then $\alpha u \leq \alpha v$

for all $u, v, w \in V$ and $\alpha \geq 0$. If the partial ordering is a lattice, then the vector space is called a vector lattice or **Riesz space**.



A **cone** is a subset C of a vector space satisfying

- $C + C \subseteq C$
- $\alpha C \subseteq C$ for all $\alpha \geq 0$
- $C \cap (-C) = \{0\}$

If V is a partially ordered vector space, then the set $V^+ = \{v \in V : v \geq 0\}$ is a cone, called the **positive cone**. Conversely, given any cone C for a vector space V, we can define a partial ordering on V by $u \leq v$ iff $v - u \in C$.

Given a countable set $U = \{u_1, u_2, \ldots\}$ of vectors, the cone generated by U is the set $C_U = \{v : v = \sum_i \alpha_i u_i\}$ for some $\alpha_i \geq 0$.

3 Context Vector Cones

We can define a positive cone on $\mathcal{A}$ to be the cone generated by all context vectors, $\hat{A} = \{\hat{x} : x \in A^*\}$. There are two questions we can ask:

- Is the ordering interesting, or is likely to turn out to be trivial (for example, all strings have meet 0)?
- Does the ordering define a lattice? We need the meet operation in order to define a degree of entailment between strings.

In fact it seems that we can't have our cake and eat it, as if the first question is true, it seems likely the second is false, and vice versa. If the set $\hat{A}$ is linearly independent, then it seems that the first is true, and the second false, while the converse holds in general otherwise. (At the moment, I only have an intuition for this — I've yet to prove it).

4 Plane Orderings

References

Daoud Clarke. *Context-theoretic Semantics for Natural Language: an Algebraic Framework*. PhD thesis, Department of Informatics, University of Sussex, 2007.

Daoud Clarke. A context-theoretic framework for compositionality in distributional semantics. *Computational Linguistics*, 38(1):41–71, 2012.