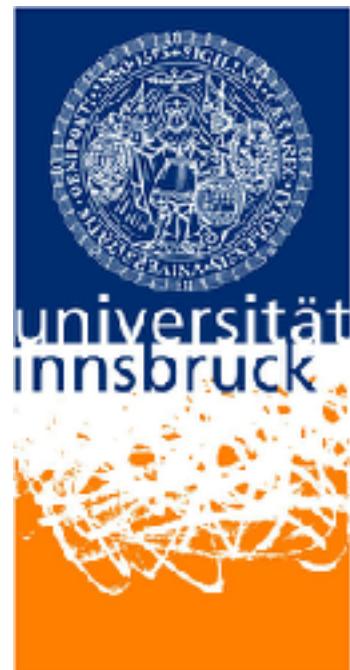


LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

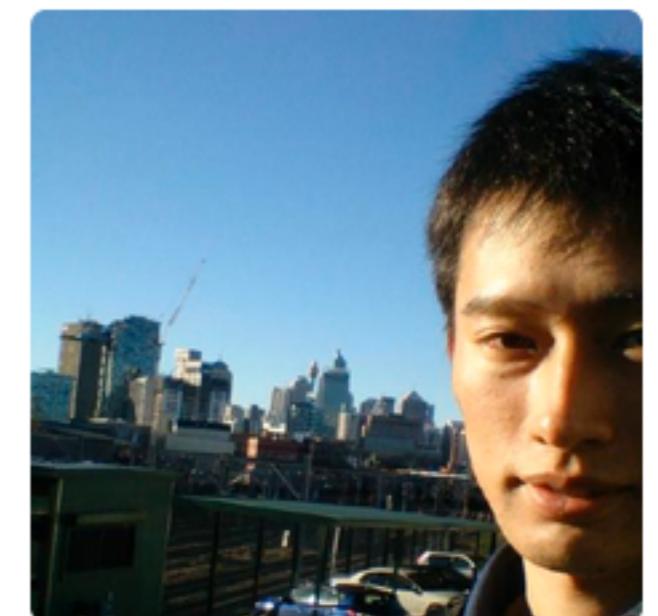
This work was supported by the project AI&Reasoning (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000466).



Yutaka Nagashima
University of Innsbruck
Czech Technical University



**CZECH INSTITUTE
OF INFORMATICS
ROBOTICS AND
CYBERNETICS
CTU IN PRAGUE**



Yutaka Ng

<https://twitter.com/YutakangE>

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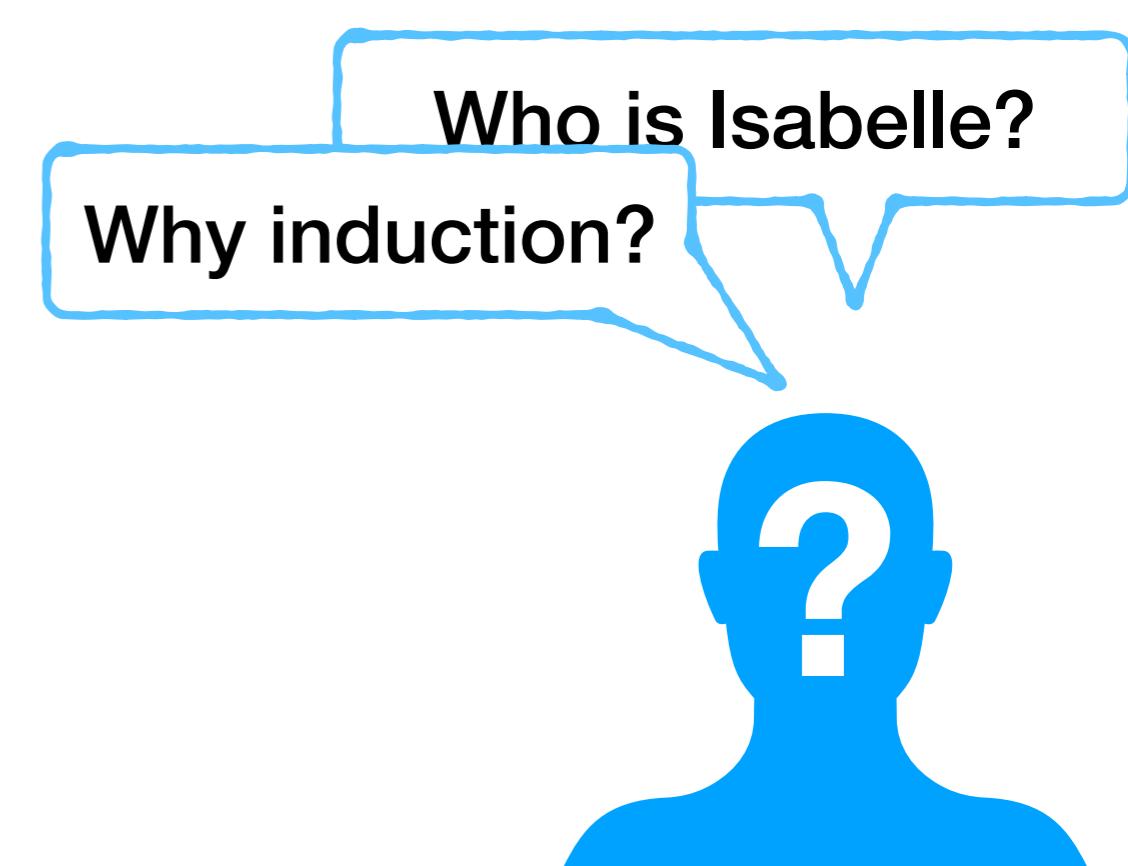
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Who is Isabelle?



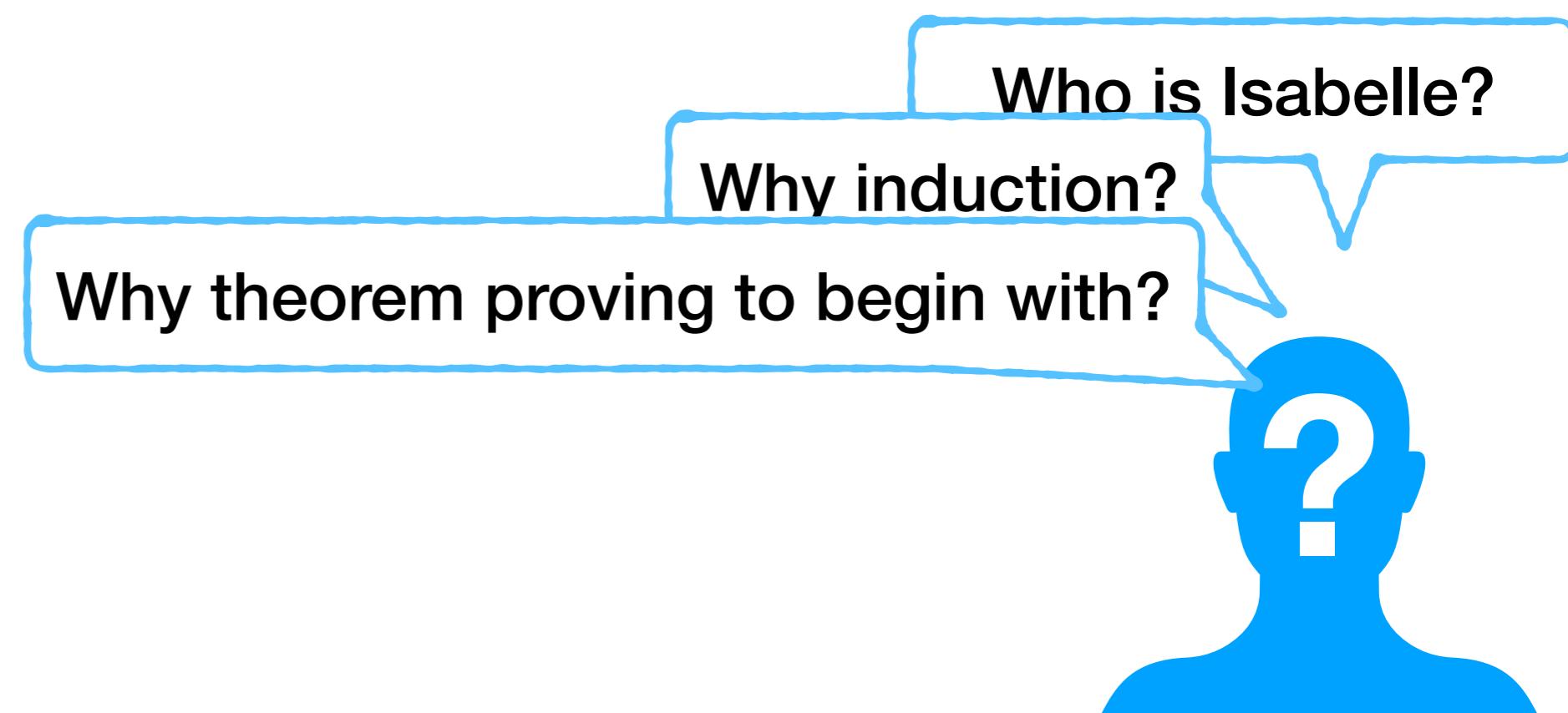
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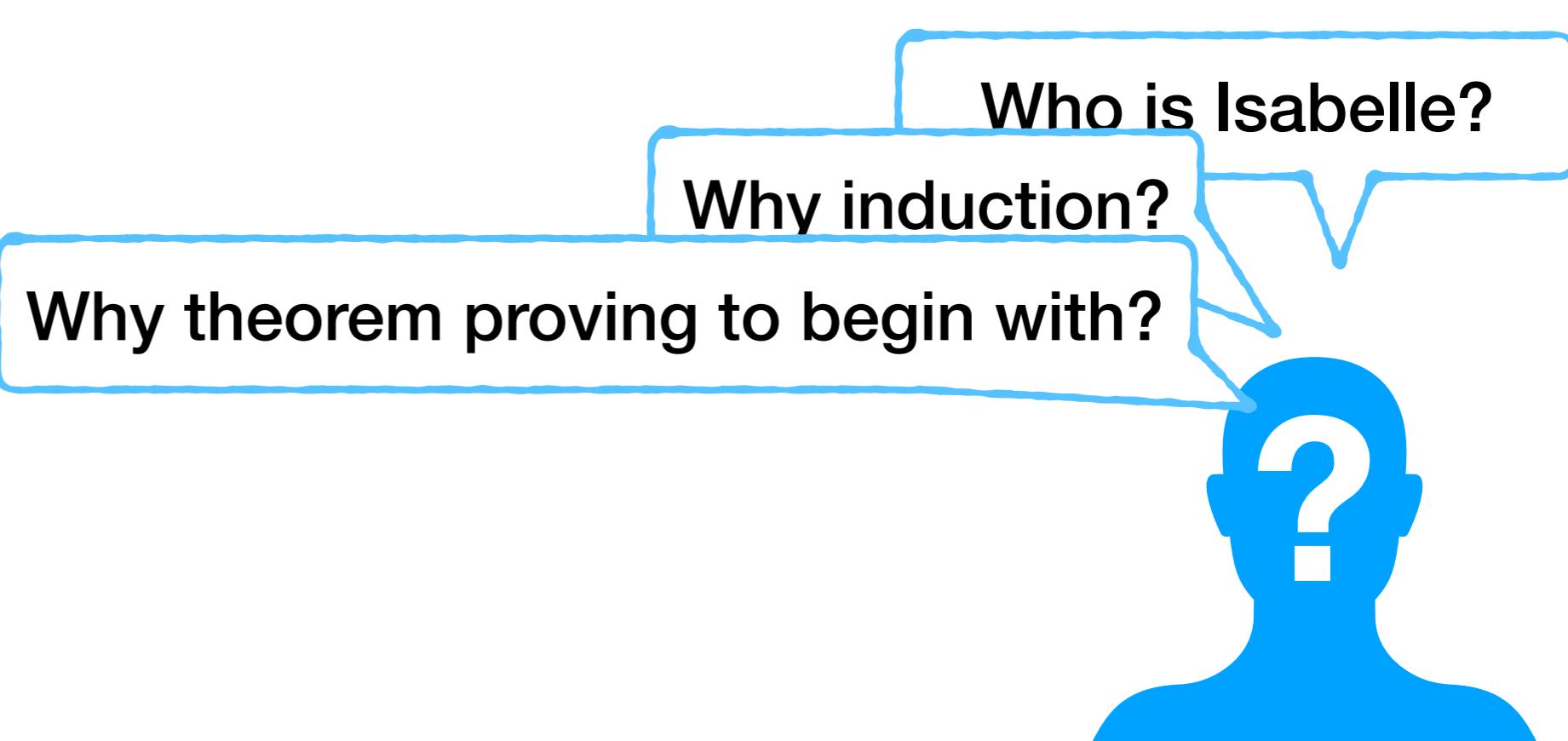
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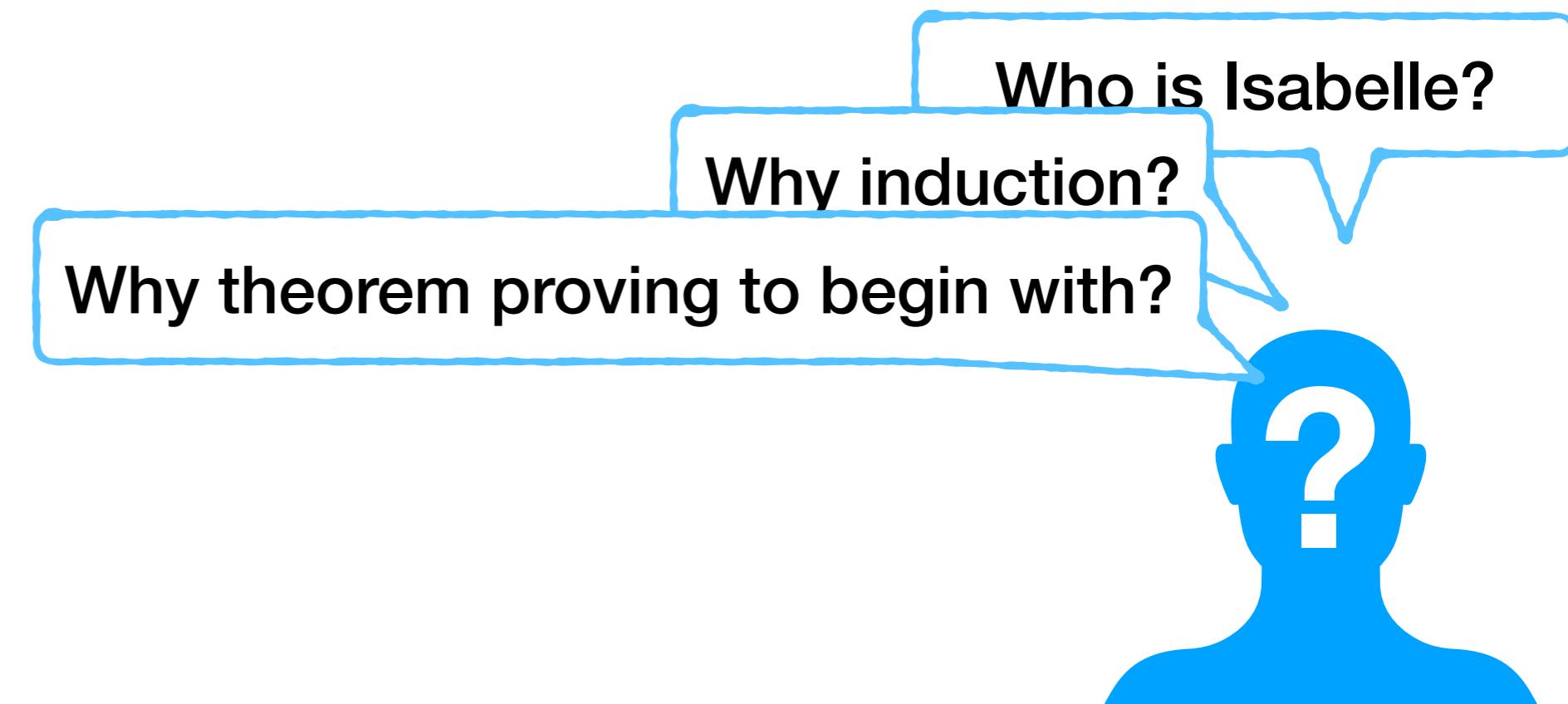
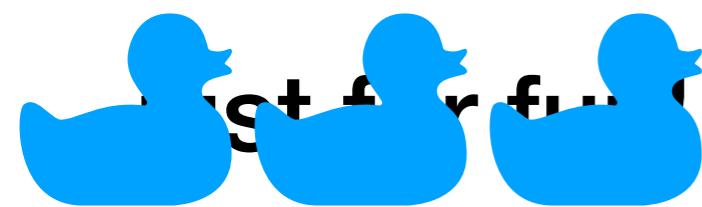
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just for fun!



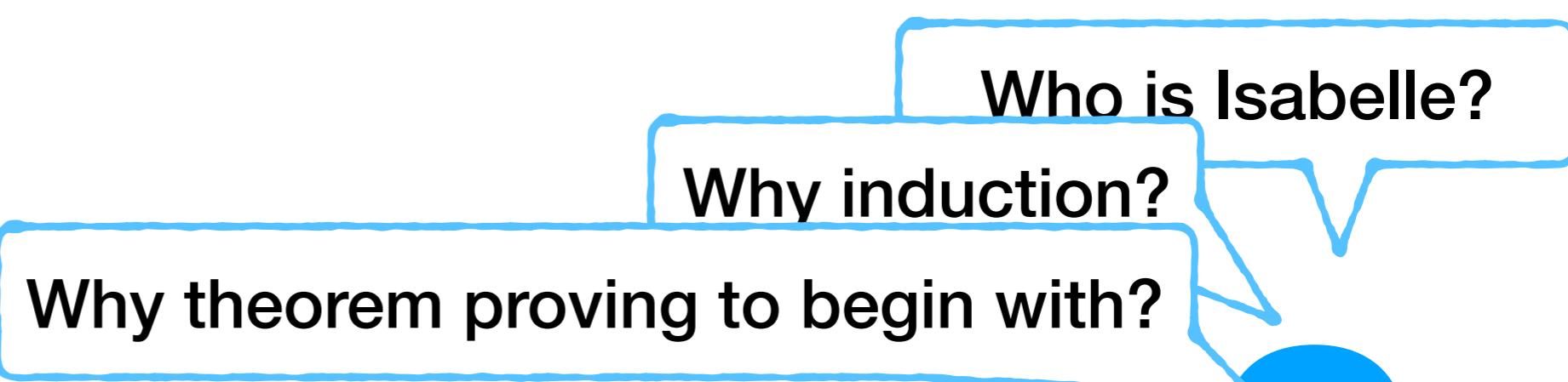
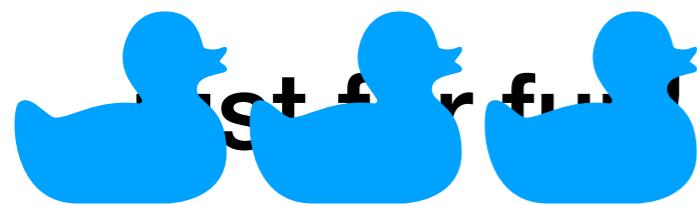
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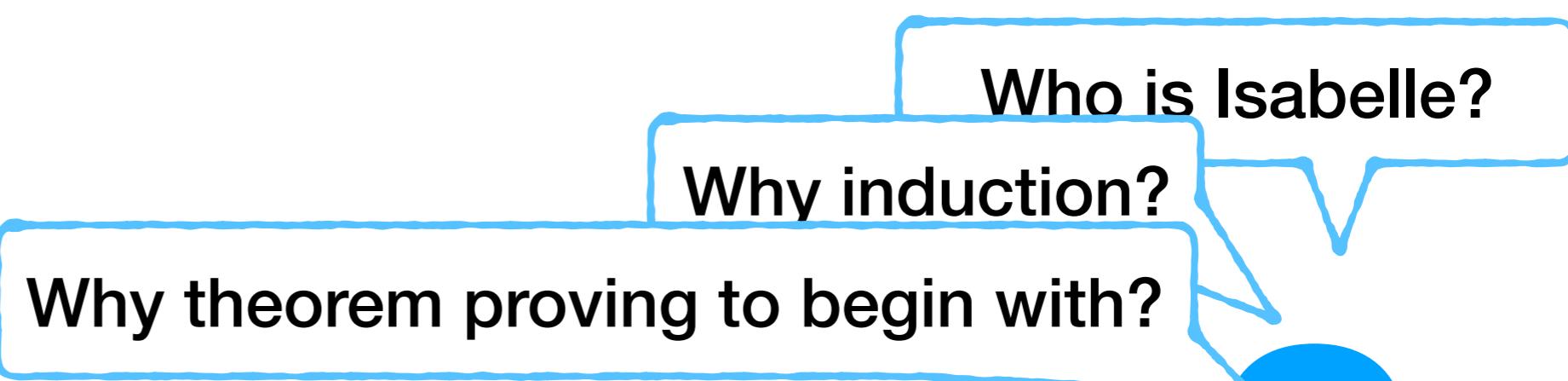
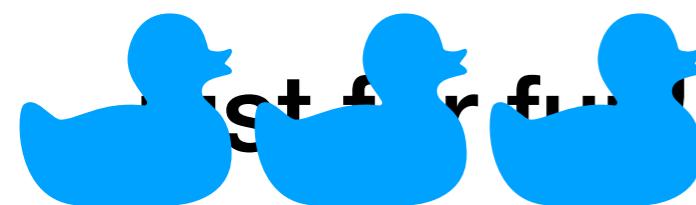


for complete formal verification!



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for complete formal verification!

developer

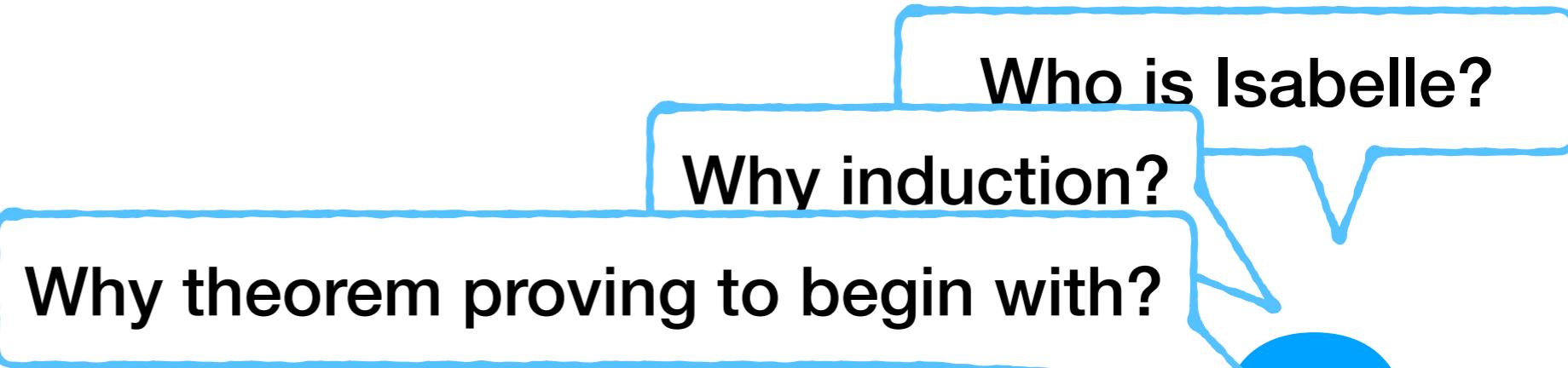
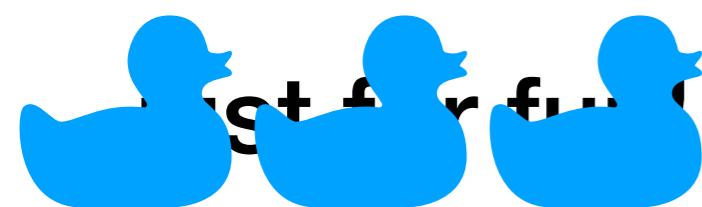
proof assistant / ITP

implementation



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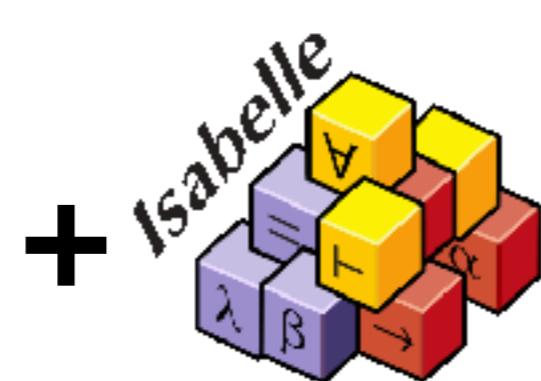


for complete formal verification!

developer



proof assistant / ITP



Gewin Klein et. al

Isabelle/HOL

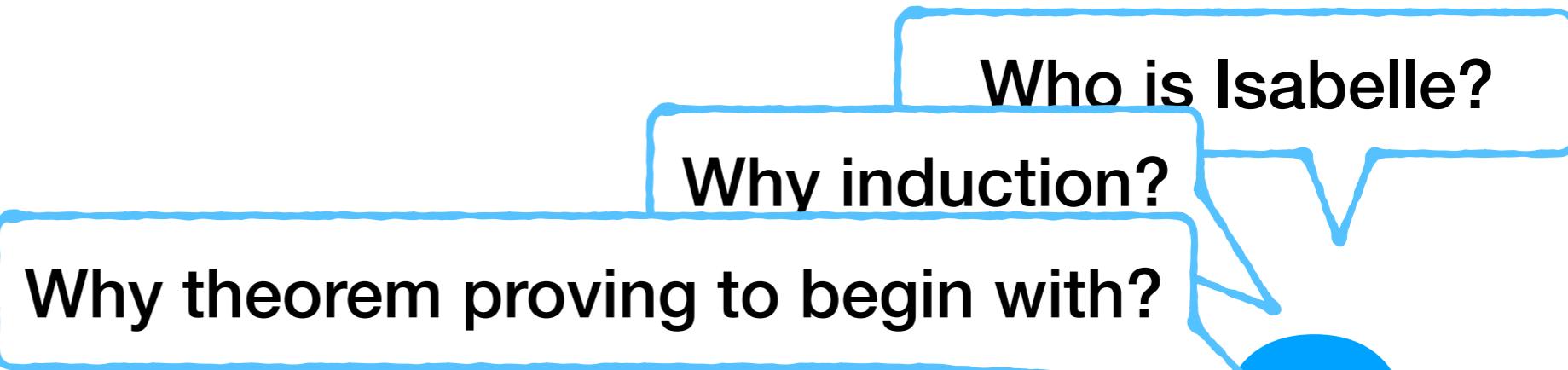
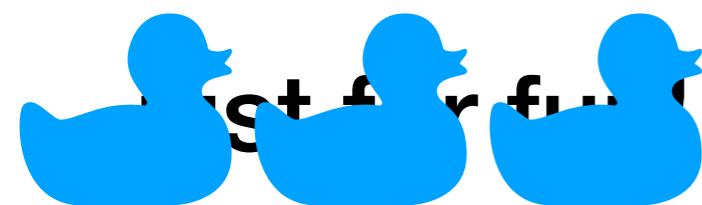
implementation



verified micro kernel,
seL4

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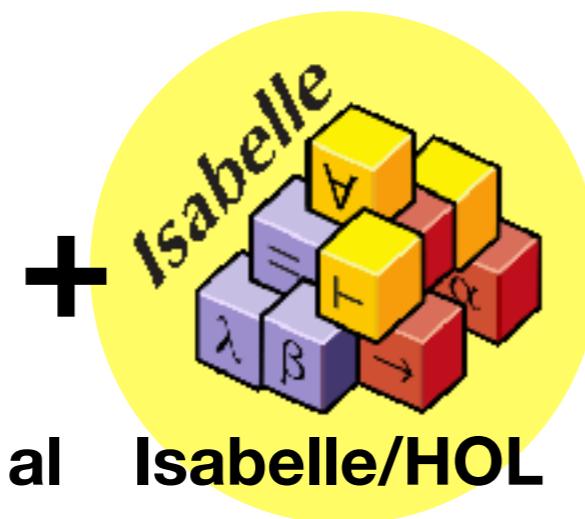


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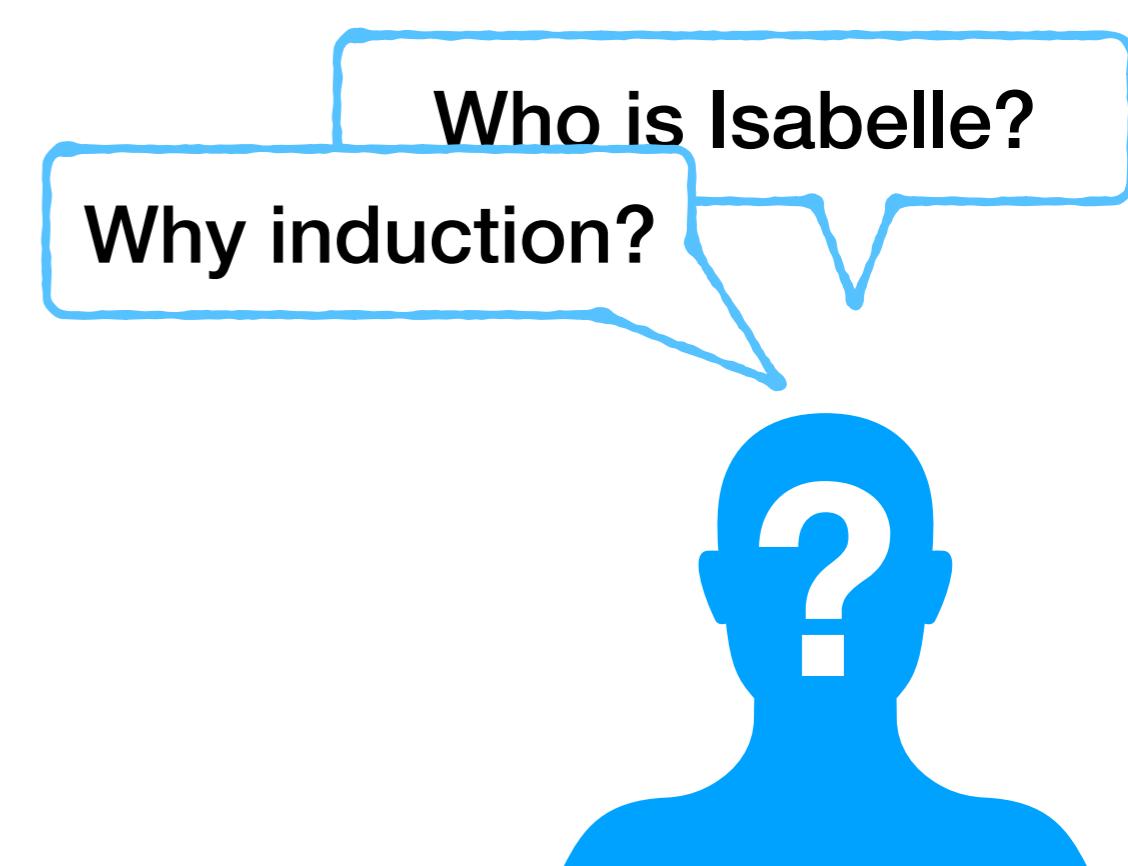
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ITP (Inductive Theorem Proving) problems are at the heart of many verification and reasoning tasks in computer science.



Who is Isabelle?
Why induction?



Prof. Bernhard Gramlich
<https://www.logic.at/staff/gramlich/>

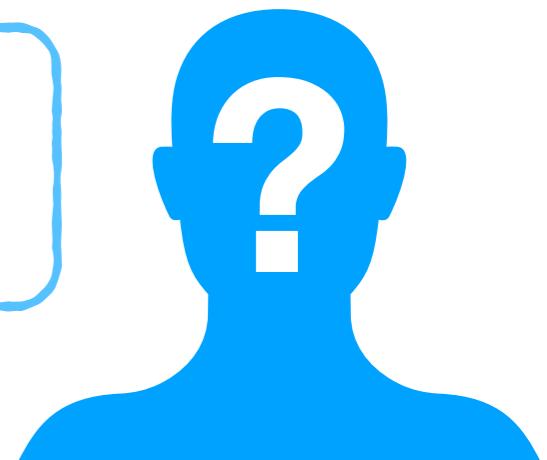
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we are convinced that substantial progress in ITP will take time.

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spectacular breakthroughs are unrealistic, in view of the enormous problems and the inherent difficulty of inductive theorem proving.

Prof. Bernhard Gramlich

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Why theorem proving to be

we are convinced that substantial progress in ITP will take time.

spectacular breakthroughs

unrealistic, in view of the problems and the inherent inductive theorem pro



Who is Isabelle?

Why induction?

Challenge accepted!



Prof. Bernhard Gramlich

<https://www.logic.at/staff/gramlich/>

Strategic Issues, Problems and Challenges in Induc

Yutaka Ng
yutakang

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ITP (Inductive Theorem Proving) problems are at the heart of many verification and reasoning tasks in

The time has come!

Who is Isabelle?

Why induction?

Challenge accepted!

substantial
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Yutaka Ng

yutakang

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Challenge accepted!

The time has
come!

...or is coming
soon.

unrealistic, in view of the
problems and the inherent
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Prof. Bernhard Gramlich

<https://www.logic.at/staff/gramlich/>

Yutaka Ng
[yutakang](https://twitter.com/yutakang)

<https://twitter.com/YutakangE>

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What I did before APLAS2020...

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What I did before APLAS2020

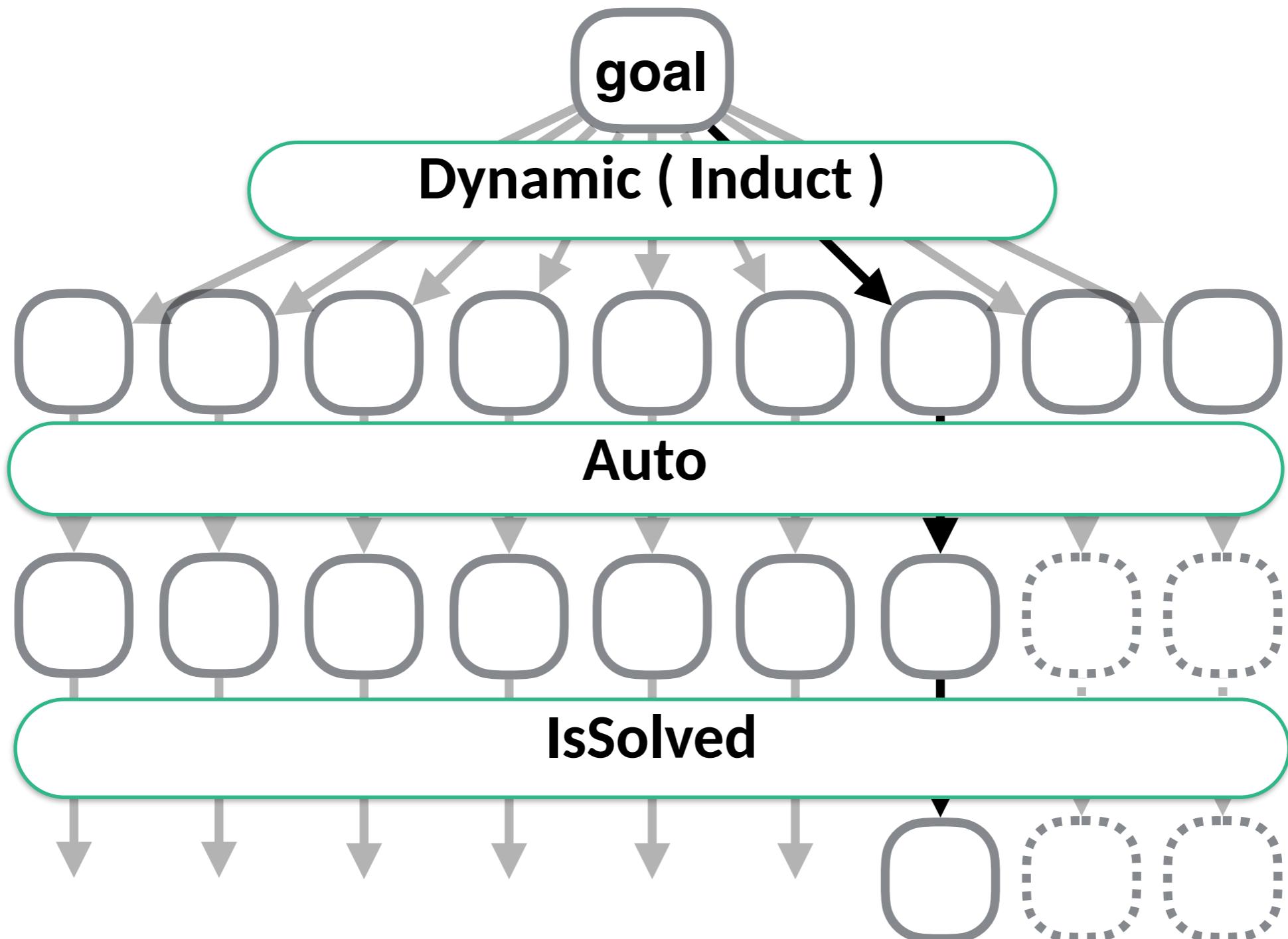
DEMO!

<https://twitter.com/YutakangE>

<https://tinyurl.com/up7zhyk>

What I did before APLAS2020

```
lemma "map f (sep x x) = map f x" apply (rule refl)
find_proof DInd(*= Thens [DInd, Induct, Auto, IsSolved]*)
```

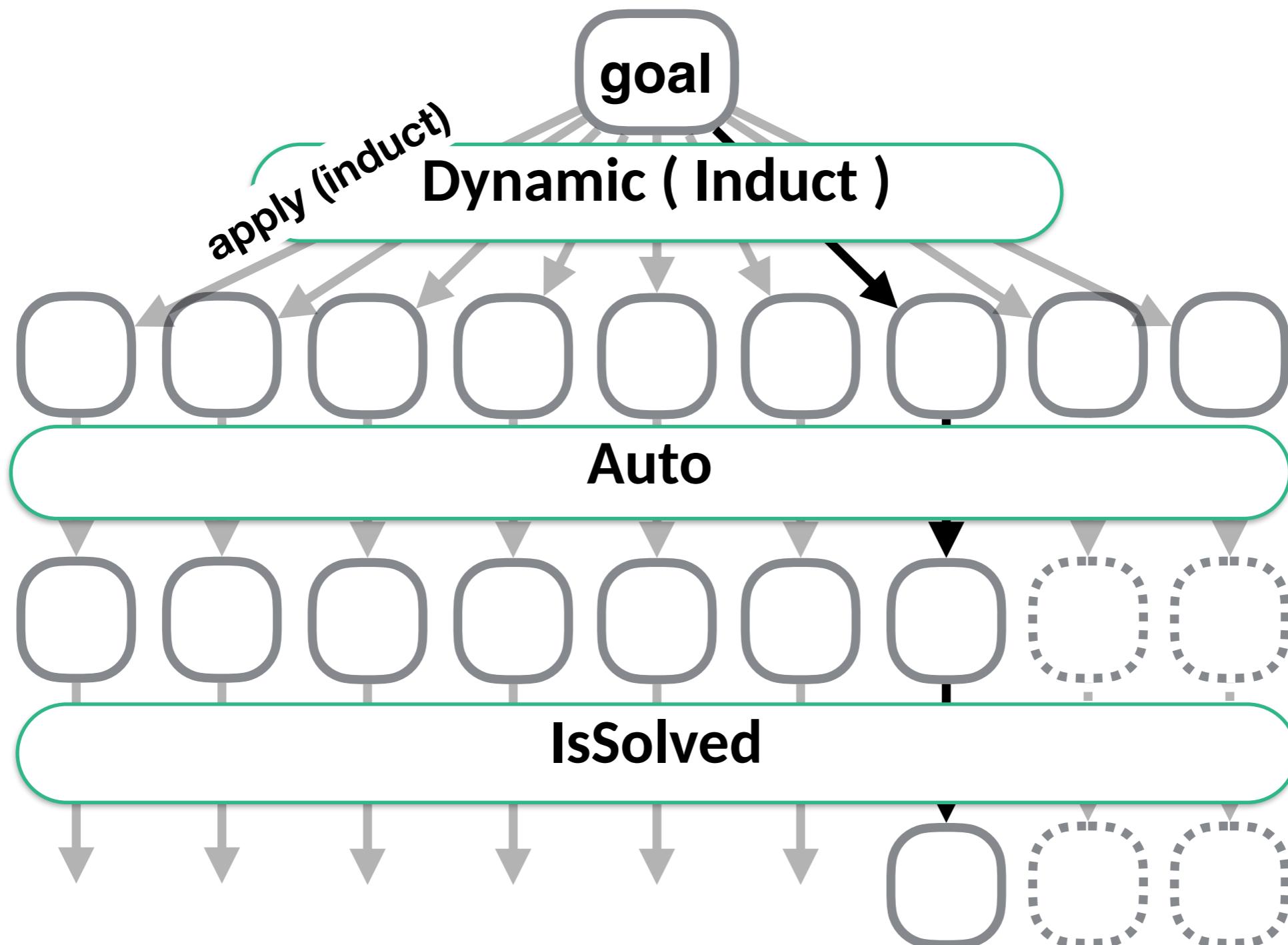


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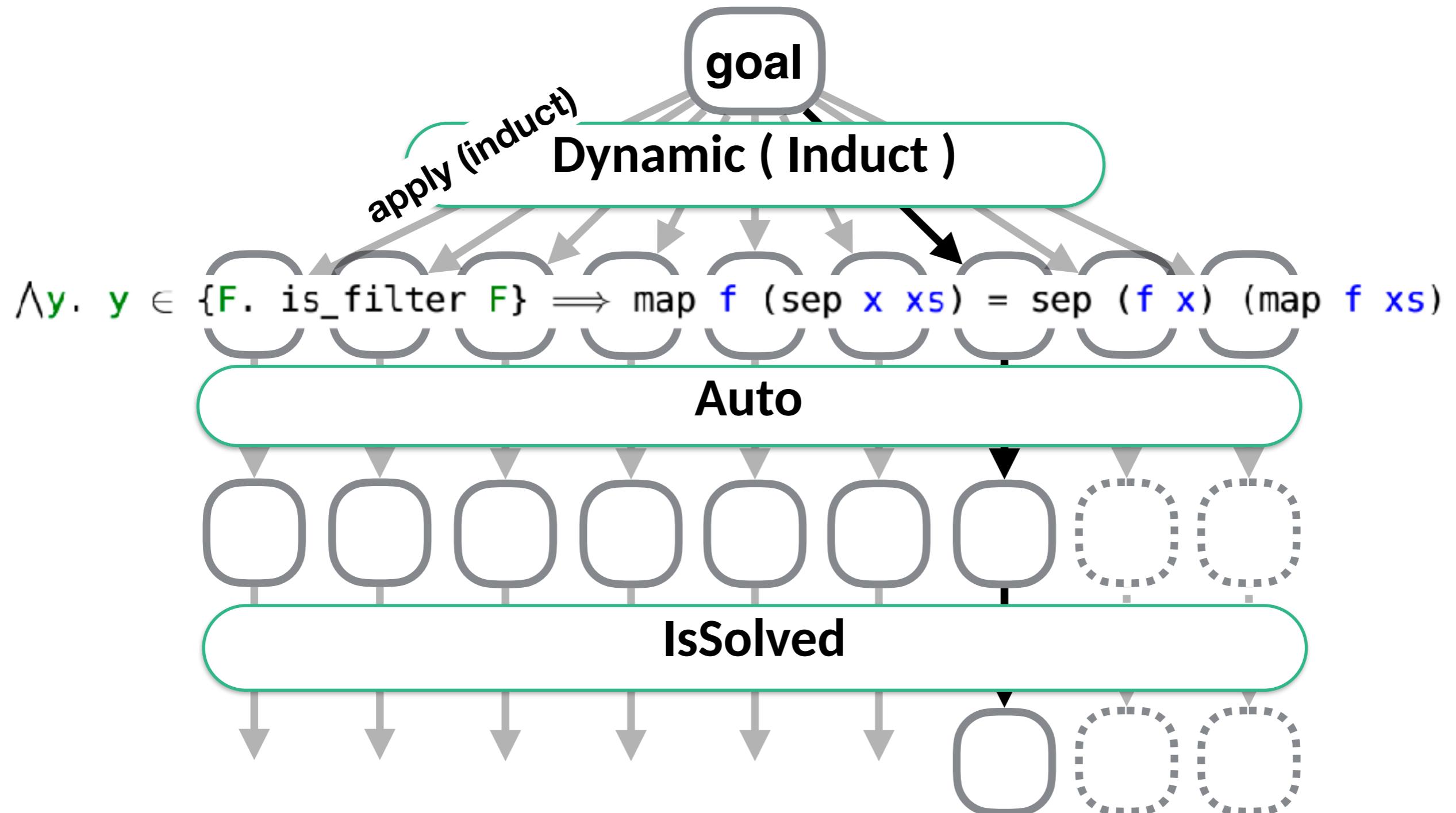
```
lemma "map f (sep x xs) = map f (x # xs)"  
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```



What I did before APLAS2020

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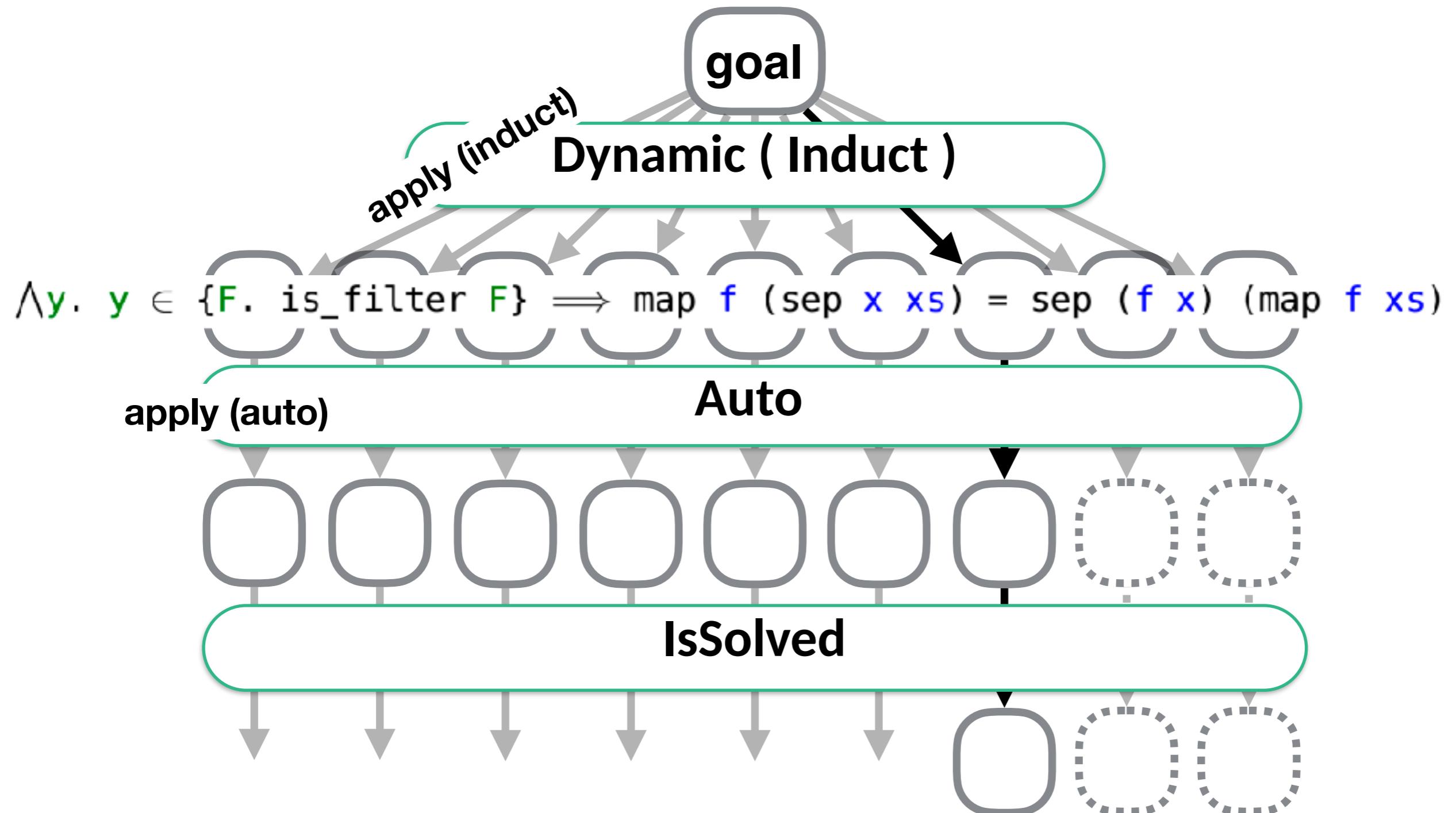
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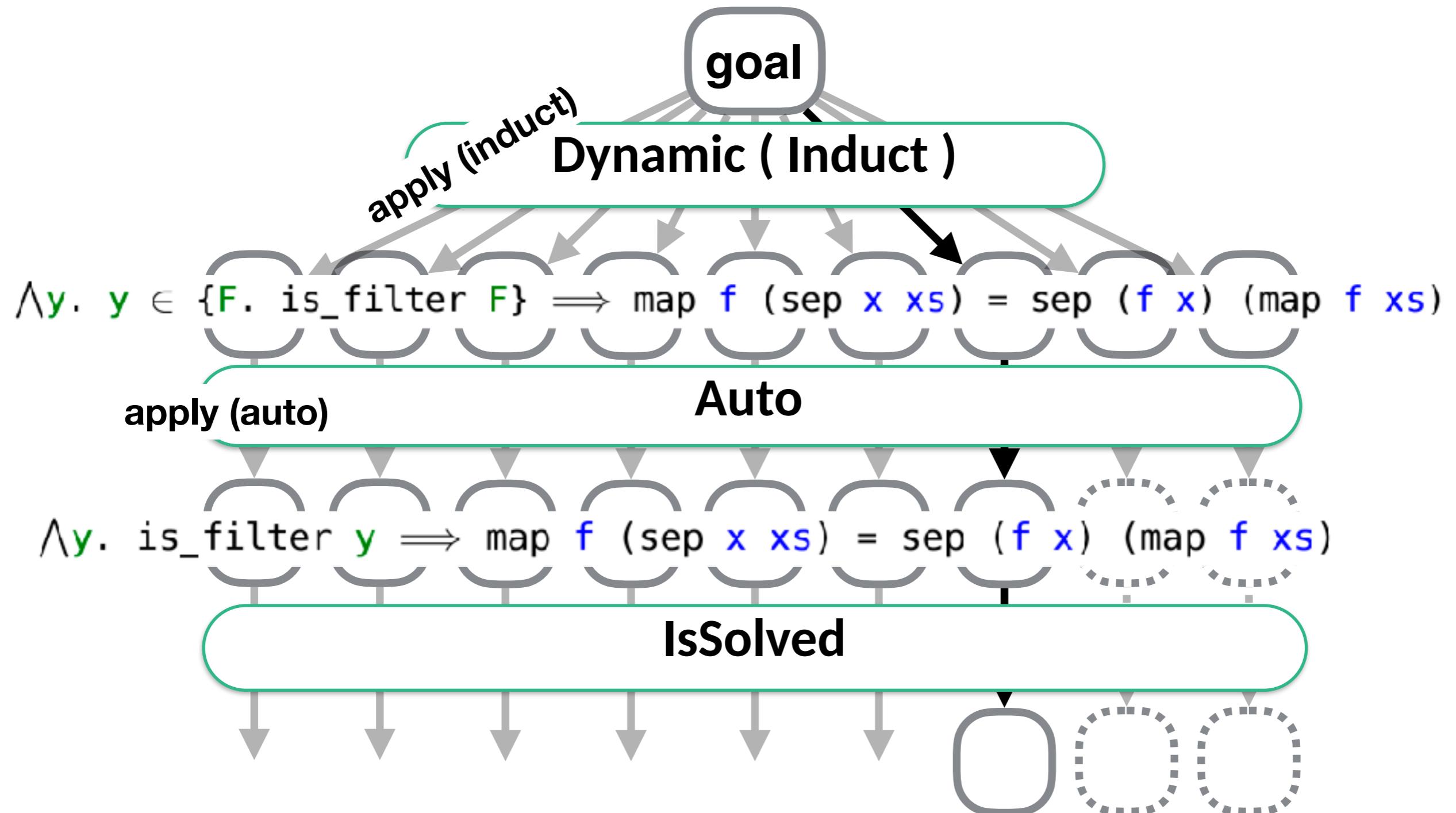
DEMO!



What I did before APLAS2020

DEMO!

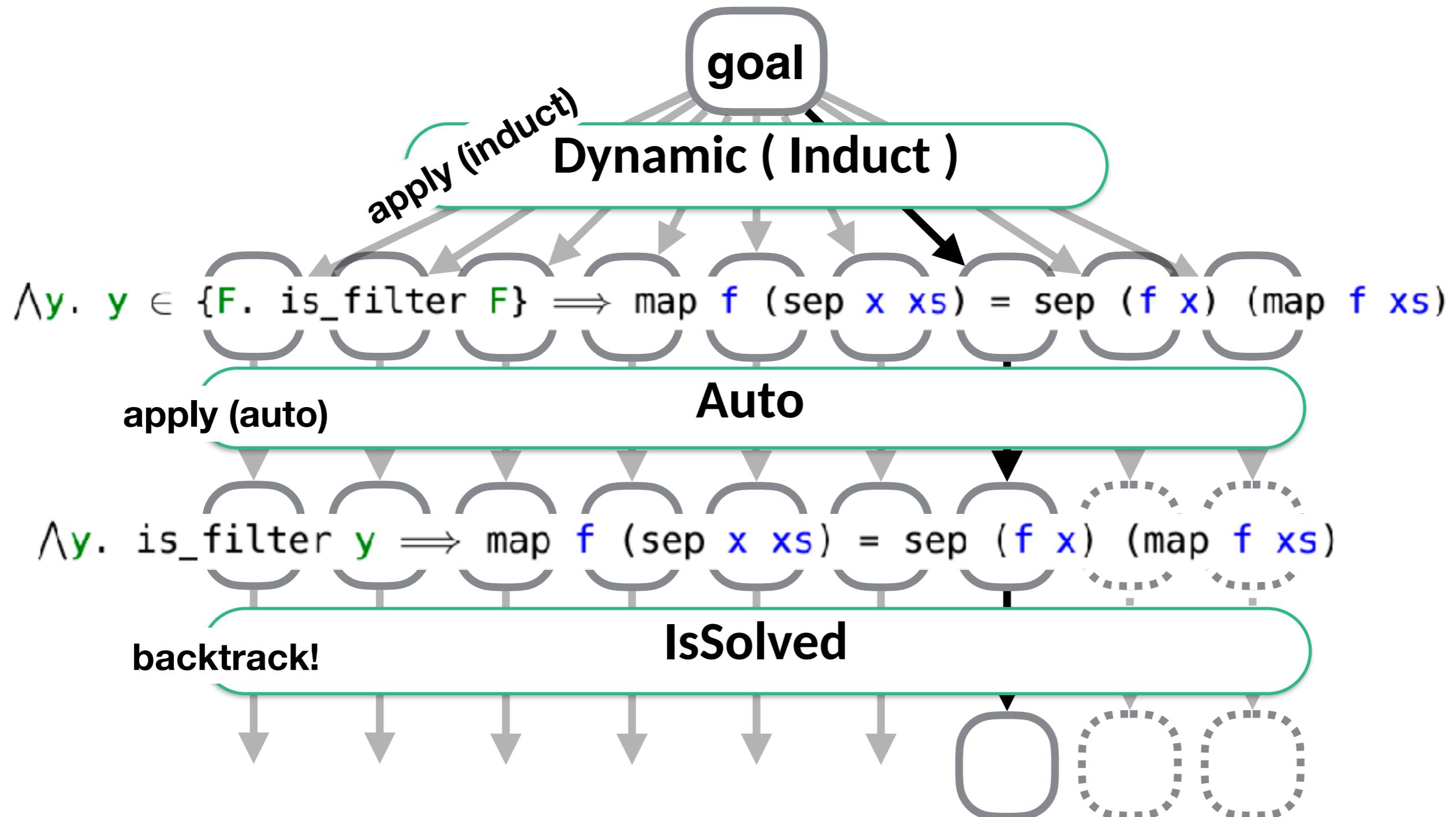
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```



What I did before APLAS2020

DEMO!

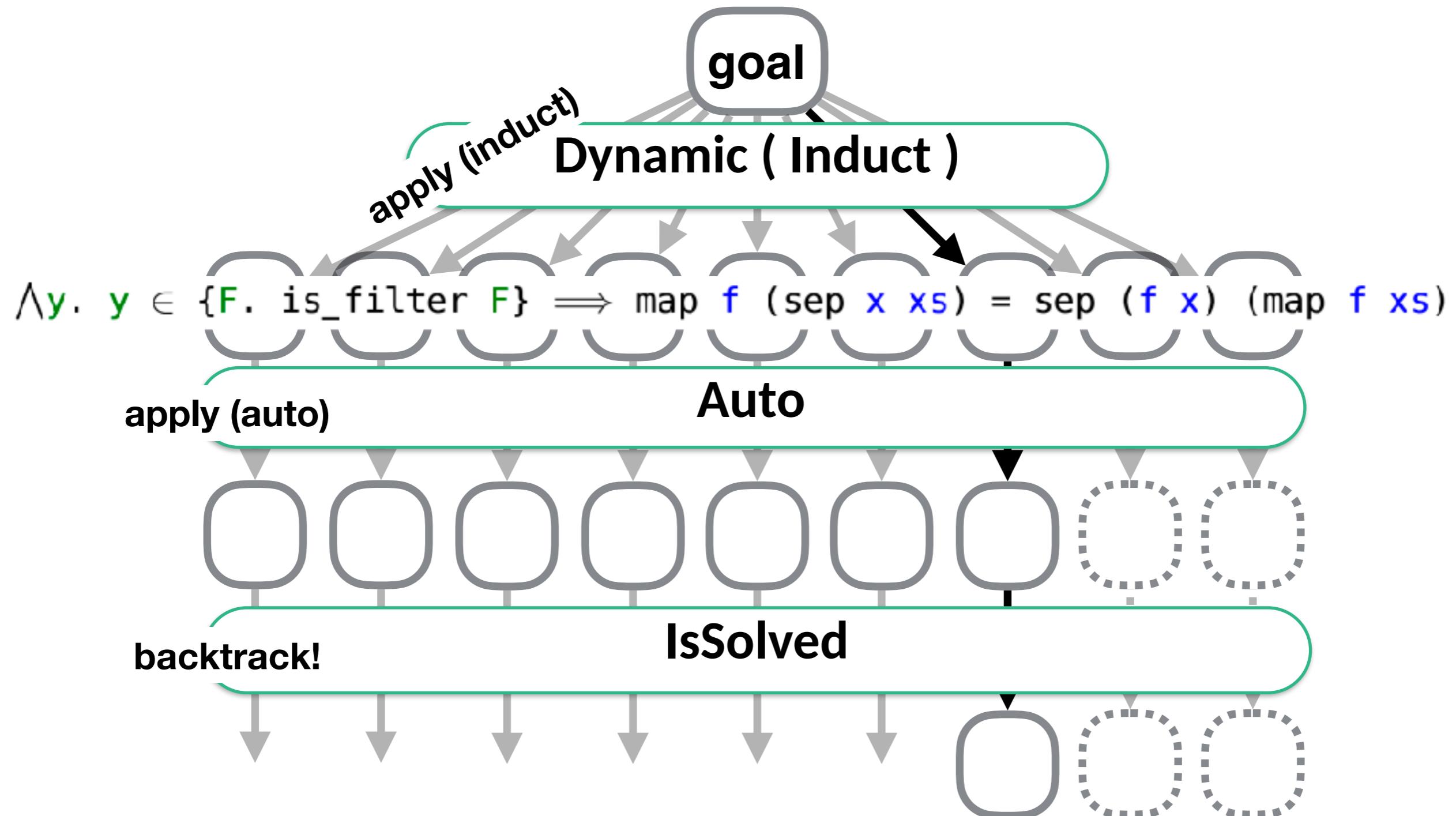
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DEMO!



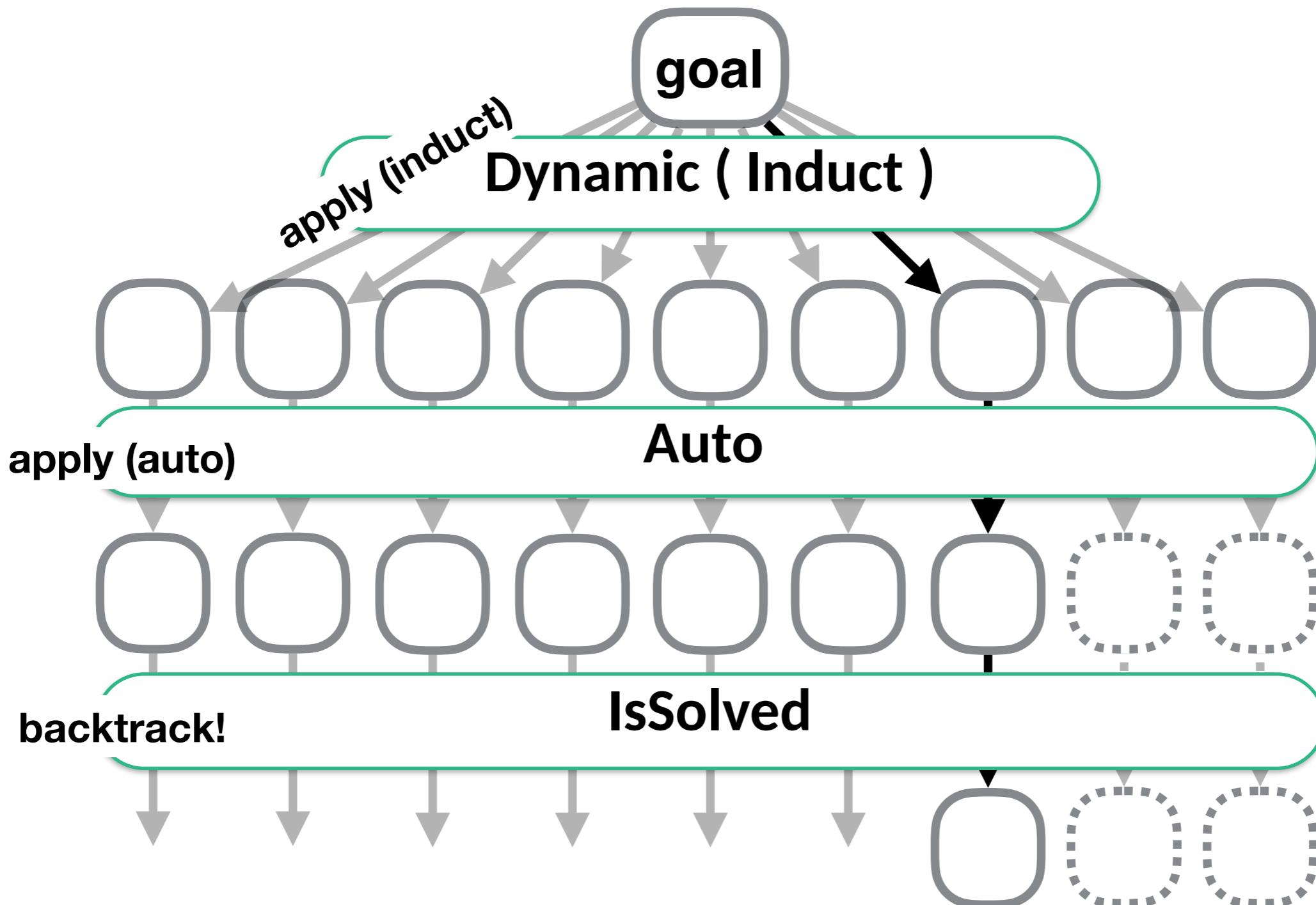
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What I did before APLAS2020

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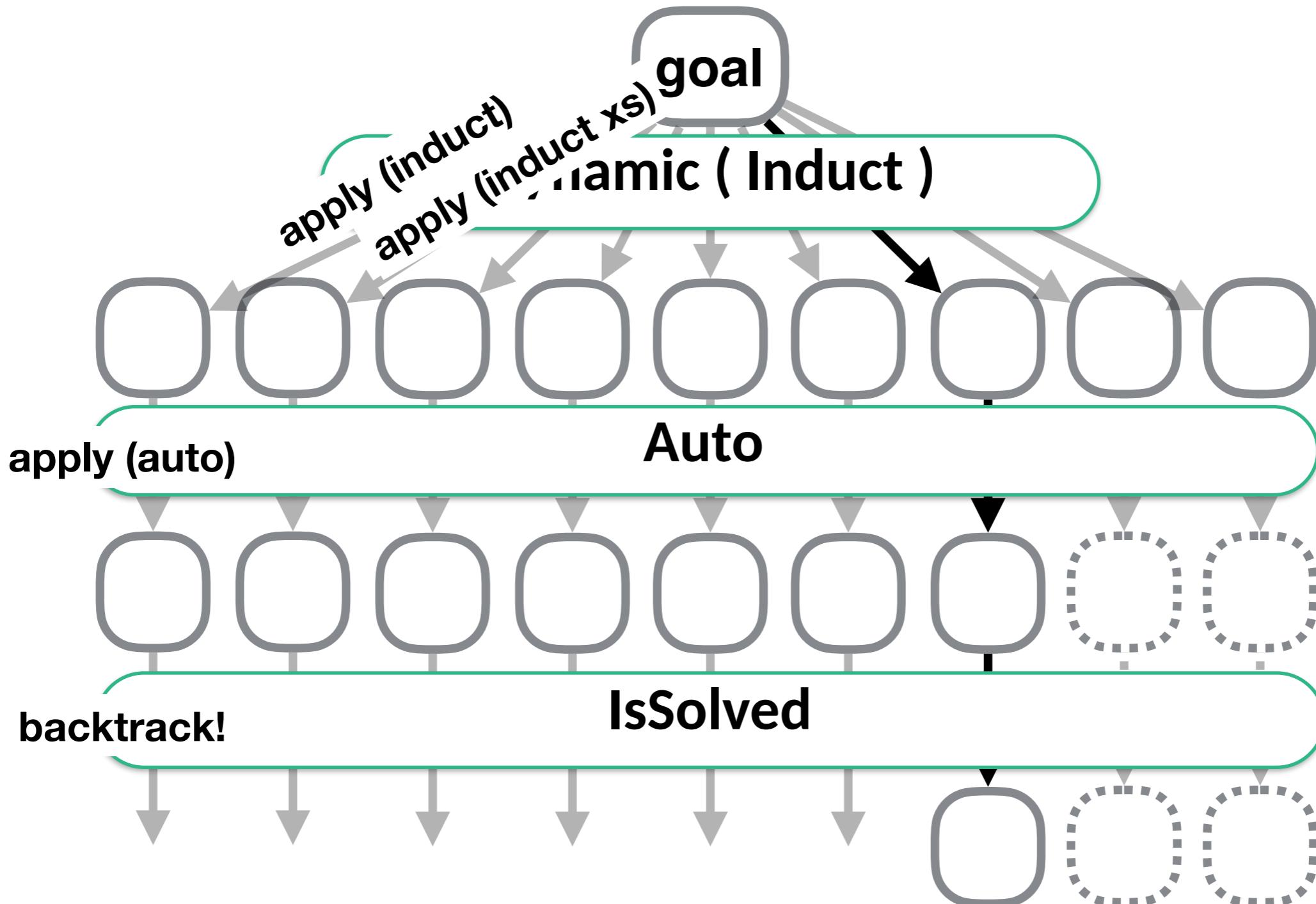
DEMO!



What I did before APLAS2020

DEMO!

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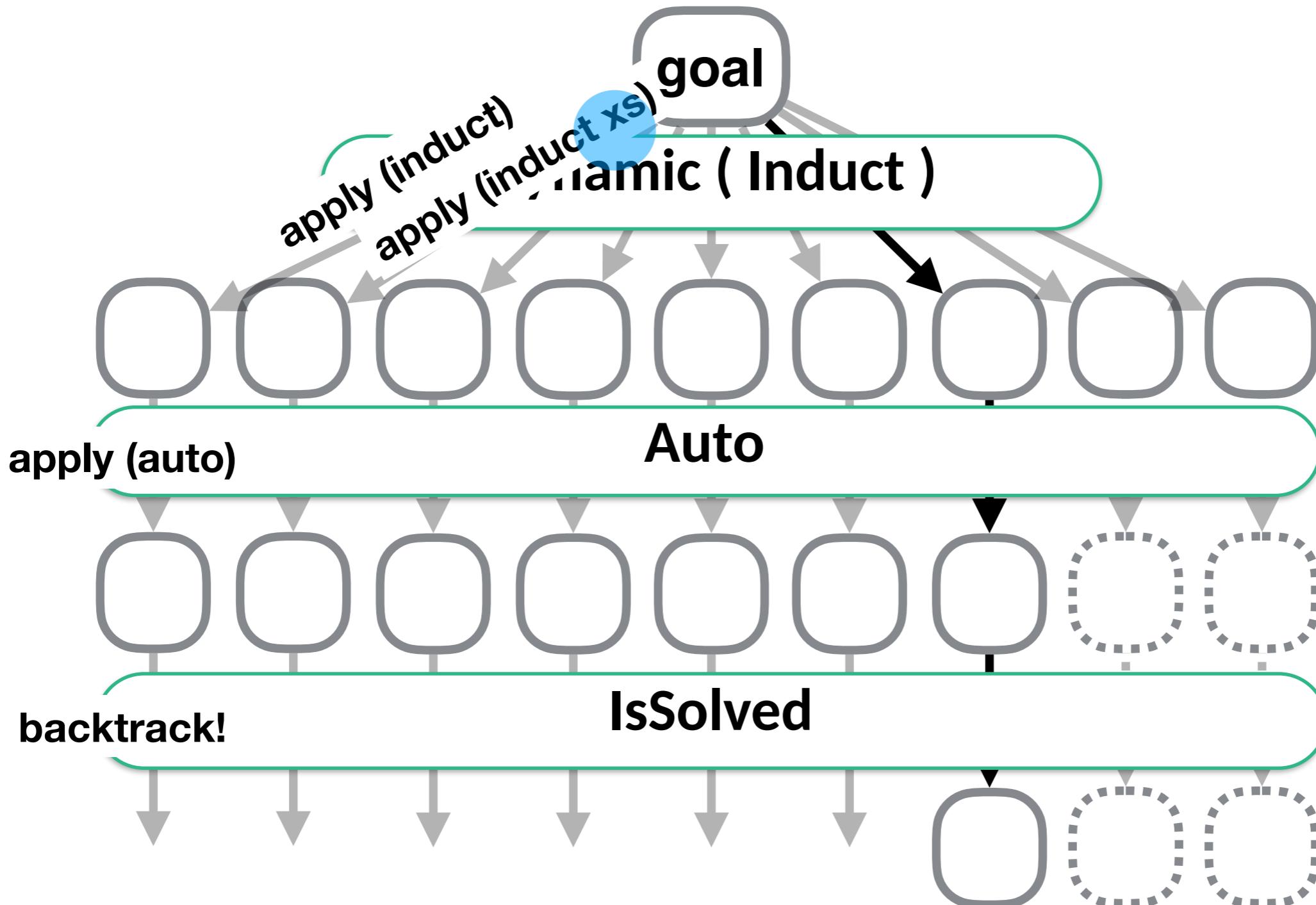
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What I did before APLAS2020

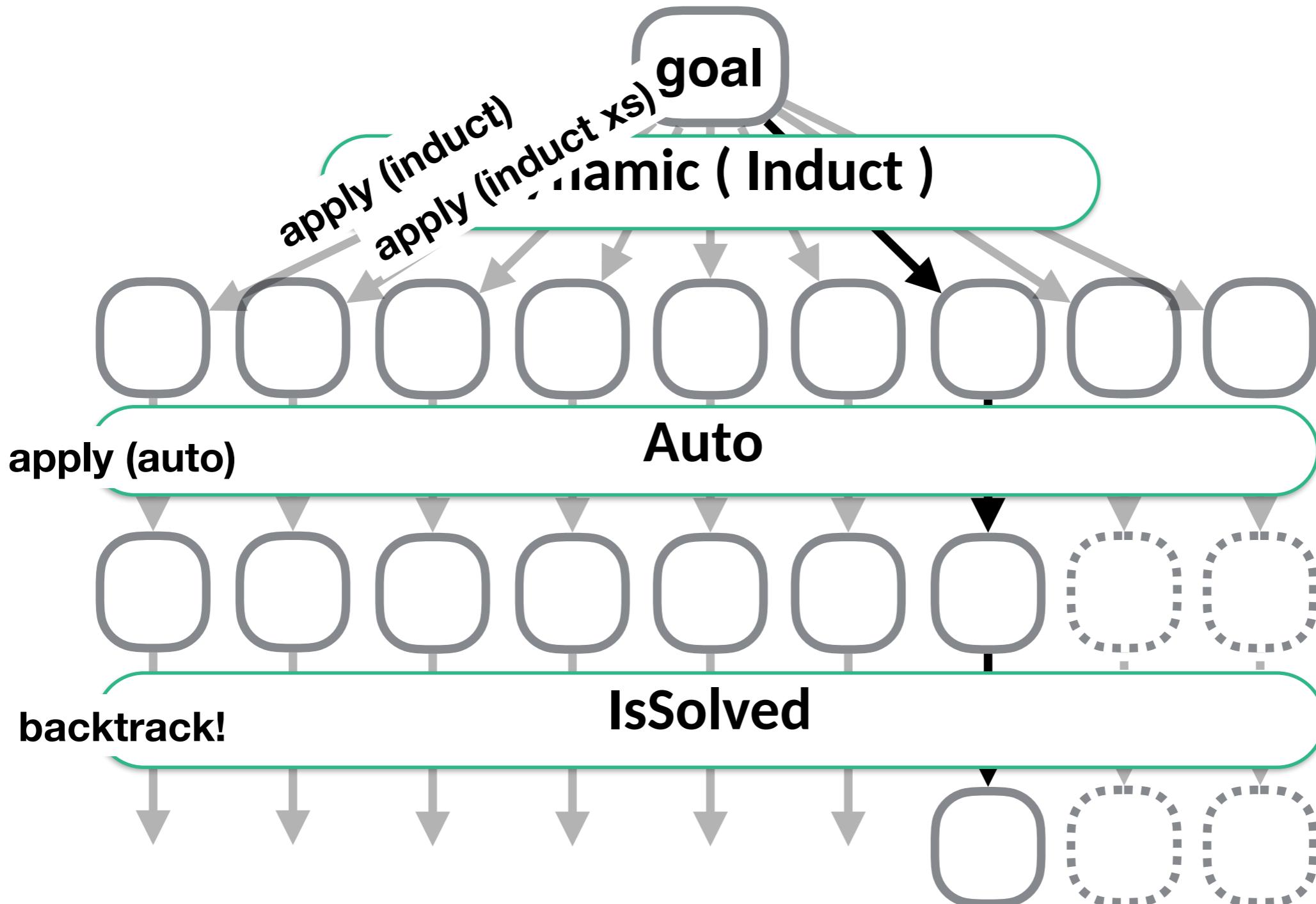
```
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find_proof DInd(*= Thens ... tactic [Auto, ...])
```

DEMO!



What I did before APLAS2020

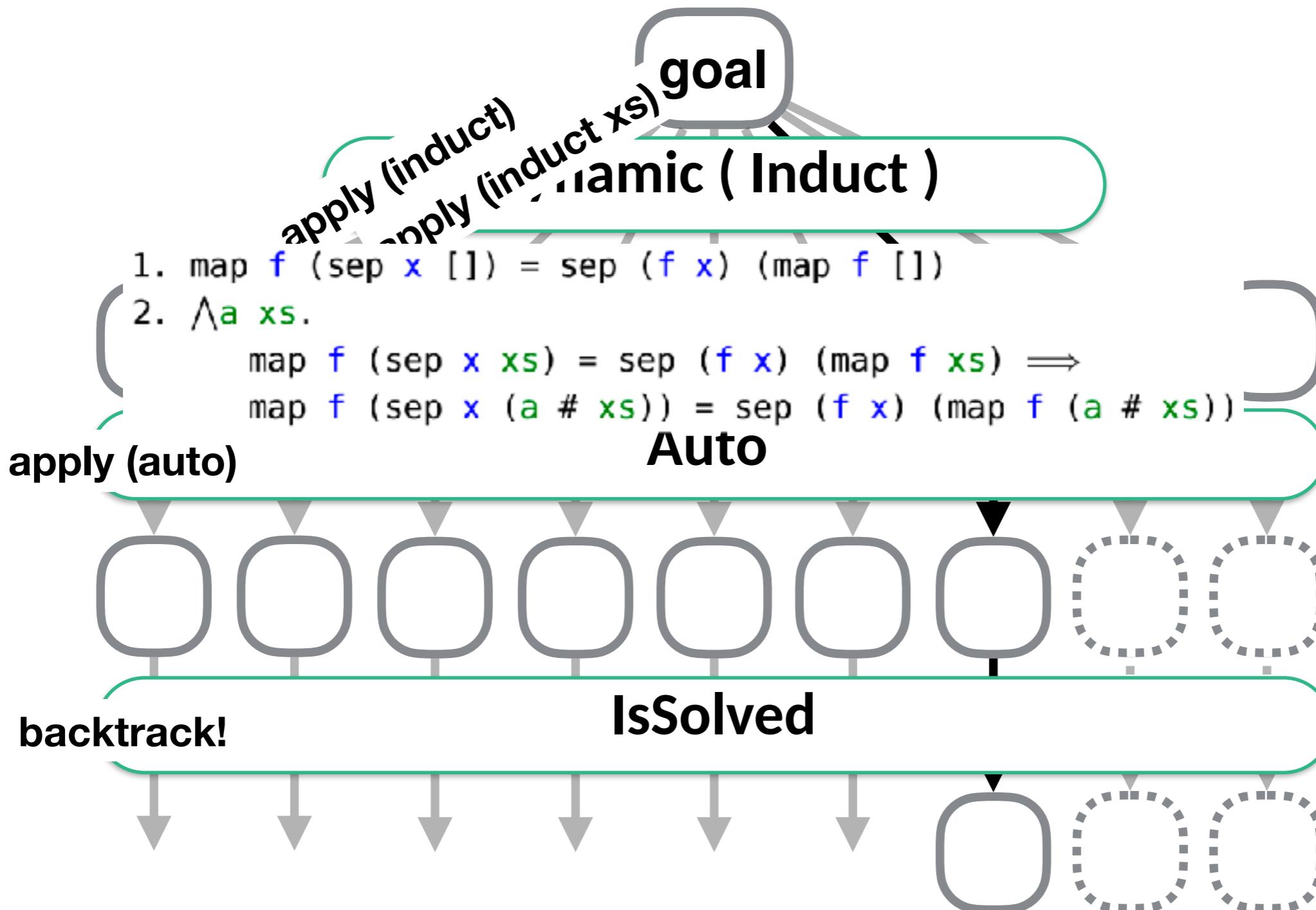
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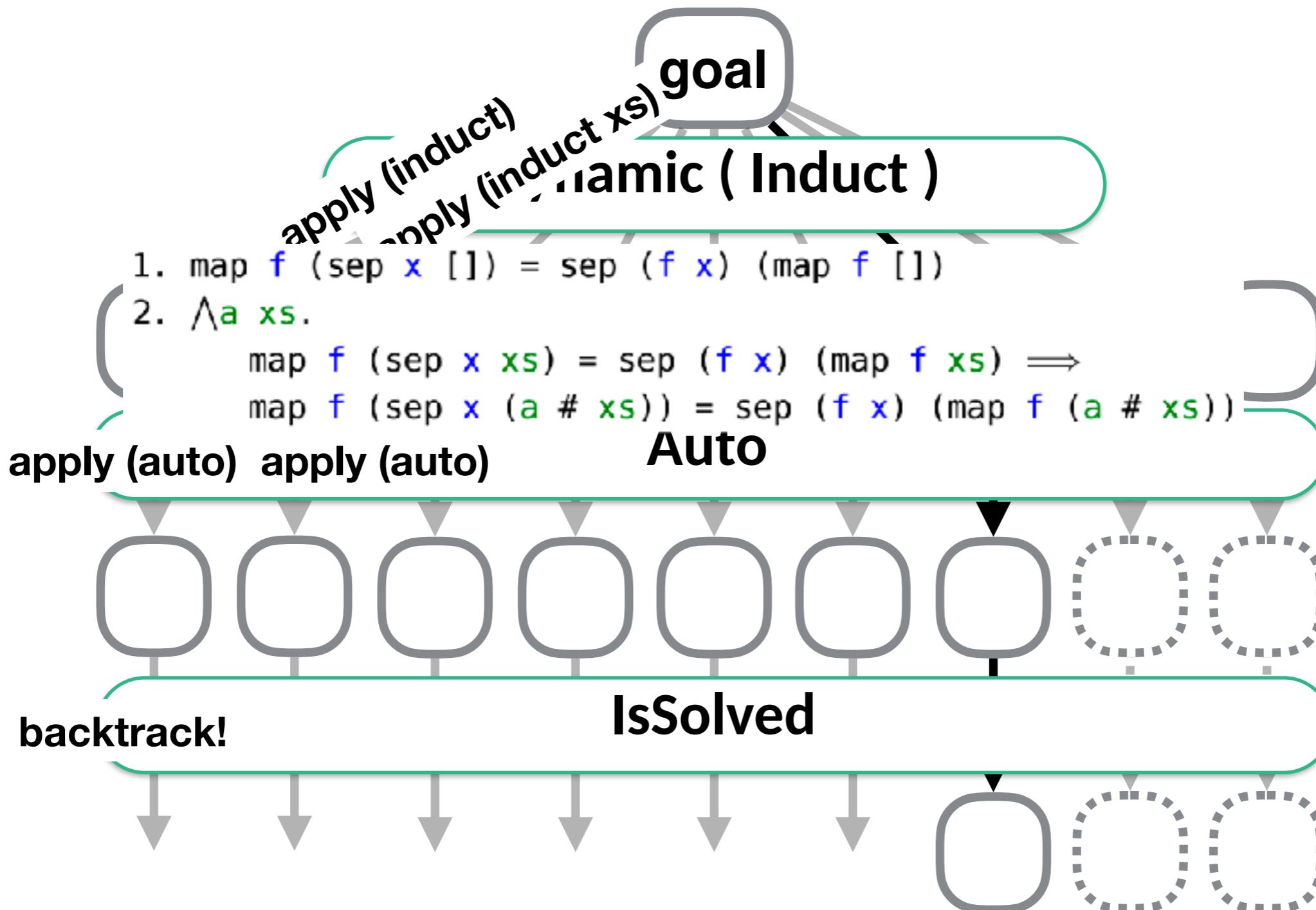
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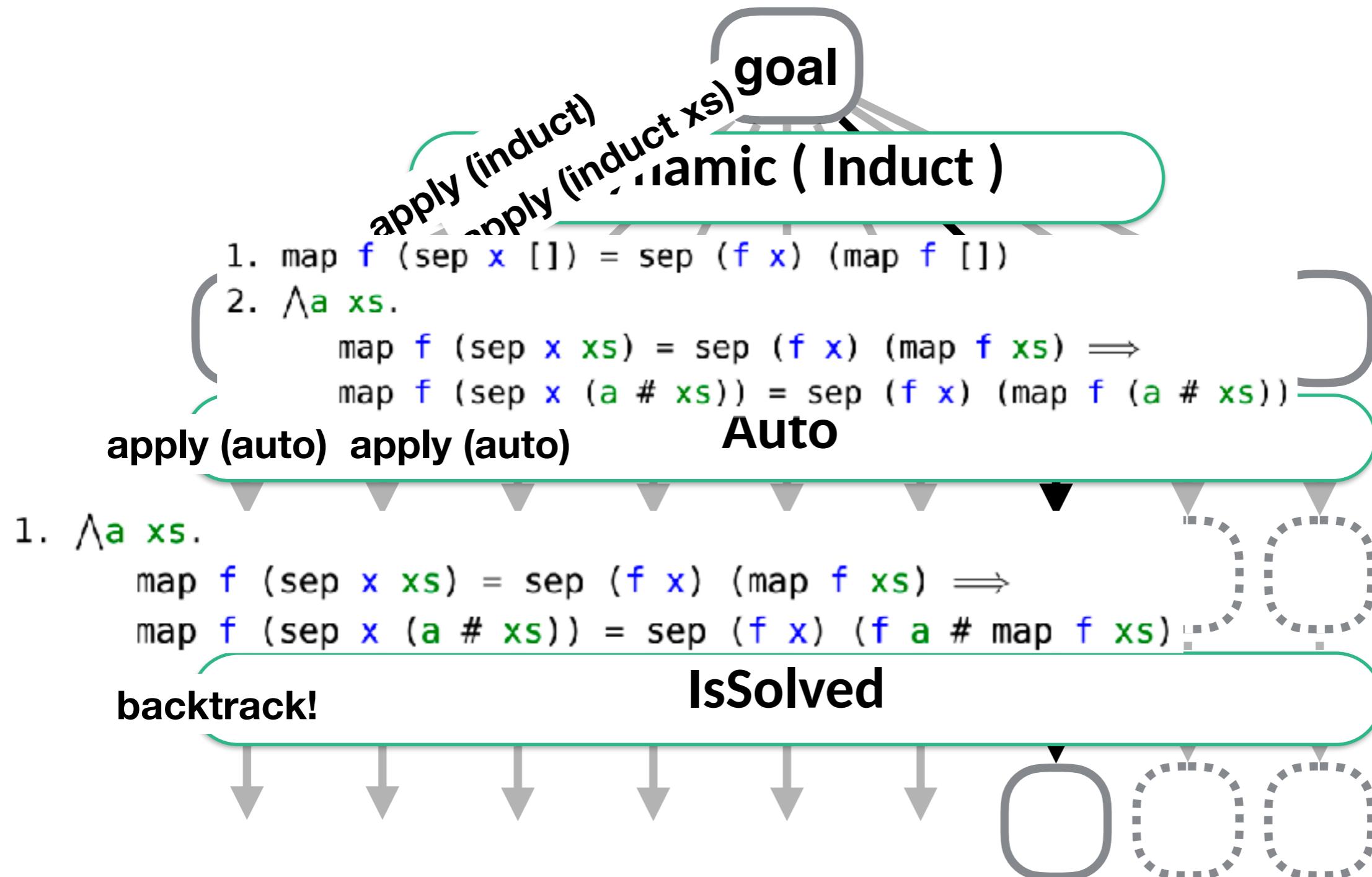
DEMO!



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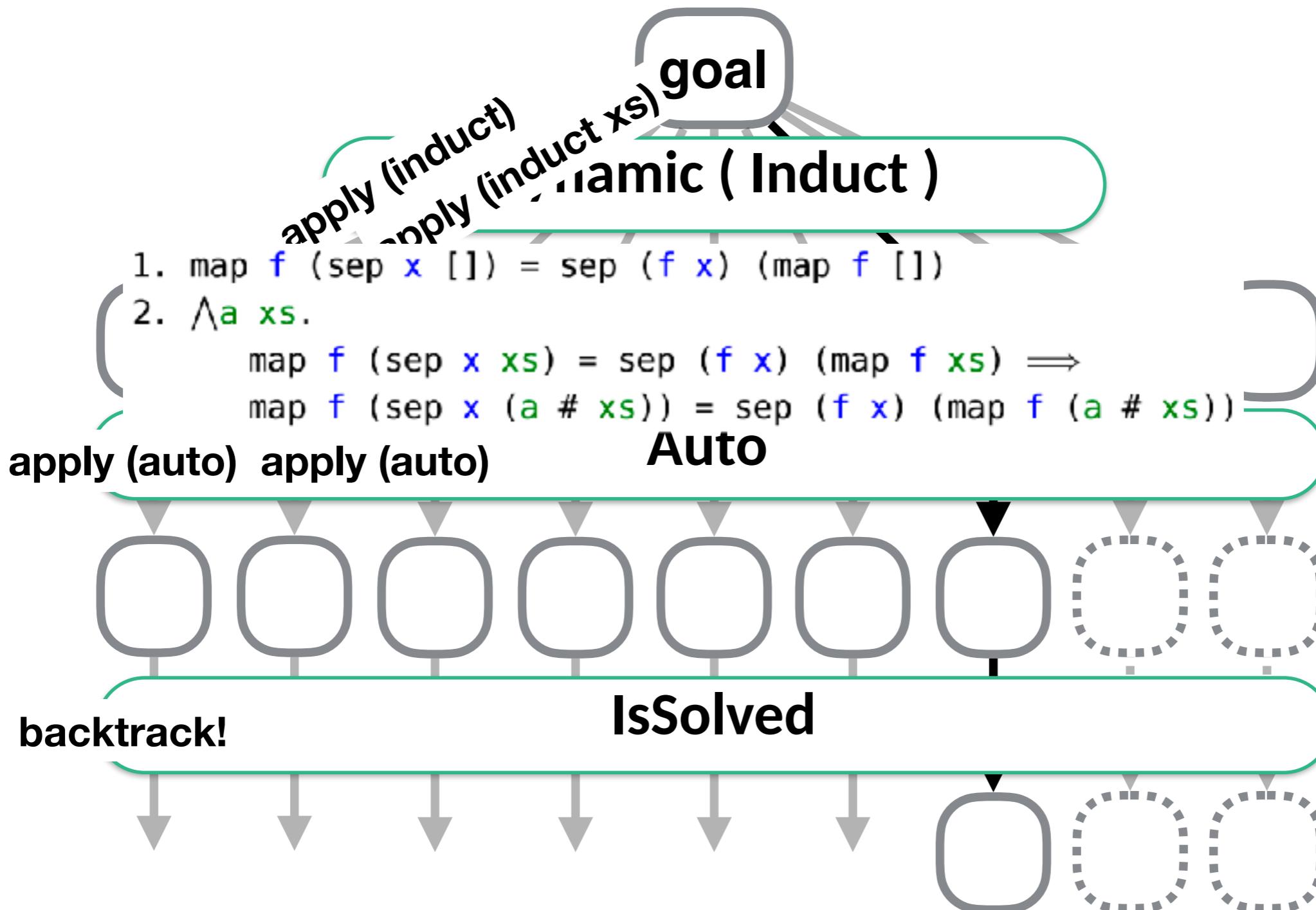
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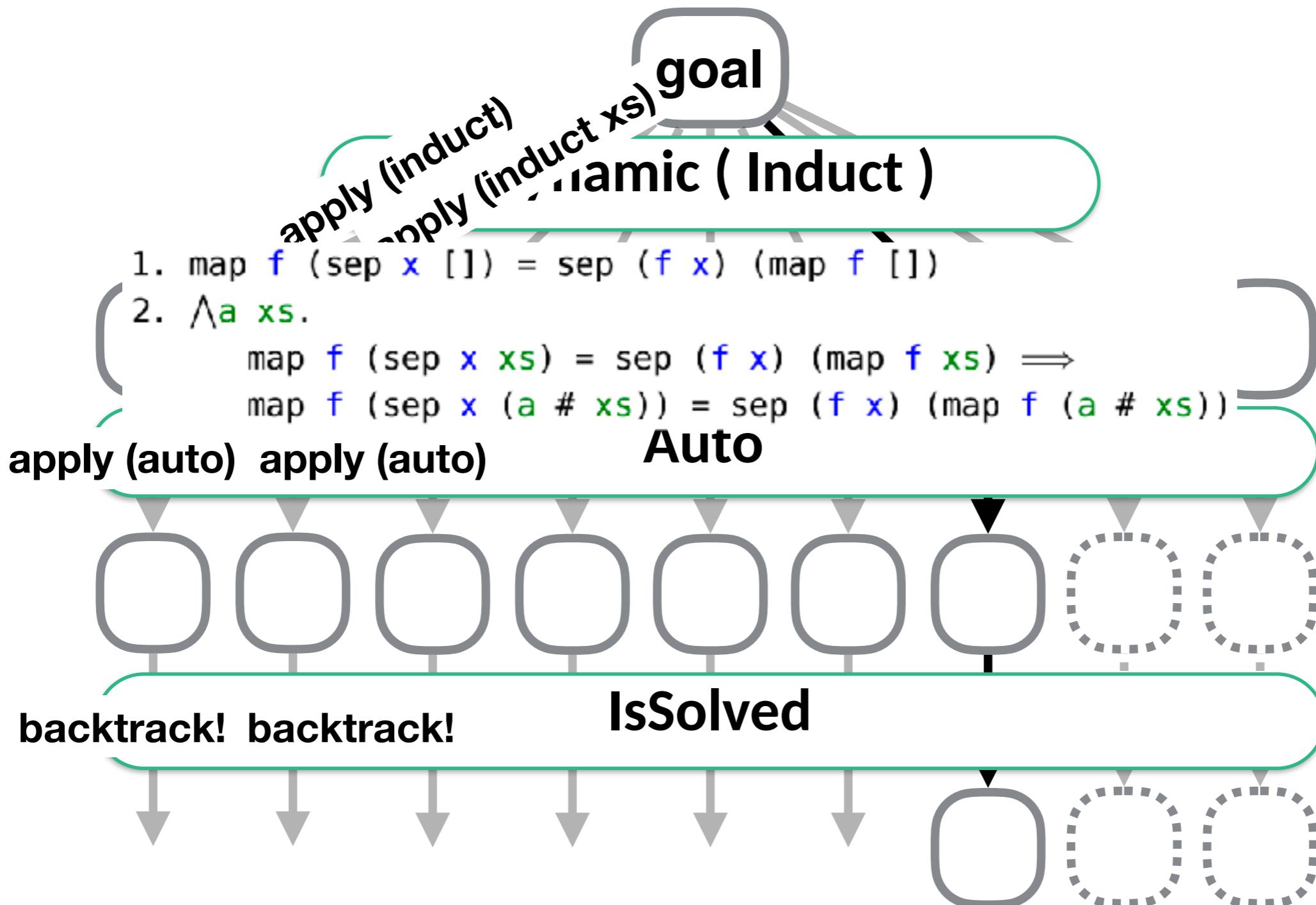
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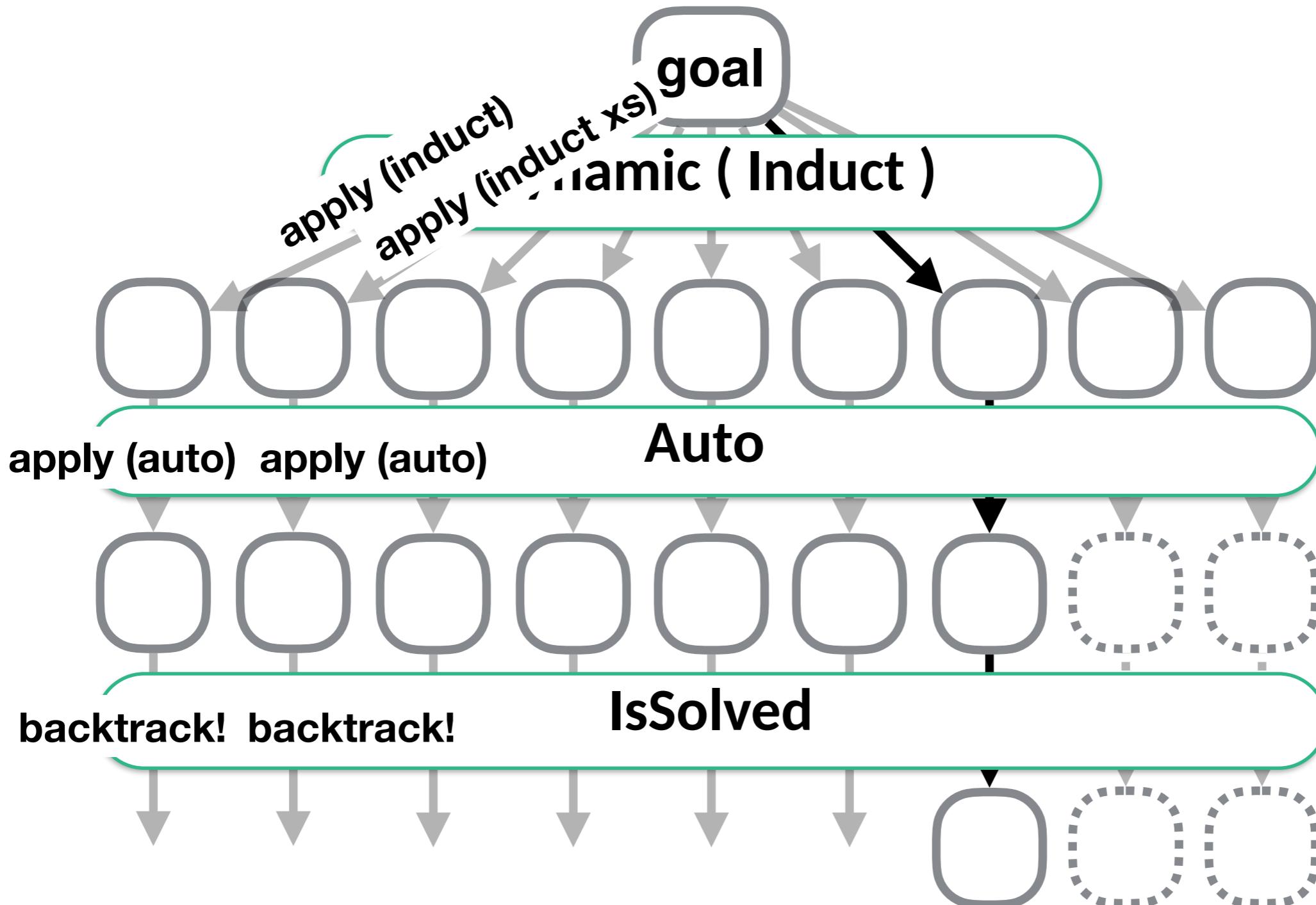
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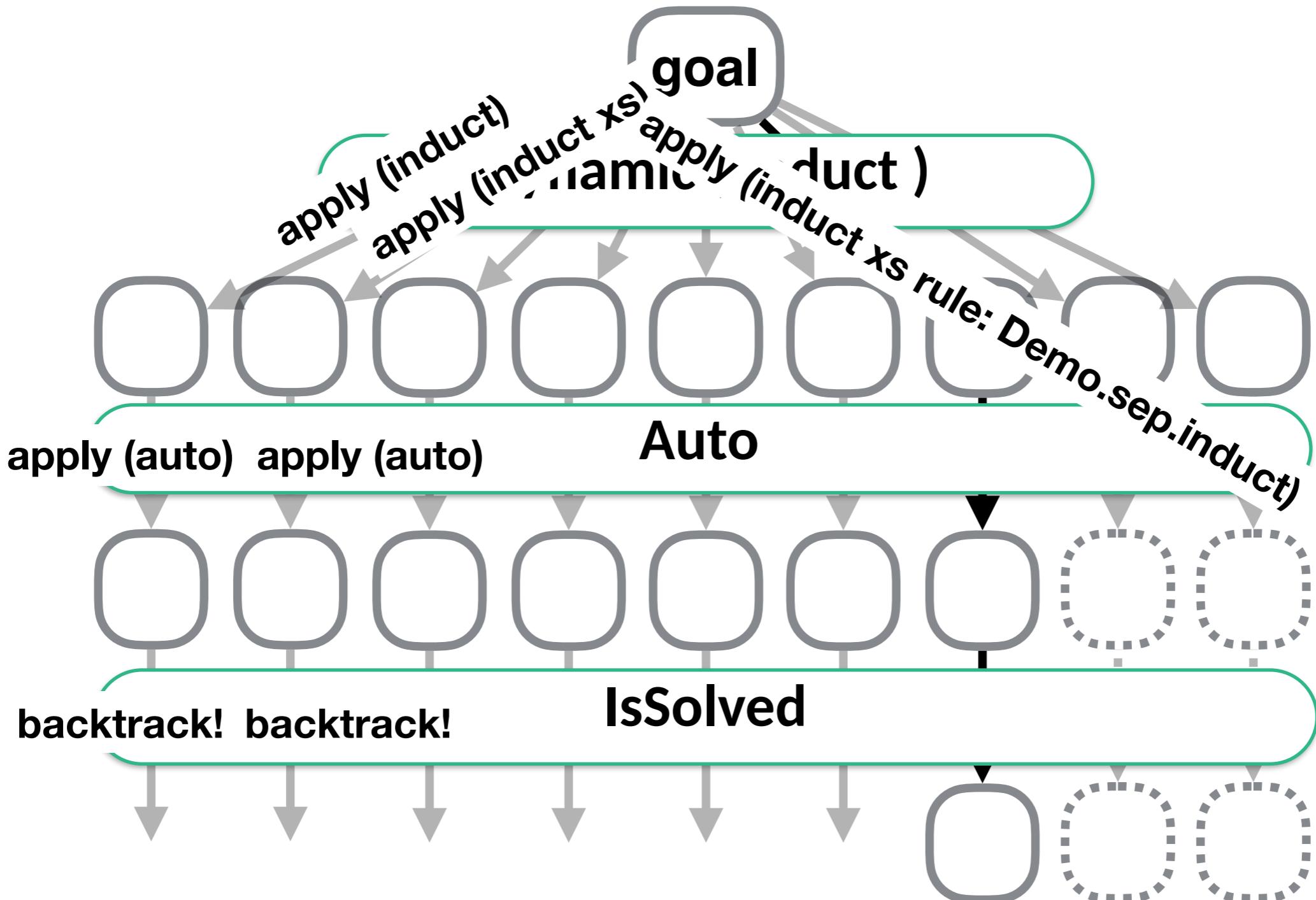


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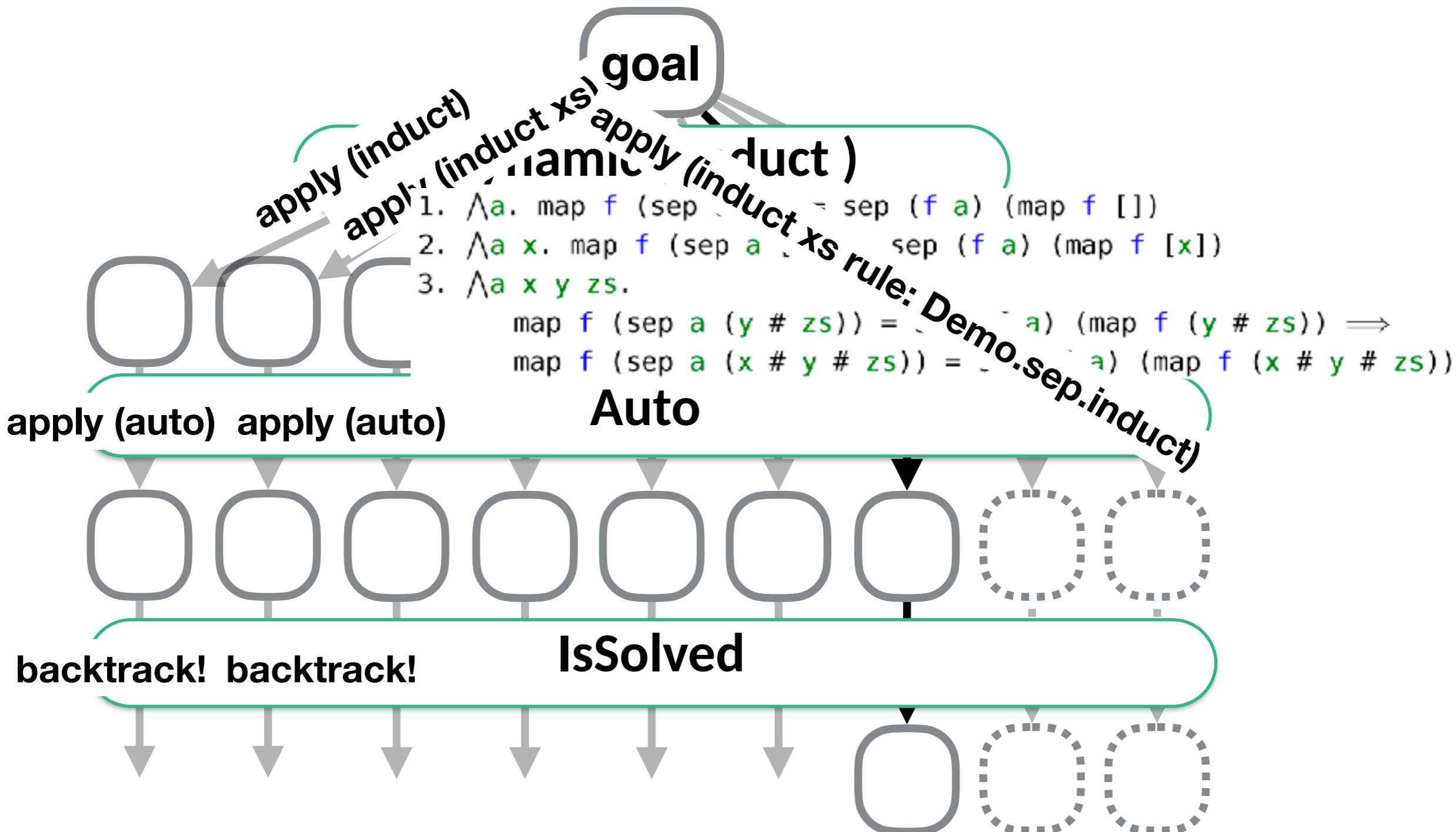
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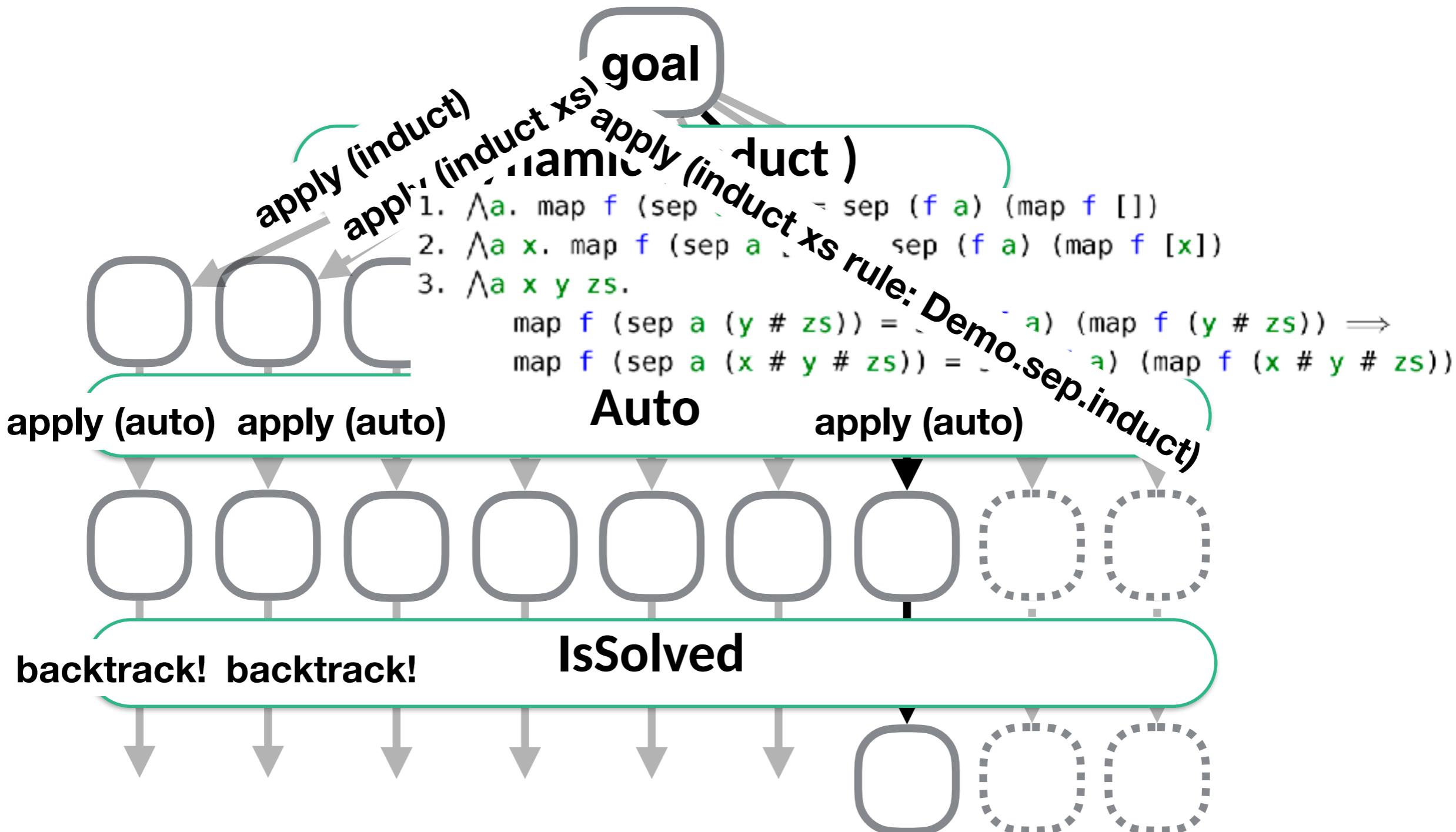
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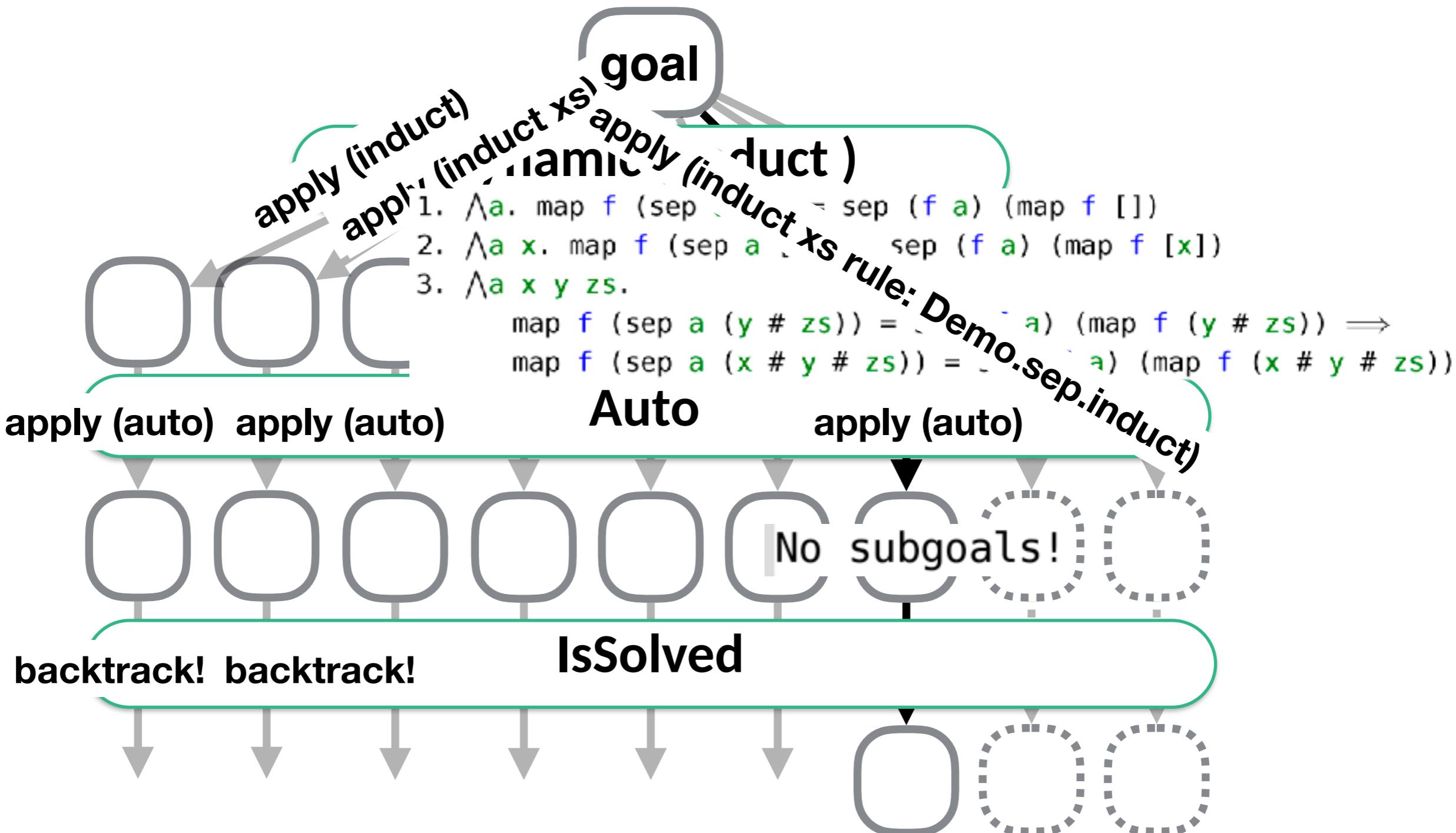
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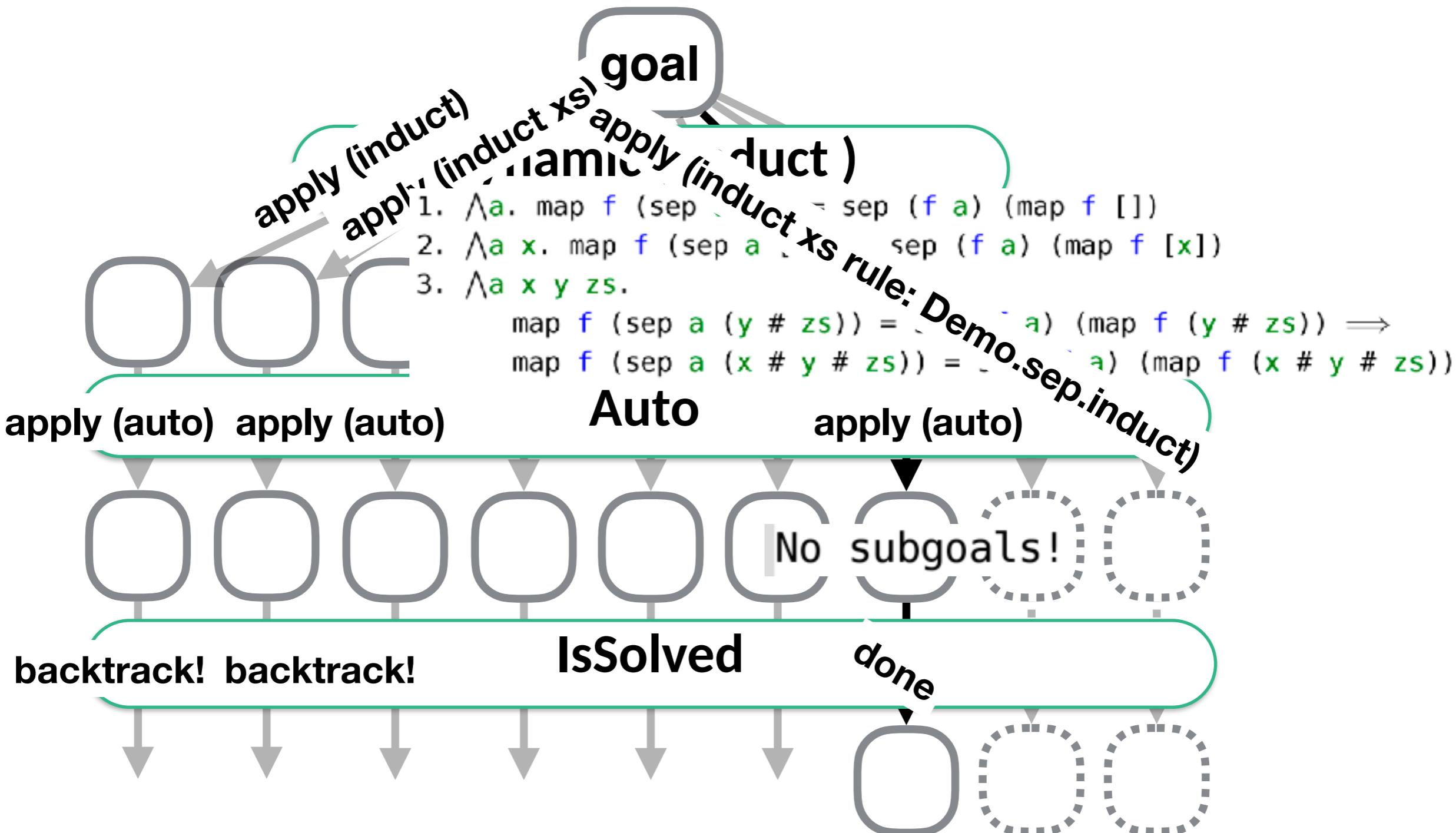
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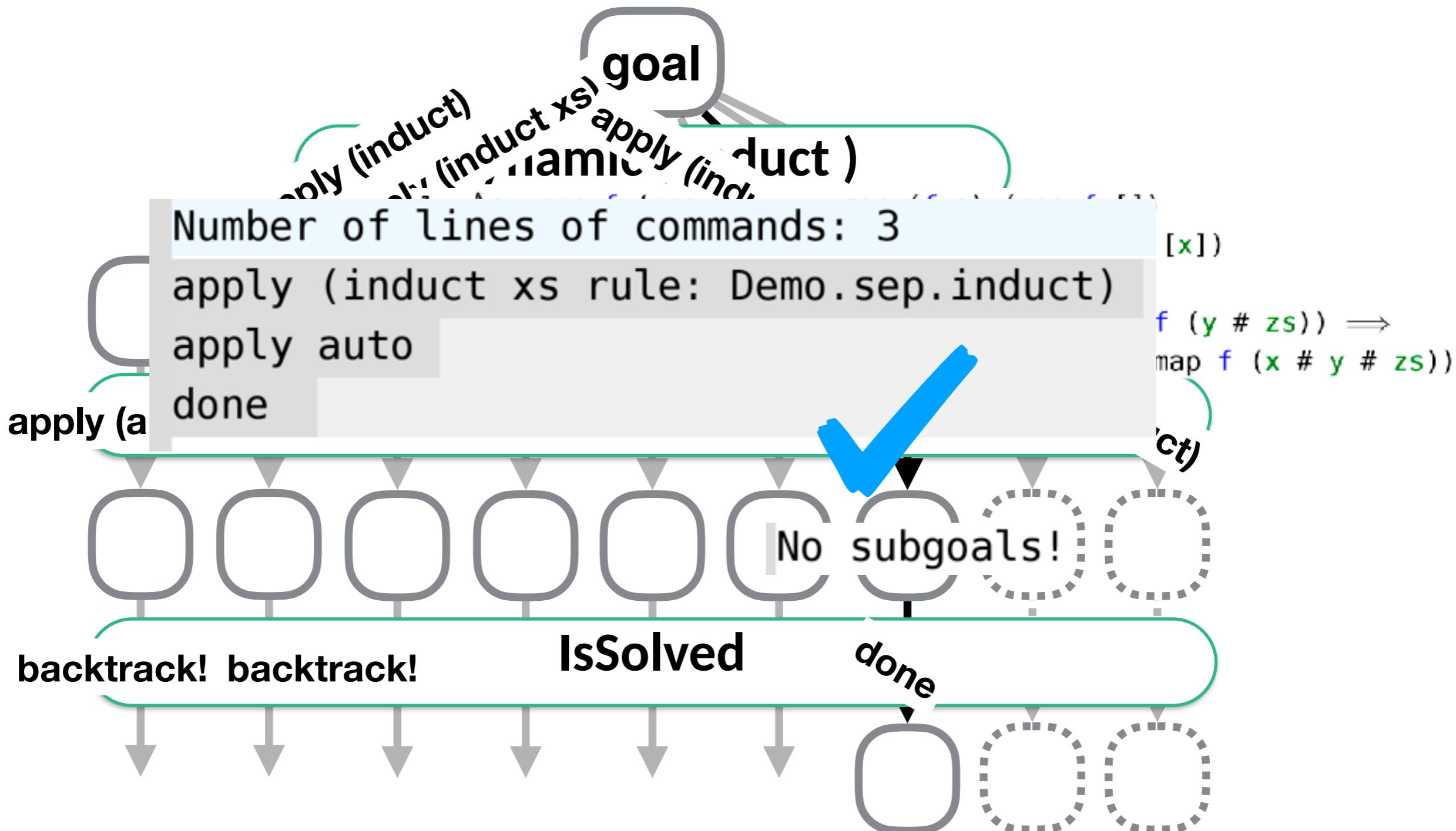
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```

DEMO!



What I did before APLAS2020

```
lemma "map f (sep x x) = map f x" (map f xs)"  
find_proof DInd(*= Thens  
  [DInd, Induct, Auto, IsSolved]*))
```

2020 DEMO!

apply (induct)
apply (induct xs)
apply (induct)

Number of lines of commands: 3

```
apply (induct xs rule: Demo.sep.induct)
```

apply auto

done

apply (a

[x])

f (**y** # **zs**)) \Rightarrow

```
map f (x # y # zs)
```

Ctrl

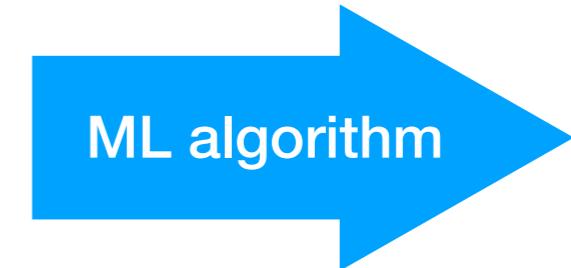
Difficult proof goals? Search space?



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Introduction to Machine Learning in 10 seconds

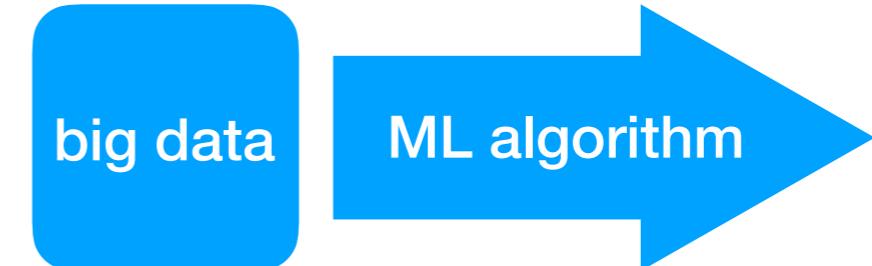


<https://duckduckgo.com/?q=cat&t=ffab&iar=images&iax=images&ia=images>

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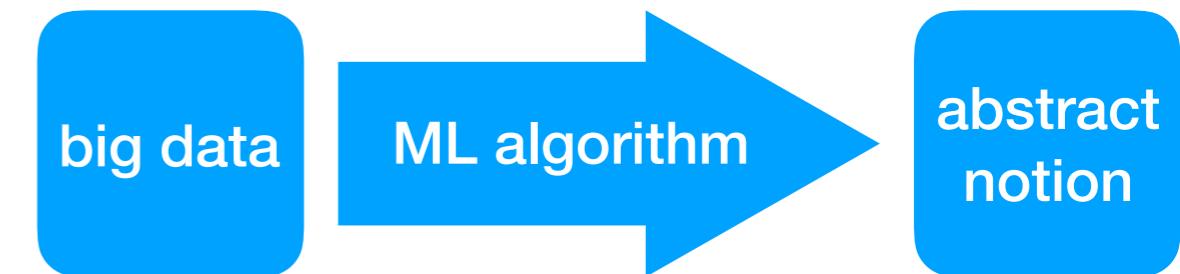


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Introduction to Machine Learning in 10 seconds



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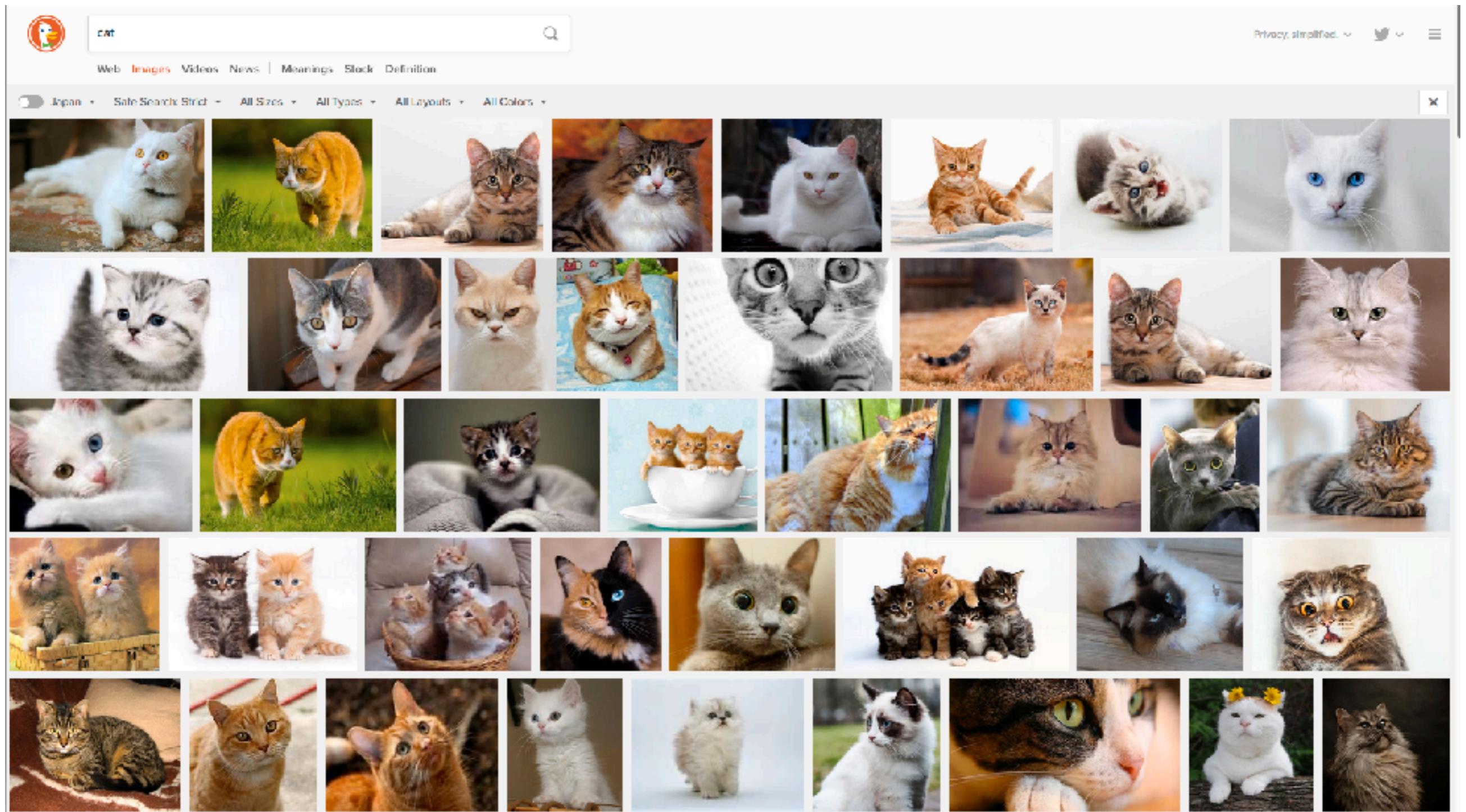
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Introduction to Machine Learning in 10 seconds

big data

ML algorithm

abstract notion



<https://duckduckgo.com/?q=cat&t=ffab&iar=images&iax=images&ia=images>

<https://twitter.com/YutakangE>

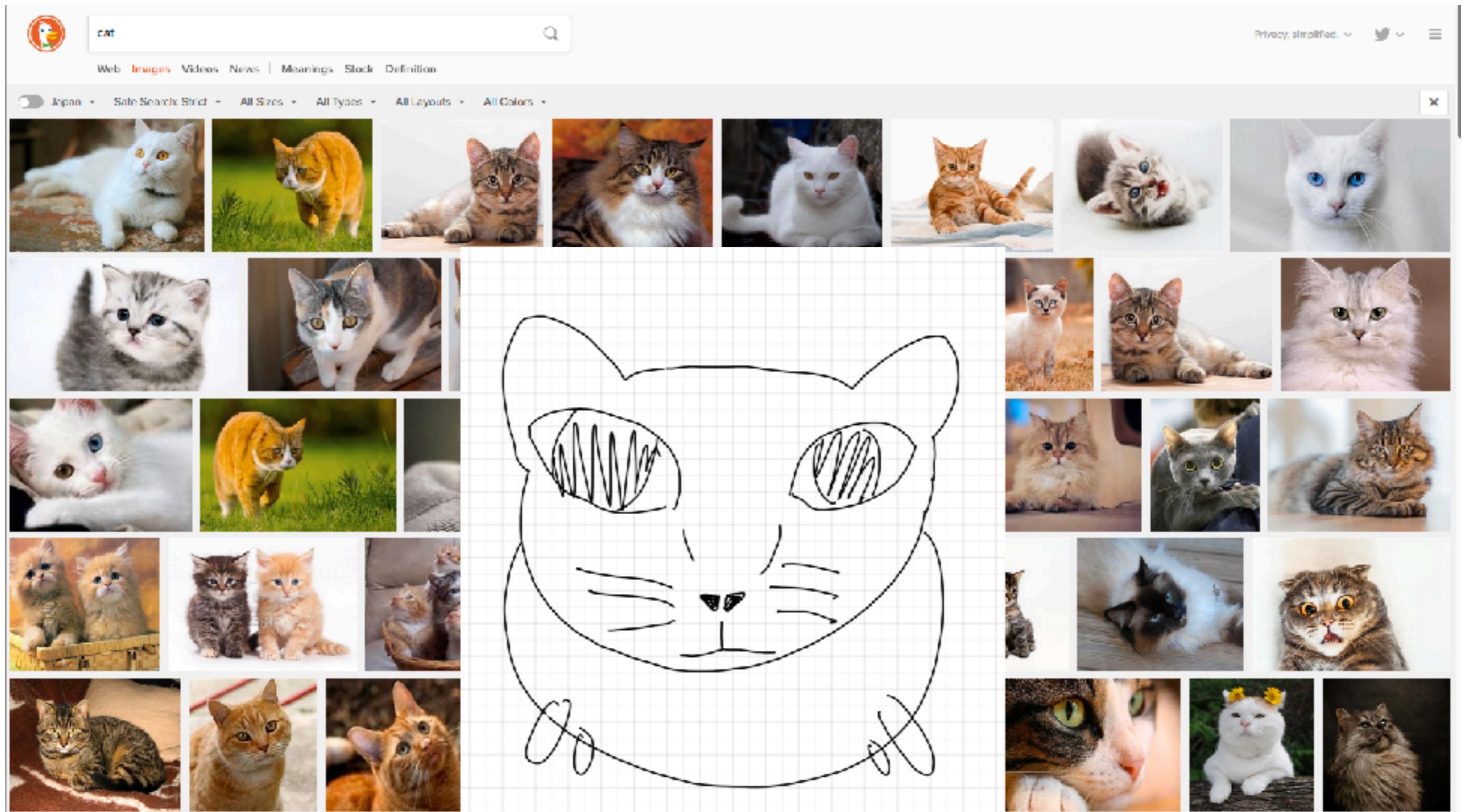
<https://tinyurl.com/up7zhyk>

Introduction to Machine Learning in 10 seconds

big data

ML algorithm

abstract notion



<https://duckduckgo.com/?q=cat&t=ffab&iar=images&iax=images&ia=images>

ML for Inductive Theorem Proving the BAD

```
Lemma "itrev xs ys = rev xs @ ys"
```

← one abstract representation

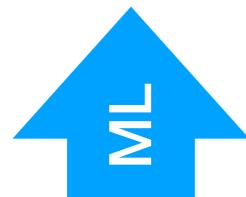
```
lemma "itrev [] = rev [] @ []" by auto
lemma "itrev [1,2] = rev [1,2] @ []" by auto
lemma "itrev [1,2,3] = rev [1,2,3] @ []" by auto
lemma "'a', 'b'" [] = rev ['a', 'b'] @ []" by auto
lemma "[x,y,z]" [] = rev [x,y,z] @ []" by auto
```

← many concrete cases

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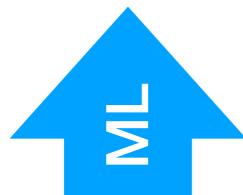
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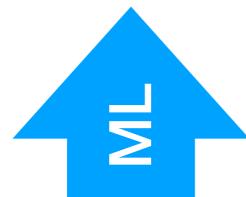
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Failed to apply proof method△:
goal (1 subgoal):
1. itrev xs ys = rev xs @ ys



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ML for Inductive Theorem Proving the BAD

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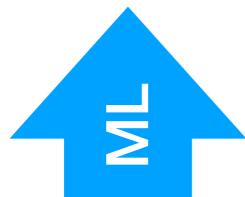
← one abstract representation

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by(induct xs ys rule:"itrev.induct") auto
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Failed to apply proof method△:

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```
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ML for Inductive Theorem Proving the BAD

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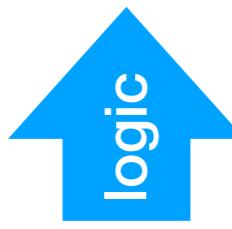
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ML for Inductive Theorem Proving the BAD

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<- abstraction using expressive logic

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ML for Inductive Theorem Proving the BAD

polymorphism

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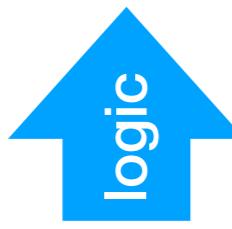
ML for Inductive Theorem Proving the BAD

polymorphism

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ML for Inductive Theorem Proving the BAD

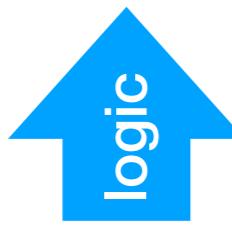
polymorphism

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ML for Inductive Theorem Proving the BAD

Higher-Order functions

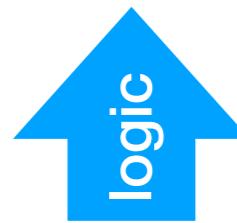
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<- many concrete cases

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

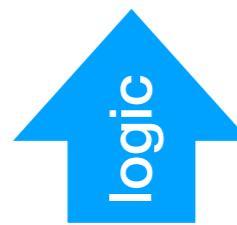
type class

universal quantifier

lambda abstraction

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<- one abstract representation



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<- many concrete cases

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

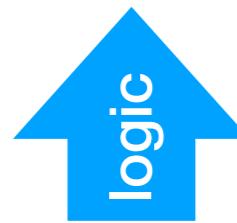
universal quantifier

lambda abstraction

concise formula that can cover
many concrete cases

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<- many concrete cases

ML for Inductive Theorem Proving the BAD

Higher-Order functions

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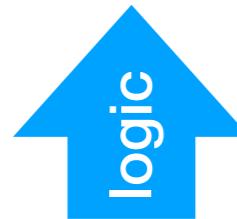
lambda abstraction

concise formula that can cover
many concrete cases

different proof for general case

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<- one abstract representation



<- abstraction using expressive logic

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<- many concrete cases

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

universal quantifier

lambda abstraction

concise formula that can cover
many concrete cases

different proof for general case

A small data set is not a failure
but an achievement!

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<- one abstract representation



<- abstraction using expressive logic

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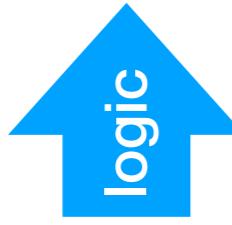
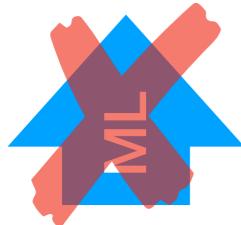
<- many concrete cases

Grand Challenge: Abstract Abstraction

Lemma "itrev xs ys = rev xs @ ys"

by(induct xs ys rule:"itrev.induct") auto

<- one abstract representation



<- abstraction using expressive logic

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<- many concrete cases

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 **Lemma** "star r x y \Rightarrow star r y z \Rightarrow star r x z"
by(induction rule: star.induct)(auto simp: step)

 **lemma** "exec (is1 @ is2) s stk =
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← small dataset about
different domains

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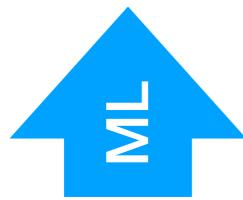


← abstraction using expressive logic

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Grand Challenge: Abstract Abstraction



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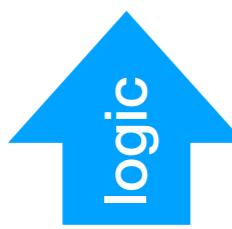
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<- small dataset about different domains



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<- abstraction using expressive logic

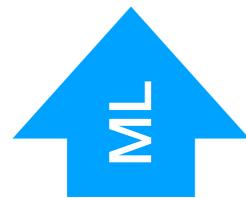
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<- many concrete cases

Grand Challenge: Abstract Abstraction

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract



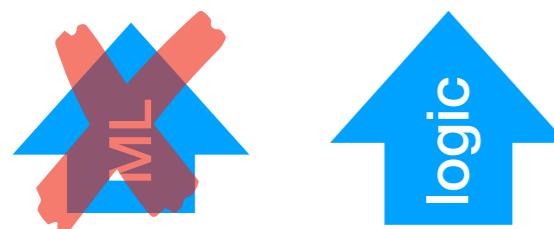
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<- abstraction using expressive logic

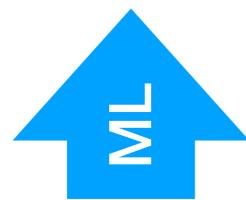
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← pros: good at ambiguity (heuristics)



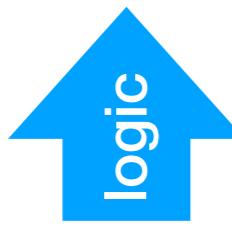
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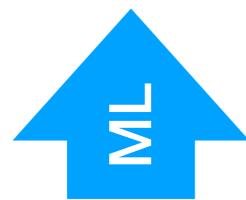
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Grand Challenge: Abstract Abstraction

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

← even more abstract



← pros: good at ambiguity (heuristics)



← cons: bad at reasoning & abstraction



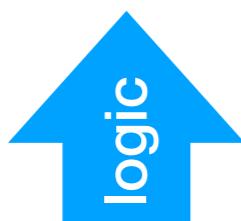
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← many concrete cases

Many key challenges remain

Unsupervised Learning

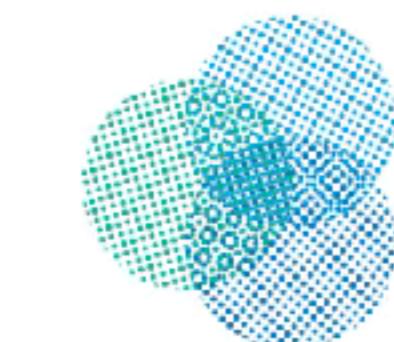
Memory and one-shot learning

Imagination-based Planning with
Generative Models

Learning Abstract Concepts

Transfer Learning

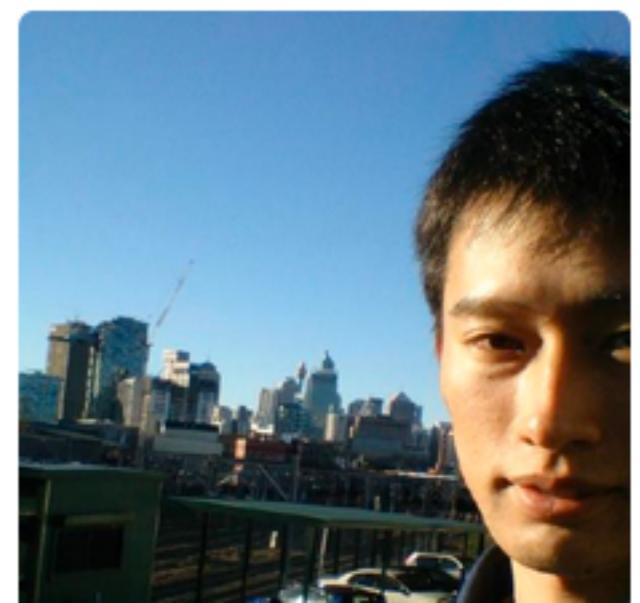
Language understanding



CENTER FOR
Brains
Minds+
Machines

March 20, 2019

The Power of
Self



Many key challenges remain

Unsupervised Learning

Memory and one-shot learning

Imagination-based Planning with
Generative Models

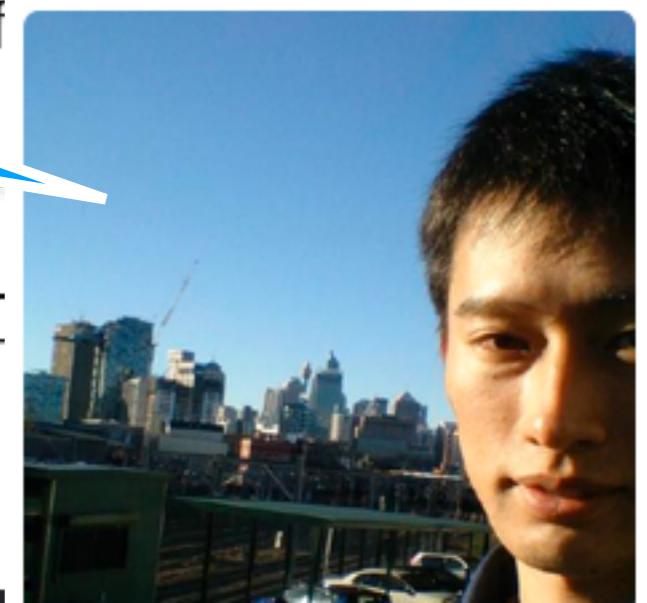
Learning Abstract Concepts

Abstract concepts?



March 20, 2019

The Power of
Self



Many key challenges remain

Unsupervised Learning

Memory and one-shot learning

Imagination-based Planning with
Generative Models

Learning Abstract Concepts

Abstract concepts?

Why not logic?
(LiFtEr = Logical Feature Extraction)



March 20, 2019

The Power of
Self



Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

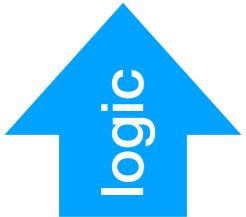
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← small dataset about
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← one abstract representation



← abstraction using expressive logic

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Abstract notion of “good” application of induction.
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Big Picture

Lifter

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logic

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← many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

Lifter

← pros: good at rigorous abstraction



 **Lemma** "star r x y \Rightarrow star r y z \Rightarrow star r x z"
by(induction rule: star.induct)(auto simp: step)

 **lemma** "exec (is1 @ is2) s stk =
exec is2 s (exec is1 s stk)"
by(induct is1 s stk rule:exec.induct) auto

← small dataset about
different domains

 **Lemma** "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto

← one abstract representation

logic

← abstraction using expressive logic

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Abstract notion of “good” application of induction.
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Big Picture

ML

Lifter

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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

ML

← pros: good at ambiguity (heuristics)



Lifter

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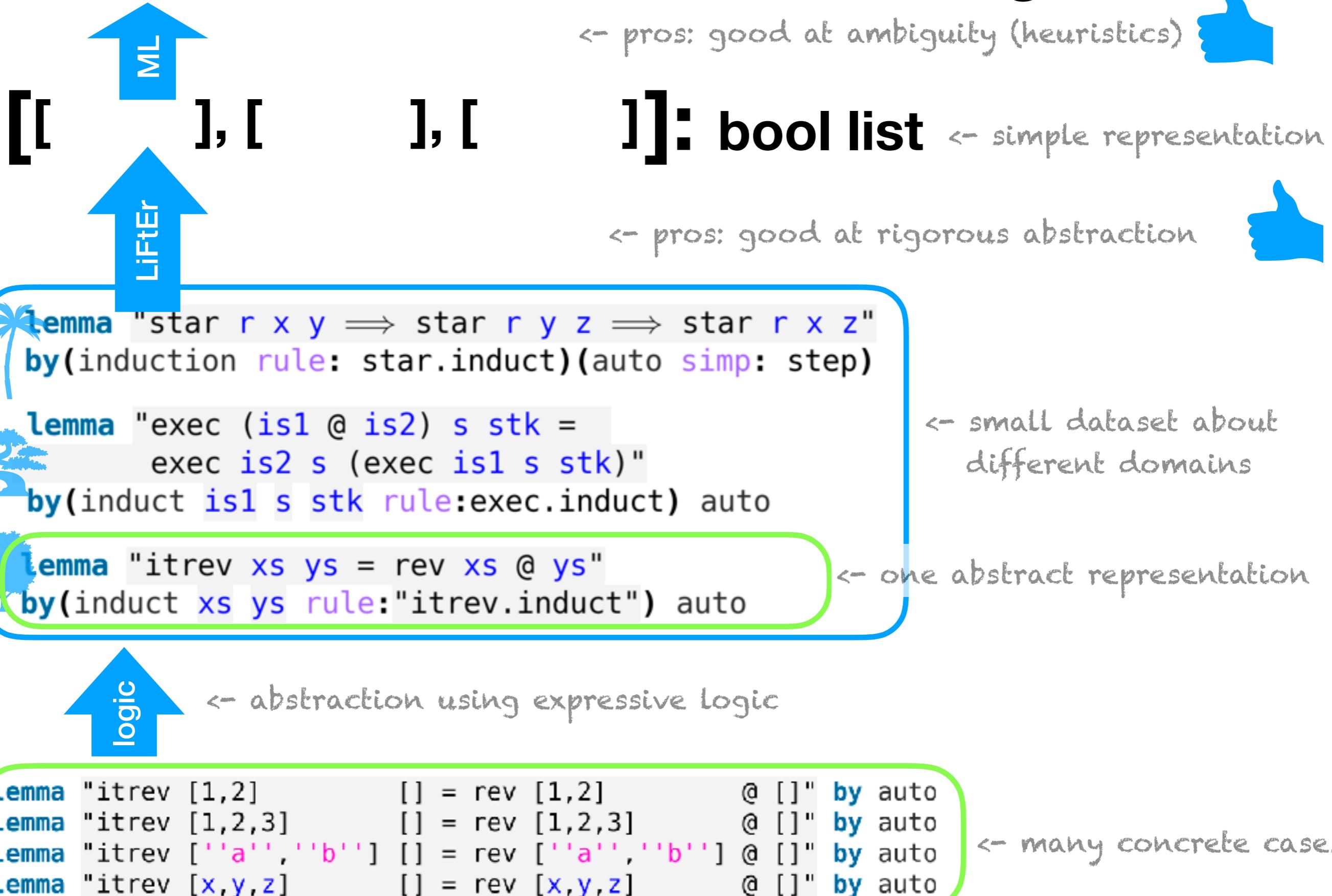
← abstraction using expressive logic

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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture



Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture



[[] , [] , []] : bool list

<- pros: good at ambiguity (heuristics)



]]: bool list

<- simple representation



<- pros: good at rigorous abstraction



```
lemma "star r x y ==> star r y z ==> star r x z"  
by(induction rule: star.induct)(auto simp: step)
```

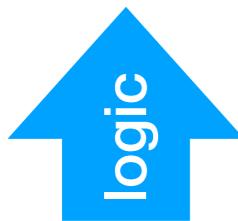
<- small dataset about different domains

```
lemma "exec (is1 @ is2) s stk =  
      exec is2 s (exec is1 s stk)"  
by(induct is1 s stk rule:exec.induct) auto
```



```
lemma "itrev xs ys = rev xs @ ys"  
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



<- abstraction using expressive logic

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lemma "itrev [x,y,z] [] = rev [x,y,z] @ []" by auto
```

<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

ML

[[], []]

LiftEr¹

<- pros: good at ambiguity (heuristics)



]]: bool list

<- simple representation



<- pros: good at rigorous abstraction

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<- small dataset about
different domains

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<- one abstract representation

logic

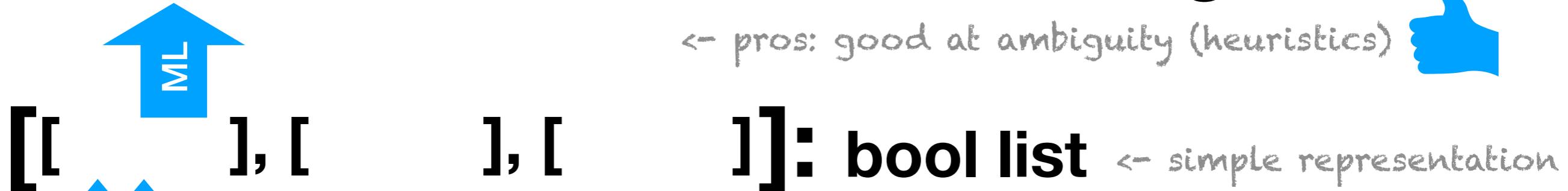
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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture



Lemma "star r x y \Rightarrow star r y z \Rightarrow star r x z"
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← pros: good at rigorous abstraction

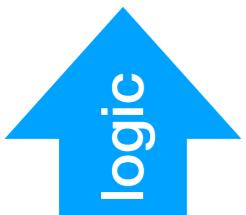


lemma "exec (is1 @ is2) s stk =
exec is2 s (exec is1 s stk)"
by(induct is1 s stk rule:exec.induct) auto

← small dataset about
different domains

lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto

← one abstract representation



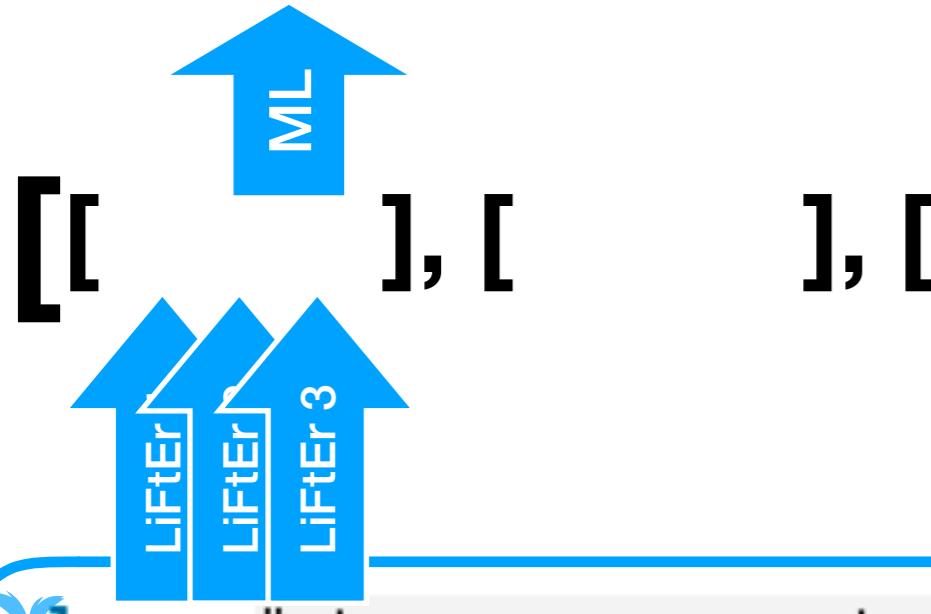
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← many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture



← pros: good at ambiguity (heuristics)



← simple representation

← pros: good at rigorous abstraction



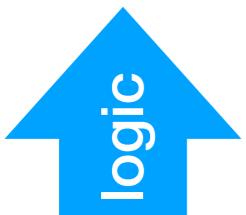
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different domains

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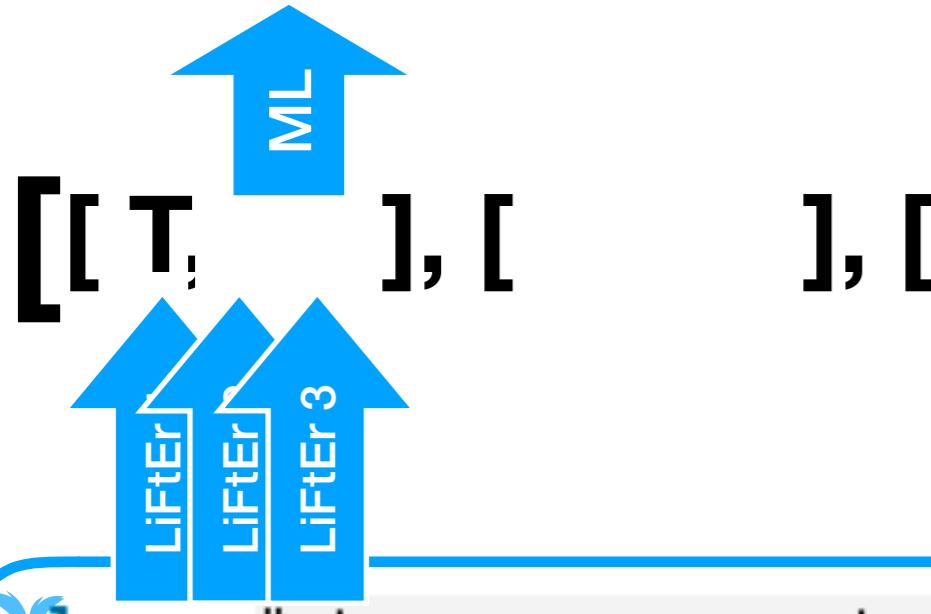
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← many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture



`[[T,], [], []]]: bool list` \leftarrow simple representation

\leftarrow pros: good at rigorous abstraction



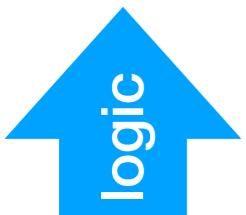
 **Lemma** "star r x y \Rightarrow star r y z \Rightarrow star r x z"
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\leftarrow small dataset about different domains

 **lemma** "exec (is1 @ is2) s stk =
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\leftarrow one abstract representation

 **Lemma** "itrev xs ys = rev xs @ ys"
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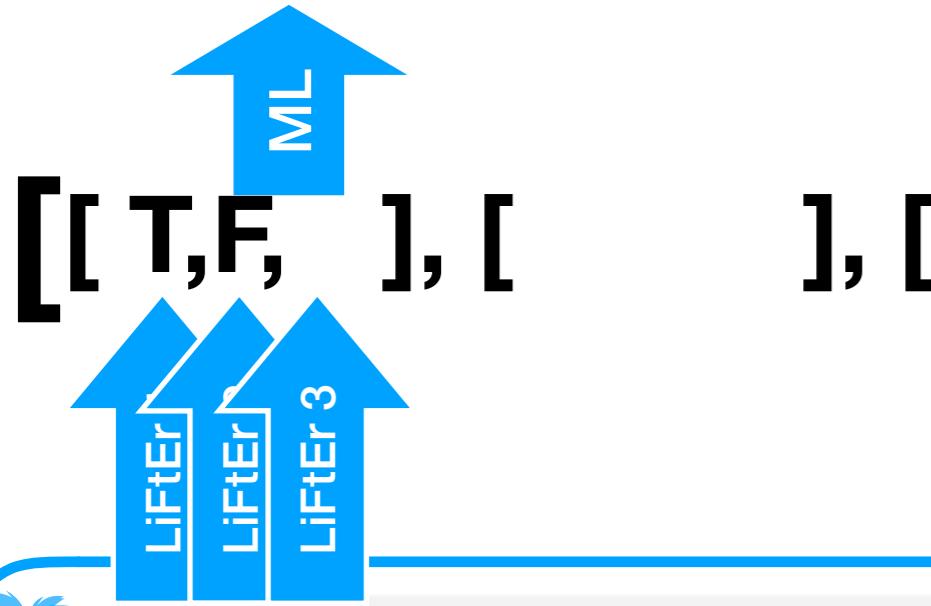
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\leftarrow many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture



← pros: good at ambiguity (heuristics)



[[T,F,], []], []]: bool list

← simple representation

← pros: good at rigorous abstraction



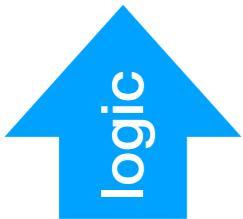
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← abstraction using expressive logic

lemma "itrev [1,2] [] = rev [1,2] @ []" **by** auto
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← many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,FT], []], []]: bool list

<- pros: good at ambiguity (heuristics)



LiftEr
LiftEr
LiftEr 3

<- simple representation



<- pros: good at rigorous abstraction

Lemma "star r x y \Rightarrow star r y z \Rightarrow star r x z"
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<- small dataset about different domains

lemma "exec (is1 @ is2) s stk =
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Lemma "itrev xs ys = rev xs @ ys"
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logic

<- abstraction using expressive logic

lemma "itrev [1,2] [] = rev [1,2] @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

 [[T,F,T], [T,T,T], [

← pros: good at ambiguity (heuristics)



]]: bool list

← simple representation

 **Lemma** "star r y z == star r x z"
by(induction r) star.induct)(auto simp: step)

← pros: good at rigorous abstraction

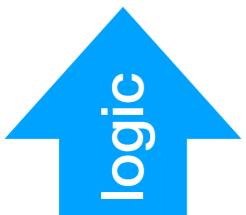


 **lemma** "exec (is1 @ is2) s stk =
exec is2 s (exec is1 s stk)"
by(induct is1 s stk rule:exec.induct) auto

← small dataset about
different domains

 **Lemma** "itrev xs ys = rev xs @ ys"
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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

ML

[[T,F,T], [T,T,T], [F,T,T]]: bool list

← pros: good at ambiguity (heuristics)



← simple representation

LiftEr
LiftEr
LiftEr 3
LiftEr 1
LiftEr 2
LiftEr 3
star r
ar.in
star r x z
simp: step

← pros: good at rigorous abstraction



Lemma "star r by(induction r)"
lemma "exec (is1 @ is2) s exec is2 s (exec is1 s) suct) auto
by(induct is1 s stk rule:e)

← small dataset about different domains

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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

ML

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← pros: good at ambiguity (heuristics)



← simple representation



← pros: good at rigorous abstraction

Lemma "star r
by(induction r)"

LiftEr 1
LiftEr 2
LiftEr 3

LiftEr 1
LiftEr 2
LiftEr 3

LiftEr 1
LiftEr 2
LiftEr 3

star r x z"
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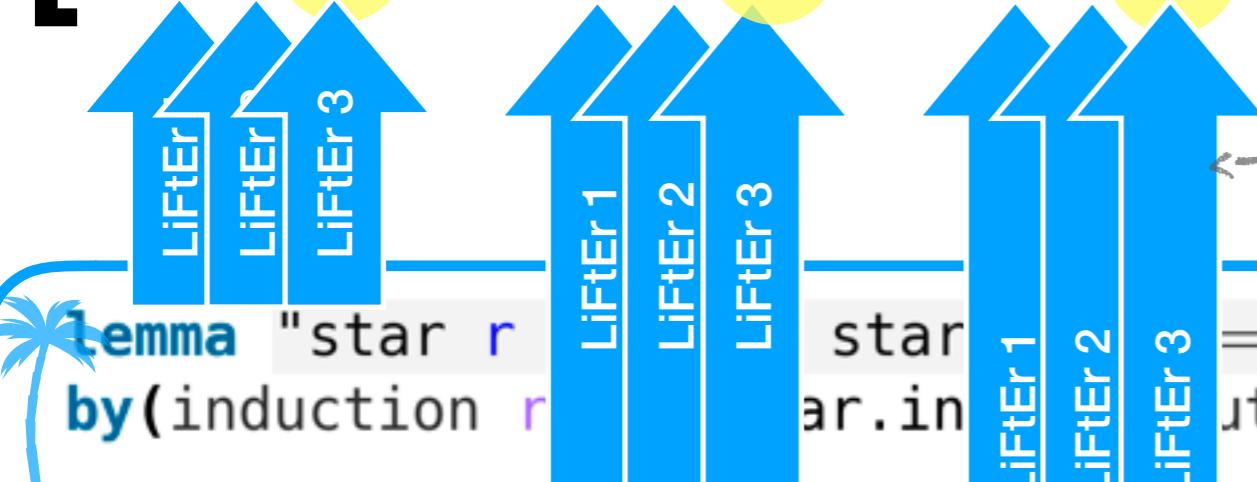
Big Picture

ML

← pros: good at ambiguity (heuristics)

[[T,F,T], [T,T,T], [F,T,T]]: bool list

← simple representation



← pros: good at rigorous abstraction

Lemma "star r
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← small dataset about different domains

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← many concrete cases

Abstract Syntax of LiFtEr

```
assertion := atomic | connective | quantifier | ( assertion )
type := term | term_occurrence | rule | number
modifier_term := induction_term | arbitrary_term
quantifier :=  $\exists x : type . assertion$ 
             |  $\forall x : type . assertion$ 
             |  $\exists x : term \in modifier\_term . assertion$ 
             |  $\forall x : term \in modifier\_term . assertion$ 
             |  $\exists x : rule . assertion$ 
             |  $\exists x : term\_occurrence \in y : term . assertion$ 
             |  $\forall x : term\_occurrence \in y : term . assertion$ 
connective := True | False | assertion  $\vee$  assertion | assertion  $\wedge$  assertion
            | assertion  $\rightarrow$  assertion |  $\neg$  assertion
pattern := all_only_var | all_constructor | mixed
```

atomic :=

```
rule is_rule_of term_occurrence
| term_occurrence term_occurrence_is_of_term term
| are_same_term ( term_occurrence , term_occurrence )
| term_occurrence is_in_term_occurrence term_occurrence
| is_atomic term_occurrence
| is_constant term_occurrence
| is_recursive_constant term_occurrence
| is_variable term_occurrence
| is_free_variable term_occurrence
| is_bound_variable term_occurrence
| is_lambda term_occurrence
| is_application term_occurrence
| term_occurrence is_an_argument_of term_occurrence
| term_occurrence is_nth_argument_of term_occurrence
```

Abstract Syntax of LiFtEr

assertion := atomic | connective | quantifier | (assertion)

type := term | term_occurrence | rule | number

modifier_term := induction_term | arbitrary_term

quantifier := $\exists x : type . assertion$

| $\forall x : type . assertion$

| $\exists x : term \in modifier_term . assertion$

| $\forall x : term \in modifier_term . assertion$

| $\exists x : rule . assertion$

| $\exists x : term_occurrence \in y : term . assertion$

| $\forall x : term_occurrence \in y : term . assertion$

connective := True | False | assertion \vee assertion

 | assertion \rightarrow assertion | \neg assertion

pattern := all_only_var | all_constructor

atomic :=

 rule is_rule_of term_occurrence

 | term_occurrence term_occurrence term_occurrence

 | are_same_term (term_occurrence , term_occurrence)

 | term_occurrence is_in_term_occurrence term_occurrence

 | is_atomic term_occurrence

 | is_constant term_occurrence

 | is_recursive_constant term_occurrence

 | is_variable term_occurrence

 | is_free_variable term_occurrence

 | is_bound_variable term_occurrence

 | is_lambda term_occurrence

 | is_application term_occurrence

 | term_occurrence is_an_argument_of term_occurrence

 | term_occurrence is_nth_argument_of term_occurrence

Example Assertion in LiFtEr (in Abstract Syntax)

```
∃ r1 : rule. True
→
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
      ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
            ∧
            t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication



```
→ ∃ r1 : rule. True  
  ∃ r1 : rule.  
    ∃ t1 : term.  
      ∃ to1 : term_occurrence ∈ t1 : term.  
        r1 is_rule_of to1  
        ∧  
        ∀ t2 : term ∈ induction_term.  
          ∃ to2 : term_occurrence ∈ t2 : term.  
            ∃ n : number.  
              is_nth_argument_of (to2, n, to1)  
              ∧  
              t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication

$$\exists r1 : \text{rule}. \text{True} \rightarrow \exists r1 : \text{rule}. \exists t1 : \text{term}. \exists to1 : \text{term_occurrence} \in t1 : \text{term}. r1 \text{ is_rule_of } to1 \wedge \forall t2 : \text{term} \in \text{induction_term}. \exists to2 : \text{term_occurrence} \in t2 : \text{term}. \exists n : \text{number}. \text{is_nth_argument_of} (to2, n, to1) \wedge t2 \text{ is_nth_induction_term } n$$

Example Assertion in LiFtEr (in Abstract Syntax)

implication

↓
→ $\exists r1 : \text{rule}. \text{True}$ variable for auxiliary lemmas

$\exists r1 : \text{rule}.$ ←

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge ← conjunction

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of } (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

Example Assertion in LiFtEr (in Abstract Syntax)

implication

↓
 $\exists r1 : \text{rule}. \text{True}$ → variable for auxiliary lemmas
 $\exists r1 : \text{rule}.$ ←
 $\exists t1 : \text{term}.$ ← variable for terms
 $\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$
 $r1 \text{ is_rule_of } to1$
 \wedge ← conjunction
 $\forall t2 : \text{term} \in \text{induction_term}.$
 $\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$
 $\exists n : \text{number}.$
 $\text{is_nth_argument_of } (to2, n, to1)$
 \wedge
 $t2 \text{ is_nth_induction_term } n$

Example Assertion in LiFtEr (in Abstract Syntax)

implication

↓
 $\exists r1 : \text{rule}. \text{True}$ → variable for auxiliary lemmas
 $\exists r1 : \text{rule}.$ ←
 $\exists t1 : \text{term}.$ ← variable for terms
 $\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$ ← variable for term occurrences
 $r1 \text{ is_rule_of } to1$ ← conjunction
 \wedge ←
 $\forall t2 : \text{term} \in \text{induction_term}.$
 $\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$
 $\exists n : \text{number}.$
 $\text{is_nth_argument_of } (to2, n, to1)$
 \wedge
 $t2 \text{ is_nth_induction_term } n$

Example Assertion in LiFtEr (in Abstract Syntax)

implication

↓
 $\exists r1 : \text{rule}. \text{True}$ → variable for auxiliary lemmas
→
 $\exists r1 : \text{rule}.$ ←
 $\exists t1 : \text{term}.$ ← variable for terms
 $\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$ ← variable for term occurrences
 $r1 \text{ is_rule_of } to1$ ←
Λ ← conjunction
 $\forall t2 : \text{term} \in \text{induction_term}.$
 $\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$
 $\exists n : \text{number}.$ ← variable for natural numbers
 $\text{is_nth_argument_of } (to2, n, to1)$
Λ
 $t2 \text{ is_nth_induction_term } n$

Example Assertion in LiFtEr (in Abstract Syntax)

implication

↓
 $\exists r1 : \text{rule}. \text{True}$ → variable for auxiliary lemmas
↳
 $\exists r1 : \text{rule}.$ ↳
 $\exists t1 : \text{term}.$ ↳ variable for terms
 $\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$ ↳ variable for term occurrences
 $r1 \text{ is_rule_of } to1$ ↳ conjunction
 \wedge ↳
 $\forall t2 : \text{term} \in \text{induction_term}.$
 $\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$
 $\exists n : \text{number}.$ ↳ variable for natural numbers
 $\text{is_nth_argument_of } (to2, n, to1)$
 \wedge
 $t2 \text{ is_nth_induction_term } n$

universal quantifier

Example Assertion in LiFtEr (in Abstract Syntax)

implication existential quantifier

$\exists r1 : \text{rule}. \text{True}$ variable for auxiliary lemmas

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$ variable for terms

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$ variable for term occurrences

$r1 \text{ is_rule_of } to1$

\wedge conjunction

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$ variable for natural numbers

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

Example Assertion in LiFtEr (in Abstract Syntax)

LiFtEr assertion: (proof goal * induction arguments) -> bool

implication existential quantifier
↓ ↓
 $\exists r1 : \text{rule}. \text{True}$ variable for auxiliary lemmas
→
 $\exists r1 : \text{rule}.$ ← variable for terms
 $\exists t1 : \text{term}.$ ← variable for term occurrences
 $\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$ ← variable for natural numbers
 $r1 \text{ is_rule_of } to1$ conjunction
 \wedge universal quantifier
 $\forall t2 : \text{term} \in \text{induction_term}.$
 $\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$
 $\exists n : \text{number}.$ ← variable for induction arguments
 $\text{is_nth_argument_of} (to2, n, to1)$
 \wedge
 $t2 \text{ is_nth_induction_term } n$

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)
```

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

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```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
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fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

→

$\exists r1 : \text{rule}.$

($r1 = \text{itrev.induct}$)

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

r1

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

($r1 = \text{itrev.induct}$)

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

r1

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys"
  "itrev (x#xs) ys" = itrev xs (x#ys)"
```

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

($r1 = \text{itrev.induct}$)

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

$r1$

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```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

r1

(r1 = itrev.induct)

(t1 = itrev)

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```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]
```

```
fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
```

```
apply(induct xs ys rule:"itrev.induct")
apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

($r1 = \text{itrev.induct}$)
($t1 = \text{itrev}$)
($to1 = \text{itrev}$)

r1

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]
```

```
fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
```

```
apply(induct xs ys rule:"itrev.induct")
apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

($r1 = \text{itrev.induct}$)

($t1 = \text{itrev}$)

($to1 = \text{itrev}$)

r1

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
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  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

($r1 = \text{itrev.induct}$)

$\exists t1 : \text{term}.$

($t1 = \text{itrev}$)

$\exists tol : \text{term_occurrence} \in t1 : \text{term}.$

($tol = \text{itrev}$)

r1 is_rule_of tol True! $r1 (= \text{itrev.induct})$ is a lemma about $tol (= \text{itrev})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of}(to2, n, tol)$

\wedge

$t2 \text{ is_nth_induction_term } n$

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

($r1 = \text{itrev.induct}$)

$\exists t1 : \text{term}.$

($t1 = \text{itrev}$)

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

($to1 = \text{itrev}$)

 $r1 \text{ is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

($r1 = \text{itrev.induct}$)

$\exists t1 : \text{term}.$

($t1 = \text{itrev}$)

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

($to1 = \text{itrev}$)

$r1 \text{is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{is_nth_induction_term } n$

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys"
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

($r1 = \text{itrev.induct}$)

($t1 = \text{itrev}$)

($to1 = \text{itrev}$)

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{is_rule_of } to1 \quad \text{True! } r1 (= \text{itrev.induct}) \text{ is a lemma about } to1 (= \text{itrev}).$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of } (to2, n, to1)$

\wedge

$t2 \text{is_nth_induction_term } n$

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys"
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$(t2 = xs \text{ and } ys)$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]
```

```
fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys |
  "itrev (x#xs) ys" = itrev xs (x#ys)"
```

to1

to2

lemma "itrev xs ys = rev xs @ ys"

apply(induct xs ys rule:"itrev.induct")
apply auto done

t2

r1

$\exists r1 : \text{rule}. \text{True}$

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.

^

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

^

$t2 \text{ is_nth_induction_term } n$

($r1 = \text{itrev.induct}$)

($t1 = \text{itrev}$)

($to1 = \text{itrev}$)

($t2 = \text{xs and ys}$)

($to2 = \text{xs and ys}$)

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```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys"
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.



$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

($r1 = \text{itrev.induct}$)

($t1 = \text{itrev}$)

($to1 = \text{itrev}$)

($t2 = \text{xs and ys}$)

($to2 = \text{xs and ys}$)

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```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys"
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

($r1 = \text{itrev.induct}$)

($t1 = \text{itrev}$)

($to1 = \text{itrev}$)

($t2 = \text{xs and ys}$)

($to2 = \text{xs and ys}$)

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```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys"
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r_1 : \text{rule}. \text{True}$

\rightarrow

$\exists r_1 : \text{rule}.$

$\exists t_1 : \text{term}.$

$\exists t_{01} : \text{term_occurrence} \in t_1 : \text{term}.$

$r_1 \text{ is_rule_of } t_{01}$ True! $r_1 (= \text{itrev.induct})$ is a lemma about $t_{01} (= \text{itrev})$.

\wedge

$\forall t_2 : \text{term} \in \text{induction_term}.$

$\exists t_{02} : \text{term_occurrence} \in t_2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of } (t_{02}, n, t_{01})$

\wedge

$t_2 \text{ is_nth_induction_term } n$

($r_1 = \text{itrev.induct}$)

($t_1 = \text{itrev}$)

($t_{01} = \text{itrev}$)

($t_2 = \text{xs}$ and ys)

($t_{02} = \text{xs}$ and ys)

when t_2 is xs ($n = 1$)?

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```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys" = ys"
  "itrev (x#xs) ys" = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r_1 : \text{rule}. \text{True}$

\rightarrow

$\exists r_1 : \text{rule}.$

$\exists t_1 : \text{term}.$

$\exists t_{01} : \text{term_occurrence} \in t_1 : \text{term}.$

$r_1 \text{ is_rule_of } t_{01}$ True! $r_1 (= \text{itrev.induct})$ is a lemma about $t_{01} (= \text{itrev})$.

\wedge

$\forall t_2 : \text{term} \in \text{induction_term}.$

$\exists t_{02} : \text{term_occurrence} \in t_2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of } (t_{02}, n, t_{01})$

\wedge

$t_2 \text{ is_nth_induction_term } n$

t_{01}

first

t_{02}

t_2

first

r_1

($r_1 = \text{itrev.induct}$)

($t_1 = \text{itrev}$)

($t_{01} = \text{itrev}$)

($t_2 = \text{xs}$ and ys)

($t_{02} = \text{xs}$ and ys)

when t_2 is xs ($n = 1$)

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []      = []" |
  "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev []      ys = ys" |
  "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

($r1 = \text{itrev.induct}$)

($t1 = \text{itrev}$)

($to1 = \text{itrev}$)

($t2 = xs \text{ and } ys$)

($to2 = xs \text{ and } ys$)

when $t2$ is xs ($n = 1$)

when $t2$ is ys ($n = 2$) ?

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []      = []" |
  "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev []      ys = ys" |
  "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{itrev.induct})$ is a lemma about $to1 (= \text{itrev})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

($r1 = \text{itrev.induct}$)

($t1 = \text{itrev}$)

($to1 = \text{itrev}$)

($t2 = xs$ and ys)

($to2 = xs$ and ys)

when $t2$ is xs ($n = 1$)

when $t2$ is ys ($n = 2$)

<https://twitter.com/YutakangE>

```
primrec rev :: "'a list ⇒ 'a list" where
  "rev []"      = []
  "rev (x # xs)" = rev xs @ [x]
```

```
fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
```

```
  "itrev [] ys" = ys |
```

```
  "itrev (x#xs) ys" = itrev xs (x#ys)"
```

t_{01} → t_{02}

```
lemma "itrev xs ys = rev xs @ ys"
```

```
apply(induct xs ys rule:"itrev.induct")
```

```
apply auto done
```

t_2 → t_1
first → second
second → first

r_1

$\exists r_1 : \text{rule}. \text{True}$

→

$\exists r_1 : \text{rule}.$

$\exists t_1 : \text{term}.$

$\exists t_{01} : \text{term_occurrence} \in t_1 : \text{term}.$

$r_1 \text{ is_rule_of } t_{01}$ True! $r_1 (= \text{itrev.induct})$ is a lemma about $t_{01} (= \text{itrev})$.

^

$\forall t_2 : \text{term} \in \text{induction_term}.$

$\exists t_{02} : \text{term_occurrence} \in t_2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (t_{02}, n, t_{01})$

^

$t_2 \text{ is_nth_induction_term } n$

($r_1 = \text{itrev.induct}$)

($t_1 = \text{itrev}$)

($t_{01} = \text{itrev}$)

($t_2 = xs$ and ys)

($t_{02} = xs$ and ys)

when t_2 is xs ($n = 1$)



when t_2 is ys ($n = 2$)



```
 $\exists r1 : \text{rule}. \text{True}$ 
```

\rightarrow

```
 $\exists r1 : \text{rule}.$ 
```

```
 $\exists t1 : \text{term}.$ 
```

```
 $\exists to1 : \text{term\_occurrence} \in t1 : \text{term}.$ 
```

```
 $r1 \text{ is\_rule\_of } to1$ 
```

\wedge

```
 $\forall t2 : \text{term} \in \text{induction\_term}.$ 
```

```
 $\exists to2 : \text{term\_occurrence} \in t2 : \text{term}.$ 
```

```
 $\exists n : \text{number}.$ 
```

```
 $\text{is\_nth\_argument\_of} (to2, n, to1)$ 
```

\wedge

```
 $t2 \text{ is\_nth\_induction\_term } n$ 
```

the same LIFTER assertion



$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

datatype instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

the same LIFTER assertion



$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype      instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

the same LIFTER assertion

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```



$\exists r1 : \text{rule}. \text{True}$

\rightarrow

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
  "exec1 (LOADI n) s stk = n # stk" |  
  "exec1 (LOAD x) s stk = s(x) # stk" |  
  "exec1 ADD s (j#i#stk) = (i + j) # stk"
```

the same LIFTER assertion

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
  "exec [] s stk = stk" |  
  "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

↓
new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
a model proof -> apply(induct is1 s stk rule:exec.induct)
 $\exists r1 : \text{rule}. \text{True}$ apply auto done

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule. } \text{True }$ apply auto done

r1

→

$\exists r1 : \text{rule. } (\text{r1} = \text{exec.induct})$

$\exists t1 : \text{term. }$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term. }$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term. }$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term. }$

$\exists n : \text{number. }$

$\text{is_nth_argument_of } (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

r1

$\rightarrow \exists r1 : \text{rule}.$

(r1 = exec.induct)

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

r1

$\rightarrow \exists r1 : \text{rule}.$

(r1 = exec.induct)

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

r1

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

(r1 = exec.induct)
(t1 = exec)

r1 is_rule_of to1

^

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

is_nth_argument_of (to2, n, to1)

^

t2 is_nth_induction_term n

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

to1

r1

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

$r1 \text{ is_rule_of } to1$

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

to1

r1

→

$\exists r1 : \text{rule}.$

(r1 = exec.induct)

$\exists t1 : \text{term}.$

(t1 = exec)

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$

(to1 = exec)

$r1 \text{ is_rule_of } to1$

^

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

^

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

to1

r1

$\rightarrow \exists r1 : \text{rule}.$

($r1 = \text{exec.induct}$)

($t1 = \text{exec}$)

($to1 = \text{exec}$)

$\exists t1 : \text{term}.$

True! $r1 (= \text{exec.induct})$ is a lemma about $to1 (= \text{exec})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of}(to2, n, to1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

to1

r1

$\rightarrow \exists r1 : \text{rule}.$

($r1 = \text{exec.induct}$)

($t1 = \text{exec}$)

$\exists t1 : \text{term}.$ ($t1 = \text{exec}$)

$r1 \text{ is_rule_of } t1 \quad \text{True! } r1 (= \text{exec.induct}) \text{ is a lemma about } t1 (= \text{exec}).$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists t2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (t2, n, t1)$

\wedge

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

to1

r1

$\exists r1 : \text{rule}.$

(r1 = exec.induct)

(t1 = exec)

$\exists t1 : \text{term}.$ (t1 = exec)

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}.$ (to1 = exec)



r1 is_rule_of to1 True! r1 (= exec.induct) is a lemma about to1 (= exec).

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

is_nth_argument_of (to2, n, to1)

\wedge

t2 is_nth_induction_term n

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> apply(induct is1 s stk rule:exec.induct)

$\exists r1 : \text{rule}. \text{True}$ apply auto done

to1

r1

$\exists r1 : \text{rule}.$

($r1 = \text{exec.induct}$)

($t1 = \text{exec}$)

$\exists t1 : \text{term}.$ ($t1 = \text{exec}$)

$r1 \text{ is_rule_of } t1$ True! $r1 (= \text{exec.induct})$ is a lemma about $t1 (= \text{exec})$.

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists t2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

is_nth_argument_of (t2, n, to1)

\wedge

t2 is_nth_induction_term n

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{True}$ **apply auto done**

b2

to1

r1

$\exists r1 : \text{rule}.$

(r1 = exec.induct)

(t1 = exec)

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}. (to1 = exec)$

r1 is_rule_of to1 True! r1 (= exec.induct) is a lemma about to1 (= exec).

^

$\forall t2 : \text{term} \in \text{induction_term}.$

(t2 = is1, s, and stk)

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}.$

$\exists n : \text{number}.$

is_nth_argument_of (to2, n, to1)

^

t2 is_nth_induction_term n

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
a model proof -> **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{True}$ **apply auto done**

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}. \quad (to1 = \text{exec})$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{exec.induct})$ is a lemma about $to1 (= \text{exec})$.

^

$\forall t2 : \text{term} \in \text{induction_term}. \quad (t2 = \text{is1, s, and stk})$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}. \quad (to2 = \text{is1, s, and stk})$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$

^

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{True}$ **apply auto done**



$\rightarrow \exists r1 : \text{rule}.$

($r1 = \text{exec.induct}$)

($t1 = \text{exec}$)

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}. (to1 = \text{exec})$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{exec.induct})$ is a lemma about $to1 (= \text{exec})$.



$\forall t2 : \text{term} \in \text{induction_term}. (t2 = \text{is1, s, and stk})$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}. (to2 = \text{is1, s, and stk})$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$



$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{True}$ **apply auto done**



$\rightarrow \exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}. \quad (to1 = \text{exec})$

$r1 \text{ is_rule_of } to1 \quad \text{True! } r1 (= \text{exec.induct}) \text{ is a lemma about } to1 (= \text{exec}).$



$\wedge \forall t2 : \text{term} \in \text{induction_term}.$

$(t2 = \text{is1, s, and stk})$

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}. \quad (to2 = \text{is1, s, and stk})$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (to2, n, to1)$



$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{True}$

apply auto done



$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists t01 : \text{term_occurrence} \in t1 : \text{term}. \quad (t01 = \text{exec})$

$r1 \text{ is_rule_of } t01 \quad \text{True! } r1 (= \text{exec.induct}) \text{ is a lemma about } t01 (= \text{exec}).$



$\forall t2 : \text{term} \in \text{induction_term}.$

$(t2 = \text{is1, s, and stk})$

$\exists t02 : \text{term_occurrence} \in t2 : \text{term}. \quad (t02 = \text{is1, s, and stk})$

$\exists n : \text{number}.$

$\text{is_nth_argument_of } (t02, n, t01) \text{ when } t2 \text{ is } \text{is1} (n \rightarrow 1) ?$



$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> ~~lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"~~

a model proof -> ~~apply(induct is1 s stk rule:exec.induct)~~

$\exists r1 : \text{rule}. \text{True}$

apply auto done



$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists t01 : \text{term_occurrence} \in t1 : \text{term}. \quad (\text{t01} = \text{exec})$

$r1 \text{ is_rule_of } t01 \quad \text{True! } r1 (= \text{exec.induct}) \text{ is a lemma about } t01 (= \text{exec}).$



$\forall t2 : \text{term} \in \text{induction_term}.$

$\exists t02 : \text{term_occurrence} \in t2 : \text{term}. \quad (\text{t02} = \text{is1, s, and stk})$

$\exists n : \text{number}.$

$\text{is_nth_argument_of } (t02, n, t01) \text{ when } t2 \text{ is } \text{is1} (n \rightarrow 1)$



$t2 \text{ is_nth_induction_term } n$



new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{True}$

apply auto done



$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists to1 : \text{term_occurrence} \in t1 : \text{term}. (to1 = \text{exec})$

$r1 \text{ is_rule_of } to1$ True! $r1 (= \text{exec.induct})$ is a lemma about $to1 (= \text{exec})$.



first
second (r1 = exec.induct)

(t1 = exec)

(to1 = exec)

$\forall t2 : \text{term} \in \text{induction_term}.$

(t2 = is1, s, and stk)

$\exists to2 : \text{term_occurrence} \in t2 : \text{term}. (to2 = is1, s, and stk)$

$\exists n : \text{number}.$

$\text{is_nth_argument_of } (to2, n, to1)$

when $t2$ is $is1$ ($n \rightarrow 1$)



when $t2$ is s ($n \rightarrow 2$) ?

$t2 \text{ is_nth_induction_term } n$

new types ->

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where  
"exec1 (LOADI n) _ stk = n # stk" |  
"exec1 (LOAD x) s stk = s(x) # stk" |  
"exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

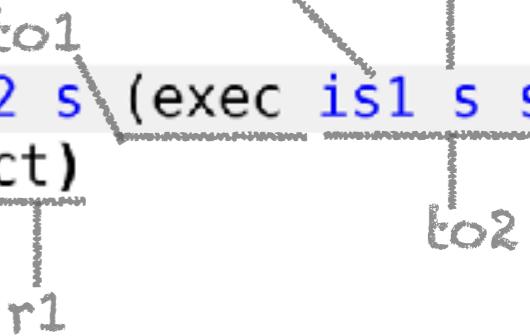
```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where  
"exec [] _ stk = stk" |  
"exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma -> **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof -> **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{True}$

apply auto done



$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists t01 : \text{term_occurrence} \in t1 : \text{term}. (t01 = \text{exec})$

$r1 \text{ is_rule_of } t01$ True! $r1 (= \text{exec.induct})$ is a lemma about $t01 (= \text{exec})$.



first
 $\text{second} (r1 = \text{exec.induct})$

$(t1 = \text{exec})$

$r1$

$t01$

$t02$

\wedge

$\forall t2 : \text{term} \in \text{induction_term}.$

$(t2 = \text{is1, s, and stk})$

$\exists t02 : \text{term_occurrence} \in t2 : \text{term}. (t02 = \text{is1, s, and stk})$

$\exists n : \text{number}.$

$\text{is_nth_argument_of} (t02, n, t01)$

when $t2$ is is1 ($n \rightarrow 1$)

\wedge

$t2 \text{ is_nth_induction_term } n$

when $t2$ is s ($n \rightarrow 2$)



new types \rightarrow

```
datatype      instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants \rightarrow

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
  "exec1 (LOADI n) _ stk = n # stk" |
  "exec1 (LOAD x) _ s stk = s(x) # stk" |
  "exec1 ADD (j#i#stk) = (i + j) # stk"
```

```

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
  "exec []      _ stk = stk" | first third
  "exec (i#is) s stk = exec is s (exec i s stk)" second
                                                is2 is1 stk
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

```

new lemma \rightarrow **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

a model proof → **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{ True}$

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists \text{tol} : \text{term_occurrence} \in t1 : \text{term}. \quad (\text{tol} = \text{exec})$

`r1 is_rule_of t01` True! `r1` (`= exec.induct`) is a lemma about `t01` (`= exec`).

8

$\forall t2 : \text{term} \in \text{induction_term}.$

(No = in 1, and etc)

$\exists \text{ } to2 : \text{term occurrence} \in t2 : \text{term}.$

(to2 = is1, s, and stk)

$\exists n : \text{number}.$

is_nth_argument_of (to2, n, to1)

△

t2 is_nth_induction_term *n*

when t_2 is $i \neq 1$ ($n \rightarrow 1$)

when t_2 is s ($n \rightarrow 2$)

when t_2 is stk ($n \rightarrow 3$) ?

new types →

```
datatype      instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants \rightarrow

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
  "exec1 (LOADI n) _ stk = n # stk" |
  "exec1 (LOAD x) _ s stk = s(x) # stk" |
  "exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
  "exec [] _ stk = stk" | first third
  "exec (i#is) s stk = exec is s (exec1 i s stk)" second
t2 t01
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"

```

~~new lemma \rightarrow lemma "exec (is1 @ is2) s\ stk = exec is2 s\ (exec is1 s\ stk)"~~

a model proof \rightarrow **apply(induct is1 s stk rule:exec.induct)**

$\exists r1 : \text{rule}. \text{ True}$

→

$\exists r1 : \text{rule}.$

$\exists t1 : \text{term}.$

$\exists \text{tol} : \text{term_occurrence} \in t1 : \text{term}. \quad (\text{tol} = \text{exec})$

`r1 is_rule_of t01` True! `r1` (`= exec.induct`) is a lemma about `t01` (`= exec`).

8

$\forall t2 : \text{term} \in \text{induction_term}.$

(E_2 = E_1 , ..., and E_k)

$\exists \text{ } to2 \text{ : term occurrence} \in t2 \text{ : term.}$

($t_{02} = is1$, s , and stk)

$\exists n : \text{number}.$

is_nth_argument_of (to2, n, to1)

when t_2 is i_1 ($n \rightarrow 1$)

△

t2 is_nth_induction_term *n*

when t_2 is s ($n \rightarrow 2$)

new types →

```
datatype instr = LOADI val | LOAD vname | ADD  
type_synonym stack = "val list"
```

new constants \rightarrow

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
  "exec1 (LOADI n) _ stk = n # stk" |
  "exec1 (LOAD x) _ stk = s(x) # stk" |
  "exec1 ADD _ (j#i#stk) = (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
  "exec [] _ stk = stk" | first
  "exec (i#is) s stk = exec is s (exec1 i s stk)" second
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

new lemma → **lemma** "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 a model proof → **apply(induct is1 s stk rule:exec.induct)**
 : rule. True **apply auto done**

 : rule.
 t1 : term.
 $\exists t01 : \text{term_occurrence} \in t1 : \text{term. } (t01 = \text{exec})$

`r1 is_rule_of tol` True! r1 (= exec.induct) is a lemma about tol (= exec).

\wedge
 $\forall t_2 : \text{term} \in \text{induction_term}. \quad (\text{t2} = \text{is1}, \text{s}, \text{and } \text{stk})$
 $\exists t_{o2} : \text{term_occurrence} \in t_2 : \text{term}. \quad (\text{to2} = \text{is1}, \text{s}, \text{and } \text{stk})$
 $\exists n : \text{number}.$

`is_nth_argument_of (to2, n, to1)`

when t_2 is $i \neq 1$ ($n \rightarrow 1$)

t2 is_nth_induction_term *n*

when t_2 is s ($n \rightarrow 2$)

**Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.**

Big Picture

The diagram illustrates the relationship between logic, abstraction, and ML, with examples of Isabelle proofs and their abstractions.

ML (top) is connected to **simple representation** (yellow circles) and **abstraction** (blue arrows).

simple representation (yellow circles) is connected to **Pros: good at ambiguity (heuristics)** (blue thumbs up) and **Cons: bad at abstraction** (red line through blue thumbs up).

abstraction (blue arrows) is connected to **Pros: good at rigorous abstraction** (blue thumbs up) and **Cons: bad at ambiguity (heuristics)** (red line through blue thumbs up).

logic (bottom) is connected to **Pros: good at abstraction using expressive logic** (blue thumbs up).

Isabelle Proofs:

- Lemma "star r by(induction r)"** (Lifter 1, Lifter 2, Lifter 3)
- Lemma "exec (is1 @ is2) s exec is2 s (exec is1 s)" by(induct is1 s stk rule:exec.induct)"** (Lifter 1, Lifter 2, Lifter 3)
- Lemma "itrev xs ys = rev xs @ ys" by(induct xs ys rule:"itrev.induct") auto** (Lifter 1, Lifter 2, Lifter 3)

```
lemma "itrev [1,2]      [] = rev [1,2]      @ []" by auto
lemma "itrev [1,2,3]    [] = rev [1,2,3]    @ []" by auto
lemma "itrev ['`a`', `b`] [] = rev ['`a`', `b`] @ []" by auto
lemma "itrev [x,y,z]    [] = rev [x,y,z]    @ []" by auto
```

← many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [F,T,T]]: bool list

WIP

← pros: good at ambiguity (heuristics)

LiftEr 1 LiftEr 2 LiftEr 3
LiftEr 1 LiftEr 2 LiftEr 3
LiftEr 1 LiftEr 2 LiftEr 3

Lemma "star r by(induction r)"

lemma "exec (is1 @ is2) s
exec is2 s (exec is1 s)"
by(induct is1 s stk rule:euct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto

← pros: good at rigor

← cons: bad at ambiguity (heuristics)



← abstraction using expressive logic

lemma "itrev [1,2] [] = rev [1,2] @ []" by auto
lemma "itrev [1,2,3] [] = rev [1,2,3] @ []" by auto
lemma "itrev [''a'', ''b''] [] = rev [''a'', ''b''] @ []" by auto
lemma "itrev [x,y,z] [] = rev [x,y,z] @ []" by auto

← many concrete cases

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL



3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL



Secret 1

Many induction heuristics are simple.

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL



Secret 1

Many induction heuristics are simple.

Secret 2

Proofs can be more abstract than theorems.

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

APLAS2019

Secret 1

Many induction heuristics are simple.

Secret 2

Proofs can be more abstract than theorems.

Secret 3

LiFtEr is not good enough because ...

LiFtEr almost ignores the definitions of constants.



$[[T,F,T], [T,T,T], [F,T,T]]$: bool list

\leftarrow simple representation

```
lemma "star r x y ==> star r x z"  
by(induction rule: star.induct) auto simp: step)
```

\leftarrow small dataset about different domains

```
lemma "exec (is1 @ is2) s ==>  
      exec is2 s (exec is1 s)"  
by(induct is1 s stk rule: exec.induct) auto
```

\leftarrow one abstract representation

```
lemma "itrev xs ys = rev xs @ ys"  
by(induct xs arbitrary: ys) auto
```

```
lemma "itrev xs ys = rev xs @ ys"  
by(induct xs ys rule:"itrev.induct") auto
```

$[[T,F,T], [T,T,T], [F,T,T]]$: bool list

\leftarrow simple representation

```
lemma "star r x y ==> star r x z"  
by(induction rule: star.induct) auto simp: step)
```

\leftarrow small dataset about different domains

```
lemma "exec (is1 @ is2) s ==>  
      exec is2 s (exec is1 s)"  
by(induct is1 s stk rule: exec.induct) auto
```

```
lemma "itrev xs ys = rev xs @ ys"  
by(induct xs arbitrary: ys) auto
```

\leftarrow one abstract representation

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lemma "itrev xs ys = rev xs @ ys"  
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```

```
primrec rev :: "'a list => 'a list" where  
"rev [] = []" |  
"rev (x # xs) = rev xs @ [x]"
```

\leftarrow relevant definitions

```
fun itrev :: "'a list => 'a list => 'a list" where  
"itrev [] ys = ys" |  
"itrev (x#xs) ys = itrev xs (x#ys)"
```

$[[T,F,T], [T,T,T], [F,T,T]]$: bool list

```
lemma "star r x y ==> star r x z"  
by(induction rule: star.induct) auto simp: step)  
  
lemma "exec (is1 @ is2) s ==>  
      exec is2 s (exec is1 s)"  
by(induct is1 s stk rule: exec.induct) auto  
  
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← simple representation

← small dataset about different domains

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 $\leftarrow$  nested assertions to examine the "semantics" of constants (rev and itrev)
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LiFtEr: (proof goal * induction arguments) -> bool

[[T,F,T], [T,T,T], [F,T,T]]: bool list

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← relevant definitions

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LiFtEr₂ (proof goal * induction arguments) -> bool
* relevant definitions

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LiFtEr₂ (proof goal * induction arguments) -> bool
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WIP

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fin.