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LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

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University of Innsbruck
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Yutaka Ng
yutakang

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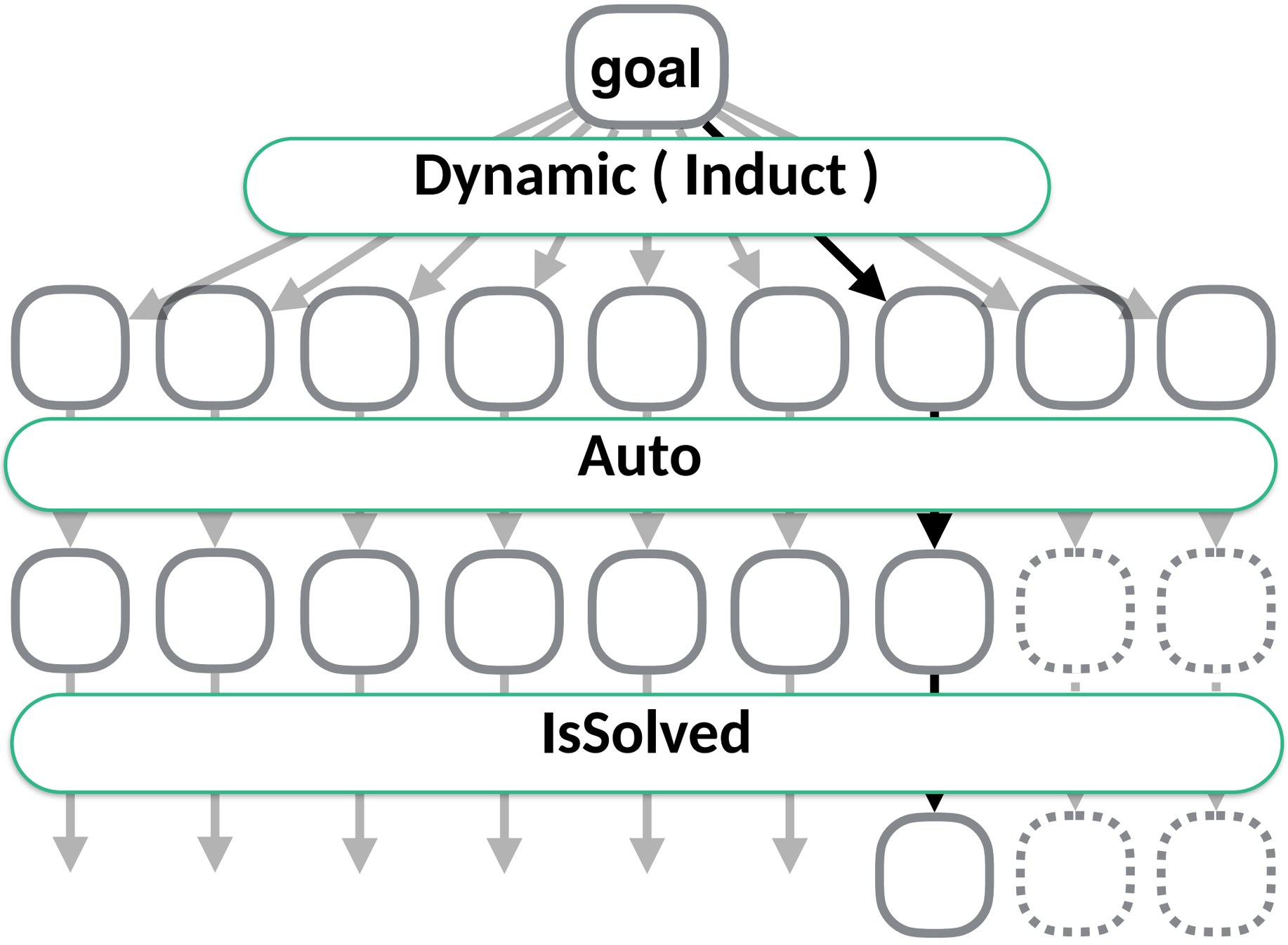

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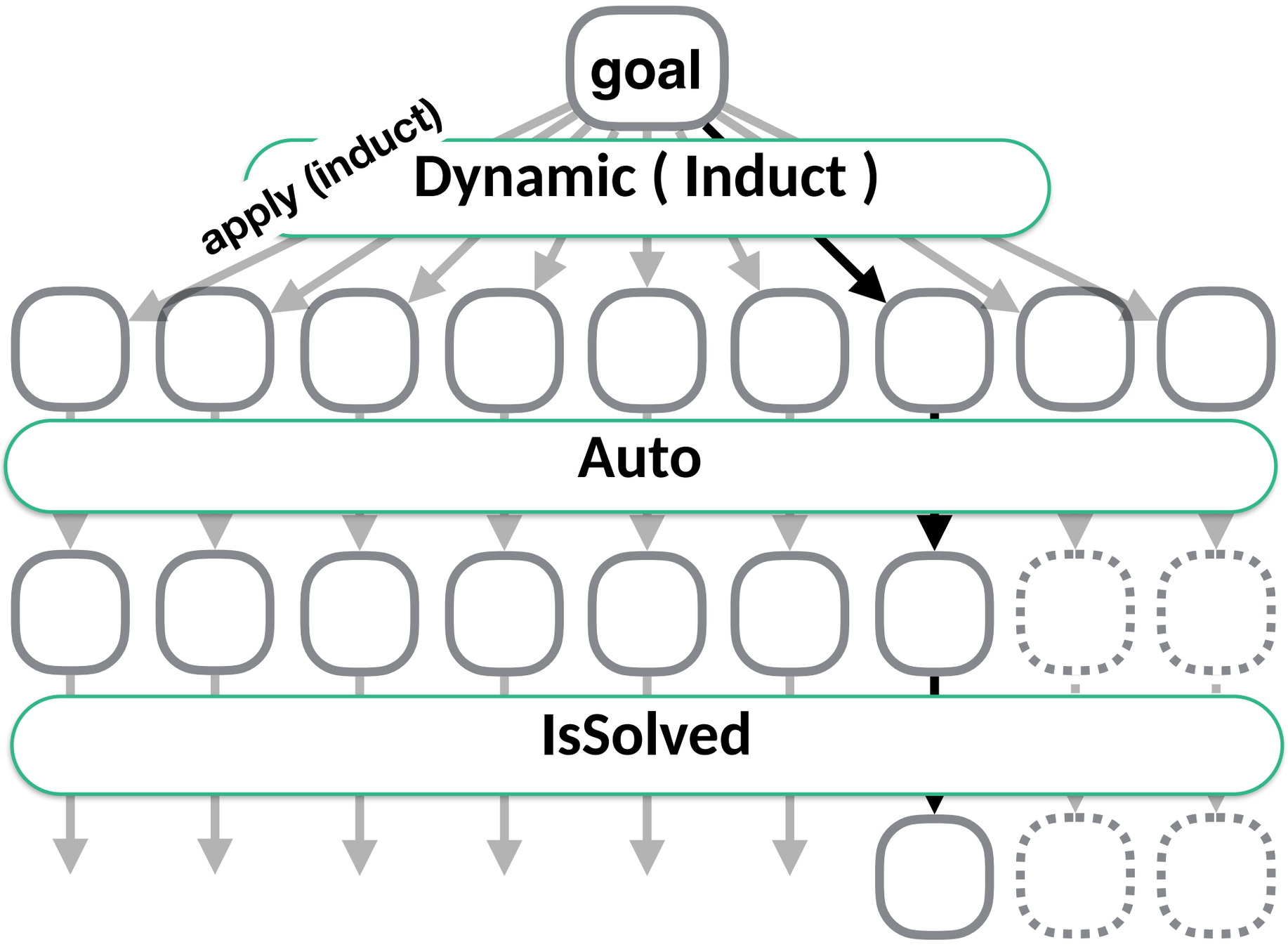

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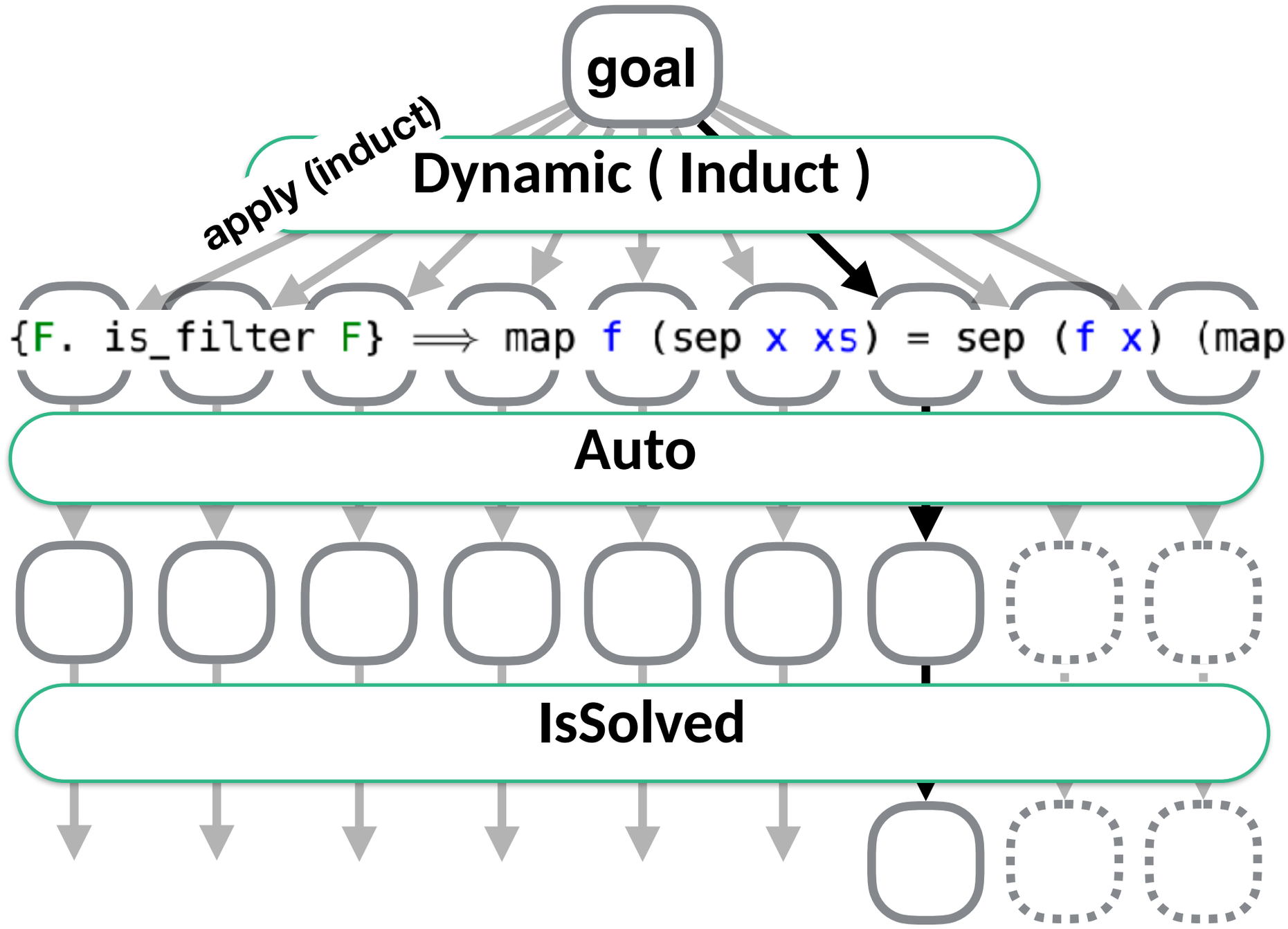


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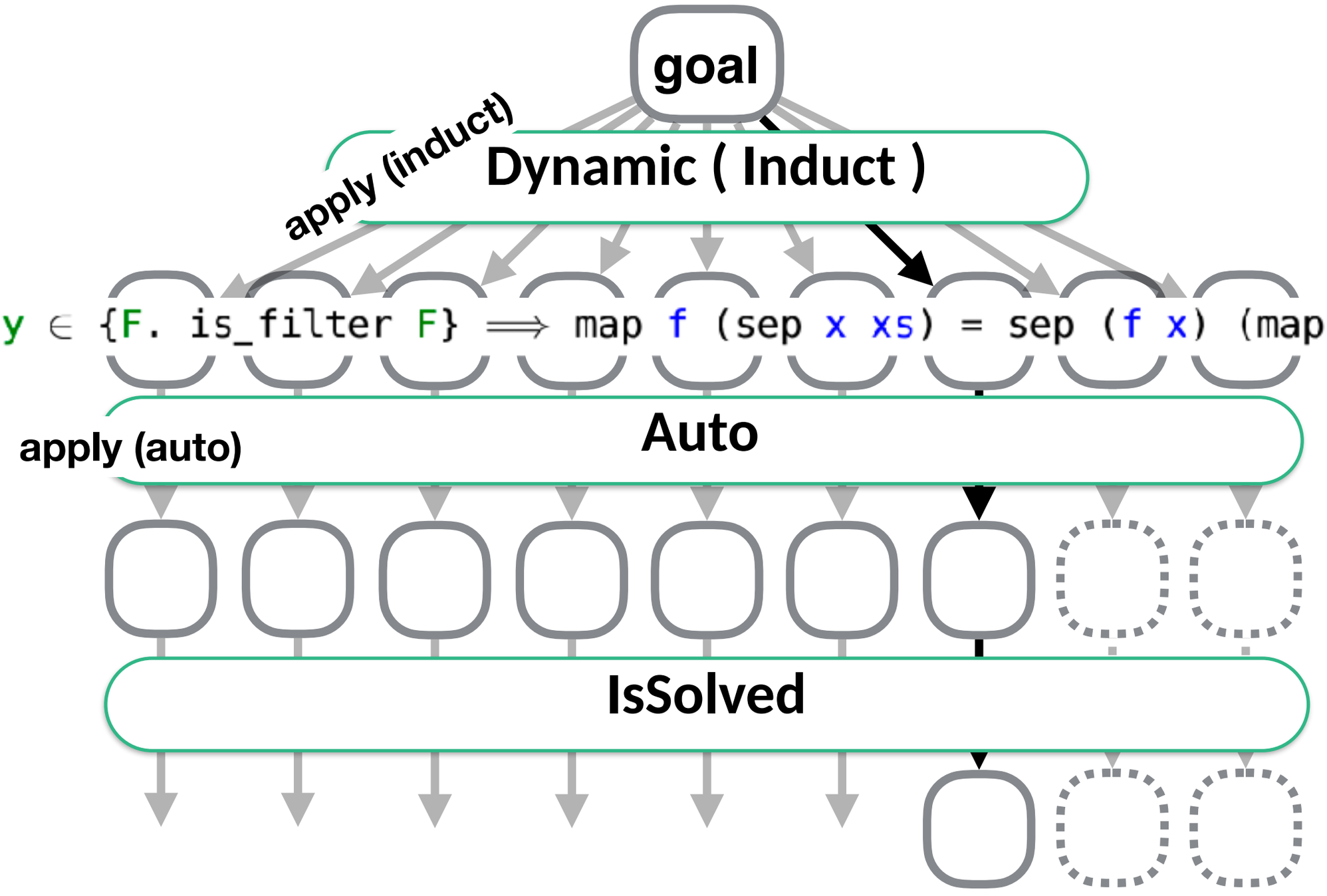


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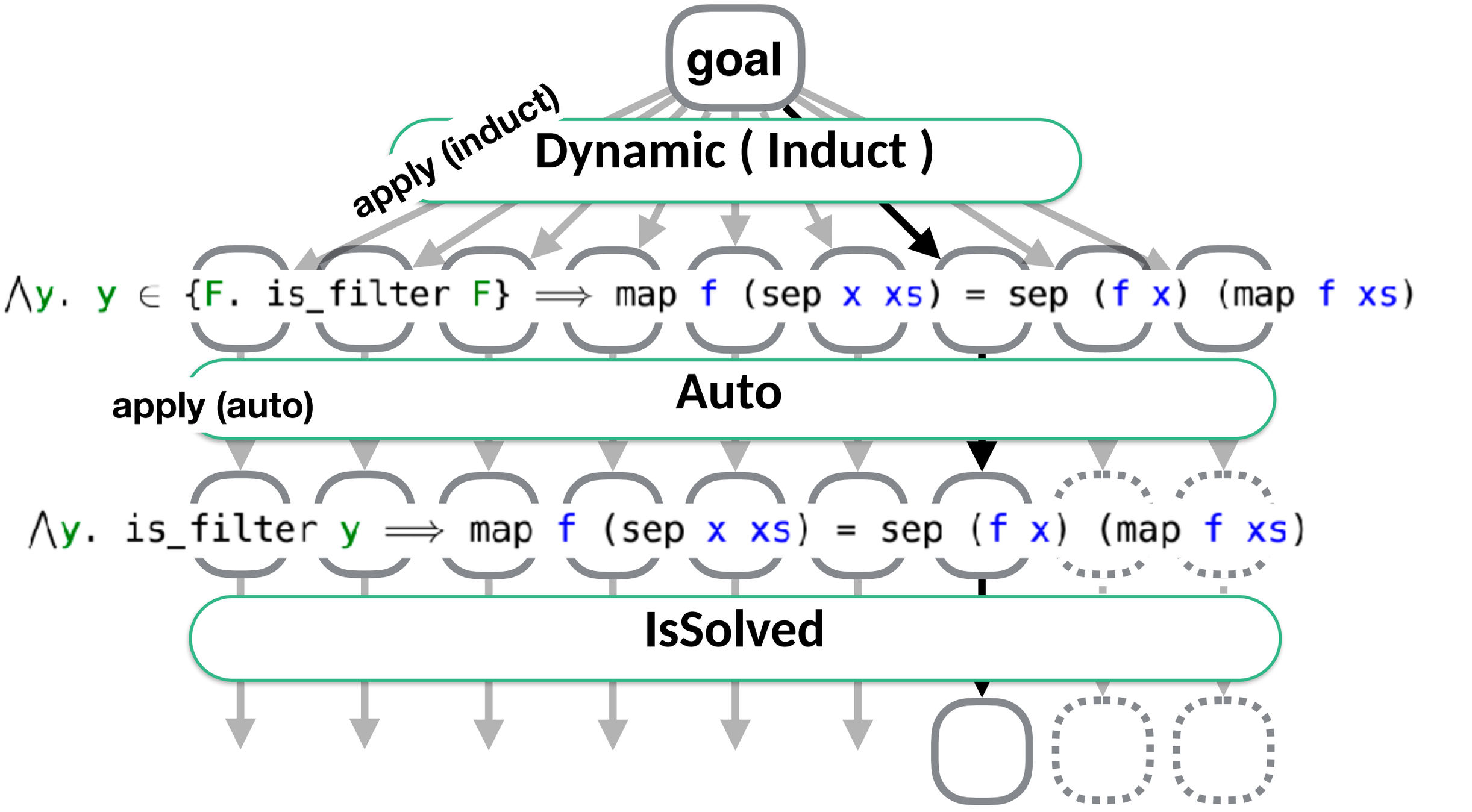

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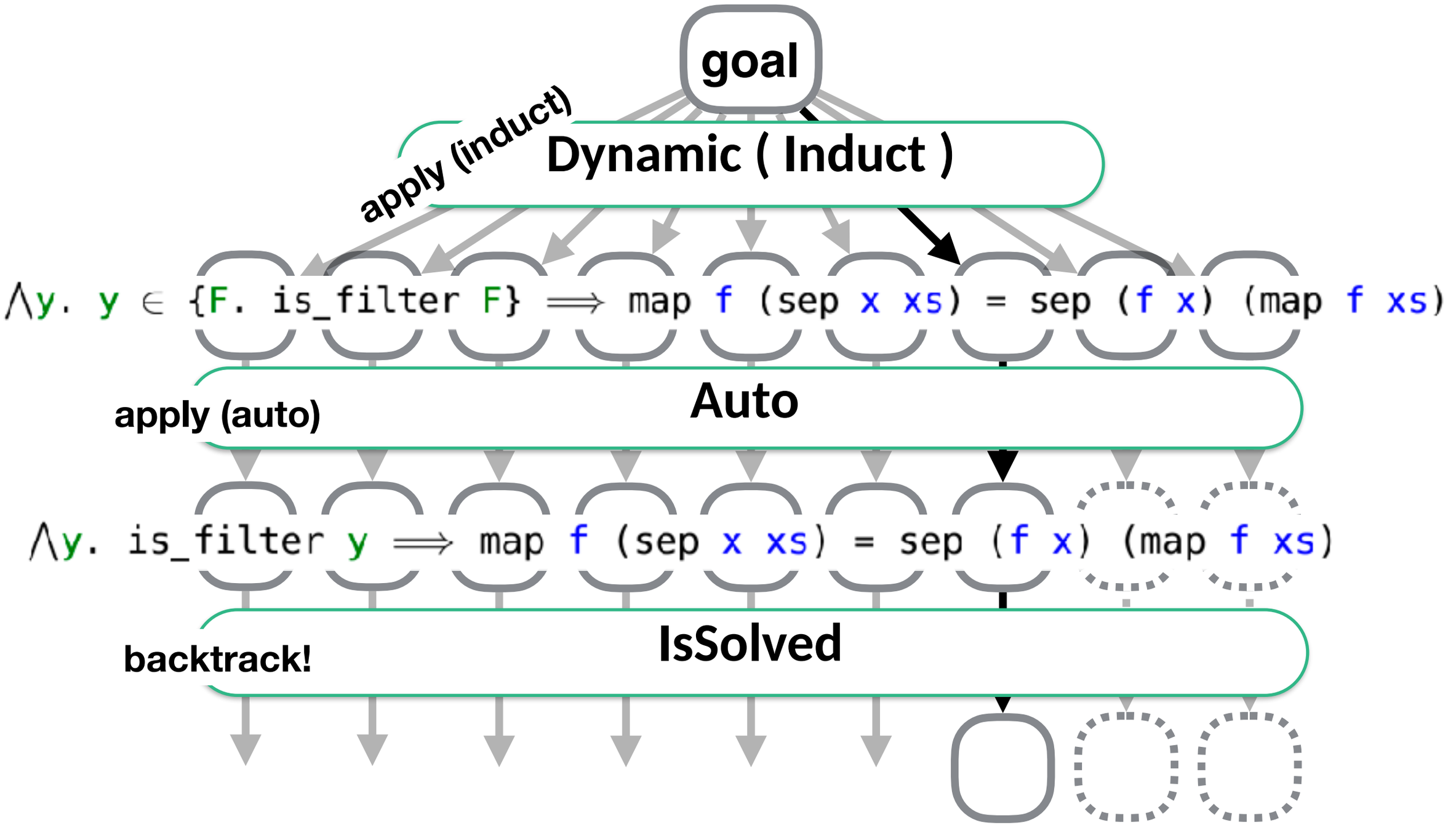

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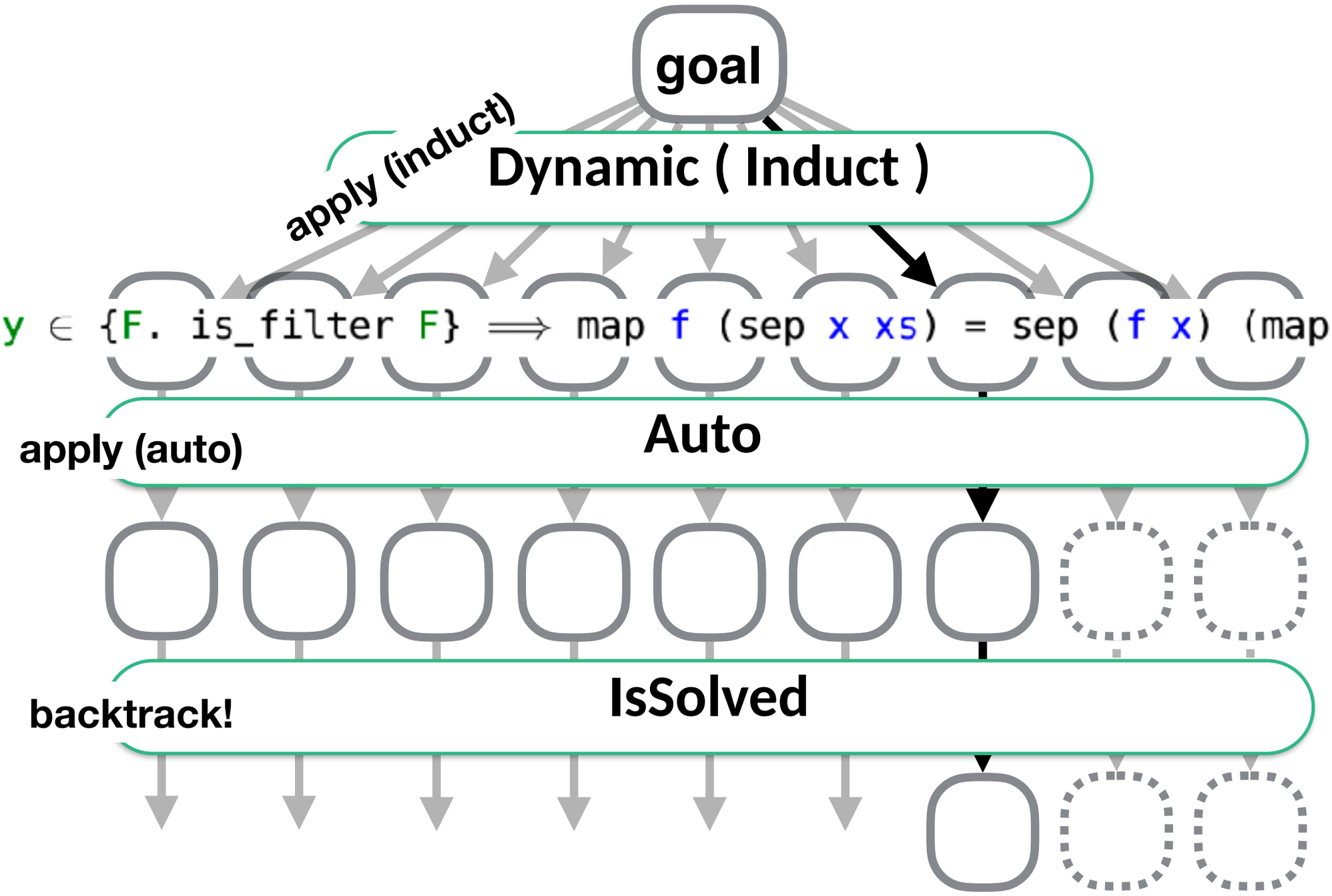


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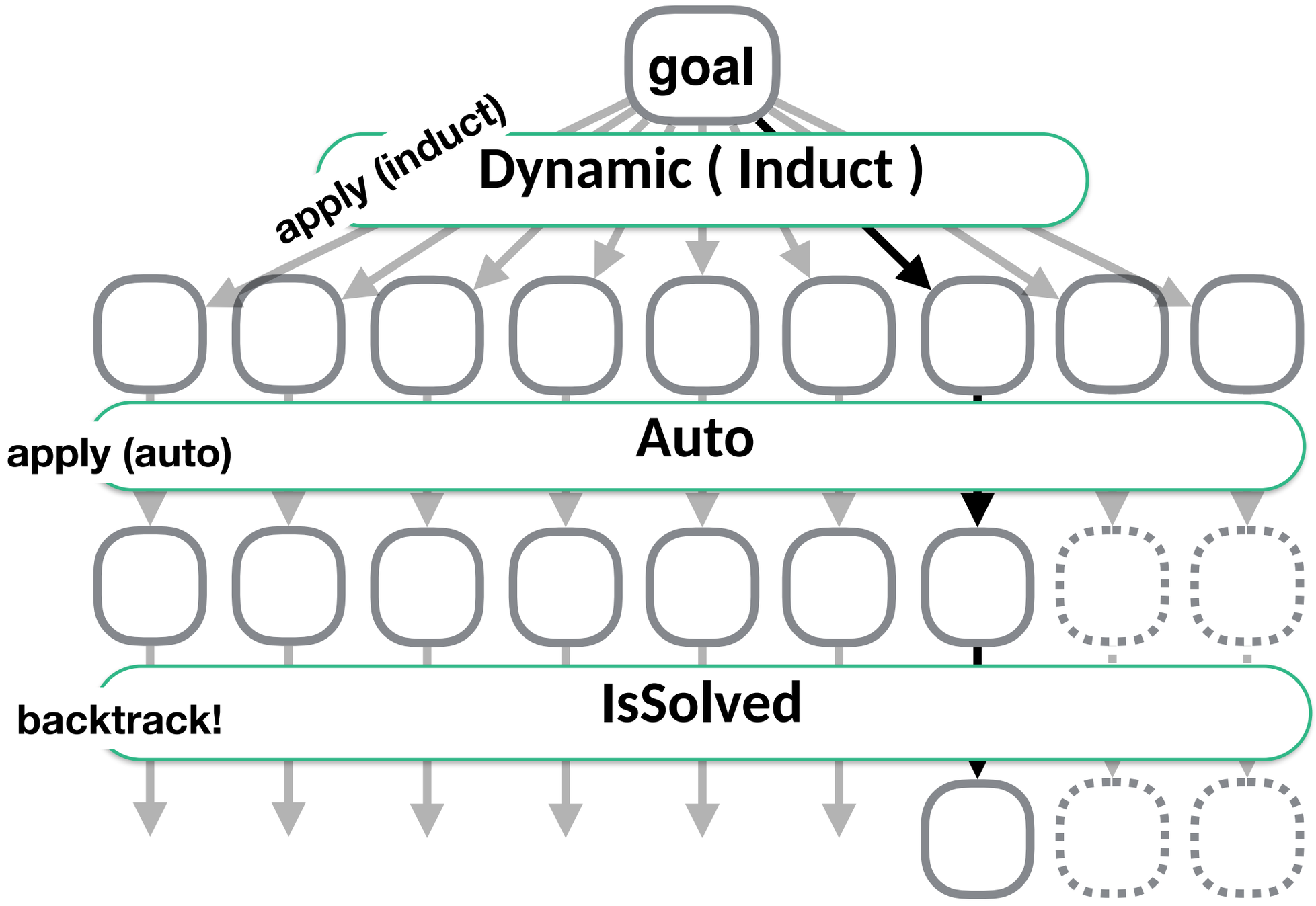

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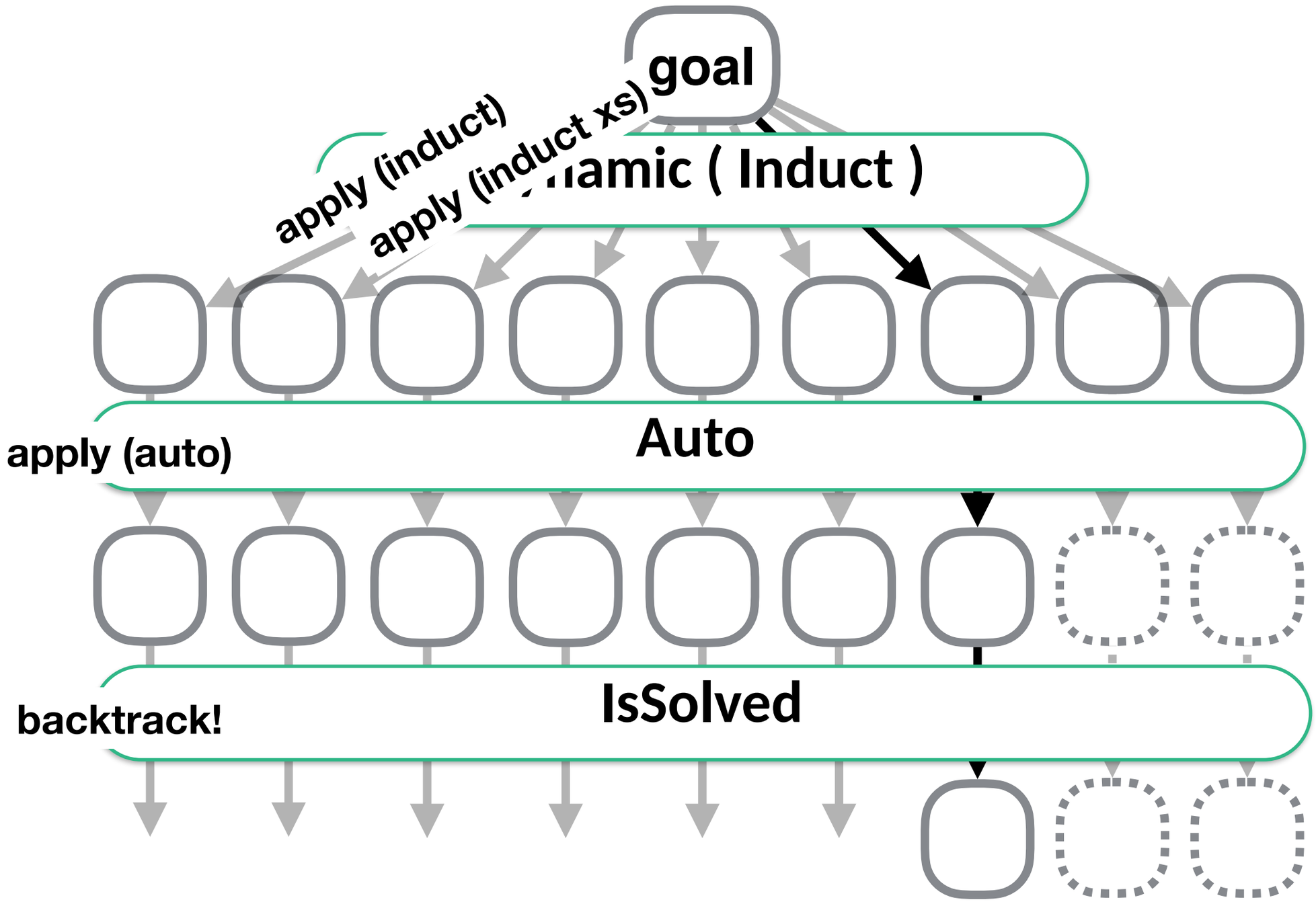

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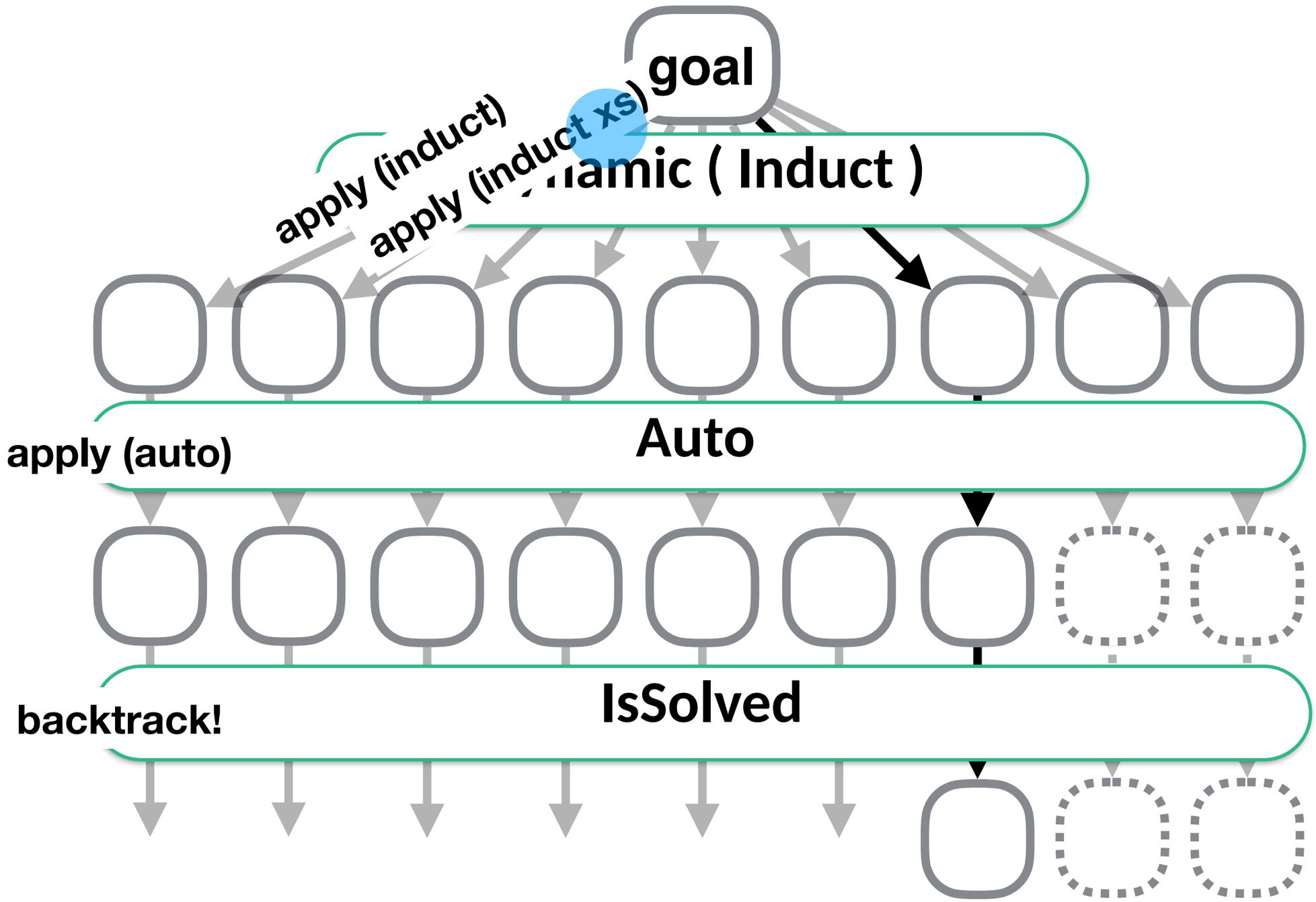

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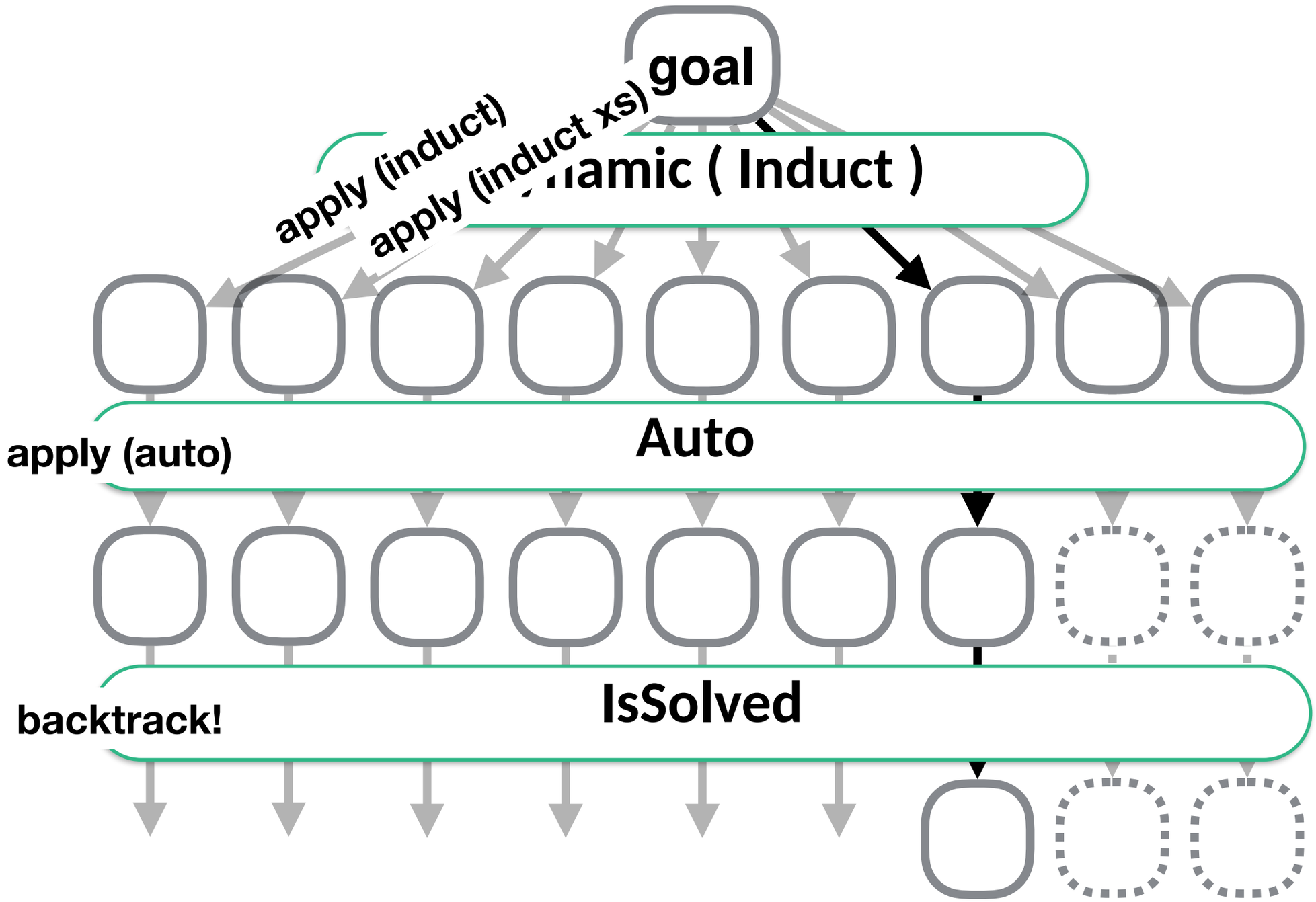

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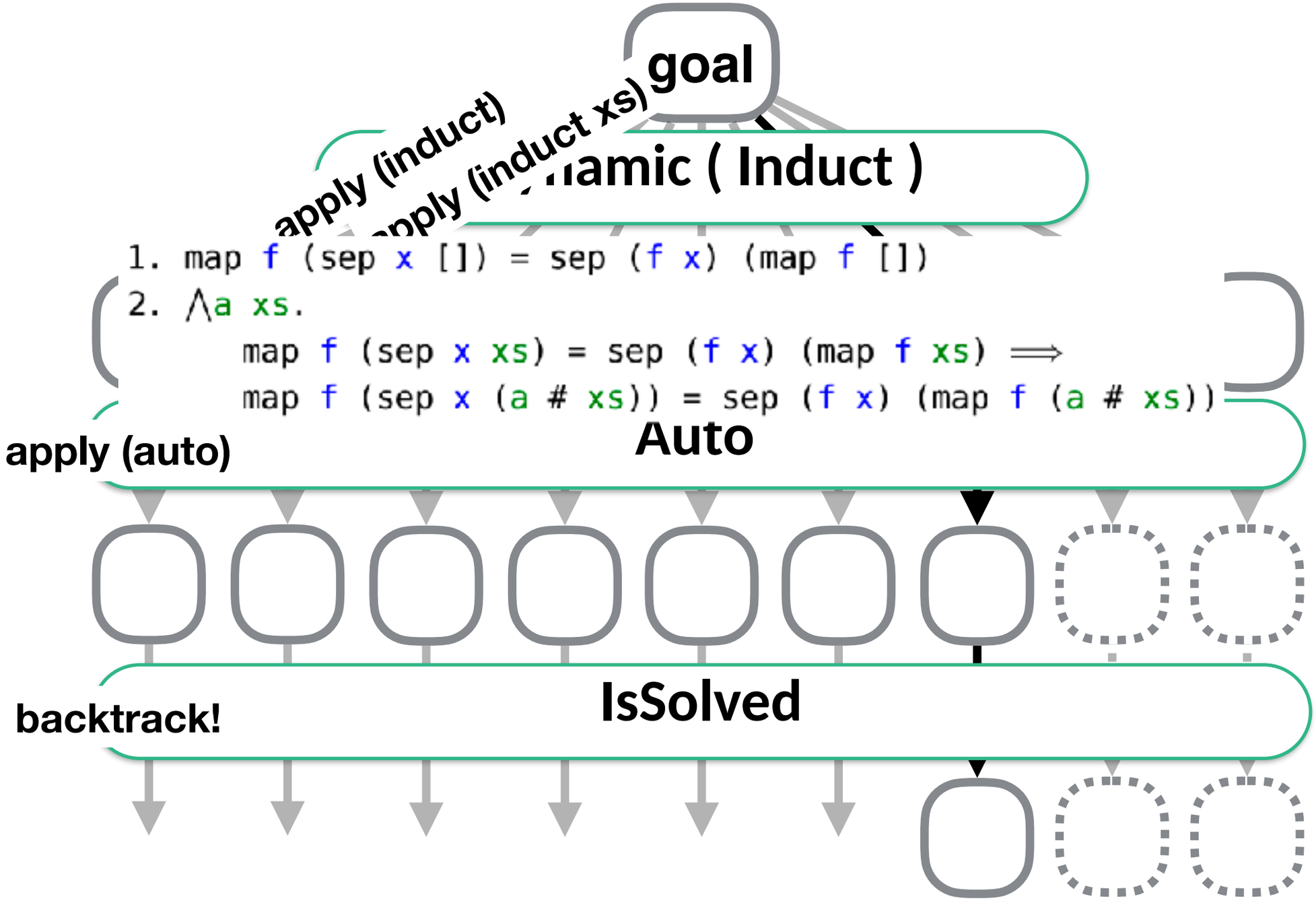

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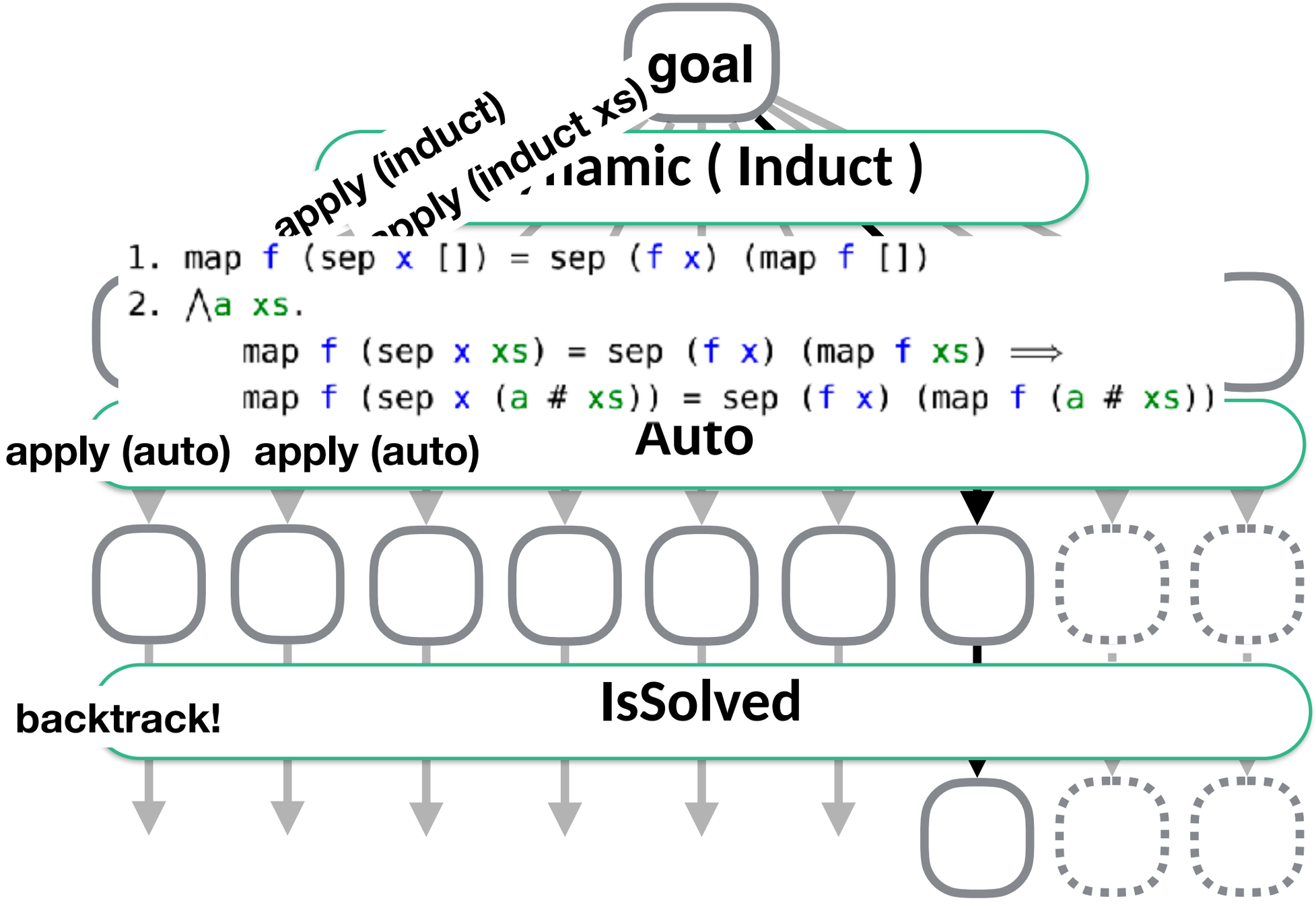

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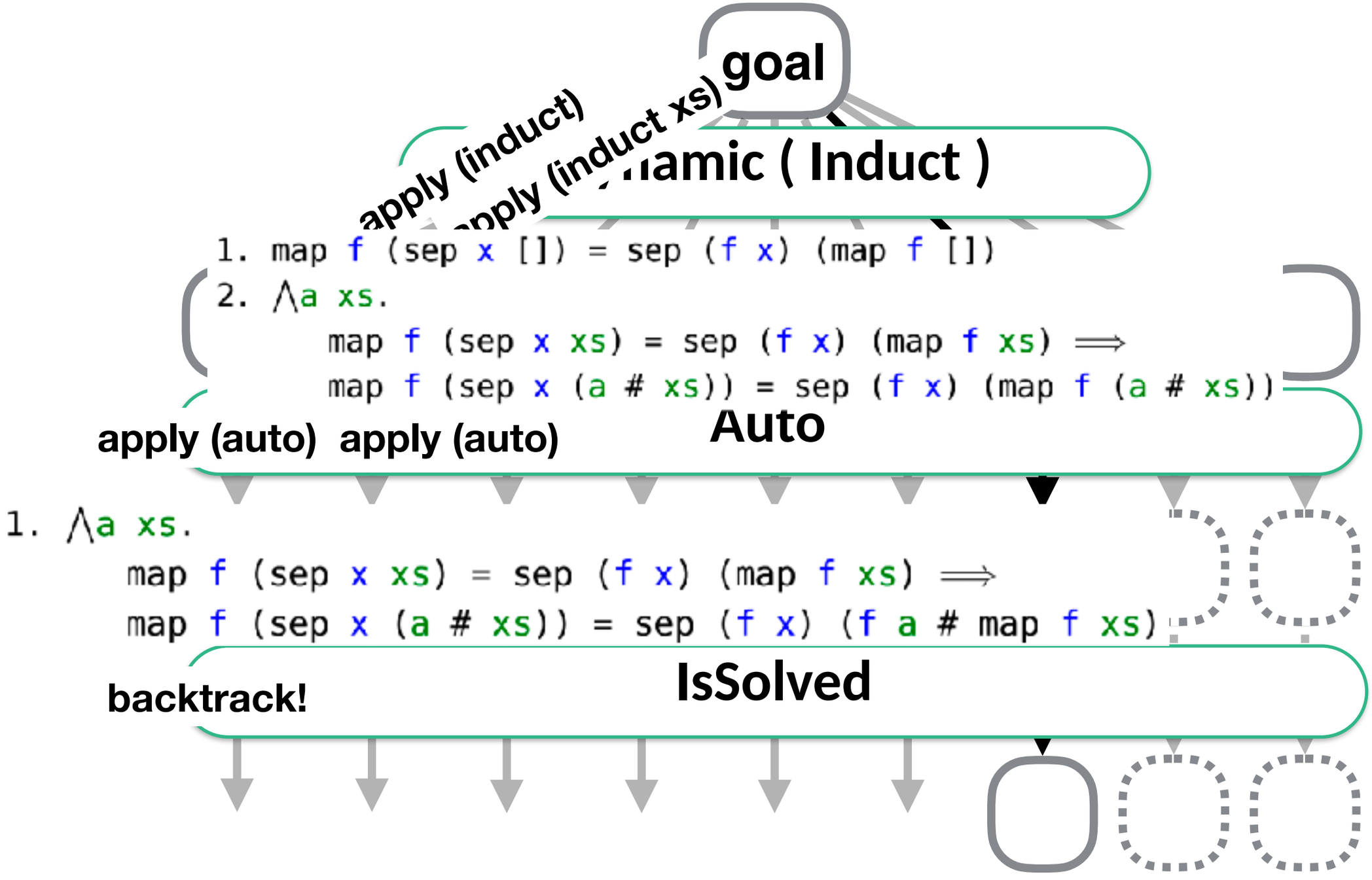

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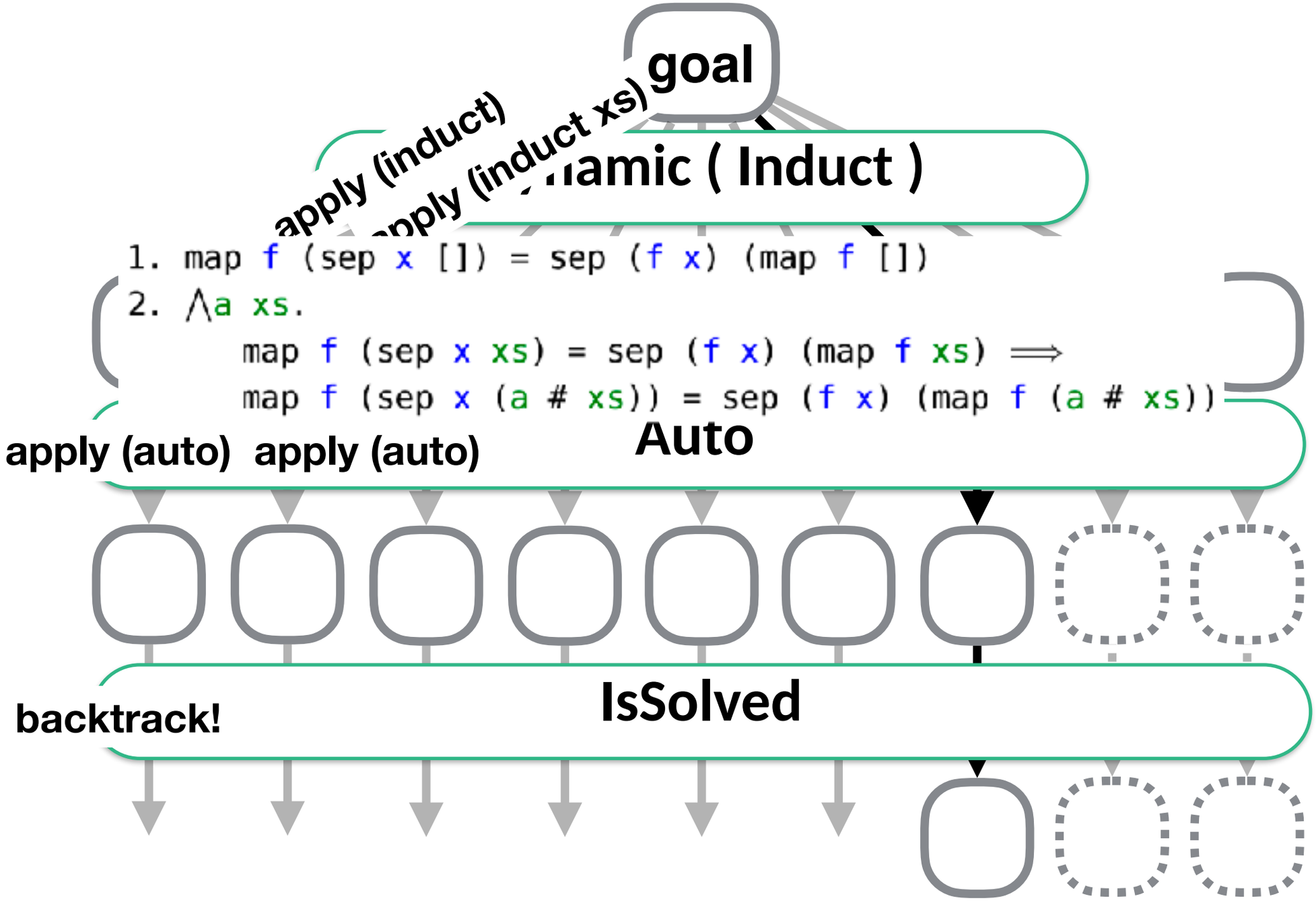

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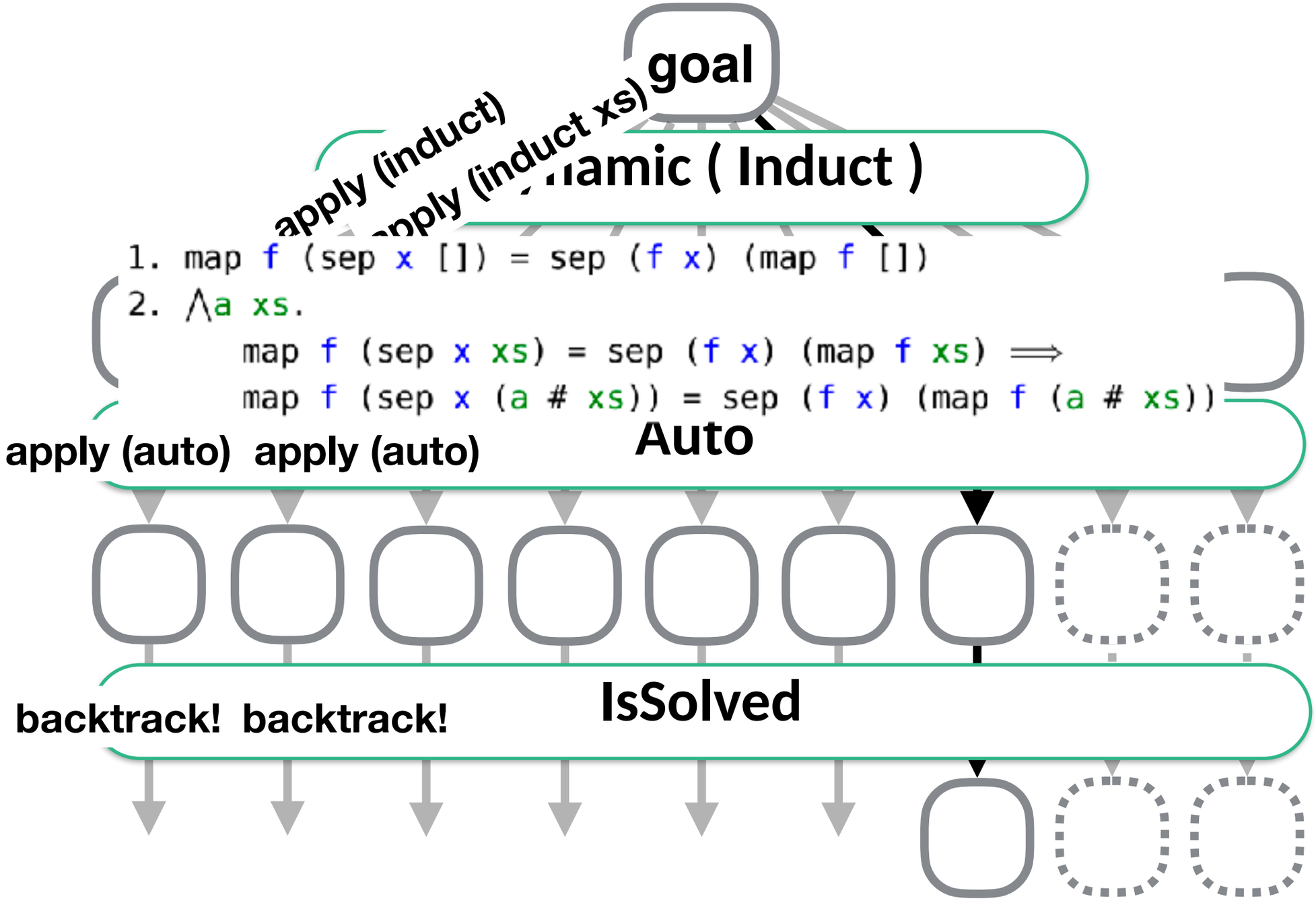

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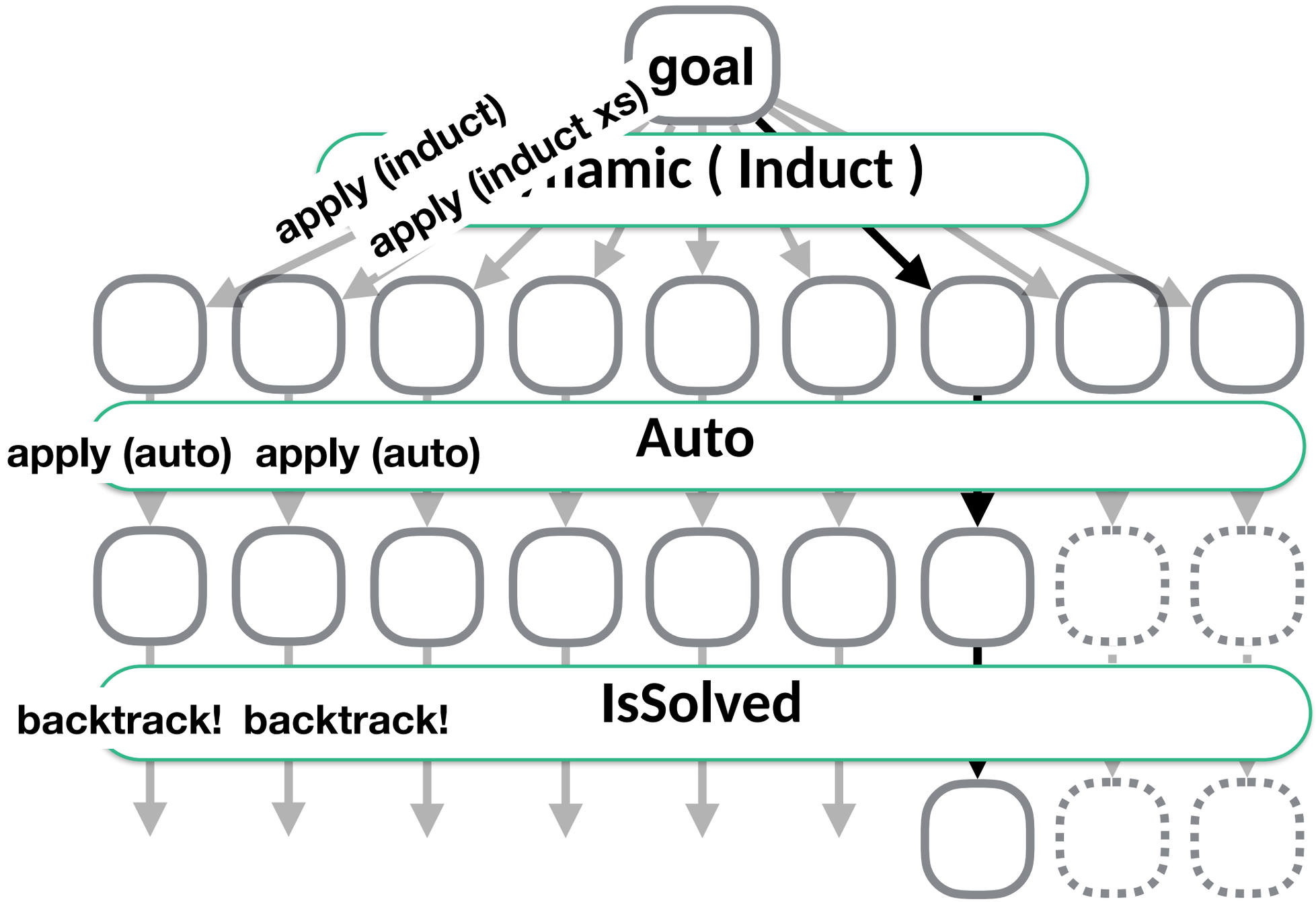

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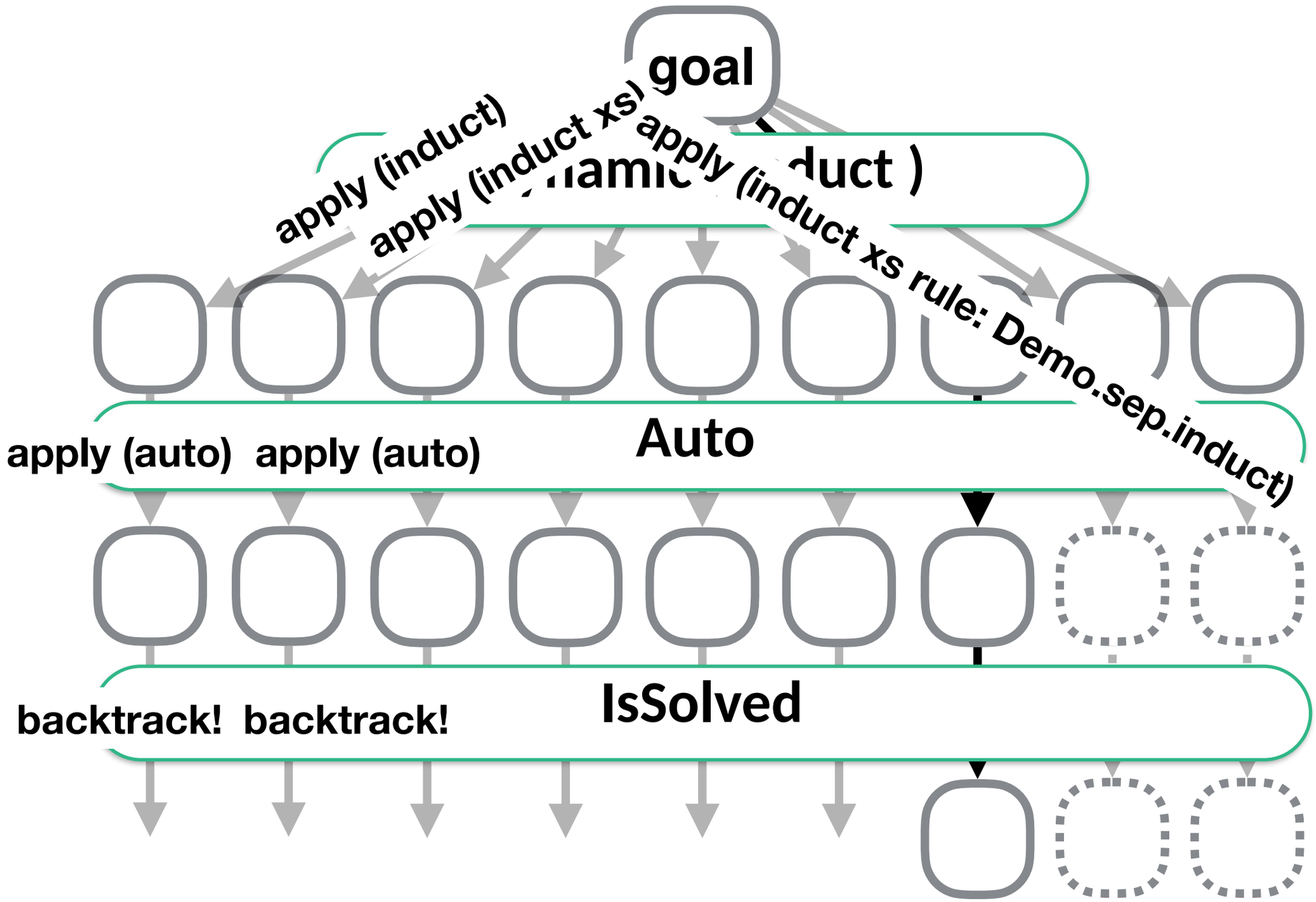

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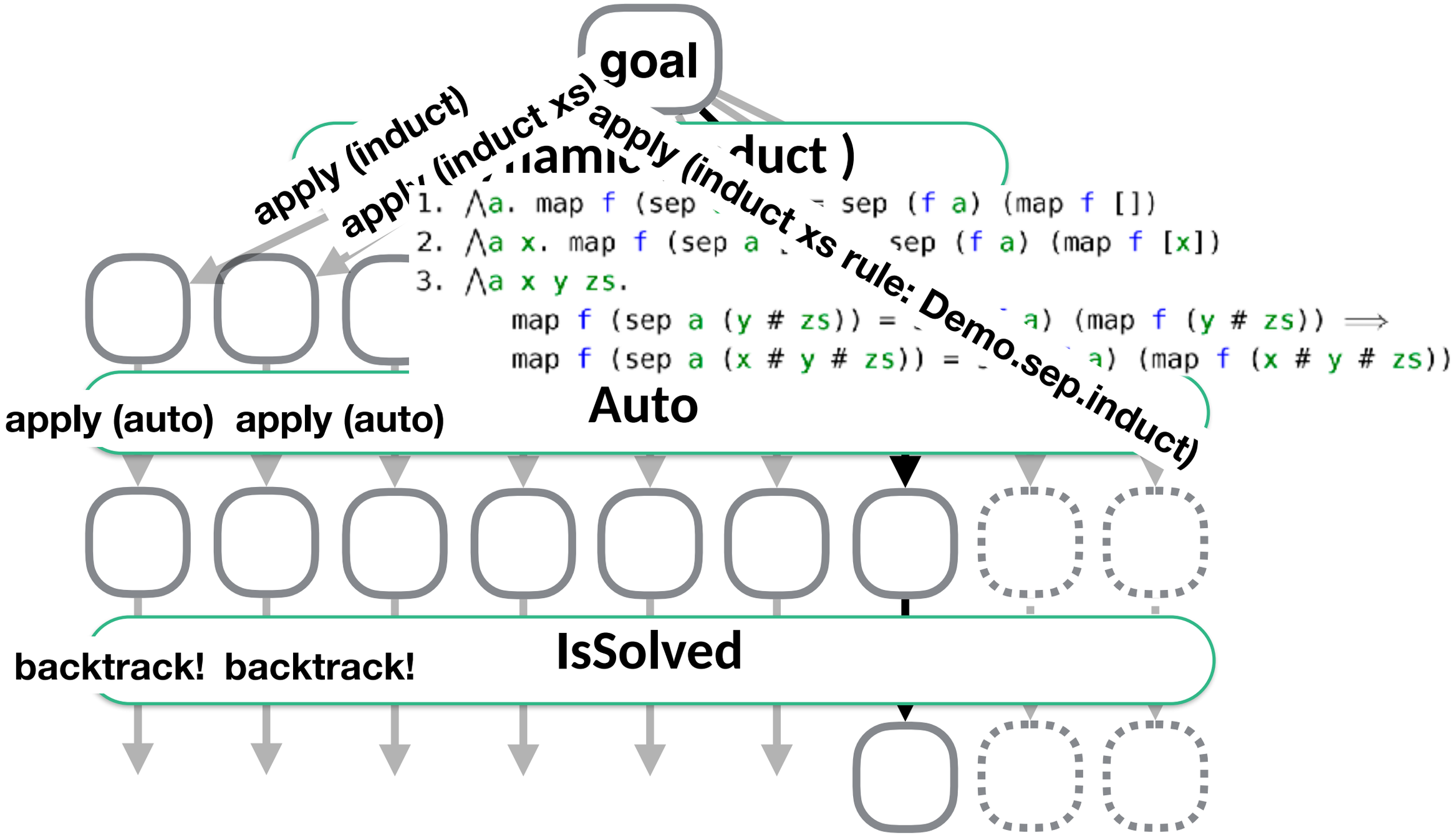

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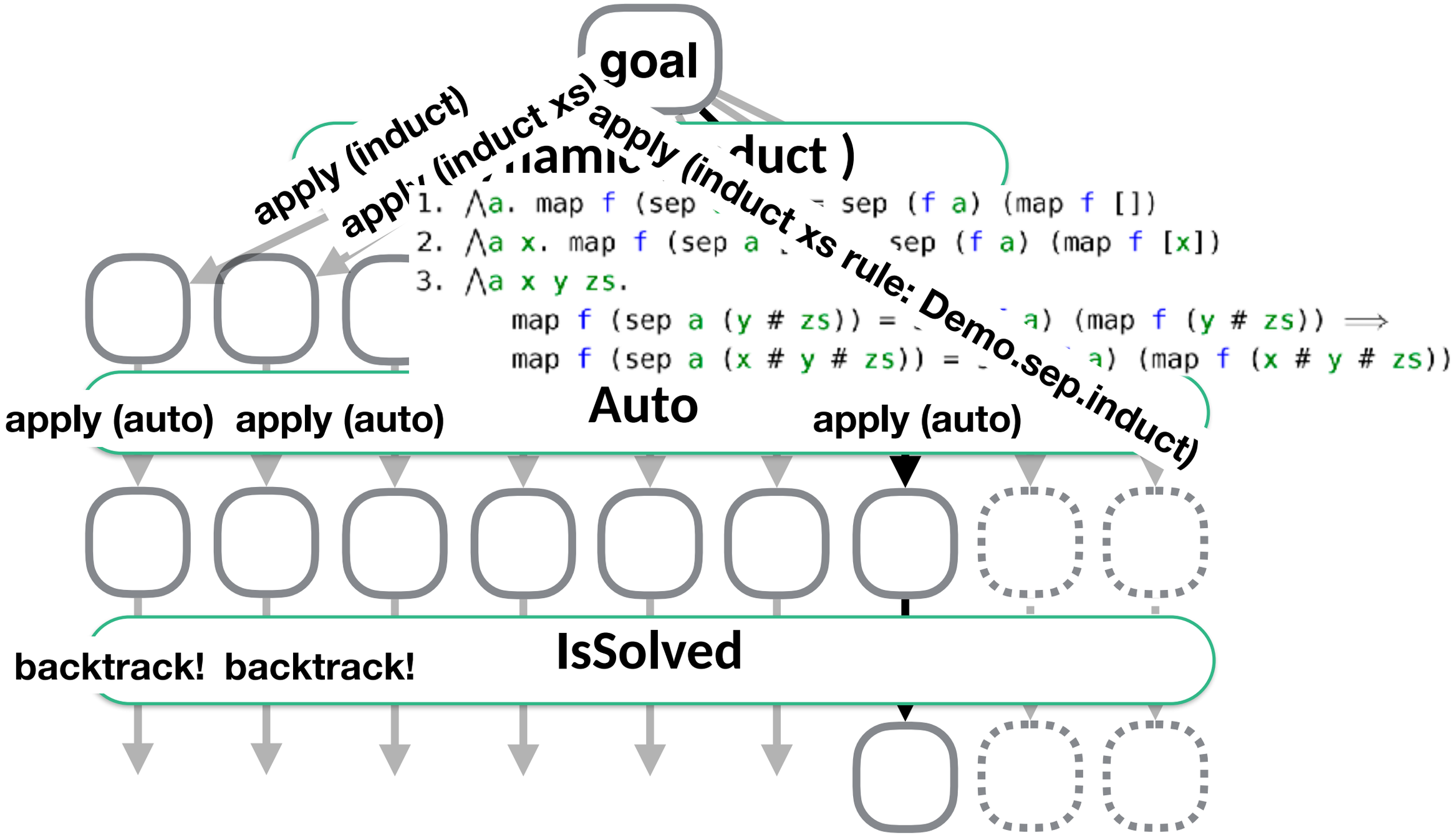

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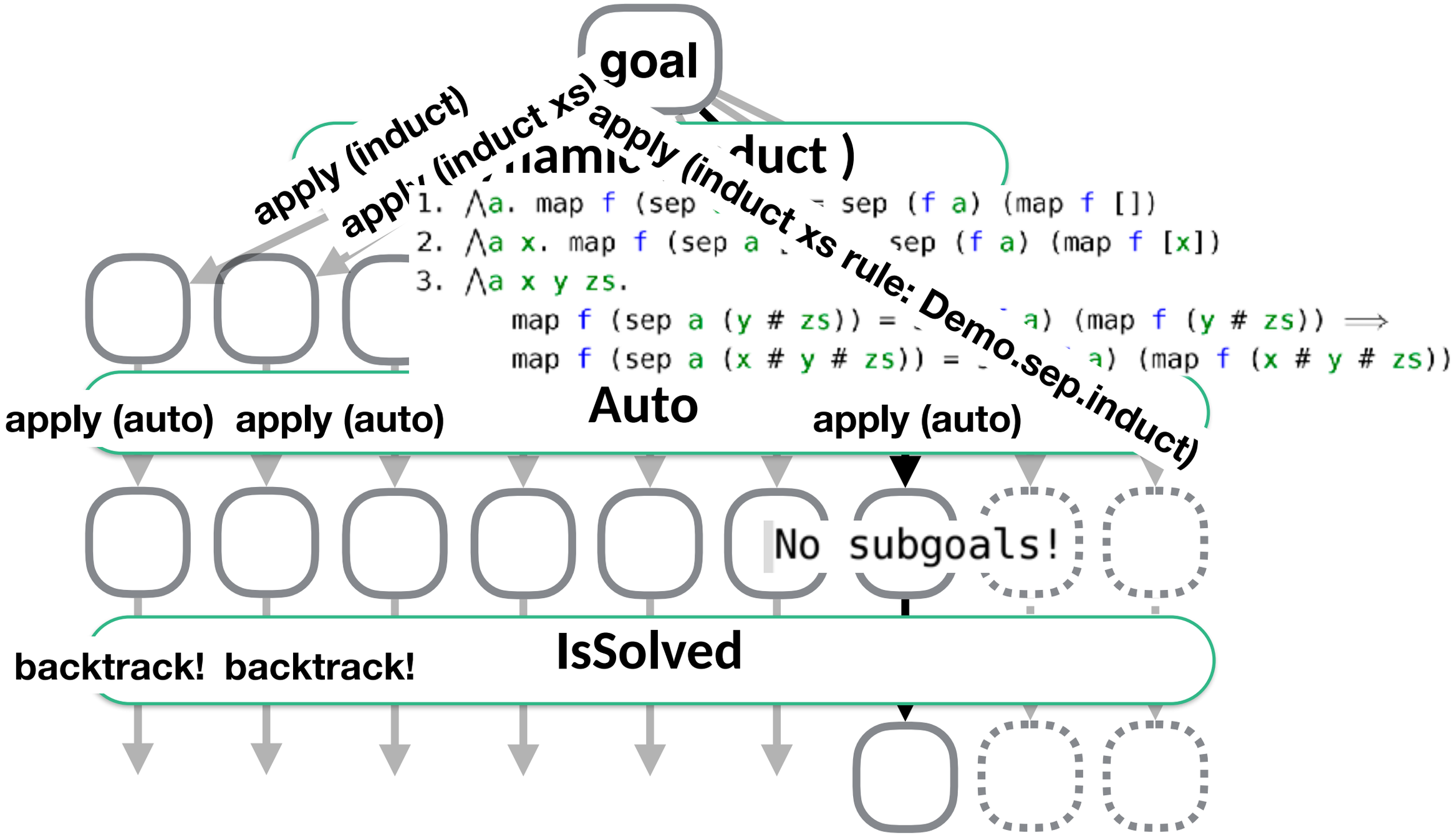

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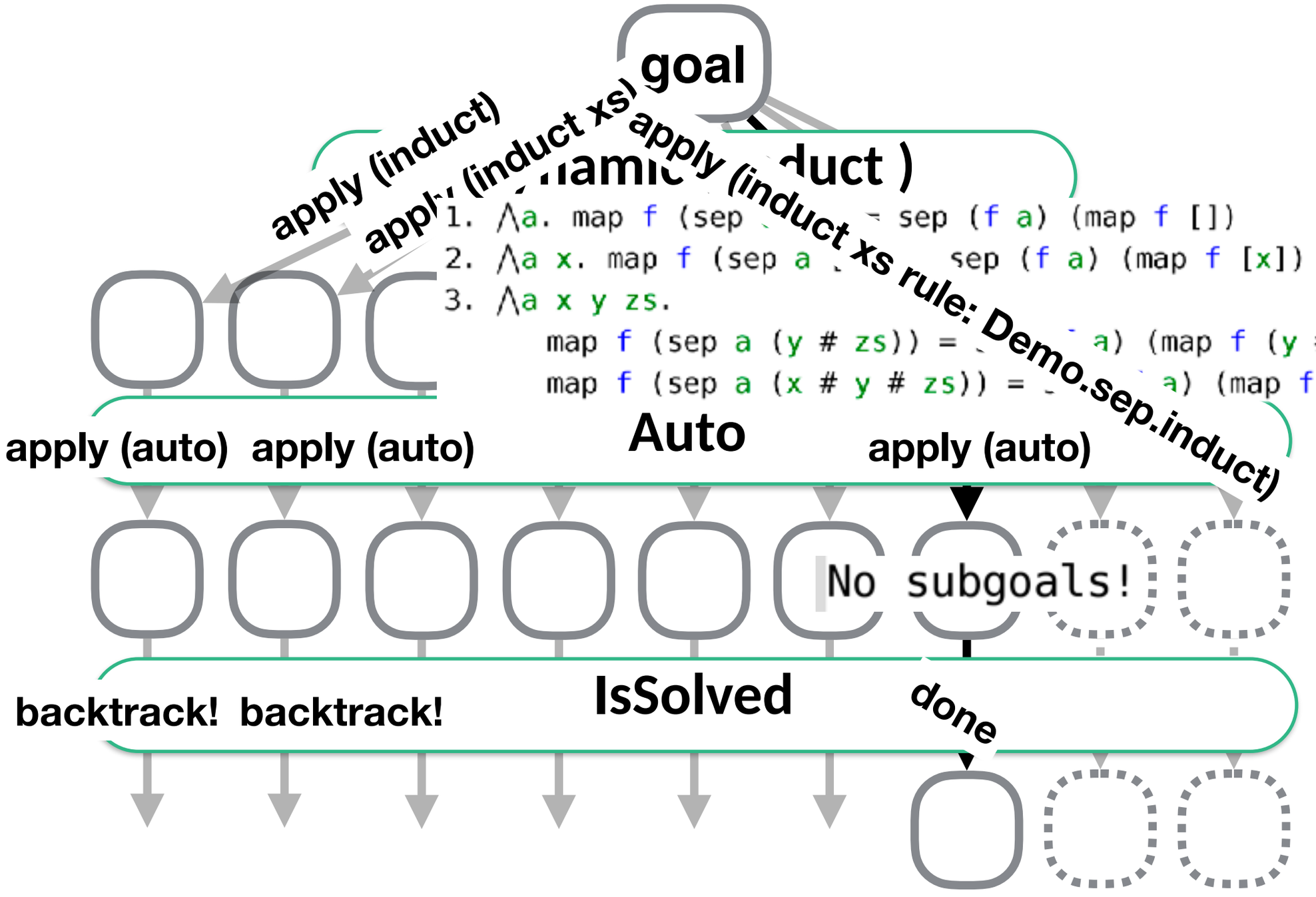

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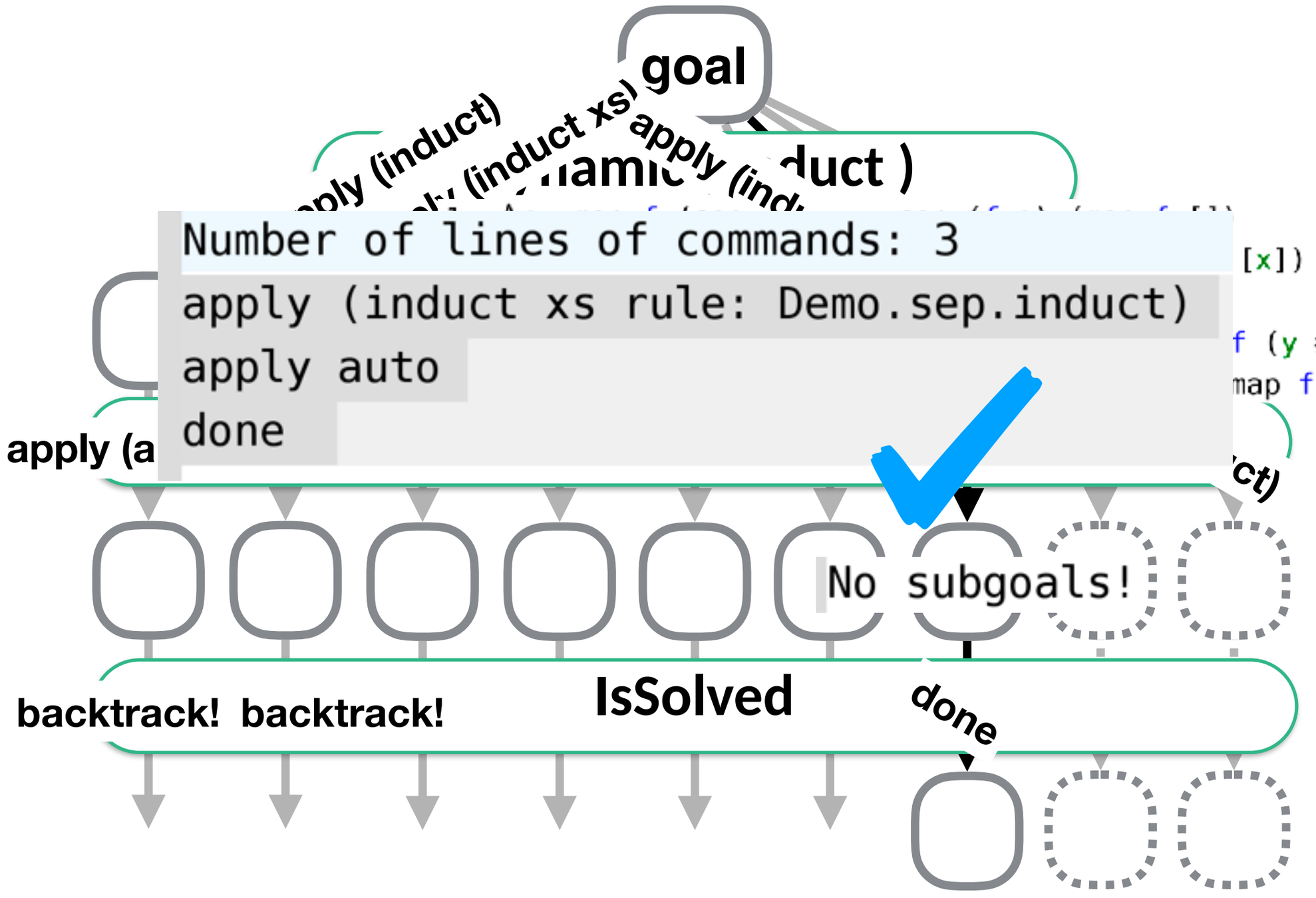

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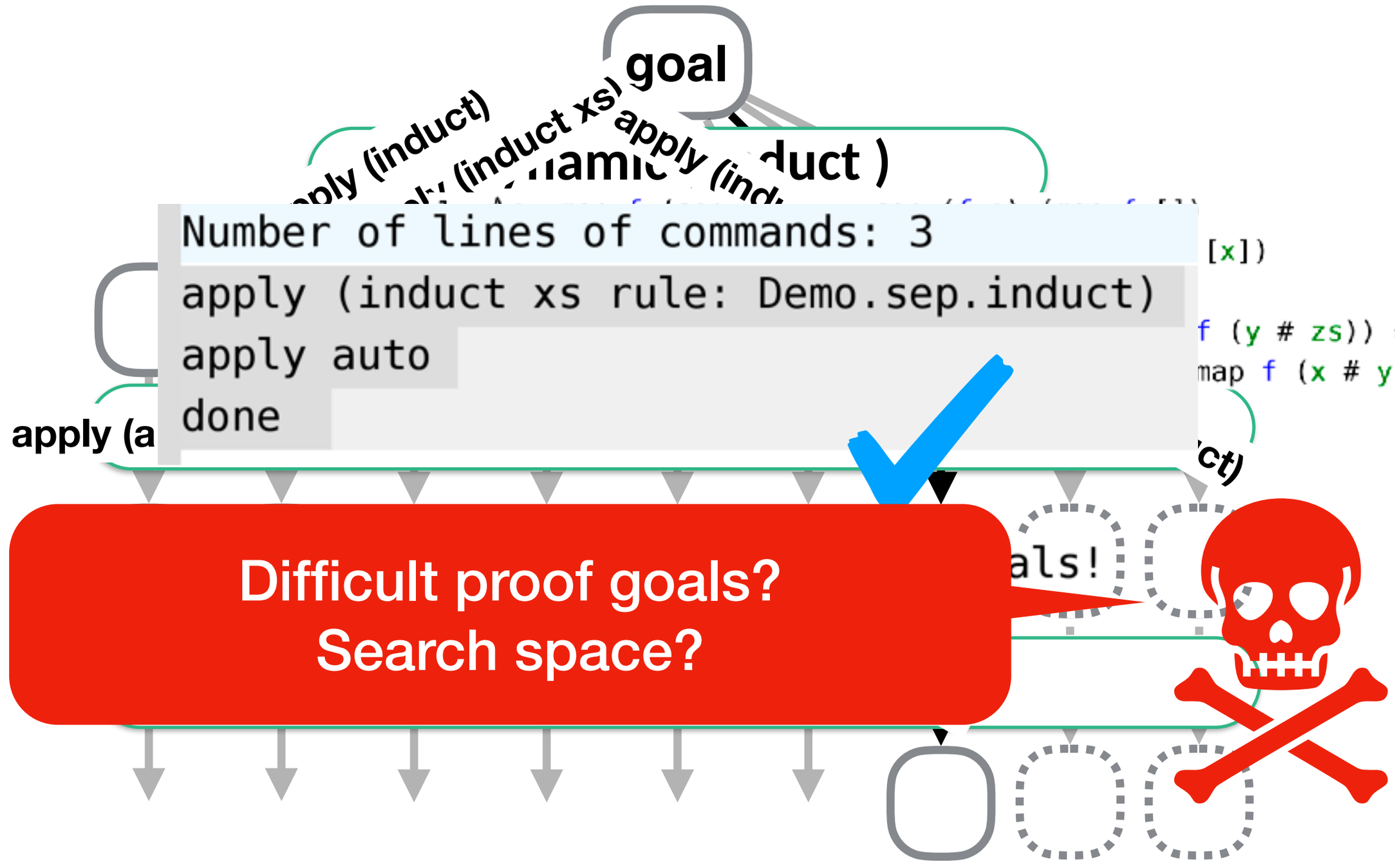

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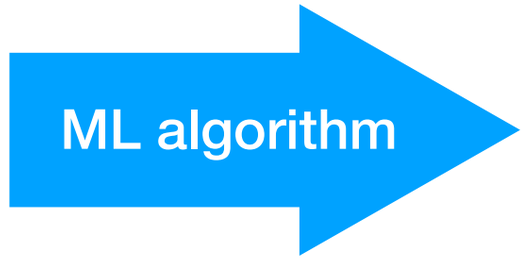

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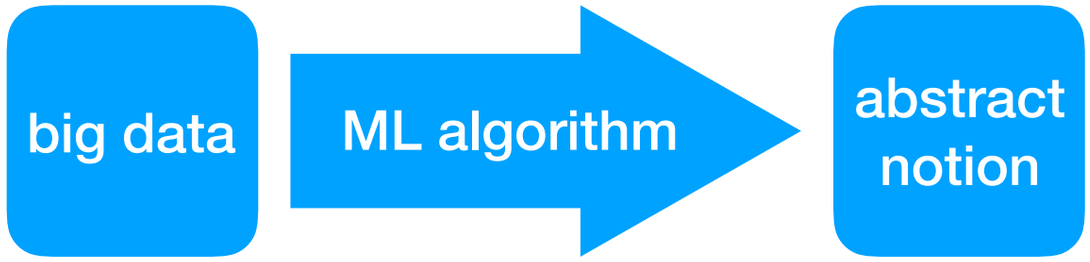

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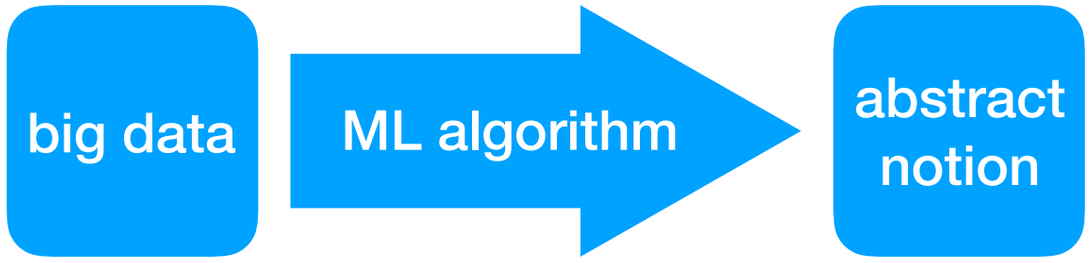

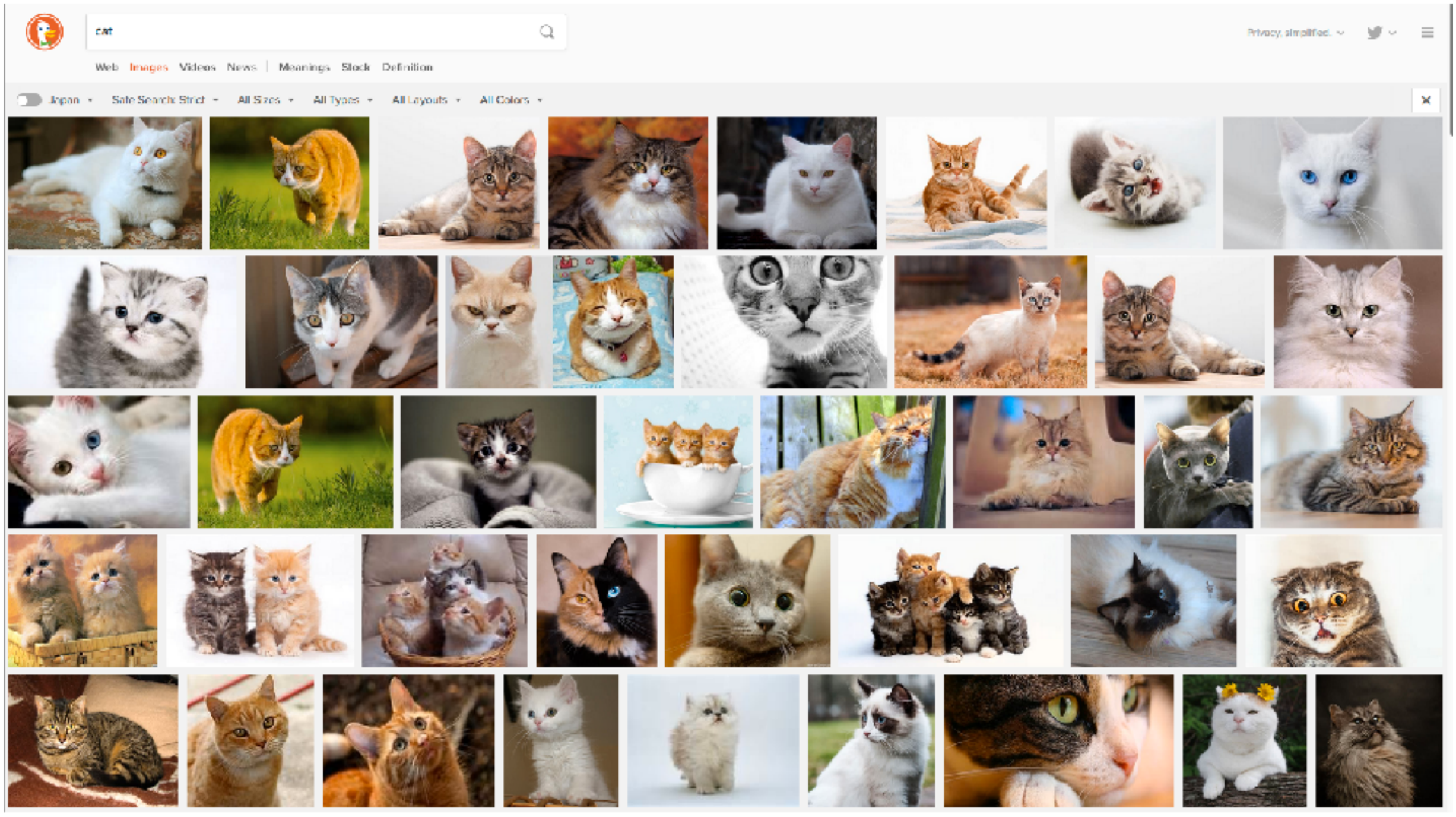

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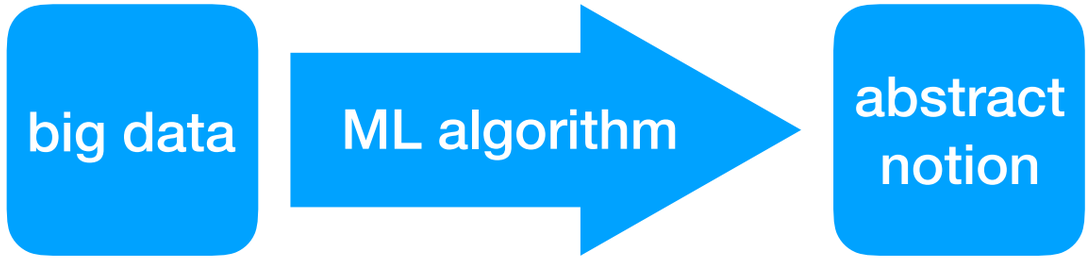

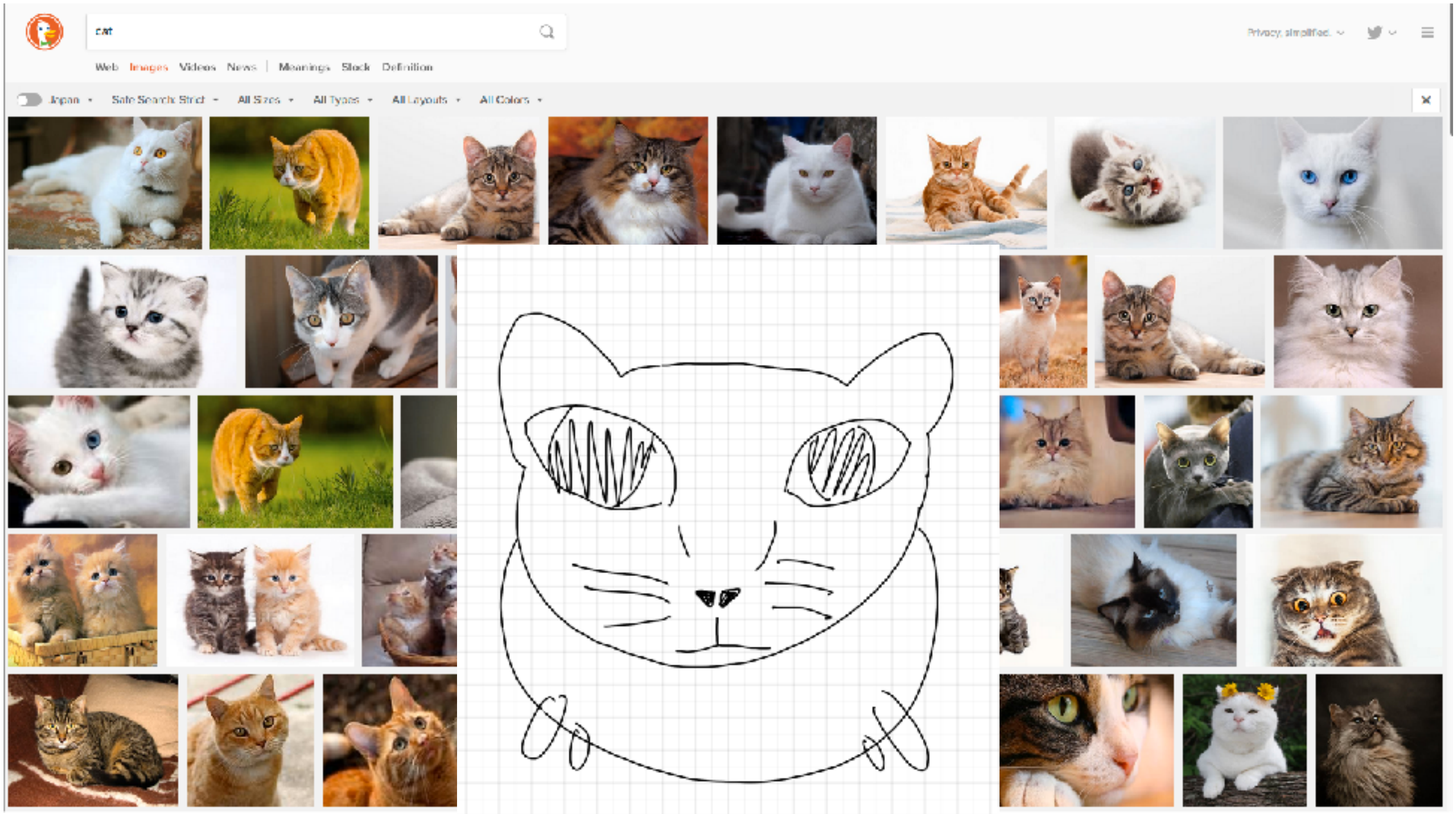

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ML for Inductive Theorem Proving the BAD

```
lemma "itrev xs ys = rev xs @ ys"
```

<- one abstract representation

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
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<- many concrete cases

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lemma "itrev xs ys = rev xs @ ys"  by auto
by(induct xs ys rule:"itrev.induct") auto
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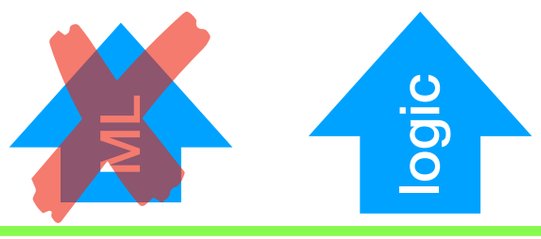
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

https://twitter.com/YutakangE

ML for Inductive Theorem Proving the BAD

polymorphism

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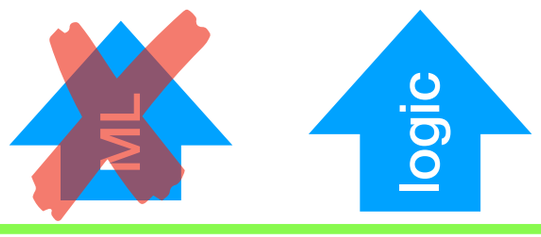
```

<- one abstract representation



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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

https://twitter.com/YutakangE

ML for Inductive Theorem Proving the BAD

polymorphism **type class**

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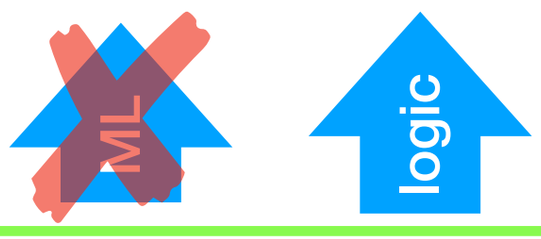
<- one abstract representation



<- abstraction using expressive logic

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<- many concrete cases

https://twitter.com/YutakangE

ML for Inductive Theorem Proving the BAD

polymorphism

type class

universal quantifier

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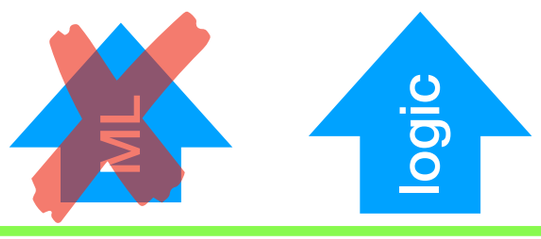

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<- many concrete cases

https://twitter.com/YutakangE

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism **type class**

universal quantifier

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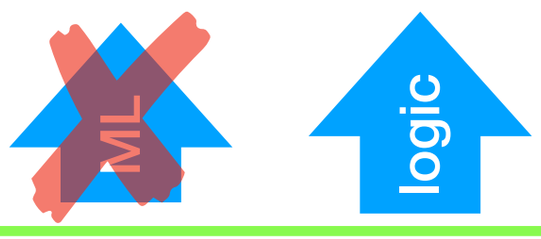
```

<- one abstract representation



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<- many concrete cases

https://twitter.com/YutakangE

ML for Inductive Theorem Proving the BAD

Higher-Order functions

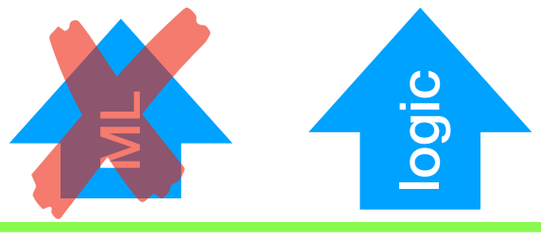
polymorphism **type class**

universal quantifier

lambda abstraction

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



<- abstraction using expressive logic

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ML for Inductive Theorem Proving the BAD

Higher-Order functions

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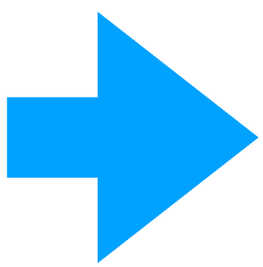

concise formula that can cover many concrete cases

lambda abstraction

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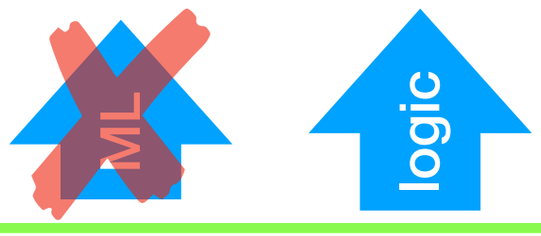
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



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<- many concrete cases

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ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

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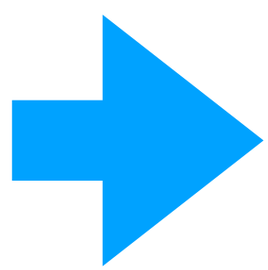

concise formula that can cover many concrete cases

different proof for general case

lambda abstraction

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lemma "itrev xs ys = rev xs @ ys"
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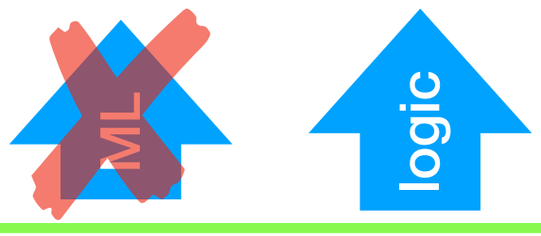
<- one abstract representation



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<- many concrete cases

https://twitter.com/YutakangE

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

universal quantifier

lambda abstraction

concise formula that can cover many concrete cases

different proof for general case

A small data set is not a failure but an achievement!

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
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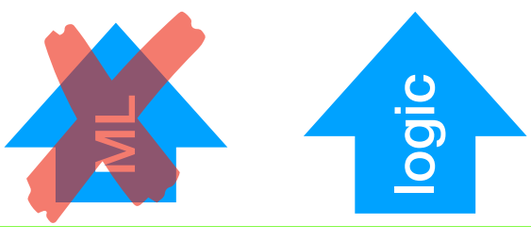
<- one abstract representation



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<- many concrete cases

https://twitter.com/YutakangE

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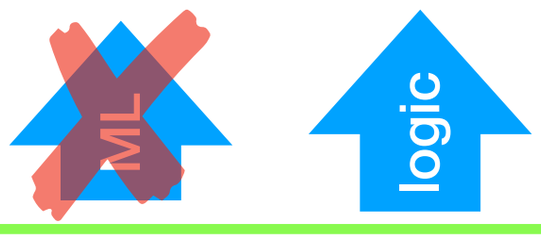

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<- many concrete cases

https://twitter.com/YutakangE

Grand Challenge: Abstract Abstraction

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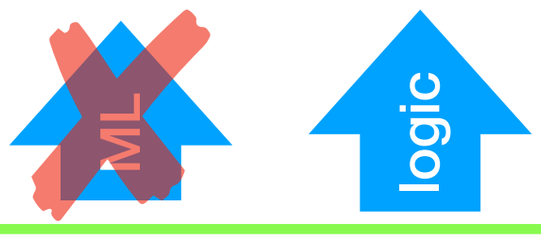
<- one abstract representation



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<- many concrete cases

https://twitter.com/YutakangE

Grand Challenge: Abstract Abstraction

```
lemma "star r x y ⟹ star r y z ⟹ star r x z"
by(induction rule: star.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
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by(induct is1 s stk rule:exec.induct) auto

lemma "itrev xs ys = rev xs @ ys"
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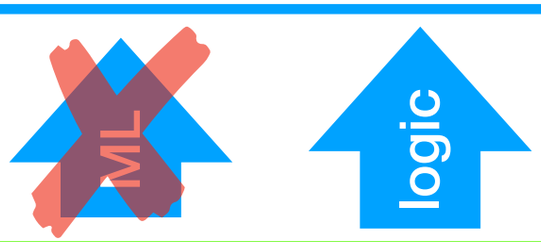
```

<- small dataset about different domains

<- one abstract representation



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https://twitter.com/YutakangE

Grand Challenge: Abstract Abstraction

ML

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lemma "itrev xs ys = rev xs @ ys"
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<- small dataset about different domains

<- one abstract representation

~~ML~~

logic

<- abstraction using expressive logic

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<- many concrete cases

https://twitter.com/YutakangE

Grand Challenge: Abstract Abstraction

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract

ML

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lemma "star r x y ⟹ star r y z ⟹ star r x z"
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Grand Challenge: Abstract Abstraction

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

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<- pros: good at ambiguity (heuristics)

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ML

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Grand Challenge: Abstract Abstraction

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<- even more abstract

ML

<- pros: good at ambiguity (heuristics)

<- cons: bad at reasoning & abstraction

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ML

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<- many concrete cases

https://twitter.com/YutakangE

Many key challenges remain

Unsupervised Learning

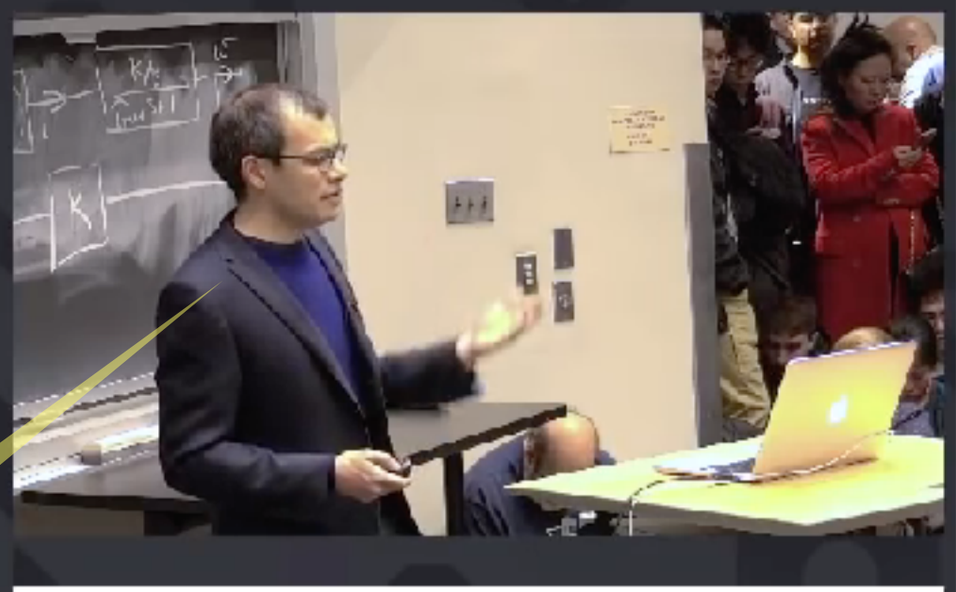
Memory and one-shot learning

Imagination-based Planning with Generative Models

Learning Abstract Concepts

Transfer Learning

Language understanding



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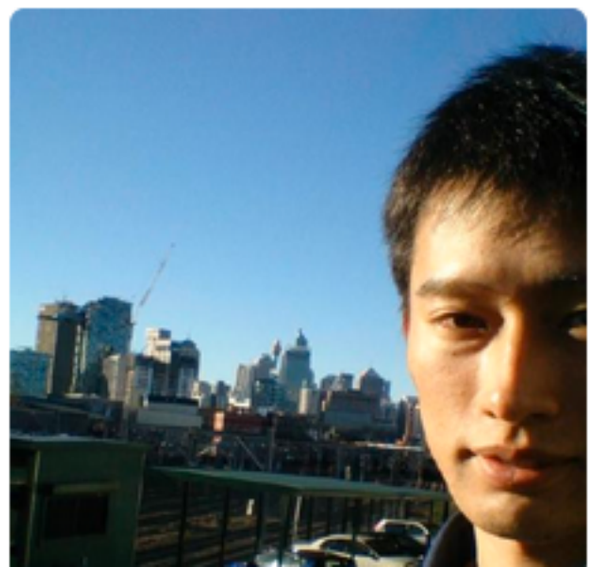
March 20, 2019

The Power of

Self



https://cbmm.mit.edu/video/power-self-learning-systems

https://twitter.com/YutakangE

Many key challenges remain

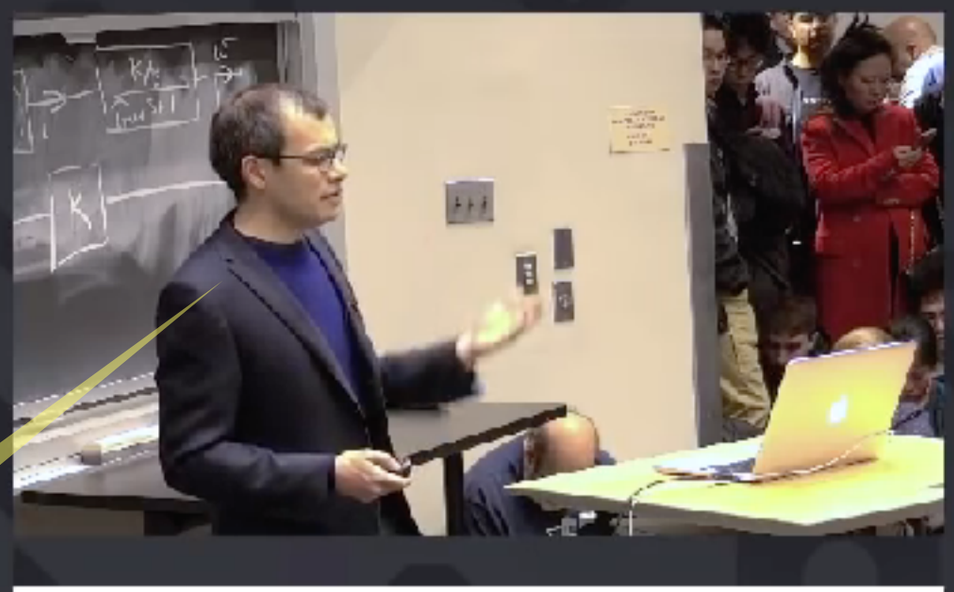
Unsupervised Learning

Memory and one-shot learning

Imagination-based Planning with Generative Models

Learning Abstract Concepts

Abstract concepts?



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March 20, 2019

The Power of

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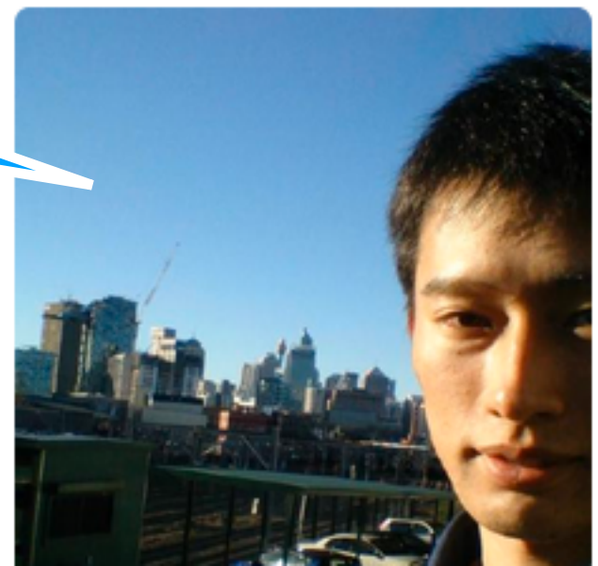

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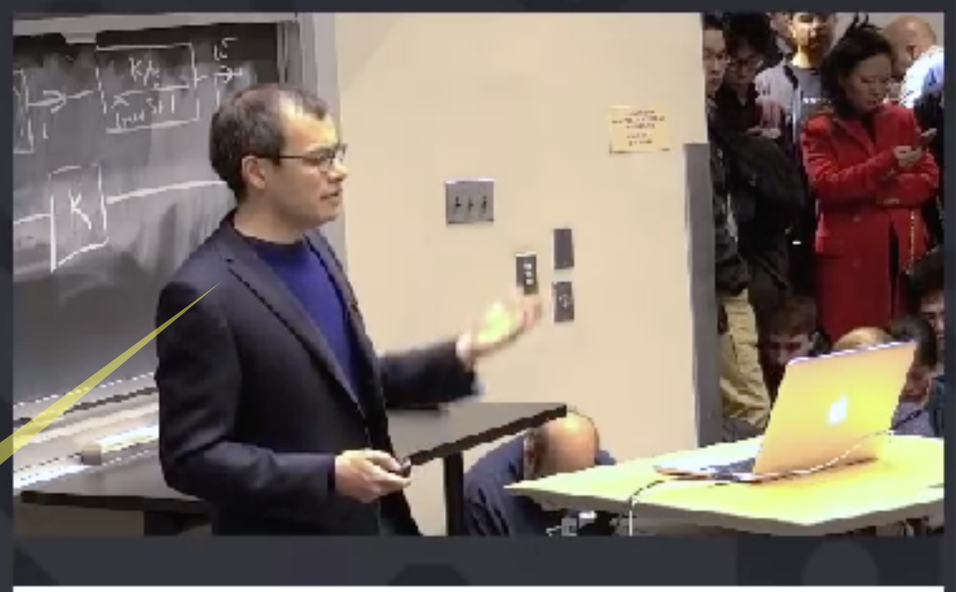

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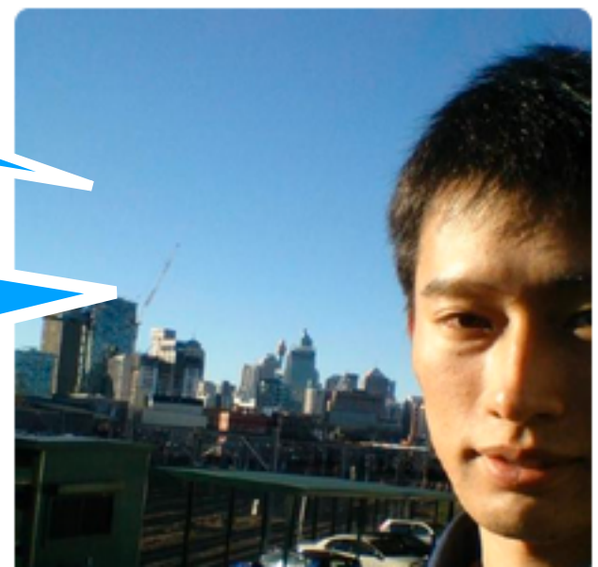
March 20, 2019

Abstract concepts?

why not logic? (LiFtEr)

The Power of

Self

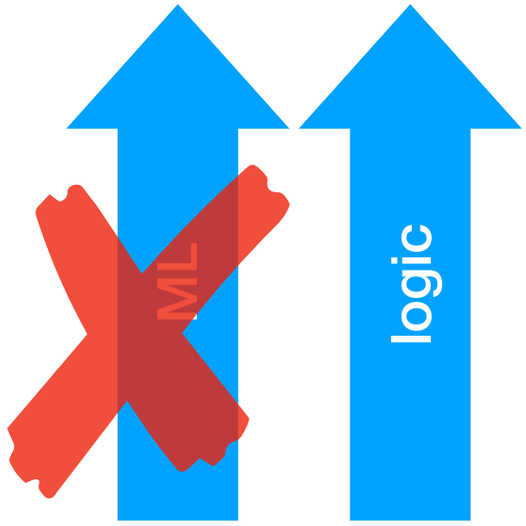


https://cbmm.mit.edu/video/power-self-learning-systems

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract

Big Picture



abstraction using
another logic (LiFtEr)

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<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

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<- many concrete cases

Abstract notion of “good” application of induction.
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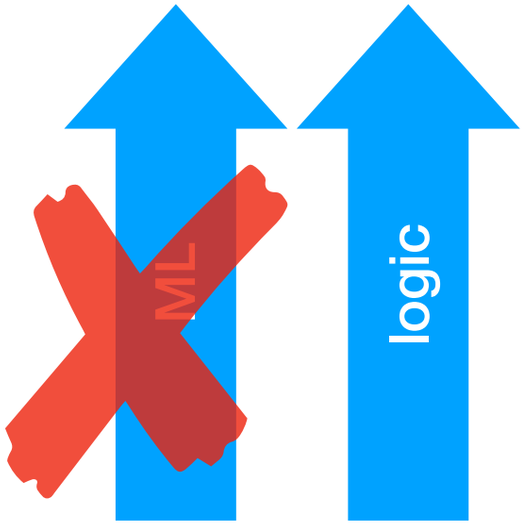
Big Picture



abstraction using
another logic (LiFtEr)

<- pros: good at rigorous abstraction

```
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lemma "itrev xs ys = rev xs @ ys"
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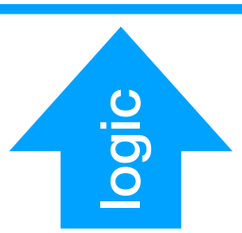
```

<- small dataset about different domains

<- one abstract representation



<- abstraction using expressive logic

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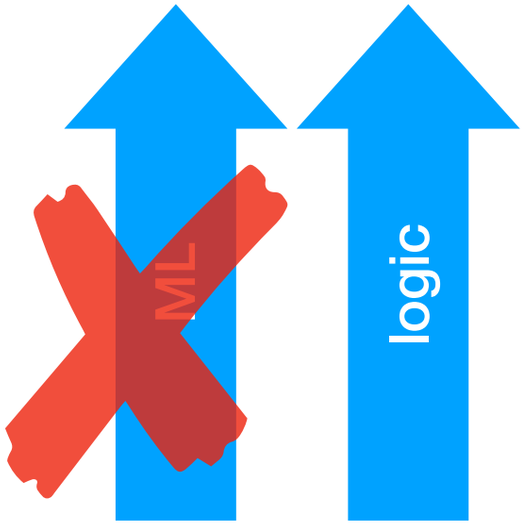
<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract

Big Picture



abstraction using another logic (LiFtEr)

<- pros: good at rigorous abstraction
<- cons: bad at ambiguity (heuristics)

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lemma "star r x y ⟹ star r y z ⟹ star r x z"
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logic

<- abstraction using expressive logic

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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

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Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

LiFtEr

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Big Picture

LiFtEr

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Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

LiFtEr

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LiFtEr

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LiFtEr

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logic

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Heuristics that are valid across problem domains.

ML

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LiFtEr 1

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Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

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LiFtEr

LiFtEr 2

<- pros: good at rigorous abstraction

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lemma "star r x y ⟹ star r y z ⟹ star r x z"
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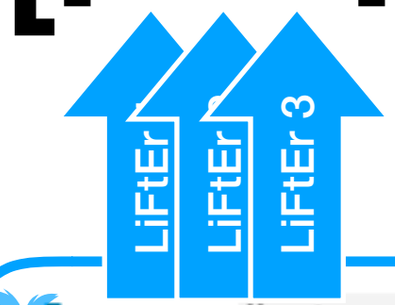

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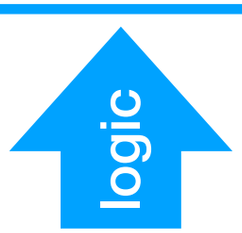

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Abstract notion of “good” application of induction.
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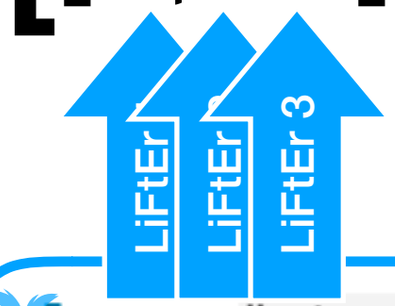

[[T,], [ ], [ ]]: bool list

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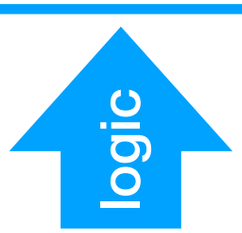

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Abstract notion of “good” application of induction.
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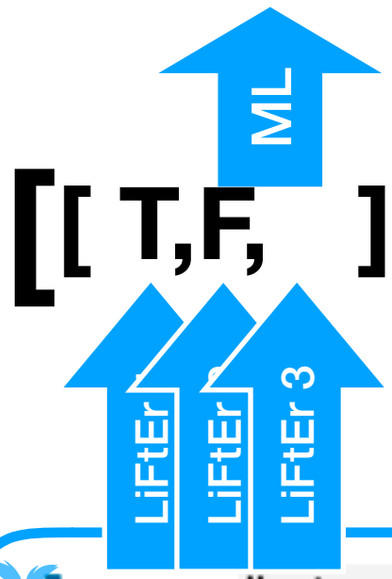

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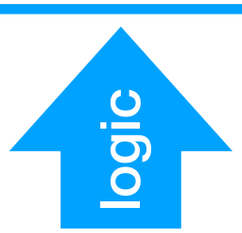

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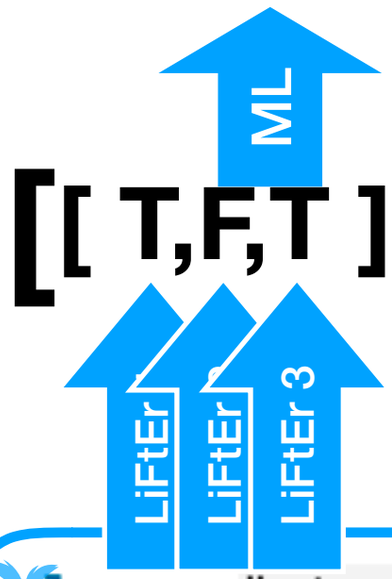
Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture



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Heuristics that are valid across problem domains.

Big Picture

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ML

[[T,F,T], [T,T,T], [ ]]: bool list <- simple representation

LiFtEr 3 LiFtEr 2 LiFtEr 1 LiFtEr 3 LiFtEr 2 LiFtEr 1

<- pros: good at rigorous abstraction

```
lemma "star r x y ∧ star r y z ⟹ star r x z"
by(induction r rule:star.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
       exec is2 s (exec is1 s stk)"
by(induct is1 s stk rule:exec.induct) auto

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LiFtEr

LiFtEr

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

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lemma "star r                star                ⟹ star r x z"
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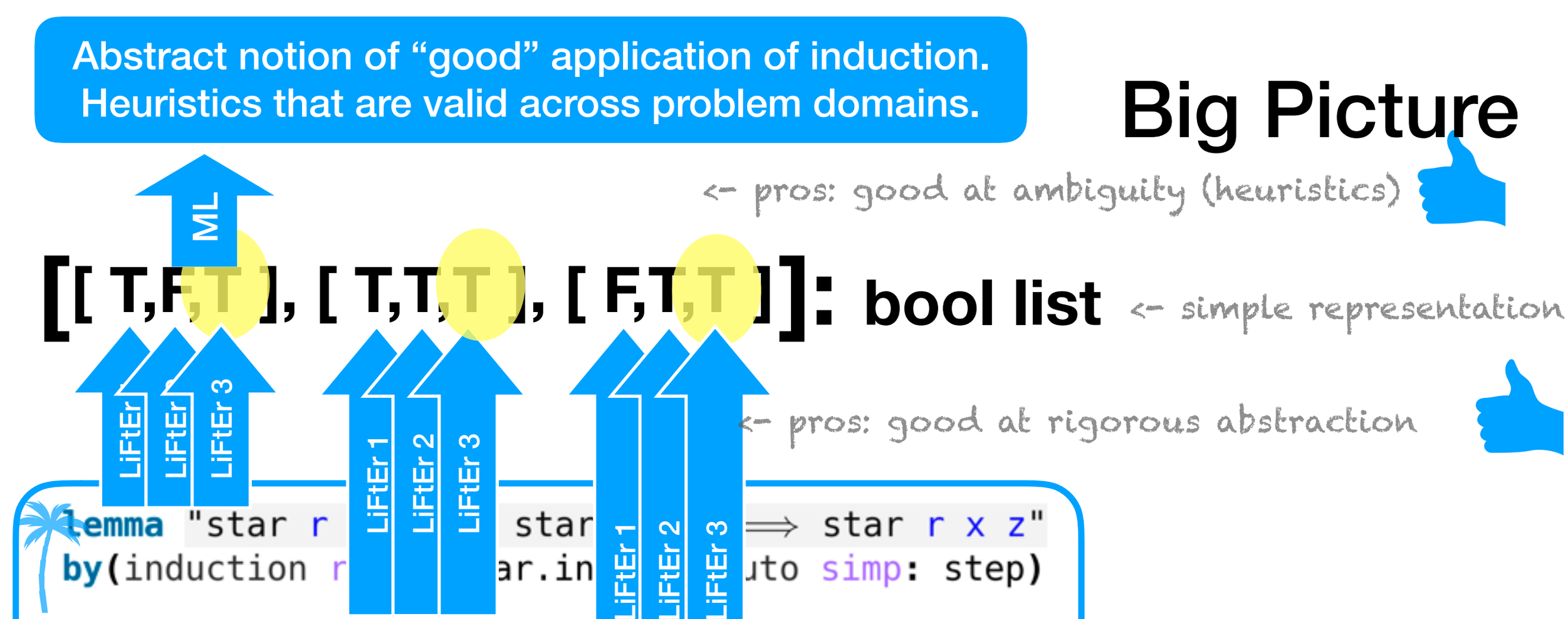

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LiFtEr

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LiFtEr 3

<- pros: good at rigorous abstraction

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LiFtEr 2

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example?

<- small dataset about different domains

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lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

https://twitter.com/YutakangE

Example Assertion in LiFtEr (in Abstract Syntax)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

https://twitter.com/YutakangE

Example Assertion in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

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Example Assertion in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧                                  conjunction
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
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Example Assertion in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
   ∃ r1 : rule.
     ∃ t1 : term.
       ∃ to1 : term_occurrence ∈ t1 : term.
           r1 is_rule_of to1
         ∧
           ∀ t2 : term ∈ induction_term.
             ∃ to2 : term_occurrence ∈ t2 : term.
               ∃ n : number.
                   is_nth_argument_of (to2, n, to1)
                 ∧
                   t2 is_nth_induction_term n
```

variable for auxiliary lemmas

conjunction

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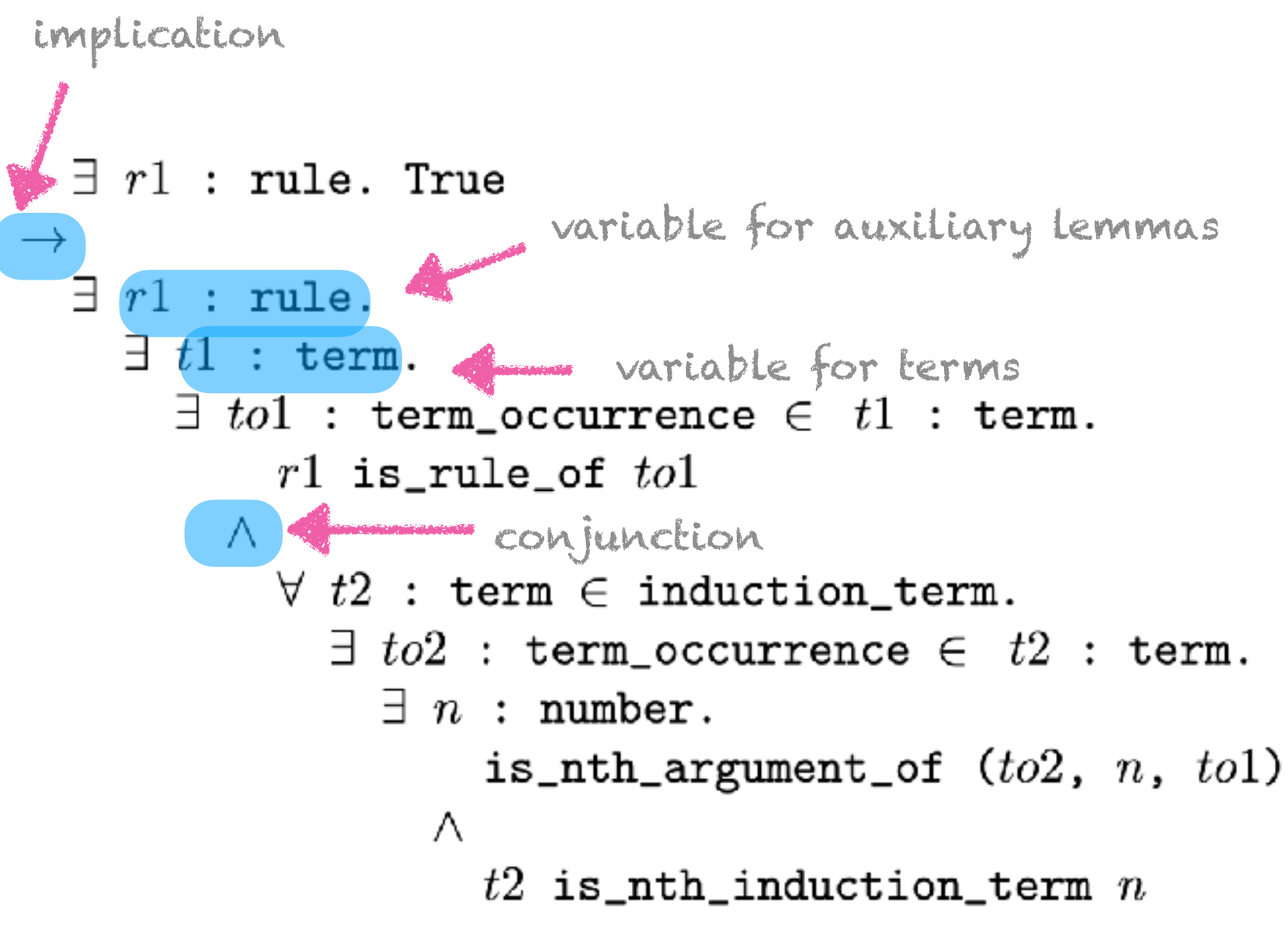
Example Assertion in LiFtEr (in Abstract Syntax)



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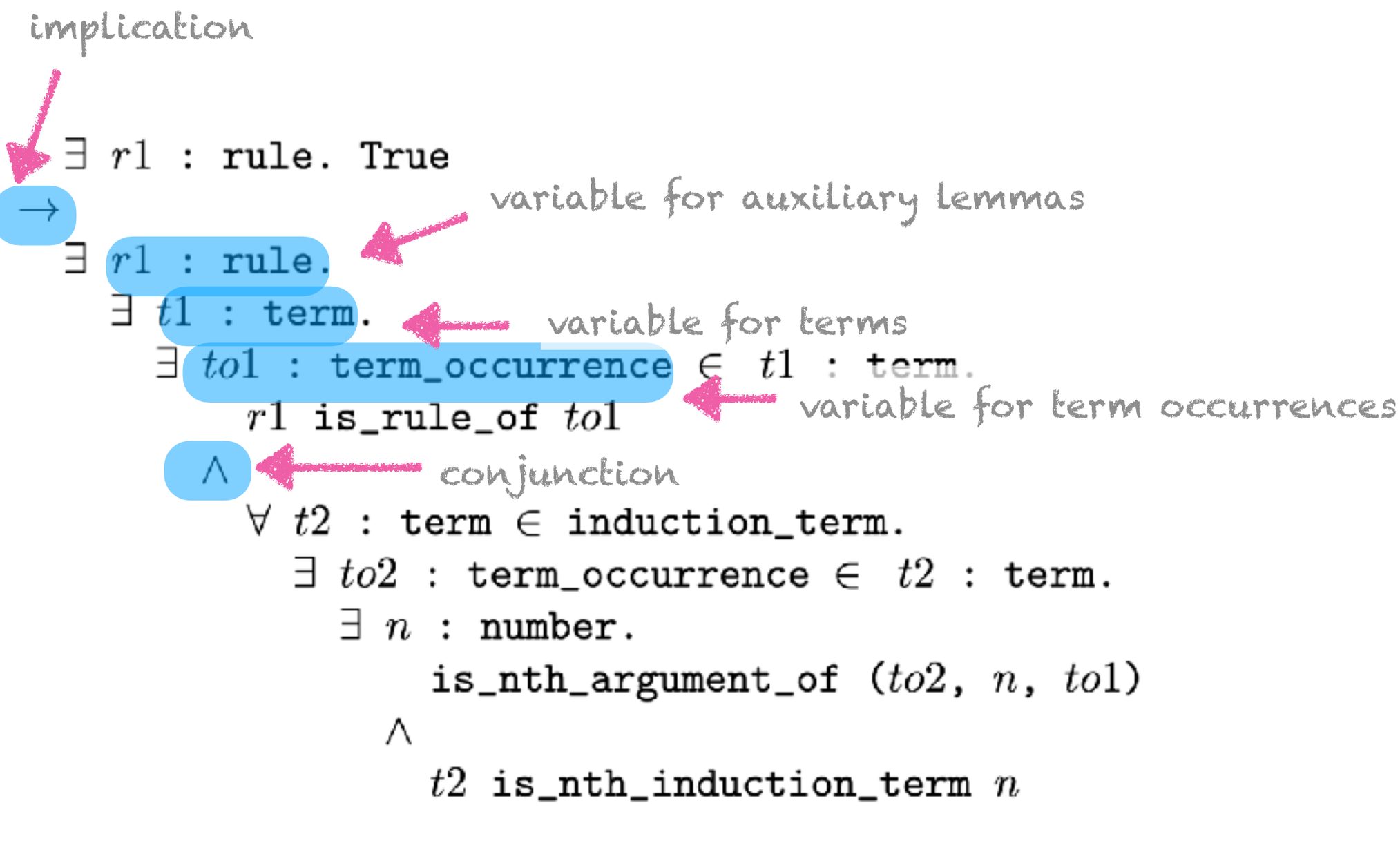
Example Assertion in LiFtEr (in Abstract Syntax)



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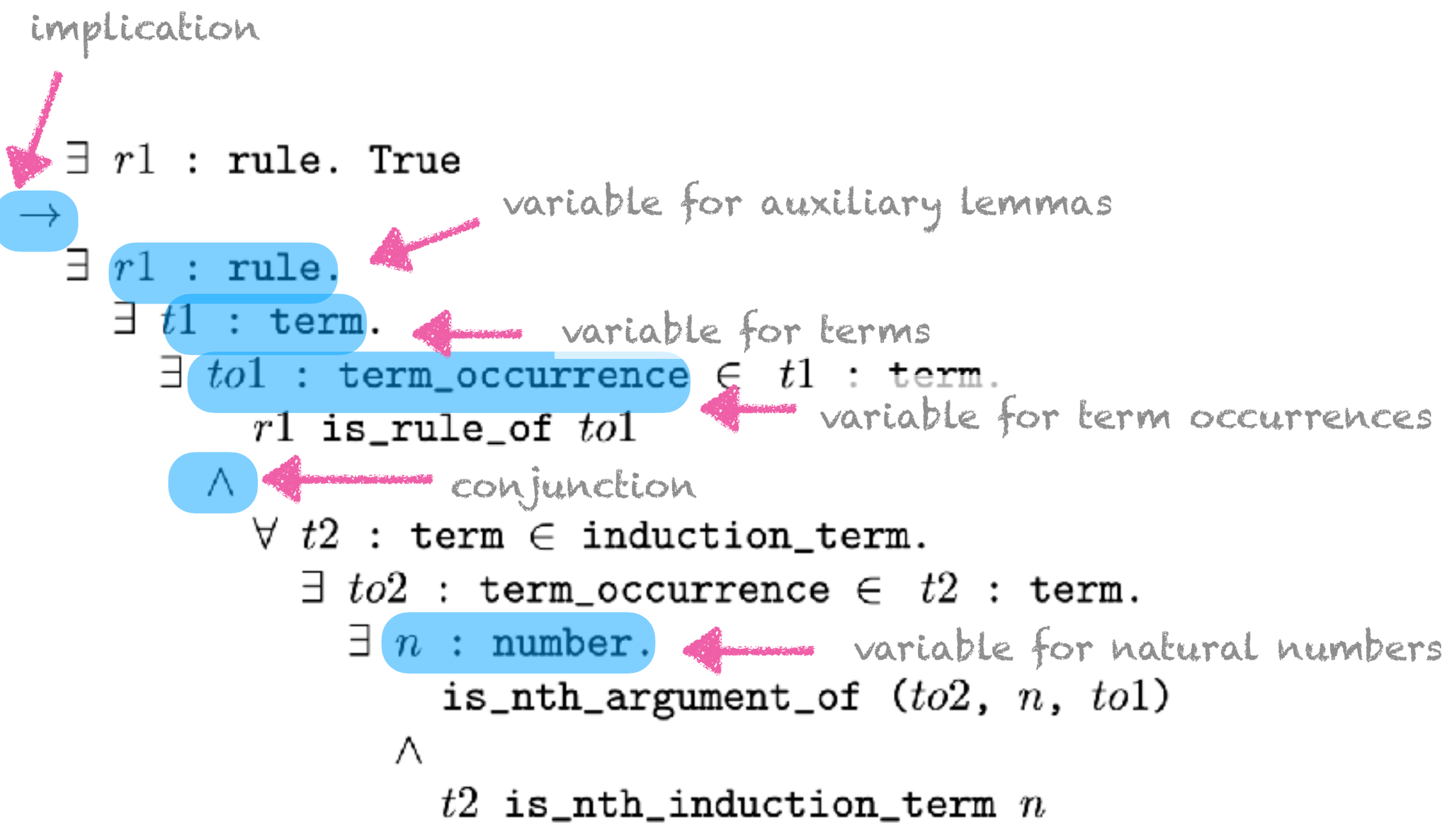

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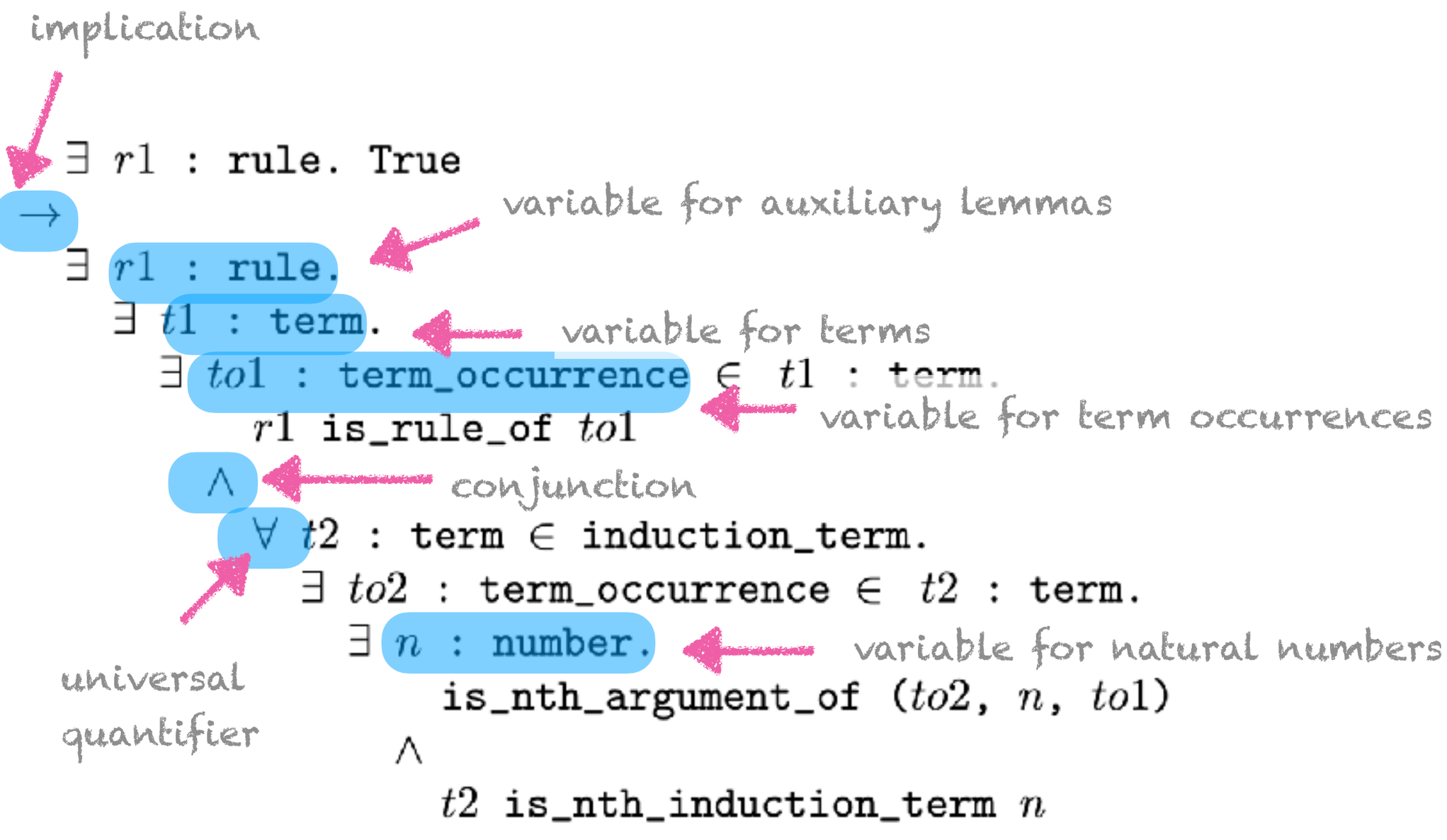
Example Assertion in LiFtEr (in Abstract Syntax)



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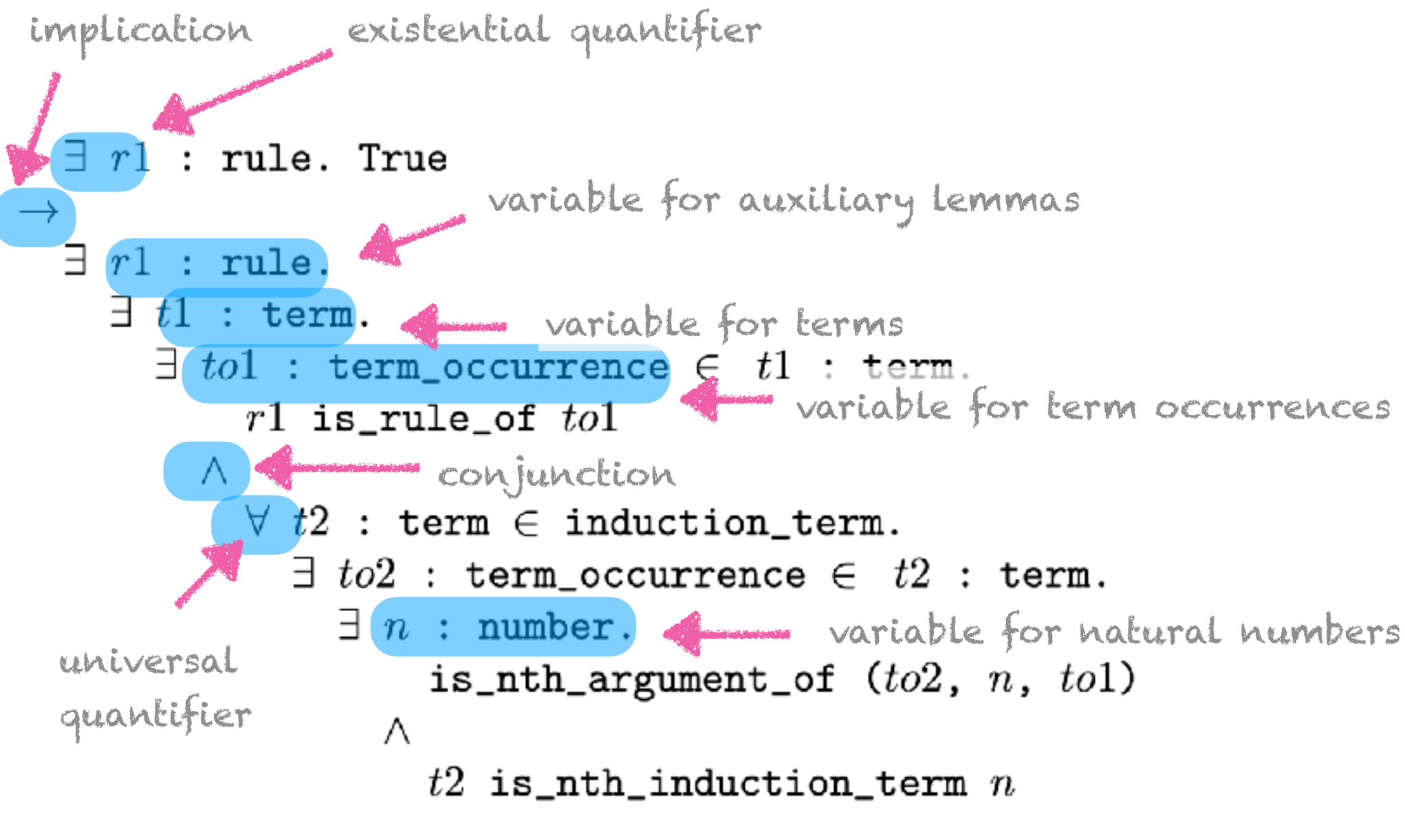
Example Assertion in LiFtEr (in Abstract Syntax)



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```
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 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

$\exists$ $r1$: rule. True

$\rightarrow$

$\exists$ $r1$: rule.
  $\exists$ $t1$: term.
    $\exists$ $to1$: term_occurrence $\in$ $t1$: term.
      $r1$ is_rule_of $to1$
    $\wedge$
      $\forall$ $t2$: term $\in$ induction_term.
        $\exists$ $to2$: term_occurrence $\in$ $t2$: term.
          $\exists$ n : number.
            is_nth_argument_of ($to2$, n, $to1$)
          $\wedge$
            $t2$ is_nth_induction_term n

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```
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
```

→ r1

(r1 = itrev.induct)

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
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```
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lemma "itrev xs ys = rev xs @ ys"
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  apply auto done
```

∃ r1 : rule. True

→ r1

∃ r1 : rule.

(r1 = itrev.induct)

∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
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```
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∃ r1 : rule. True
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∃ r1 : rule.
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      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
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(r1 = itrev.induct)

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```
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→
∃ r1 : rule.
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          ∧
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```

t1 r1

(r1 = itrev.induct)
(t1 = itrev)

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```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

t1 r1

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

```
∃ r1 : rule. True
→
 ∃ r1 : rule.
  ∃ t1 : term.
   ∃ to1 : term_occurrence ∈ t1 : term.
     r1 is_rule_of to1
   ∧
     ∀ t2 : term ∈ induction_term.
       ∃ to2 : term_occurrence ∈ t2 : term.
         ∃ n : number.
             is_nth_argument_of (to2, n, to1)
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t1 r1

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
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  apply auto done
```

t1 r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                    ( r1 = itrev.induct )
    ∃ t1 : term.                                  ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.        ( to1 = itrev )
        r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
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```

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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to1

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  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

t1 r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                    ( r1 = itrev.induct )
    ∃ t1 : term.                                  ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.        ( to1 = itrev )
        r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
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```

t1 r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                    ( r1 = itrev.induct )
    ∃ t1 : term.                                  ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.        ( to1 = itrev )
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
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to1

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lemma "itrev xs ys = rev xs @ ys"
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```

t1 r1

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
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      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
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```

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

t2

t1 r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                          ( r1 = itrev.induct )
    ∃ t1 : term.                                        ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.              ( to1 = itrev )
          r1 is_rule_of to1   True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
        ∧
          ∀ t2 : term ∈ induction_term.                          ( t2  = xs and ys )
            ∃ to2 : term_occurrence ∈ t2 : term.
                ∃ n : number.
                    is_nth_argument_of (to2, n, to1)
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  apply auto done
```

to1 to2 t2 t1 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
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∃ n : number.
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to1 to2 t2 t1 r1

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→

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∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
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∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1)
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→

∃ r1 : rule. (r1 = itrev.induct)
  ∃ t1 : term. (t1 = itrev)
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    ∧
      ∀ t2 : term ∈ induction_term. (t2 = xs and ys)
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first to2 t2 first t1 r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                          ( r1 = itrev.induct )
    ∃ t1 : term.                                        ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.              ( to1 = itrev )
          r1 is_rule_of to1   True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
        ∧
          ∀ t2 : term ∈ induction_term.                 ( t2  = xs and ys )
            ∃ to2 : term_occurrence ∈ t2 : term.        ( to2 = xs and ys )
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)      when t2 is xs (n = 1) ?
                ∧
                  t2 is_nth_induction_term n
```

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```
primrec rev :: "'a list ⇒ 'a list" where
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to1 first to2 t2 first t1 r1

```
∃ r1 : rule. True
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  ∃ r1 : rule.
    ∃ t1 : term.
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          r1 is_rule_of to1
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            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

(t2 = xs and ys)
(to2 = xs and ys)

when t2 is xs (n = 1)

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```
primrec rev :: "'a list ⇒ 'a list" where
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first second to2 t2 second first t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
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(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

(t2 = xs and ys)
(to2 = xs and ys)

when t2 is xs (n = 1) ?

when t2 is ys (n = 2) ?

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```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first second to2

t2 second t1 r1

first

∃ r1 : rule. True

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1 ∧
    ∀ t2 : term ∈ induction_term.
     ∃ to2 : term_occurrence ∈ t2 : term.
      ∃ n : number.
       is_nth_argument_of (to2, n, to1) ∧
       t2 is_nth_induction_term n

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

(t2 = xs and ys)
(to2 = xs and ys)

when t2 is xs (n = 1)

when t2 is ys (n = 2)

https://twitter.com/YutakangE

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first second to2 t2 second first t1 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1)
∧
t2 is_nth_induction_term n when t2 is ys (n = 2)

$\exists$ $r1$: rule. True

$\rightarrow$

$\exists$ $r1$: rule.
$\exists$ $t1$: term.
$\exists$ $to1$: term_occurrence $\in$ $t1$: term.
$r1$ is_rule_of $to1$
$\wedge$
$\forall$ $t2$: term $\in$ induction_term.
$\exists$ $to2$: term_occurrence $\in$ $t2$: term.
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

the same LiFtEr assertion

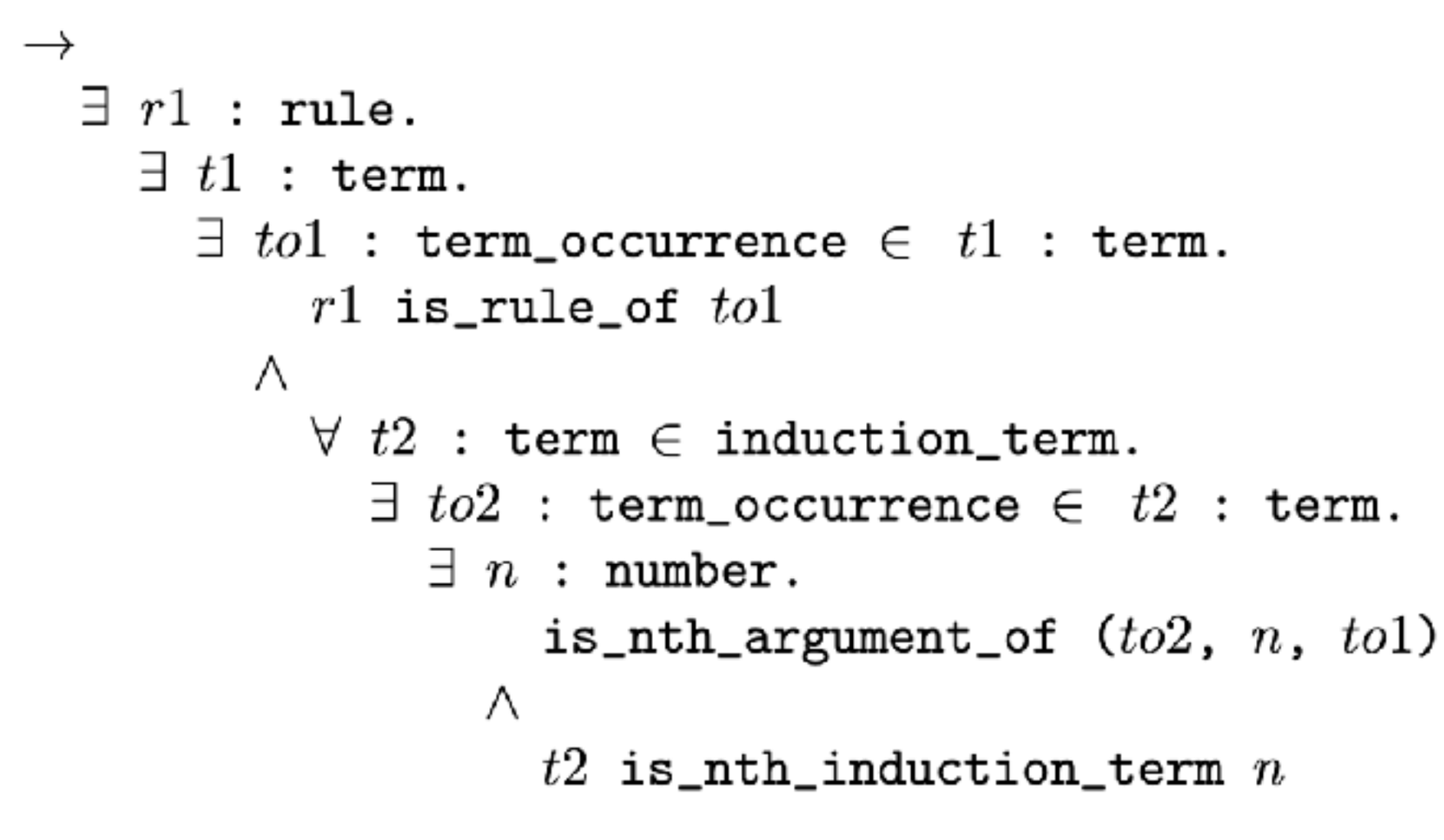
```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

```
datatype      instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
```



```
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

∃ r1 : rule. True

r1

(r1 = exec.induct)

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

r1

(r1 = exec.induct)

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

r1

(r1 = exec.induct)

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
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 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

t1 r1

∃ r1 : rule.
  ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
    r1 is_rule_of to1
  ∧
    ∀ t2 : term ∈ induction_term.
      ∃ to2 : term_occurrence ∈ t2 : term.
        ∃ n : number.
          is_nth_argument_of (to2, n, to1)
        ∧
          t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
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fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
  ∧
   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
     ∧
      t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

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 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

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 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

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 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

∃ $r1$: rule.
  ∃ $t1$: term.
    ∃ $to1$: term_occurrence ∈ $t1$: term.
      $r1$ is_rule_of $to1$
    ∧
      ∀ $t2$: term ∈ induction_term.
        ∃ $to2$: term_occurrence ∈ $t2$: term.
          ∃ n : number.
            is_nth_argument_of ($to2$, n, $to1$)
          ∧
            $t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)  =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
  ∧
   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
     ∧
      t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 t1 r1

∃ r1 : rule. True

→

∃ $r1$: rule.
  ∃ $t1$: term.
    ∃ $to1$: term_occurrence ∈ $t1$: term.
      $r1$ is_rule_of $to1$
    ∧
      ∀ $t2$: term ∈ induction_term.
        ∃ $to2$: term_occurrence ∈ $t2$: term.
          ∃ n : number.
            is_nth_argument_of ($to2$, n, $to1$)
          ∧
            $t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
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new types ->

new constants ->

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 "exec []       _ stk = stk" |
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new lemma ->

a model proof ->

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t2 t01 t02 t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
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    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

(r1 = exec.induct)
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True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
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new types ->

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new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first

t2 t01 first t01 t02

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1 ∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1) ∧ t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first

t2

to1

to2

t1

r1

first

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

∧

∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)

new types ->

new constants ->

new lemma ->

a model proof ->

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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first / second / t2 / to1 / t1 / r1 / to2

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1 ∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1) ∧
t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?
when t2 is s (n -> 2) ?

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first, second, t2, to1, t1, r1, to2

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

∧

∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)

when t2 is s (n -> 2)

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
  ∧
   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
     ∧
      t2 is_nth_induction_term n

first second third t2

first second third to1

t1 r1

to2

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?
when t2 is s (n -> 2) ?
when t2 is stk (n -> 3) ?

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

first, second, third, t1, r1, to1, to2, t2

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
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when t2 is is1 (n -> 1)
when t2 is s (n -> 2)
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new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
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```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
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   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
     ∧
      t2 is_nth_induction_term n

first second third t2

first second third to1

t1 r1

to2

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk) ✓ ✓ ✓
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)
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when t2 is stk (n -> 3)

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

<- pros: good at ambiguity (heuristics)

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in               uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                        )"
by(induct is1 s stk rule:e                        uct) auto
```

<- small dataset about different domains

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]          [] = rev [1,2]          @ []" by auto
lemma "itrev [1,2,3]        [] = rev [1,2,3]        @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]        [] = rev [x,y,z]        @ []" by auto
```

<- many concrete cases

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

[[T,F,T], [T,T,T], [F,T,T]]: bool list

<- pros: good at ambiguity (heuristics)

<- pros: good at rigor

~~<- cons: bad at ambiguity (heuristics)~~

WIP

LiFtEr

LiFtEr

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

```
lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in              uto simp: step)

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lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

https://twitter.com/YutakangE

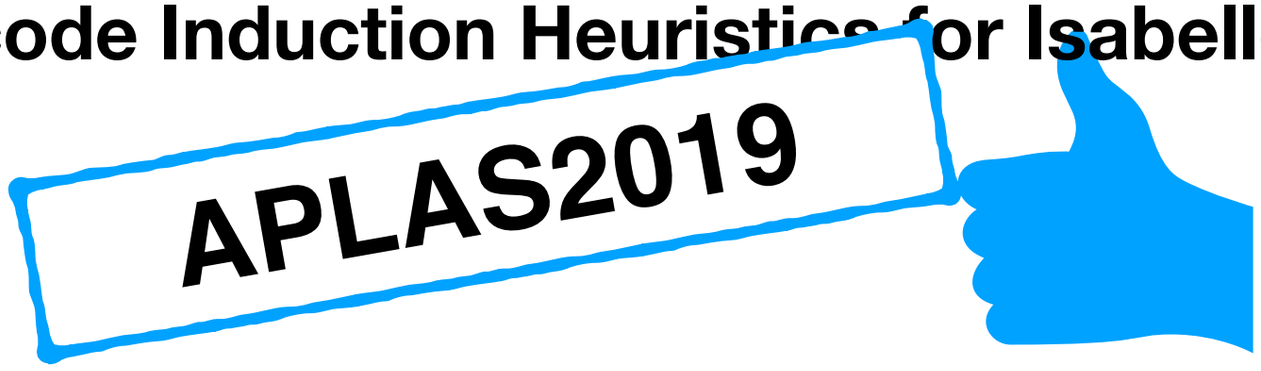
3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

https://twitter.com/YutakangE

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL



https://twitter.com/YutakangE

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

APLAS2019

Secret 1

Many induction heuristics are simple.

https://twitter.com/YutakangE

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

APLAS2019

Secret 1

Many induction heuristics are simple.

Secret 2

Proofs can be more abstract than theorems.

https://twitter.com/YutakangE https://github.com/data61/PSL/slide/2019_ps.pdf

3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

APLAS2019

Secret 1

Many induction heuristics are simple.

Secret 2

Proofs can be more abstract than theorems.

Secret 3

LiFtEr is not good enough because …

LiFtEr almost ignores the definitions of constants.

[T,F,T], [T,T,T], [F,T,T]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in              uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

<- small dataset about different domains

<- one abstract representation

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
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LiFtEr LiFtEr LiFtEr

```
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<- small dataset about different domains

<- one abstract representation

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in              uto simp: step)

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<- small dataset about different domains

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 "itrev   []     ys = ys" |
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```

<- relevant definitions

[T,F,T], [T,T,T], [F,T,T]: bool list <- simple representation

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in              uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr LiFtEr LiFtEr

<- small dataset about different domains

<- one abstract representation

LiFtEr 2 LiFtEr 2

```
lemma "itrev xs ys = rev xs @
by(induct xs ys rule:"itrev.i                ) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

LiFtEr: (proof goal * induction arguments) -> bool

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in             uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr LiFtEr LiFtEr

<- small dataset about different domains

<- one abstract representation

LiFtEr 2 LiFtEr 2

```
lemma "itrev xs ys = rev xs @
by(induct xs ys rule:"itrev.i                ) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev   []       = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in              uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
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LiFtEr LiFtEr LiFtEr

<- small dataset about different domains

<- one abstract representation

LiFtEr 2 LiFtEr 2

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<- nested assertions to examine the "semantics" of constants (rev and itrev)

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 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
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<- relevant definitions

LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

WIP

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fin.