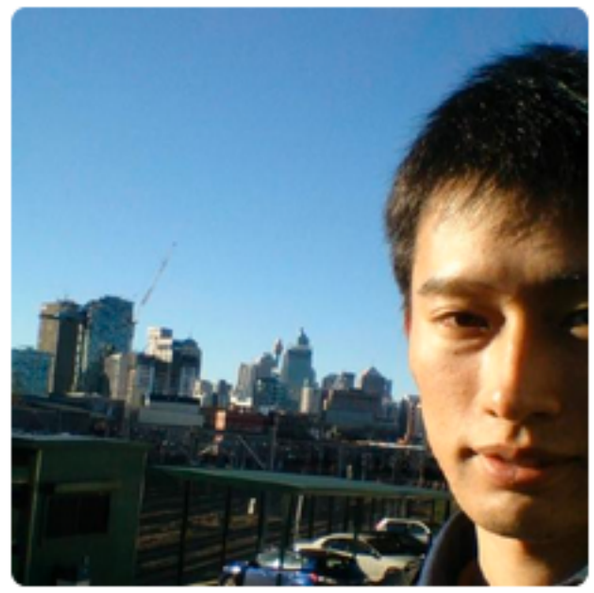
https://twitter.com/YutakangE https://tinyurl.com/up7zhyk

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

This work was supported by the project AI&Reasoning (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000466).

Yutaka Nagashima
University of Innsbruck
Czech Technical University



Yutaka Ng

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slides

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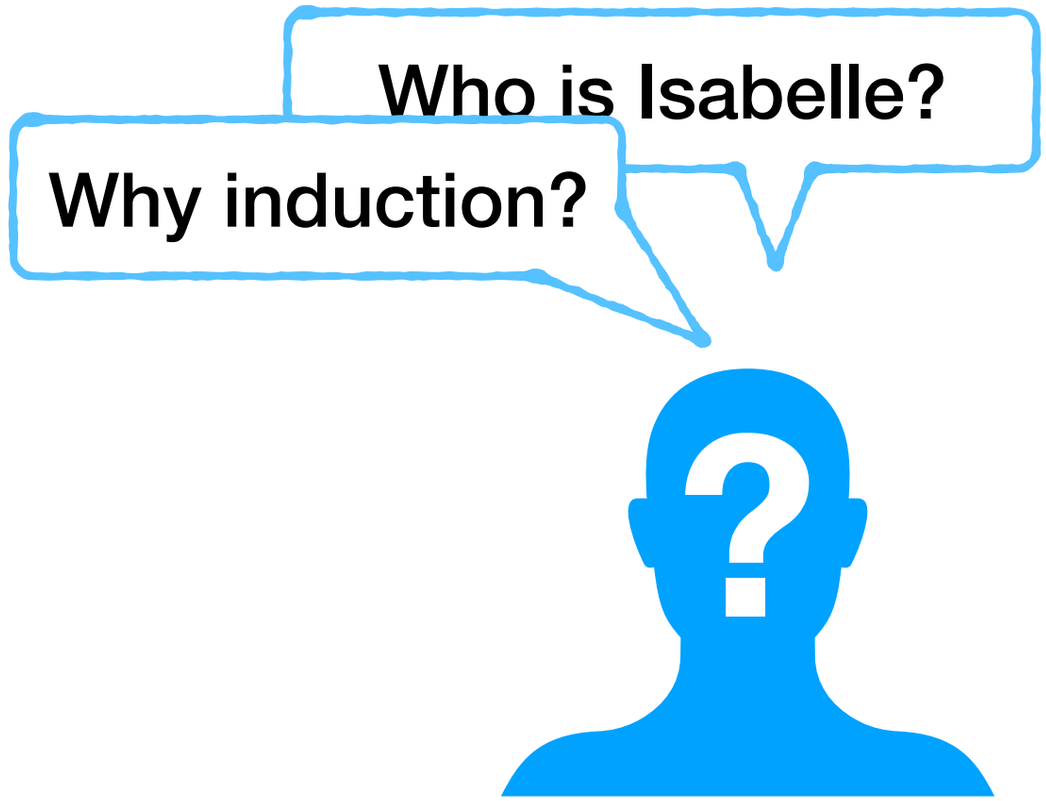

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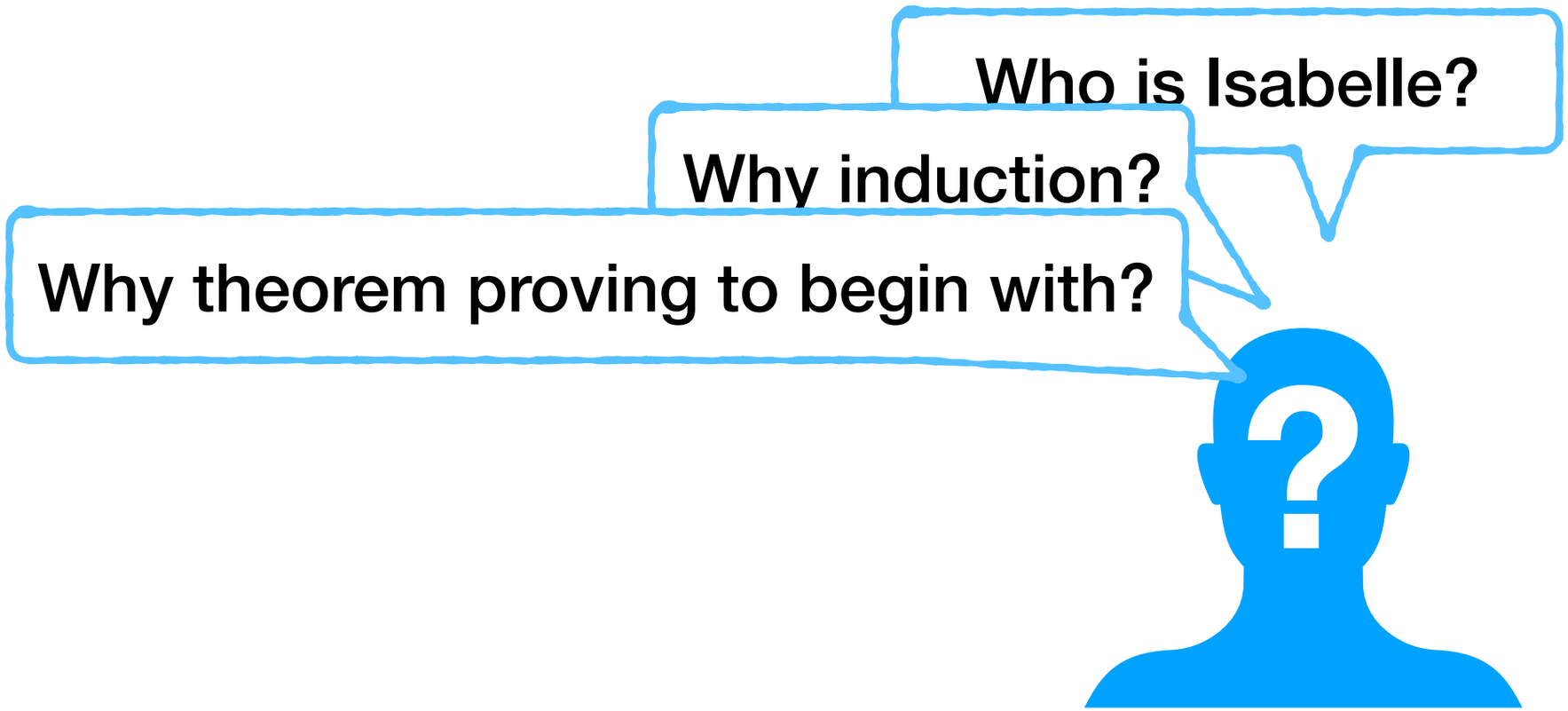

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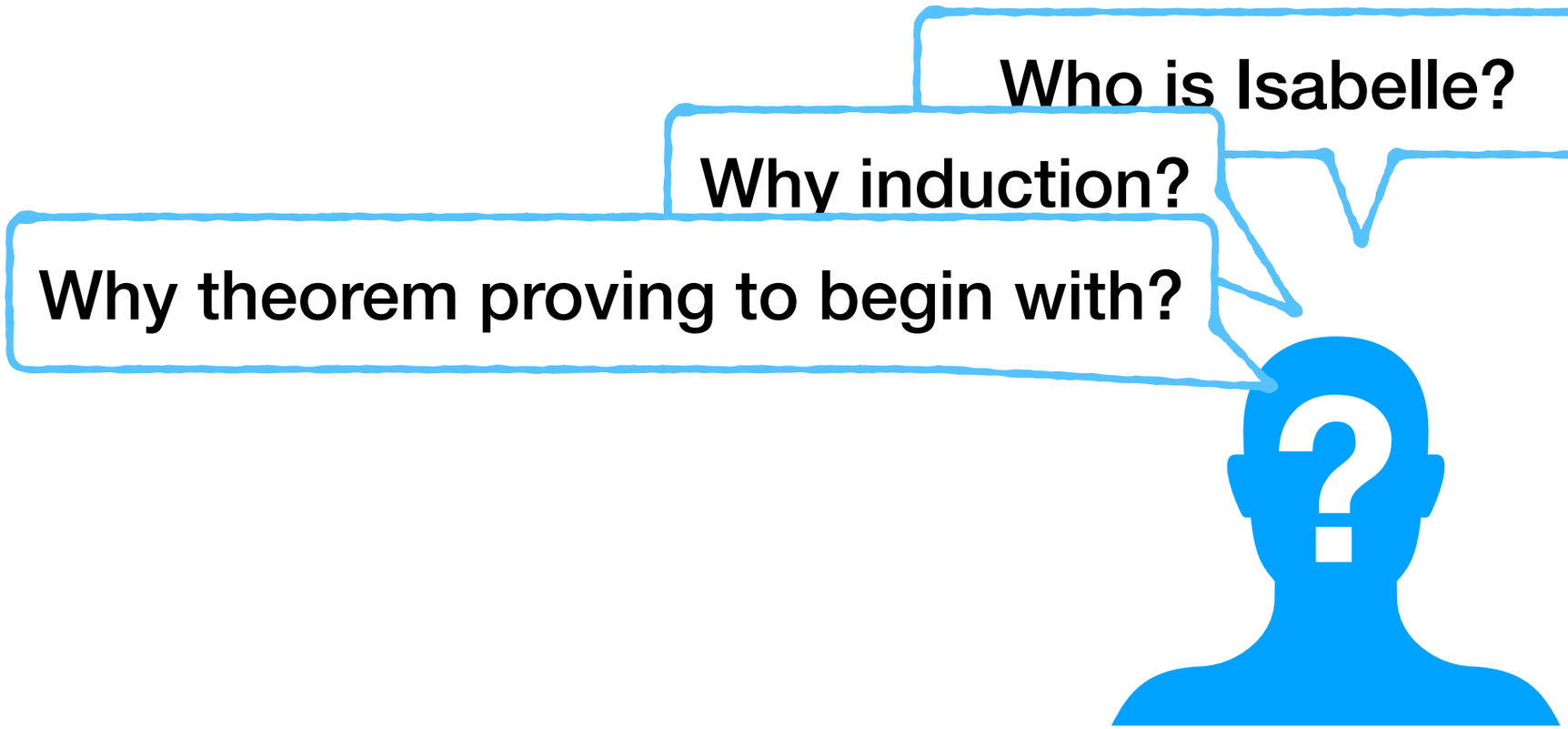
just for fun!



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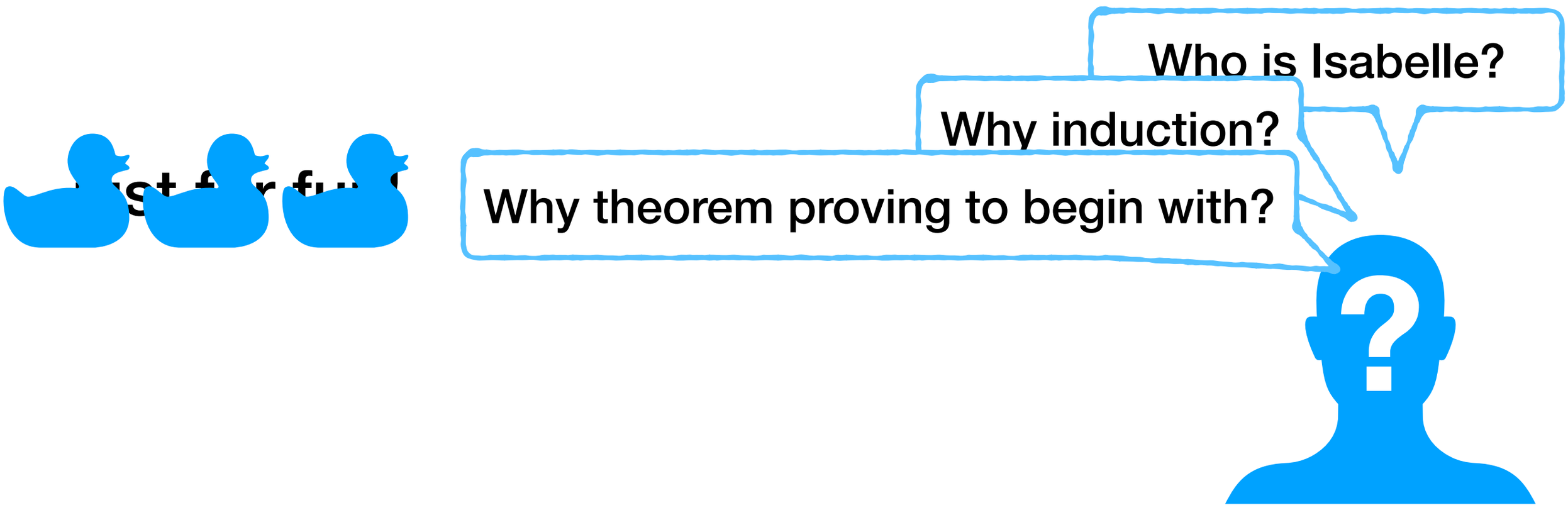

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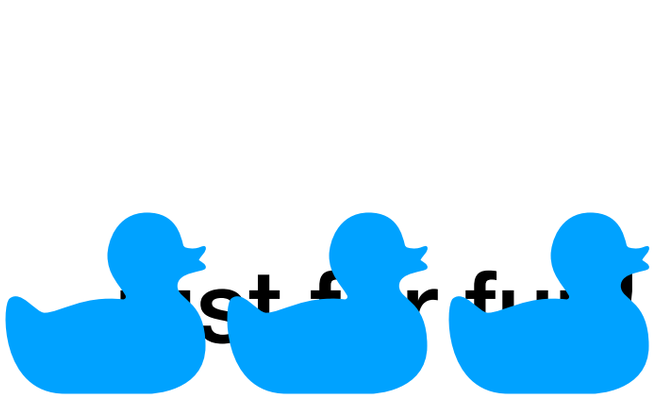

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Why induction?

Why theorem proving to begin with?

for complete formal verification!

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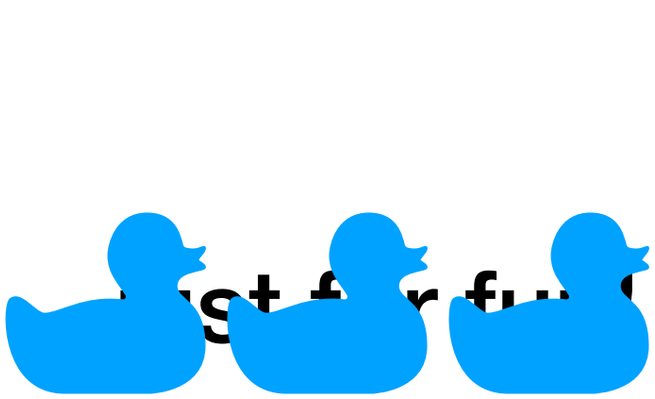

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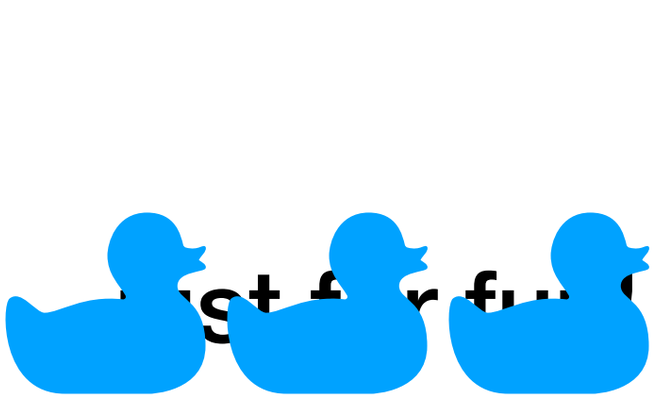

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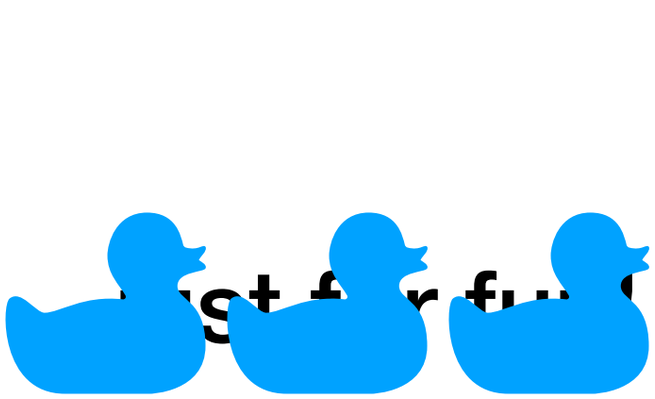

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	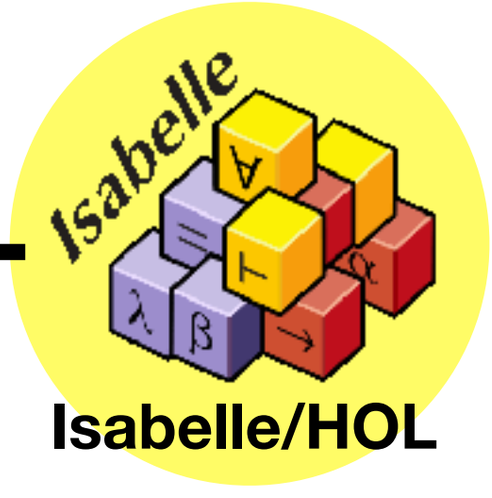	=	
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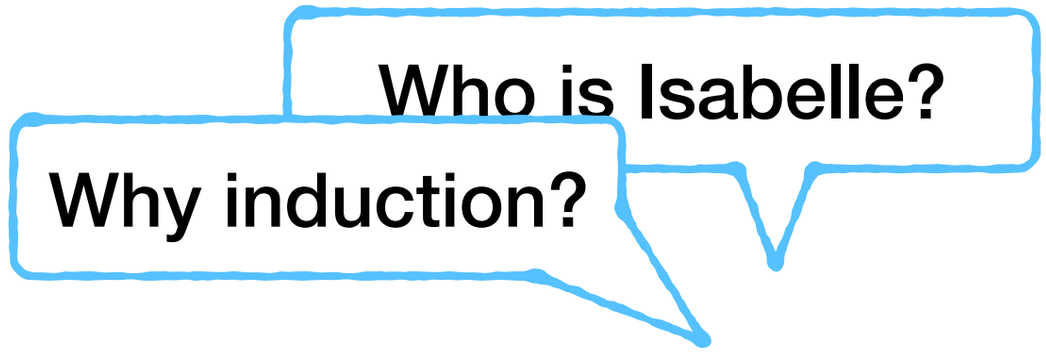

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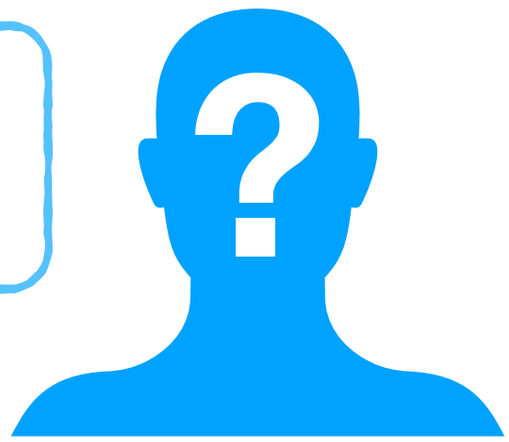

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Strategic Issues, Problems and Challenges in Inductive Theorem Proving

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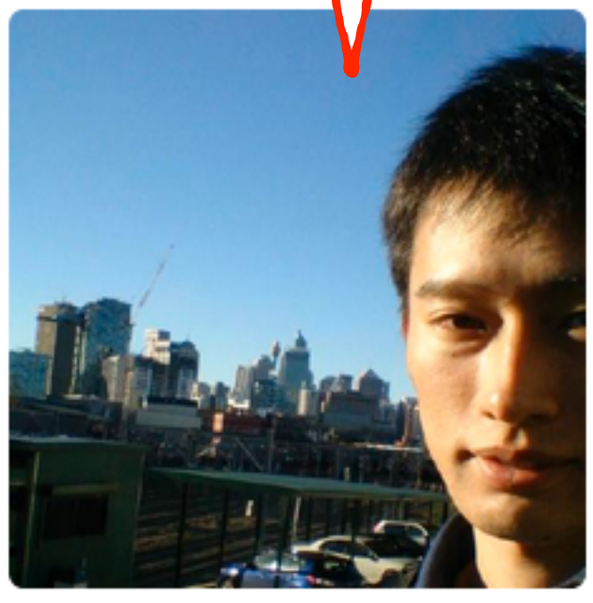

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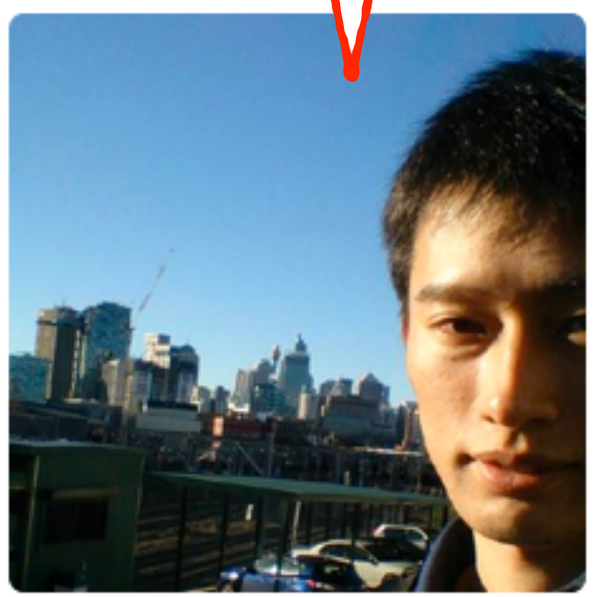

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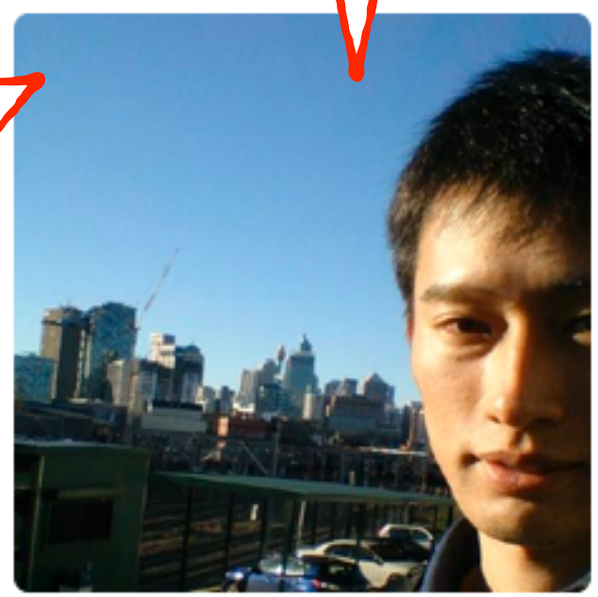
...or is coming soon.



Yutaka Ng
yutakang

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What I did before APLAS2020…

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What I did before APLAS2020…

DEMO!

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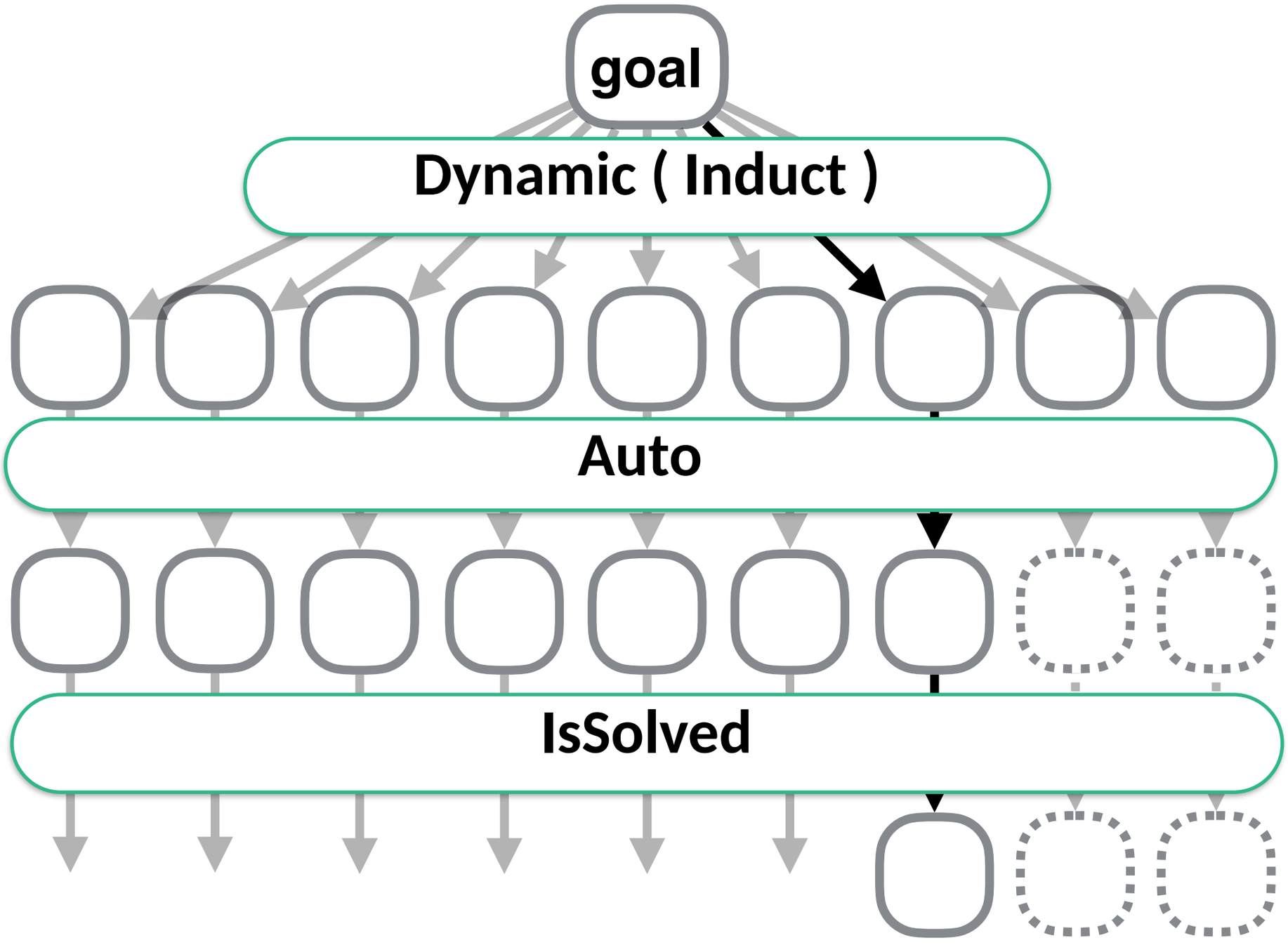

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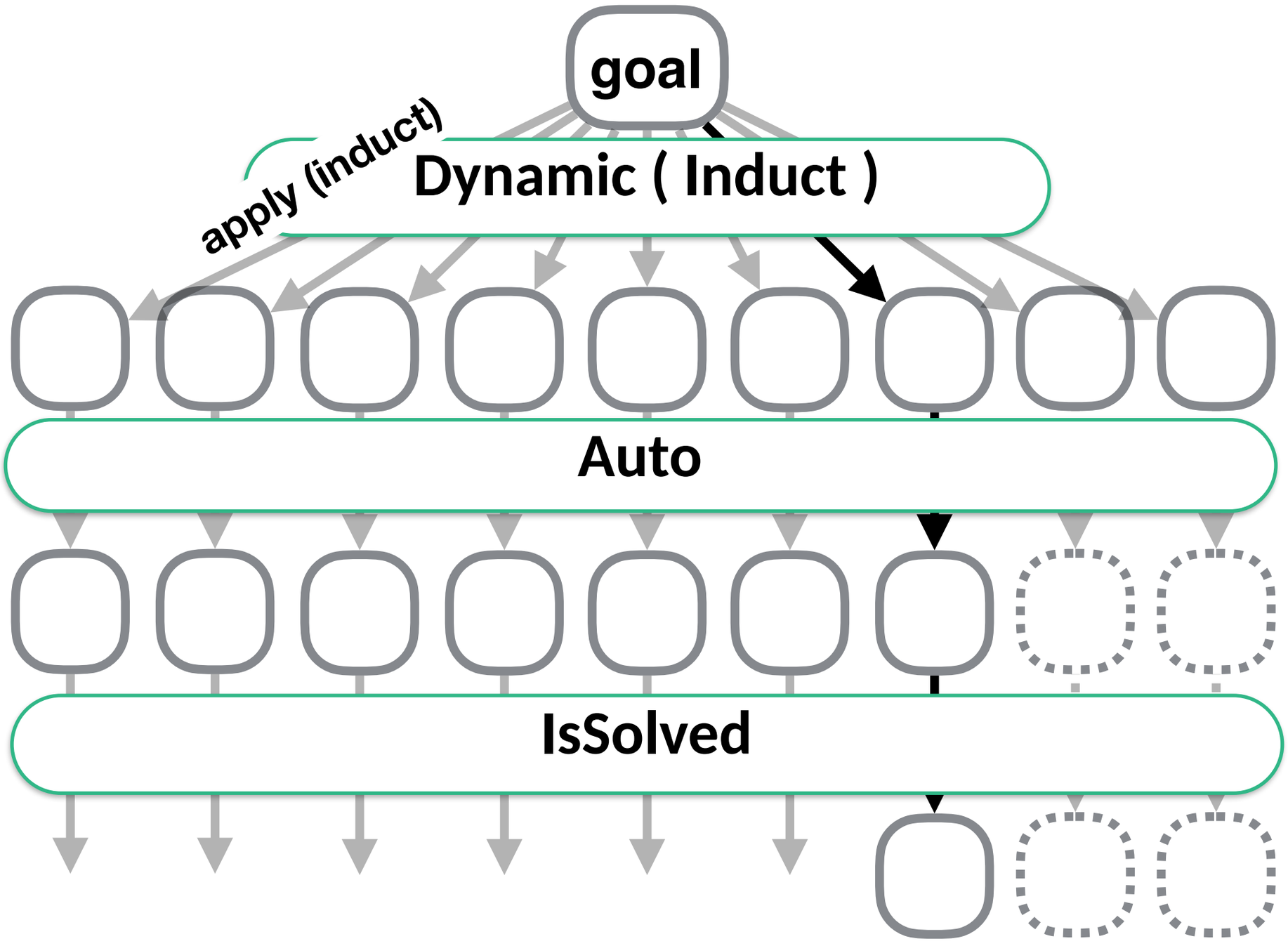

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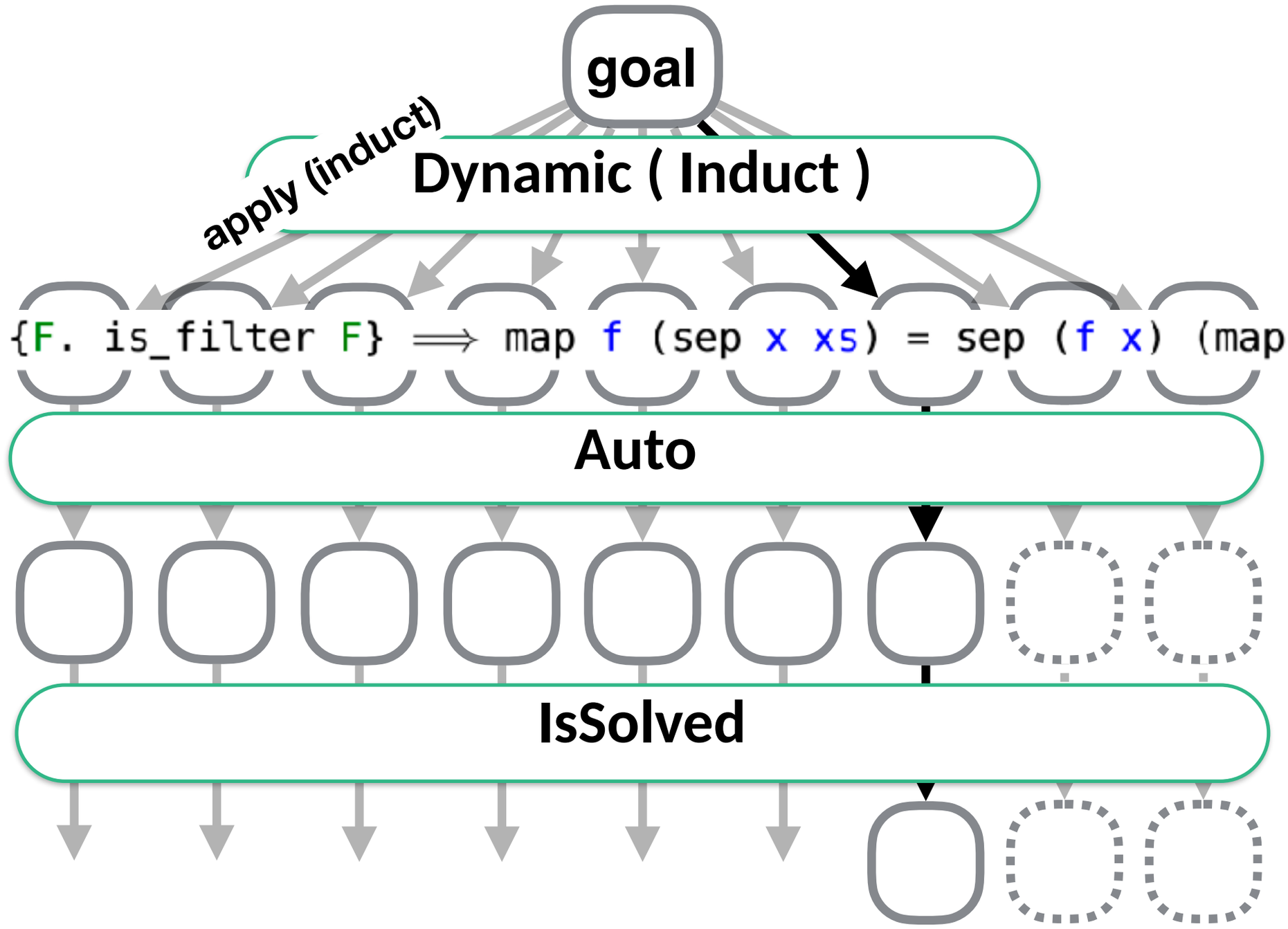

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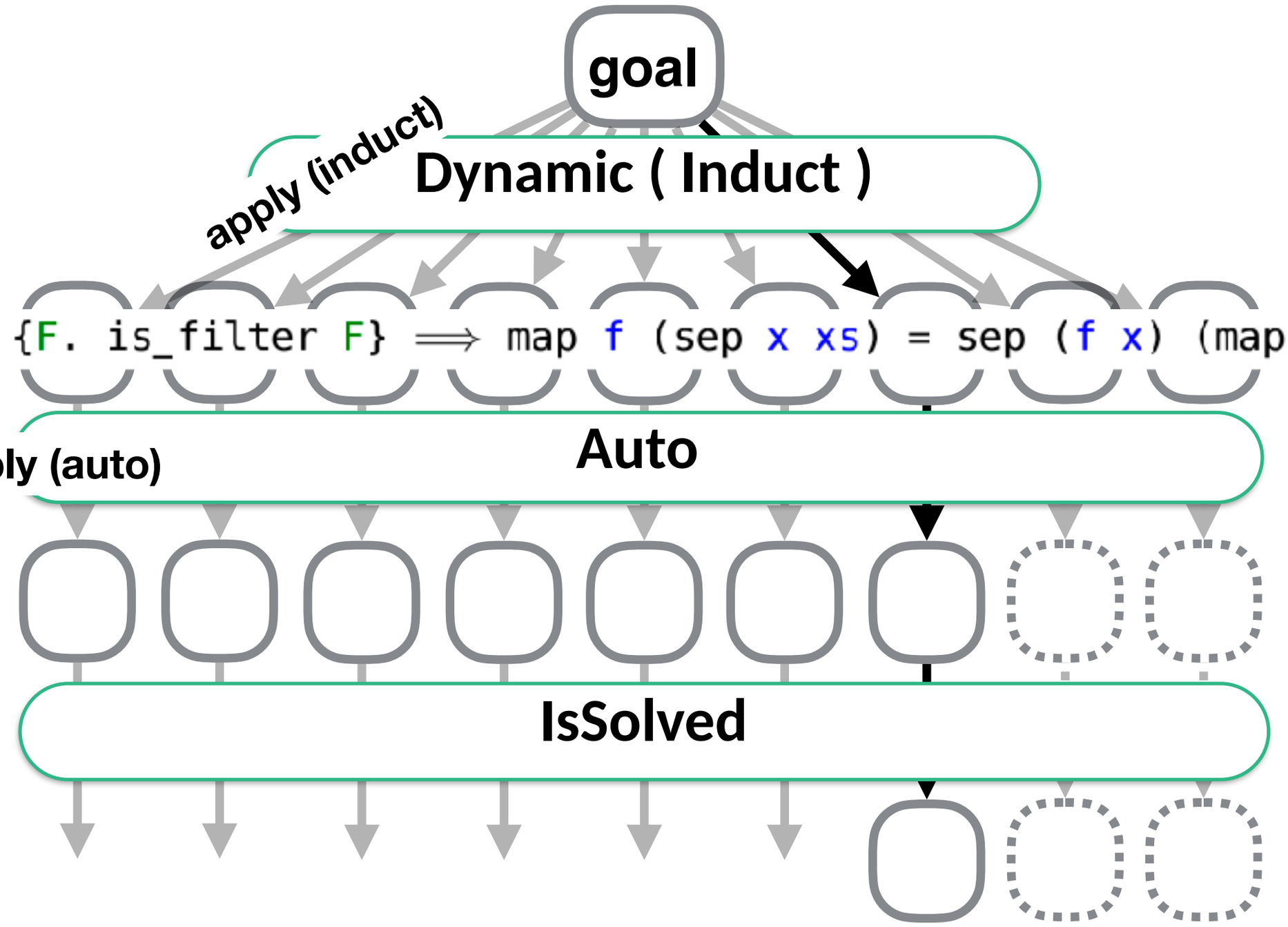

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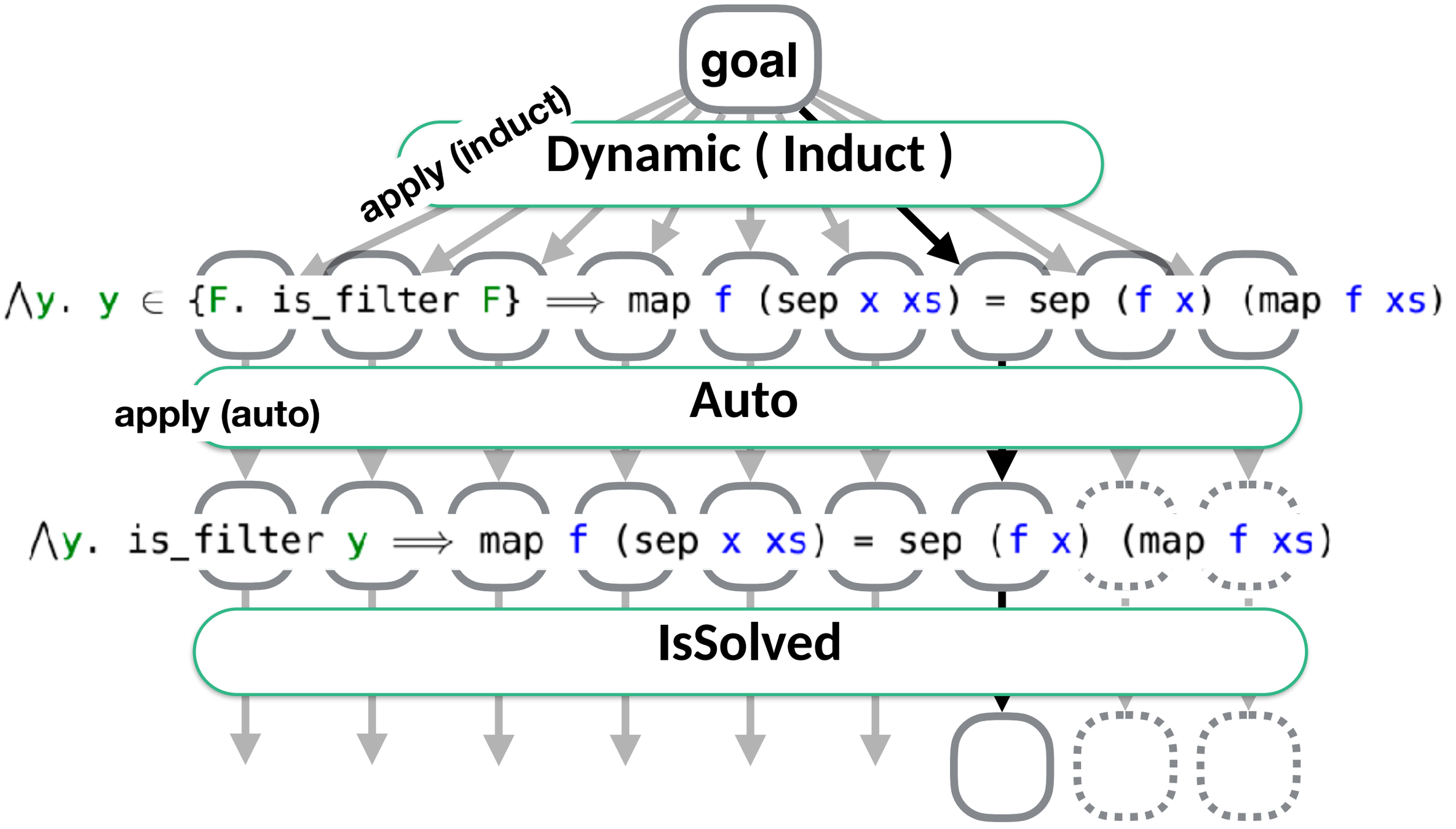

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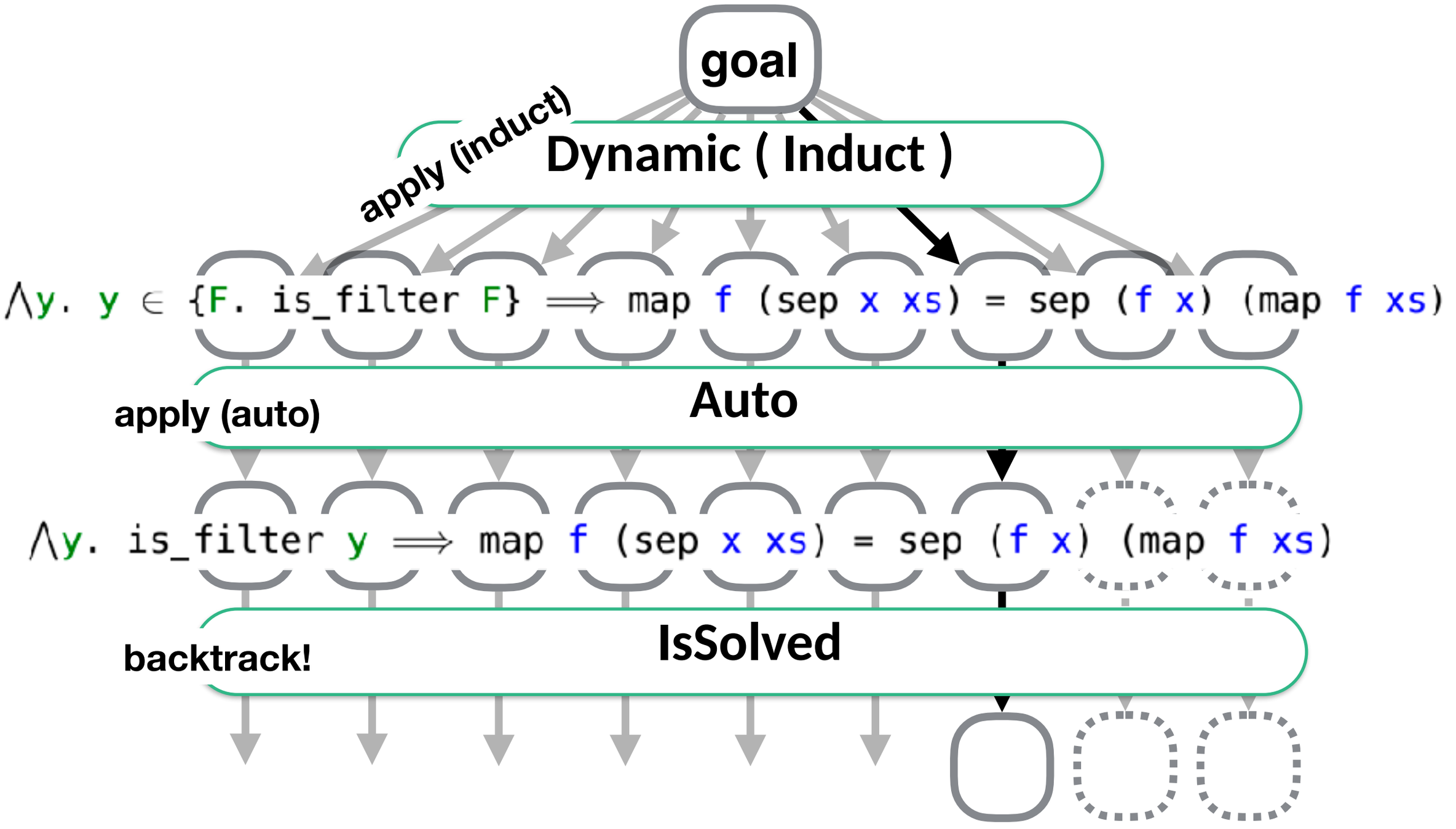

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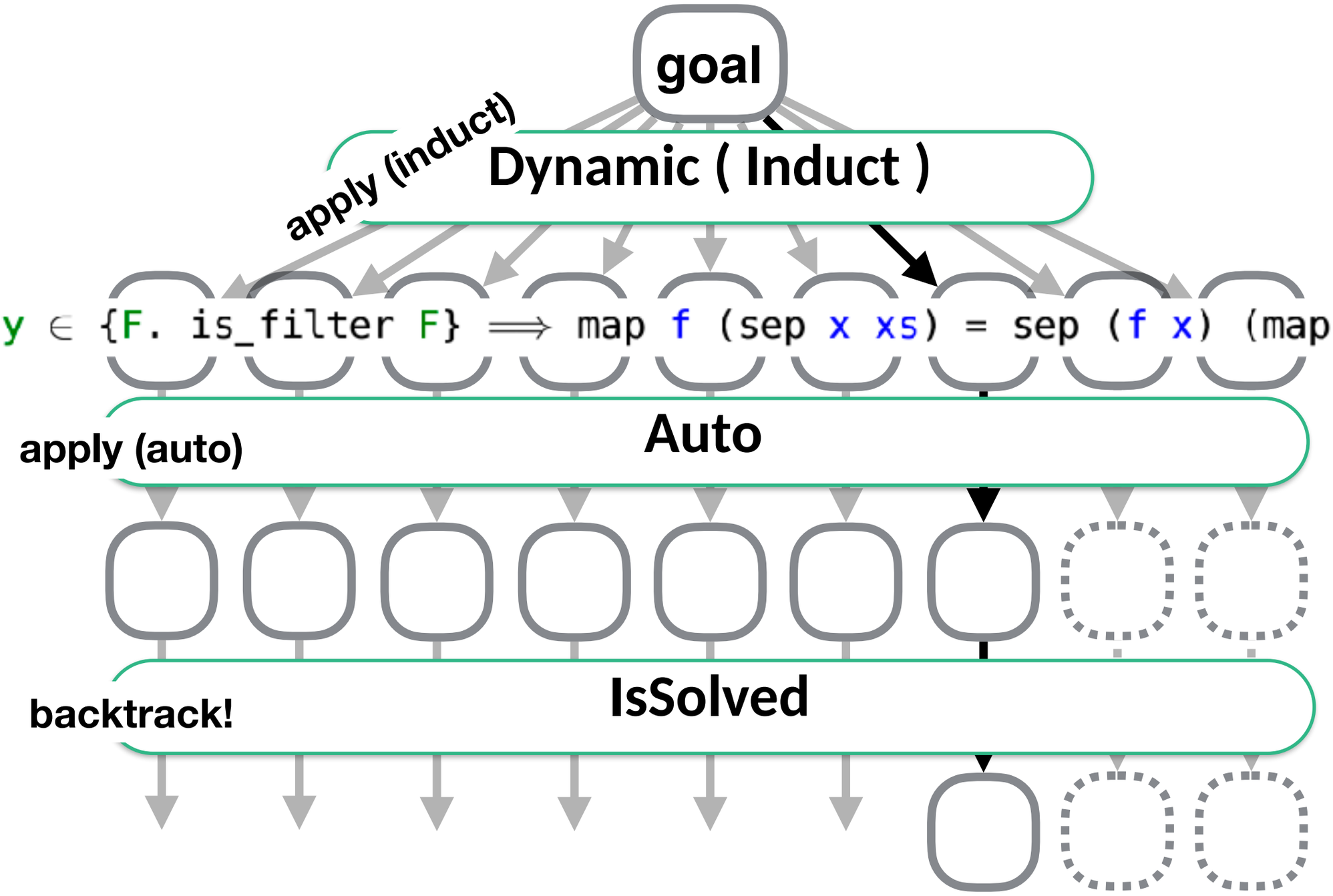

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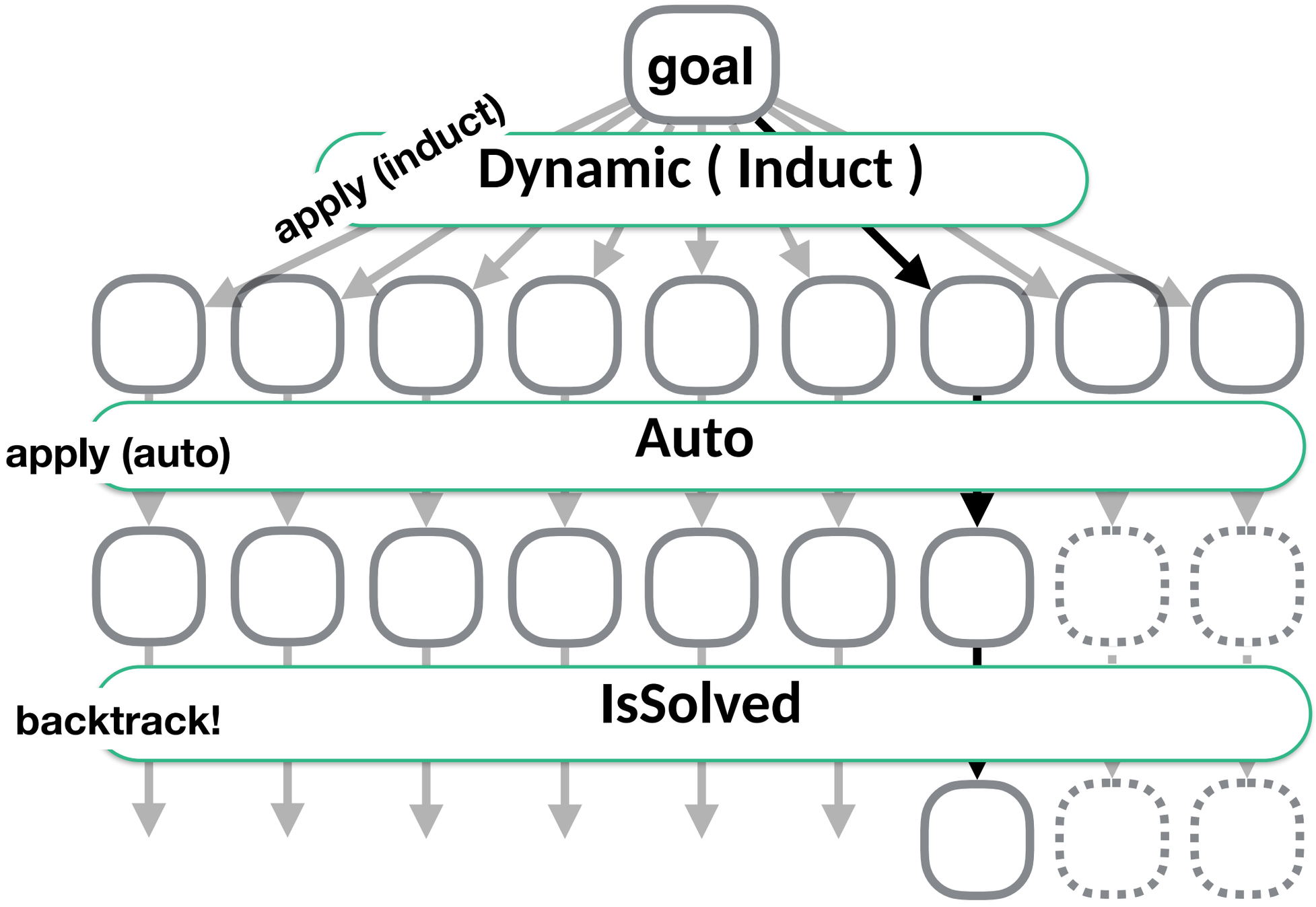

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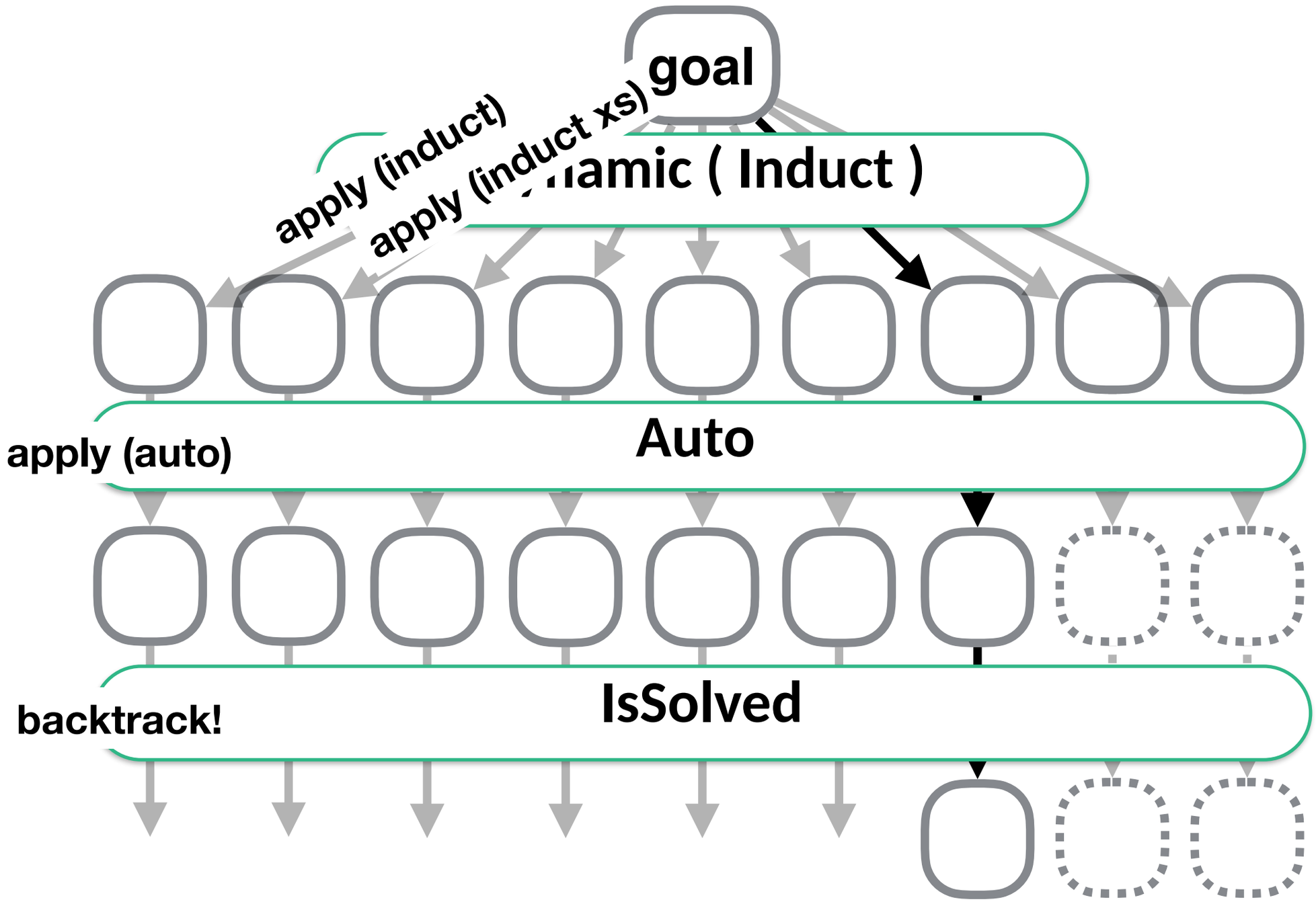

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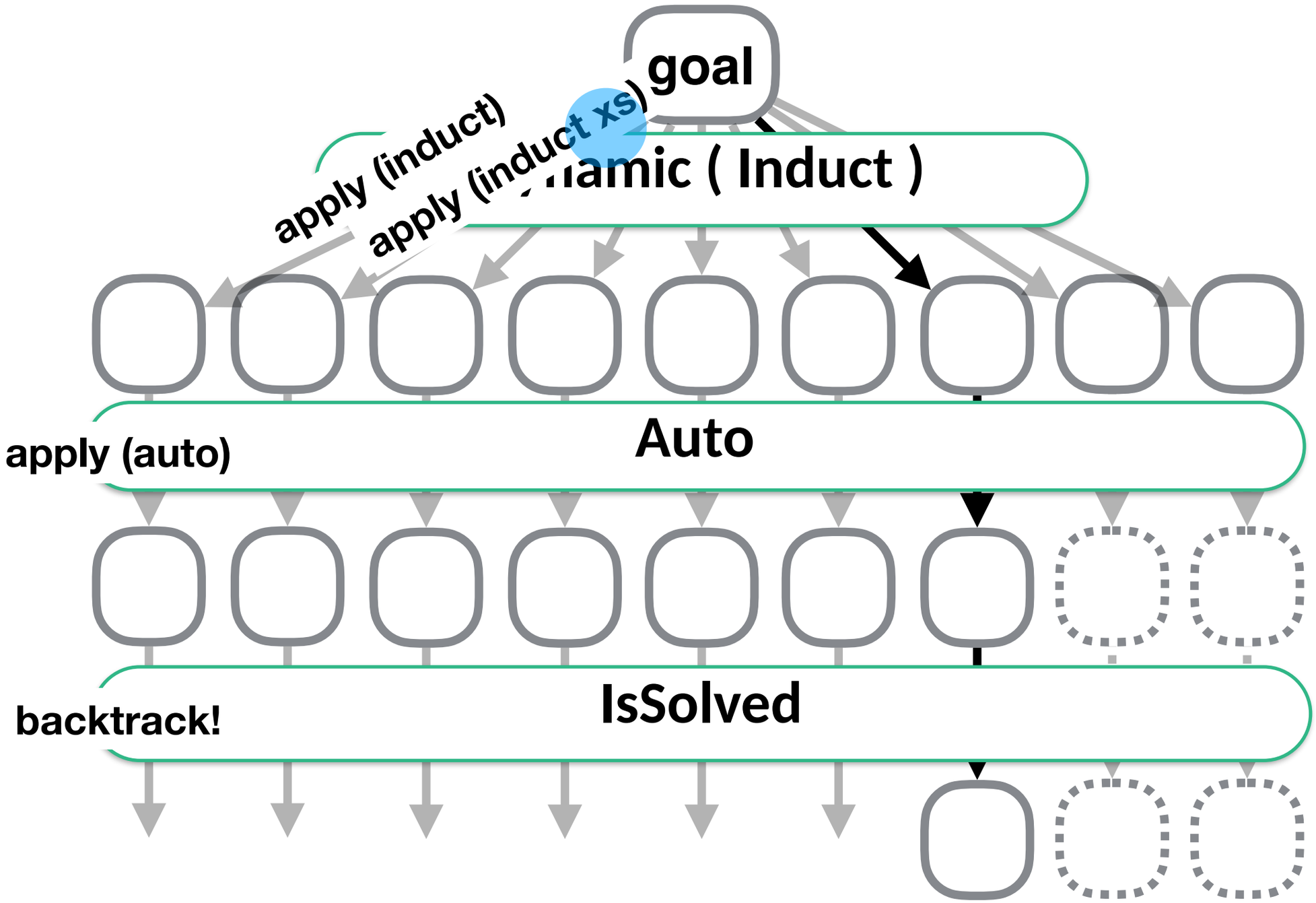

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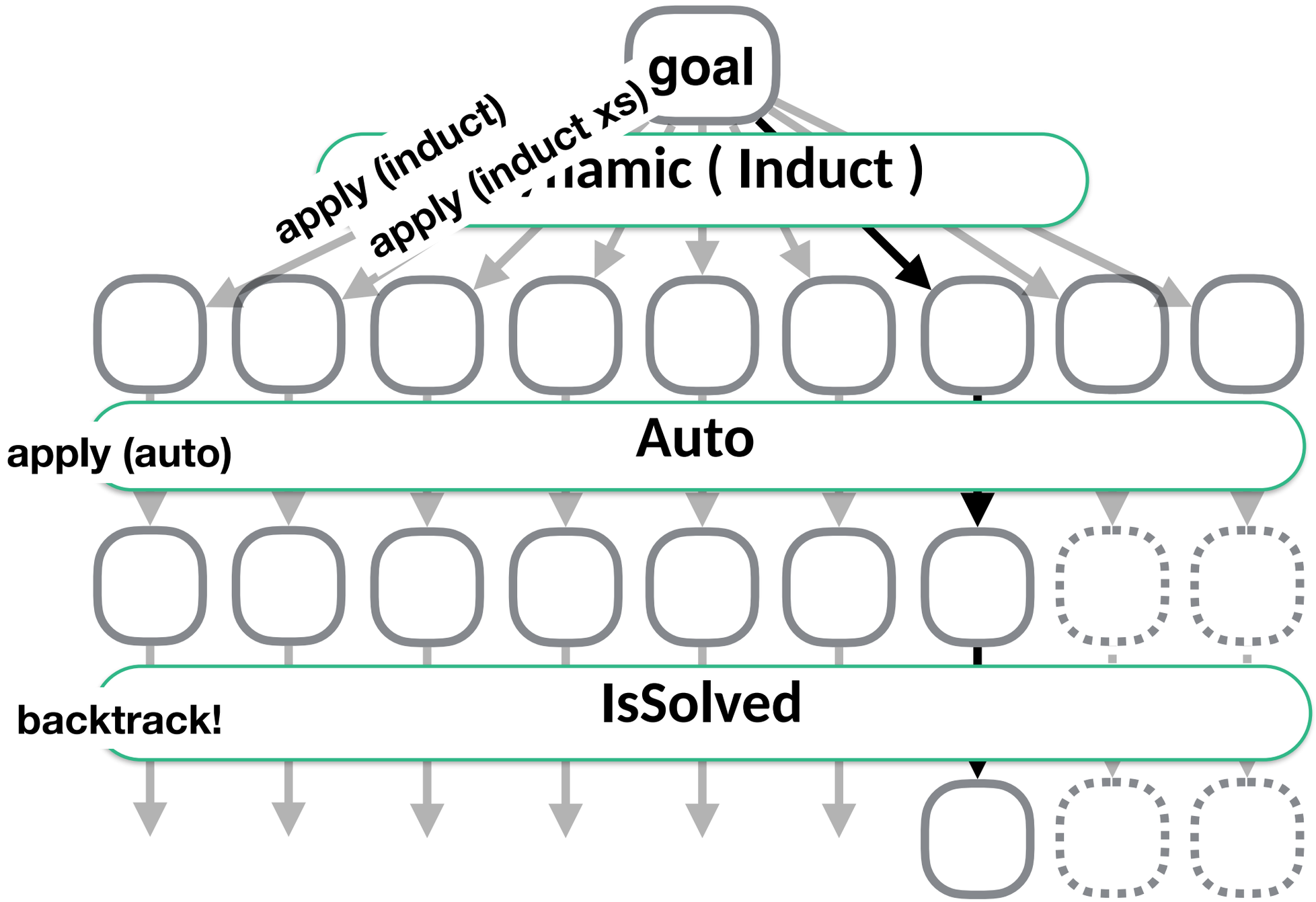

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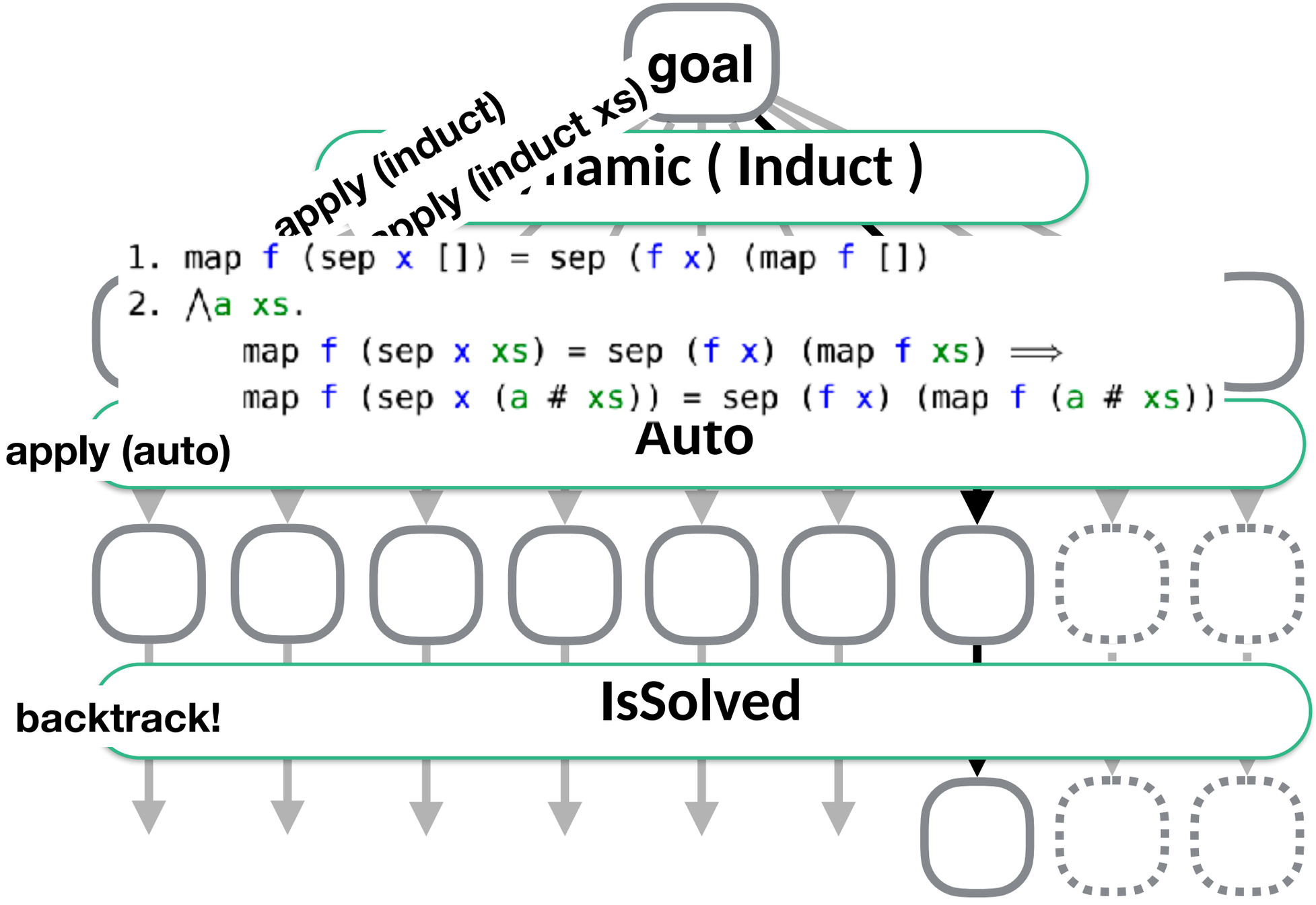

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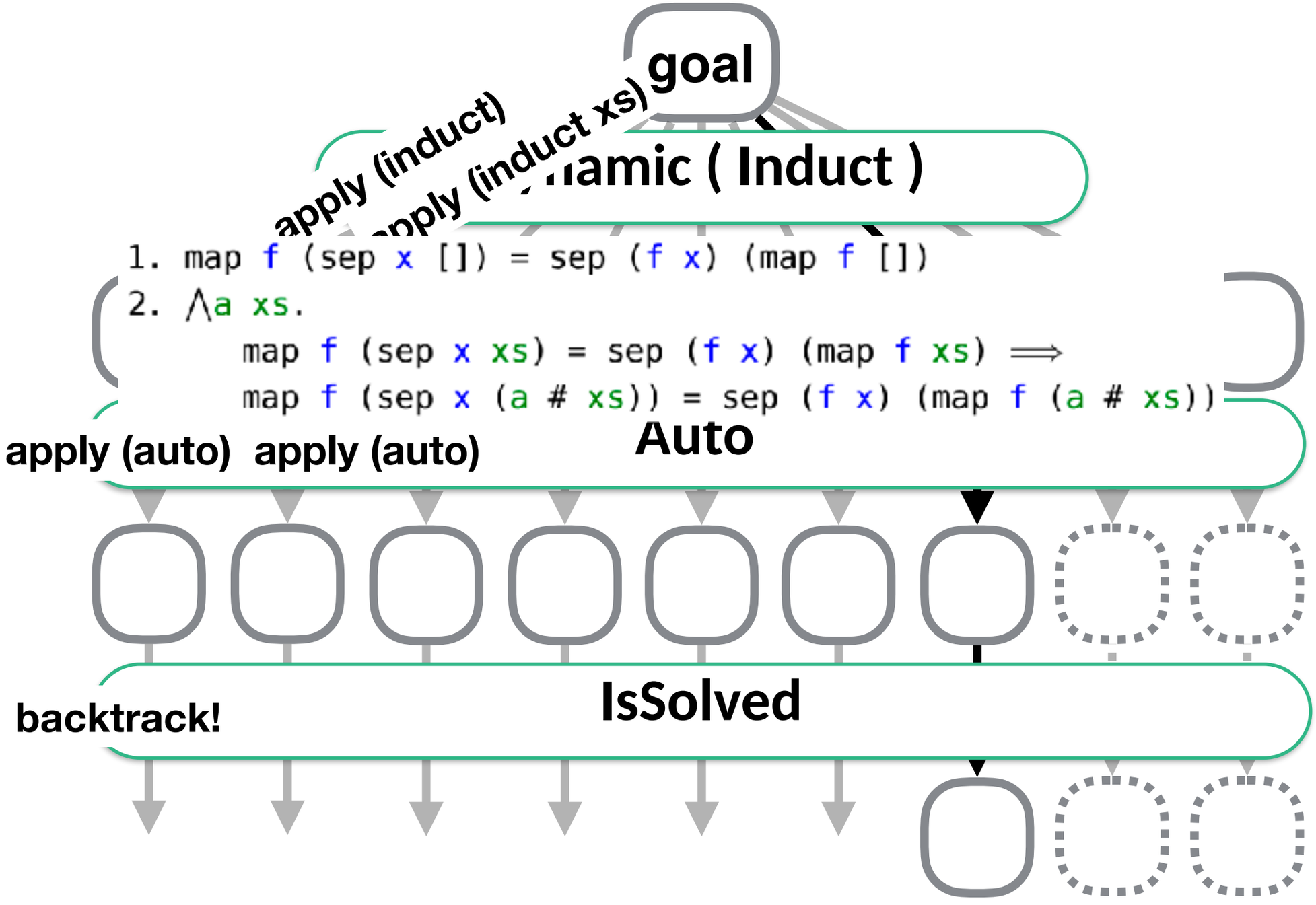

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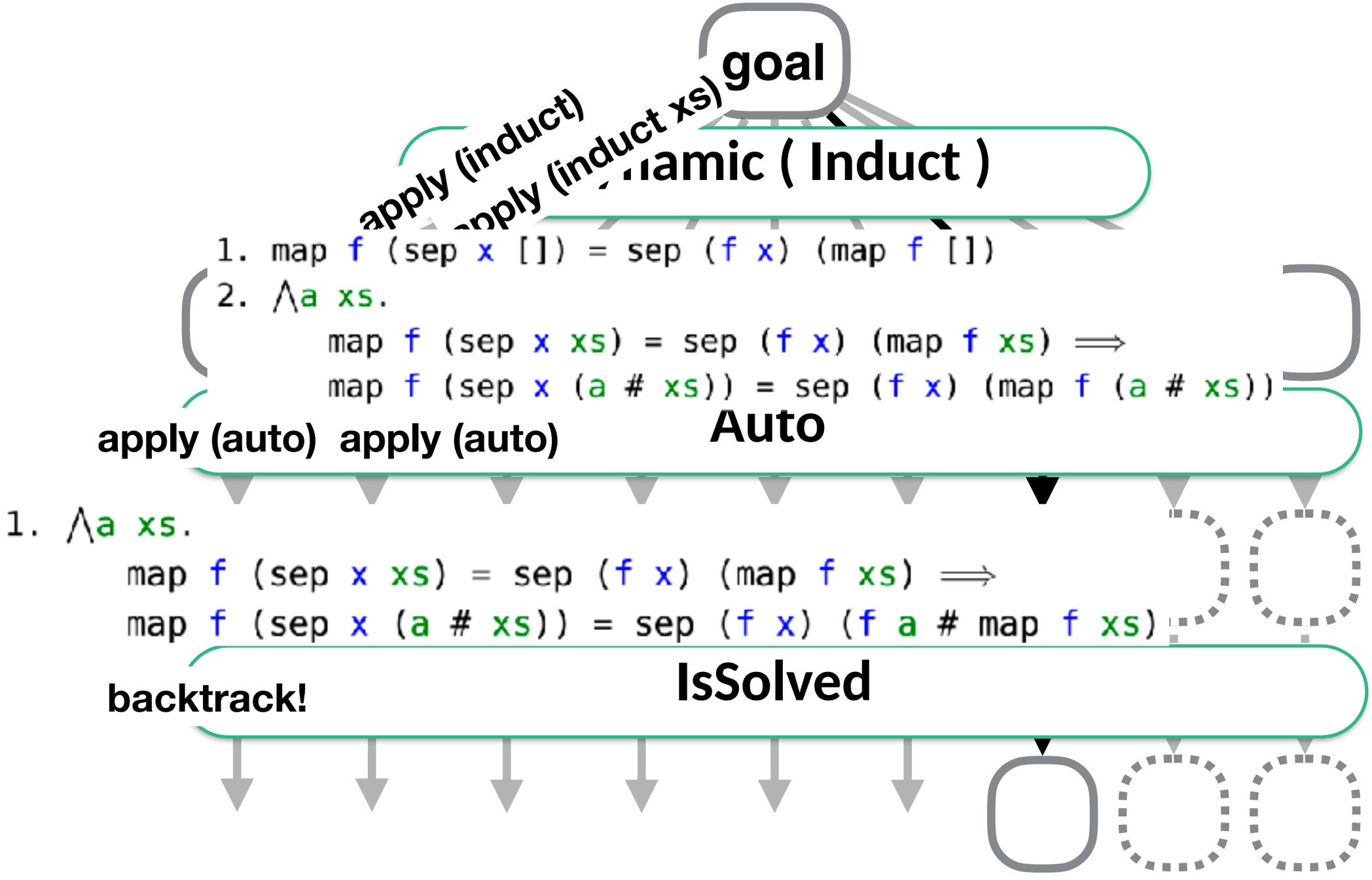

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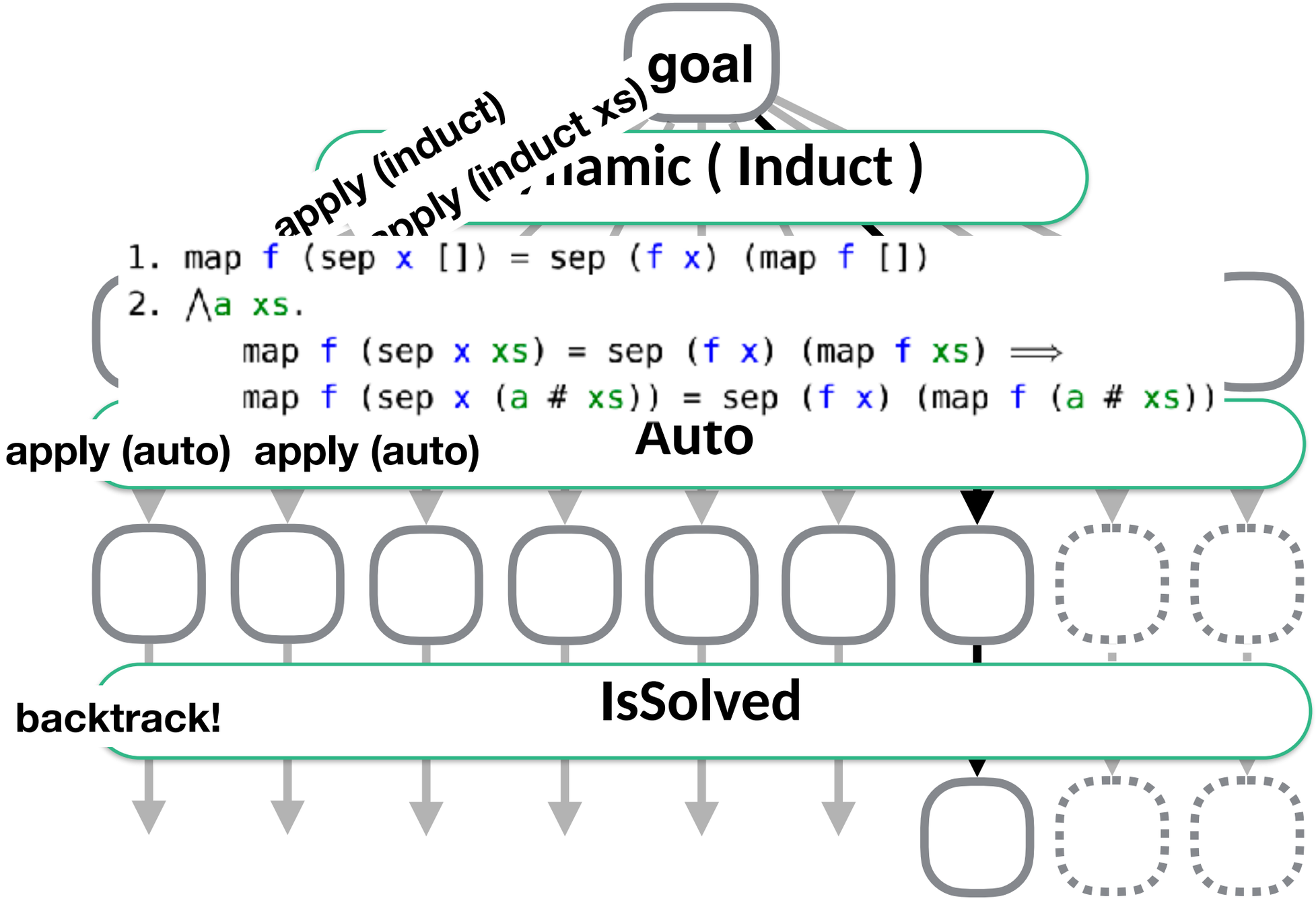

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lemma "map f (sep x xs) = sep (f x) (map f xs)"
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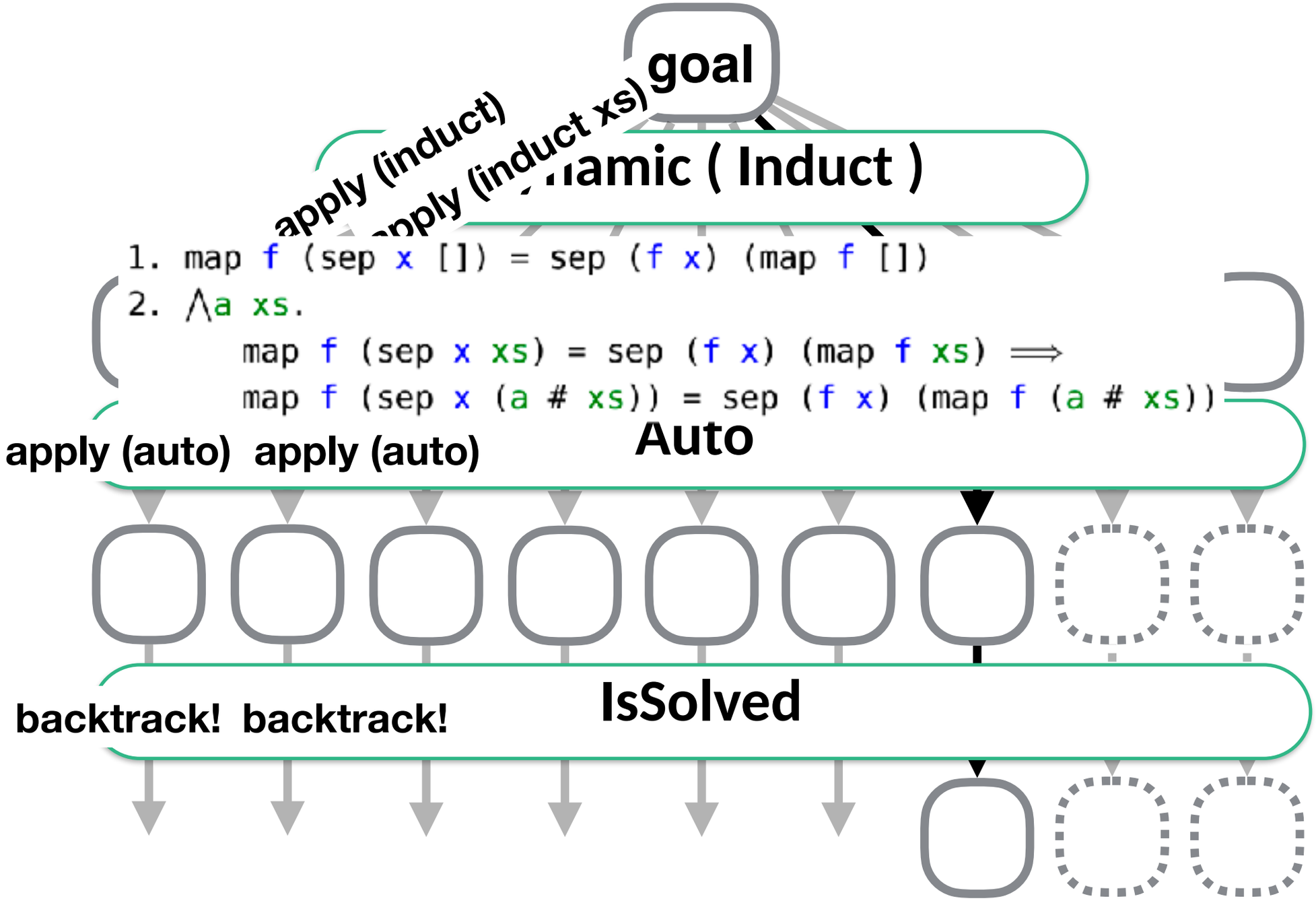

https://twitter.com/YutakangE https://tinyurl.com/up7zhyk

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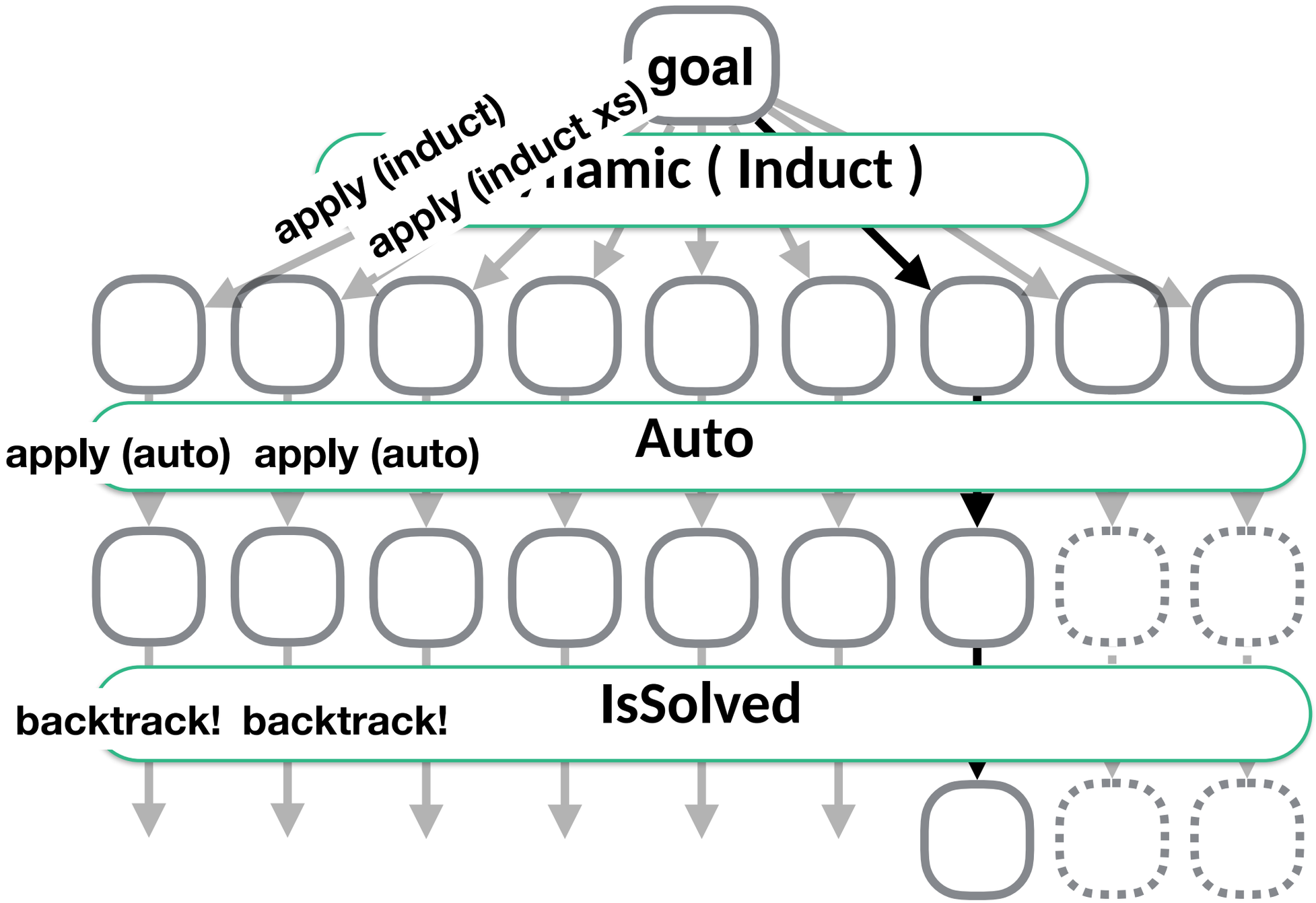

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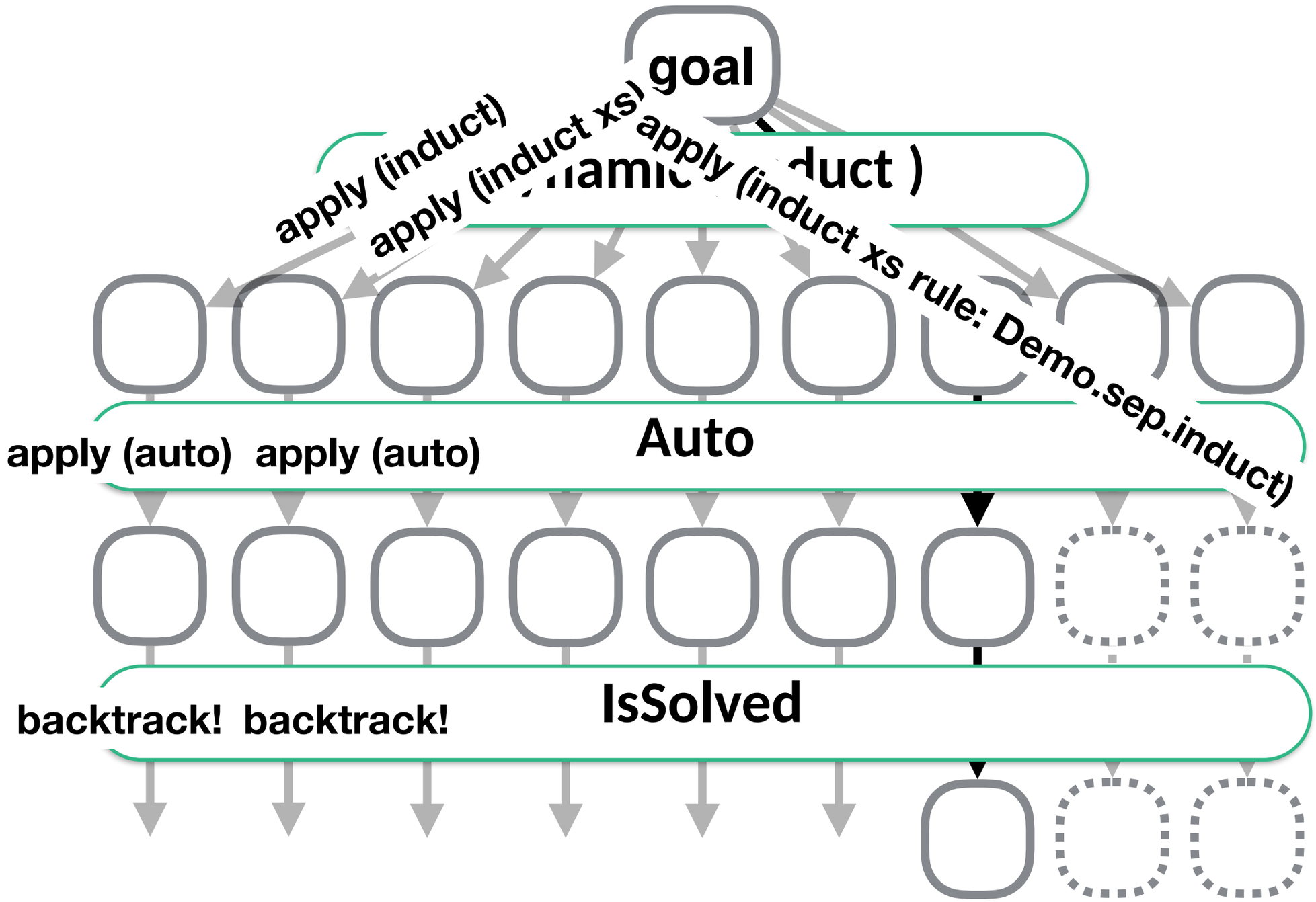

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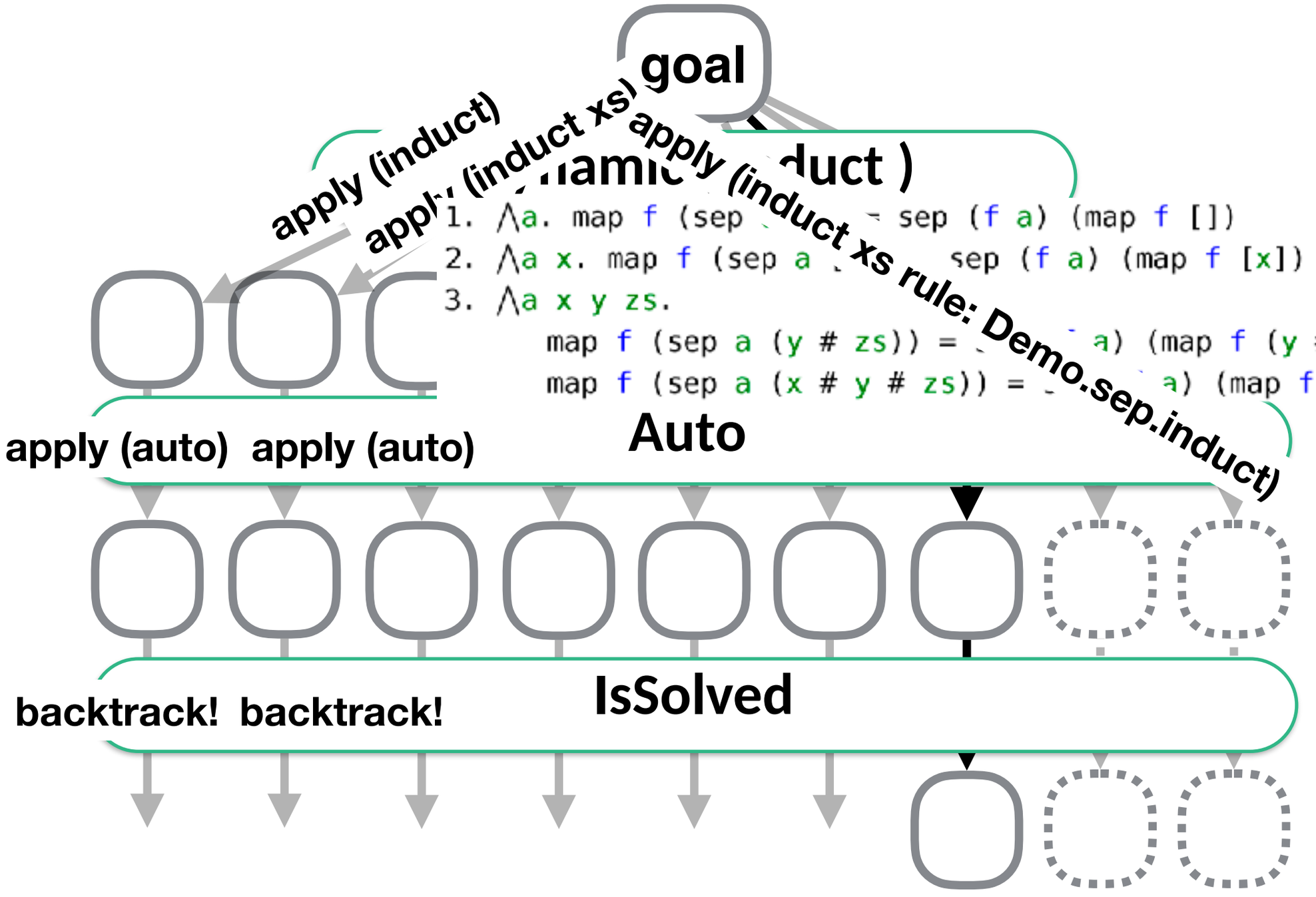

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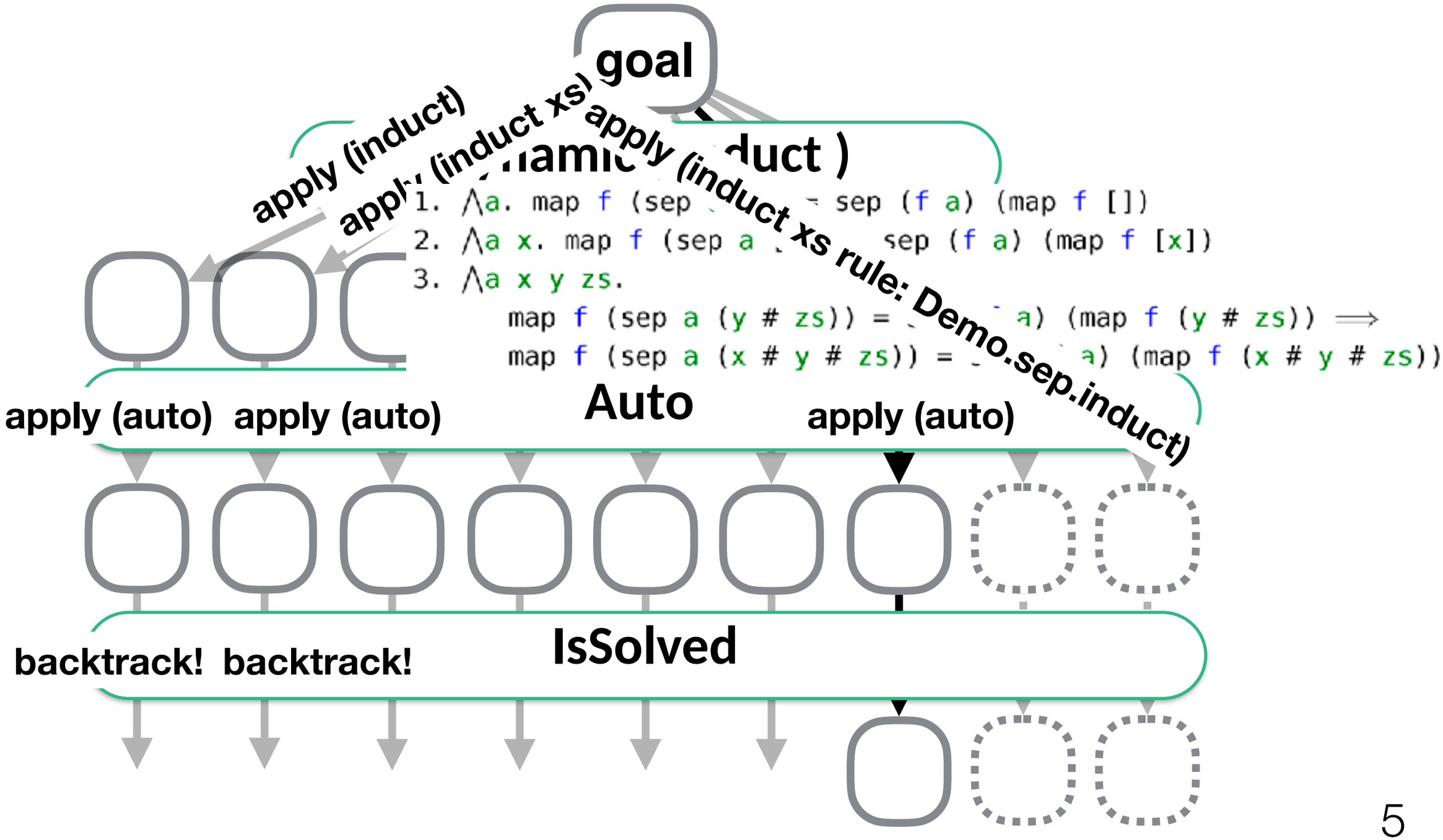

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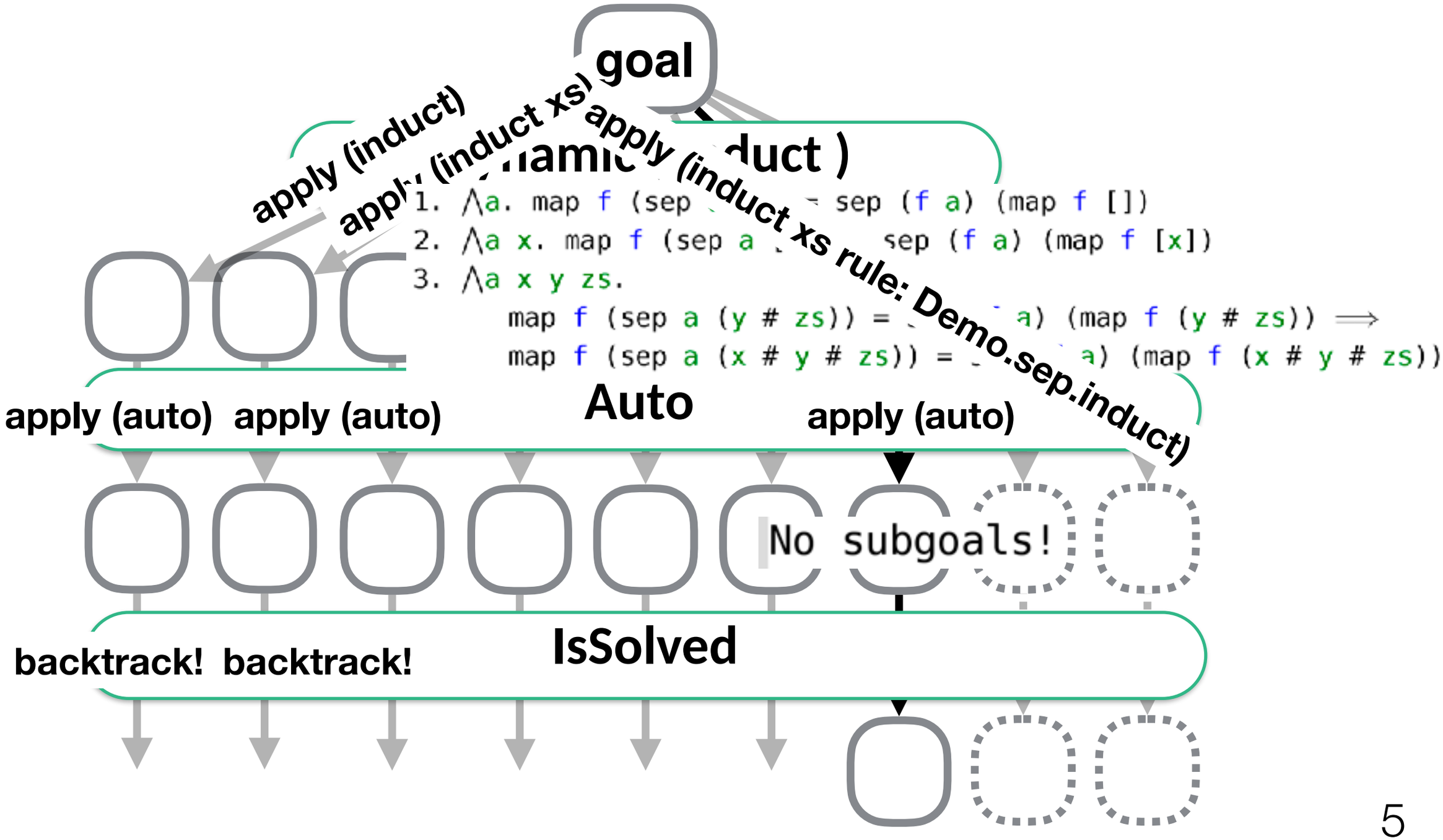

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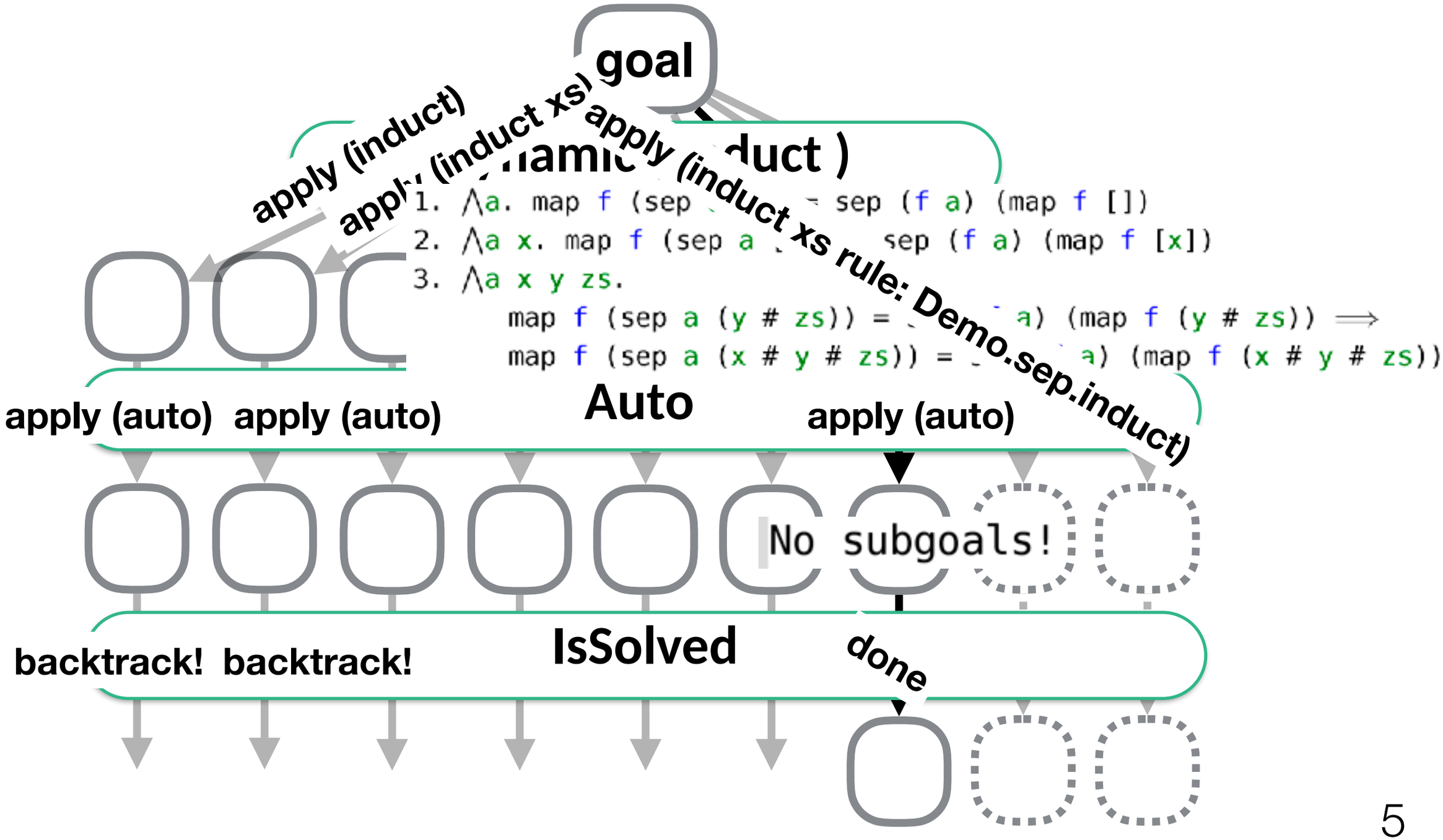

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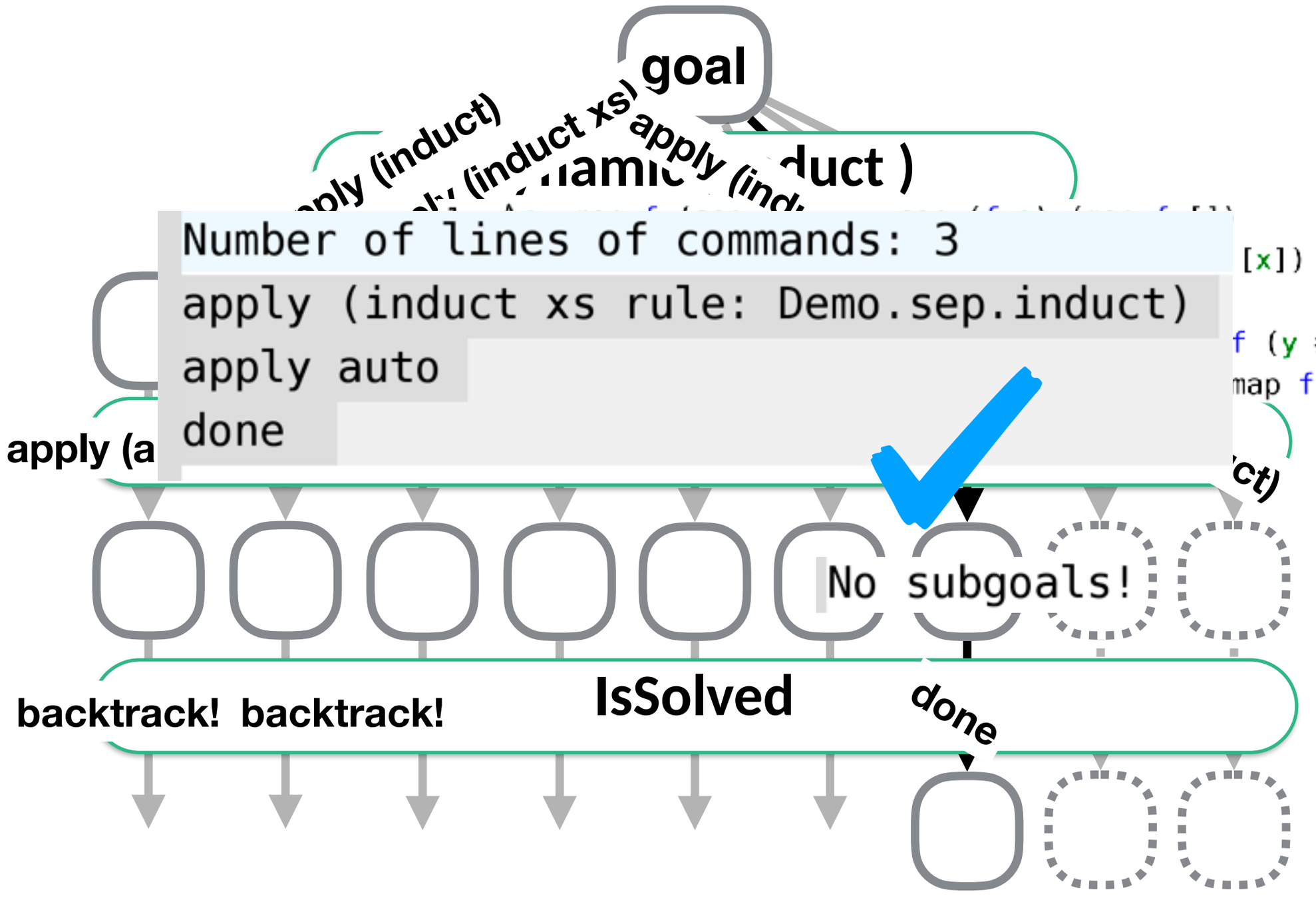

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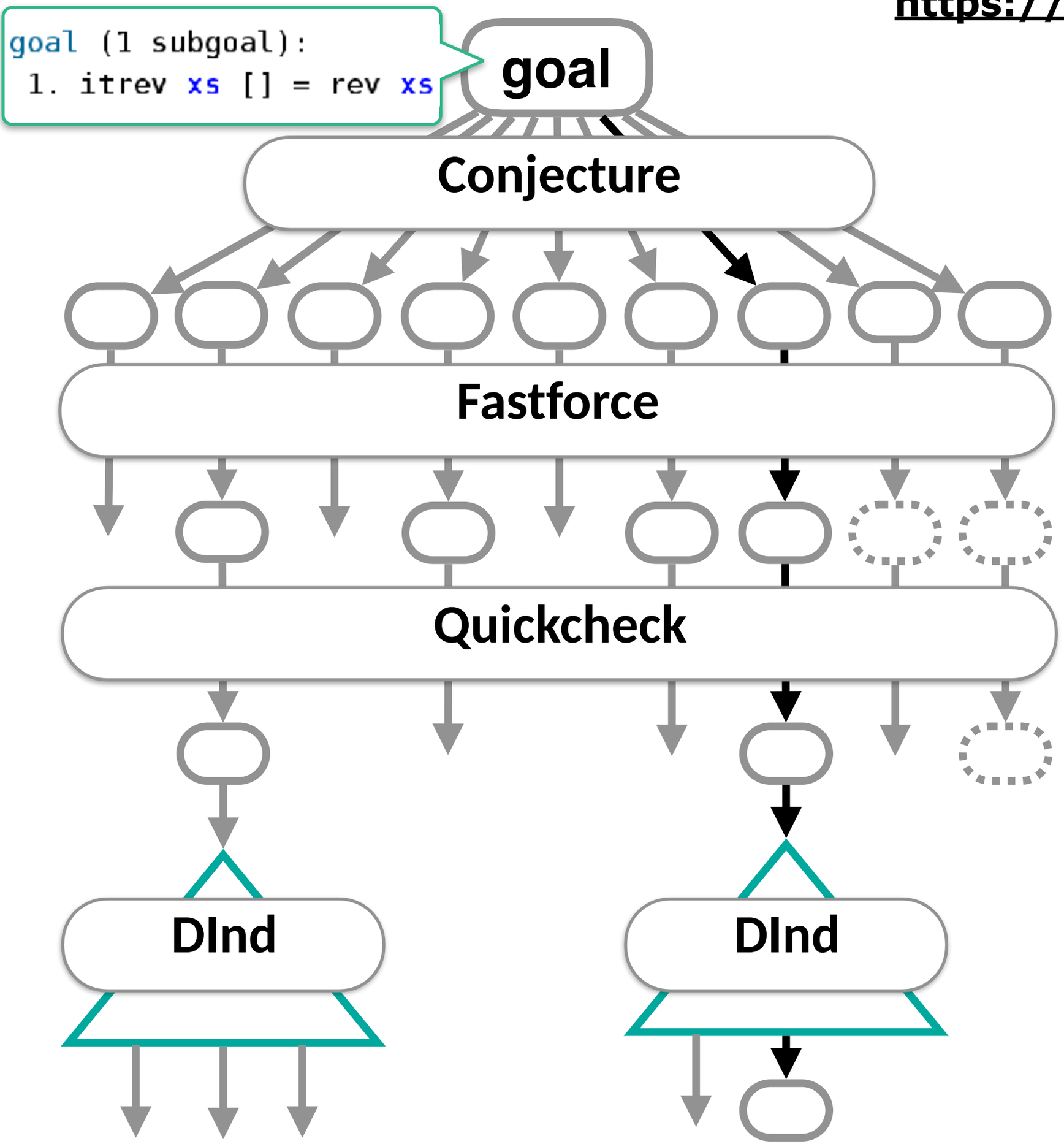

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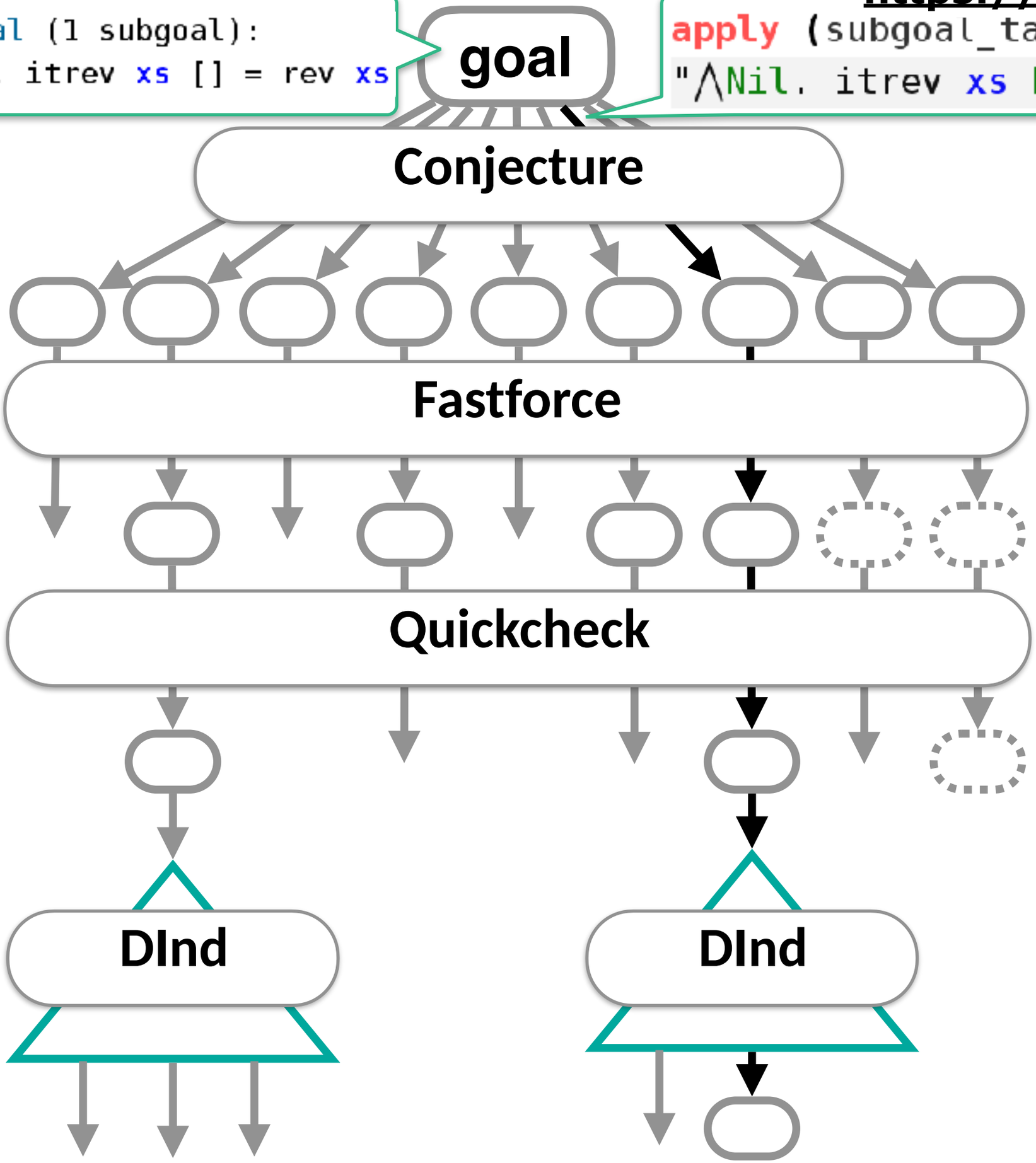

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Conjecture

Fastforce

Quickcheck

DInd

DInd

https://tinyurl.com/up7zhyk

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Fastforce

```
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```
Number of lines of commands: 5
apply (subgoal_tac "⋀Nil. itrev xs Nil = rev xs @ Nil")
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Nil

Nil

DInd

DInd

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Nil

Nil

Difficult proof goals?
Search space?

```
theorem itrev ?xs [] = rev ?xs
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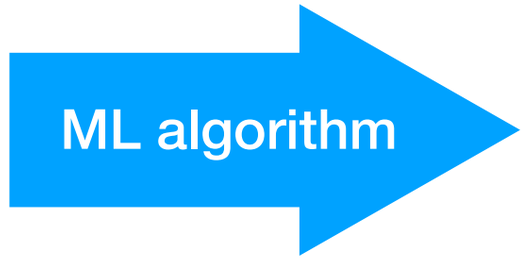
Introduction to Machine Learning in 10 seconds



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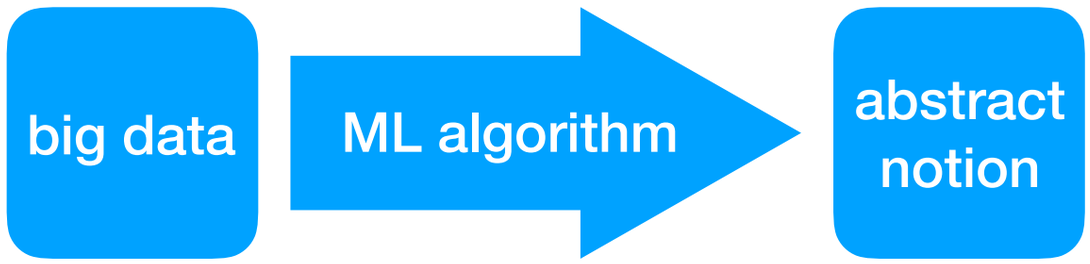

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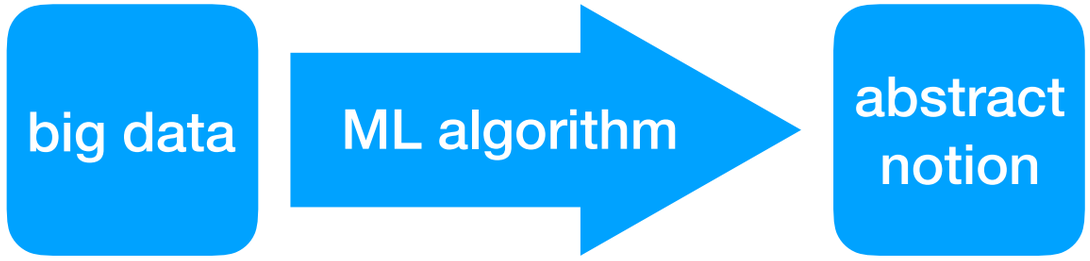

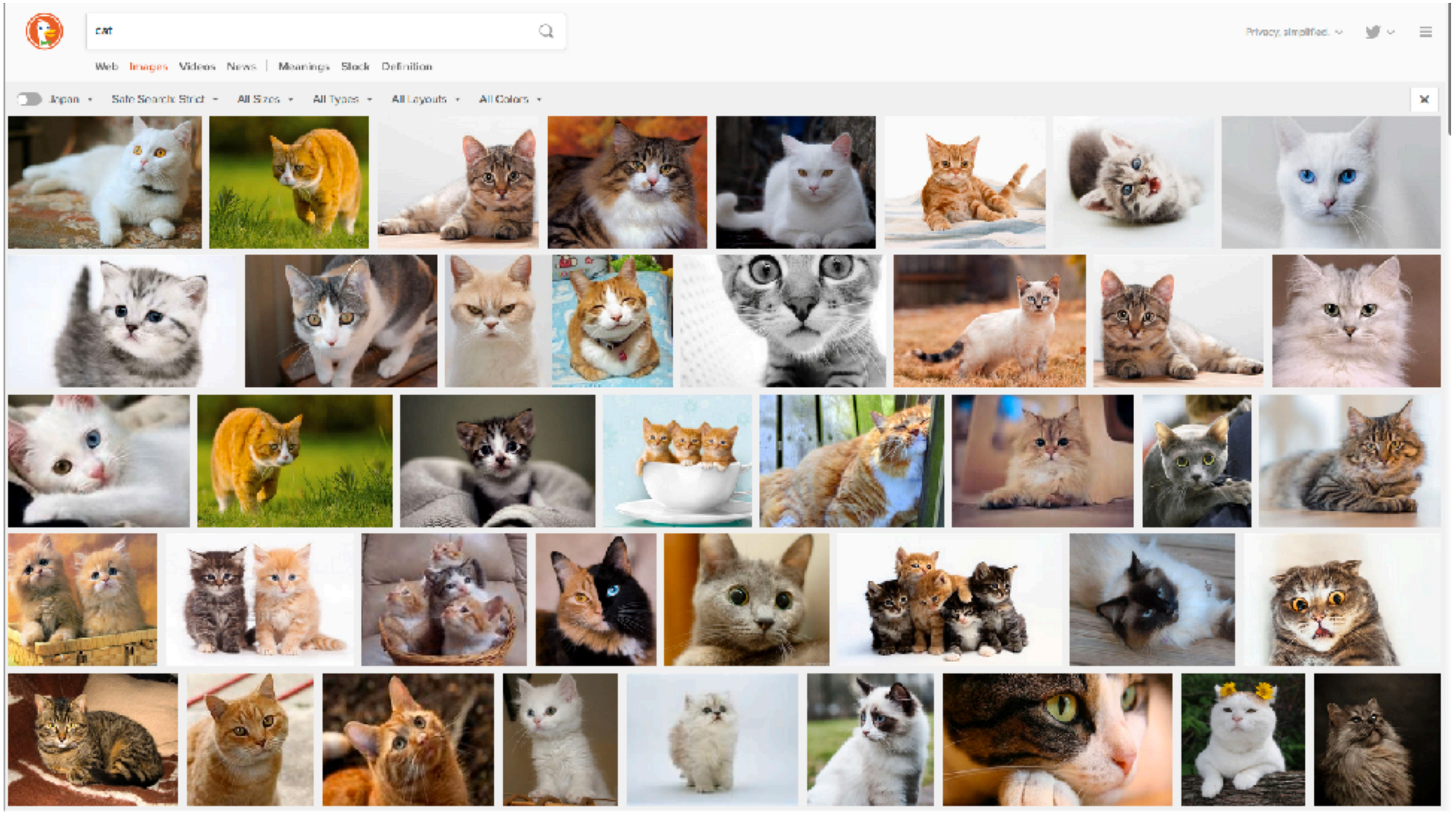

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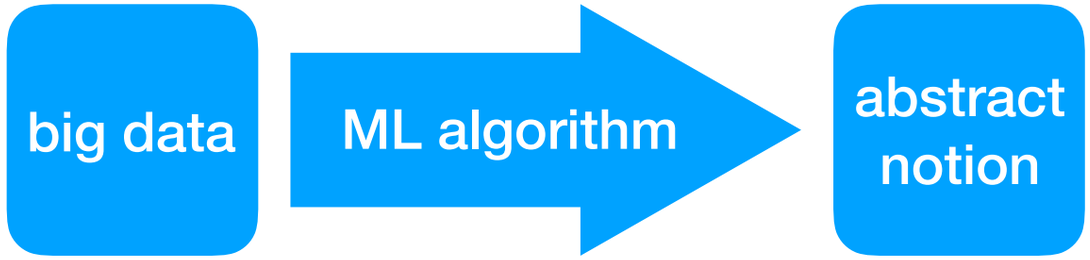

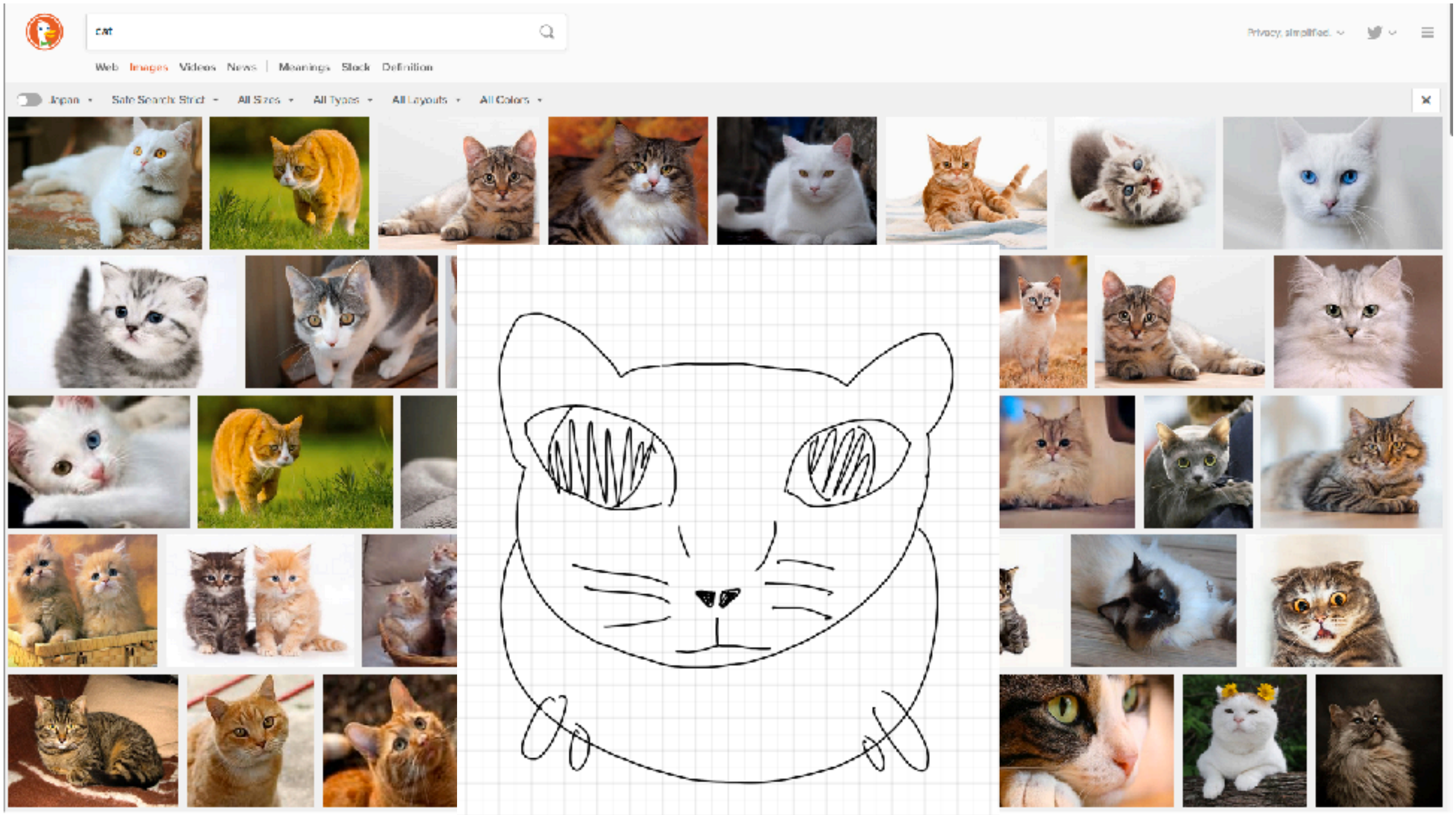

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ML for Inductive Theorem Proving the BAD

```
lemma "itrev xs ys = rev xs @ ys"
```

<- one abstract representation

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

https://twitter.com/YutakangE https://tinyurl.com/up7zhyk

ML for Inductive Theorem Proving
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ML for Inductive Theorem Proving the BAD

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lemma "itrev xs ys = rev xs @ ys"  by auto
by(induct xs ys rule:"itrev.induct") auto
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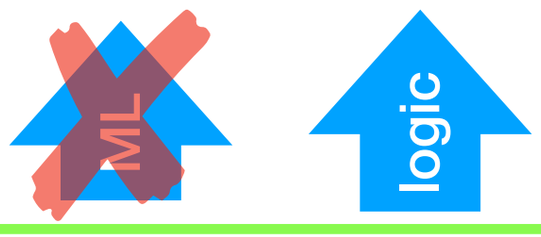

<- abstraction using expressive logic

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ML for Inductive Theorem Proving the BAD

polymorphism

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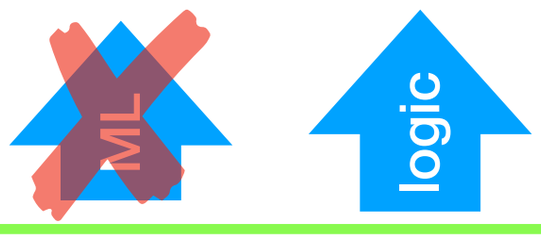

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https://twitter.com/YutakangE https://tinyurl.com/up7zhyk

ML for Inductive Theorem Proving the BAD

polymorphism **type class**

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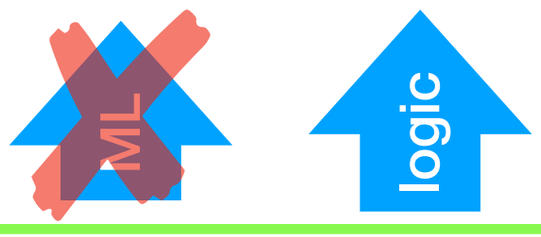

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ML for Inductive Theorem Proving the BAD

polymorphism **type class**

universal quantifier

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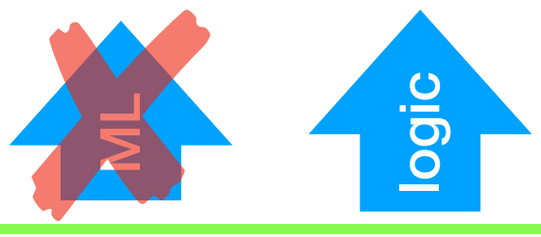

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ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism **type class**

universal quantifier

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lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```
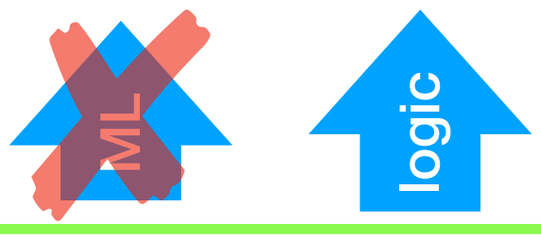

<- one abstract representation



<- abstraction using expressive logic

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<- many concrete cases

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism **type class**

universal quantifier

lambda abstraction

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by(induct xs ys rule:"itrev.induct") auto
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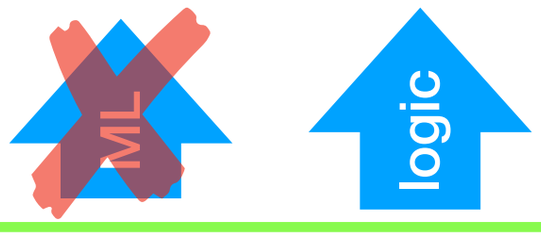

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https://twitter.com/YutakangE https://tinyurl.com/up7zhyk

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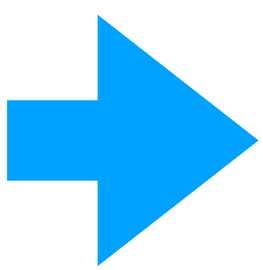

concise formula that can cover many concrete cases

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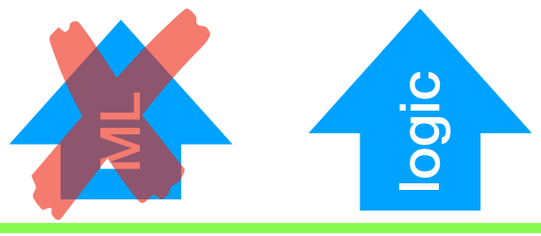

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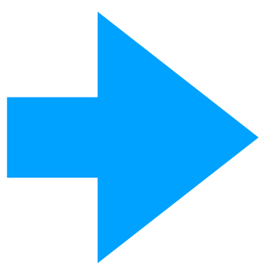

concise formula that can cover many concrete cases

different proof for general case

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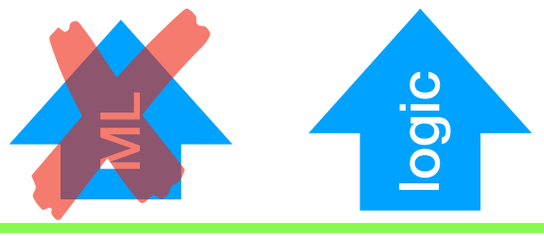

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ML for Inductive Theorem Proving the BAD

Higher-Order functions

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concise formula that can cover many concrete cases

different proof for general case

A small data set is not a failure but an achievement!

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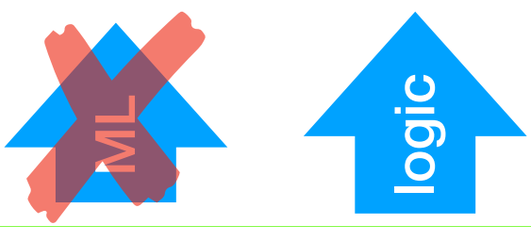

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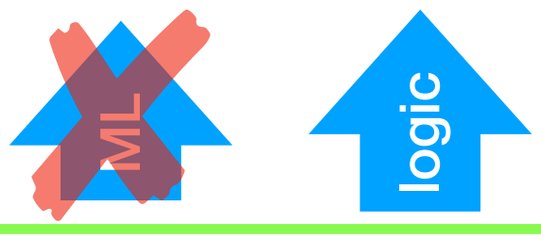

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Grand Challenge: Abstract Abstraction

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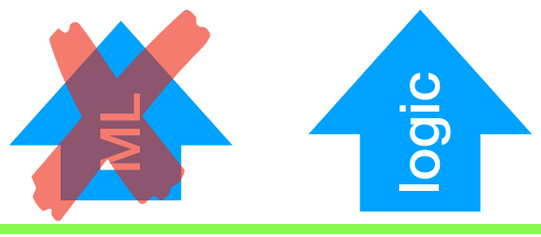

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<- small dataset about different domains

<- one abstract representation

ML

logic

<- abstraction using expressive logic

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ML

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~~ML~~ logic

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Grand Challenge: Abstract Abstraction

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract

ML

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ML (crossed out)

logic

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<- cons: bad at reasoning & abstraction

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~~ML~~

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Many key challenges remain

Unsupervised Learning

Memory and one-shot learning

Imagination-based Planning with Generative Models

Learning Abstract Concepts

Transfer Learning

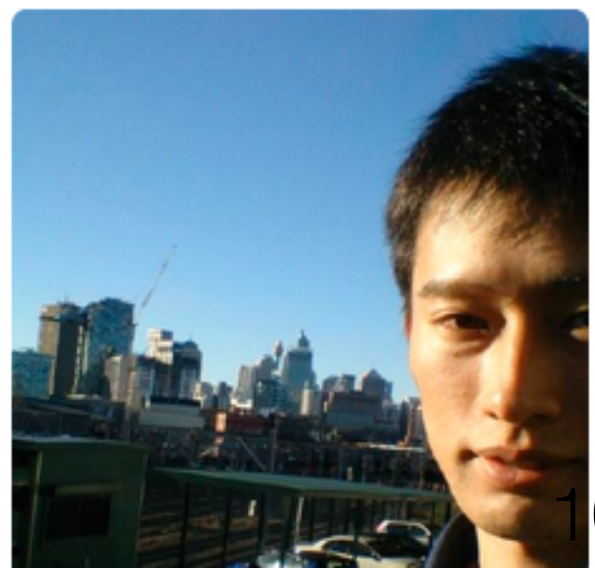
Language understanding

CENTER FOR
Brains
Minds+
Machines

March 20, 2019

The Power of

Self



https://cbmm.mit.edu/video/power-self-learning-systems

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Abstract concepts?

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Brains
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Machines

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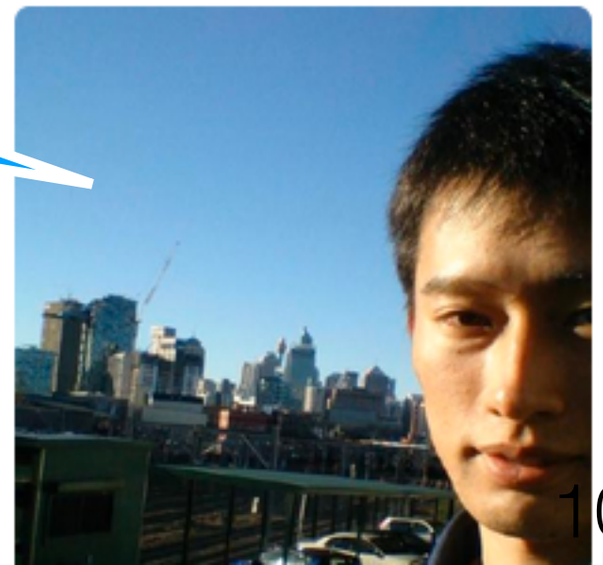

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Brains
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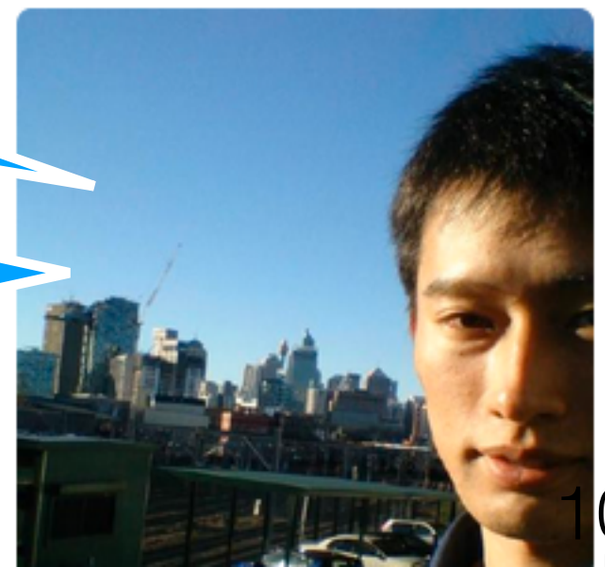
March 20, 2019

Abstract concepts?

Why not logic?
(LiFtEr = Logical Feature Extraction)

The Power of

Self



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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

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<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

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Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

<- pros: good at ambiguity (heuristics)

[[ ], [ ], [ ]]: bool list <- simple representation

LiFtEr

LiFtEr 2

<- pros: good at rigorous abstraction

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lemma "star r x y ⟹ star r y z ⟹ star r x z"
by(induction rule: star.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
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lemma "itrev xs ys = rev xs @ ys"
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<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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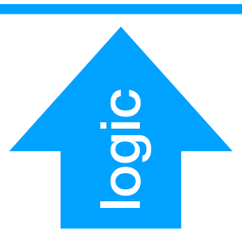

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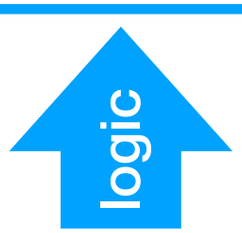

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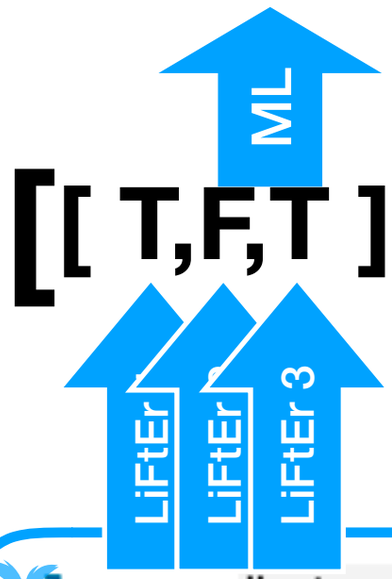

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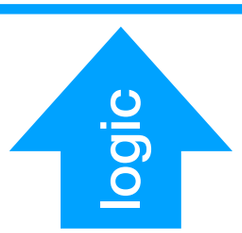

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LiFtEr

LiFtEr

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

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Abstract Syntax of LiFtEr

assertion := *atomic* | *connective* | *quantifier* | (*assertion*)
type := `term` | `term_occurrence` | `rule` | `number`
modifier_term := `induction_term` | `arbitrary_term`
quantifier := $\exists x$: *type* . *assertion*
| $\forall x$: *type* . *assertion*
| $\exists x$: `term` $\in$ *modifier_term* . *assertion*
| $\forall x$: `term` $\in$ *modifier_term* . *assertion*
| $\exists x$: `rule` . *assertion*
| $\exists x$: `term_occurrence` $\in y$: `term` . *assertion*
| $\forall x$: `term_occurrence` $\in y$: `term` . *assertion*
connective := `True` | `False` | *assertion* $\vee$ *assertion* | *assertion* $\wedge$ *assertion*
assertion $\rightarrow$ *assertion* | $\neg$ *assertion*
pattern := `all_only_var` | `all_constructor` | `mixed`
atomic :=
rule `is_rule_of` *term_occurrence*
| *term_occurrence* `term_occurrence_is_of_term` *term*
| `are_same_term` (*term_occurrence* , *term_occurrence*)
| *term_occurrence* `is_in_term_occurrence` *term_occurrence*
| `is_atomic` *term_occurrence*
| `is_constant` *term_occurrence*
| `is_recursive_constant` *term_occurrence*
| `is_variable` *term_occurrence*
| `is_free_variable` *term_occurrence*
| `is_bound_variable` *term_occurrence*
| `is_lambda` *term_occurrence*
| `is_application` *term_occurrence*
| *term_occurrence* `is_an_argument_of` *term_occurrence*
| *term_occurrence* `is_nth_argument_of` *term_occurrence*

Abstract Syntax of LiFtEr

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| $\exists x$: `rule` . *assertion*
| $\exists x$: `term_occurrence` $\in y$: `term` . *assertion*
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| `is_recursive_constant` *term_occurrence*
| `is_variable` *term_occurrence*
| `is_free_variable` *term_occurrence*
| `is_bound_variable` *term_occurrence*
| `is_lambda` *term_occurrence*
| `is_application` *term_occurrence*
| *term_occurrence* `is_an_argument_of` *occurrence*
| *term_occurrence* `is_nth_argument_of` *occurrence*

Example Assertion in LiFtEr (in Abstract Syntax)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication

$\exists\ r1\ :\ \texttt{rule}.\ \texttt{True}$

$\rightarrow$

$\exists\ r1\ :\ \texttt{rule}.$
$\quad\exists\ t1\ :\ \texttt{term}.$
$\quad\quad\exists\ to1\ :\ \texttt{term\_occurrence} \in\ t1\ :\ \texttt{term}.$
$\quad\quad\quad r1\ \texttt{is\_rule\_of}\ to1$
$\quad\quad\quad\wedge$
$\quad\quad\quad\forall\ t2\ :\ \texttt{term} \in \texttt{induction\_term}.$
$\quad\quad\quad\quad\exists\ to2\ :\ \texttt{term\_occurrence} \in\ t2\ :\ \texttt{term}.$
$\quad\quad\quad\quad\quad\exists\ n\ :\ \texttt{number}.$
$\quad\quad\quad\quad\quad\quad\texttt{is\_nth\_argument\_of}\ (to2,\ n,\ to1)$
$\quad\quad\quad\quad\quad\quad\wedge$
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Example Assertion in LiFtEr (in Abstract Syntax)

implication

$$
\begin{array}{l}
\exists\ r1\ :\ \texttt{rule}.\ \ \texttt{True} \\
\rightarrow \\
\exists\ r1\ :\ \texttt{rule}. \\
\quad \exists\ t1\ :\ \texttt{term}. \\
\quad\quad \exists\ to1\ :\ \texttt{term\_occurrence} \in\ t1\ :\ \texttt{term}. \\
\quad\quad\quad r1\ \texttt{is\_rule\_of}\ to1 \\
\quad\quad \wedge \quad \text{conjunction} \\
\quad\quad\quad \forall\ t2\ :\ \texttt{term} \in \texttt{induction\_term}. \\
\quad\quad\quad\quad \exists\ to2\ :\ \texttt{term\_occurrence} \in\ t2\ :\ \texttt{term}. \\
\quad\quad\quad\quad\quad \exists\ n\ :\ \texttt{number}. \\
\quad\quad\quad\quad\quad\quad \texttt{is\_nth\_argument\_of}\ (to2,\ n,\ to1) \\
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variable for auxiliary lemmas

conjunction

Example Assertion in LiFtEr (in Abstract Syntax)

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variable for auxiliary lemmas

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implication

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

Example Assertion in LiFtEr (in Abstract Syntax)

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implication

variable for auxiliary lemmas

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variable for natural numbers

Example Assertion in LiFtEr (in Abstract Syntax)

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implication

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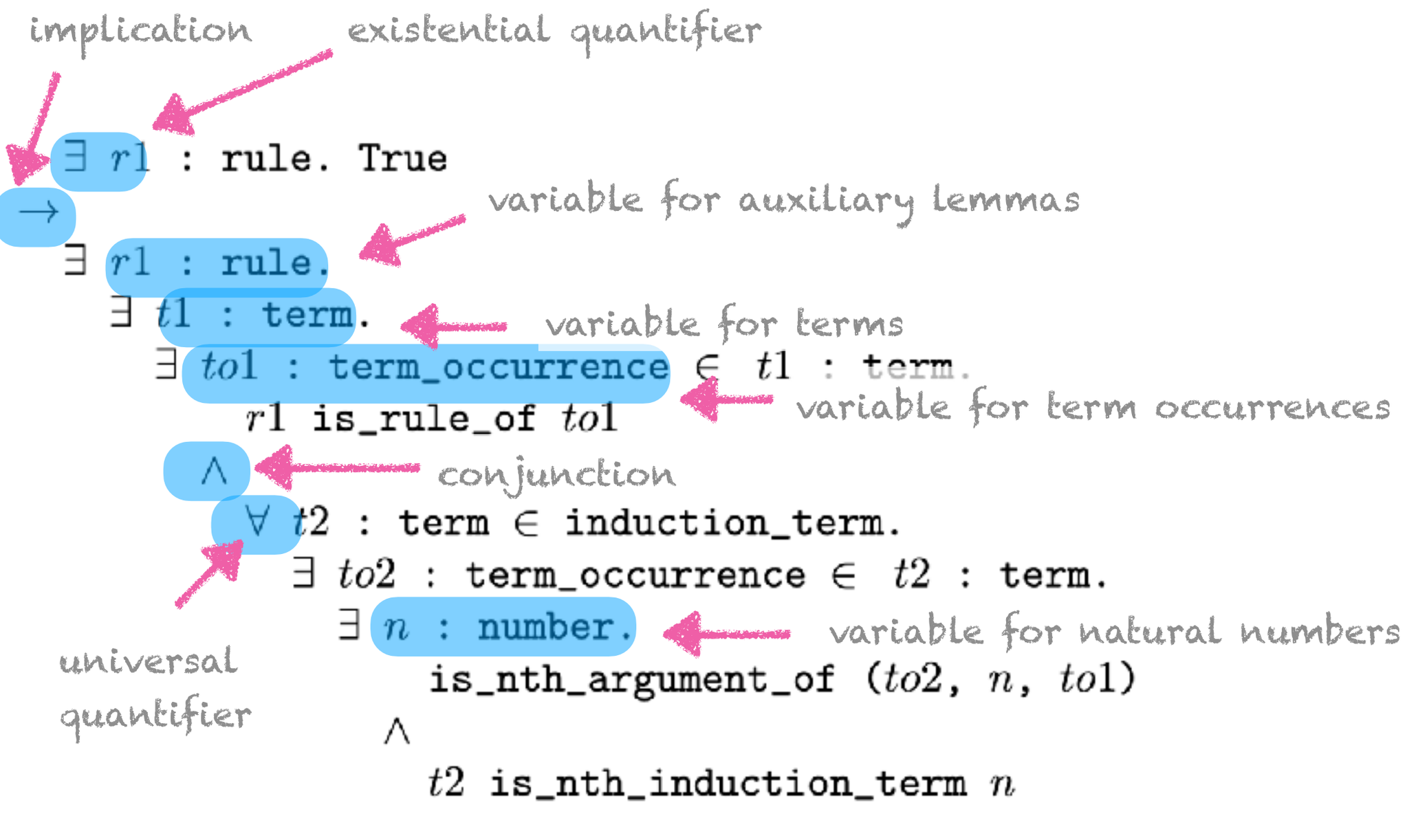
universal quantifier

variable for natural numbers

Example Assertion in LiFtEr (in Abstract Syntax)



Example Assertion in LiFtEr (in Abstract Syntax)

LiFtEr assertion: (proof goal * induction arguments) -> bool

implication

existential quantifier

```
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```

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

universal quantifier

variable for natural numbers

https://twitter.com/YutakangE

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
```

→

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```
∃ r1 : rule. True
```

→ r1

(r1 = itrev.induct)

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```

```
∃ r1 : rule. True
```

→ r1

(r1 = itrev.induct)

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

https://twitter.com/YutakangE

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
```

→ r1

```
∃ r1 : rule.
  ∃ t1 : term.
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```

(r1 = itrev.induct)
(t1 = itrev)

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```
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```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
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            is_nth_argument_of (to2, n, to1)
          ∧
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```

(r1 = itrev.induct)
(t1 = itrev)
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to1

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```

r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                    ( r1 = itrev.induct )
    ∃ t1 : term.                                  ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.        ( to1 = itrev )
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
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                  is_nth_argument_of (to2, n, to1)
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  ∃ r1 : rule.                                    ( r1 = itrev.induct )
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True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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r1

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  ∃ r1 : rule.                                    ( r1 = itrev.induct )
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to1

```
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r1

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∃ r1 : rule. True
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→

```
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```

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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to1

```
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  apply(induct xs ys rule:"itrev.induct")
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```

r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                        ( r1 = itrev.induct )
    ∃ t1 : term.                                      ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.            ( to1 = itrev )
          r1 is_rule_of to1    True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
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```

to1

t2

r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
  ∃ t1 : term. (t1 = itrev)
    ∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
      r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
    ∧
      ∀ t2 : term ∈ induction_term. (t2 = xs and ys)
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 to2 t2 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
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∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1)
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to1 to2 t2 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
  ∃ t1 : term. (t1 = itrev)
    ∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
      r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 to2 t2 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
  ∃ t1 : term. (t1 = itrev)
    ∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
      r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
    ∧
      ∀ t2 : term ∈ induction_term. (t2 = xs and ys)
        ∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
          ∃ n : number.
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fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1 first to2

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

t2 r1

first

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                          ( r1 = itrev.induct )
    ∃ t1 : term.                                        ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.              ( to1 = itrev )
          r1 is_rule_of to1    True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
        ∧
          ∀ t2 : term ∈ induction_term.                      ( t2  = xs and ys )
            ∃ to2 : term_occurrence ∈ t2 : term.             ( to2 = xs and ys )
              ∃ n : number.
                is_nth_argument_of (to2, n, to1)   when t2 is xs (n = 1) ?
              ∧
                t2 is_nth_induction_term n
```

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```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first to2 t2 first r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1)
∧
t2 is_nth_induction_term n

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```
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 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first second to2 t2 second first r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1) ?
∧
t2 is_nth_induction_term n when t2 is ys (n = 2) ?

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```
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  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 · first · second · to2 · t2 · second · first · r1

$\exists$ *r1* : rule. True

$\rightarrow$

$\exists$ *r1* : rule. (r1 = itrev.induct)
  $\exists$ *t1* : term. (t1 = itrev)
    $\exists$ *to1* : term_occurrence $\in$ *t1* : term. (to1 = itrev)
      r1 is_rule_of *to1* True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
    $\wedge$
      $\forall$ *t2* : term $\in$ induction_term. (t2 = xs and ys)
        $\exists$ *to2* : term_occurrence $\in$ *t2* : term. (to2 = xs and ys)
          $\exists$ *n* : number.
            is_nth_argument_of (*to2*, *n*, *to1*) when t2 is xs (n = 1)
          $\wedge$
            t2 is_nth_induction_term *n* when t2 is ys (n = 2)

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```
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first second to2 t2 second first r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys ✓ ✓)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1)
∧
t2 is_nth_induction_term n when t2 is ys (n = 2)

$\exists$ $r1$: rule. True

$\rightarrow$

$\exists$ $r1$: rule.
$\exists$ $t1$: term.
$\exists$ $to1$: term_occurrence $\in$ $t1$: term.
$r1$ is_rule_of $to1$
$\wedge$
$\forall$ $t2$: term $\in$ induction_term.
$\exists$ $to2$: term_occurrence $\in$ $t2$: term.
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
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```

new types ->

```
datatype      instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
→
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    ∃ t1 : term.
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              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
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  ∃ r1 : rule.
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 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
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new types ->

new constants ->

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new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

r1

(r1 = exec.induct)

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

r1

(r1 = exec.induct)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
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 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

r1

(r1 = exec.induct)
(t1 = exec)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
   ∃ to1 : term_occurrence ∈ t1 : term.
       r1 is_rule_of to1
     ∧
       ∀ t2 : term ∈ induction_term.
         ∃ to2 : term_occurrence ∈ t2 : term.
           ∃ n : number.
               is_nth_argument_of (to2, n, to1)
             ∧
               t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->
∃ r1 : rule. True

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

$$
\begin{array}{l}
\rightarrow \\
\quad \exists\ r1\ :\ \texttt{rule}. \\
\quad\quad \exists\ t1\ :\ \texttt{term}. \\
\quad\quad\quad \exists\ to1\ :\ \texttt{term\_occurrence} \in\ t1\ :\ \texttt{term}. \\
\quad\quad\quad\quad r1\ \texttt{is\_rule\_of}\ to1 \\
\quad\quad\quad\quad \wedge \\
\quad\quad\quad\quad\quad \forall\ t2\ :\ \texttt{term} \in \texttt{induction\_term}. \\
\quad\quad\quad\quad\quad\quad \exists\ to2\ :\ \texttt{term\_occurrence} \in\ t2\ :\ \texttt{term}. \\
\quad\quad\quad\quad\quad\quad\quad \exists\ n\ :\ \texttt{number}. \\
\quad\quad\quad\quad\quad\quad\quad\quad \texttt{is\_nth\_argument\_of}\ (to2,\ n,\ to1) \\
\quad\quad\quad\quad\quad\quad\quad\quad \wedge \\
\quad\quad\quad\quad\quad\quad\quad\quad t2\ \texttt{is\_nth\_induction\_term}\ n
\end{array}
$$

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 r1

$\exists$ r1 : rule. True

$\rightarrow$

$\exists$ $r1$: rule. (r1 = exec.induct)
$\exists$ $t1$: term. (t1 = exec)
$\exists$ $to1$: term_occurrence $\in$ $t1$: term. (to1 = exec)
r1 is_rule_of $to1$ True! r1 (= exec.induct) is a lemma about to1 (= exec).
$\wedge$
$\forall$ $t2$: term $\in$ induction_term. (t2 = is1, s, and stk)
$\exists$ $to2$: term_occurrence $\in$ $t2$: term. (to2 = is1, s, and stk)
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 r1

$\exists$ r1 : rule. True

$\rightarrow$

$\exists$ $r1$: rule. (r1 = exec.induct)
$\exists$ $t1$: term. (t1 = exec)
$\exists$ $to1$: term_occurrence $\in$ $t1$: term. (to1 = exec)
$r1$ is_rule_of $to1$ True! r1 (= exec.induct) is a lemma about to1 (= exec).
$\wedge$
$\forall$ $t2$: term $\in$ induction_term. (t2 = is1, s, and stk)
$\exists$ $to2$: term_occurrence $\in$ $t2$: term. (to2 = is1, s, and stk)
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 first

to1 first

r1

to2

∃ r1 : rule. True

→

∃ $r1$: rule.
 ∃ $t1$: term.
  ∃ $to1$: term_occurrence ∈ $t1$: term.
   $r1$ is_rule_of $to1$
  ∧
   ∀ $t2$: term ∈ induction_term.
    ∃ $to2$: term_occurrence ∈ $t2$: term.
     ∃ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
     ∧
      $t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"
```

```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first, t2, to1, r1, to2

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

r1 is_rule_of to1 — True! r1 (= exec.induct) is a lemma about to1 (= exec).

∧

∀ t2 : term ∈ induction_term.

(t2 = is1, s, and stk)

∃ to2 : term_occurrence ∈ t2 : term.

(to2 = is1, s, and stk)

∃ n : number.

is_nth_argument_of (to2, n, to1) — when t2 is is1 (n -> 1)

∧

t2 is_nth_induction_term n

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first
second
t2
to1
to2
r1

$\exists$ r1 : rule. True

$\rightarrow$

$\exists$ $r1$: rule.
 $\exists$ $t1$: term.
  $\exists$ $to1$: term_occurrence $\in$ $t1$: term.
   $r1$ is_rule_of $to1$
  $\wedge$
   $\forall$ $t2$: term $\in$ induction_term.
    $\exists$ $to2$: term_occurrence $\in$ $t2$: term.
     $\exists$ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
     $\wedge$
      $t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)
when t2 is s (n -> 2) ?

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

first second

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

t2 to1 to2

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

r1

first second

$\exists$ r1 : rule. True

$\rightarrow$

$\exists$ $r1$: rule. (r1 = exec.induct)

$\exists$ $t1$: term. (t1 = exec)

$\exists$ $to1$: term_occurrence $\in$ $t1$: term. (to1 = exec)

$r1$ is_rule_of $to1$ True! r1 (= exec.induct) is a lemma about to1 (= exec).

$\wedge$

$\forall$ $t2$: term $\in$ induction_term. (t2 = is1, s, and stk)

$\exists$ $to2$: term_occurrence $\in$ $t2$: term. (to2 = is1, s, and stk)

$\exists$ n : number.

is_nth_argument_of $(to2, n, to1)$ when t2 is is1 (n -> 1)

$\wedge$ when t2 is s (n -> 2)

$t2$ is_nth_induction_term n

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
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a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

first second third — t2
first second third — to1
r1 — to1
first second third — to2

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk) ✓ ✓
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?
when t2 is s (n -> 2) ?
when t2 is stk (n -> 3) ?

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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∃ to2 : term_occurrence ∈ t2 : term.
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is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

first second third — t2

first second third — to1

r1 — to2

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```
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a model proof ->

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  apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

first second third t2

first second third to1

to2

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)
when t2 is s (n -> 2)
when t2 is stk (n -> 3)

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

<- pros: good at ambiguity (heuristics)

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in               uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                        )"
by(induct is1 s stk rule:e                        uct) auto
```

<- small dataset about different domains

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]          [] = rev [1,2]          @ []" by auto
lemma "itrev [1,2,3]        [] = rev [1,2,3]        @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]        [] = rev [x,y,z]        @ []" by auto
```

<- many concrete cases

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

[T,F,T], [T,T,T], [F,T,T]: bool list

<- pros: good at ambiguity (heuristics)

WIP

<- pros: good at rigor

~~<- cons: bad at ambiguity (heuristics)~~

LiFtEr

LiFtEr

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

```
lemma "star r                star                ⟹ star r x z"
by(induction r                 ar.in              uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                        )"
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<- small dataset about different domains

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lemma "itrev xs ys = rev xs @ ys"
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```

<- one abstract representation

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<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

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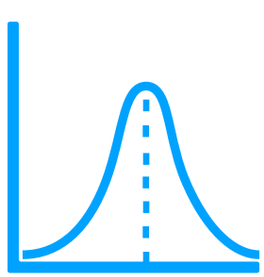

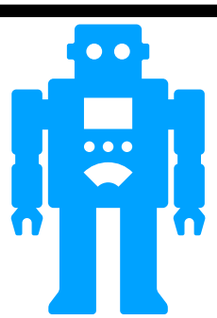

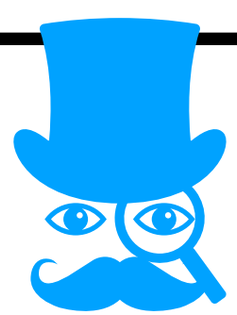

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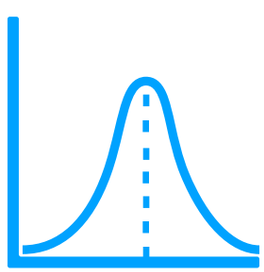
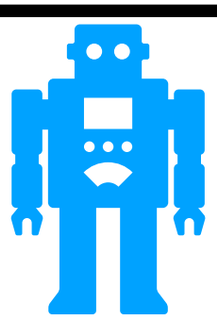
Isabelle's proof methods
(deductive reasoning)





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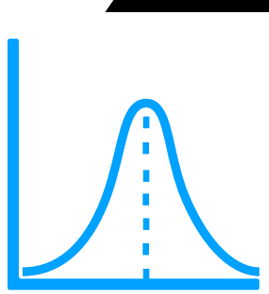
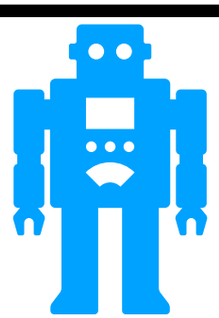
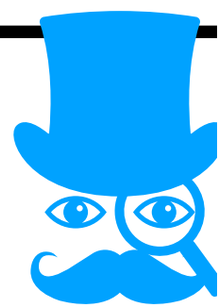
Isabelle's proof methods
(deductive reasoning)

goal-oriented conjecturing
(abductive reasoning)







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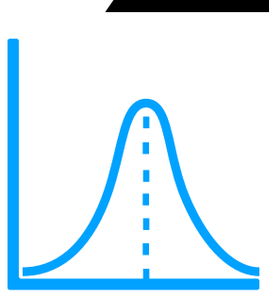
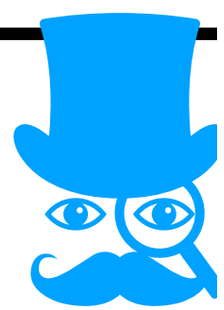
Isabelle's proof methods
(deductive reasoning)

machine learning induction
(inductive reasoning)



goal-oriented conjecturing
(abductive reasoning)



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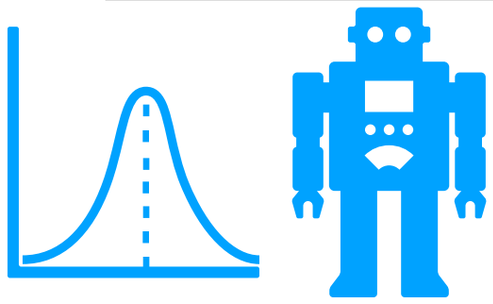
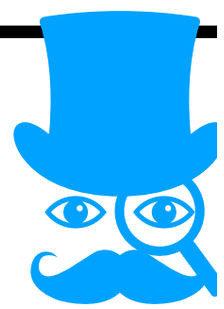
Isabelle's proof methods
(deductive reasoning)

united reasoning for automatic induction

machine learning induction
(inductive reasoning)



goal-oriented conjecturing
(abductive reasoning)



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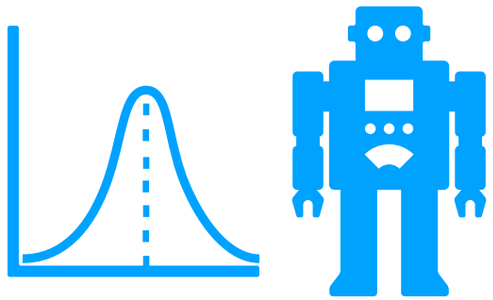
Isabelle's proof methods
(deductive reasoning)

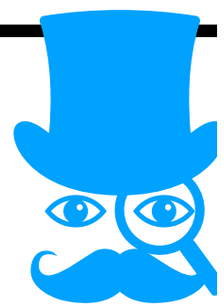
united reasoning for automatic induction

depth first search -> best first search

machine learning induction
(inductive reasoning)



goal-oriented conjecturing
(abductive reasoning)



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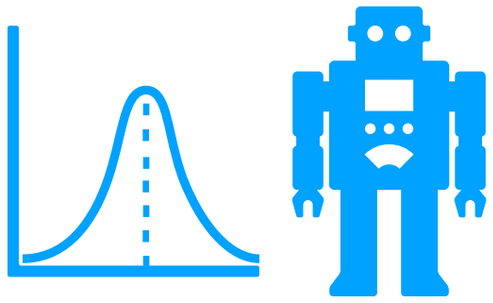
Isabelle's proof methods
(deductive reasoning)

united reasoning for automatic induction

fin.

depth first search -> best first search

machine learning induction
(inductive reasoning)



goal-oriented conjecturing
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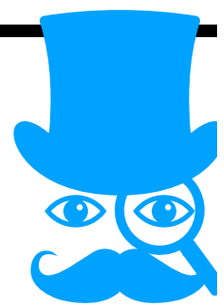

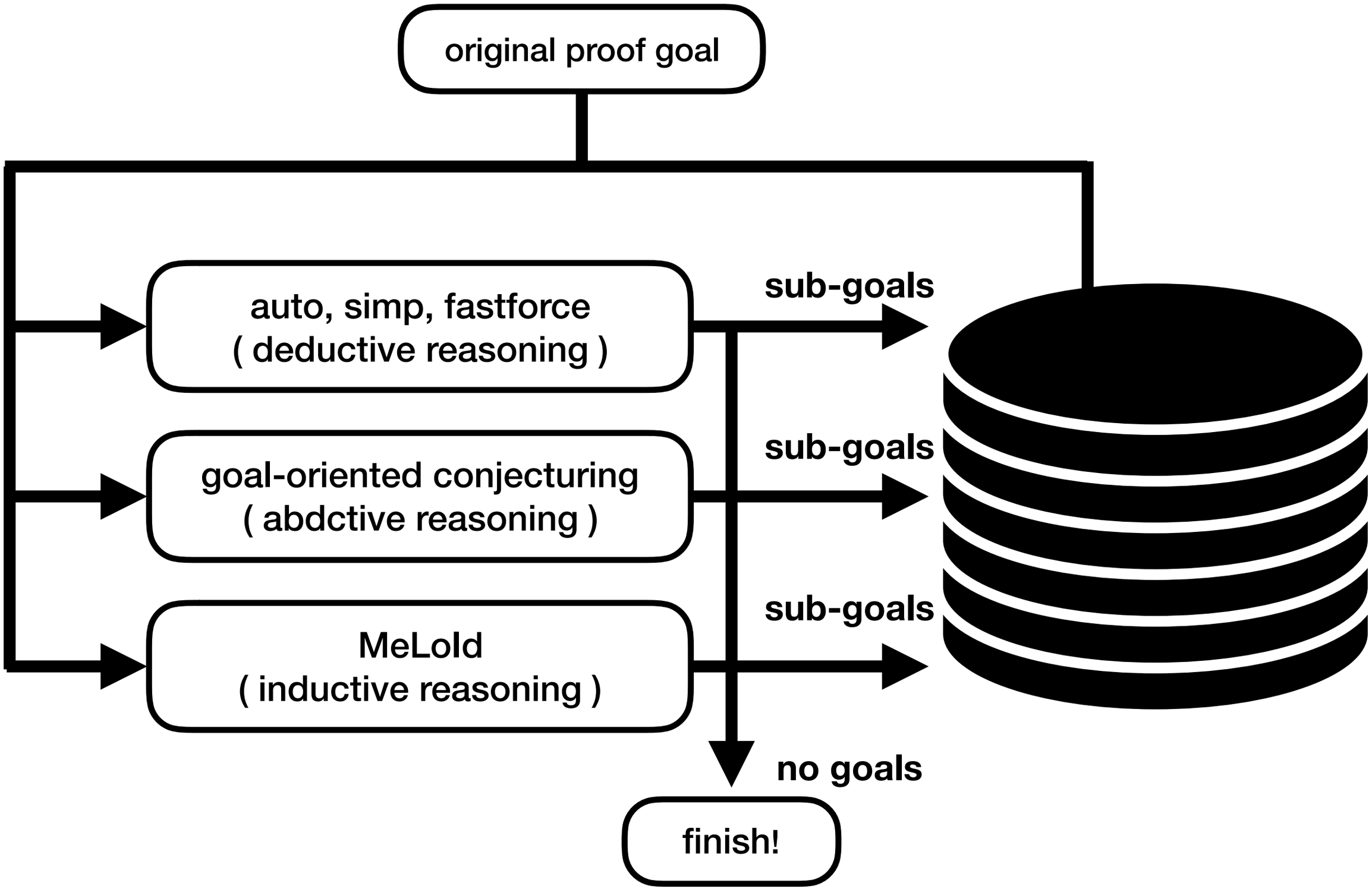
original proof goal
auto, simp, fastforce
(deductive reasoning)
goal-oriented conjecturing
(abdctive reasoning)
MeLoId
(inductive reasoning)
sub-goals
sub-goals
sub-goals
no goals
finish!

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3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

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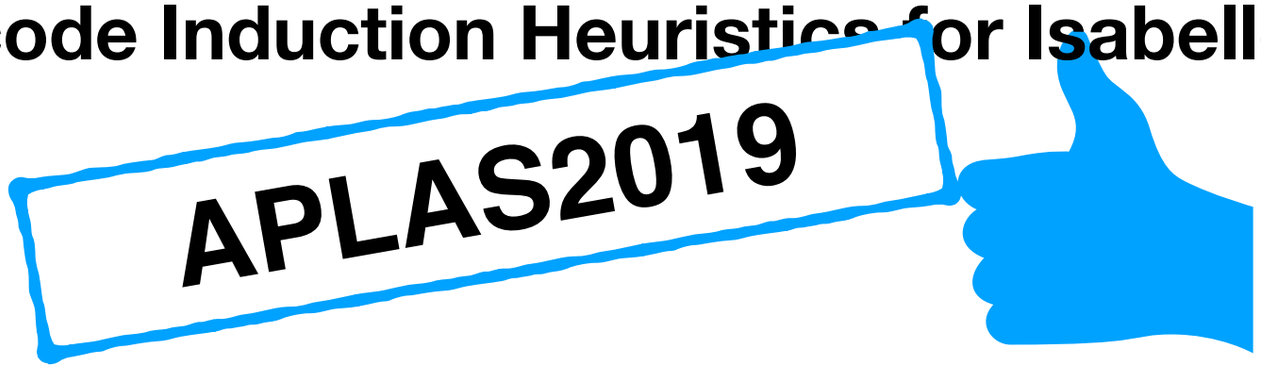
3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL



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3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

APLAS2019

Secret 1

Many induction heuristics are simple.

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3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

APLAS2019

Secret 1

Many induction heuristics are simple.

Secret 2

Proofs can be more abstract than theorems.

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3 secrets behind LiFtEr

LiFtEr: Language to Encode Induction Heuristics for Isabelle/HOL

APLAS2019

Secret 1

Many induction heuristics are simple.

Secret 2

Proofs can be more abstract than theorems.

Secret 3

LiFtEr is not good enough because …

LiFtEr almost ignores the definitions of constants.

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in              uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto
```

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

<- small dataset about different domains

<- one abstract representation

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in              uto simp: step)

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lemma "itrev xs ys = rev xs @ ys"
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```

<- small dataset about different domains

<- one abstract representation

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
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<- relevant definitions

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in              uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr LiFtEr LiFtEr

<- small dataset about different domains

<- one abstract representation

LiFtEr 2 LiFtEr 2

```
lemma "itrev xs ys = rev xs @
by(induct xs ys rule:"itrev.i                ) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []       = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

LiFtEr: (proof goal * induction arguments) -> bool

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

```
lemma "star r x y ⟹ sta                ⟹ star r x z"
by(induction rule: star.in             uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr LiFtEr LiFtEr

<- small dataset about different domains

<- one abstract representation

LiFtEr 2 LiFtEr 2

```
lemma "itrev xs ys = rev xs @
by(induct xs ys rule:"itrev.i                ) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []       = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

```
lemma "star r x y ⟹ sta                ⇒ star r x z"
by(induction rule: star.in             uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
by(induct is1 s stk rule:e                uct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr LiFtEr LiFtEr

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LiFtEr 2 LiFtEr 2

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<- relevant definitions

LiFtEr$_2$ (proof goal * induction arguments) -> bool
*** relevant definitions**

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- s

WIP

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LiFtEr LiFtEr LiFtEr

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LiFtEr 2 LiFtEr 2

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<- relevant definitions

LiFtEr$_2$ (**proof goal * induction arguments**) **-> bool**
*** relevant definitions**

WIP

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- st

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LiFtEr LiFtEr LiFtEr

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LiFtEr 2 LiFtEr 2

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fin.