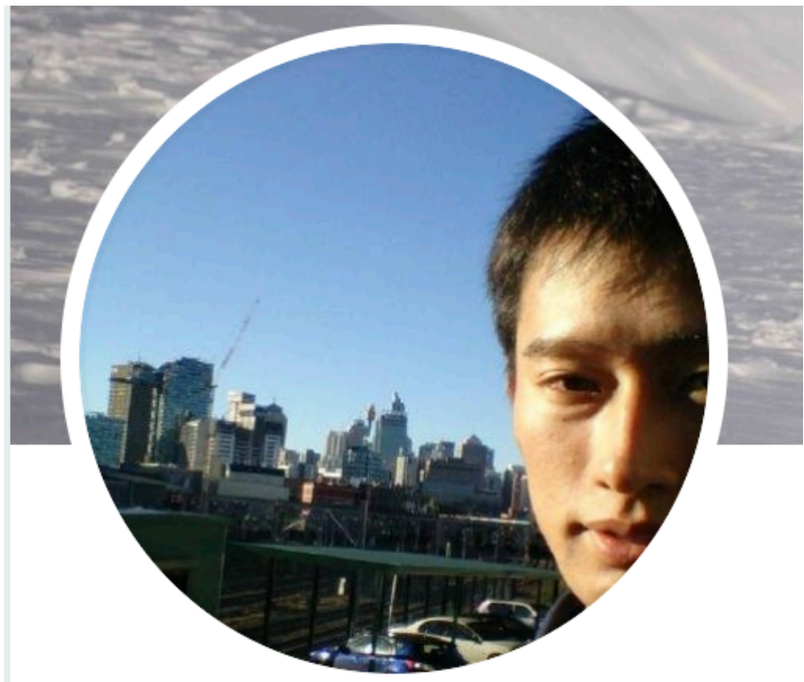
https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

AI for Isabelle/HOL

This work was supported by the project AI&Reasoning (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000466).

Yutaka Nagashima
University of Innsbruck
Czech Technical University



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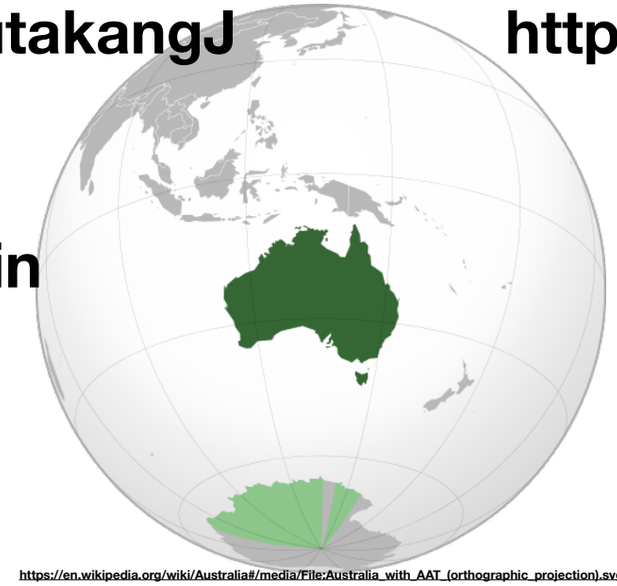
2013 ~ 2017

with Dr. Gerwin Klein



https://en.wikipedia.org/wiki/Australia#/media/File:Australia_with_AAT_(orthographic_projection).svg

Isabelle HOL

(seL4 Security. Performance. Proof.)

http://www.cse.unsw.edu.au/~kleing/

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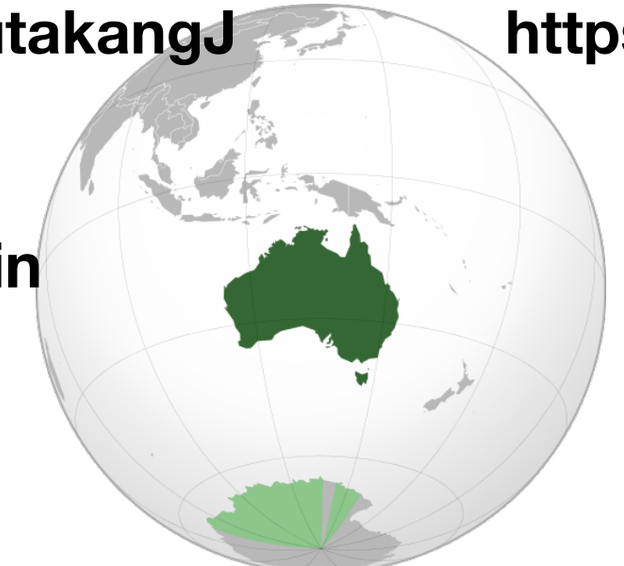
2013 ~ 2017

with Dr. Gerwin Klein

pre-PhD



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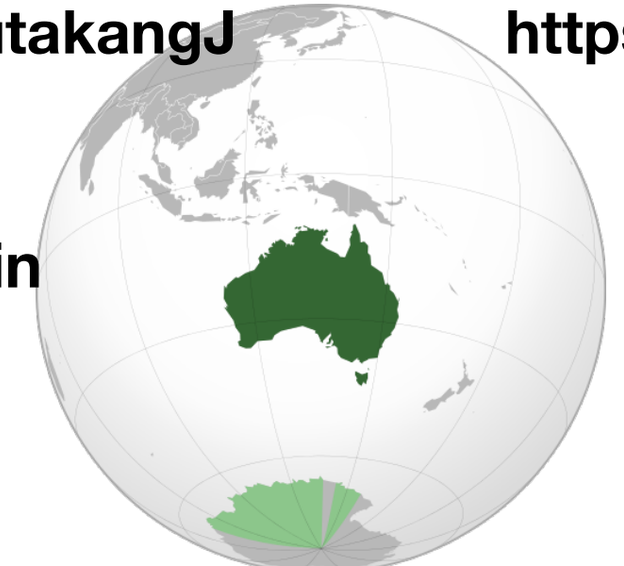
PhD in
AI for theorem proving



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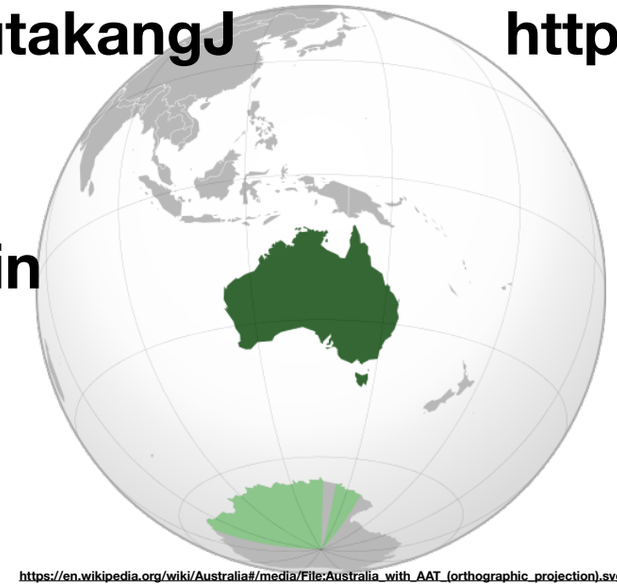

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PhD in

AI for theorem proving

2017 ~ 2018

with Prof. Cezary Kaliszyk

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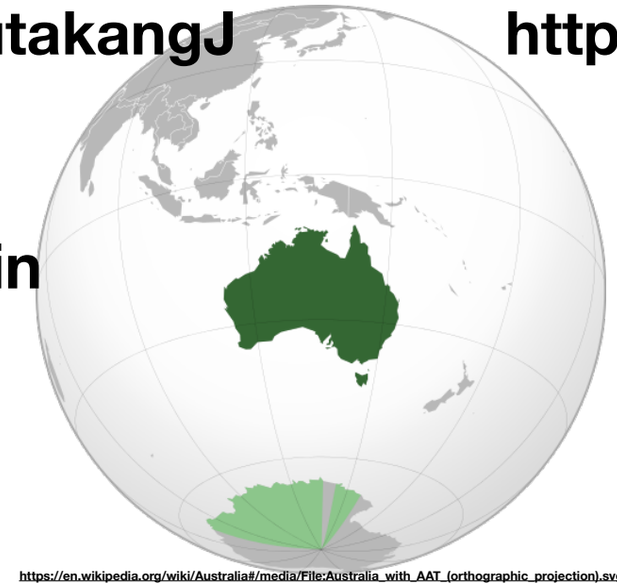
2013 ~ 2017

with Dr. Gerwin Klein



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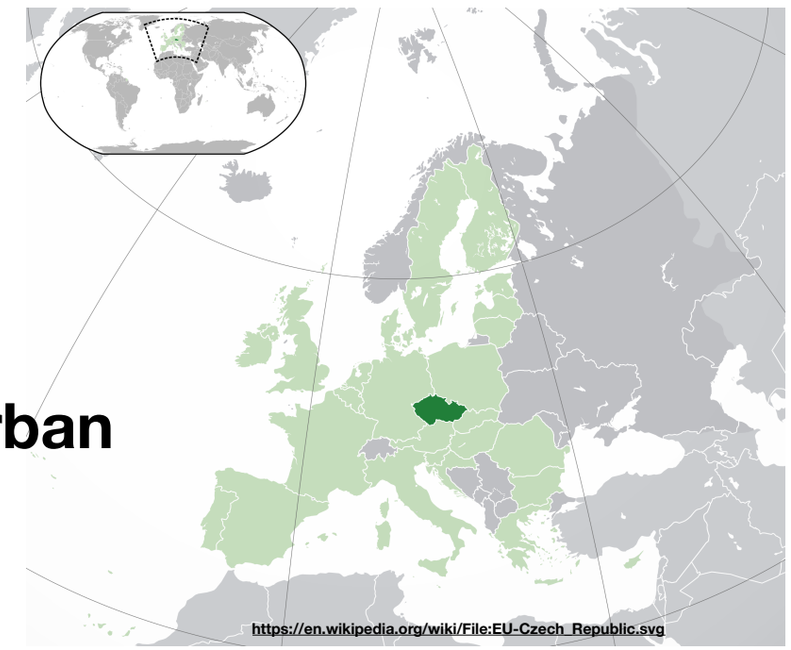
2018 ~ 2020

with Dr. Josef Urban



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http://ai4reason.org/members.html

https://twitter.com/YutakangJ

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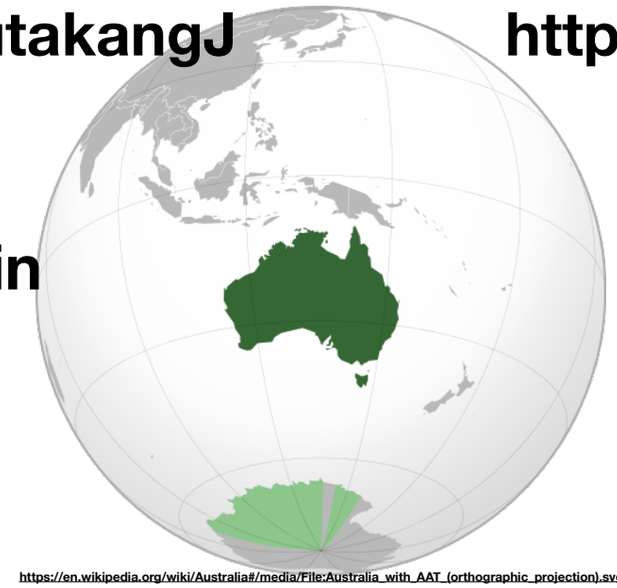
2013 ~ 2017

with Dr. Gerwin Klein



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pre-PhD

PhD in AI for theorem proving

2017 ~ 2018
2020 ~ 2021?

with Prof. Cezary Kaliszyk

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2018 ~ 2020

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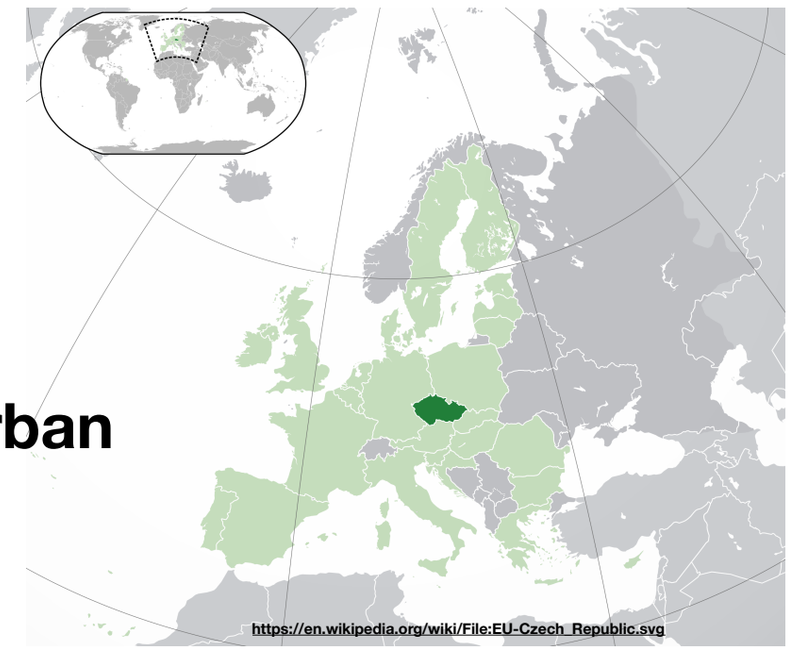

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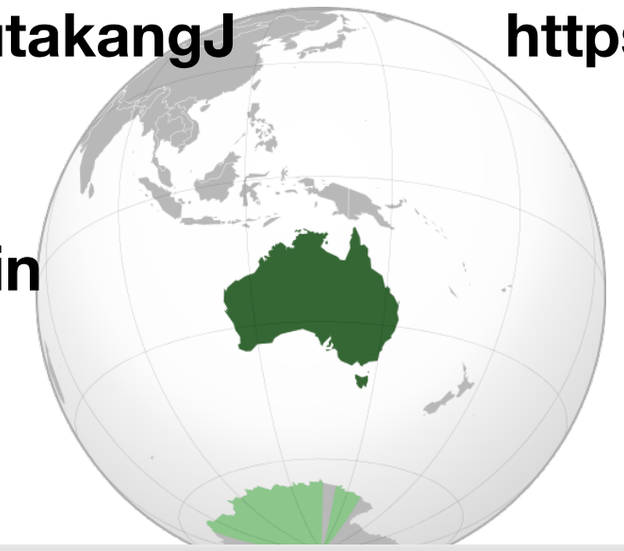

pre-PhD

Registration is now closed.

http://aitp-conference.org/2019/

Background

Large-scale semantic processing and strong computer assistance of mathematics and science is our inevitable future. New combinations of AI and reasoning methods and tools deployed over large mathematical and scientific corpora will be instrumental to this task. The AITP conference is the forum for discussing how to get there as soon as possible, and the force driving the progress towards that.

Topics

- AI and big-data methods in theorem proving and mathematics
- Collaboration between automated and interactive theorem proving
- Common-sense reasoning and reasoning in science
- Alignment and joint processing of formal, semi-formal, and informal libraries
- Methods for large-scale computer understanding of mathematics and science
- Combinations of linguistic/learning-based and semantic/reasoning methods

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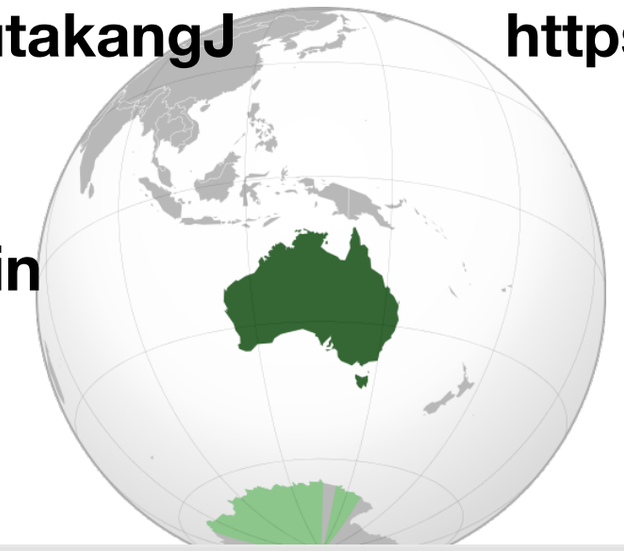
2013 ~ 2017

with Dr. Gerwin Klein



pre-PhD

4th Conference on Artificial Intelligence and Theorem Proving
AITP 2019
April 7–12, 2019, Obergurgl, Austria

Registration is now closed.

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Isabelle/HOL architecture

Isabelle/HOL architecture

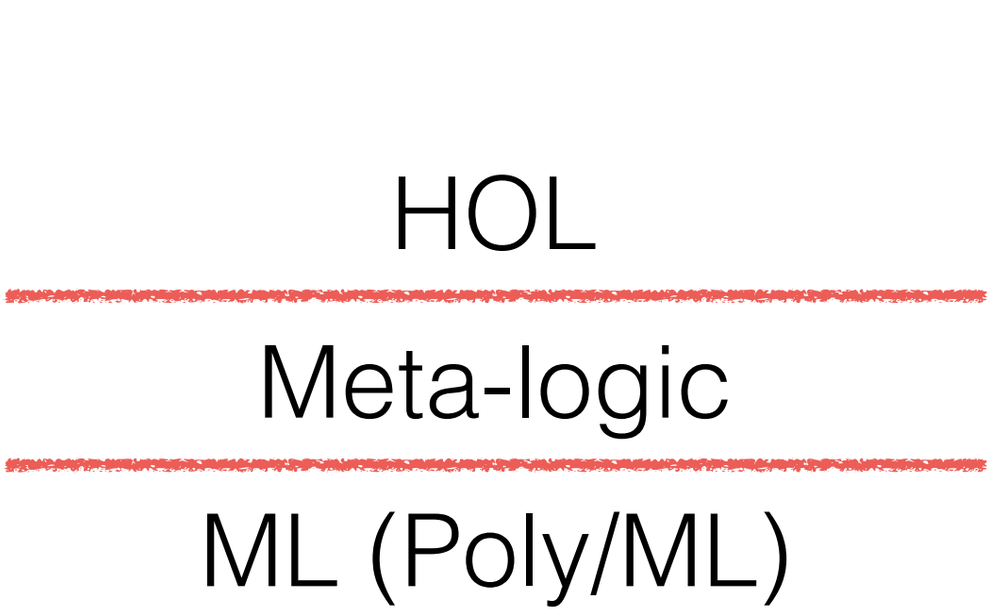
ML (Poly/ML)

Isabelle/HOL architecture

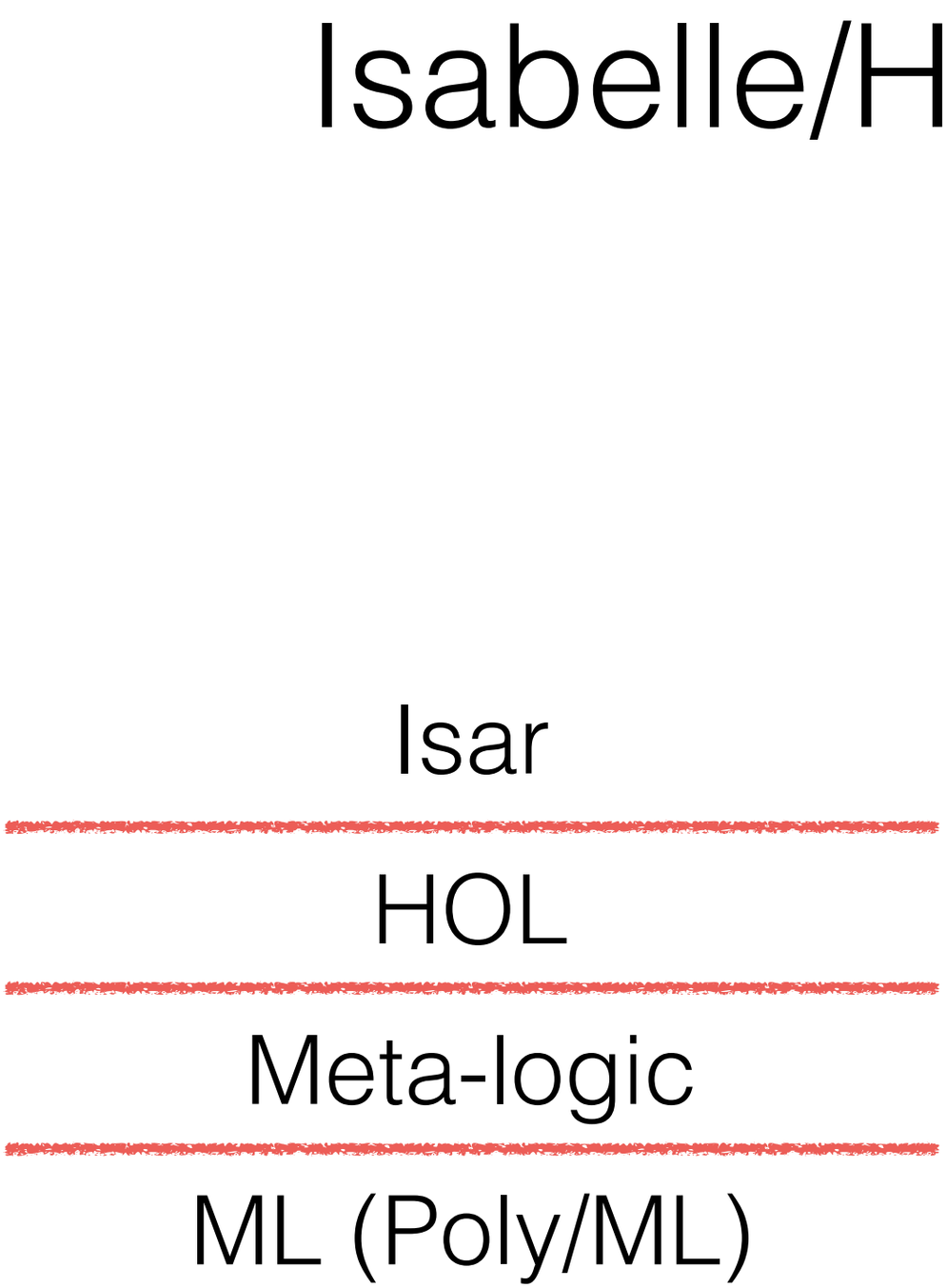
Meta-logic

ML (Poly/ML)

Isabelle/HOL architecture



Isabelle/HOL architecture



Isabelle/HOL architecture

PIDE / jEdit

Isar

HOL

Meta-logic

ML (Poly/ML)

Isabelle/HOL architecture

PIDE / jEdit

Isar

HOL

Meta-logic

ML (Poly/ML)

Isabelle/HOL architecture

PIDE / jEdit

Isar

HOL

Meta-logic

ML (Poly/ML)

You can access all the layers! :)

Isabelle/HOL architecture

PIDE / jEdit

Isar

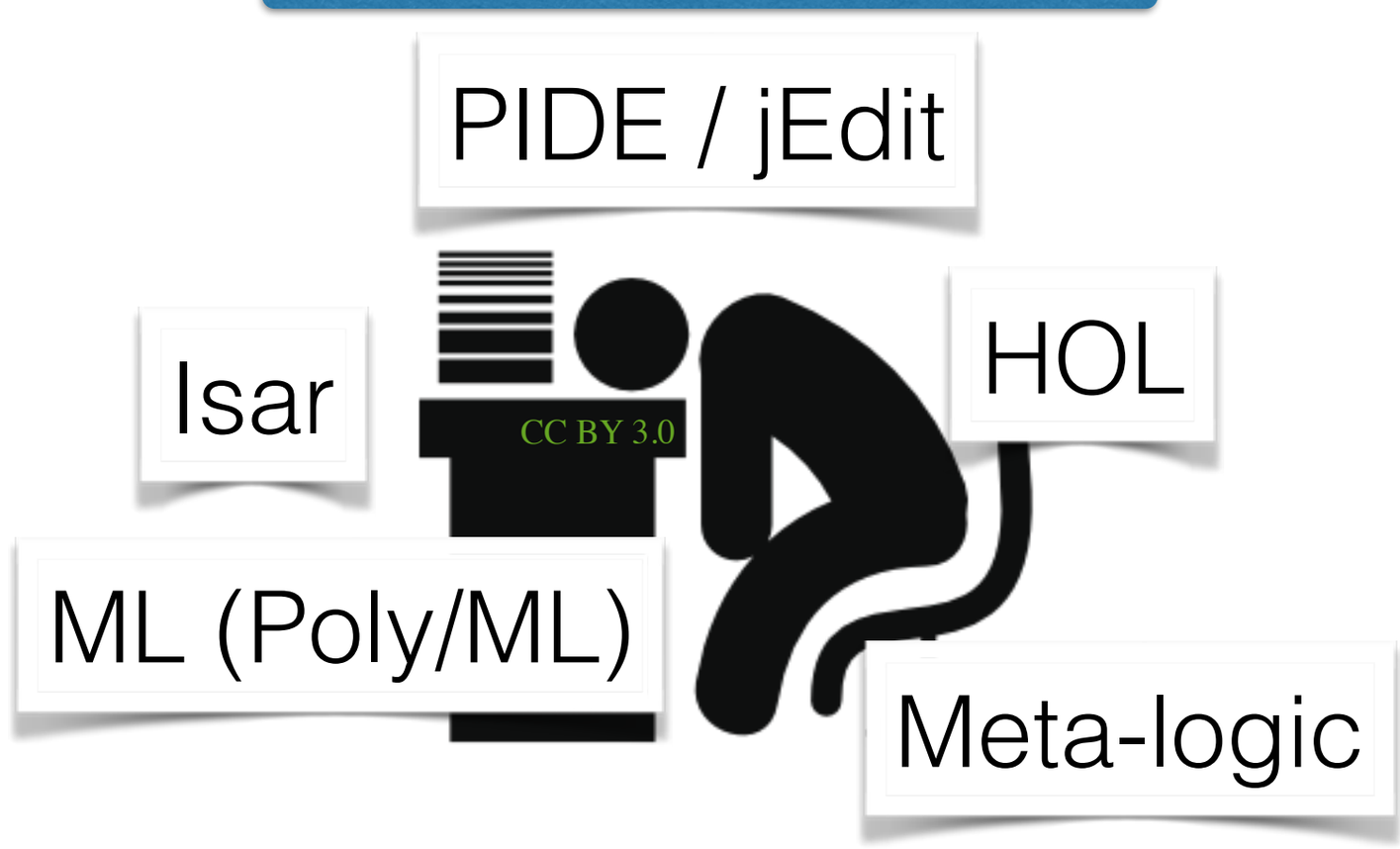
HOL

Meta-logic

ML (Poly/ML)

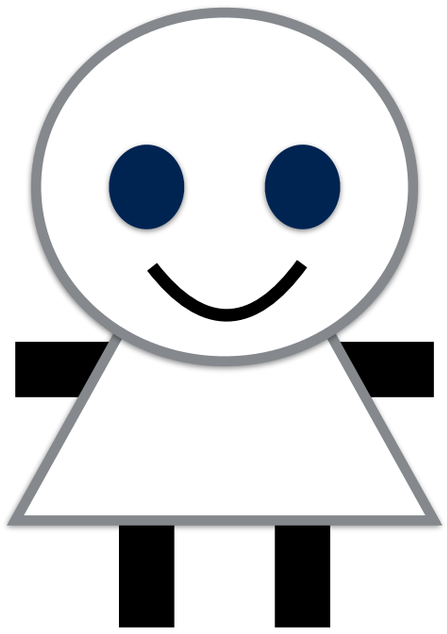
You can access all the layers!
:)

They come all together!
: (



https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Interactive theorem proving with Isabelle/HOL



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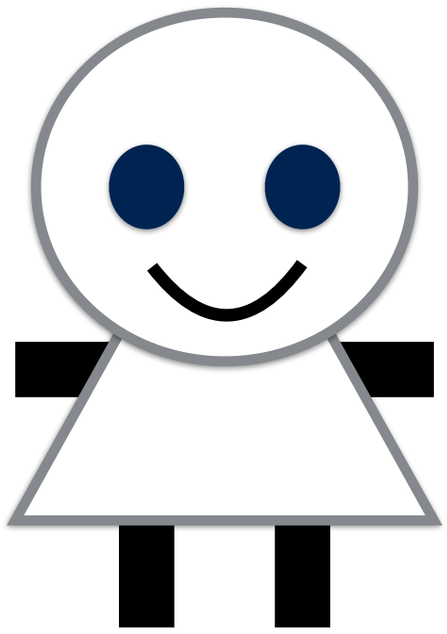
Interactive theorem proving with Isabelle/HOL

proof goal **context**

tactic / proof method



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Interactive theorem proving with Isabelle/HOL

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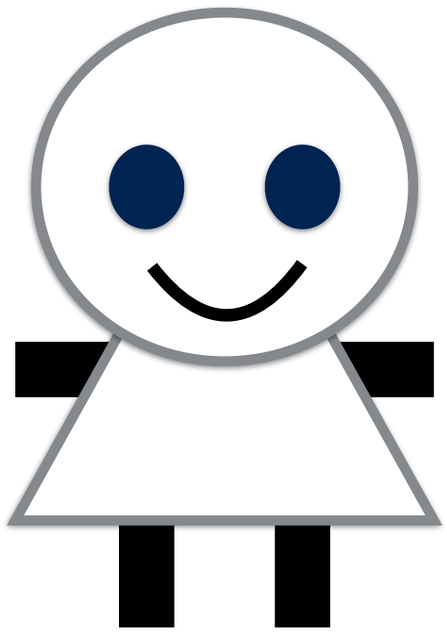

error-message

subgoals

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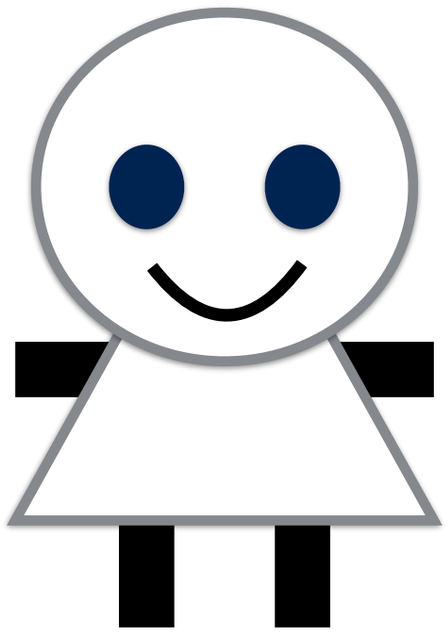
Interactive theorem proving with Isabelle/HOL

proof goal

context

tactic / proof method



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Interactive theorem proving with Isabelle/HOL

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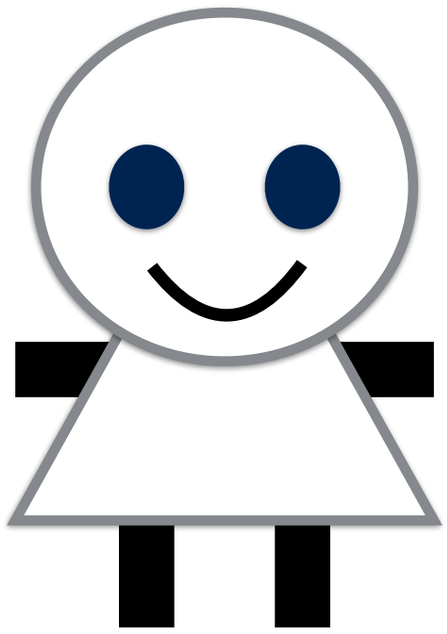

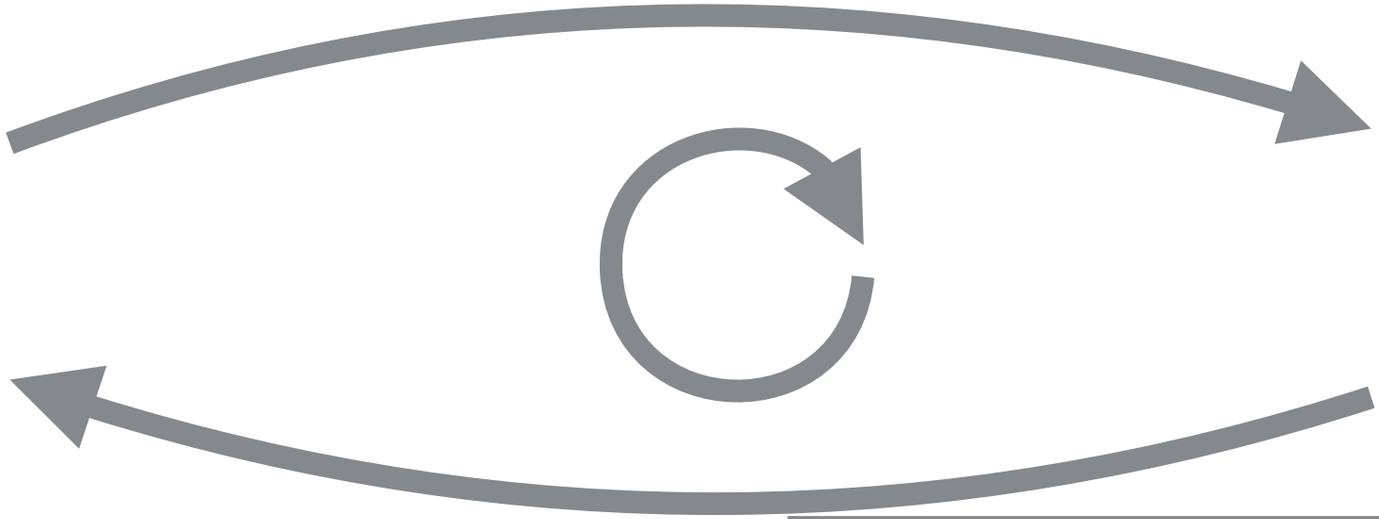

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subgoals

no sub-goal!

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Interactive theorem proving with Isabelle/HOL

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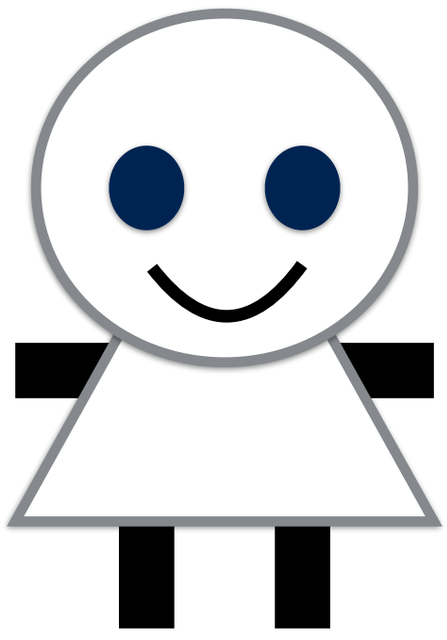

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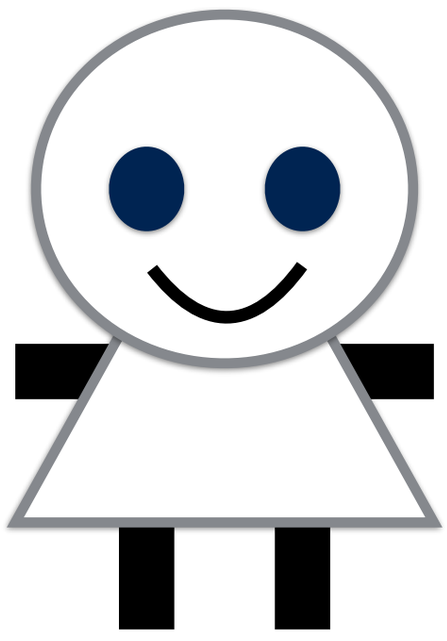
Interactive theorem proving with Isabelle/HOL



proof goal

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tactic / proof method

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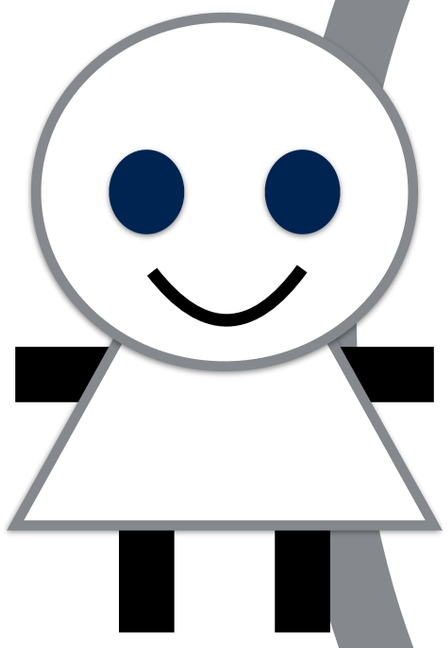
Interactive theorem proving with Isabelle/HOL



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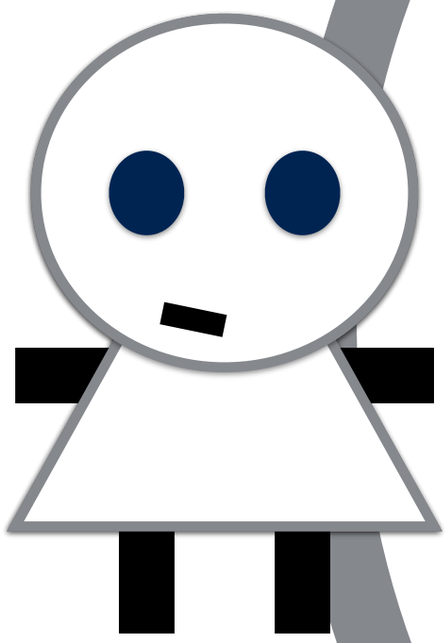

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Interactive theorem proving with Isabelle/HOL

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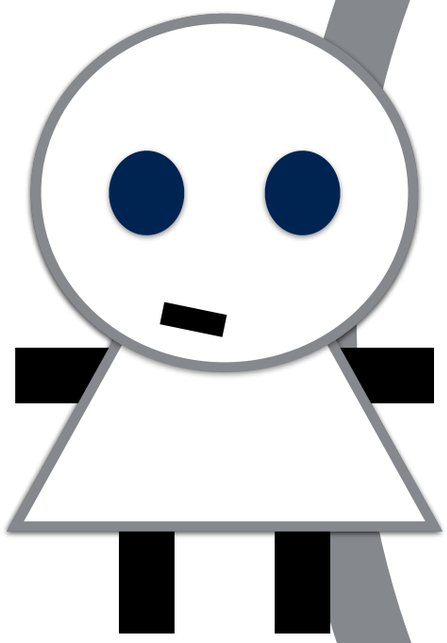

error-message

o-goal!

It's blatantly clear
You stupid machine, that what
I tell you is true
(Michael Norrish)

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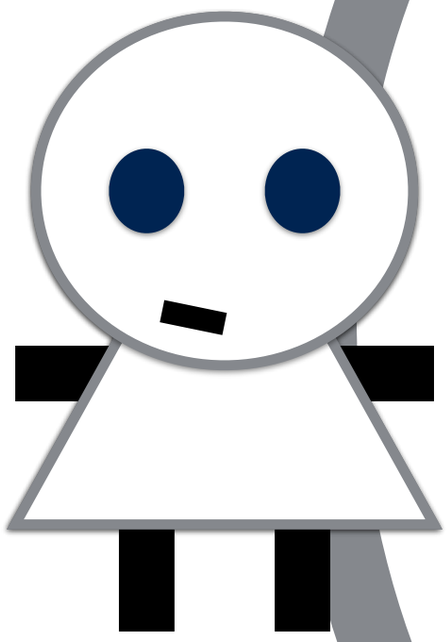
Interactive theorem proving with Isabelle/HOL

proof goal context

tactic / proof method

DEMO!



error-message

-goal!

It's blatantly clear
You stupid machine, that what
I tell you is true
(Michael Norrish)

Example proof at Data61

```
lemma performPageTableInvocationUnmap_ccorres:
  "ccorres (K (K \<bottom>) \<currency> dc) (liftxf errstate id (K ()) ret__unsigned_long_')
       (invs' and cte_wp_at' (diminished' (ArchObjectCap cap) \<circ> cteCap) ctSlot
              and (\<lambda>_. isPageTableCap cap))
       (UNIV \<inter> \<lbrace>ccap_relation (ArchObjectCap cap) \<acute>cap\<rbrace> \<inter> \<lbrace>\<acute>ctSlot
       []
       (liftE (performPageTableInvocation (PageTableUnmap cap ctSlot)))
       (Call performPageTableInvocationUnmap_'proc)"
  apply (simp only: liftE_liftM ccorres_liftM_simp)
  apply (rule ccorres_gen_asm)
  apply (cinit lift: cap_' ctSlot_')
   apply csymbr
   apply (simp del: Collect_const)
   apply (rule ccorres_split_nothrow_novcg_dc)
      apply (subgoal_tac "capPTMappedAddress cap
                        = (\<lambda>cp. if to_bool (capPTIsMapped_CL cp)
                           then Some (capPTMappedASID_CL cp, capPTMappedAddress_CL cp)
                           else None) (cap_page_table_cap_lift capa)")
       apply (rule ccorres_Cond_rhs)
        apply (simp add: to_bool_def)
        apply (rule ccorres_rhs_assoc)+
        apply csymbr
        apply csymbr
        apply csymbr
        apply csymbr
        apply (ctac add: unmapPageTable_ccorres)
          apply csymbr
          apply (simp add: storePTE_def swp_def)
          apply (ctac add: clearMemory_setObject_PTE_ccorres[unfolded dc_def])
           apply wp
        apply (simp del: Collect_const)
```

taken from:
https://github.com/seL4/seL4

Example proof at Data61

impressive! **interesting?**

```
lemma performPageTableIn[illegible]ocationUnmap_ccorres:
  "ccorres (K (K \[illegible]) \<currency> dc) ([illegible]ate id (K ()) ret__unsigned_long_')
       (invs' [illegible] (diminishe[illegible]ap cap) \<circ> cteCap) ctSlot
       [illegible]ambda>_. is[illegible]p))
       [illegible]nter> \<lbrace>cca[illegible]on (ArchObjectCap cap) \<acute>cap\<rbrace> \<inter> \<lbrace>\<acute>ctSlot

       (liftE (performPageTableInvocation (PageTableUnmap cap ctSlot)))
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  apply (simp only: liftE_liftM ccorres_liftM_simp)
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          apply csymbr
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   apply (simp del: Collect_const)
   apply (rule ccorres_split_nothrow_novcg_dc)
      apply (subgoal_tac "capPTMappedAddress cap
                          = (\<lambda>cp. if to_bool (capPTIsMapped_CL cp)
                             then Some (capPTMappedASID_CL cp, capPTMappedAddress_CL cp)
                             else None) (cap_page_table_cap_lift capa)")
        apply (rule ccorres_Cond_rhs)
         apply (simp add: to_bool_def)
         apply (rule ccorres_rhs_assoc)+
         apply csymbr
         apply csymbr
         apply csymbr
         apply csymbr
         apply (ctac add: unmapPageTable_ccorres)
           apply csymbr
           apply (simp add: storePTE_def swp_def)
           apply (ctac add: clearMemory_setObject_PTE_ccorres[unfolded dc_def])
          apply wp
         apply (simp del: Collect_const)
```

impressive!

~~interesting?~~

taken from:
https://github.com/seL4/seL4

Example proof at Data61

impressive!

~~**interesting?**~~

```
lemma performPageTableIn ocationUnmap_ccorres:
  "ccorres (K (K \  ) \<currency> dc) (             ate id (K ()) ret__unsigned_long_')
       (invs          (diminishe              ap cap) \<circ> cteCap) ctSlot
                   ambda>_. is                   p))
              nter> \<lbrace>cc               on (ArchObjectCap cap) \<acute>cap\<rbrace> \<inter> \<lbrace>\<acute>ctSlot

       (liftE (performPageTableInvocation (PageTableUnmap cap ctSlot)))
       (Call performPageTableInvocationUnmap_'proc)"
  apply (simp only: liftE_liftM ccorres_liftM_simp)
  apply (rule ccorres_gen_asm)
  apply (cinit lift: cap_' ctSlot_')
   apply csymbr
   apply (simp del: Collect_const)
   apply (rule ccorres_split_nothrow_novcg_dc)
      apply (subgoal_tac "capPTMappedAddress cap
                            = (\<lambda>cp. if to_bool (capPTIsMapped_CL cp)
                               then Some (capPTMappedASID_CL cp, capPTMappedAddress_CL cp)
                               else None) (cap_page_table_cap_lift capa)")
                ccorres_Cond_rhs)
               mp add: to_bool_def)
                ccorres_rhs_assoc)+
                br
        ply   symbr
        ply   symbr
         apply csymbr
         apply (ctac add: unmapPageTable_ccorres)
           apply csymbr
           apply (simp add: storePTE_def swp_def)
           apply (ctac add: clearMemory_setObject_PTE_ccorres[unfolded dc_def])
          apply wp
         apply (simp del: Collect_const)
```

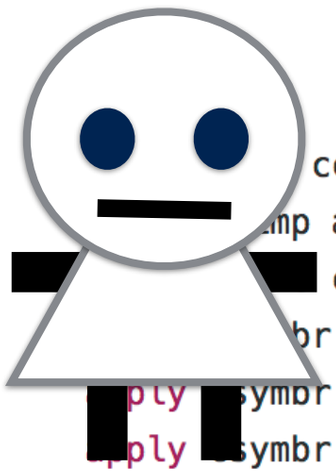
taken from:
https://github.com/seL4/seL4

Example proof at Data61

```
39  lemma performPageTableInvocationUnmap_ccorres:
40    "ccorres (K (K \<currency> dc) (                ate id (K ()) ret__unsigned_long_')
41         (invs'                (diminishe                ap cap) \<circ> cteCap) ctSlot
42                              <lambda>_. is                p))
43                        nter> \<lbrace>cc                on (ArchObjectCap cap) \<acute>cap\<rbrace> \<inter> \<lbrace>\<acute>ctSlot
44
45         (liftE (performPageTableInvocation (Pa
46         (Call performPageTableInvocationU
47    apply (simp only: liftE_liftM ccorr
48    apply (rule ccorres_gen_asm)
49    apply (cinit lift: cap_' ctSlot_')
50     apply csymbr
51     apply (simp del: Collect_const)
52    apply (rule ccorres_split_nothrow novcg_
53       apply (subgoal_tac "capPT              dress cap
54                             = (\<              a>cp. if to_bool (capPTIsMapped_CL cp)
55                                        hen Some (capPTMappedASID_CL cp, capPTMappedAddress_CL cp)
56                                   else None) (cap_page_table_cap_lift capa)")
57                  ccorres_Cond_rhs)
58                  mp add: to_bool_def)
59                  ccorres_rhs_assoc)+
60                  br
61            ply  symbr
62            ply  symbr
63          apply csymbr
64          apply (ctac add: unmapPageTable_ccorres)
65            apply csymbr
66            apply (simp add: storePTE_def swp_def)
67            apply (ctac add: clearMemory_setObject_PTE_ccorres[unfolded dc_def])
68           apply wp
69          apply (simp del: Collect_const)
```

impressive!

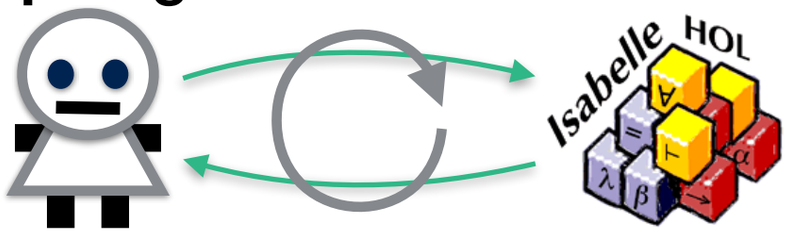
interesting?

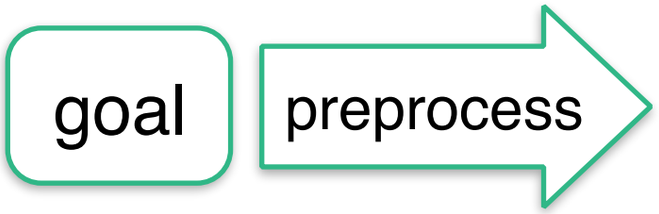


これを人間が書くべきなの…？

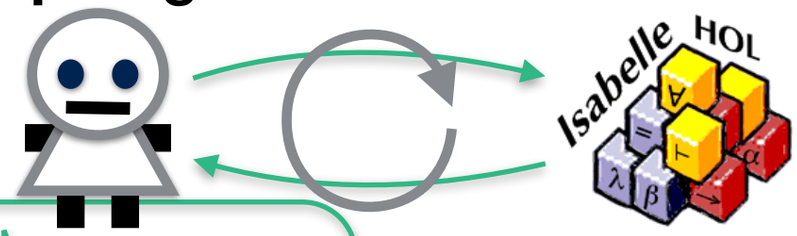
どうにかしろよ、人工知能。

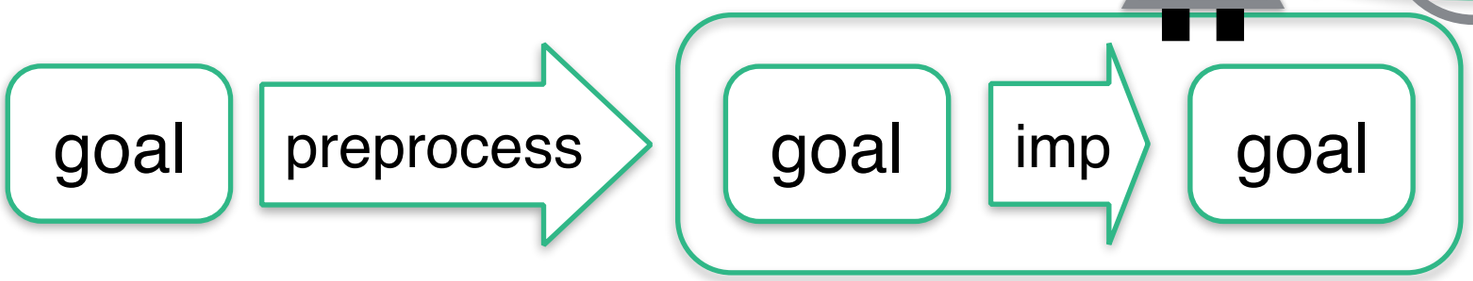
Tactics 1





Tactics 1

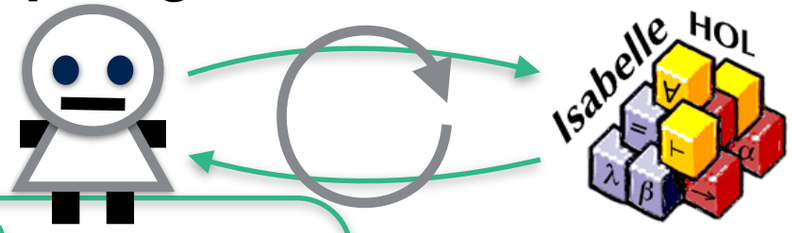




https://twitter.com/YutakangJ

https://github.com/data61/PSL/slide/2019_ps.pdf

Tactics 1



goal → preprocess → goal → imp → goal

principle of explosion

False → imp → P

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Tactics 1

goal → preprocess → goal → imp → goal

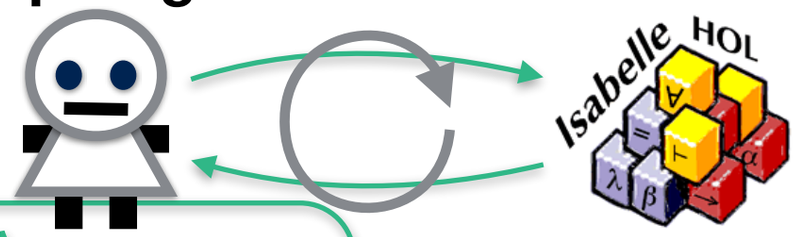
principle of explosion

False → imp → P

Case 1

new goal → imp → goal

Tactics 1



goal → preprocess → goal → imp → goal

tactic

principle of explosion

False → imp → P

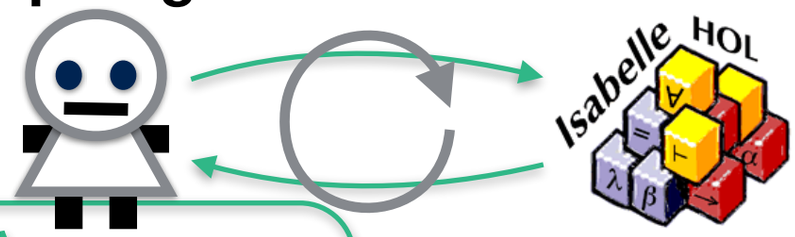
Case 1

new goal → imp → goal

Case 2

goal

Tactics 1



goal → preprocess → goal → imp → goal

tactic

principle of explosion

False → imp → P

Case 1

new goal → imp → goal

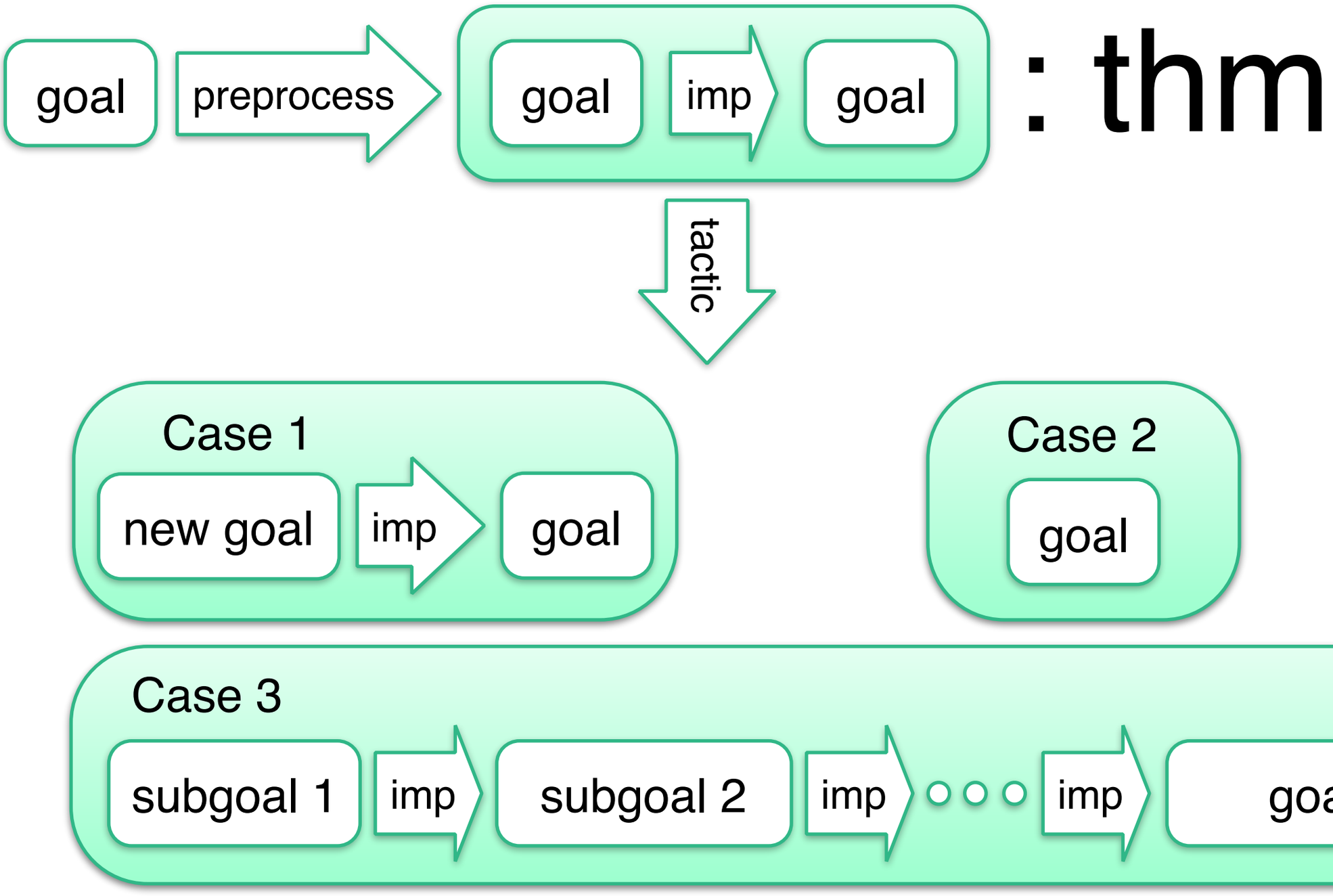
Case 2

goal

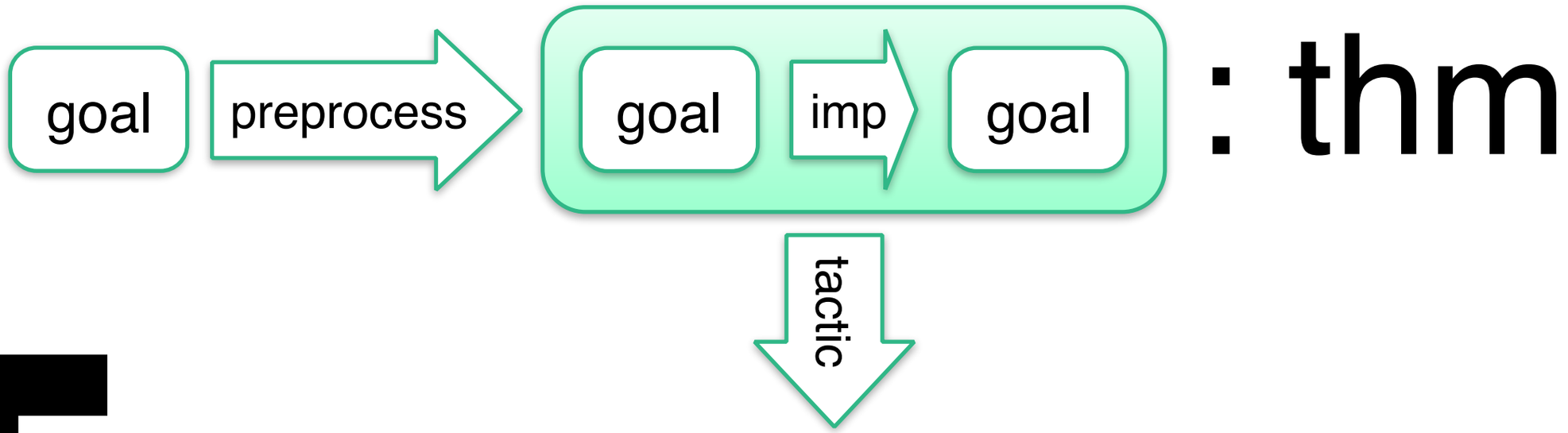
Case 3

subgoal 1 → imp → subgoal 2 → imp → ○○○ → imp → goal

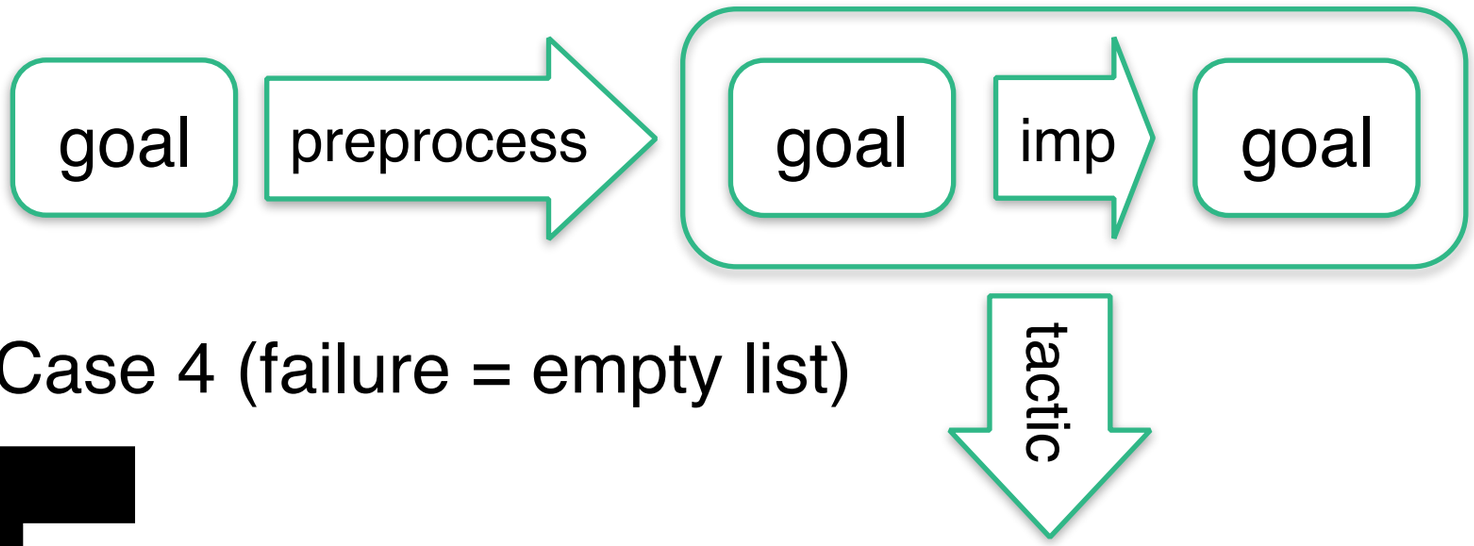
Tactics 2


goal
preprocess
goal
imp
goal
: thm
tactic
Case 1
new goal
imp
goal
Case 2
goal
Case 3
subgoal 1
imp
subgoal 2
imp
imp
goal

Tactics 2


goal
preprocess
goal
imp
goal
: thm
tactic
Case 1
new goal
imp
goal
,
Case 2
goal
,
Case 3
subgoal 1
imp
subgoal 2
imp
imp
goal

Tactics 2



Case 4 (failure = empty list)

Tactics 3

(w /\ x => y /\ z => z)

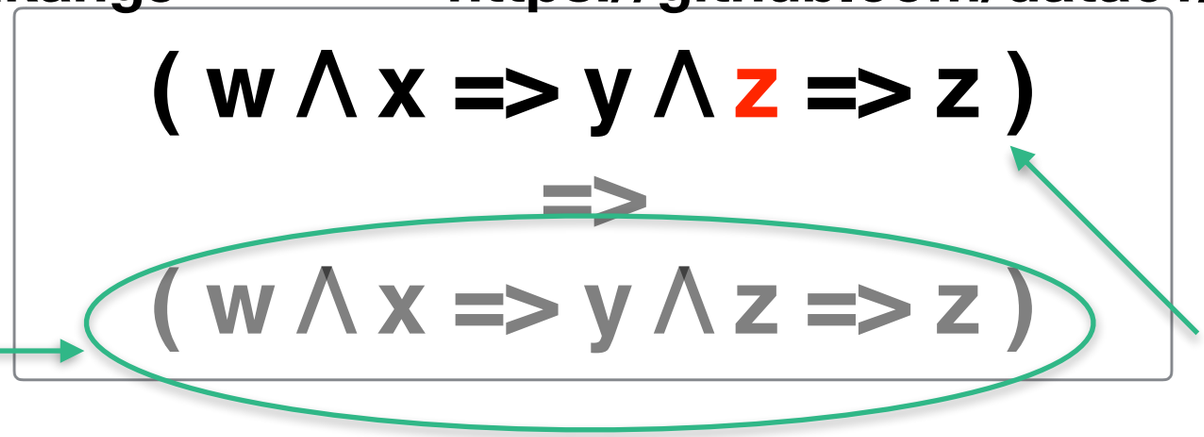
Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

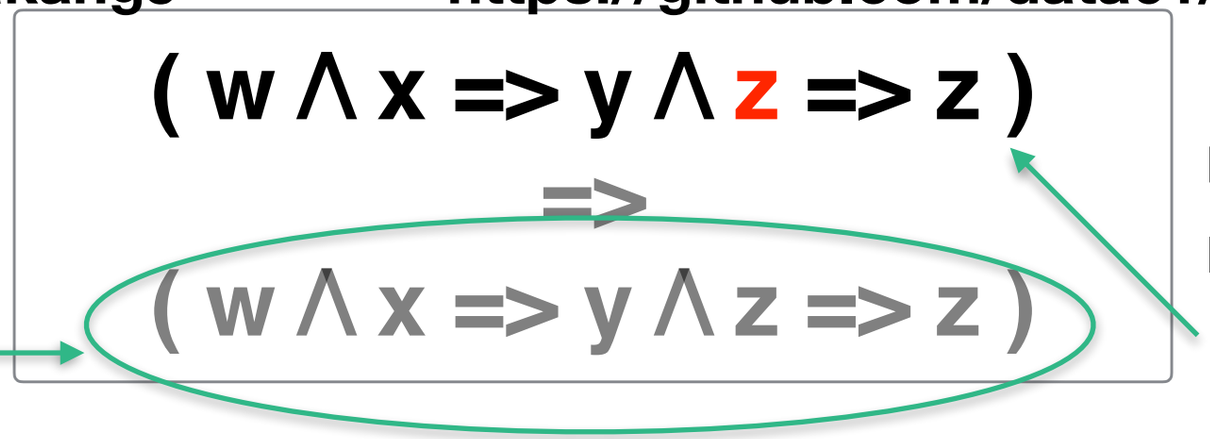
Tactics 3



our original goal

our current proof obligation

Tactics 3

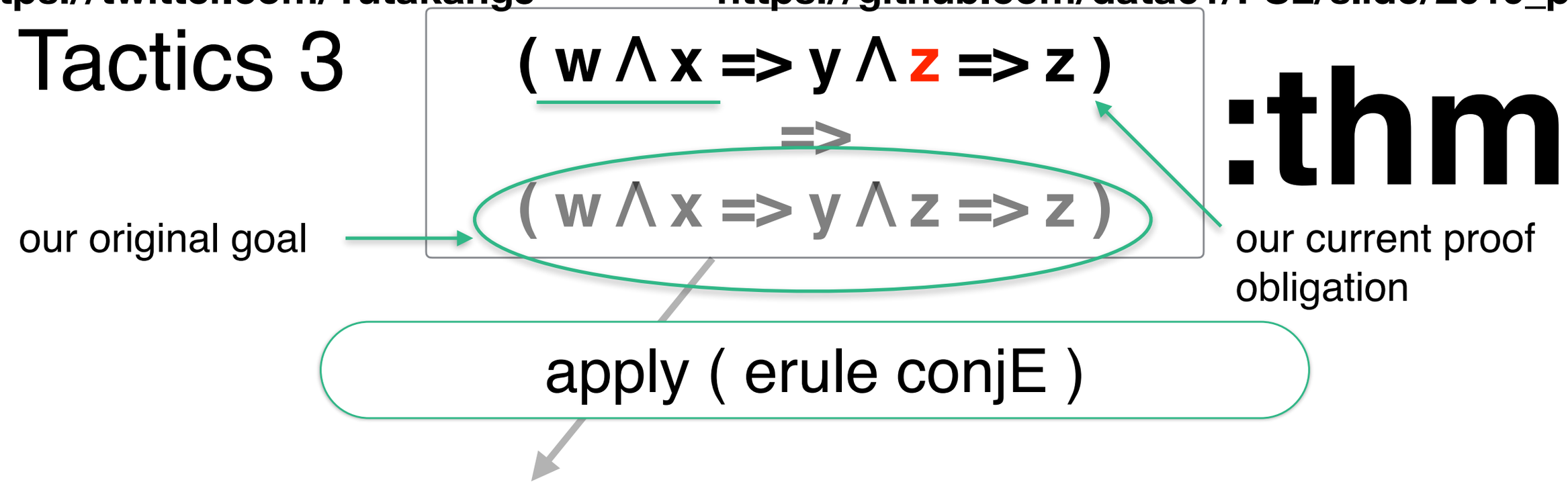


:thm

our original goal

our current proof obligation

Tactics 3


(w ∧ x => y ∧ z => z)
=>
(w ∧ x => y ∧ z => z)
:thm
our original goal
our current proof obligation
apply (erule conjE)

Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

our original goal

:thm

our current proof obligation

apply (erule conjE)

[(y ∧ z => w => x => z)

=>

(w ∧ x => y ∧ z => z) ,

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Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

:thm

our original goal

our current proof obligation

apply (erule conjE)

[(y ∧ z => w => x => z)

=>

(w ∧ x => y ∧ z => z) ,

apply (assumption)

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

:thm

our original goal

our current proof obligation

apply (erule conjE)

[(y ∧ z => w => x => z)

=>

(w ∧ x => y ∧ z => z) ,

]

apply (assumption)

[ ]

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

:thm

our original goal

our current proof obligation

apply (erule conjE)

back

[(y ∧ z => w => x => z)

=>

(w ∧ x => y ∧ z => z) ,

]

apply (assumption)

[ ]

Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

:thm

our original goal

our current proof obligation

apply (erule conjE)

back

[(y ∧ z => w => x => z)

=>

(w ∧ x => y ∧ z => z) ,

(w ∧ x => y => z => z)

=>

(w ∧ x => y ∧ z => z)]

apply (assumption)

[ ]

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

:thm

our original goal

our current proof obligation

apply (erule conjE)

back

[(y ∧ z => w => x => z)

=>

(w ∧ x => y ∧ z => z) ,

(w ∧ x => y => z => z)

=>

(w ∧ x => y ∧ z => z)]

apply (assumption)

[ ]

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Tactics 3

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

:thm

our original goal

our current proof obligation

apply (erule conjE)

back

[(y ∧ z => w => x => z)

=>

(w ∧ x => y ∧ z => z) ,

(w ∧ x => y => z => z)

=>

(w ∧ x => y ∧ z => z)]

apply (assumption)

[ ]++[(w ∧ x => y ∧ z => z)]

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Tactics 3

:thm

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

our original goal

our current proof obligati

apply (erule conjE)

[(y ∧ z => w => x => ... => y => z) ; (w ∧ ...)]

apply (rule conjE, assumption)

sequential combinator that admits backtracking (= THEN)

(y ∧ z => z) ; (assumption)

[ ]++[(w ∧ x => y ∧ z => z)]

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Tactics 3

:thm

our original goal

(w ∧ x => y ∧ z => z)

=>

(w ∧ x => y ∧ z => z)

our current proof obligati

apply (erule conjE)

[(y ∧ z => w => x => y => z)]

=>

[(w ∧ x => y ∧ z => z) ; (y ∧ z => z)]

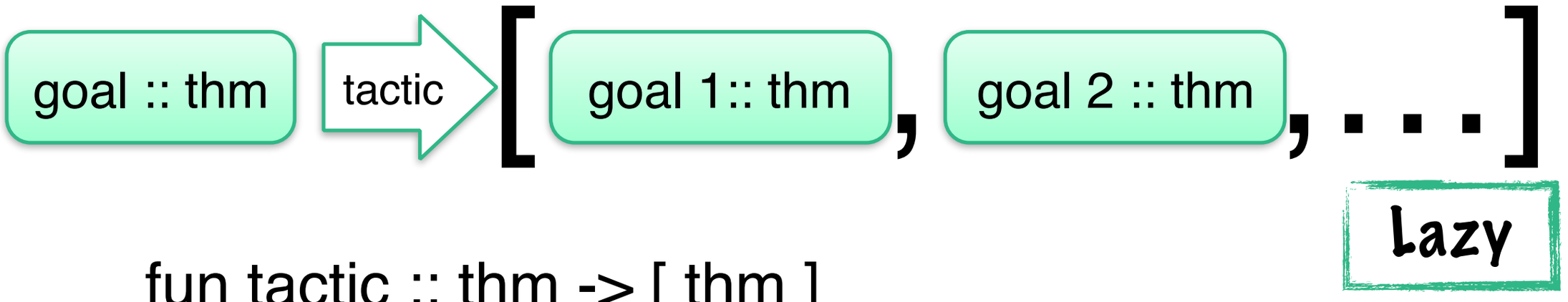
apply (rule conjE, assumption)

sequential combinator that admits backtracking (= THEN)

apply (assumption)

[ ]++[(w ∧ x => y ∧ z => z)]

Tactics 4



fun tactic :: thm -> [thm]

Tactics 4

goal :: thm → tactic → [goal 1:: thm , goal 2 :: thm , . . .]

Lazy

fun tactic :: thm -> [thm]

fail

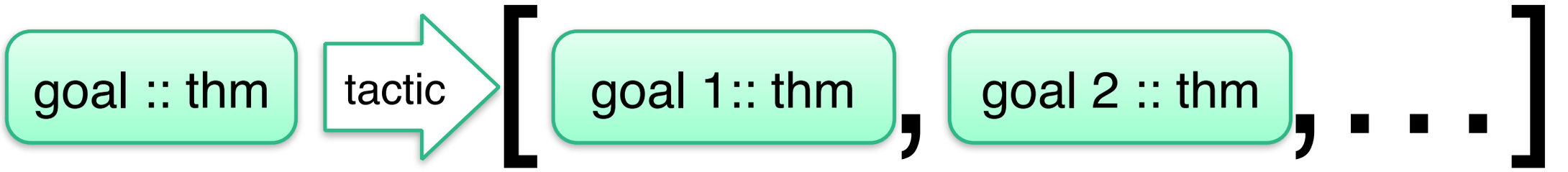
succeed

auto

simp

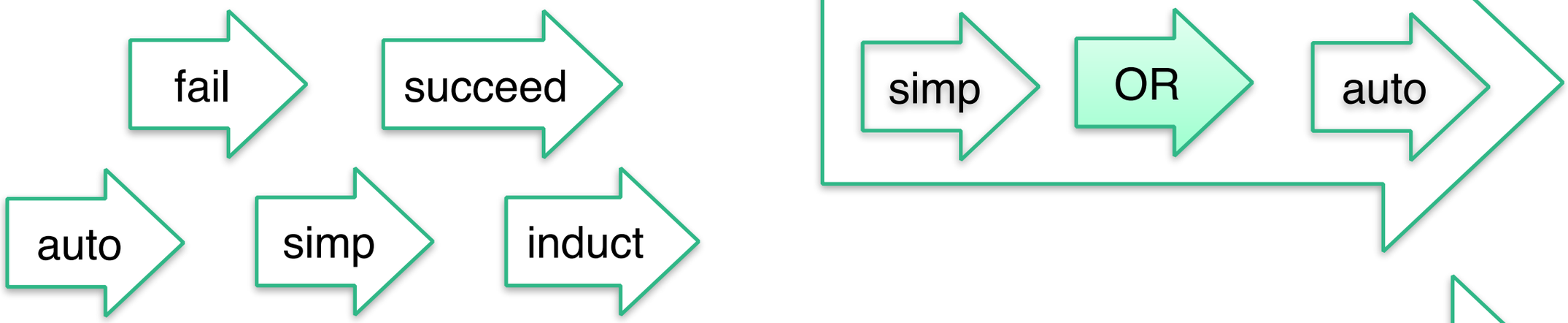
induct

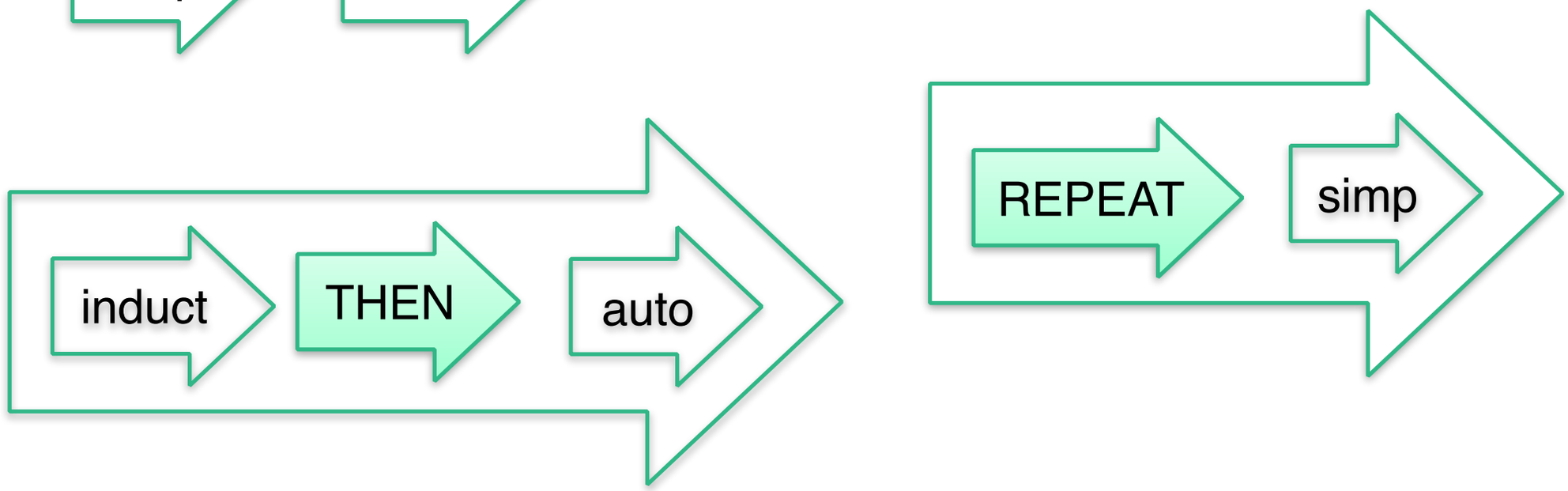
Tactics 4



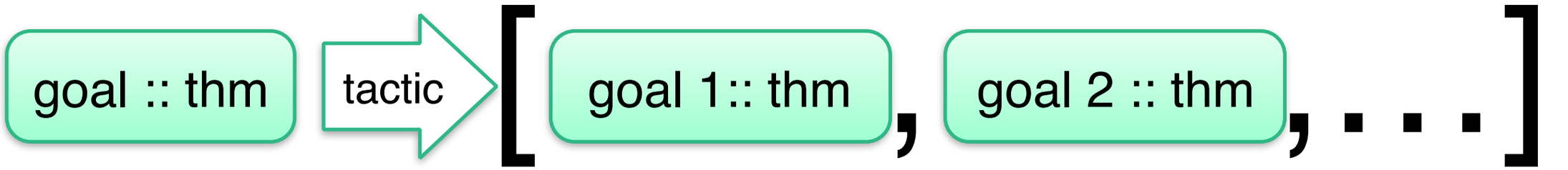
Lazy

fun tactic :: thm -> [thm]

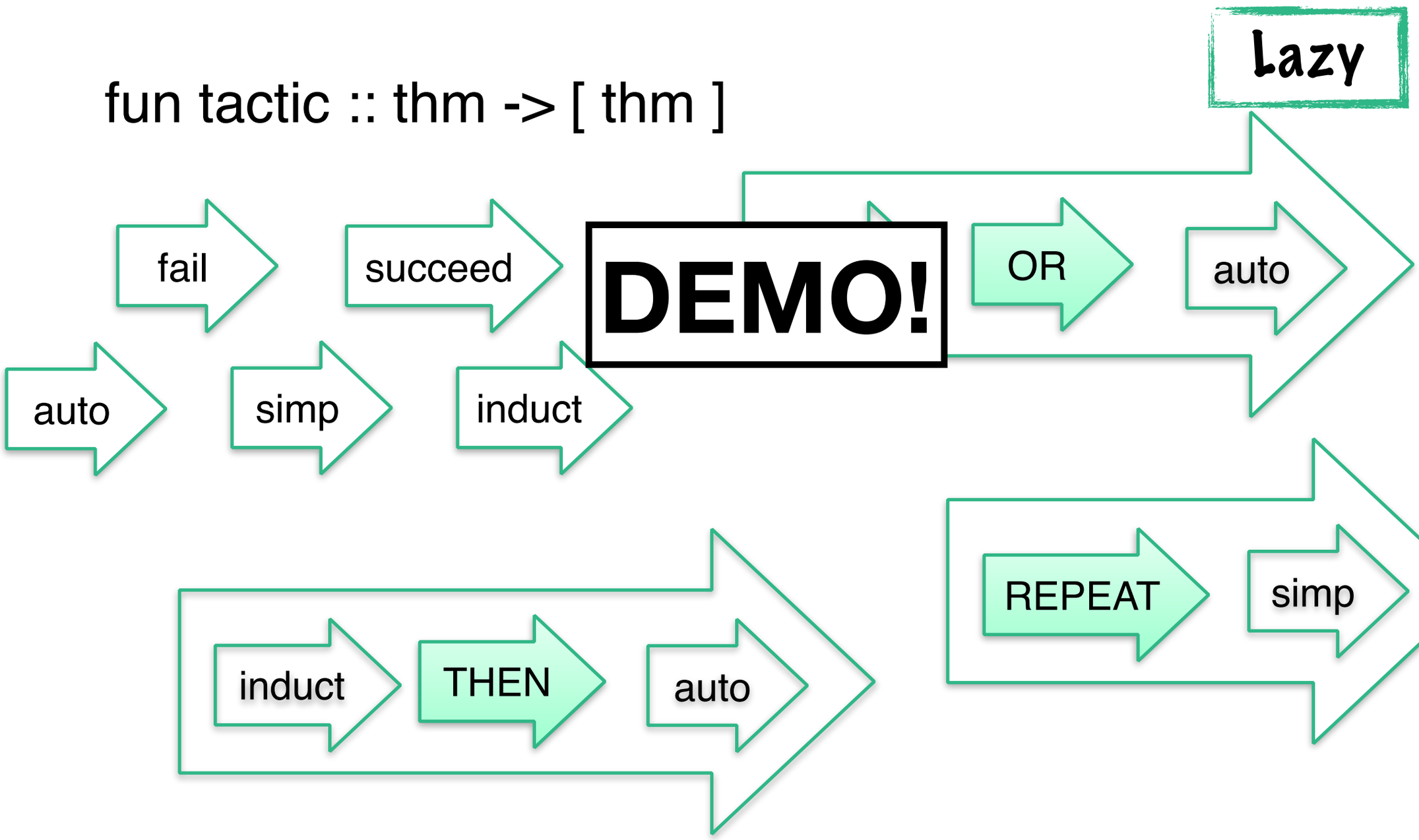




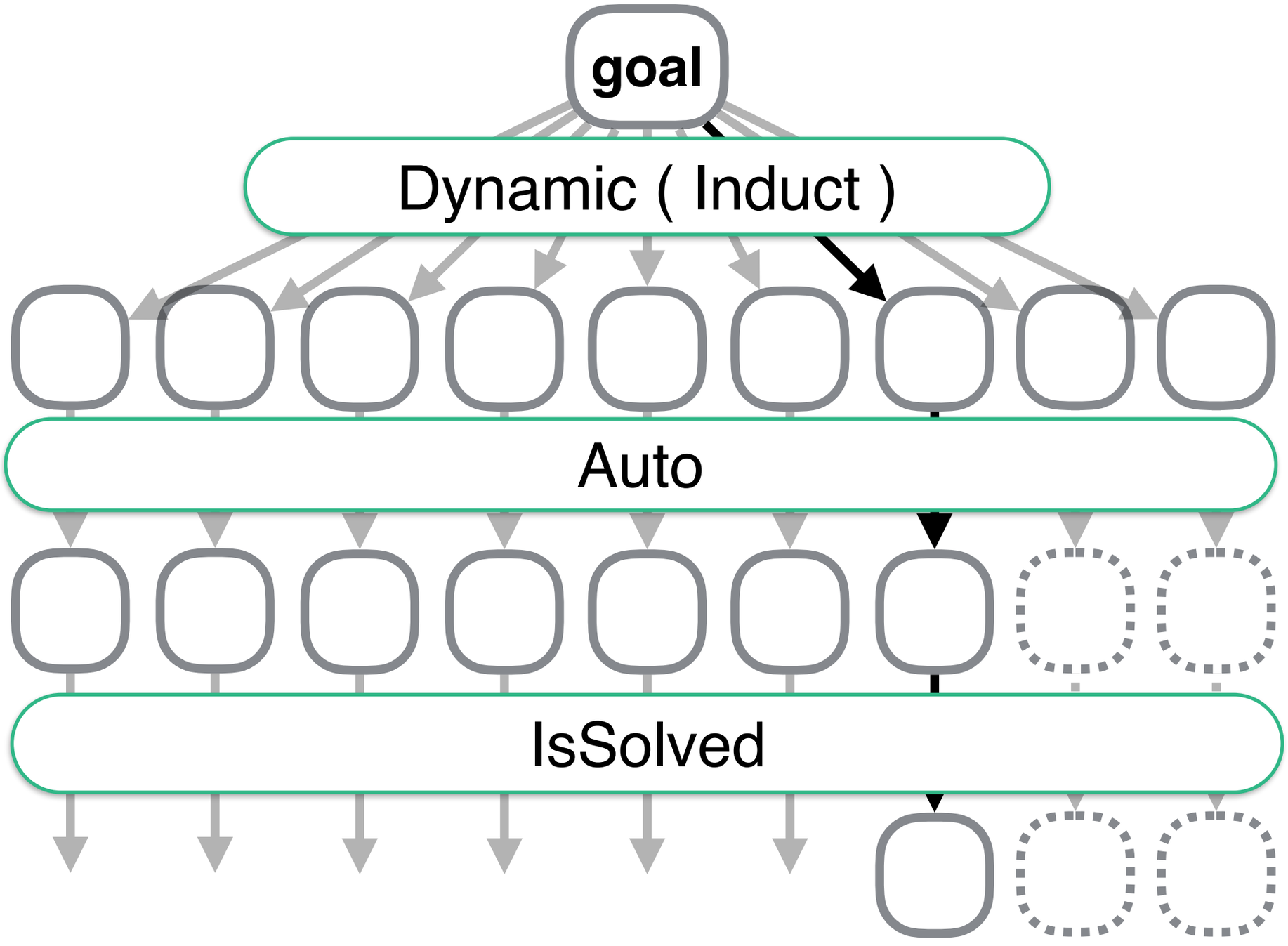
Tactics 4



fun tactic :: thm -> [thm]



```
lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
```



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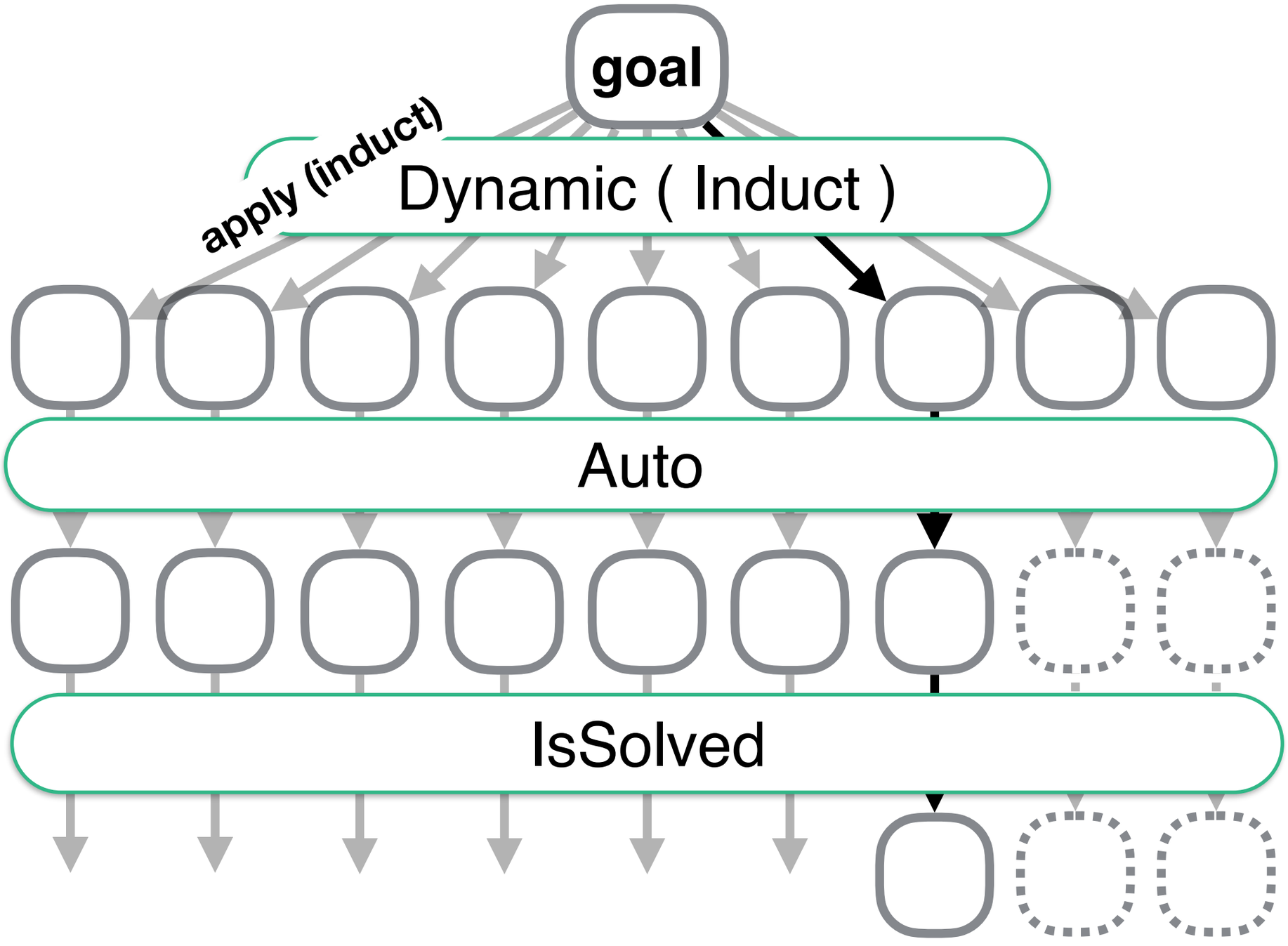

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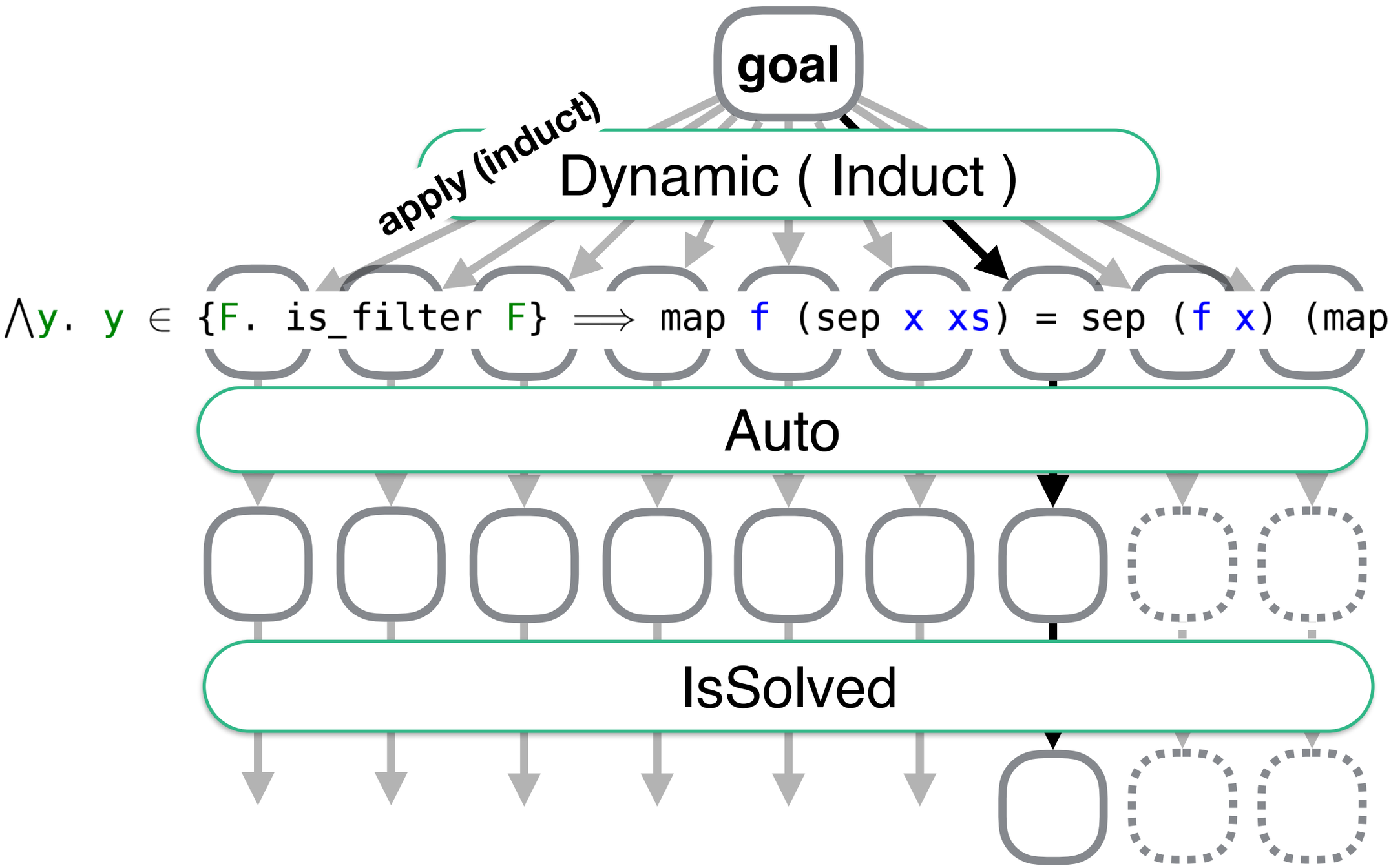

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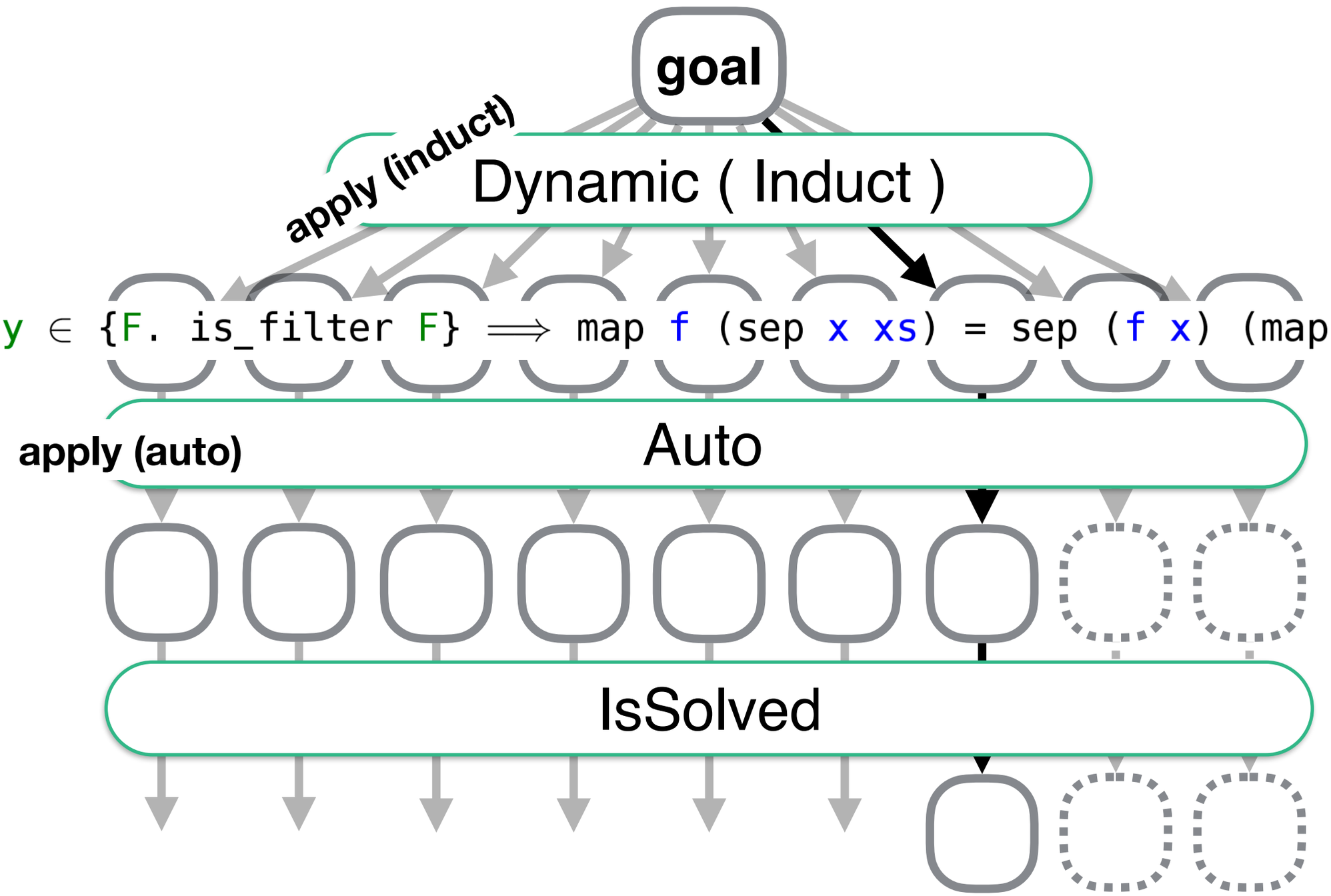

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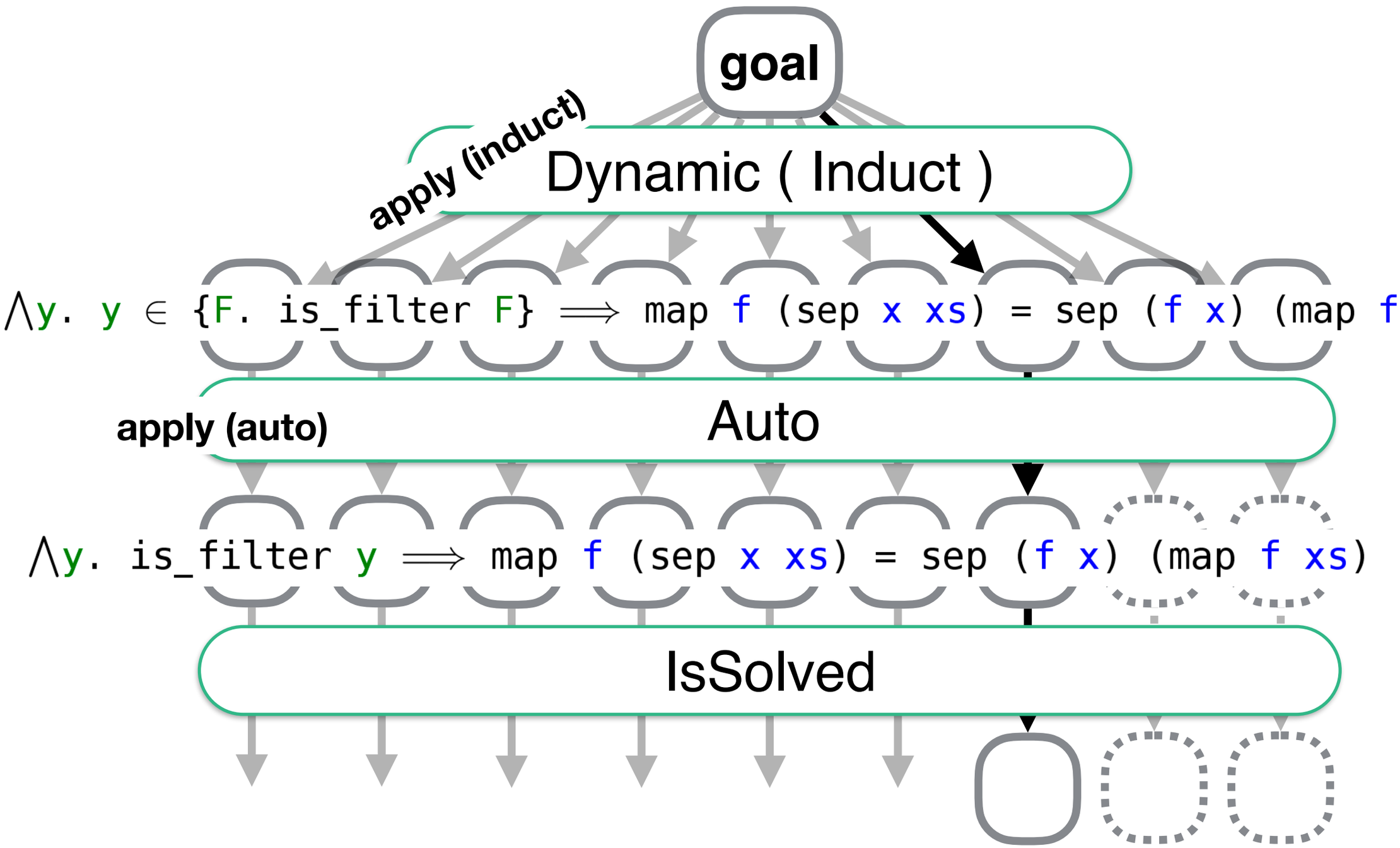
```



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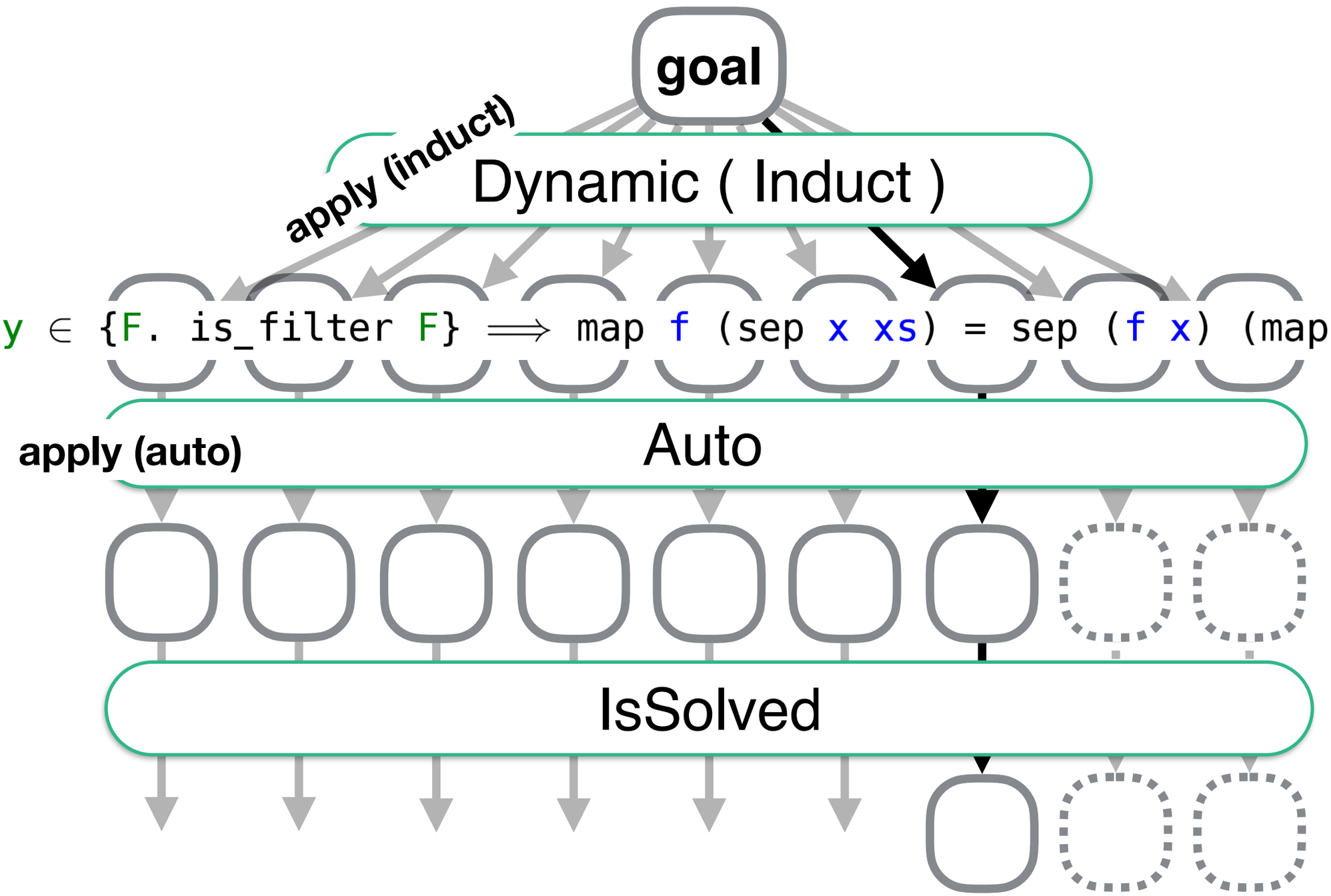
```



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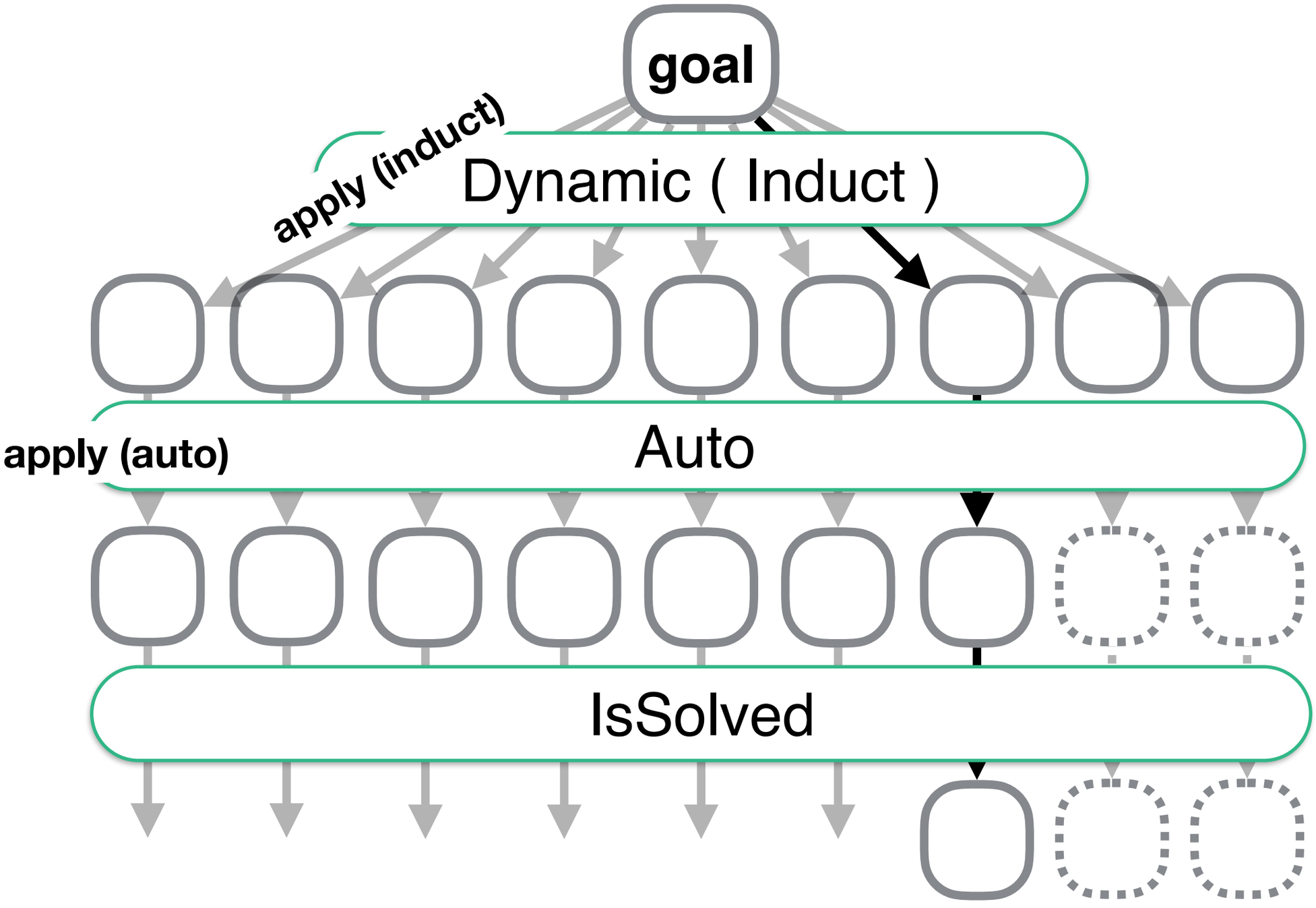

```
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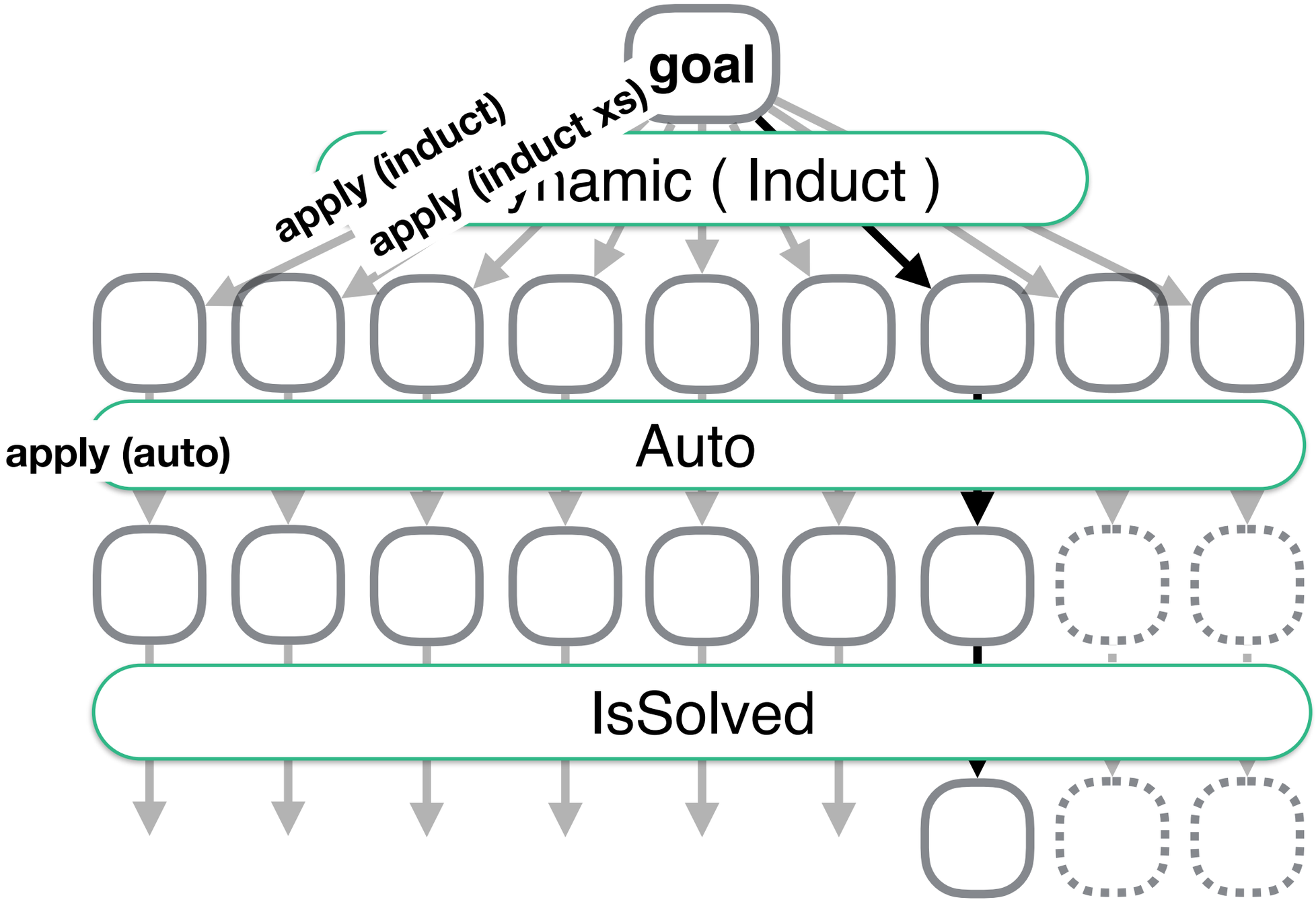
```



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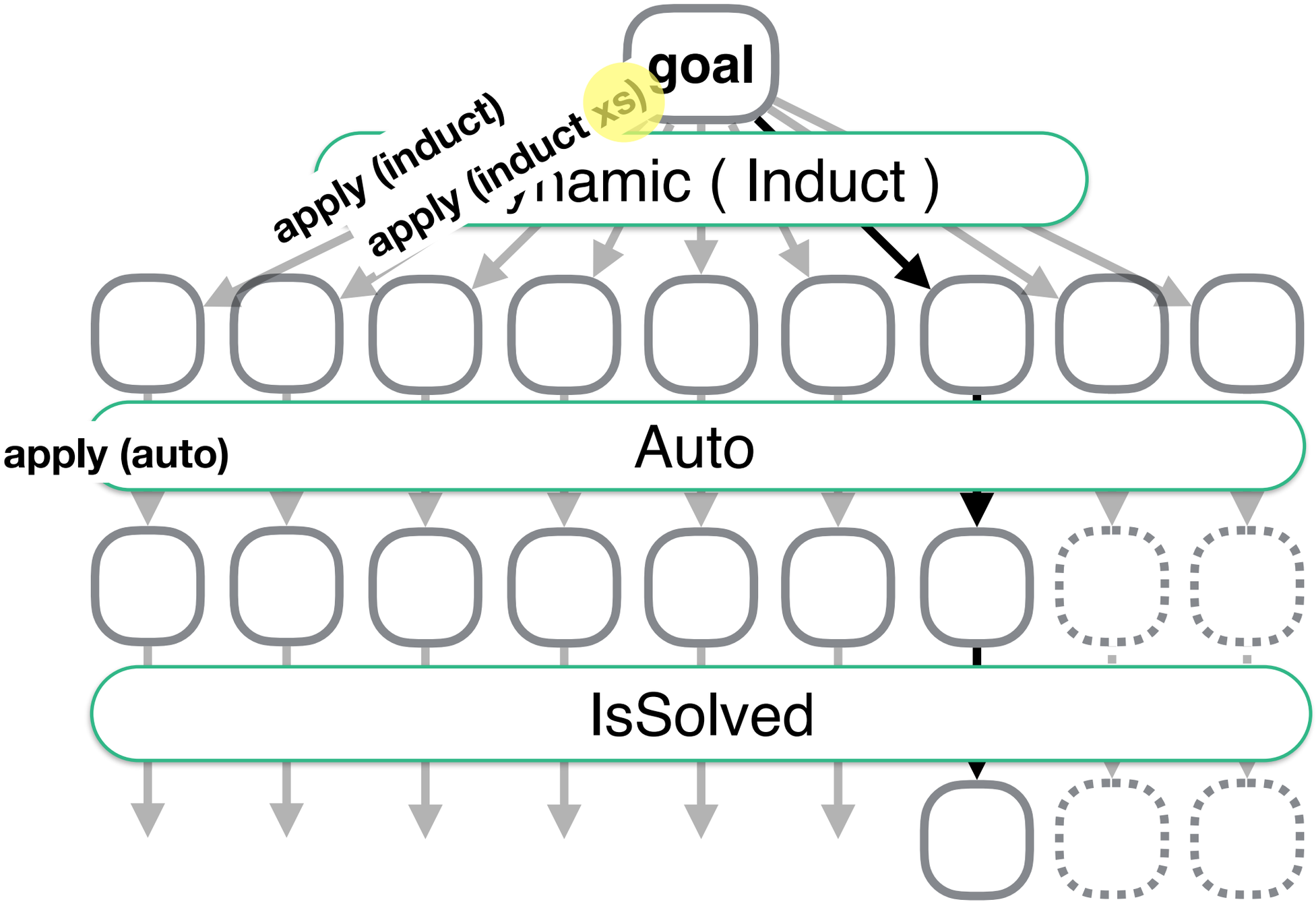
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
```



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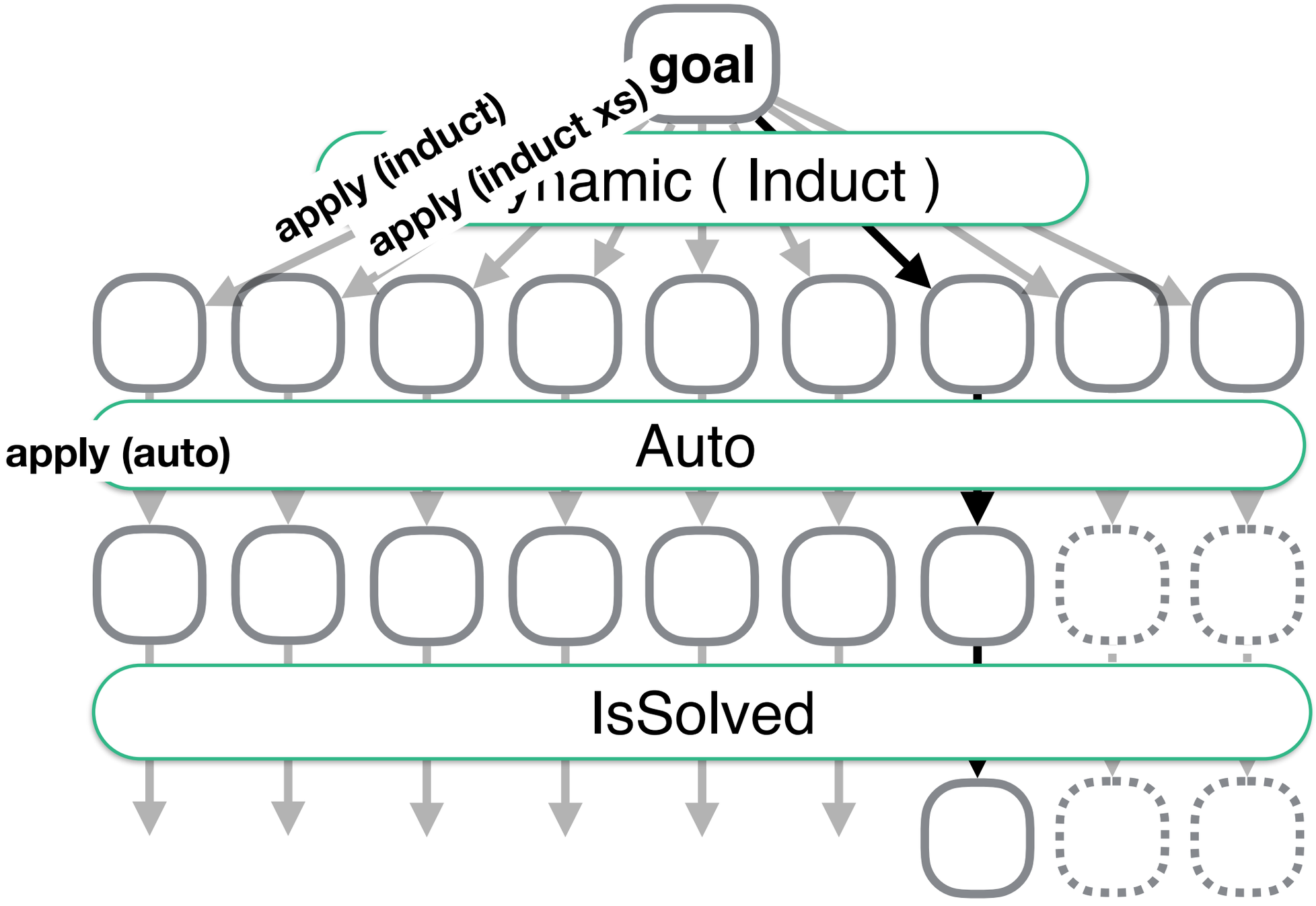

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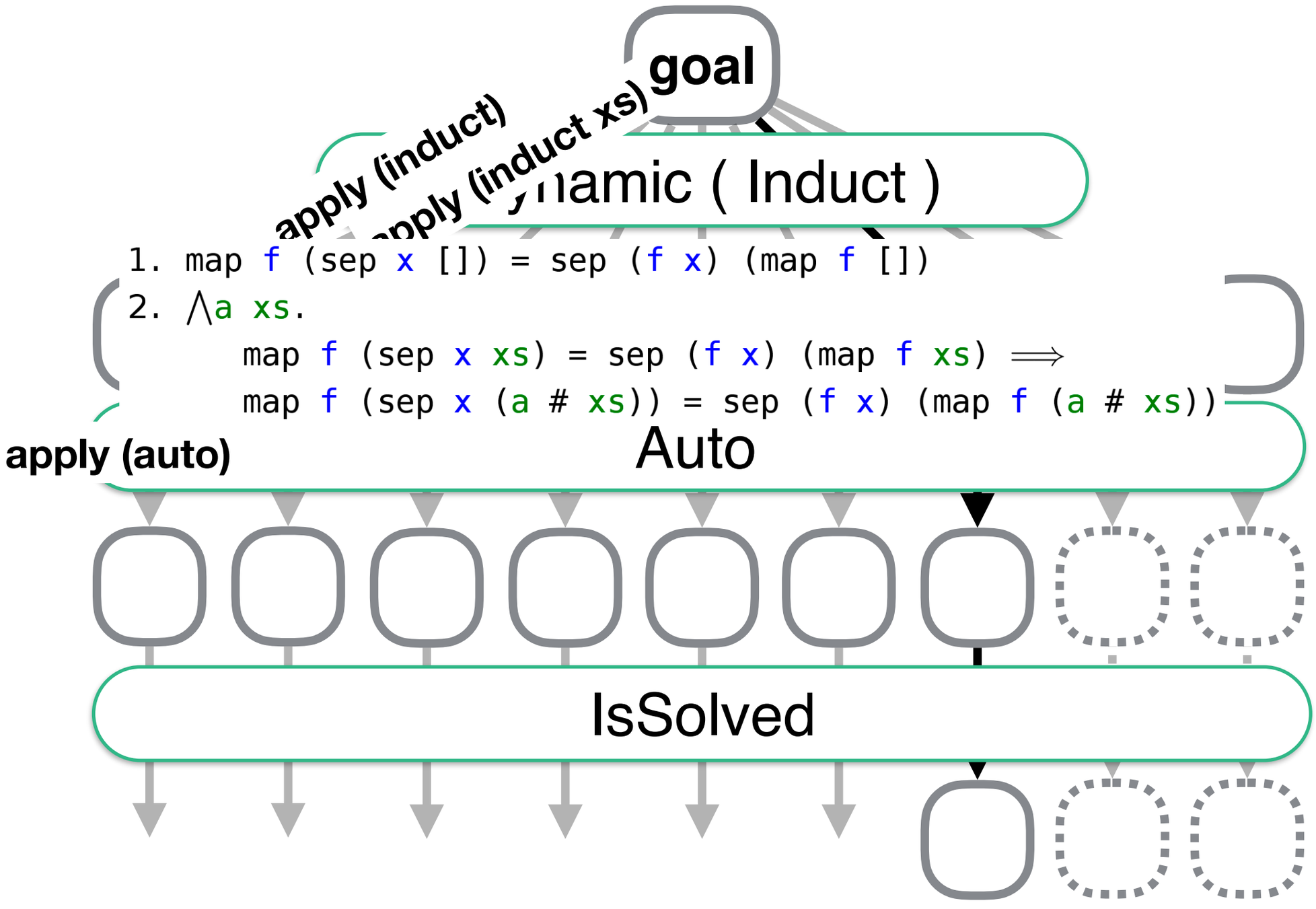

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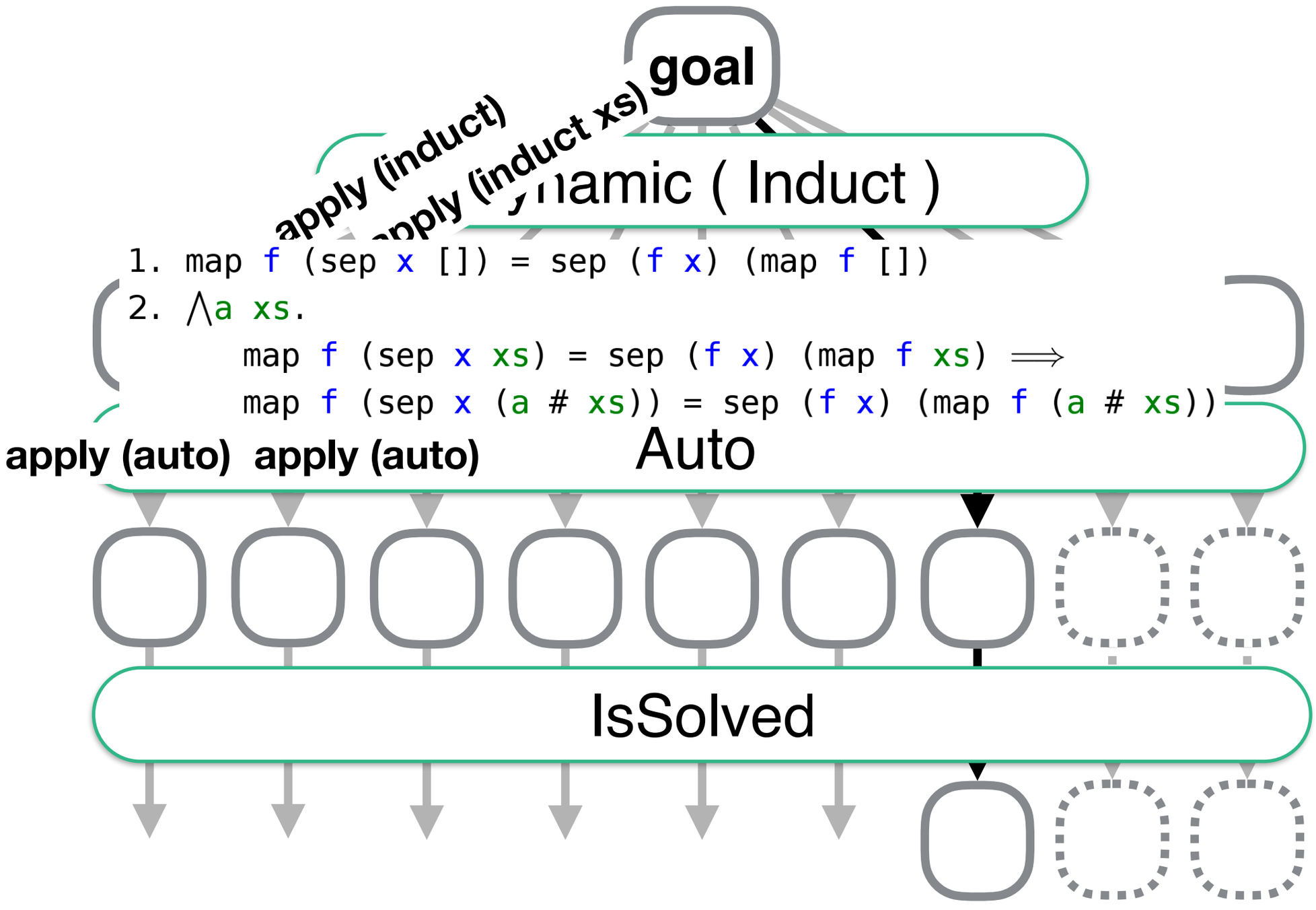

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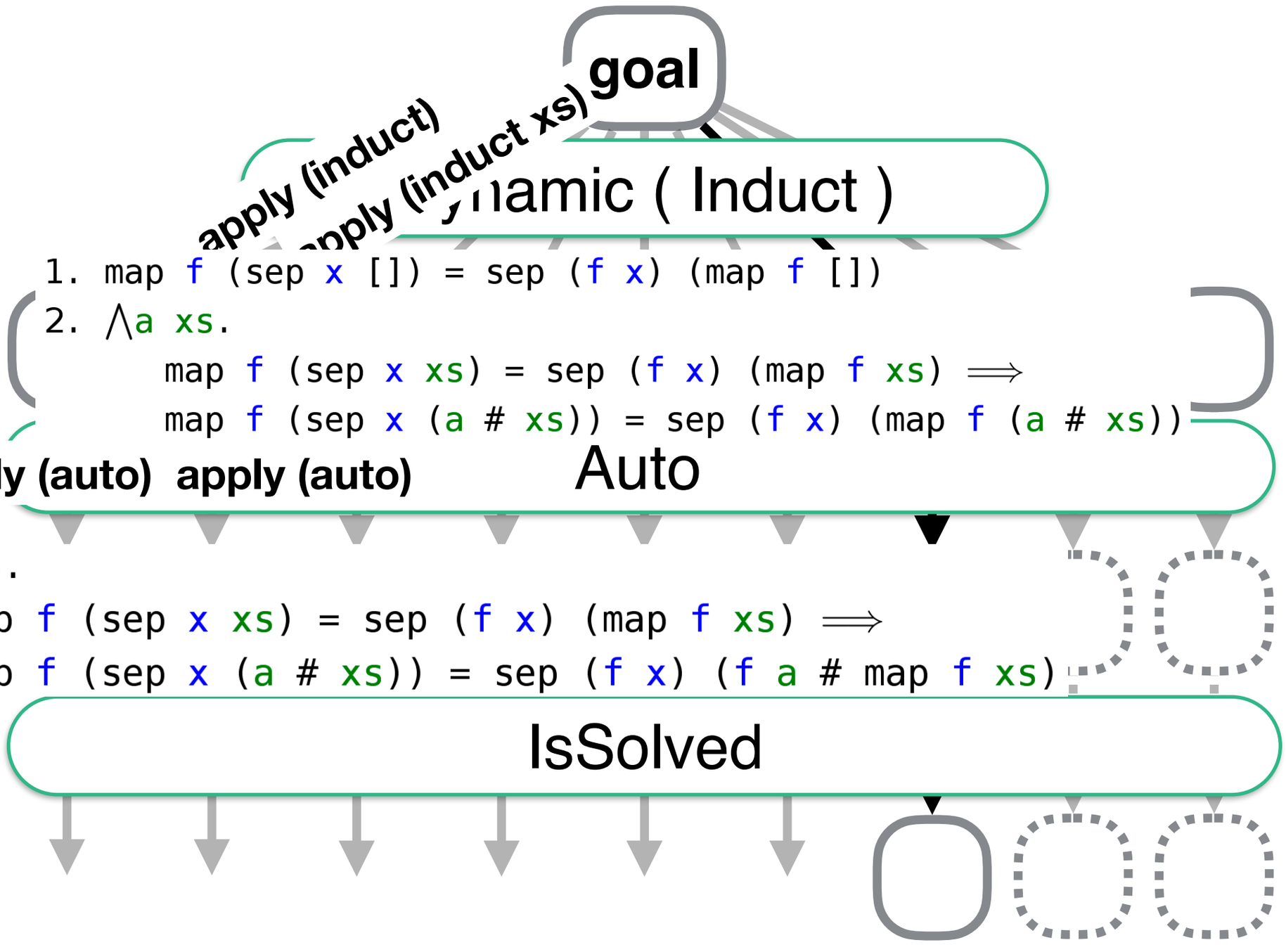

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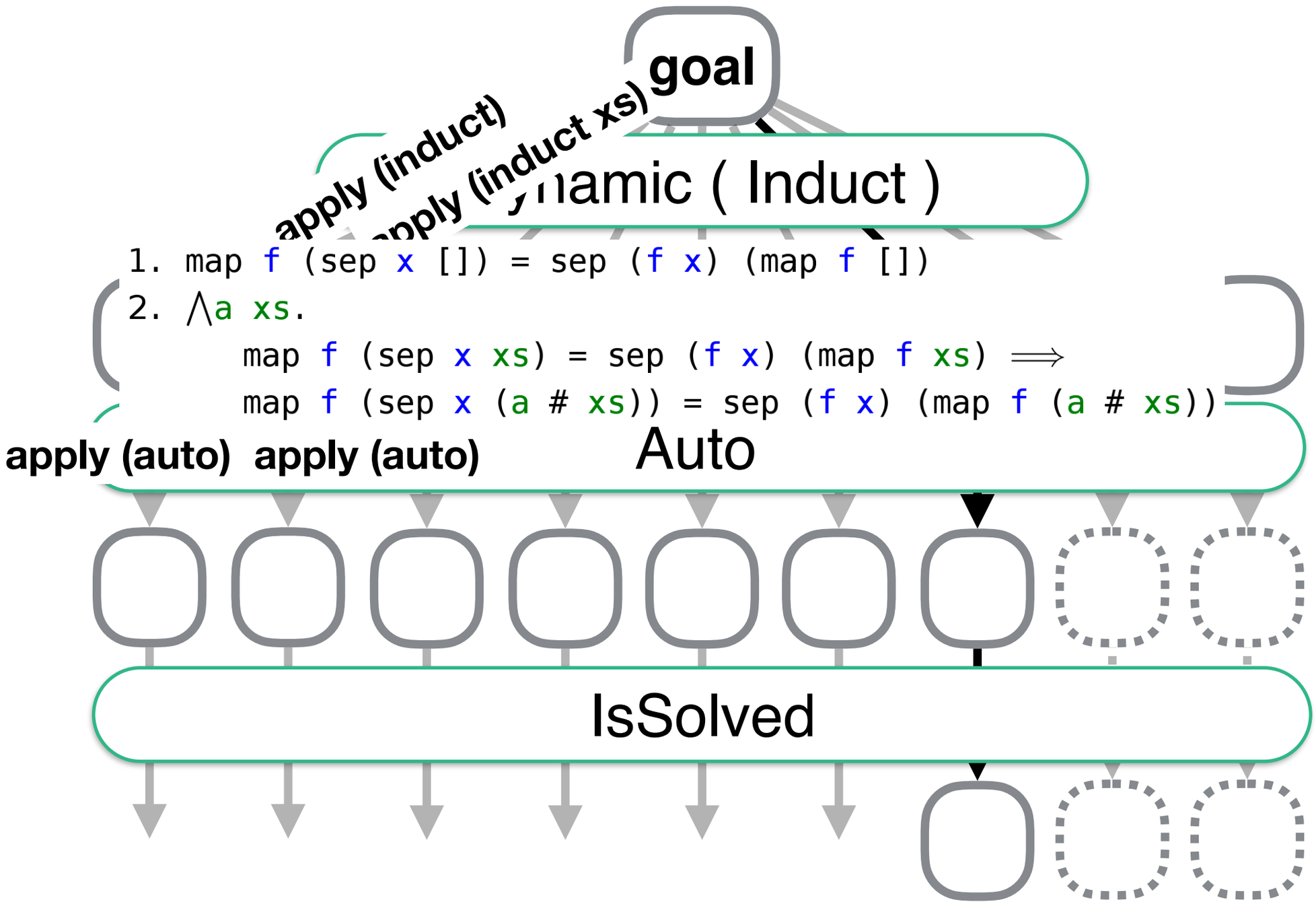

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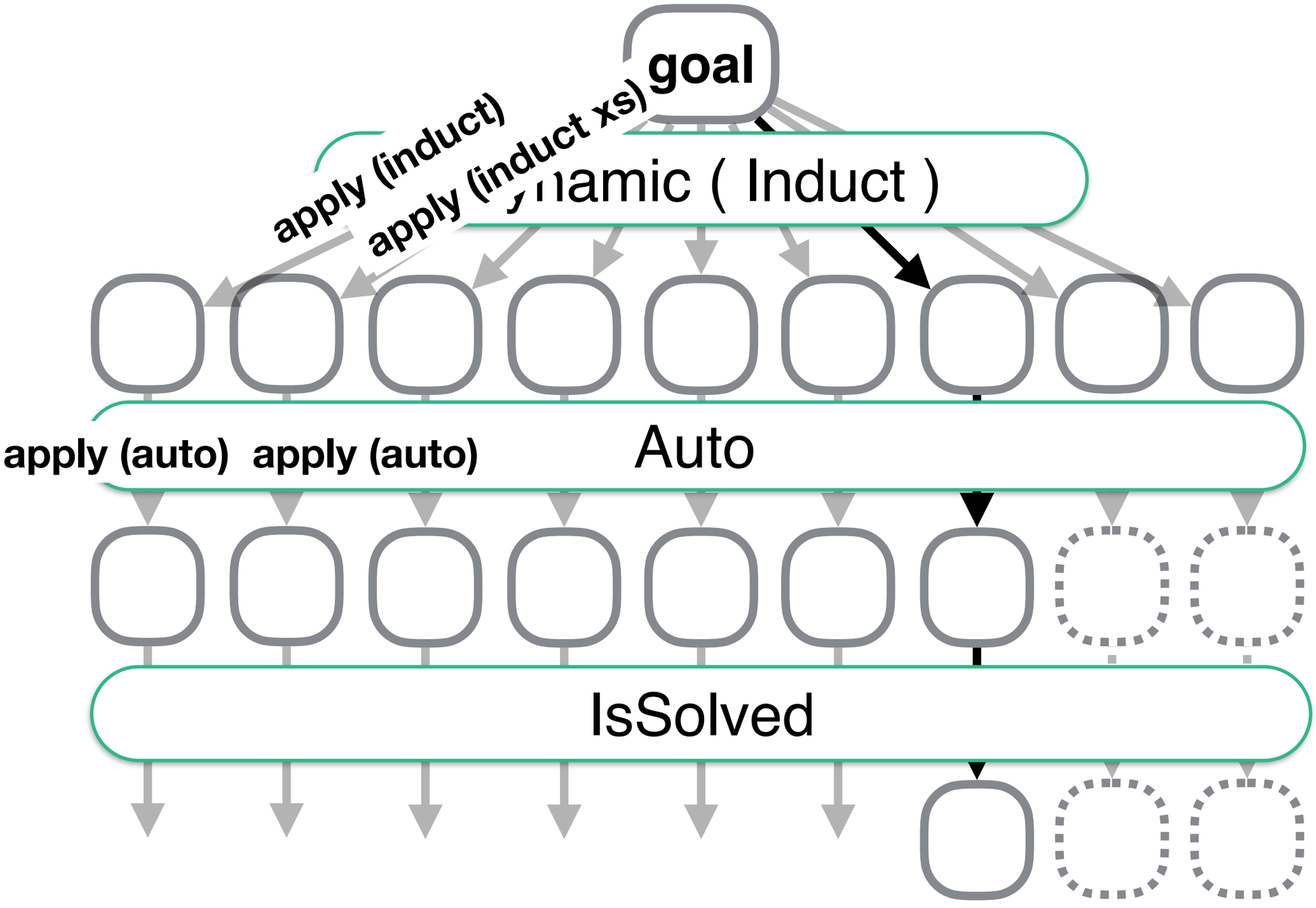

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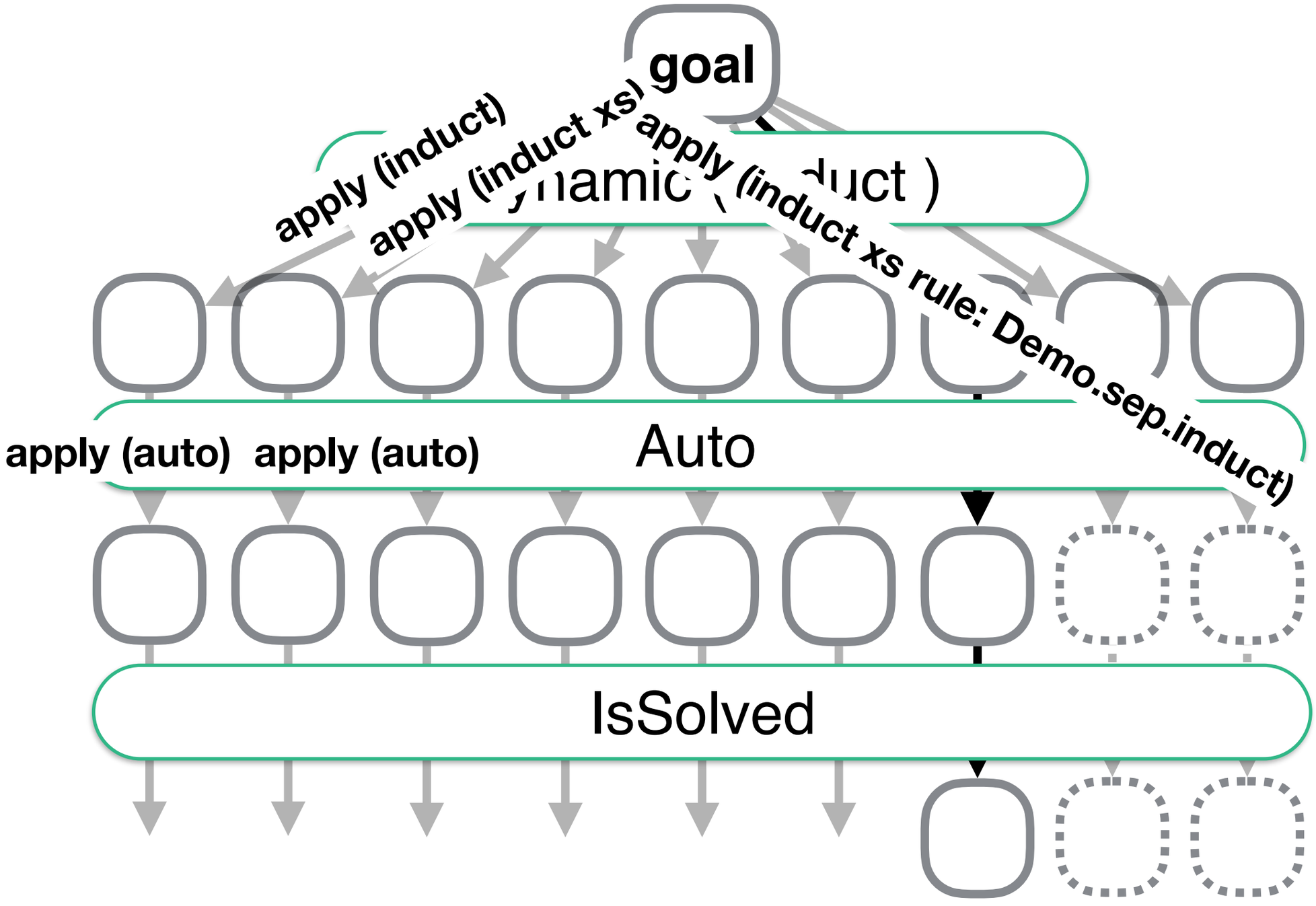

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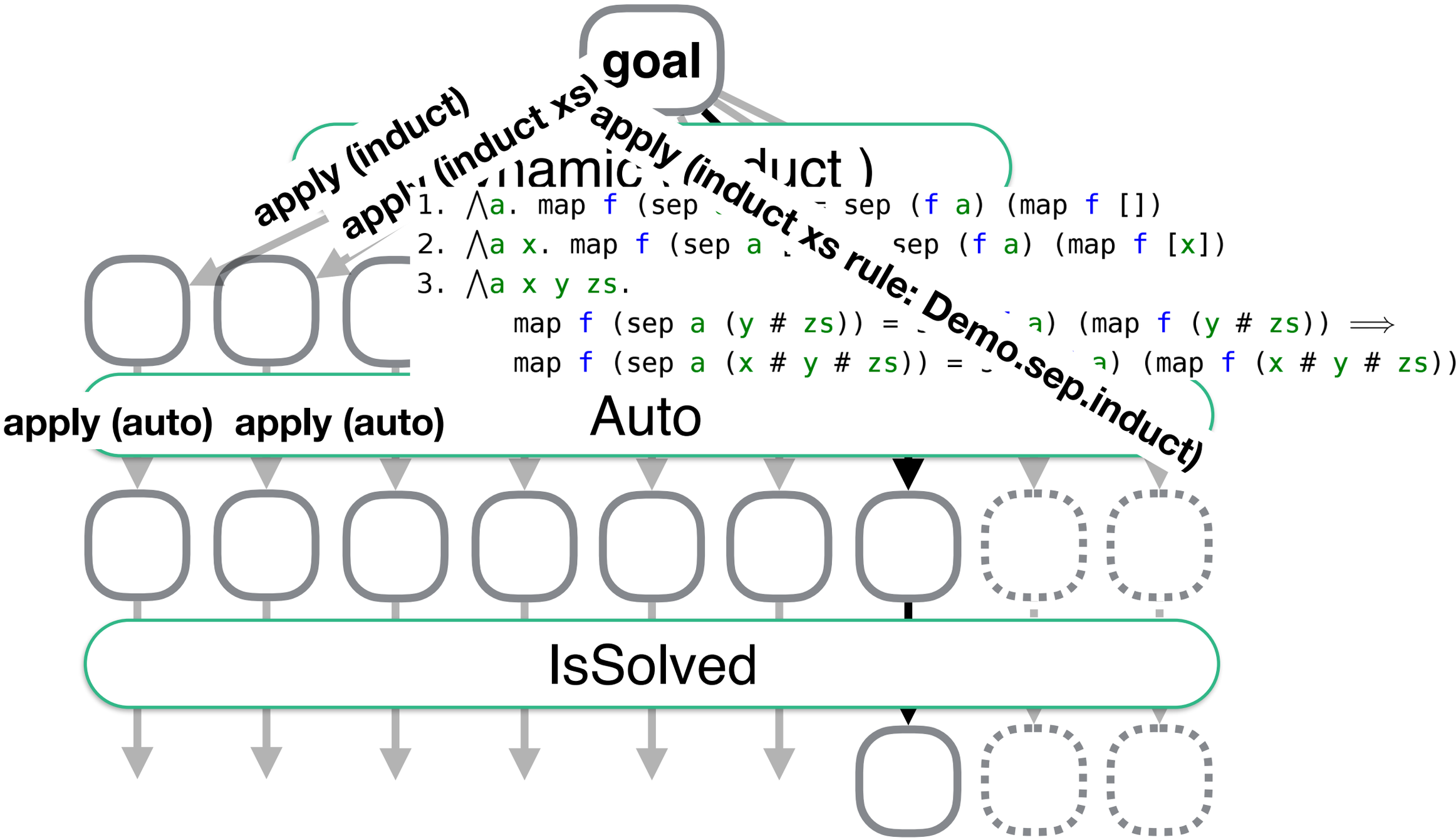

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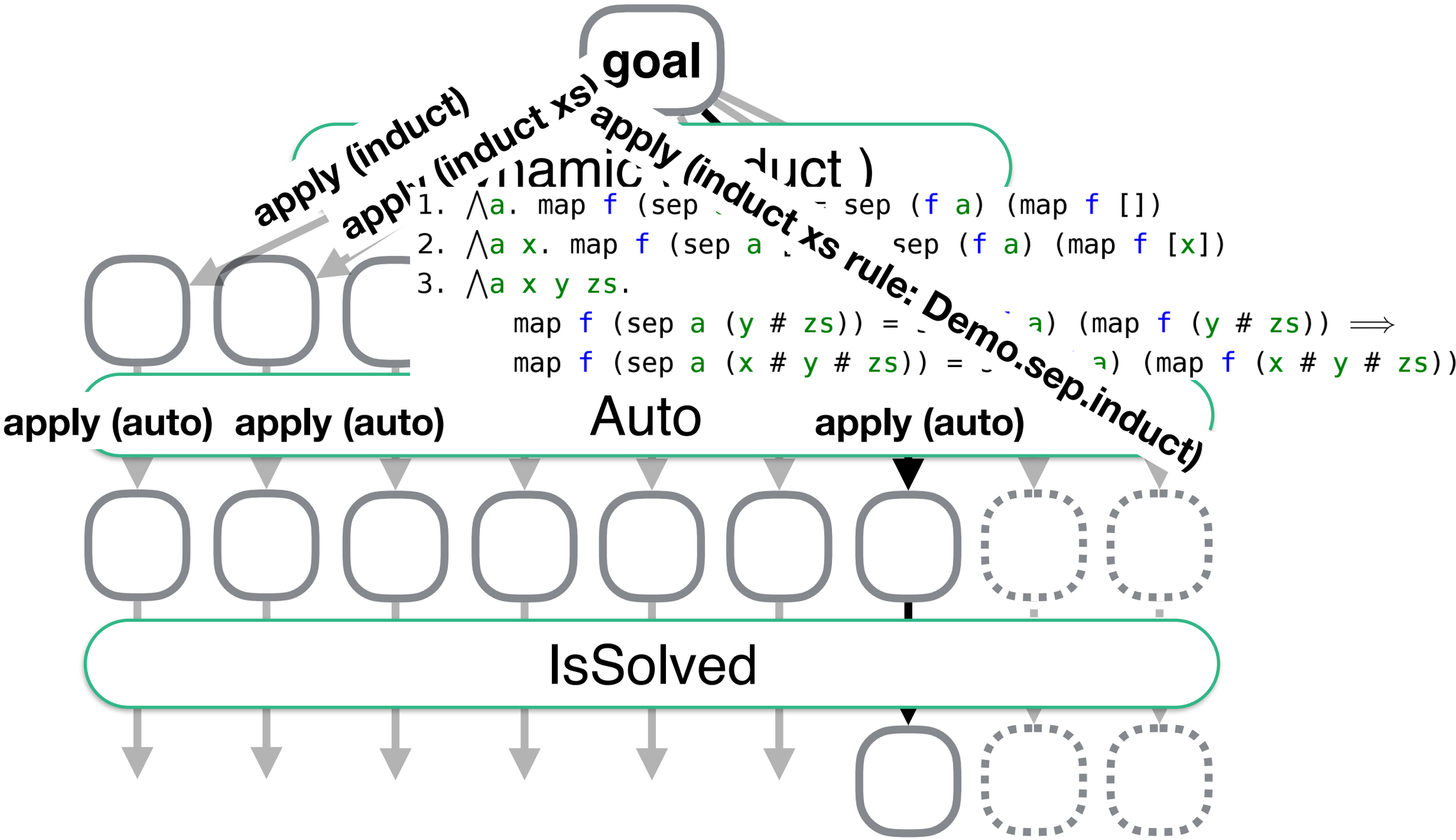

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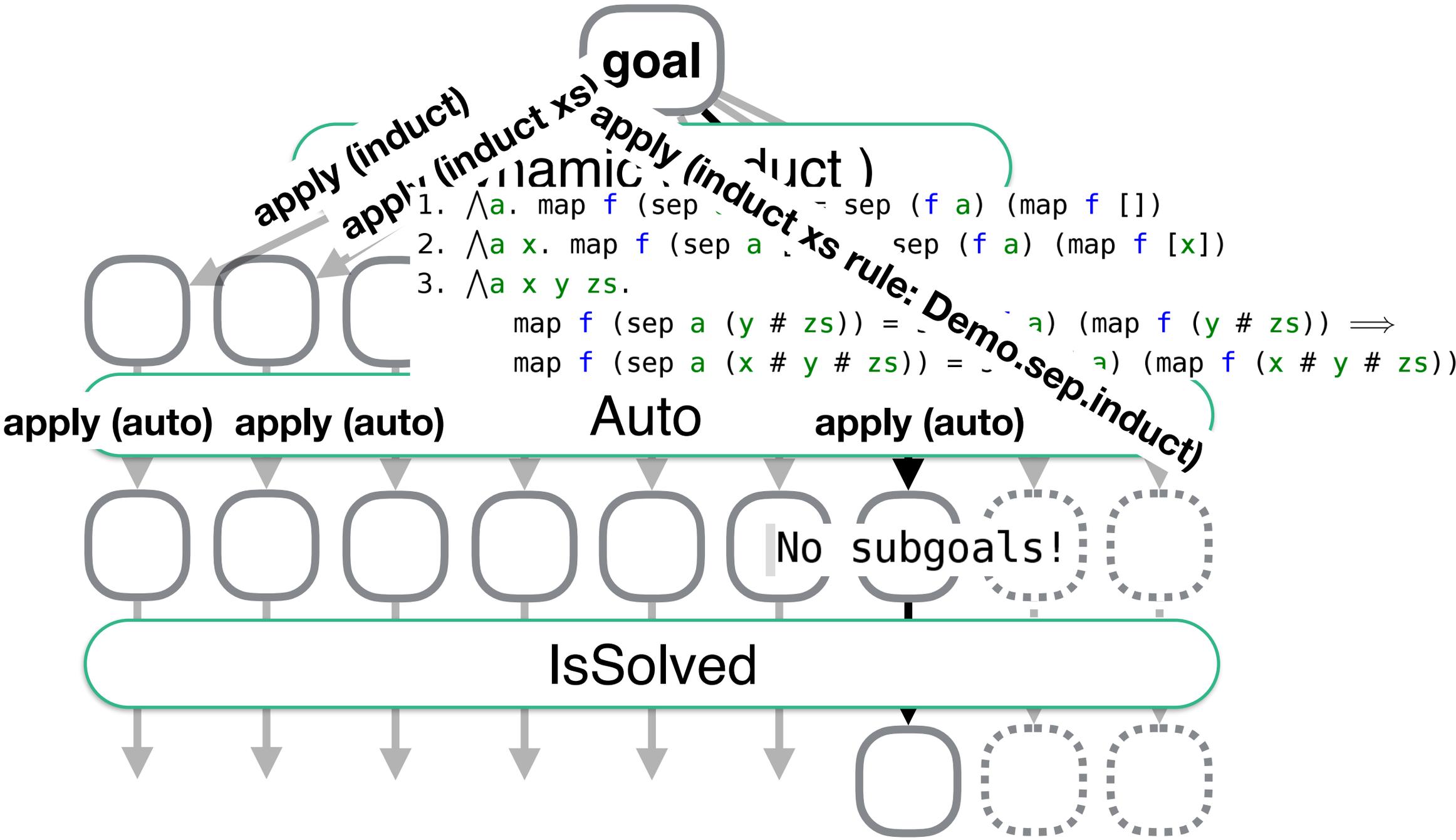

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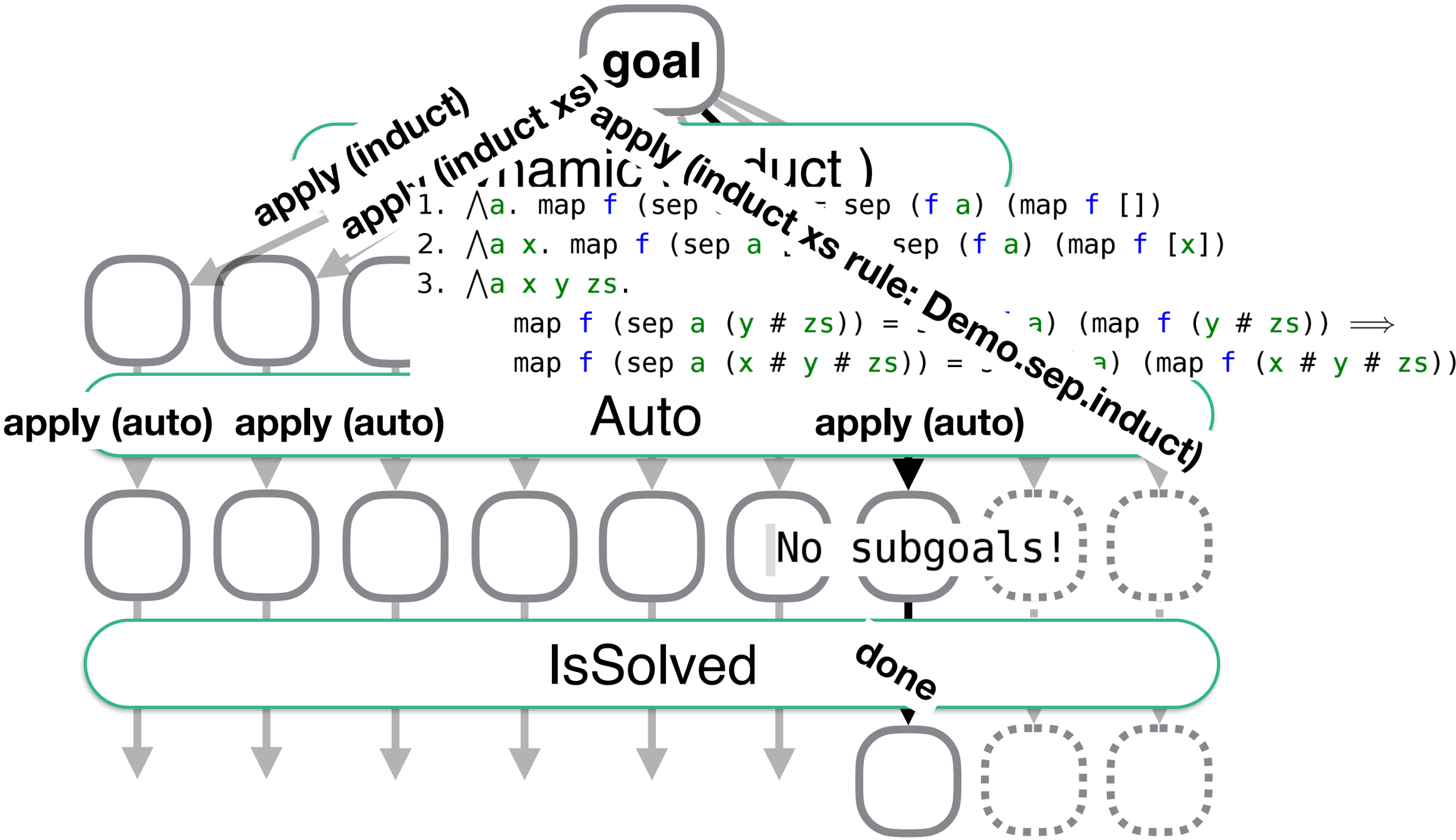

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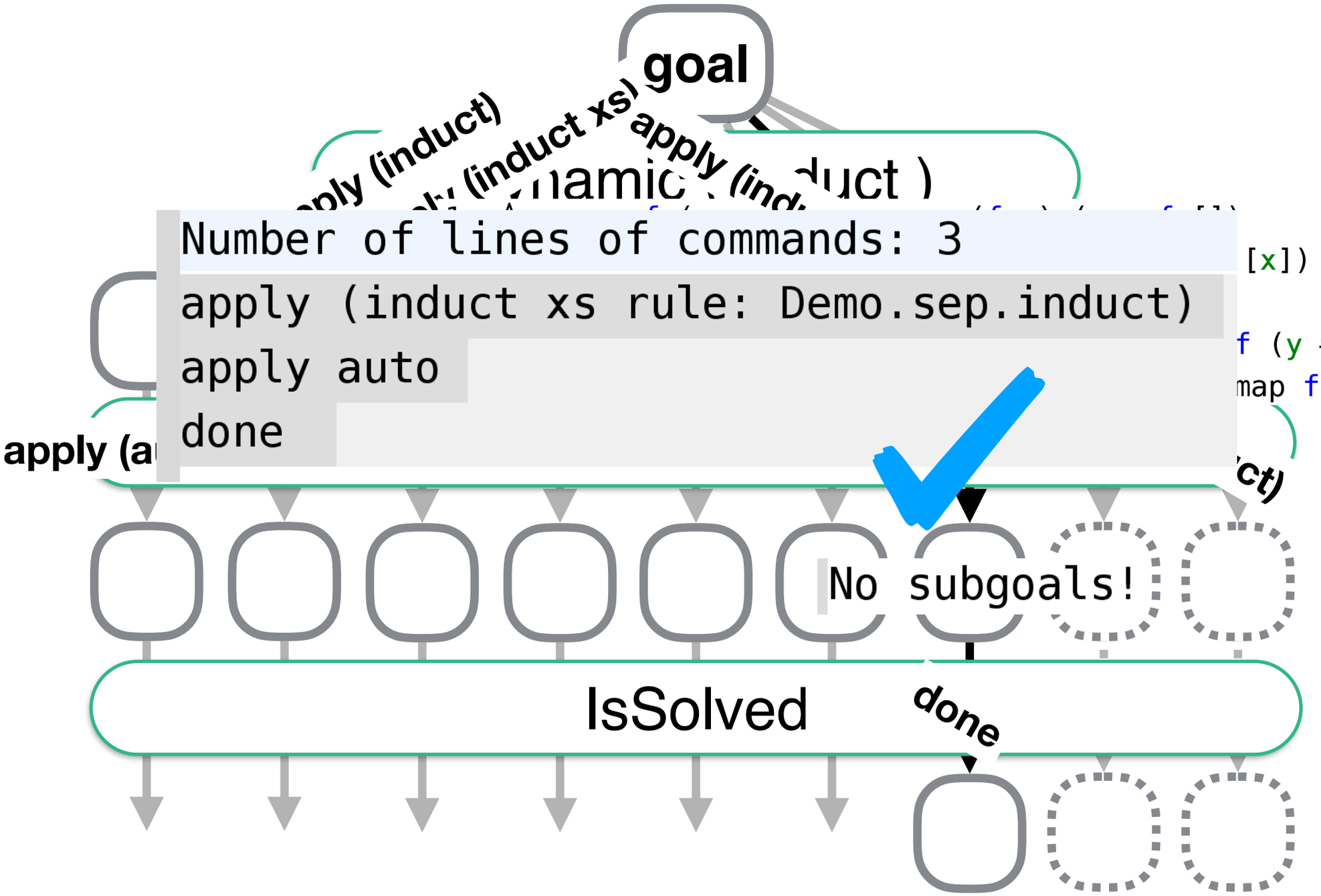


Try_Hard: the default strategy

```
strategy Try_Hard =
Ors [Thens [Subgoal, Basic],
      Thens [DInductTac, Auto_Solve],
      Thens [DCaseTac, Auto_Solve],
      Thens [Subgoal, Advanced],
      Thens [DCaseTac, Solve_Many],
      Thens [DInductTac, Solve_Many] ]
```

```
strategy Basic =
  Ors [
      Auto_Solve,
      Blast_Solve,
      FF_Solve,
      Thens [IntroClasses, Auto_Solve],
      Thens [Transfer, Auto_Solve],
      Thens [Normalization, IsSolved],
      Thens [DInduct, Auto_Solve],
      Thens [Hammer, IsSolved],
      Thens [DCases, Auto_Solve],
      Thens [DCoinduction, Auto_Solve],
      Thens [Auto, RepeatN(Hammer), IsSolved],
      Thens [DAuto, IsSolved]]
```

16 percentage point performance improvement compared to sledgehammer

but the search space explodes

PaMpeR: Proof Method Recommendation

preparation phase

How does PaMpeR work?

recommendation phase

preparation phase

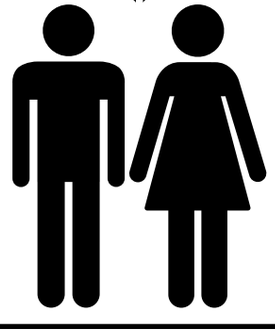
large proof corpora

AFP and standard library

How does PaMpeR work?

recommendation phase

proof state



proof engineer

preparation phase

large proof corpora

AFP and standard library

Statistics

Archive of Formal Proofs (https://www.isa-afp.org)

Statistics

Number of Articles: 468
Number of Authors: 313
Number of lemmas: ~128,900
Lines of Code: ~2,170,300

Most used AFP articles:

	Name	Used by ? articles
1.	Collections	15
2.	List-Index	14
3.	Coinductive	12

preparation phase

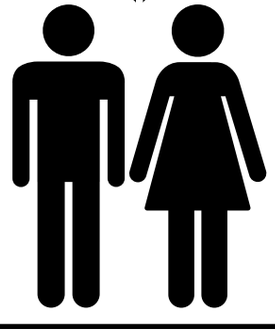
large proof corpora

AFP and standard library

How does PaMpeR work?

recommendation phase

proof state



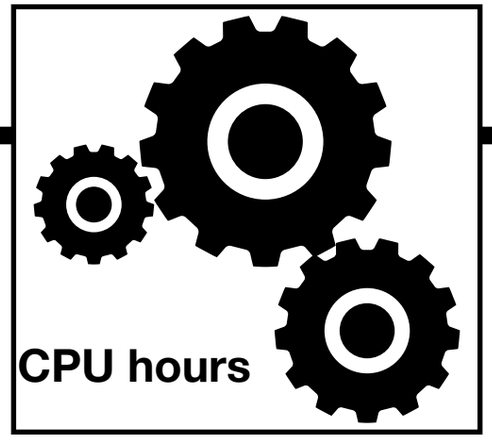
proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor



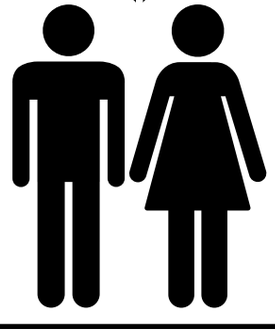
6021 CPU hours

108 assertions

How does PaMpeR work?

recommendation phase

proof state



proof engineer

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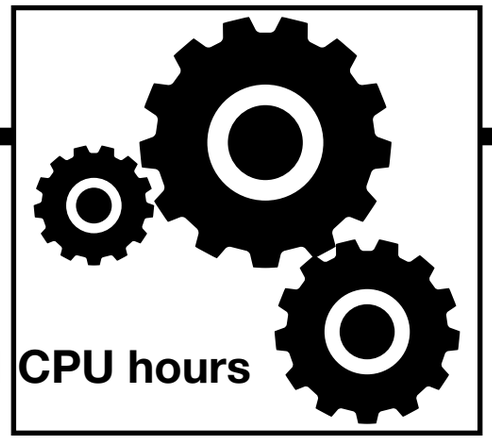
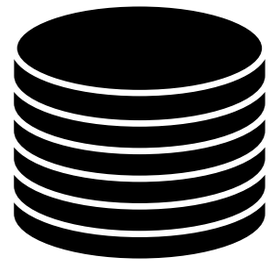


6021 CPU hours

108 assertions

database (425334 data points)

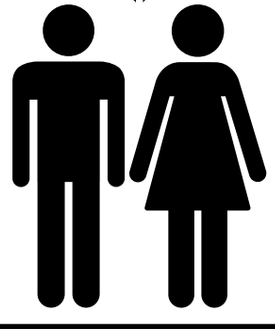


:: (tactic_name, [bool])

How does PaMpeR work?

recommendation phase

proof state



proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor

6021 CPU hours

108 assertions

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:: (tactic_name, [bool])

preprocess

decision tree construction

How does PaMpeR work?

recommendation phase

proof state

proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor

6021 CPU hours

108 assertions

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preprocess

decision tree construction

How does PaMpeR work?

recommendation phase

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proof engineer

fast feature extractor

preparation phase

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AFP and standard library

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decision tree construction

How does PaMpeR work?

recommendation phase

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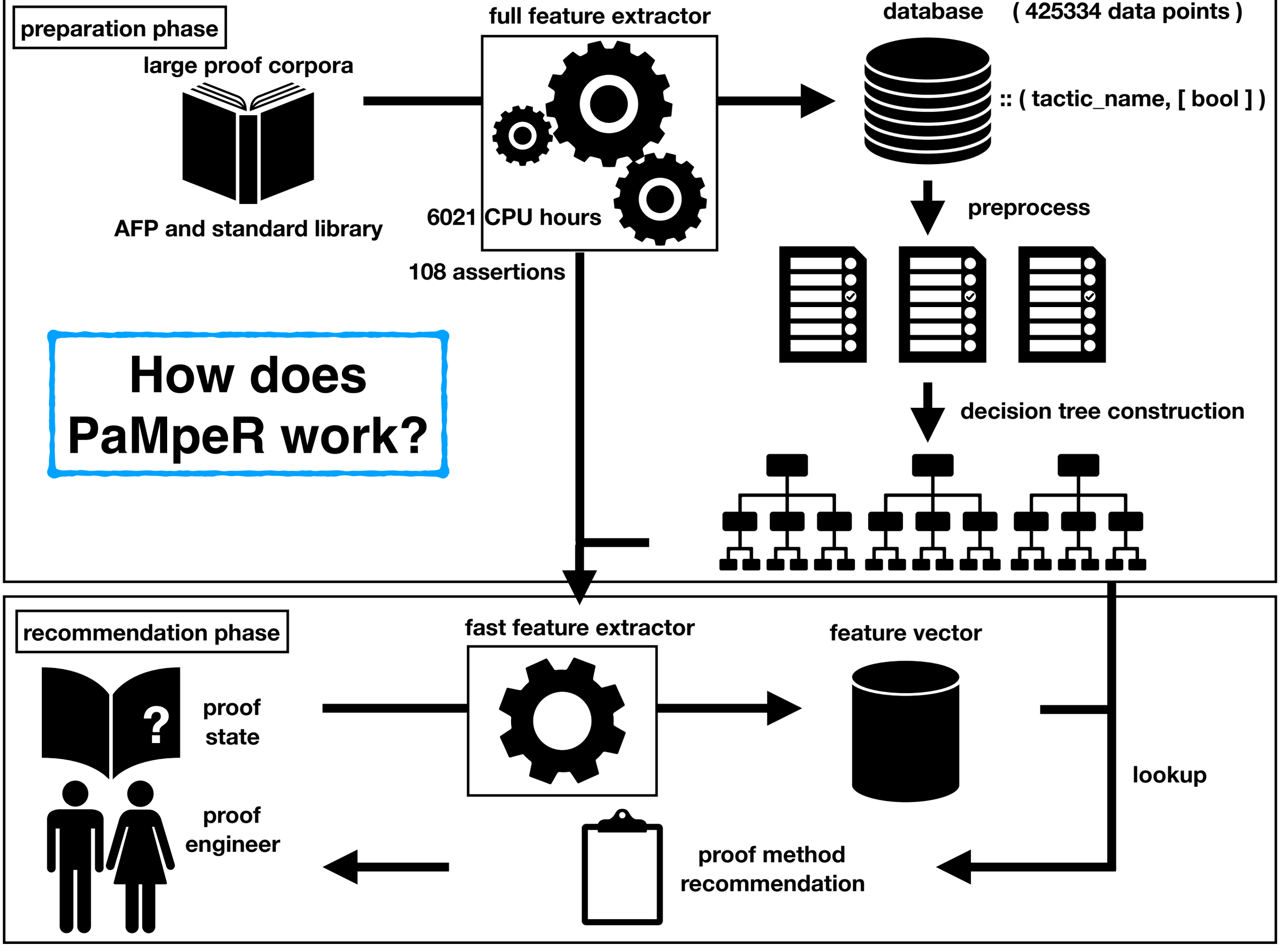
proof engineer

fast feature extractor

feature vector

How does PaMpeR work?



preparation phase

large proof corpora

AFP and standard library

full feature extractor

6021 CPU hours

database (425334 data points)

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preprocess

decision tree construction

108 assertions

?

How does PaMpeR work?

recommendation phase

proof state

proof engineer

fast feature extractor

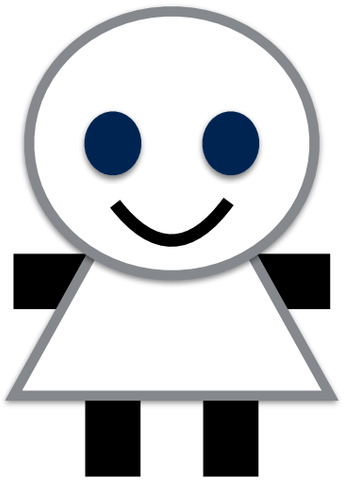
feature vector

lookup

proof method recommendation

https://twitter.com/YutakangJ

ITP2018 review

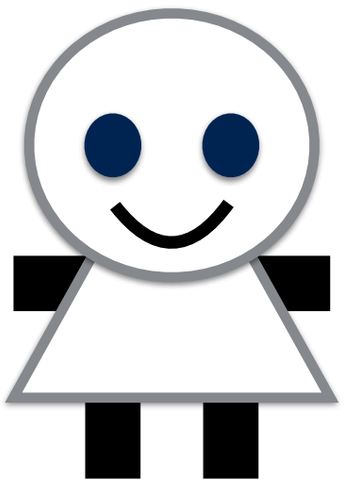


anonymous
reviewer

https://twitter.com/YutakangJ

ITP2018 review

Proof Method Recommendation, PaMpeR!



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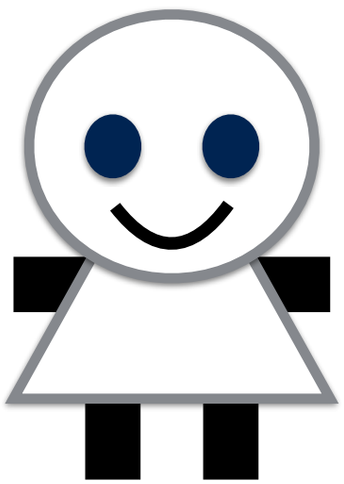
ITP2018 review

Proof Method Recommendation, PaMpeR!



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anonymous reviewer

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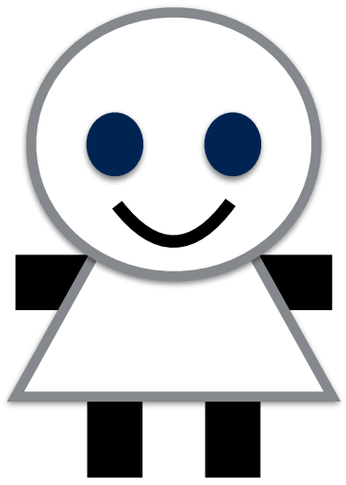
ITP2018 review

Proof Method Recommendation, PaMpeR!



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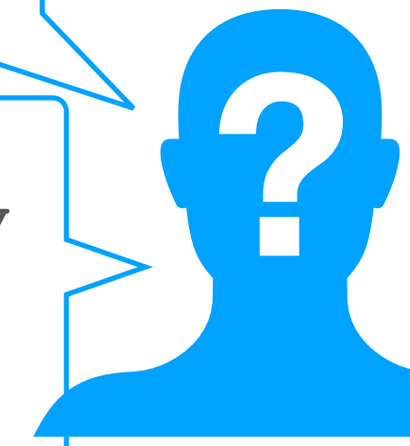
New users of Isabelle are facing many challenges from

- writing their first definitions,
- **stating suitable theorem statements….**



anonymous reviewer

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ITP2018 review

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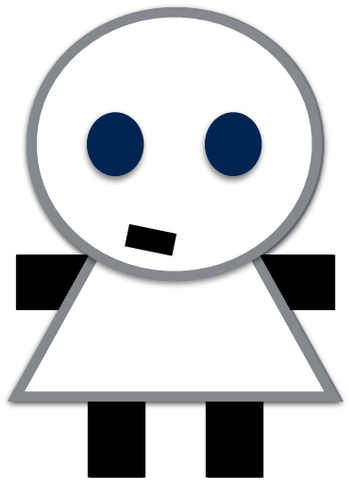

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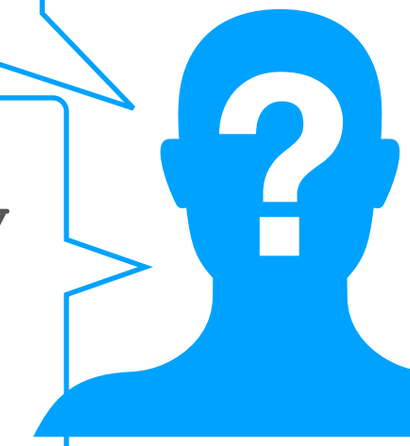

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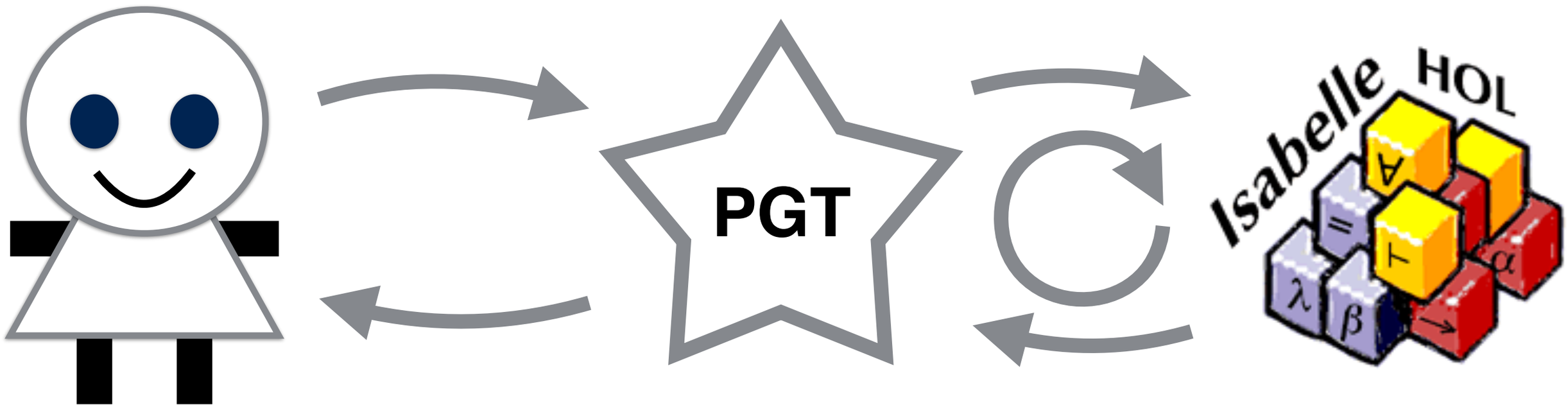
anonymous reviewer

Proof Goal Transformer, PGT!

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

PSL with PGT

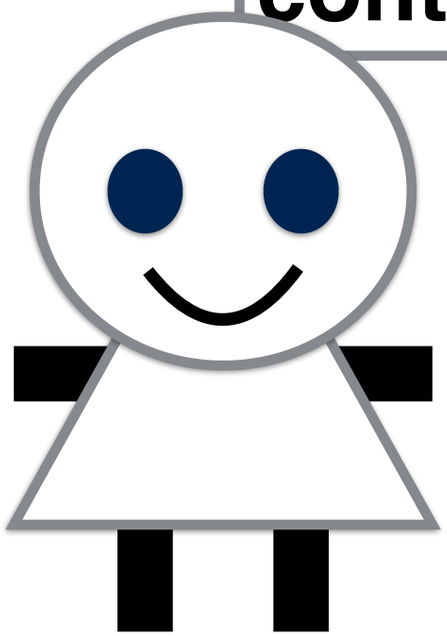


https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

PSL with PGT

proof goal <u>sub-optimal</u> for proof automation

context **PGT strategy**



PGT

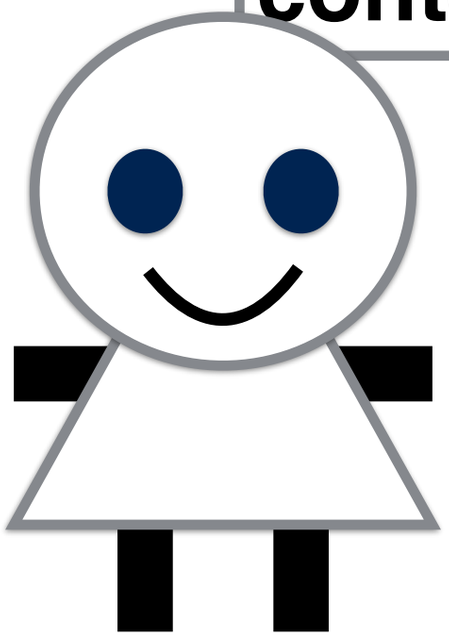
https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

PSL with PGT

proof goal sub-optimal for proof automation

context

PGT strategy



PGT

proof goal

context

tactic / sub-tool

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

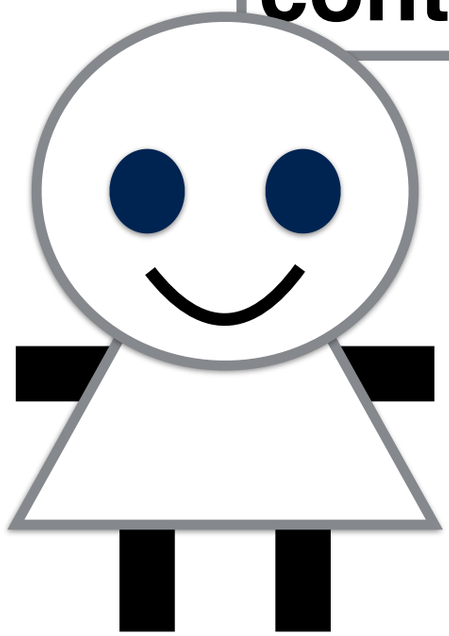
PSL with PGT

proof goal sub-optimal for proof automation

context **PGT strategy**

proof goal **context**

tactic / sub-tool



PGT

proved theorem / subgoals / message

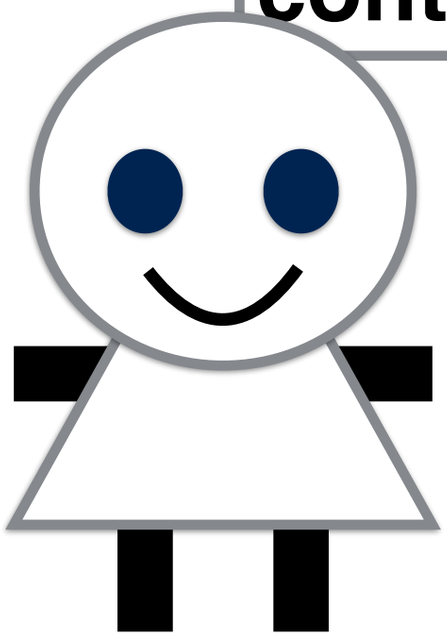
https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

PSL with PGT

proof goal sub-optimal for proof automation

context

PGT strategy



PGT

proof goal

context

tactic / sub-tool

proof for the original goal, and auxiliary lemma optimal for proof automation

proved theorem / subgoals / message

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

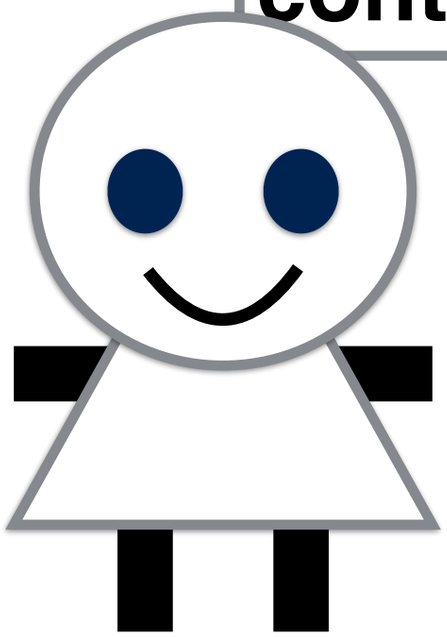
PSL with PGT

proof goal sub-optimal for proof automation

context **PGT strategy**

proof goal **context**

tactic / sub-tool

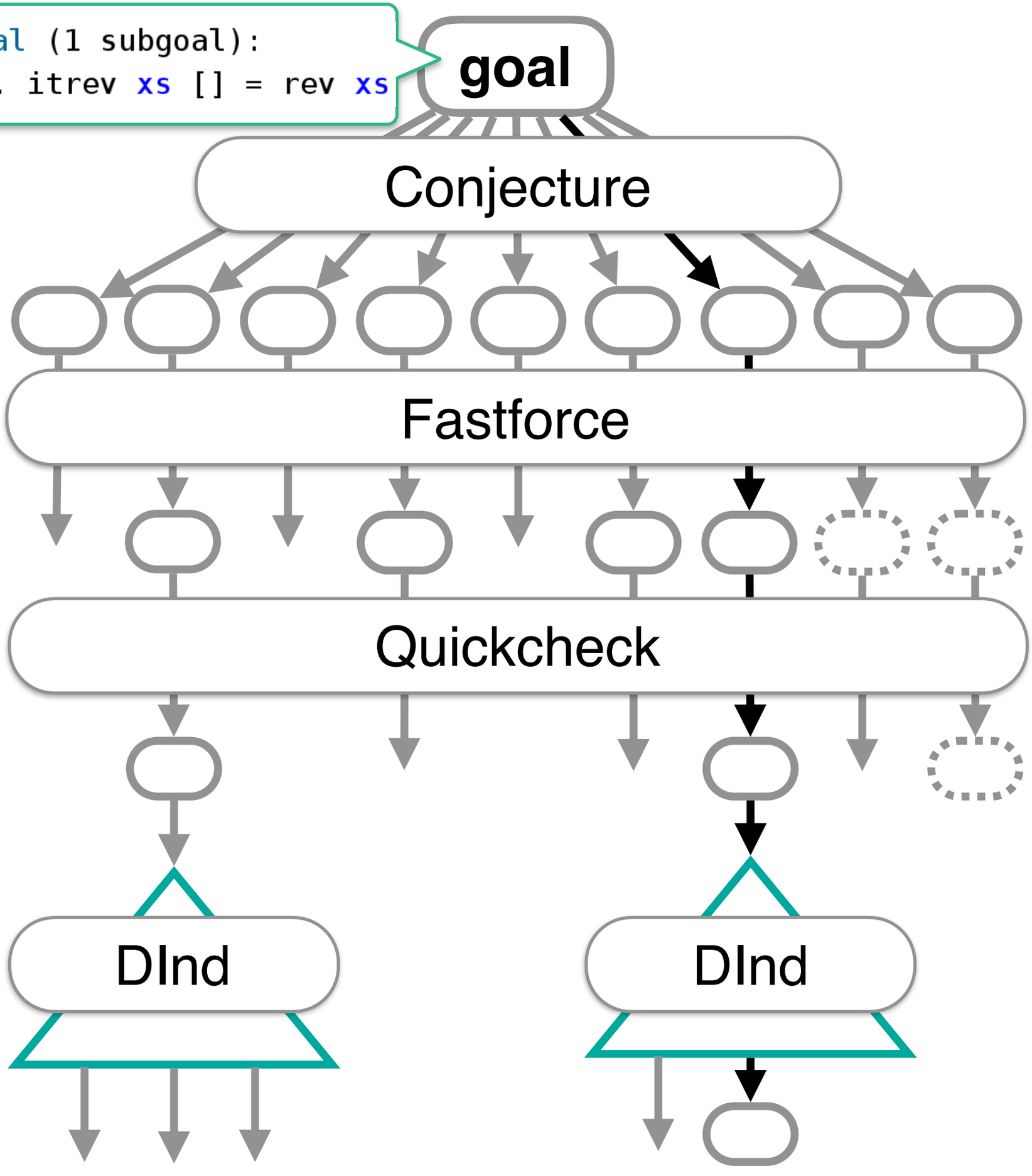


proof for the original goal, and auxiliary lemma optimal for proof automation

proved theorem / subgoals / [illegible]ssage

DEMO!

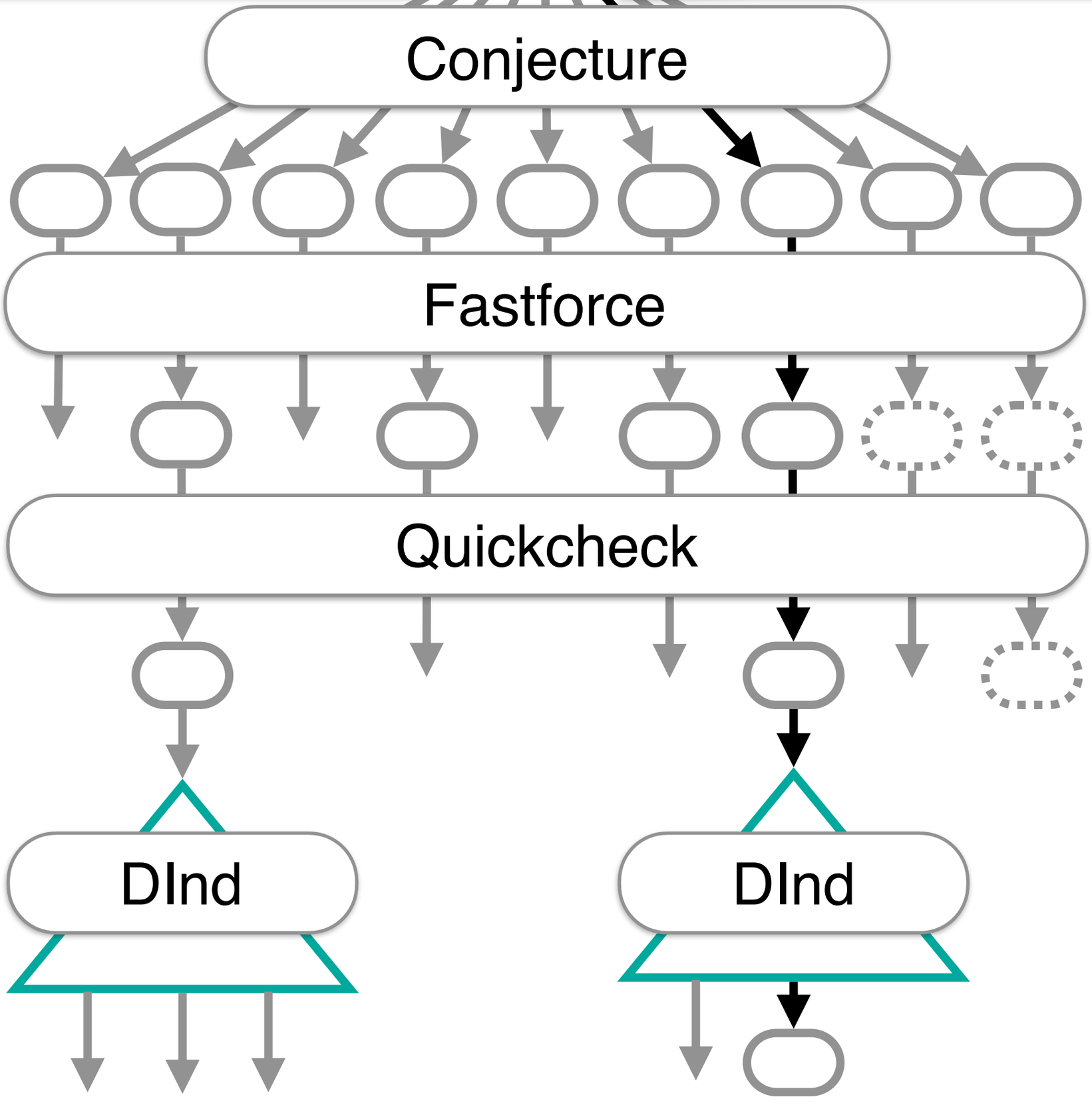
```
goal (1 subgoal):
 1. itrev xs [] = rev xs
```



```
goal (1 subgoal):
 1. itrev xs [] = rev xs
```

goal

```
apply (subgoal_tac
"⋀Nil. itrev xs Nil = rev xs @ Nil")
```



```
goal (1 subgoal):
 1. itrev xs [] = rev xs
```

goal

```
apply (subgoal_tac
"⋀Nil. itrev xs Nil = rev xs @ Nil")
```

```
goal (2 subgoals):
 1. (⋀Nil. itrev xs Nil = rev xs @ Nil) ⟹
    itrev xs [] = rev xs
 2. ⋀Nil. itrev xs Nil = rev xs @ Nil
```

Conjecture

Fastforce

Quickcheck

DInd

DInd

```
goal (1 subgoal):
 1. itrev xs [] = rev xs
```

goal

```
apply (subgoal_tac
"⋀Nil. itrev xs Nil = rev xs @ Nil")
```

```
goal (2 subgoals):
 1. (⋀Nil. itrev xs Nil = rev xs @ Nil) ⟹
    itrev xs [] = rev xs
 2. ⋀Nil. itrev xs Nil = rev xs @ Nil
```

Conjecture

Fastforce

```
apply fastforce
```

Quickcheck

DInd

DInd

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Quickcheck

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DInd

DInd

```
apply (induct xs)
apply auto
done
```

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goal (1 subgoal):
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Conjecture

Fastforce

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apply fastforce
```

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Quickcheck

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DInd

DInd

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```
theorem itrev ?xs [] = rev ?xs
```

```
goal (1 subgoal):
 1. itrev xs [] = rev xs
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goal

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Conjecture

```
apply fastforce
```

```
Number of lines of commands: 5
apply (subgoal_tac "⋀Nil. itrev xs Nil = rev xs @ Nil")
apply fastforce
apply (induct xs)
apply auto
done
```

Nil

Nil

DInd

DInd

```
apply (induct xs)
apply auto
done
```

```
theorem itrev ?xs [] = rev ?xs
```

Success story

PSL can find how to apply
induction for easy problems.

Success story

PSL can find how to apply induction for easy problems.

PaMpeR recommends which proof methods to use.

Success story

PSL can find how to apply induction for easy problems.

PaMpeR recommends which proof methods to use.

PGT produces useful auxiliary lemmas.

Success story

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CADE2017

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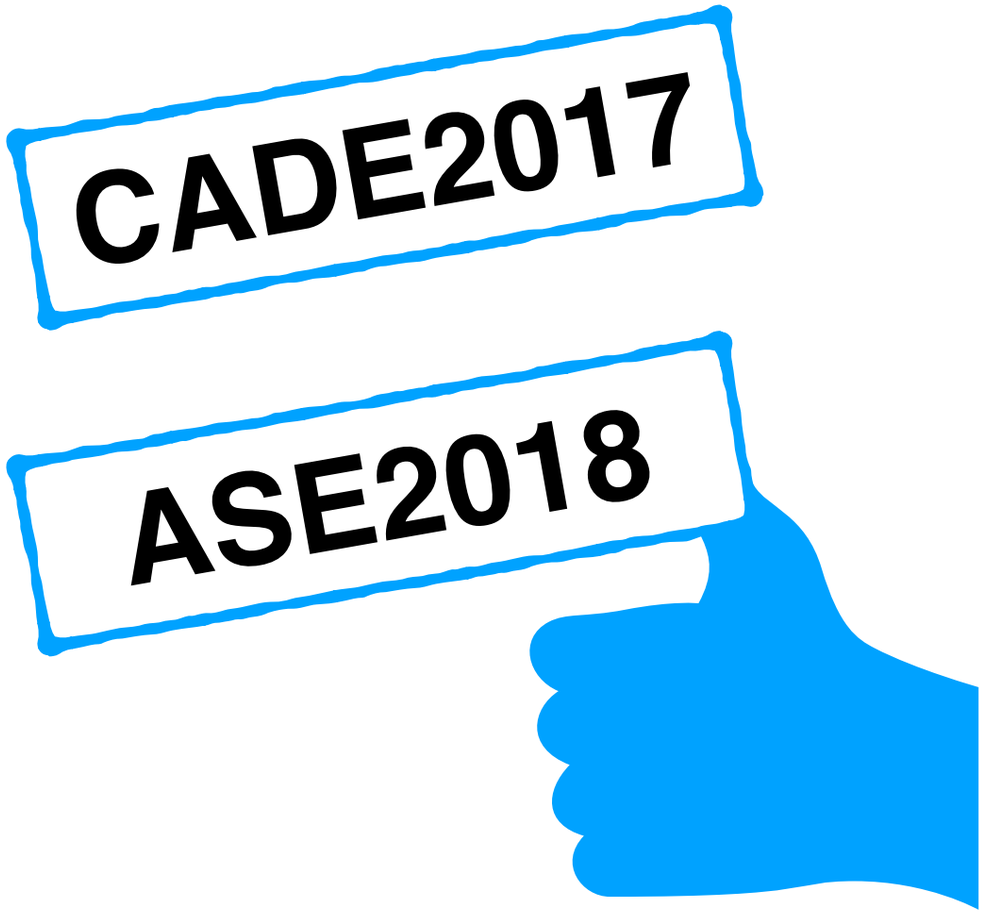
Success story

PSL can find how to apply induction for easy problems.



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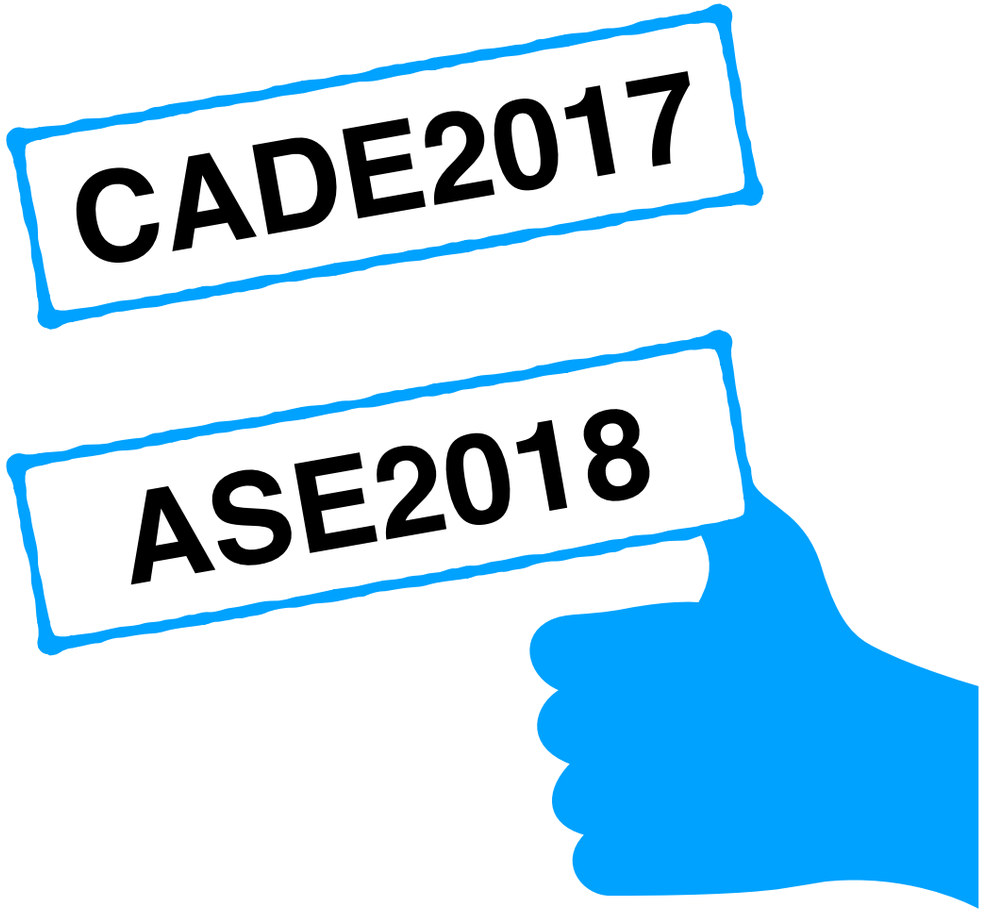

Too good to be true?

PSL can find how to apply induction for easy problems.

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PSL can find how to apply induction for easy problems

only if PSL completes a proof search

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PaMpeR recommends which proof methods to use.

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Recommend how to apply induction without completing a proof.

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MeLoId: Machine Learning Induction

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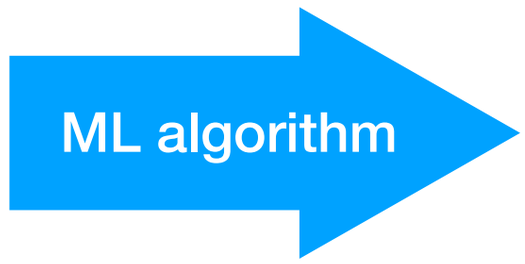
Introduction to Machine Learning in 10 seconds



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https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Introduction to Machine Learning in 10 seconds

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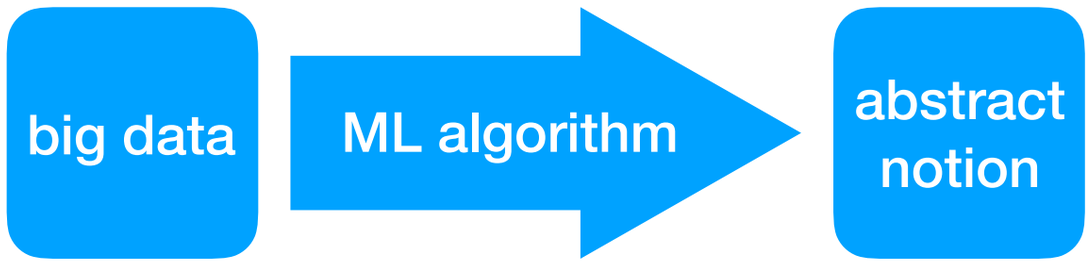
Introduction to Machine Learning in 10 seconds



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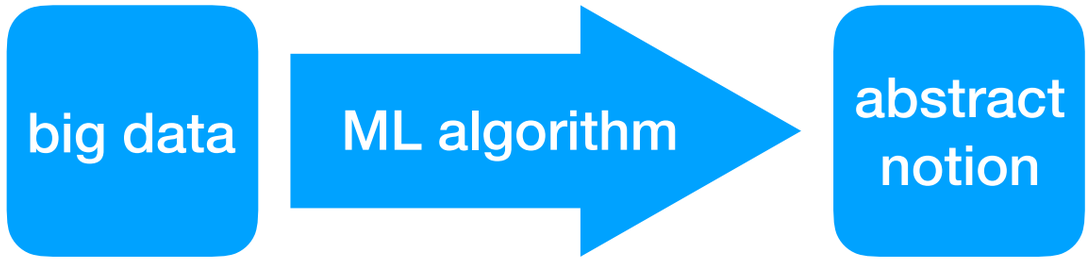
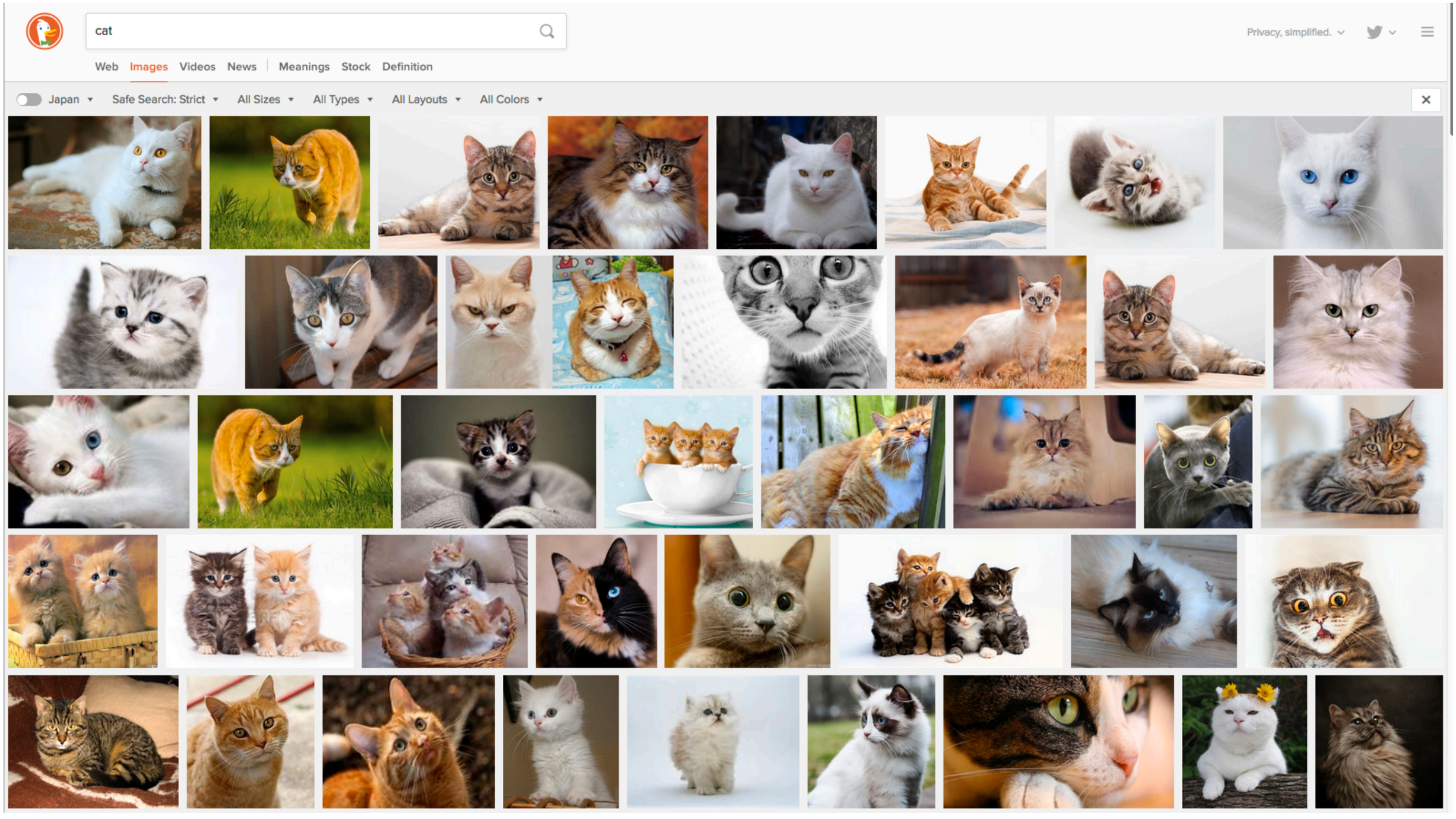
Introduction to Machine Learning in 10 seconds





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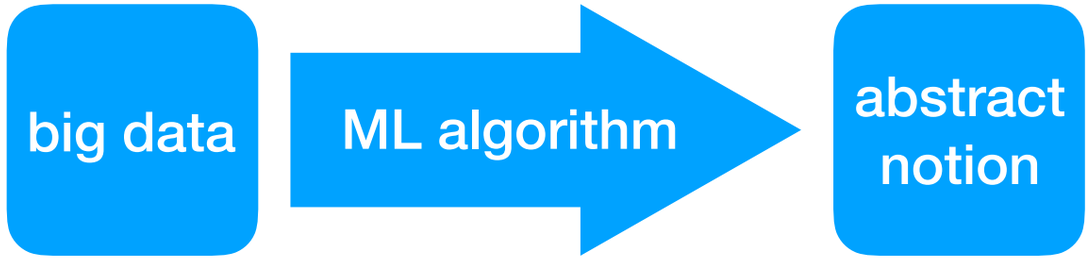
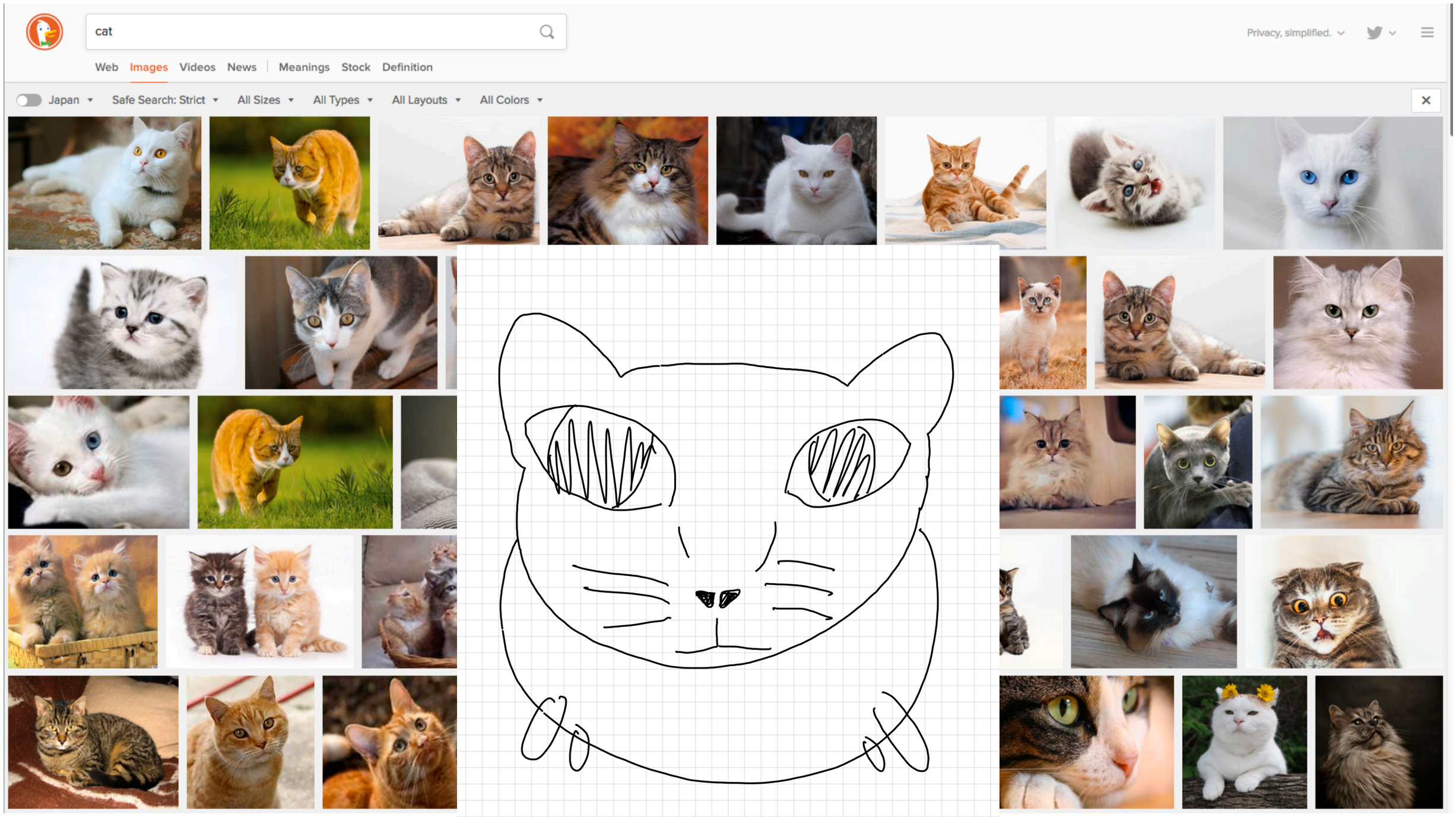
Introduction to Machine Learning in 10 seconds





https://duckduckgo.com/?q=cat&t=ffab&iar=images&iax=images&ia=images

ML for Inductive Theorem Proving the BAD

```
lemma "itrev xs ys = rev xs @ ys"
```

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
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```

<- many concrete cases

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

ML for Inductive Theorem Proving the BAD

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lemma "itrev xs ys = rev xs @ ys"
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<- one abstract representation

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<- one abstract representation

```
Failed to apply proof method⌂:
goal (1 subgoal):
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ML

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```

<- many concrete cases

ML for Inductive Theorem Proving the BAD

```
lemma "itrev xs ys = rev xs @ ys"  by auto
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation

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https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

ML for Inductive Theorem Proving
the BAD

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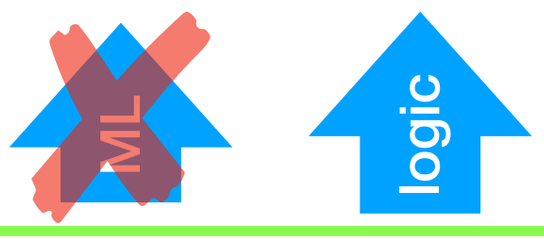

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

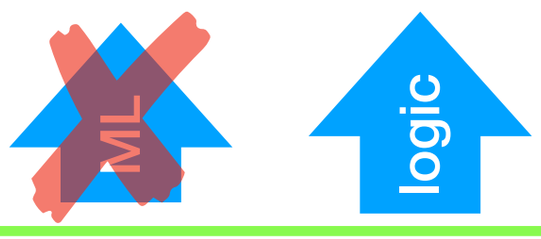
<- many concrete cases

ML for Inductive Theorem Proving the BAD

polymorphism

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



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```

<- many concrete cases

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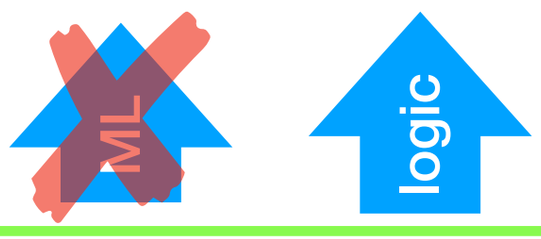
ML for Inductive Theorem Proving the BAD

polymorphism **type class**

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



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```

<- many concrete cases

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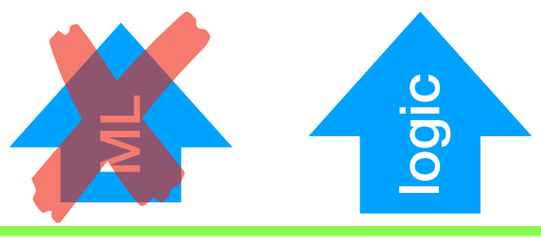
ML for Inductive Theorem Proving the BAD

polymorphism type class

universal quantifier

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



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<- many concrete cases

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ML for Inductive Theorem Proving the BAD

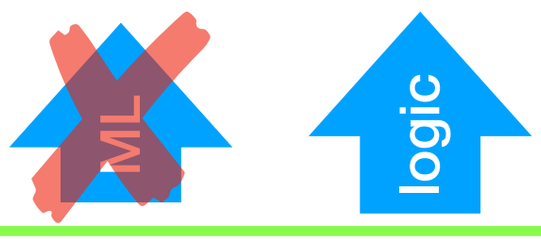
Higher-Order functions

polymorphism **type class**

universal quantifier

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



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lemma "itrev [x,y,z]        [] = rev [x,y,z]        @ []" by auto
```

<- many concrete cases

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism **type class**

universal quantifier

lambda abstraction

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lemma "itrev xs ys = rev xs @ ys"
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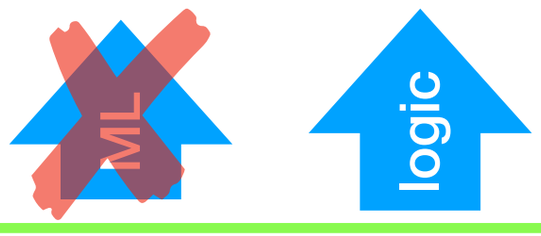
<- one abstract representation



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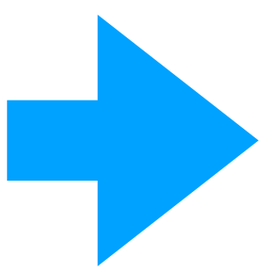
<- many concrete cases

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism **type class**

universal quantifier

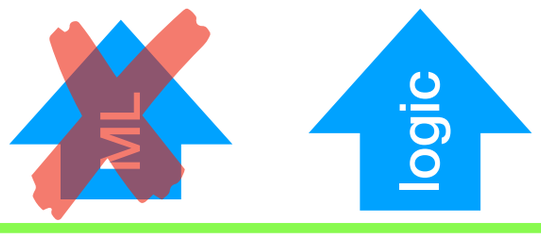


concise formula that can cover many concrete cases

lambda abstraction

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation



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<- many concrete cases

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ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

universal quantifier

concise formula that can cover many concrete cases

different proof for general case

lambda abstraction

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lemma "itrev xs ys = rev xs @ ys"
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<- many concrete cases

ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

universal quantifier

lambda abstraction

concise formula that can cover many concrete cases

different proof for general case

A small data set is not a failure but an achievement!

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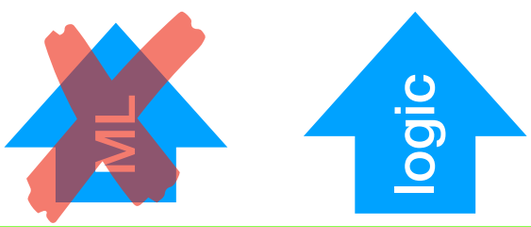

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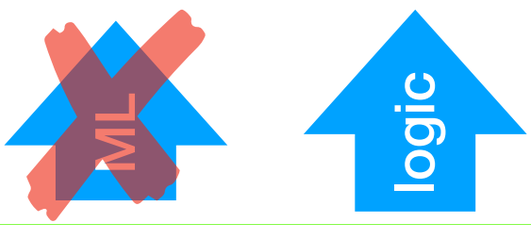

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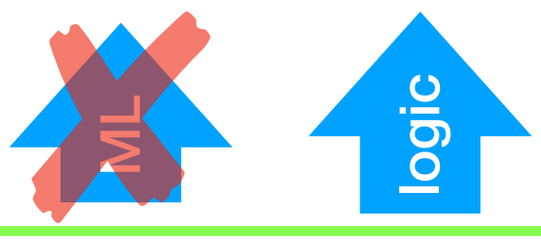

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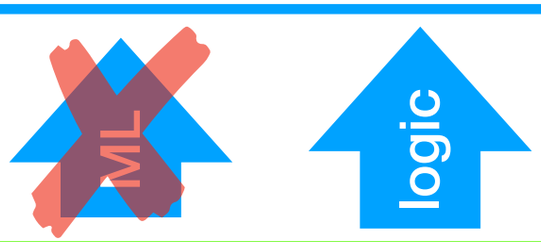

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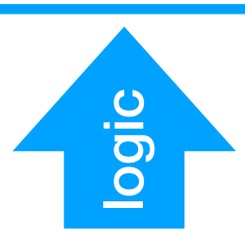

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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract

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Many key challenges remain

Unsupervised Learning

Memory and one-shot learning

Imagination-based Planning with Generative Models

Learning Abstract Concepts

Transfer Learning

Language understanding

https://cbmm.mit.edu/video/power-self-learning-systems

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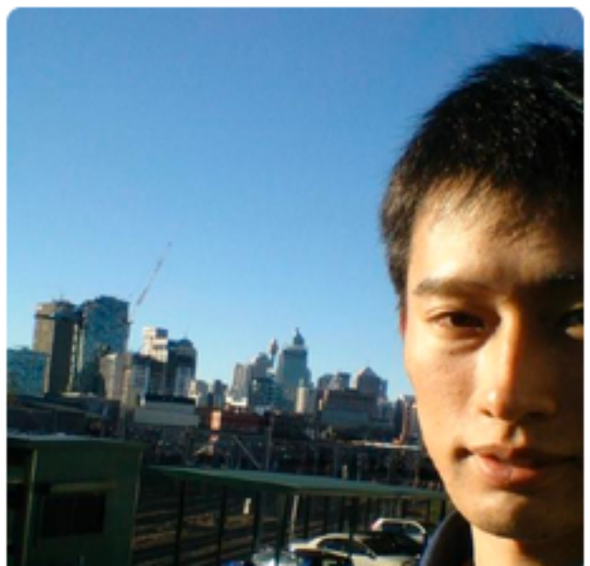
Minds+
Machines

March 20, 2019

The Power of

Self



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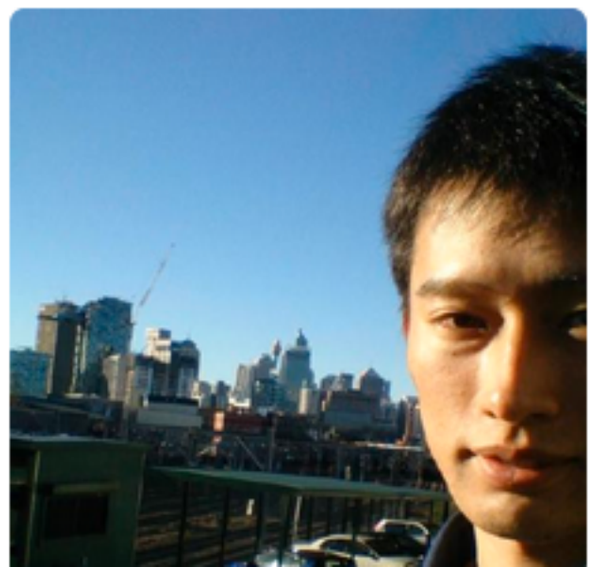

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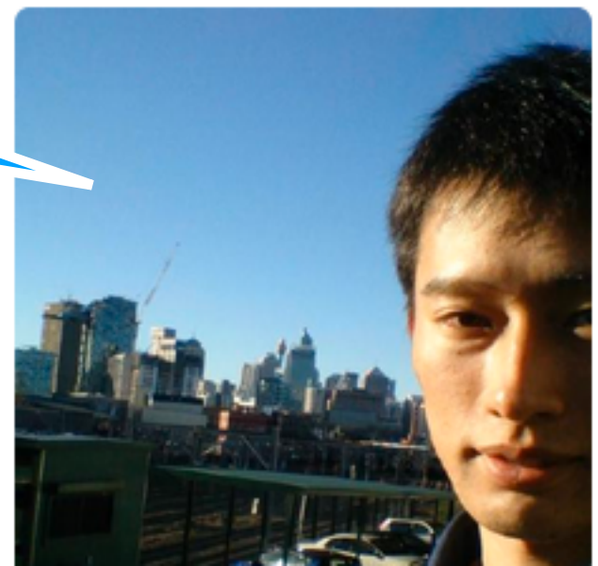

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Abstract conceptsといえば

論理でしょう。

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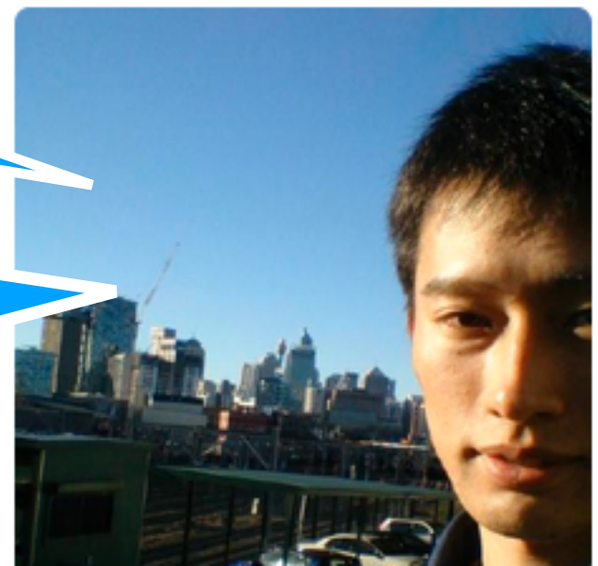

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Logic about Proofs to Abstract Abstraction

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract

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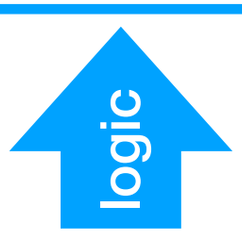

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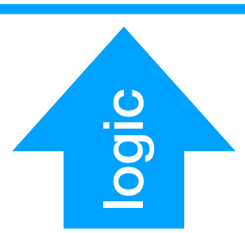

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LiFtEr 2

LiFtEr

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```
lemma "star r x y ⟹ star r y z ⟹ star r x z"
by(induction rule: star.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
       exec is2 s (exec is1 s stk)"
by(induct is1 s stk rule:exec.induct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,], [ ], [ ]]: bool list <- simple representation

LiFtEr

LiFtEr 2

abstraction using another logic (LiFtEr)

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r x y ⟹ star r y z ⟹ star r x z"
by(induction rule: star.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
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by(induct is1 s stk rule:exec.induct) auto
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<- small dataset about different domains

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lemma "itrev xs ys = rev xs @ ys"
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<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,], [ ], [ ]]: bool list <- simple representation

LiFtEr
LiFtEr
LiFtEr 3

abstraction using another logic (LiFtEr)

<- pros: good at rigorous abstraction

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```
lemma "star r x y ⟹ star r y z ⟹ star r x z"
by(induction rule: star.induct)(auto simp: step)

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lemma "itrev xs ys = rev xs @ ys"
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logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [ ], [ ]]: bool list <- simple representation

LiFtEr
LiFtEr
LiFtEr 3

abstraction using another logic (LiFtEr)

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lemma "star r x y ⟹ star r y z ⟹ star r x z"
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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [ ], [ ]]: bool list <- simple representation

LiFtEr 3 LiFtEr LiFtEr LiFtEr 1

<- abstraction using another logic (LiFtEr)

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```
lemma "star r x y ⟹ star r y z ⟹ star r x z"
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       exec is2 s (exec is1 s stk)"
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<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,], [ ]]: bool list <- simple representation

LiFtEr 3 LiFtEr LiFtEr LiFtEr 1

<- abstraction using another logic (LiFtEr)

<- pros: good at rigorous abstraction

<- cons: bad at ambiguity (heuristics)

```
lemma "star r  ⟹ star r y z ⟹ star r x z"
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<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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```

<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,], [ ]]: bool list <- simple representation

LiFtEr 3 LiFtEr LiFtEr LiFtEr 1 LiFtEr 2

...straction using other logic (LiFtEr)

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r ⟹ star r y z ⟹ star r x z"
by(induction r rule:star.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
       exec is2 s (exec is1 s stk)"
by(induct is1 s stk rule:exec.induct) auto

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<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,], [ ]]: bool list <- simple representation

LiFtEr 3 LiFtEr LiFtEr LiFtEr 1 LiFtEr 2

...traction using ... other logic (LiFtEr)

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

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       exec is2 s (exec is1 s stk)"
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<- small dataset about different domains

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lemma "itrev xs ys = rev xs @ ys"
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<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,], [ ]]: bool list <- simple representation

LiFtEr 1 LiFtEr 2 LiFtEr 3

<- abstraction using
higher logic (LiFtEr)

<- pros: good at rigorous abstraction

<- cons: bad at ambiguity (heuristics)

```
lemma "star r ⟹ star r y z ⟹ star r x z"
by(induction r rule:star.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
       exec is2 s (exec is1 s stk)"
by(induct is1 s stk rule:exec.induct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [ ]]: bool list <- simple representation

LiFtEr 1 LiFtEr 2 LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r ⟹ star r y z ⟹ star r x z"
by(induction r ... ar.induct)(auto simp: step)

lemma "exec (is1 @ is2) s stk =
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<- small dataset about different domains

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<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [ ]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3 LiFtEr 1

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r    star    y z ⟹ star r x z"
by(induction r      ar.in   t)(auto simp: step)

lemma "exec (is1 @ is2) s    =
       exec is2 s (exec is    stk)"
by(induct is1 s stk rule:e    .induct) auto

lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
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lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [F,]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3 LiFtEr 1

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

(LiFtEr)

```
lemma "star r x y ⟹ star r x z"
by(induction r ... t)(auto simp: step)

lemma "exec (is1 @ is2) s ... =
       exec is2 s (exec is1 ... stk)"
by(induct is1 s stk rule:e...induct) auto
```

<- small dataset about different domains

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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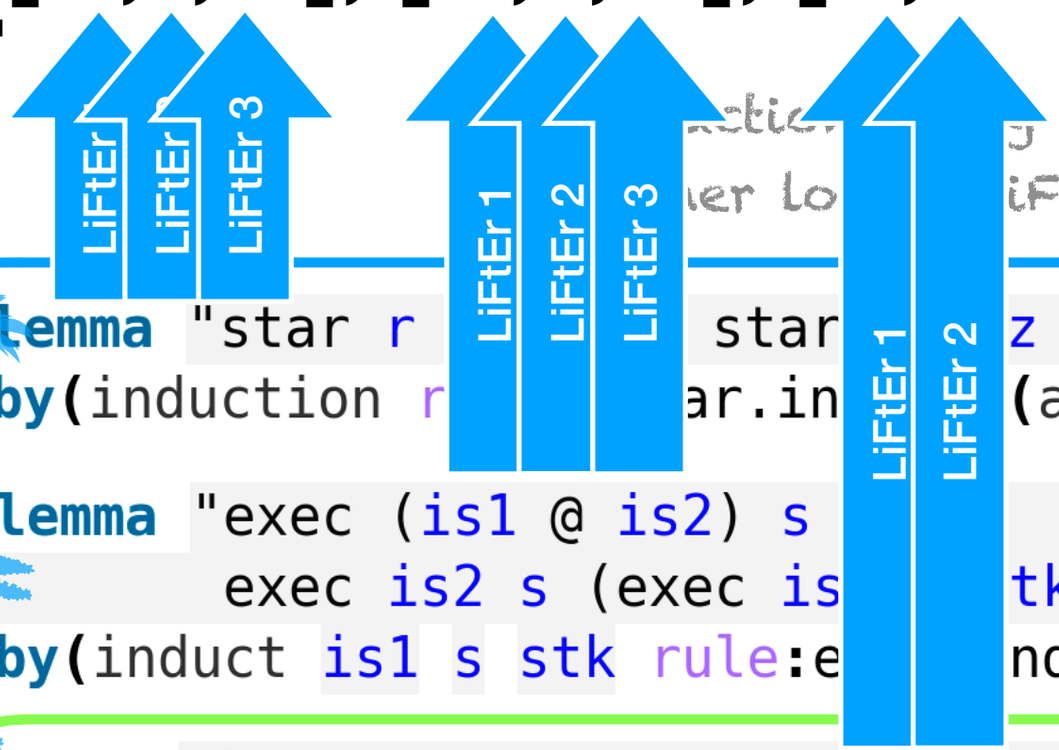
<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [F,]]: bool list <- simple representation



<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r    star    z ⟹ star r x z"
by(induction r      ar.in   (auto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is      tk)"
by(induct is1 s stk rule:e      nduct) auto

lemma "itrev xs ys = rev xs @ ys"
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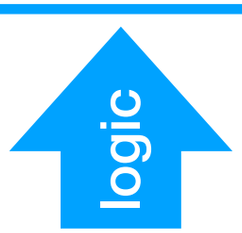
```

<- small dataset about different domains

<- one abstract representation



<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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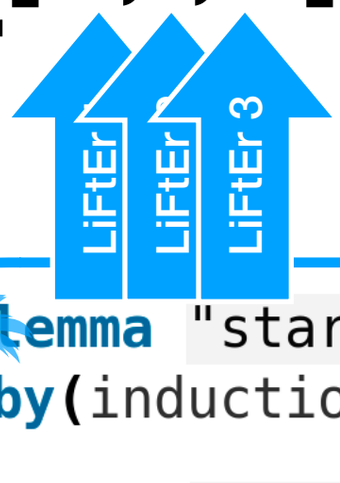
<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [F,T,]]: bool list <- simple representation



<- pros: good at rigorous abstraction

<- cons: bad at ambiguity (heuristics)

```
lemma "star r x z ⟹ star r x z"
by(induction r ar.in (auto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is tk)"
by(induct is1 s stk rule:e nduct) auto
```

<- small dataset about different domains

```
lemma "itrev xs ys = rev xs @ ys"
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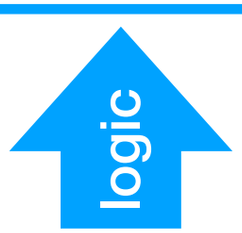
<- one abstract representation



<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [F,T,]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

LiFtEr 1 LiFtEr 2 LiFtEr 3

```
lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in              uto simp: step)

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LiFtEr 1 LiFtEr 2 LiFtEr 3

<- small dataset about different domains

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lemma "itrev xs ys = rev xs @ ys"
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<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]          [] = rev [1,2]          @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

<- pros: good at rigorous abstraction

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LiFtEr 1 LiFtEr 2 LiFtEr 3

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lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in              uto simp: step)

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LiFtEr 1 LiFtEr 2 LiFtEr 3

<- small dataset about different domains

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<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]          [] = rev [1,2]          @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

Big Picture

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

LiFtEr LiFtEr LiFtEr 3 — LiFtEr 1 LiFtEr 2 LiFtEr 3 — LiFtEr 1 LiFtEr 2 LiFtEr 3

```
lemma "star r                    star                ⟹ star r x z"
by(induction r                   ar.in               uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                           )"
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<- small dataset about different domains

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<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]          [] = rev [1,2]          @ []" by auto
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<- many concrete cases

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

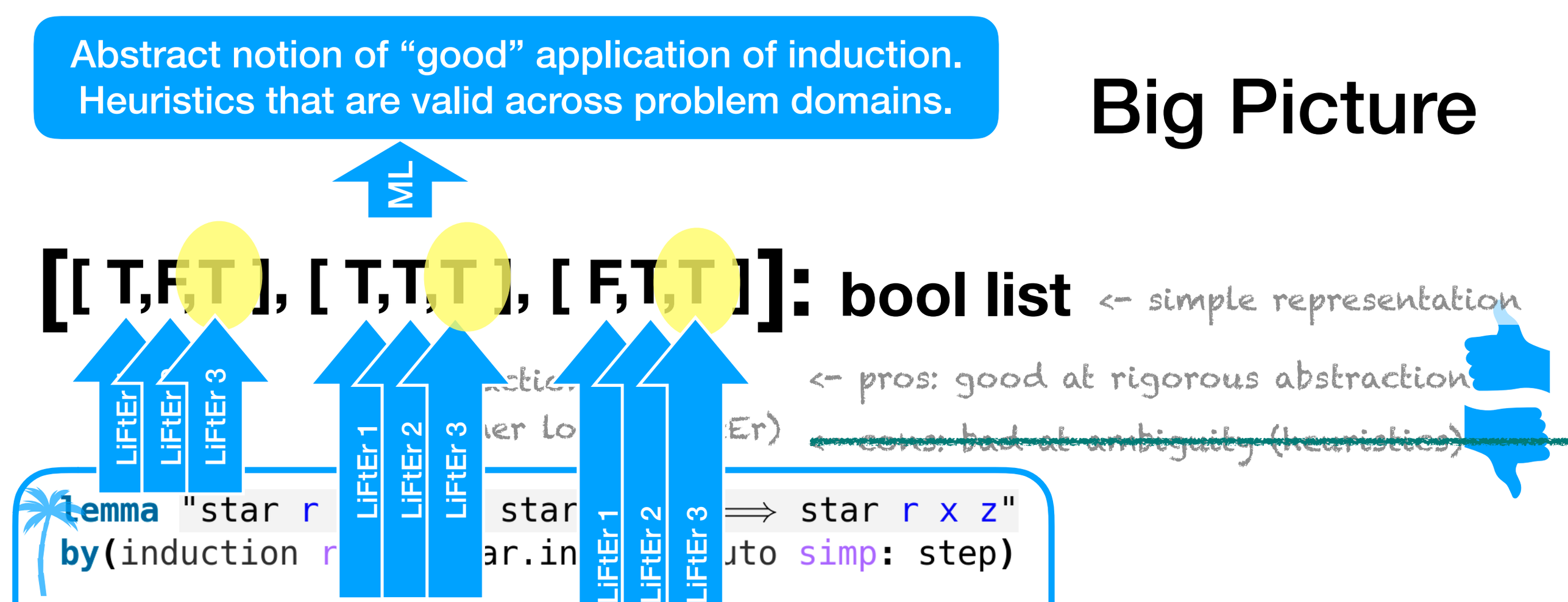
ML

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr 1 LiFtEr 2 LiFtEr 3

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~



```
lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in              uto simp: step)

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<- one abstract representation

logic

<- abstraction using expressive logic

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<- many concrete cases

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

<- pros: good at ambiguity (heuristics)

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr 1 LiFtEr 2 LiFtEr 3

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in              uto simp: step)

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<- small dataset about different domains

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logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

<- pros: good at ambiguity (heuristics)

[[T,F,T], [T,T,T], [F,T,T]]: bool list

<- simple representation

LiFtEr

LiFtEr

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

<- pros: good at rigorous abstraction

<- cons: bad at ambiguity (heuristics)

具体例?

```
lemma "star r                star                ⟹ star r x z"
by(induction r                ar.in              uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                        )"
by(induct is1 s stk rule:e                        uct) auto
```

<- small dataset about different domains

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

Example Assertion in LiFtEr (in Abstract Syntax)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication

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∃ r1 : rule. True
→
∃ r1 : rule.
  ∃ t1 : term.
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          ∃ to2 : term_occurrence ∈ t2 : term.
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Example Assertion in LiFtEr (in Abstract Syntax)

implication

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→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧                                  conjunction
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication

variable for auxiliary lemmas

conjunction

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
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Example Assertion in LiFtEr (in Abstract Syntax)

implication

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            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
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                  t2 is_nth_induction_term n
```

variable for auxiliary lemmas

variable for terms

conjunction

Example Assertion in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
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  ∃ r1 : rule.
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```

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

Example Assertion in LiFtEr (in Abstract Syntax)

```
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→
  ∃ r1 : rule.
    ∃ t1 : term.
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          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

implication

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

variable for natural numbers

Example Assertion in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

universal quantifier

variable for natural numbers

Example Assertion in LiFtEr (in Abstract Syntax)

implication

existential quantifier

```
∃ r1 : rule. True
→
```

variable for auxiliary lemmas

```
  ∃ r1 : rule.
```

variable for terms

```
    ∃ t1 : term.
```

variable for term occurrences

```
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
```

conjunction

```
        ∧
```

universal quantifier

```
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
```

variable for natural numbers

```
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

(r1 = itrev.induct)

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
```

→ r1

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = itrev.induct)

```
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 "rev  []     = []" |
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fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
```

→ r1

(r1 = itrev.induct)

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

t1 r1

(r1 = itrev.induct)
(t1 = itrev)

∃ $r1$: rule. True

→

∃ $r1$: rule.
 ∃ $t1$: term.
  ∃ $to1$: term_occurrence ∈ $t1$: term.
   $r1$ is_rule_of $to1$
  ∧
   ∀ $t2$: term ∈ induction_term.
    ∃ $to2$: term_occurrence ∈ $t2$: term.
     ∃ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
      ∧
      $t2$ is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1

t1 r1

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

∃ $r1$: rule. True

→

∃ $r1$: rule.
 ∃ $t1$: term.
  ∃ $to1$: term_occurrence ∈ $t1$: term.
   $r1$ is_rule_of $to1$
  ∧
   ∀ $t2$: term ∈ induction_term.
    ∃ $to2$: term_occurrence ∈ $t2$: term.
     ∃ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
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primrec rev :: "'a list ⇒ 'a list" where
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1

t1 r1

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

$\exists$ $r1$: rule. True

$\rightarrow$

$\exists$ $r1$: rule.
 $\exists$ $t1$: term.
  $\exists$ $to1$: term_occurrence $\in$ $t1$: term.
   $r1$ is_rule_of $to1$
  $\wedge$
   $\forall$ $t2$: term $\in$ induction_term.
    $\exists$ $to2$: term_occurrence $\in$ $t2$: term.
     $\exists$ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
     $\wedge$
      $t2$ is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
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```

to1

∃ $r1$: rule. True

→

∃ $r1$: rule.
∃ $t1$: term.
∃ $to1$: term_occurrence ∈ $t1$: term.
$r1$ is_rule_of $to1$
∧
∀ $t2$: term ∈ induction_term.
∃ $to2$: term_occurrence ∈ $t2$: term.
∃ n : number.
is_nth_argument_of ($to2$, n, $to1$)
∧
$t2$ is_nth_induction_term n

t1 r1

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).

```
primrec rev :: "'a list ⇒ 'a list" where
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to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

t1 r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                    ( r1 = itrev.induct )
    ∃ t1 : term.                                  ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.        ( to1 = itrev )
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
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```

Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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primrec rev :: "'a list ⇒ 'a list" where
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to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
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t1 r1

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
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Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 t2 t1 r1

∃ $r1$: rule. True

→

∃ $r1$: rule. (r1 = itrev.induct)
∃ $t1$: term. (t1 = itrev)
∃ $to1$: term_occurrence ∈ $t1$: term. (to1 = itrev)
$r1$ is_rule_of $to1$ Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ $t2$: term ∈ induction_term. (t2 = xs and ys)
∃ $to2$: term_occurrence ∈ $t2$: term.
∃ n : number.
is_nth_argument_of ($to2$, n, $to1$)
∧
$t2$ is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 to2 t2 t1 r1

∃ $r1$: rule. True

→

∃ $r1$: rule. (r1 = itrev.induct)
∃ $t1$: term. (t1 = itrev)
∃ $to1$: term_occurrence ∈ $t1$: term. (to1 = itrev)
$r1$ is_rule_of $to1$ Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ $t2$: term ∈ induction_term. (t2 = xs and ys)
∃ $to2$: term_occurrence ∈ $t2$: term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of ($to2$, n, $to1$)
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primrec rev :: "'a list ⇒ 'a list" where
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lemma "itrev xs ys = rev xs @ ys"
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```

to1 to2 t2 t1 r1

∃ $r1$: rule. True

→

∃ $r1$: rule. (r1 = itrev.induct)
 ∃ $t1$: term. (t1 = itrev)
  ∃ $to1$: term_occurrence ∈ $t1$: term. (to1 = itrev)
   $r1$ is_rule_of $to1$ Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).
  ∧
   ∀ $t2$: term ∈ induction_term. (t2 = xs and ys)
    ∃ $to2$: term_occurrence ∈ $t2$: term. (to2 = xs and ys)
     ∃ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
     ∧
      $t2$ is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
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 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 to2 t2 t1 r1

∃ $r1$: rule. True

→

∃ $r1$: rule. (r1 = itrev.induct)
∃ $t1$: term. (t1 = itrev)
∃ $to1$: term_occurrence ∈ $t1$: term. (to1 = itrev)
$r1$ is_rule_of $to1$ Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ $t2$: term ∈ induction_term. (t2 = xs and ys)
∃ $to2$: term_occurrence ∈ $t2$: term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of ($to2$, n, $to1$)
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lemma "itrev xs ys = rev xs @ ys"
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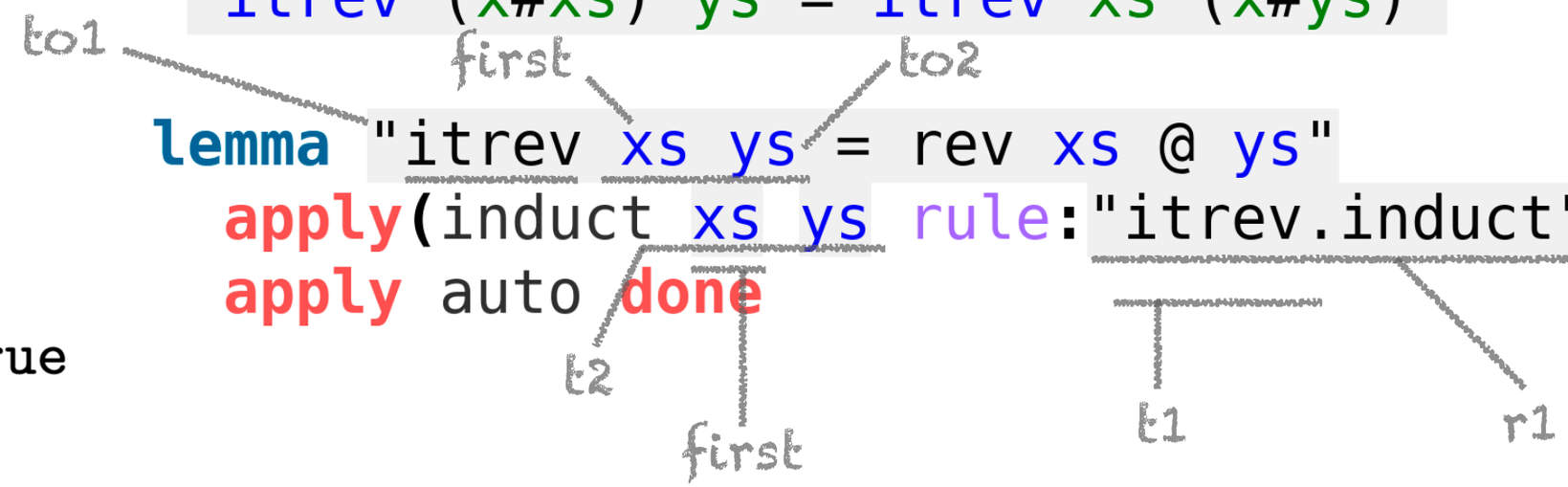
```



∃ $r1$: rule. True

→

∃ $r1$: rule.
 ∃ $t1$: term.
  ∃ $to1$: term_occurrence ∈ $t1$: term.
   $r1$ is_rule_of $to1$
  ∧
   ∀ $t2$: term ∈ induction_term.
    ∃ $to2$: term_occurrence ∈ $t2$: term.
     ∃ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
     ∧
      $t2$ is_nth_induction_term n

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).

(t2 = xs and ys)
(to2 = xs and ys)

Yes for xs (n = 1)!

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first second to2

t2 second t1 r1

first

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

Yes! r1 (= itrev.induct) is a lemma about to1 (= itrev).

(t2 = xs and ys)
(to2 = xs and ys)

Yes for xs (n = 1)!

Yes for ys (n = 2)!

$$
\begin{array}{l}
\exists\ r1\ :\ \texttt{rule}.\ \texttt{True} \\
\rightarrow \\
\quad \exists\ r1\ :\ \texttt{rule}. \\
\quad\quad \exists\ t1\ :\ \texttt{term}. \\
\quad\quad\quad \exists\ to1\ :\ \texttt{term\_occurrence} \in\ t1\ :\ \texttt{term}. \\
\quad\quad\quad\quad r1\ \texttt{is\_rule\_of}\ to1 \\
\quad\quad\quad\quad \wedge \\
\quad\quad\quad\quad \forall\ t2\ :\ \texttt{term} \in \texttt{induction\_term}. \\
\quad\quad\quad\quad\quad \exists\ to2\ :\ \texttt{term\_occurrence} \in\ t2\ :\ \texttt{term}. \\
\quad\quad\quad\quad\quad\quad \exists\ n\ :\ \texttt{number}. \\
\quad\quad\quad\quad\quad\quad\quad \texttt{is\_nth\_argument\_of}\ (to2,\ n,\ to1) \\
\quad\quad\quad\quad\quad\quad\quad \wedge \\
\quad\quad\quad\quad\quad\quad\quad t2\ \texttt{is\_nth\_induction\_term}\ n
\end{array}
$$

$\exists$ $r1$: rule. True

$\rightarrow$

$\exists$ $r1$: rule.
$\exists$ $t1$: term.
$\exists$ $to1$: term_occurrence $\in$ $t1$: term.
$r1$ is_rule_of $to1$
$\wedge$
$\forall$ $t2$: term $\in$ induction_term.
$\exists$ $to2$: term_occurrence $\in$ $t2$: term.
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

```
datatype      instr = LOADI val | LOAD vname | ADD
type_synonym  stack = "val list"
```

---

the same LiFtEr assertion

$\exists\ r1\ :\ \texttt{rule}.\ \texttt{True}$

$\rightarrow$

$\exists\ r1\ :\ \texttt{rule}.$
$\quad \exists\ t1\ :\ \texttt{term}.$
$\quad\quad \exists\ to1\ :\ \texttt{term\_occurrence} \in\ t1\ :\ \texttt{term}.$
$\quad\quad\quad r1\ \texttt{is\_rule\_of}\ to1$
$\quad\quad\quad \wedge$
$\quad\quad\quad \forall\ t2\ :\ \texttt{term} \in \texttt{induction\_term}.$
$\quad\quad\quad\quad \exists\ to2\ :\ \texttt{term\_occurrence} \in\ t2\ :\ \texttt{term}.$
$\quad\quad\quad\quad\quad \exists\ n\ :\ \texttt{number}.$
$\quad\quad\quad\quad\quad\quad \texttt{is\_nth\_argument\_of}\ (to2,\ n,\ to1)$
$\quad\quad\quad\quad\quad\quad \wedge$
$\quad\quad\quad\quad\quad\quad t2\ \texttt{is\_nth\_induction\_term}\ n$

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

new constants ->

the same LiFtEr assertion

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
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lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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 apply auto done
```

$\exists\ r1$: rule. True

$\rightarrow$

$\exists\ r1$: rule.
$\exists\ t1$: term.
$\exists\ to1$: term_occurrence $\in\ t1$: term.
$r1$ is_rule_of $to1$
$\wedge$
$\forall\ t2$: term $\in$ induction_term.
$\exists\ to2$: term_occurrence $\in\ t2$: term.
$\exists\ n$: number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

new types ->

new constants ->

```
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 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

r1

(r1 = exec.induct)

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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  apply auto done
```

r1

(r1 = exec.induct)

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

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 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

r1

(r1 = exec.induct)

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
              is_nth_argument_of (to2, n, to1)
            ∧
              t2 is_nth_induction_term n
```

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
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 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

(r1 = exec.induct)
(t1 = exec)

```
∃ r1 : rule. True
→
∃ r1 : rule.
  ∃ t1 : term.
   ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

new lemma ->

a model proof ->

```
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
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lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
   ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

new types ->

new constants ->

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datatype     instr = LOADI val | LOAD vname | ADD
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 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
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        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

Yes! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

Yes! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

new lemma ->

a model proof ->

```
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 "exec1 (LOADI n) _   stk      =  n       # stk" |
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 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
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        ∀ t2 : term ∈ induction_term.
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```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

Yes! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

Yes! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
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 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

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lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 t1 r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

Yes! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->

a model proof ->

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 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

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 "exec []     _ stk = stk" |
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t2 t01 t02 t1 r1

∃ r1 : rule. True

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∃ r1 : rule.
  ∃ t1 : term.
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              ∧
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```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

Yes! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->

a model proof ->

```
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 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

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 "exec (i#is) s stk = exec is s (exec1 i s stk)"

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```

t2 t01 t02 t1 r1

∃ r1 : rule. True

→

∃ r1 : rule.
  ∃ t1 : term.
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      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

Yes! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
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 "exec []     _ stk = stk" |
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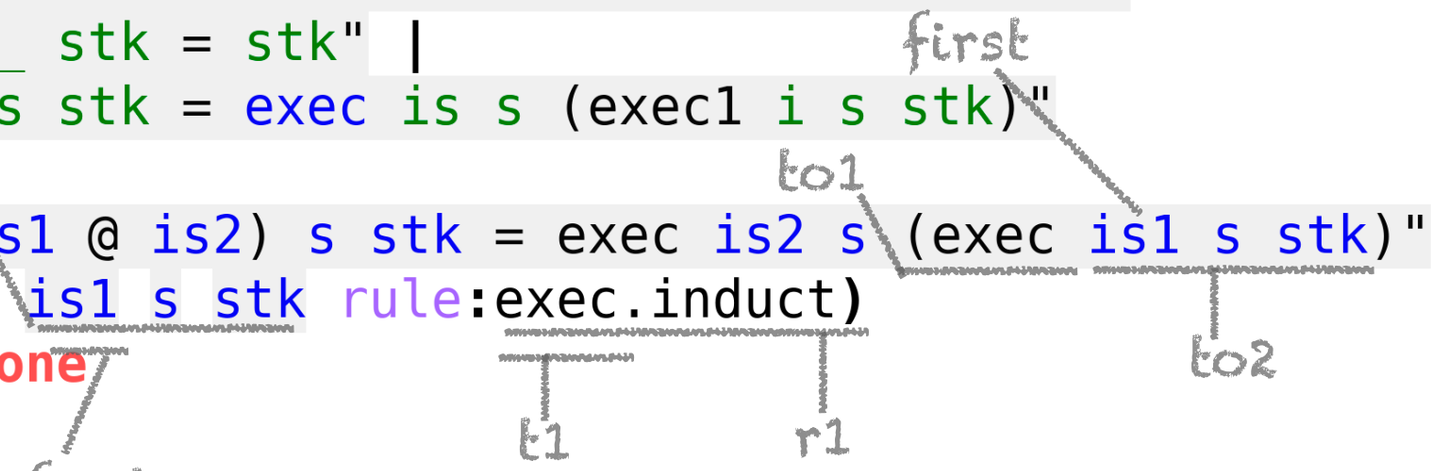
new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```




```
∃ r1 : rule. True
→
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
              is_nth_argument_of (to2, n, to1)
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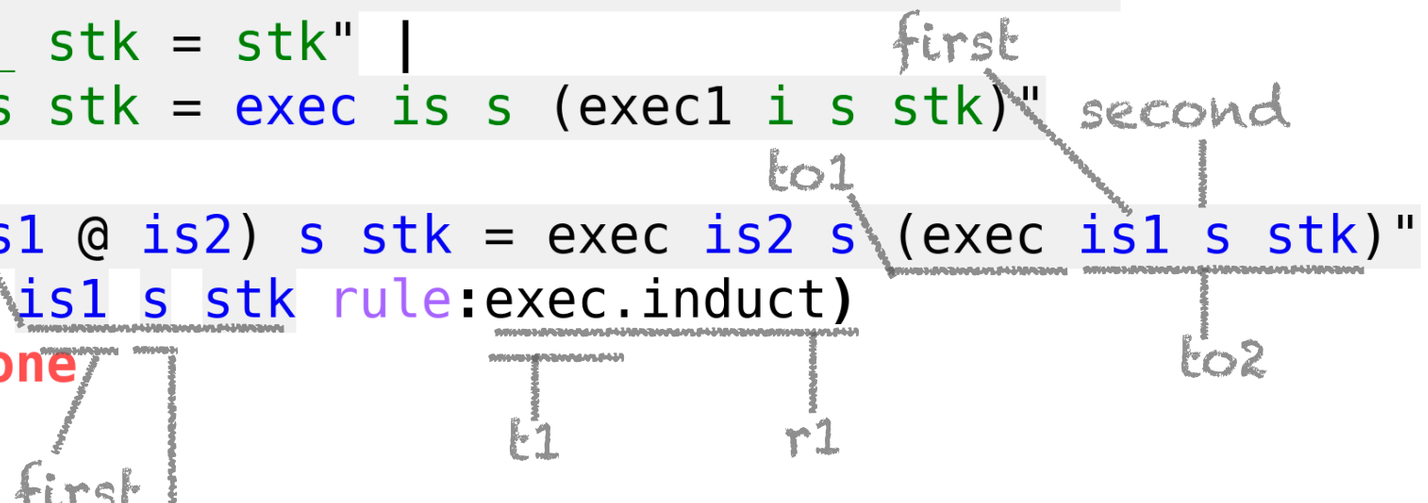


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      ∧
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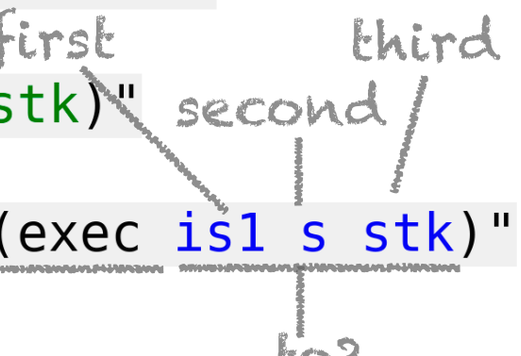

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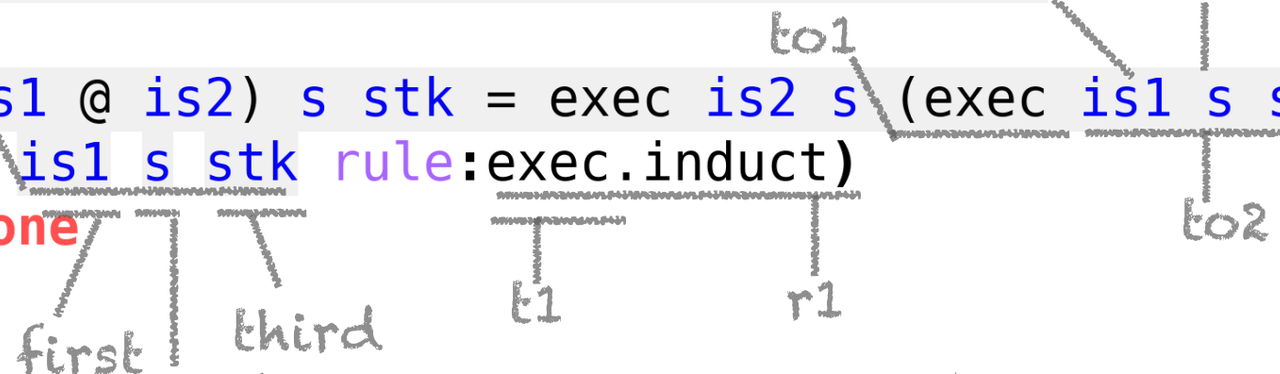

$\exists$ $r1$: rule. True

→

$\exists$ $r1$: rule. (r1 = exec.induct)
  $\exists$ $t1$: term. (t1 = exec)
    $\exists$ $to1$: term_occurrence $\in$ $t1$: term. (to1 = exec)
      $r1$ is_rule_of $to1$ Yes! r1 (= exec.induct) is a lemma about to1 (= exec).
    $\wedge$
      $\forall$ $t2$: term $\in$ induction_term. (t2 = is1, s, and stk)
        $\exists$ $to2$: term_occurrence $\in$ $t2$: term. (to2 = is1, s, and stk)
          $\exists$ n : number.
            is_nth_argument_of ($to2$, n, $to1$) Yes for is1 (n -> 1)!
          $\wedge$ Yes for ys (n -> 2)!
            $t2$ is_nth_induction_term n Yes for stk (n -> 3)!

https://github.com/data61/PSL/slide/2019_ps.pdf

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

<- pros: good at ambiguity (heuristics)

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

LiFtEr 1 LiFtEr 2 LiFtEr 3

LiFtEr 1 LiFtEr 2 LiFtEr 3

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

```
lemma "star r                    star                ⟹ star r x z"
by(induction r                   ar.in               uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                           )"
by(induct is1 s stk rule:e                           uct) auto
```

<- small dataset about different domains

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
lemma "itrev [1,2,3]       [] = rev [1,2,3]       @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]       [] = rev [x,y,z]       @ []" by auto
```

<- many concrete cases

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WIP

<- pros: good at rigor

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<- many concrete cases

https://twitter.com/YutakangJ https://github.com/data61/PSL/slide/2019_ps.pdf

Registration is now closed.

http://aitp-conference.org/2019/

Background

Large-scale semantic processing and strong computer assistance of mathematics and science is our inevitable future. New combinations of AI and reasoning methods and tools deployed over large mathematical and scientific corpora will be instrumental to this task. The AITP conference is the forum for discussing how to get there as soon as possible, and the force driving the progress towards that.

Topics

- AI and big-data methods in theorem proving and mathematics
- Collaboration between automated and interactive theorem proving
- Common-sense reasoning and reasoning in science
- Alignment and joint processing of formal, semi-formal, and informal libraries
- Methods for large-scale computer understanding of mathematics and science
- Combinations of linguistic/learning-based and semantic/reasoning methods