Automating proof by induction in Isabelle/HOL using DSLs

Yutaka Nagashima

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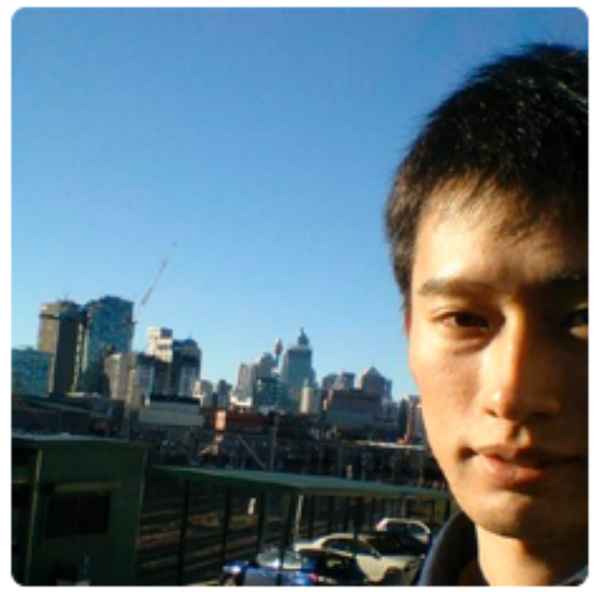
CZECH INSTITUTE OF INFORMATICS ROBOTICS AND CYBERNETICS CTU IN PRAGUE



Yutaka Ng

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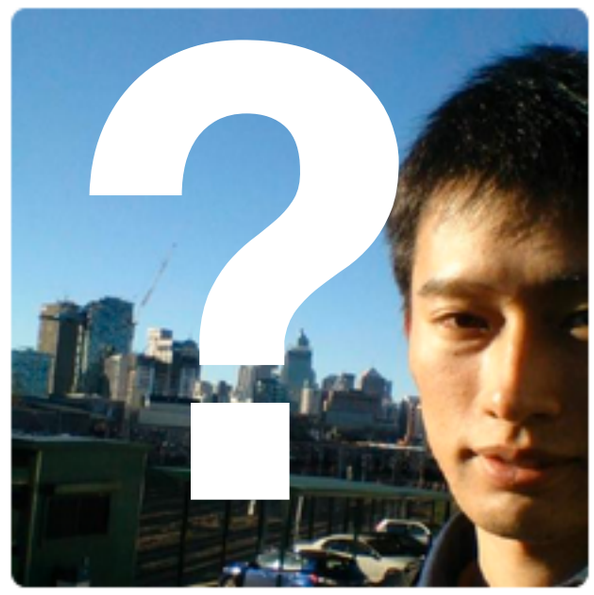

2013 ~ 2017

with Prof. Gerwin Klein

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Isabelle HOL

(seL4)

Security. Performance. Proof.

http://www.cse.unsw.edu.au/~kleing/

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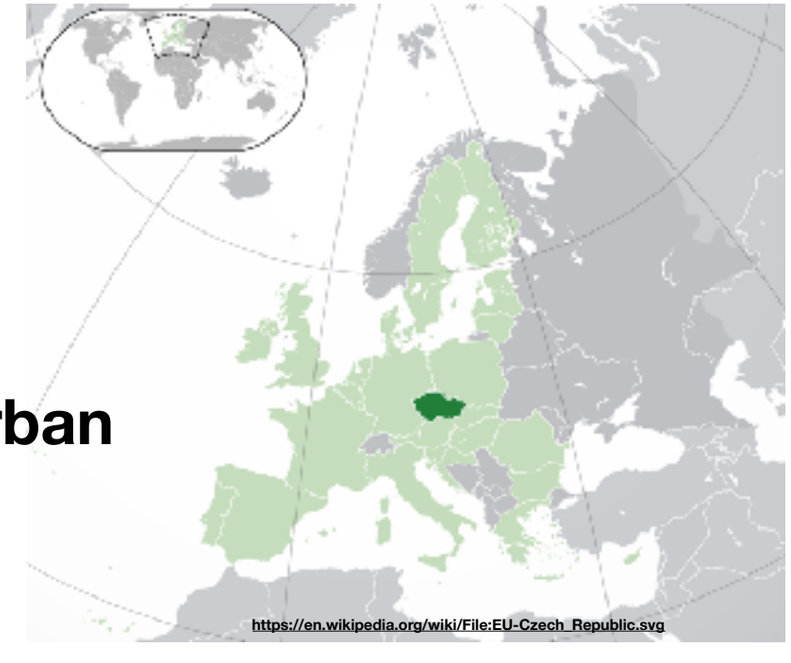
2018 ~ 2020

with Dr. Josef Urban



https://en.wikipedia.org/wiki/File:EU-Czech_Republic.svg

Isabelle HOL

http://ai4reason.org/members.html

PSL: Proof Strategy Language

PSL: Proof Strategy Lang

DEMO!

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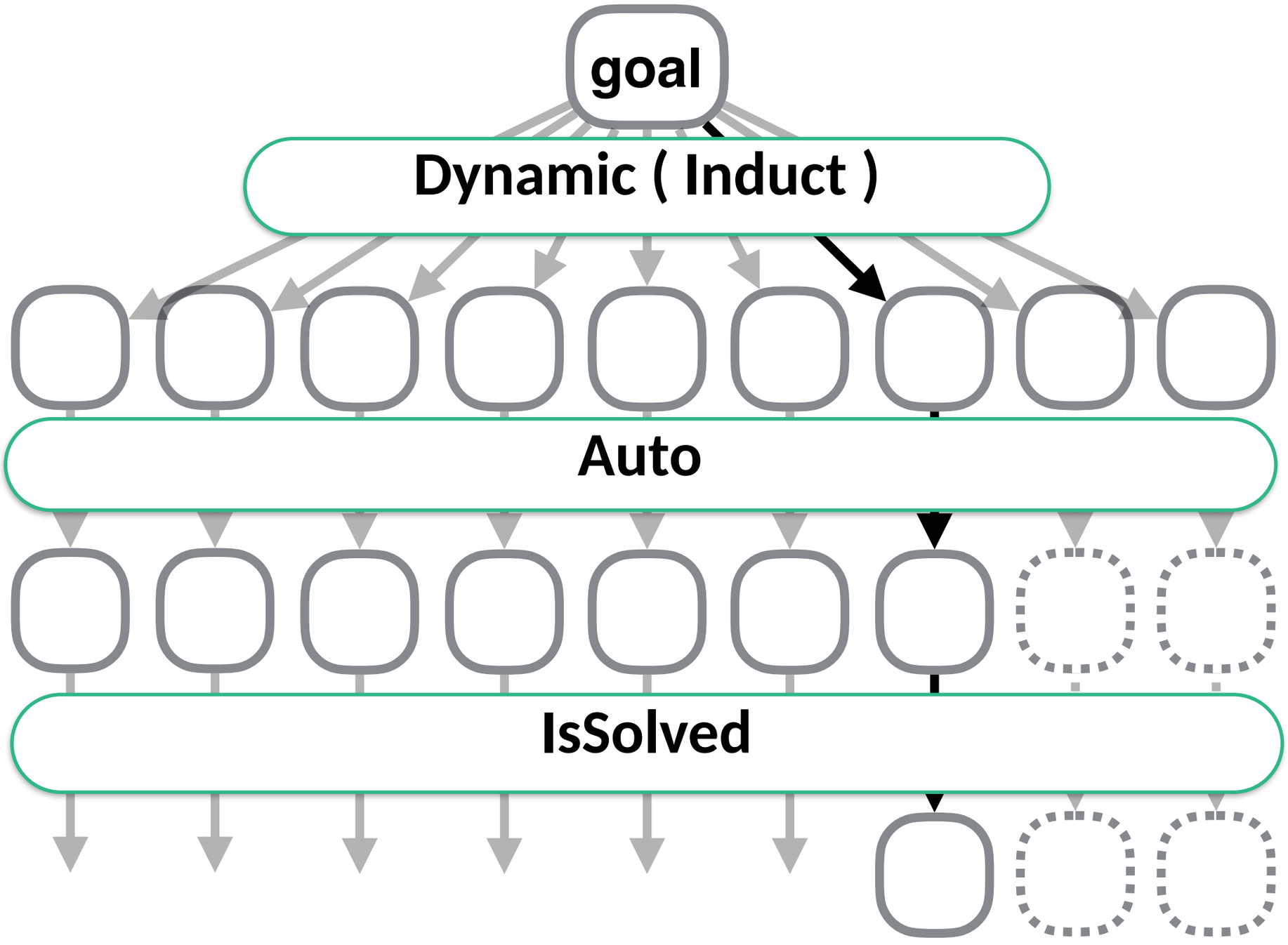

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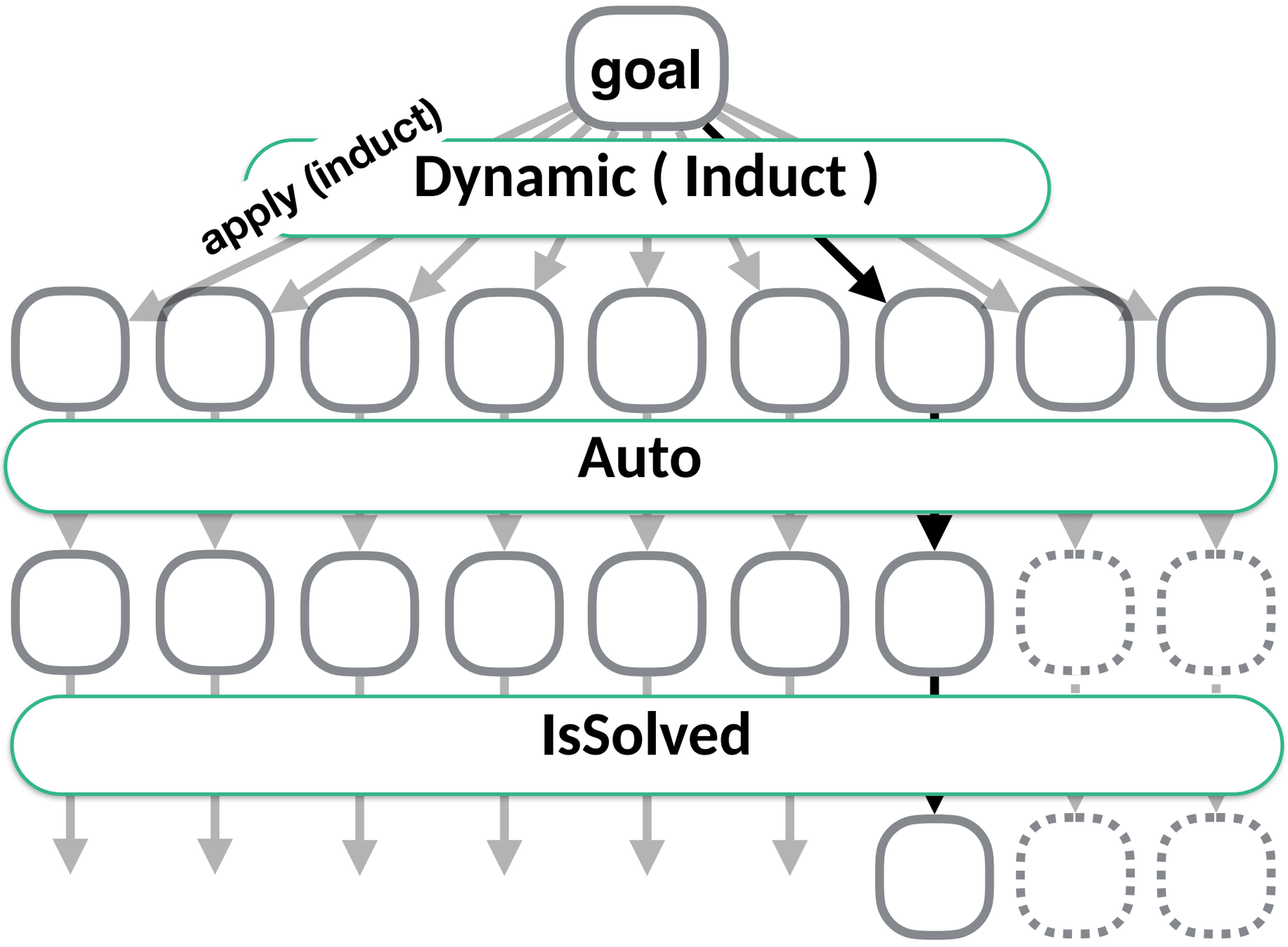

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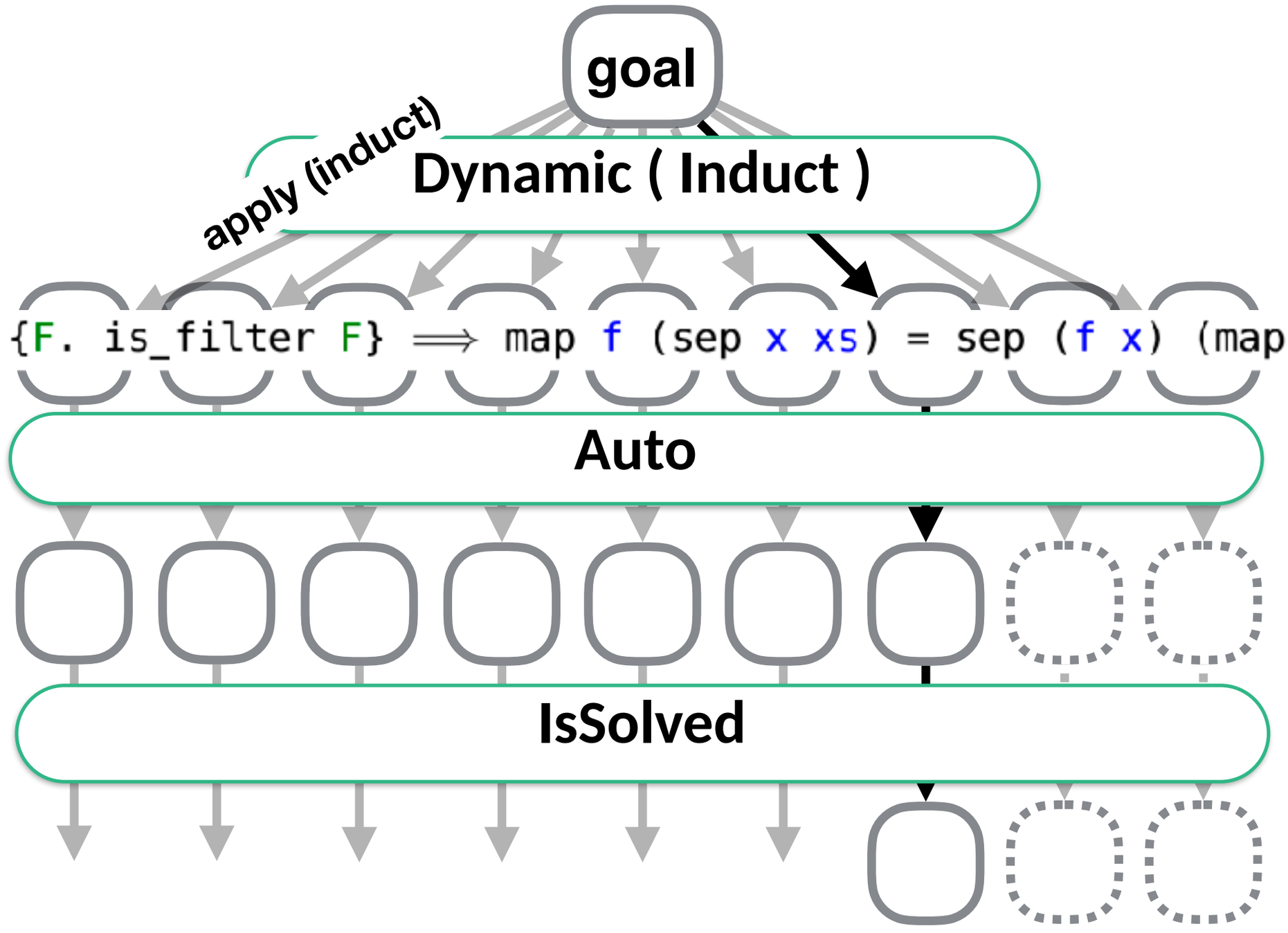


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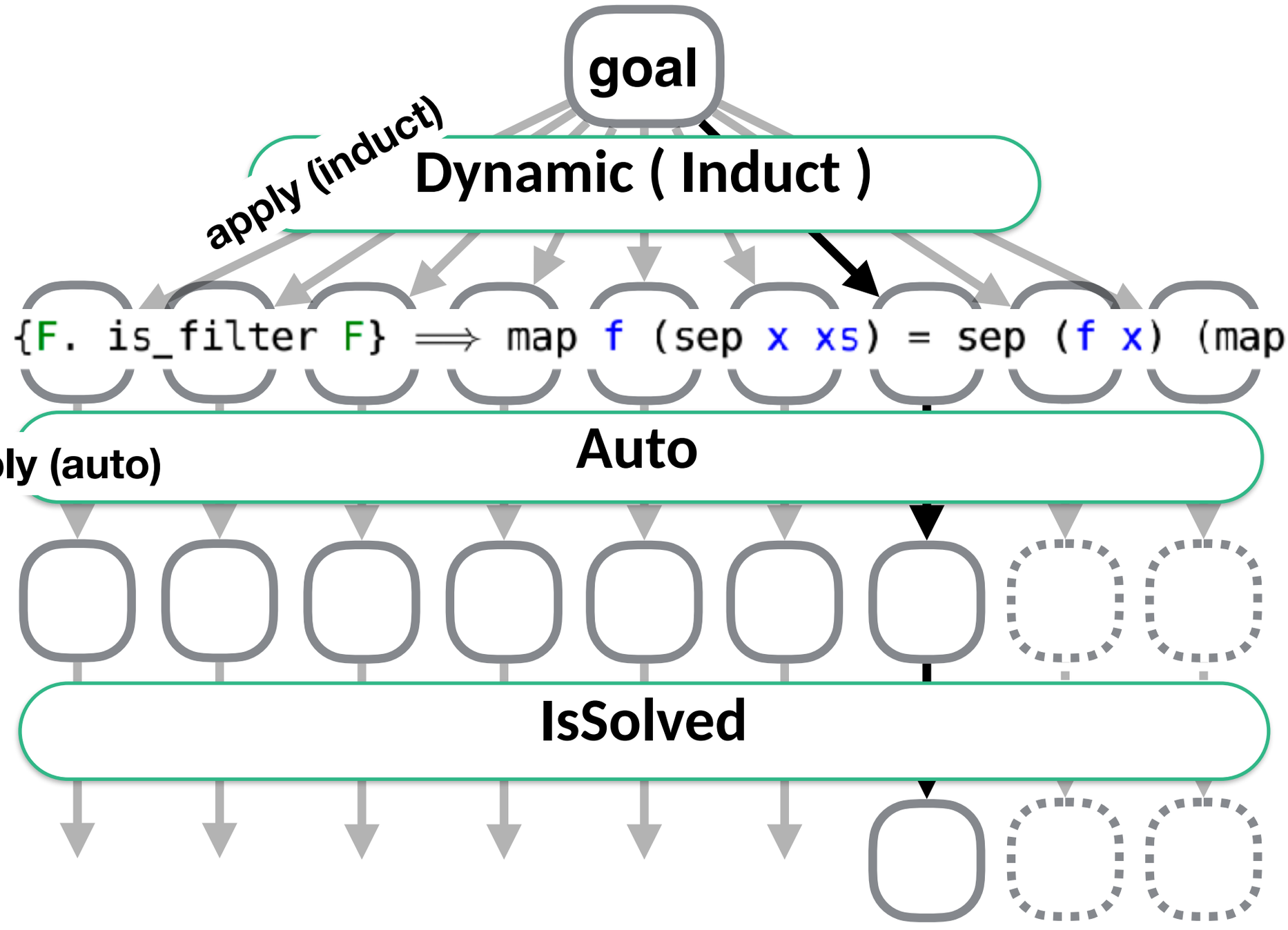


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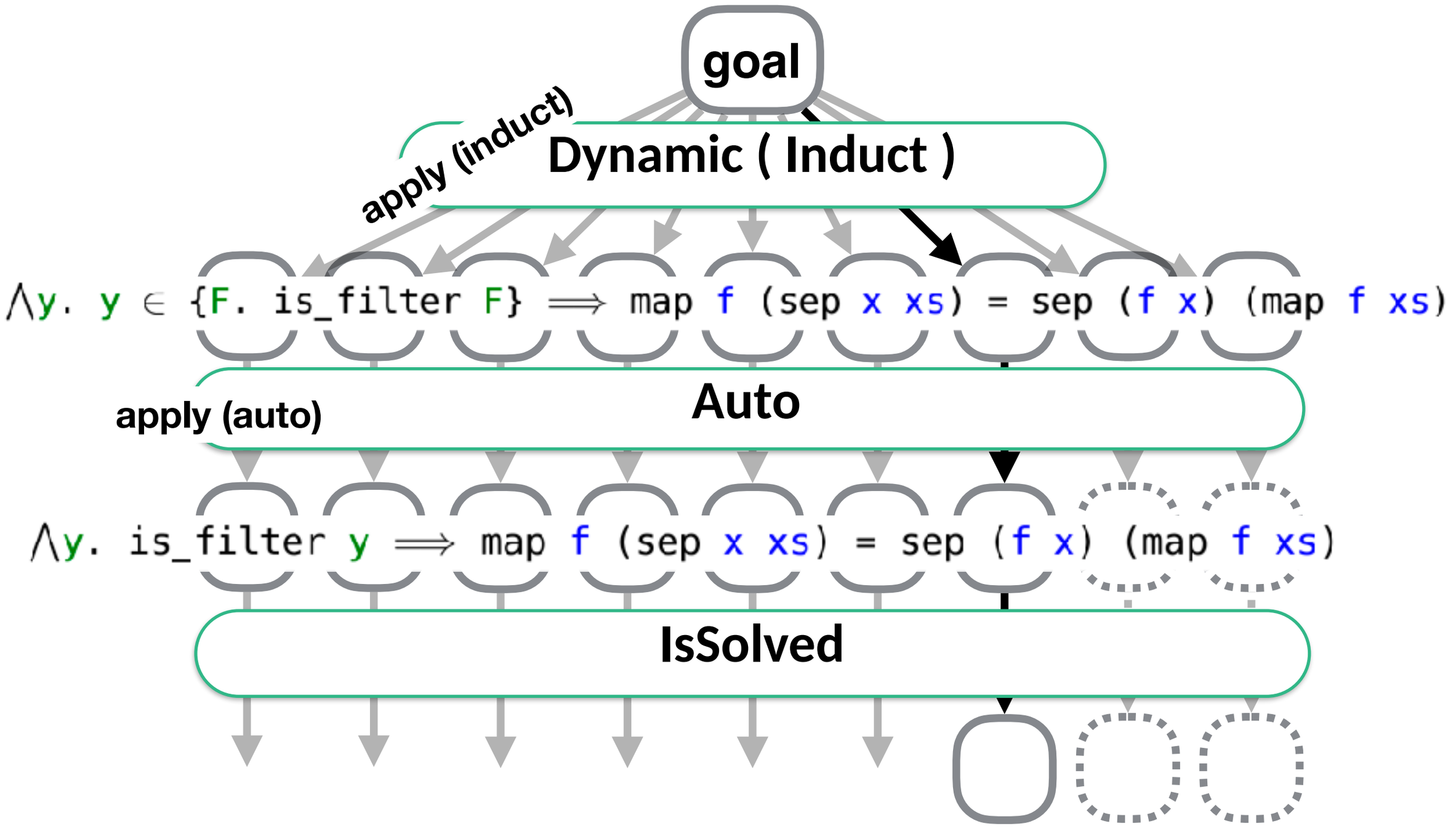

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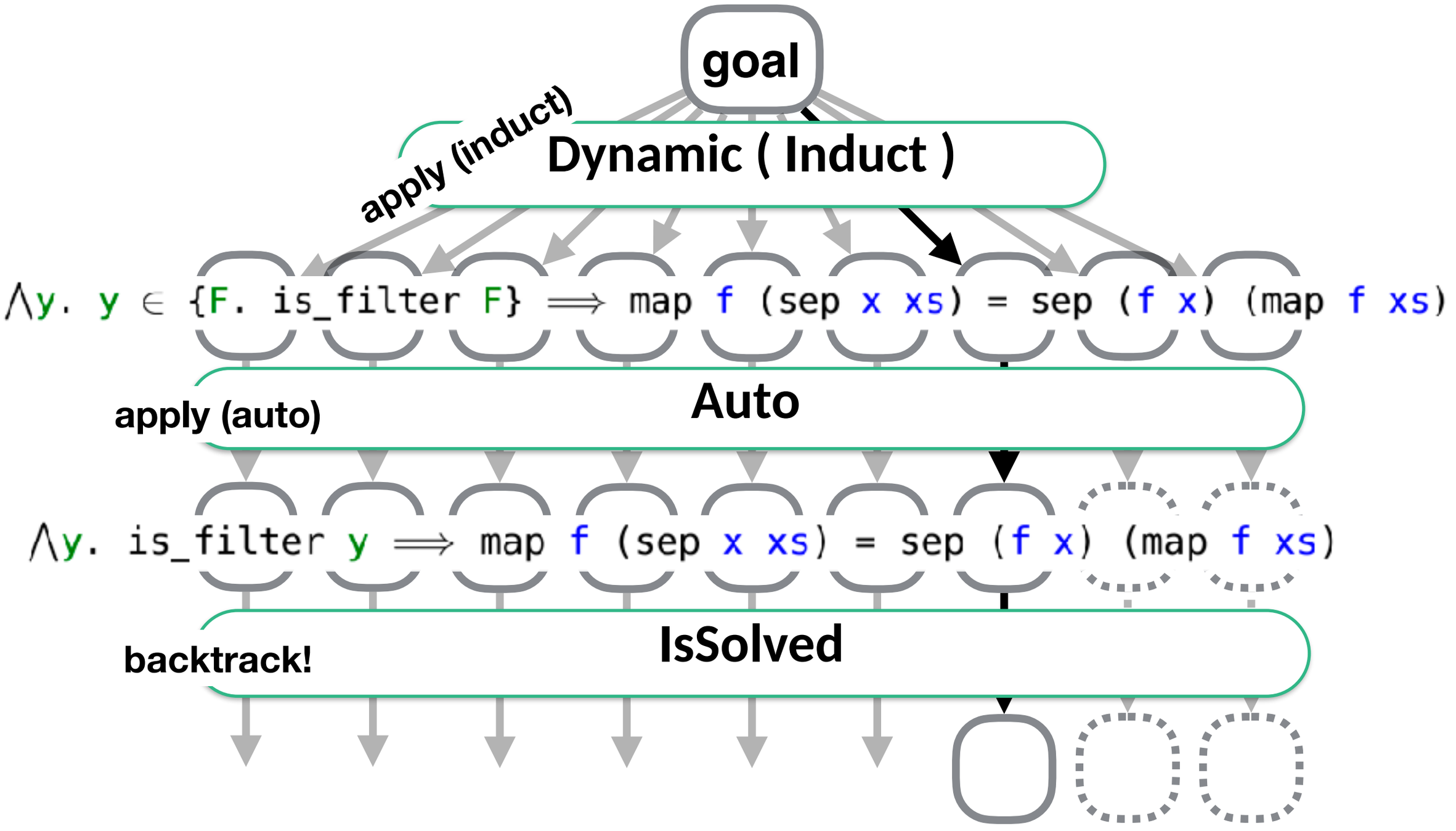

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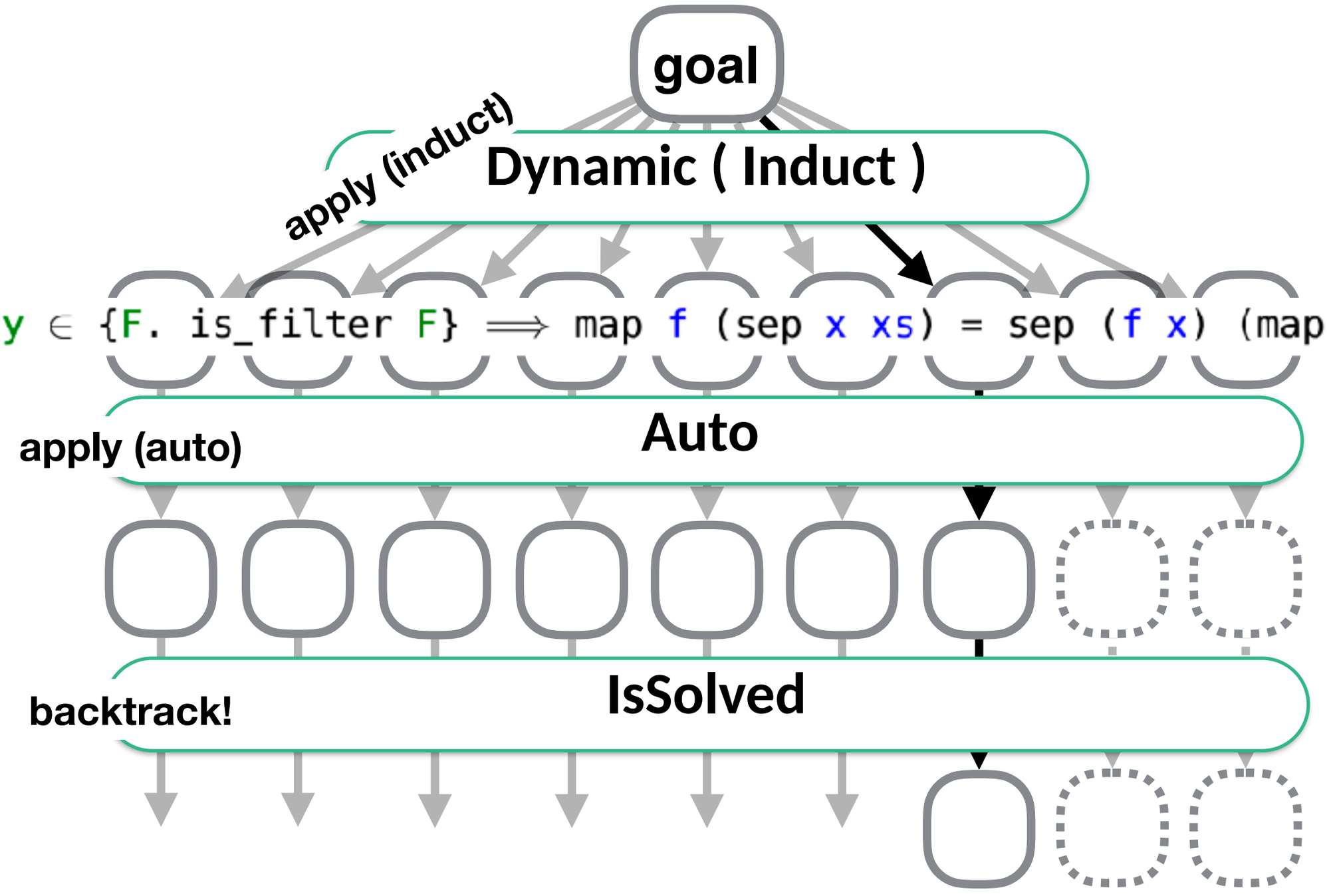

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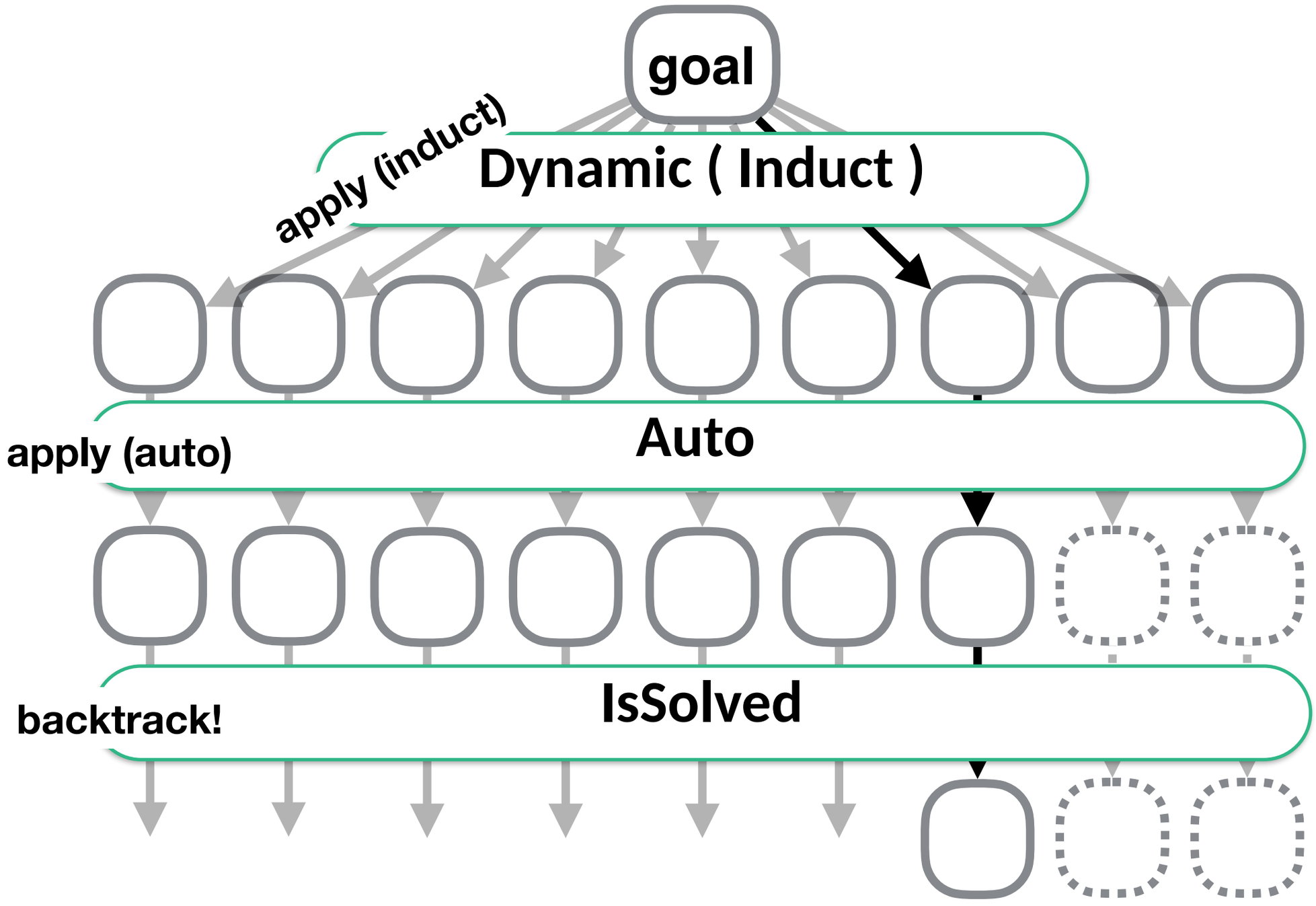

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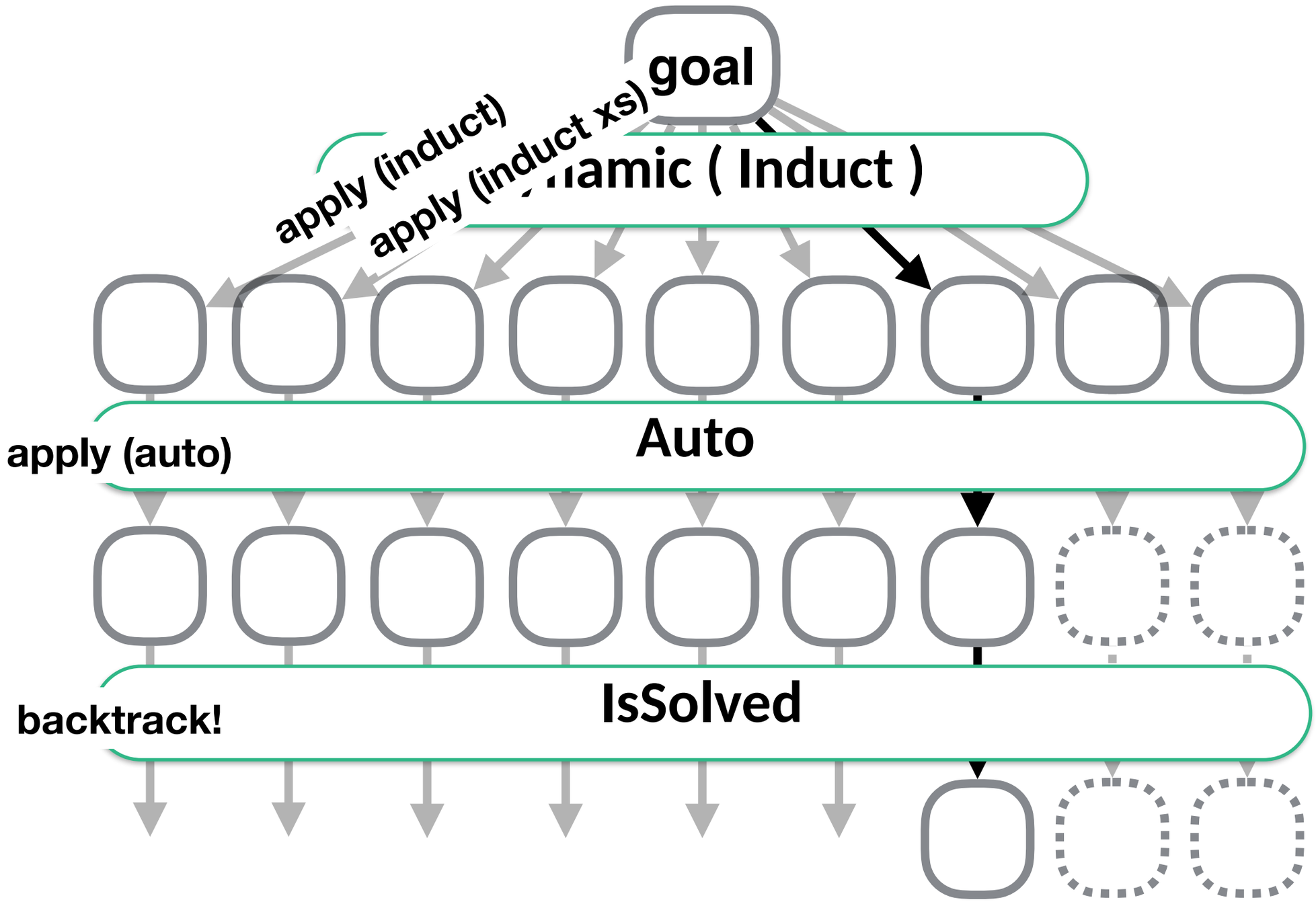

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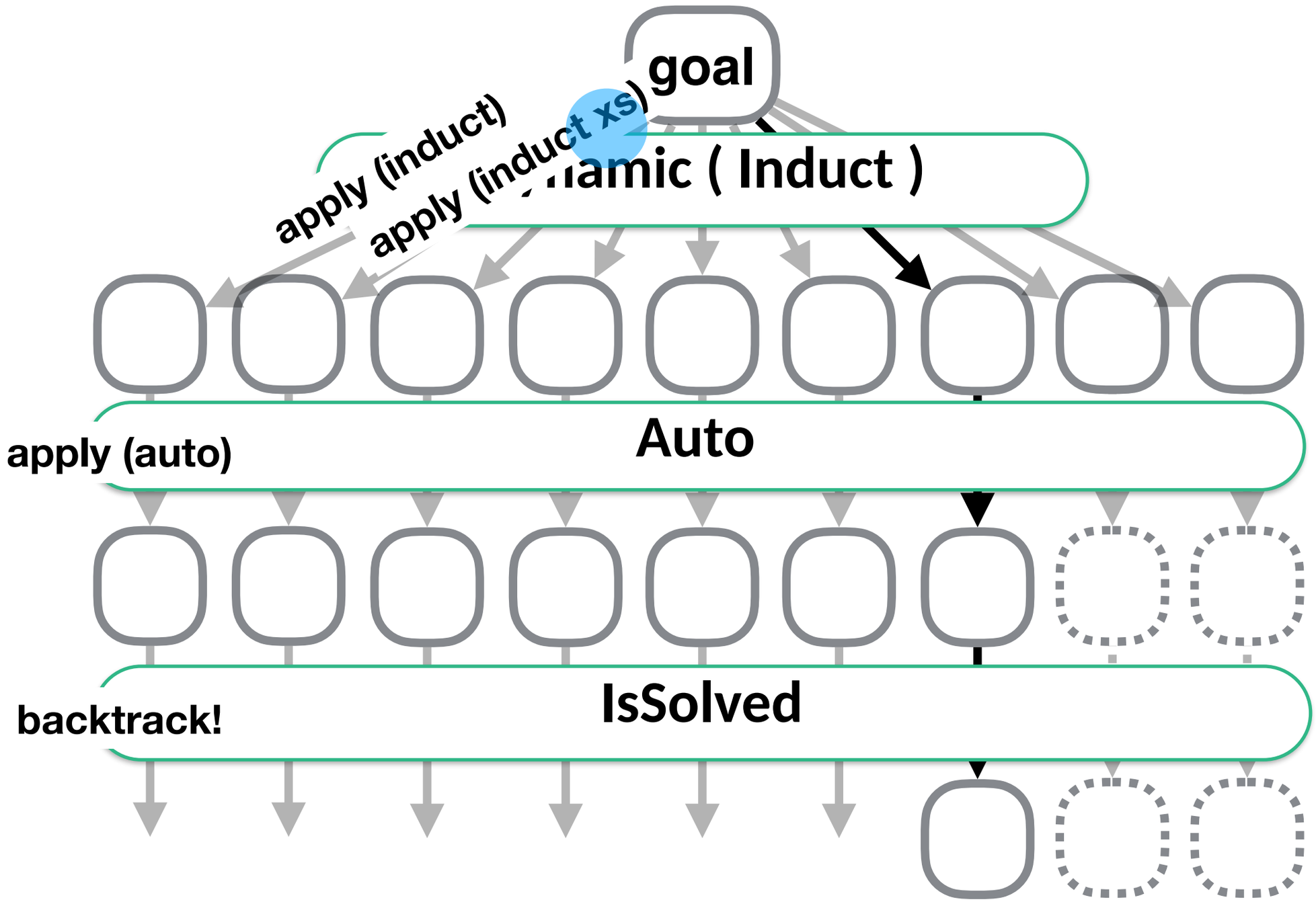

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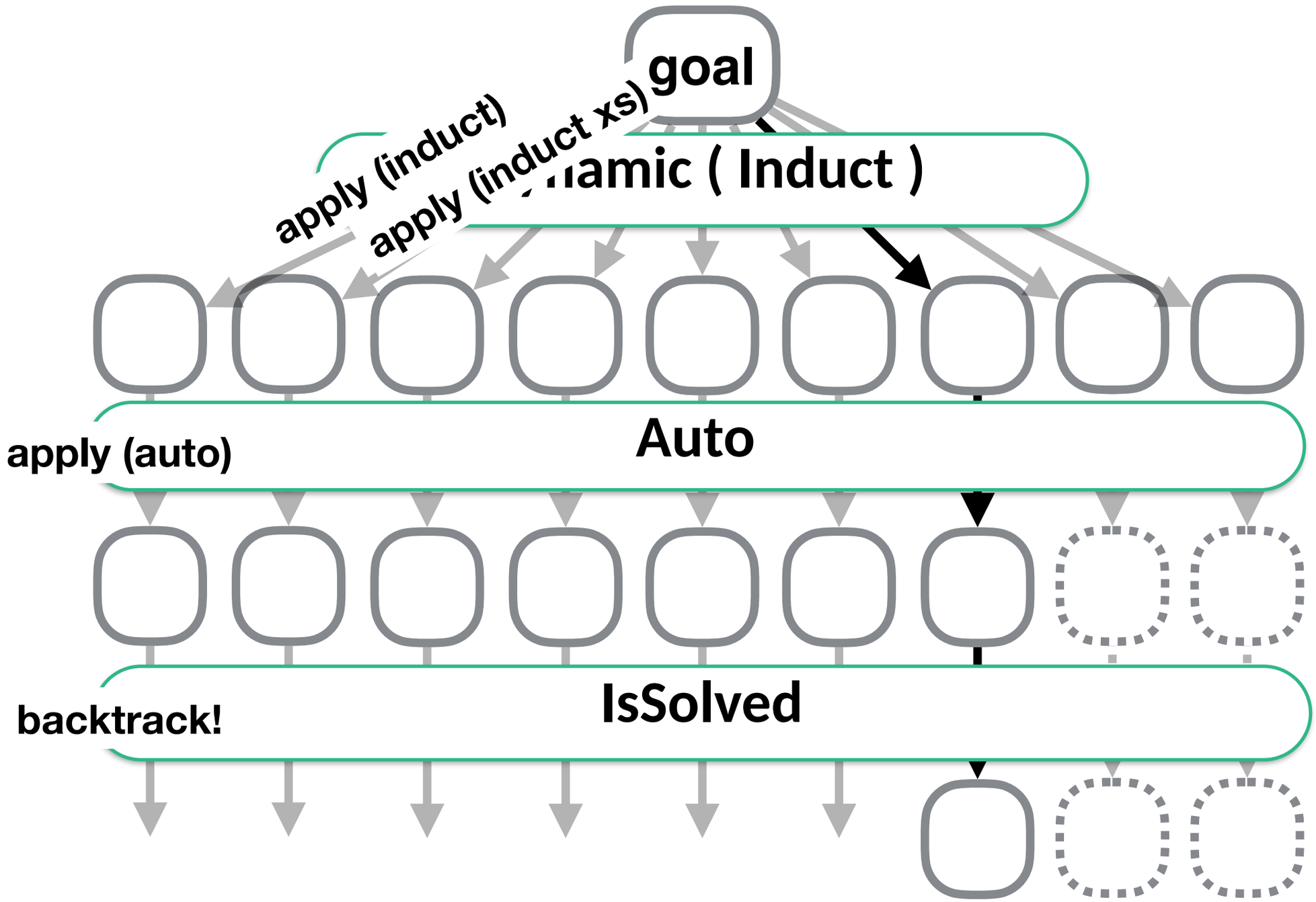

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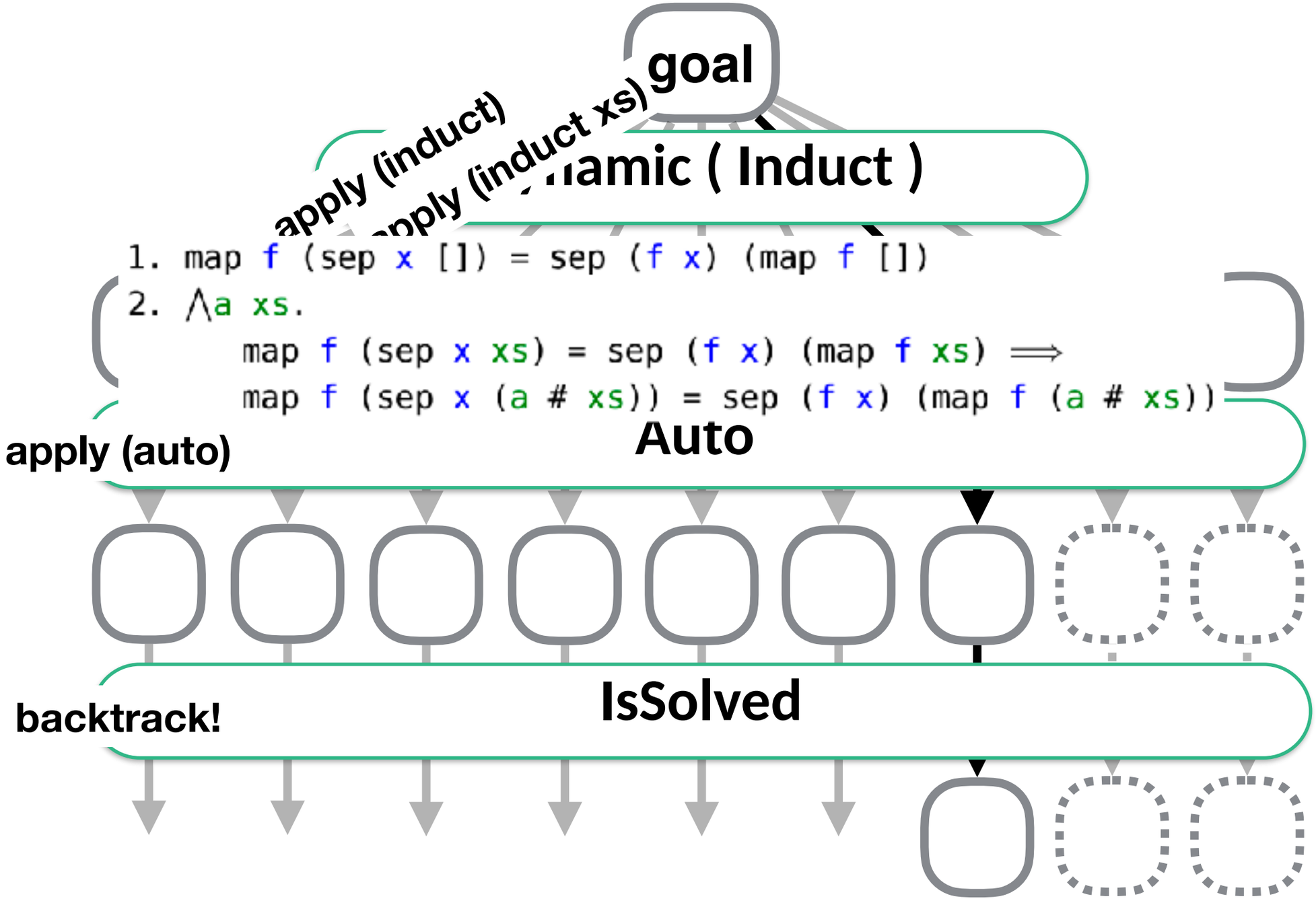

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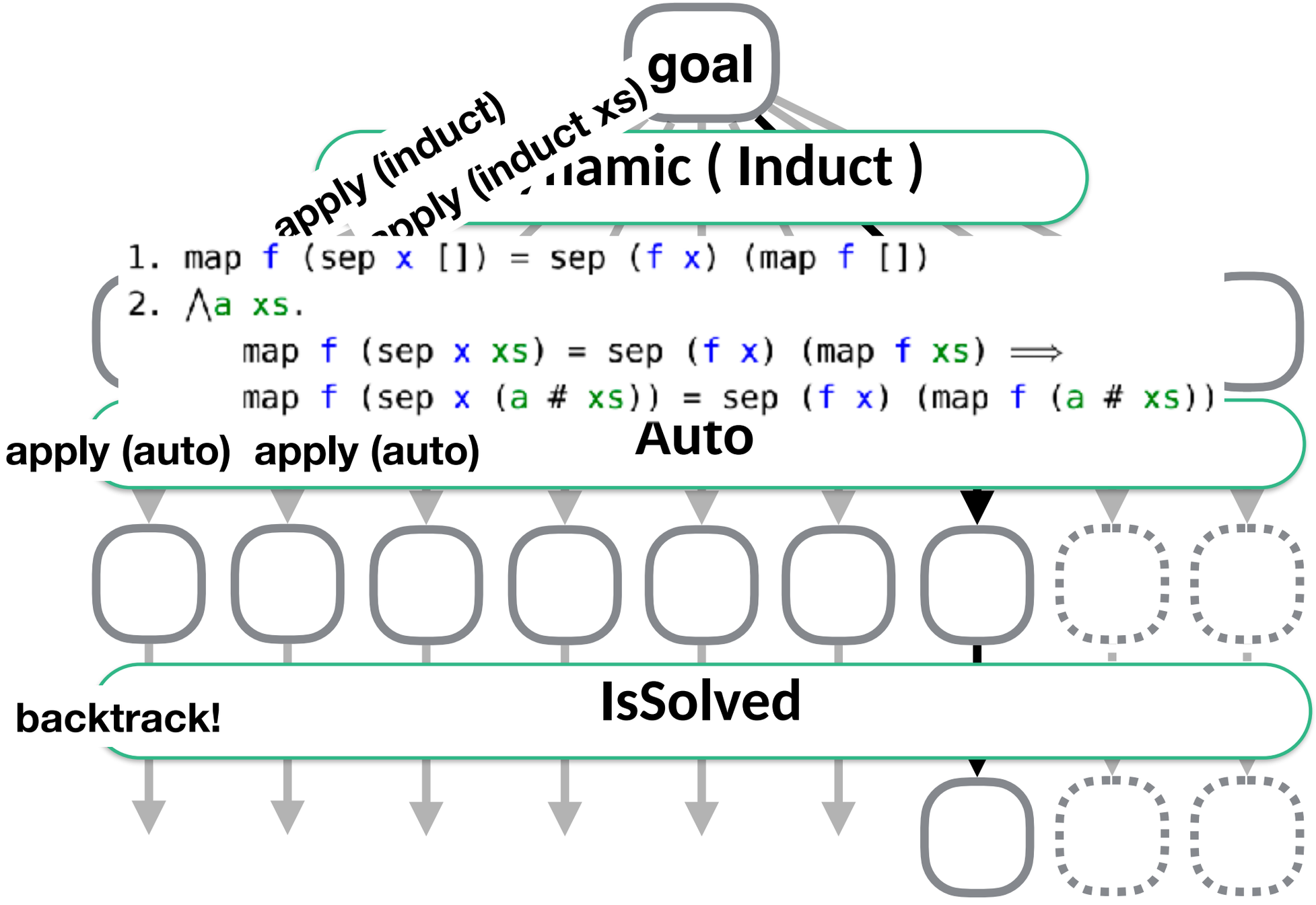

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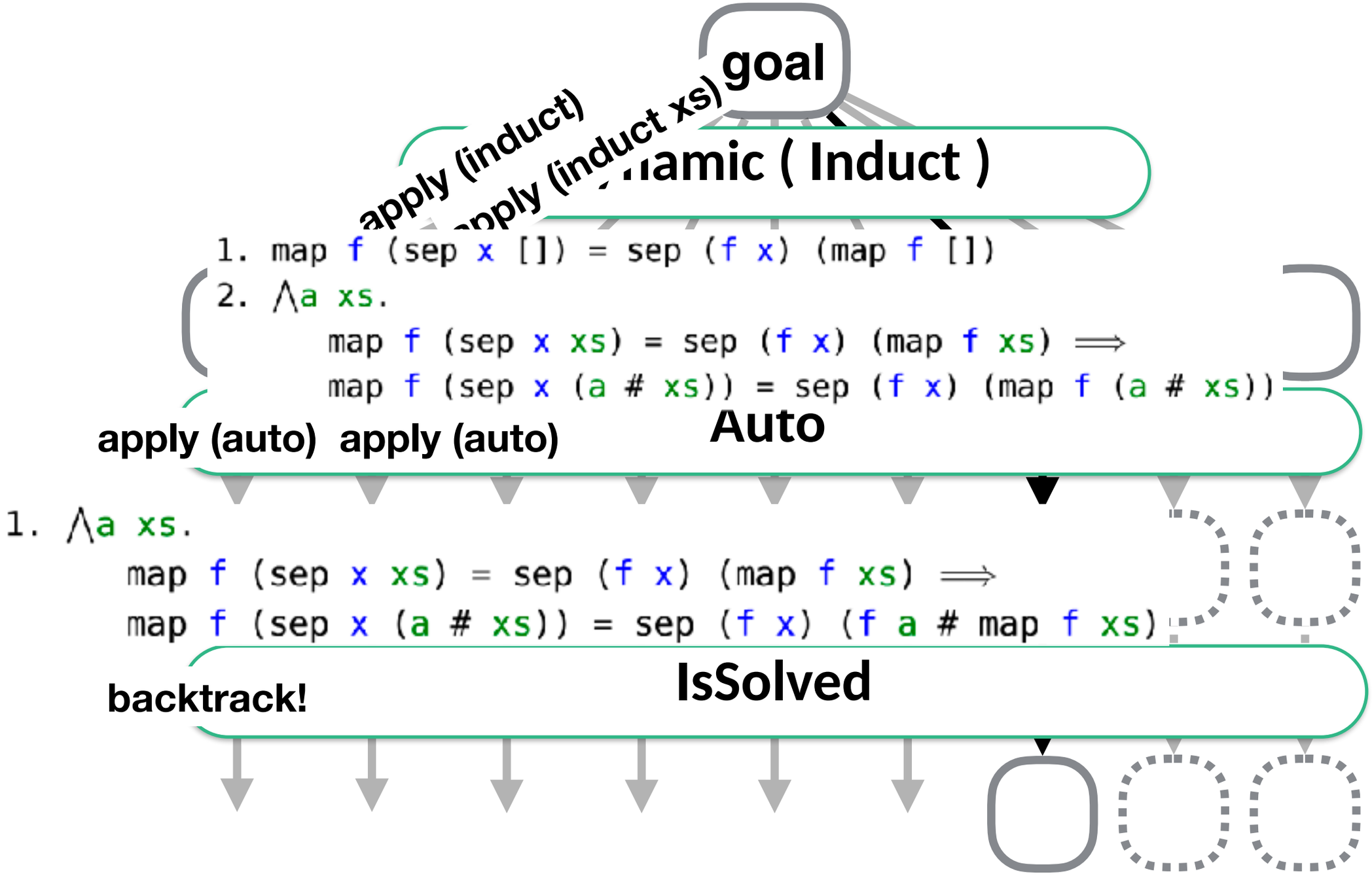

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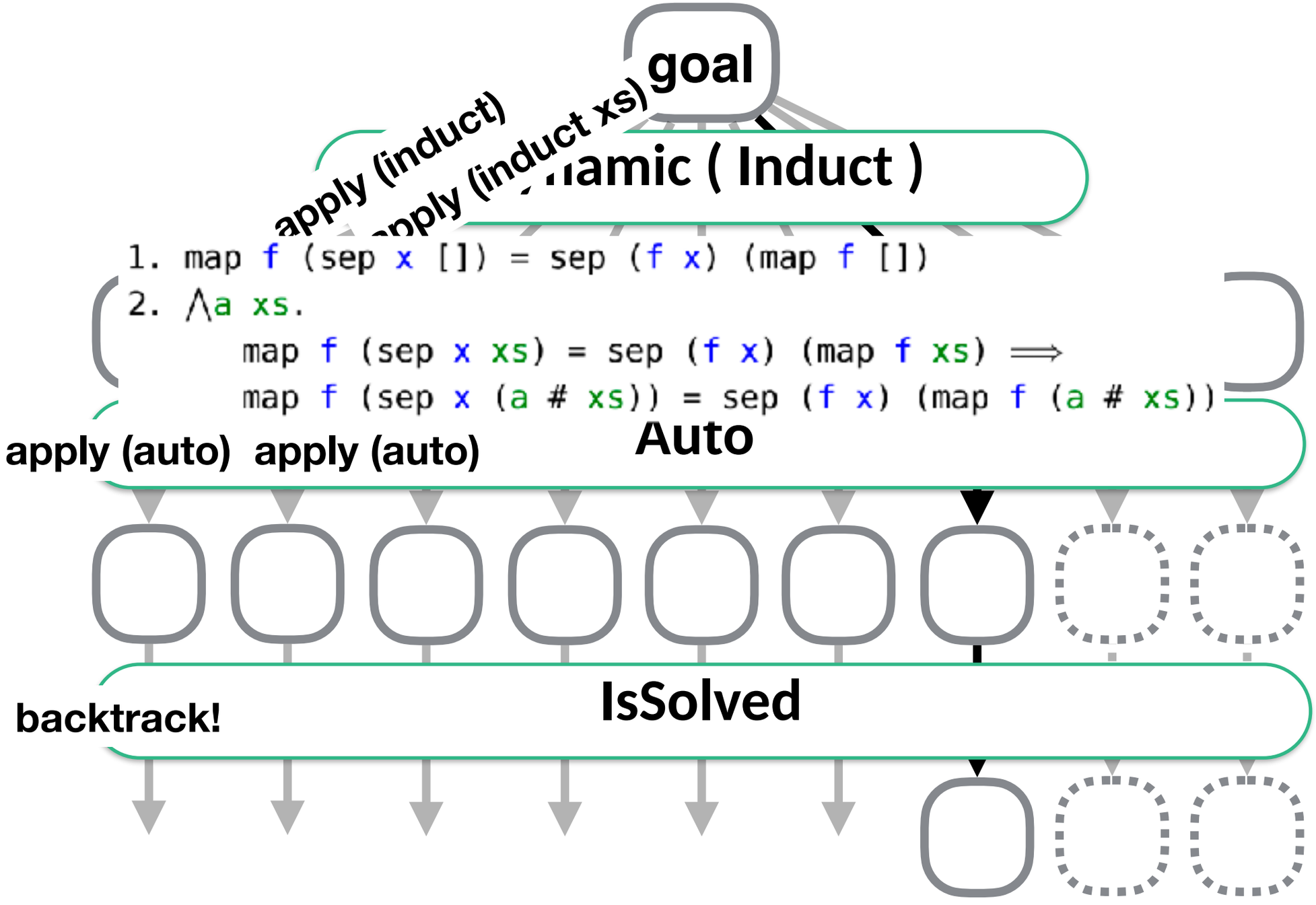

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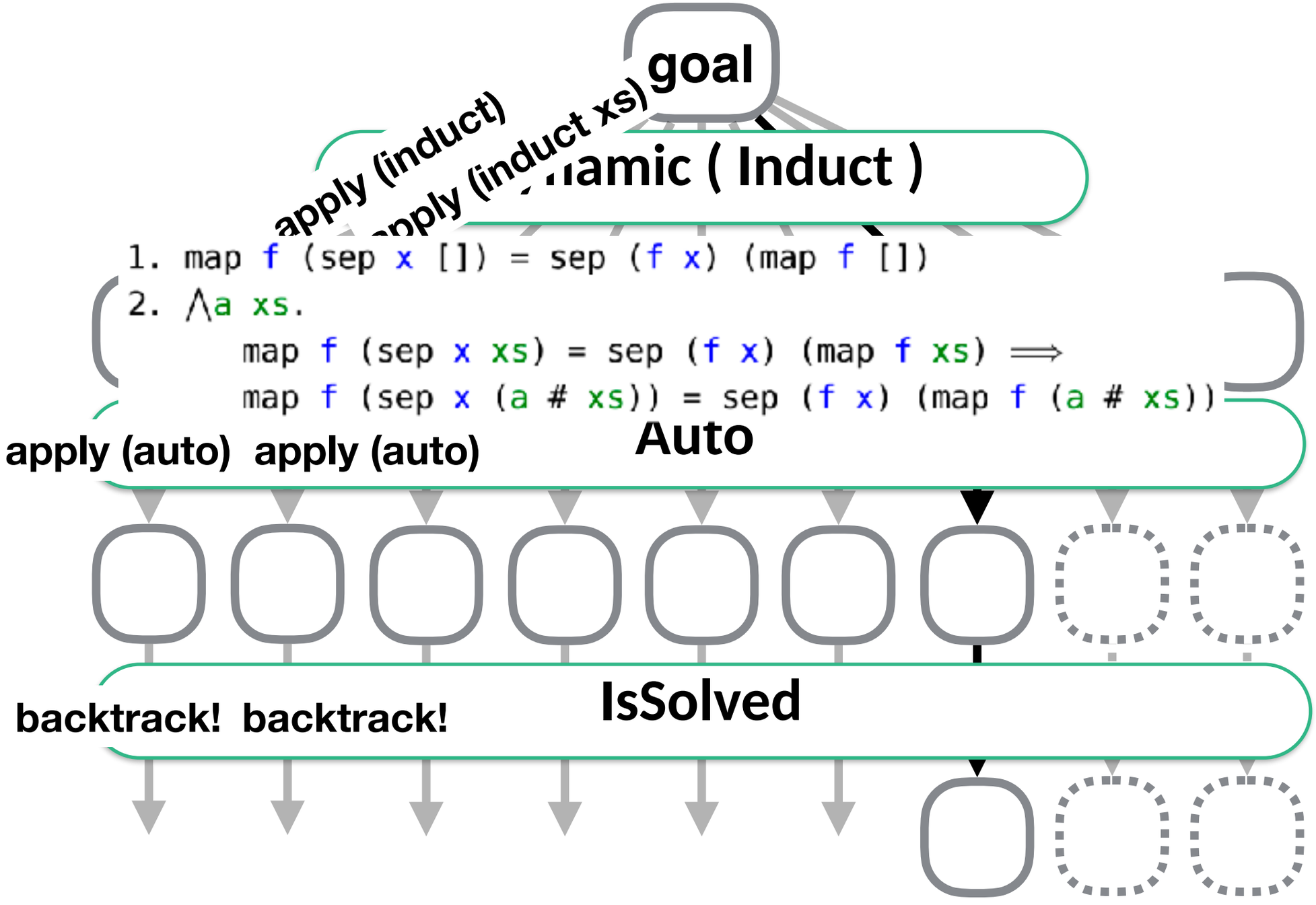

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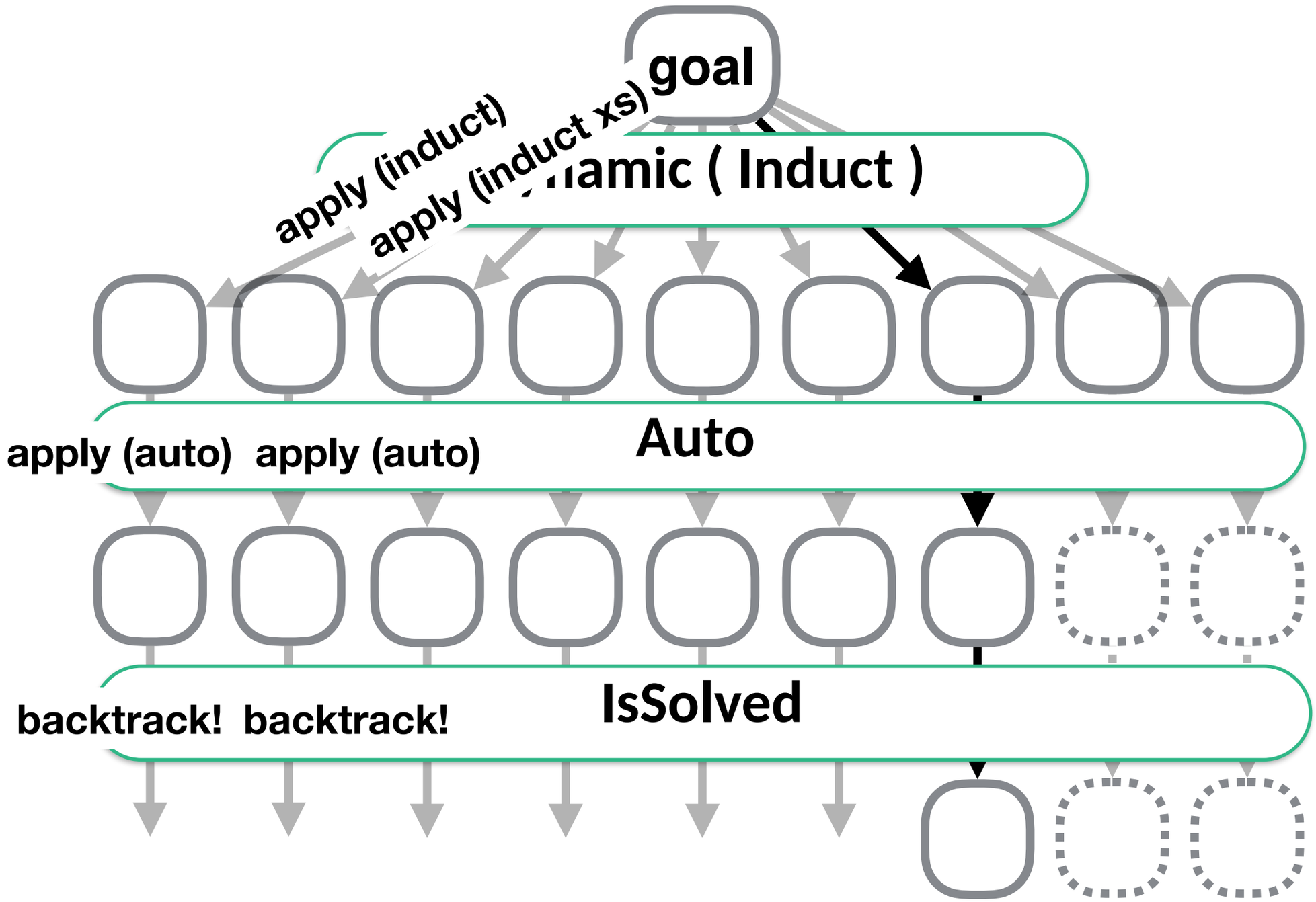

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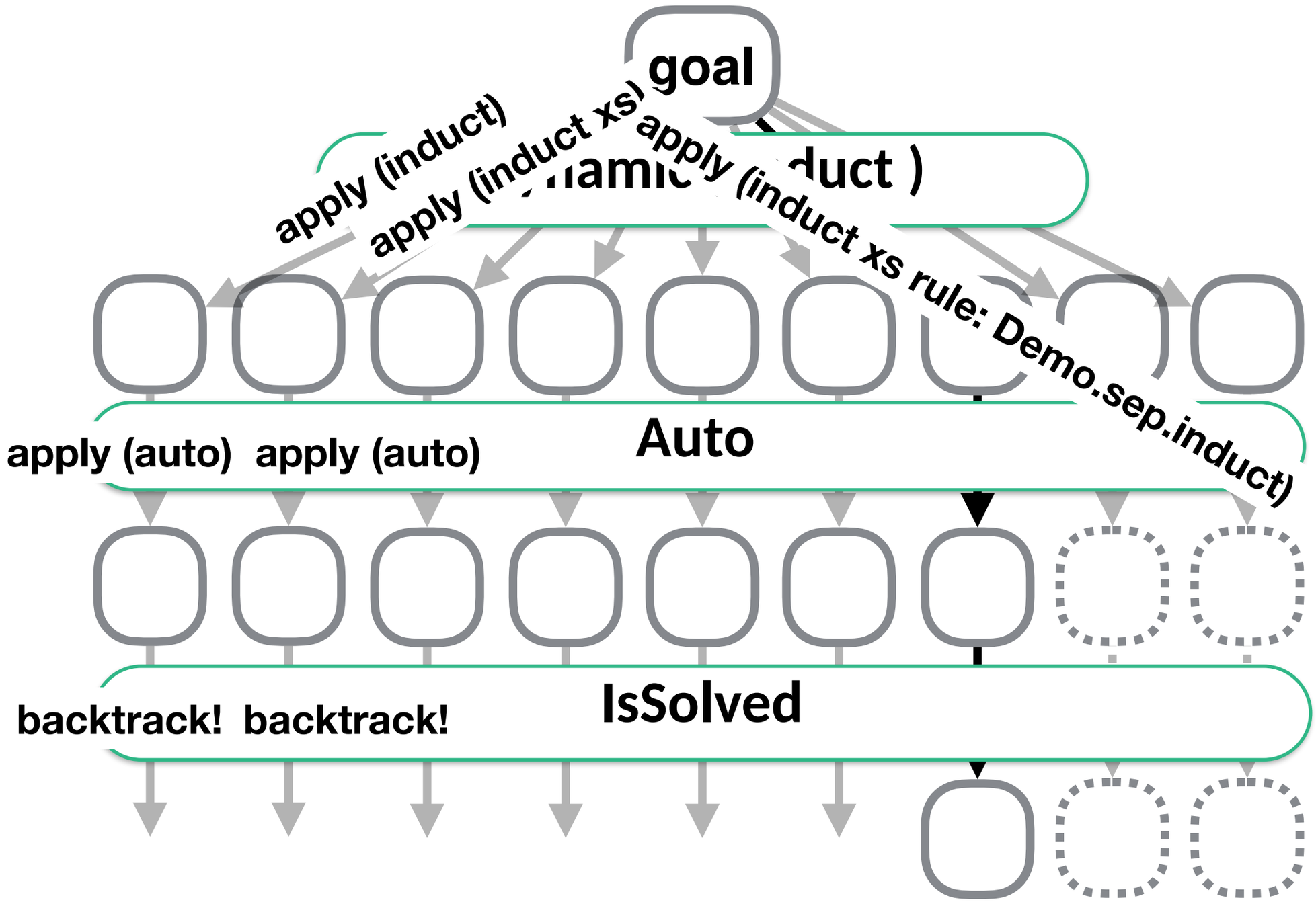

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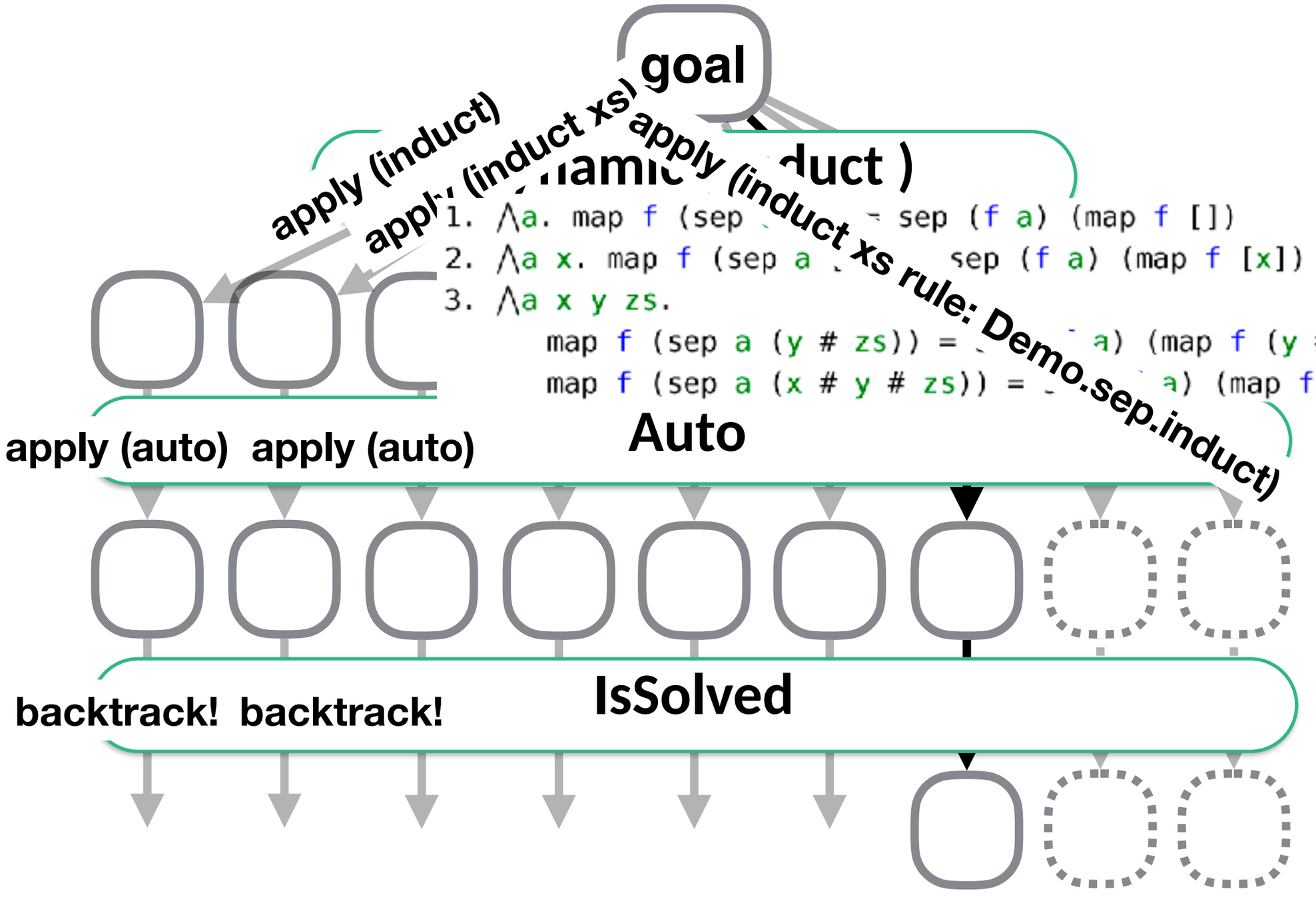

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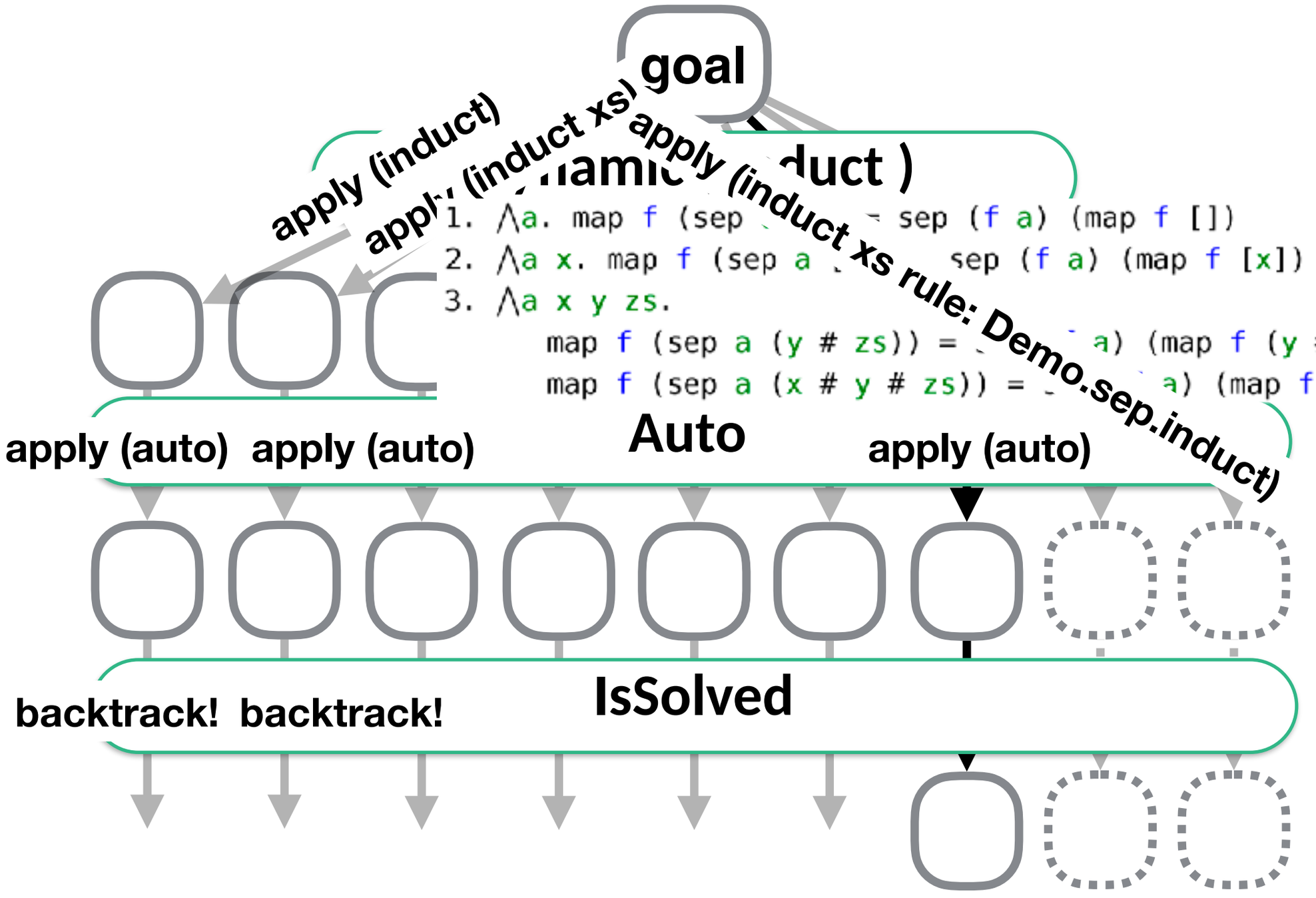

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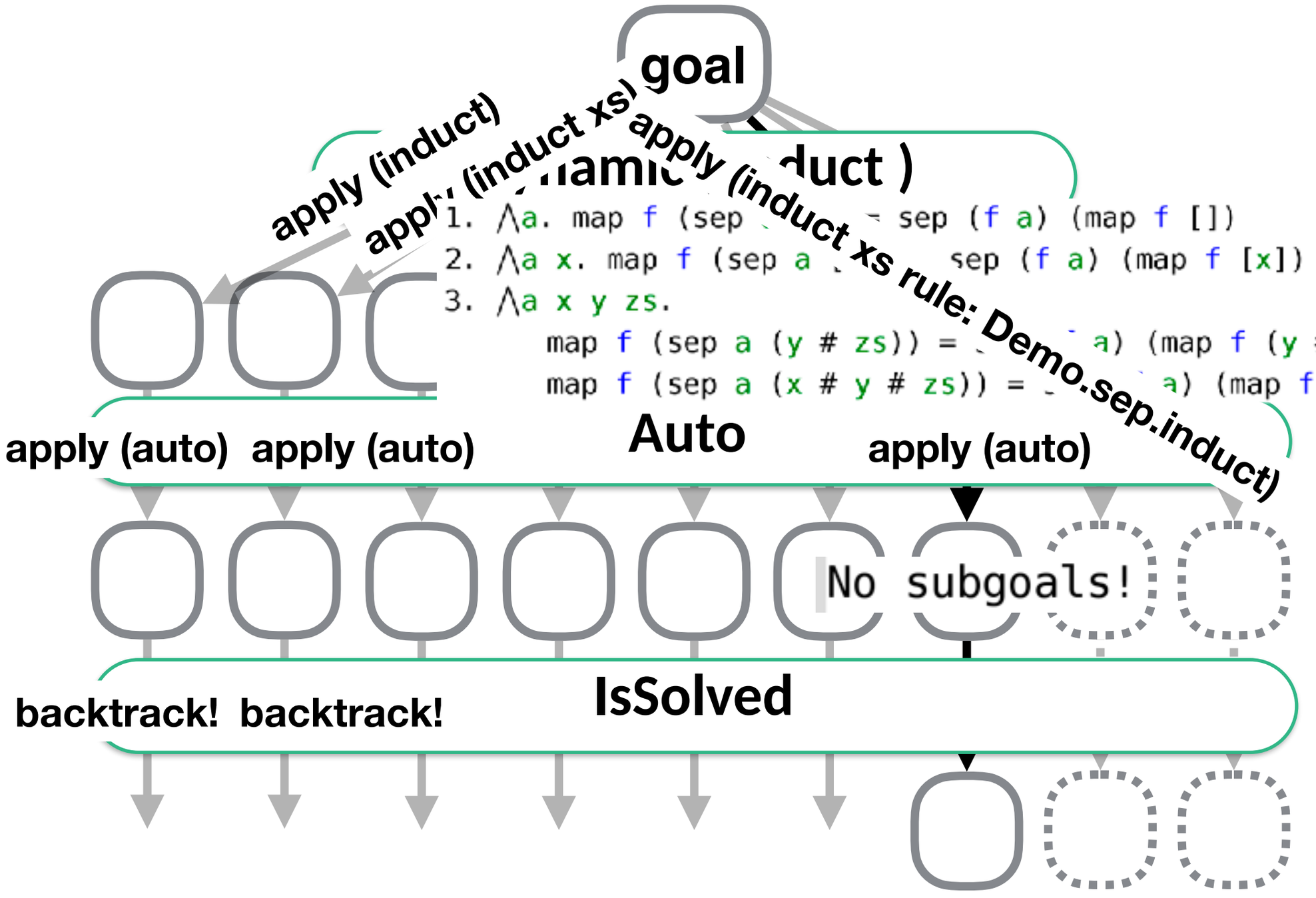

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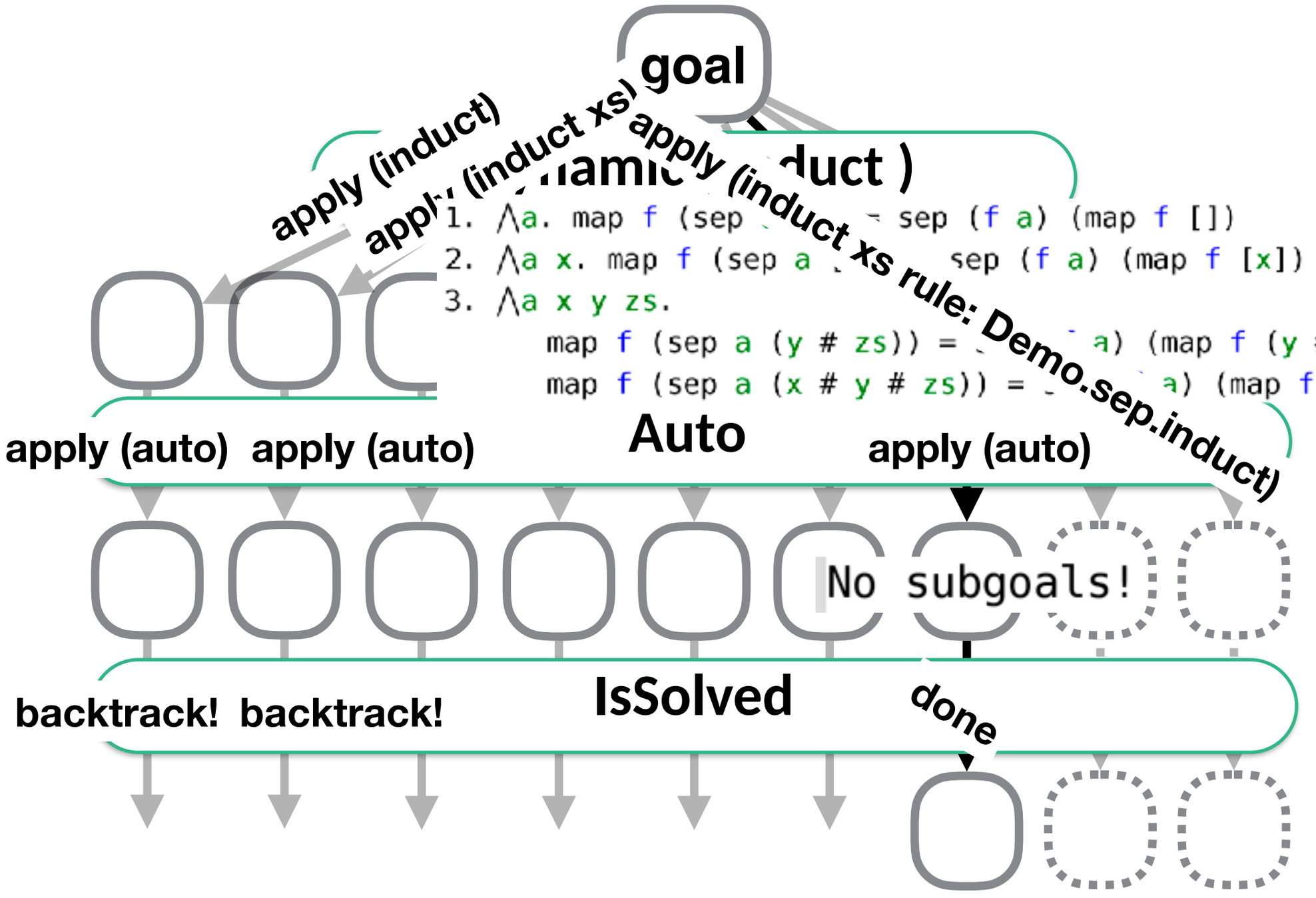

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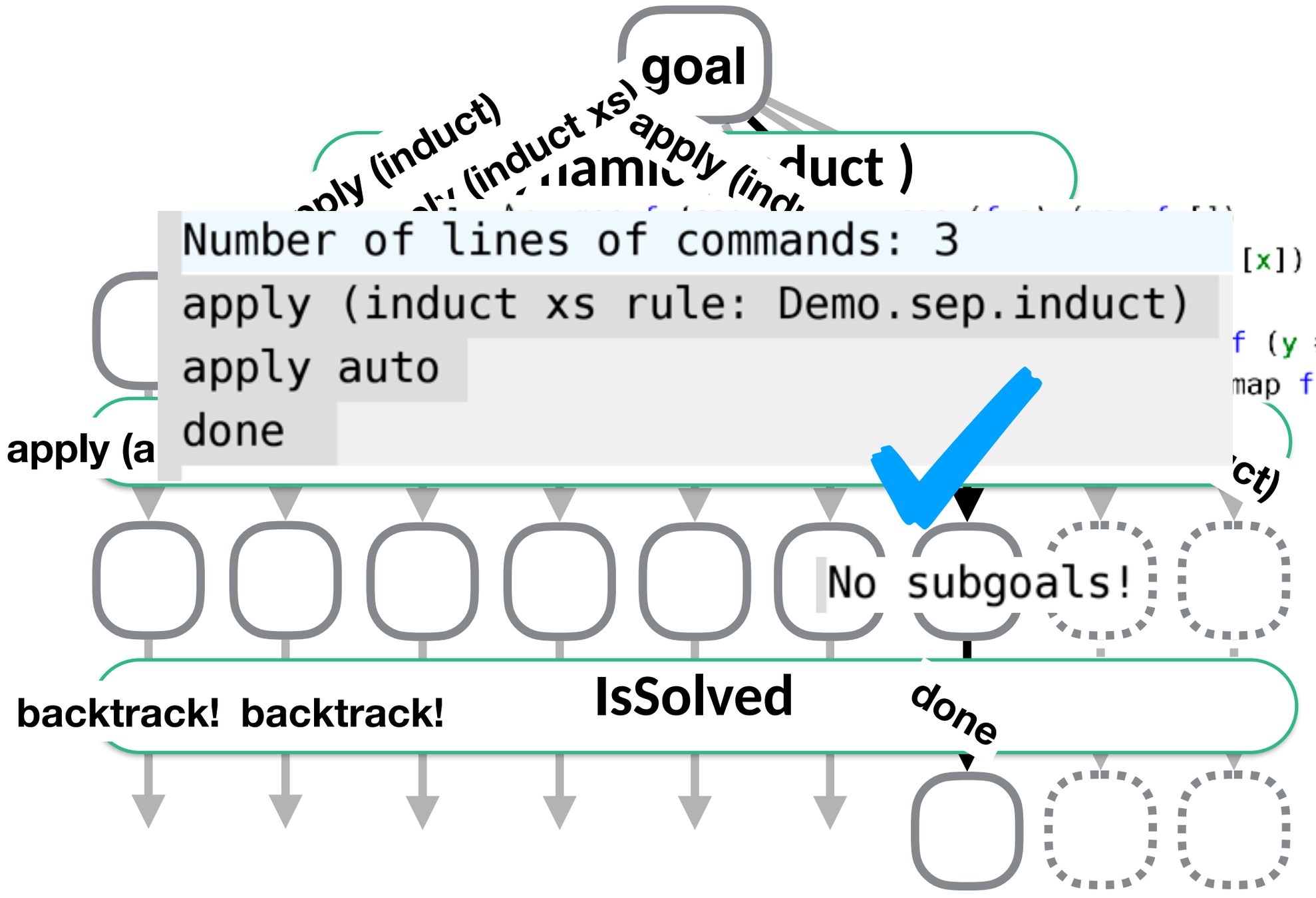

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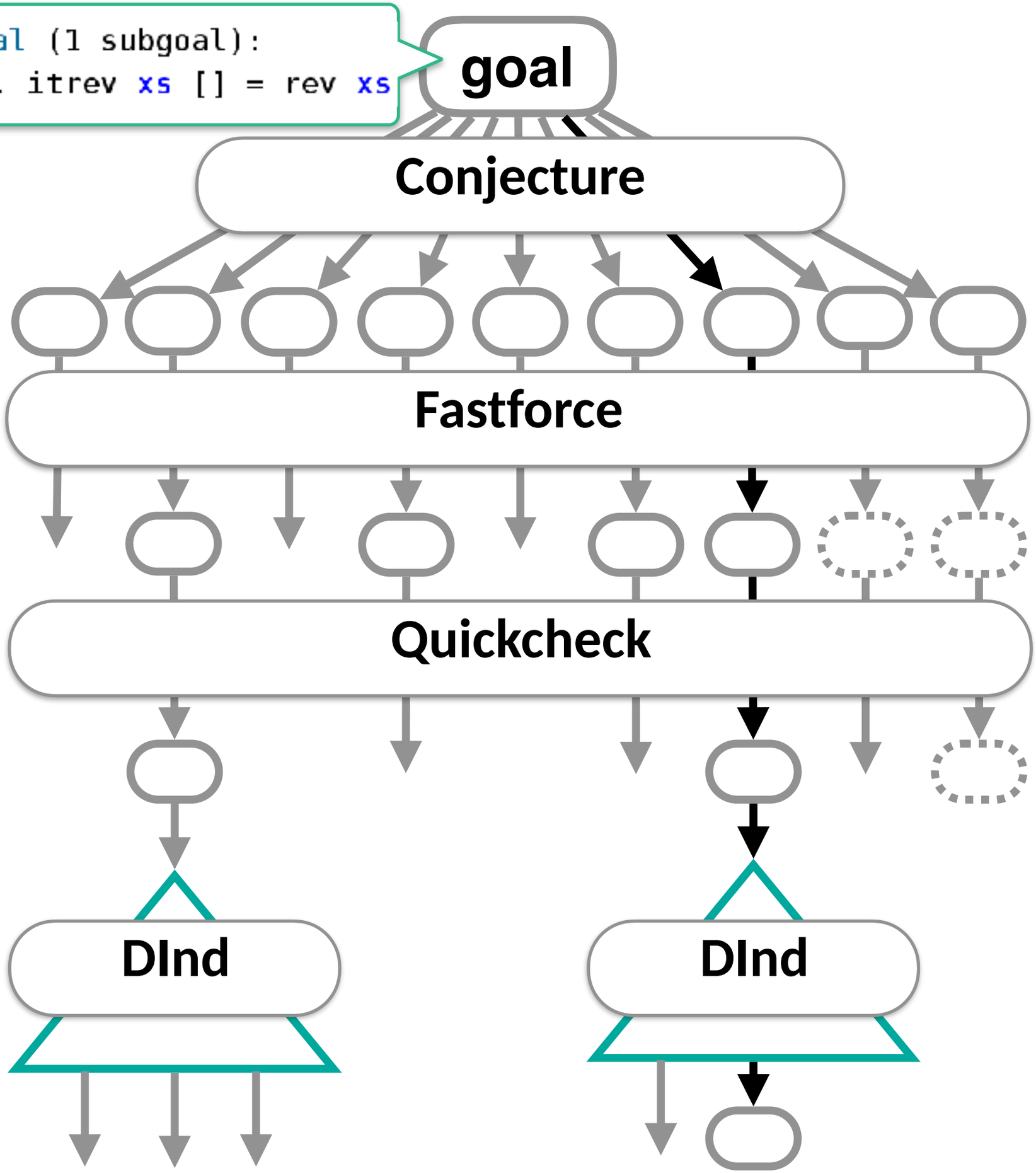

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Conjecture

Fastforce

Quickcheck

DInd

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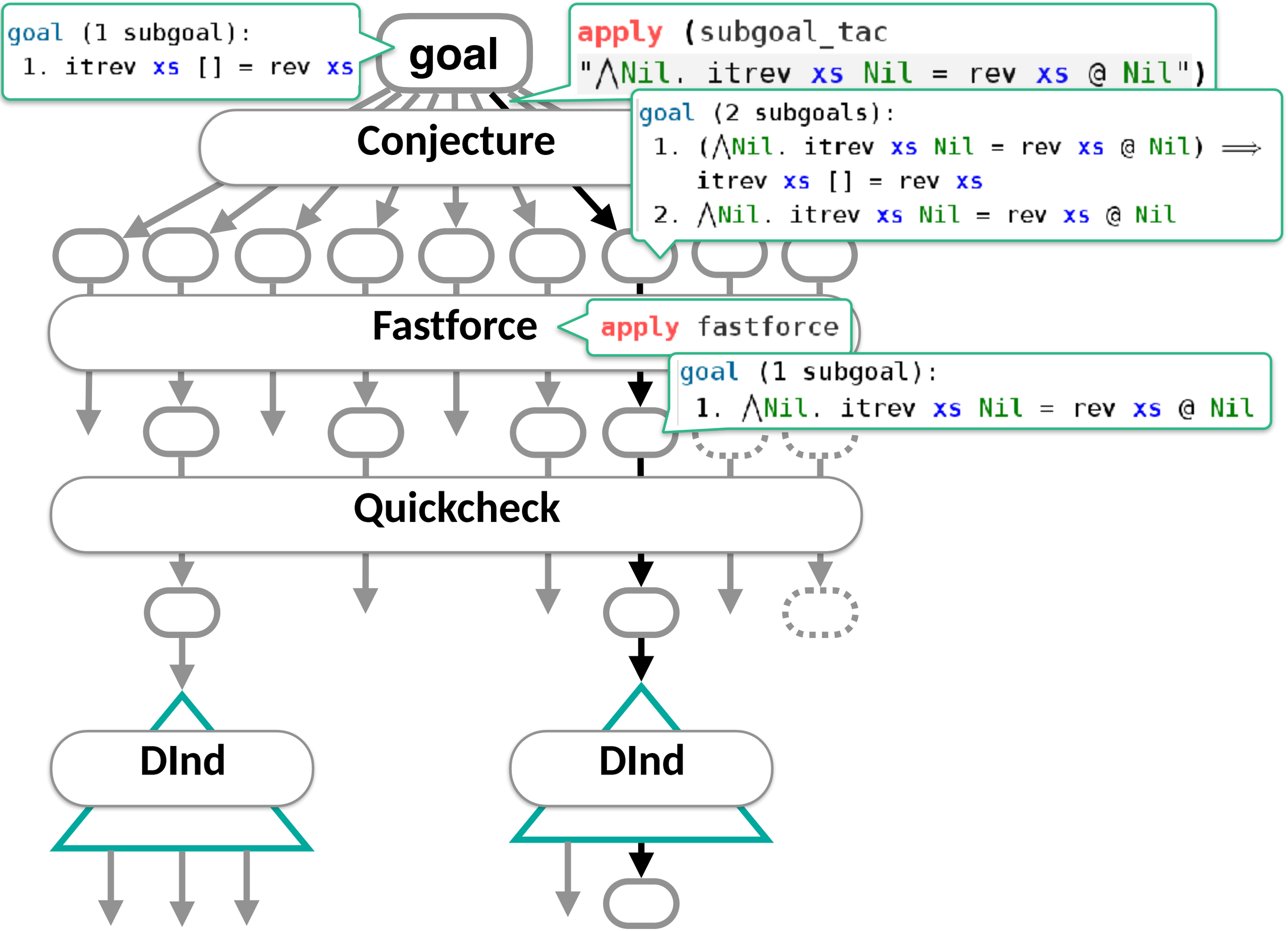
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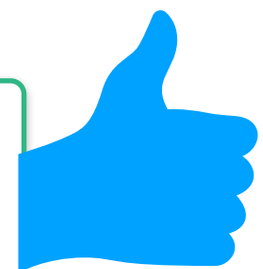

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```

Fastforce

```
apply fastforce
```

```
goal (1 subgoal):
 1. ⋀Nil. itrev xs Nil = rev xs @ Nil
```

Quickcheck

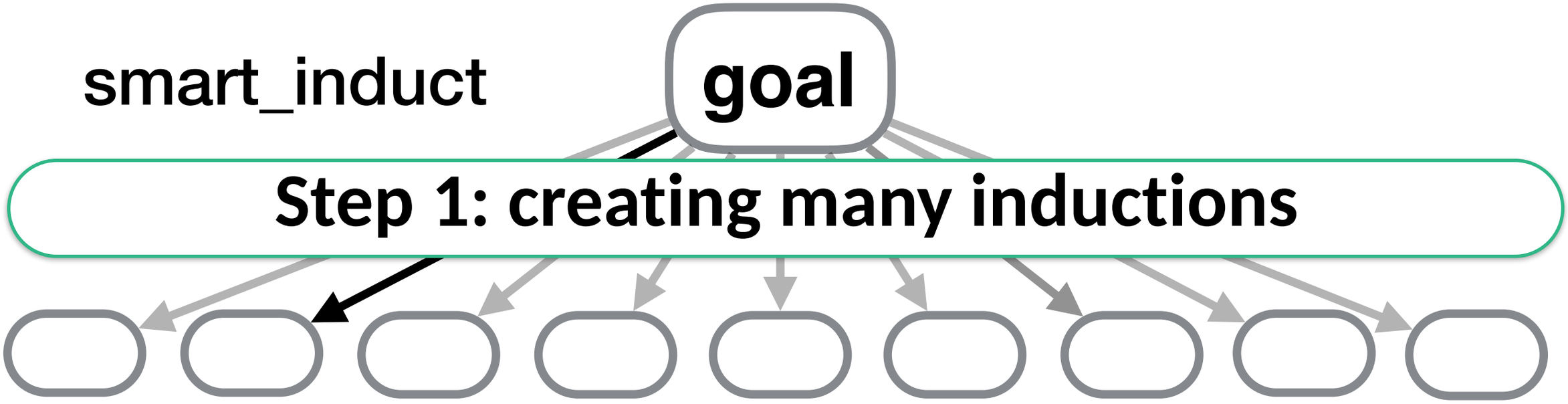
```
goal (1 subgoal):
 1. ⋀Nil. itrev xs Nil = rev xs @ Nil
```

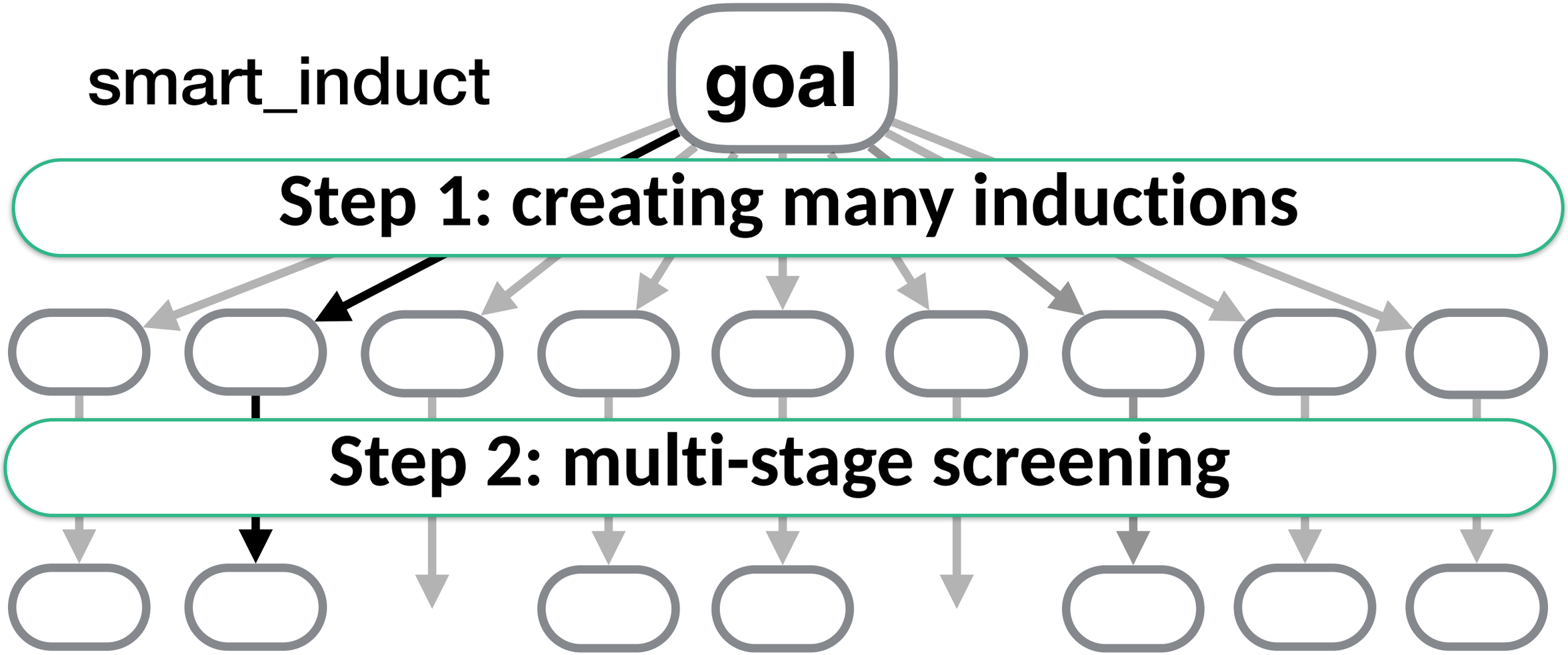
Difficult proof goals?
Search space?

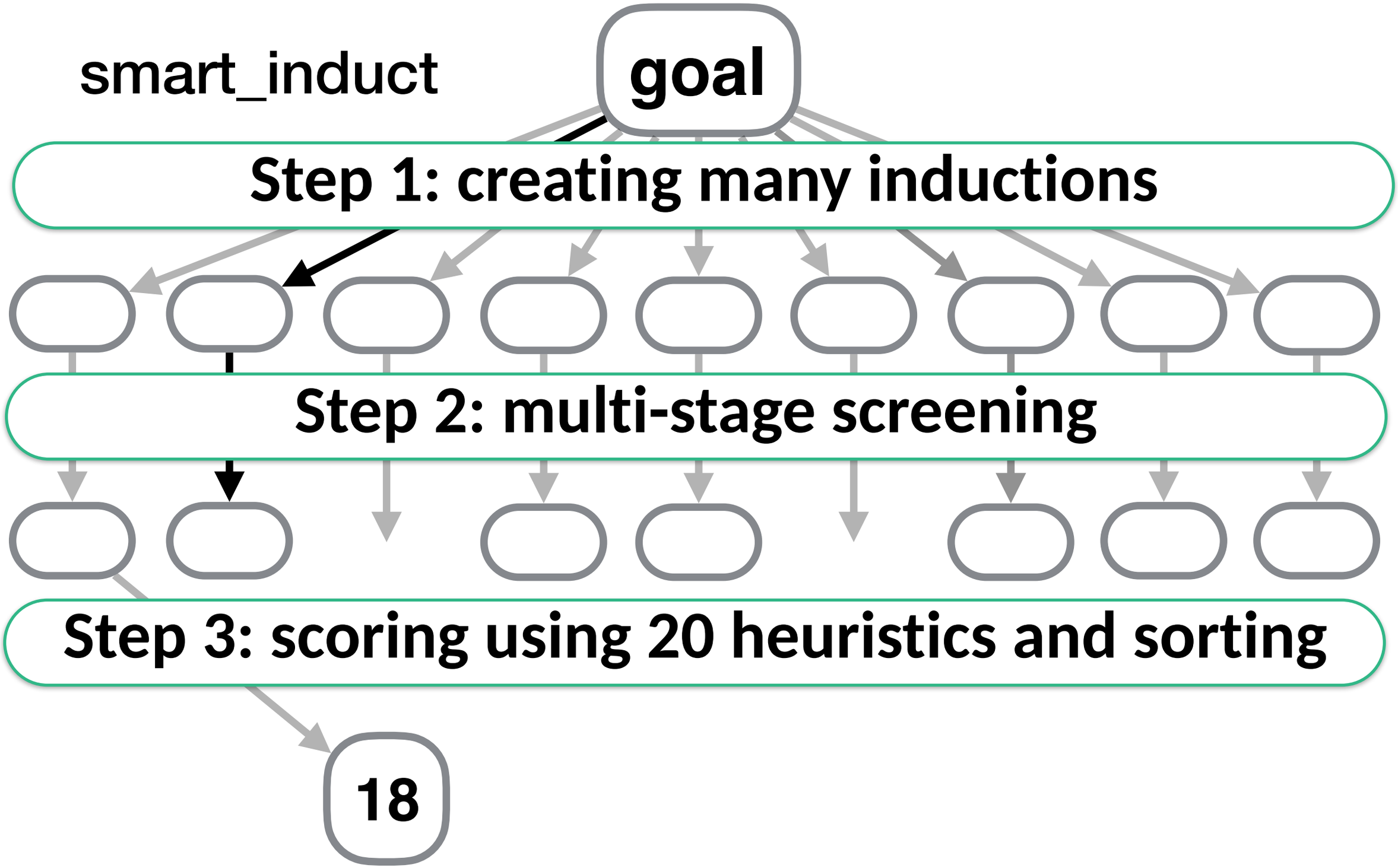
```
theorem itrev ?xs [] = rev ?xs
```

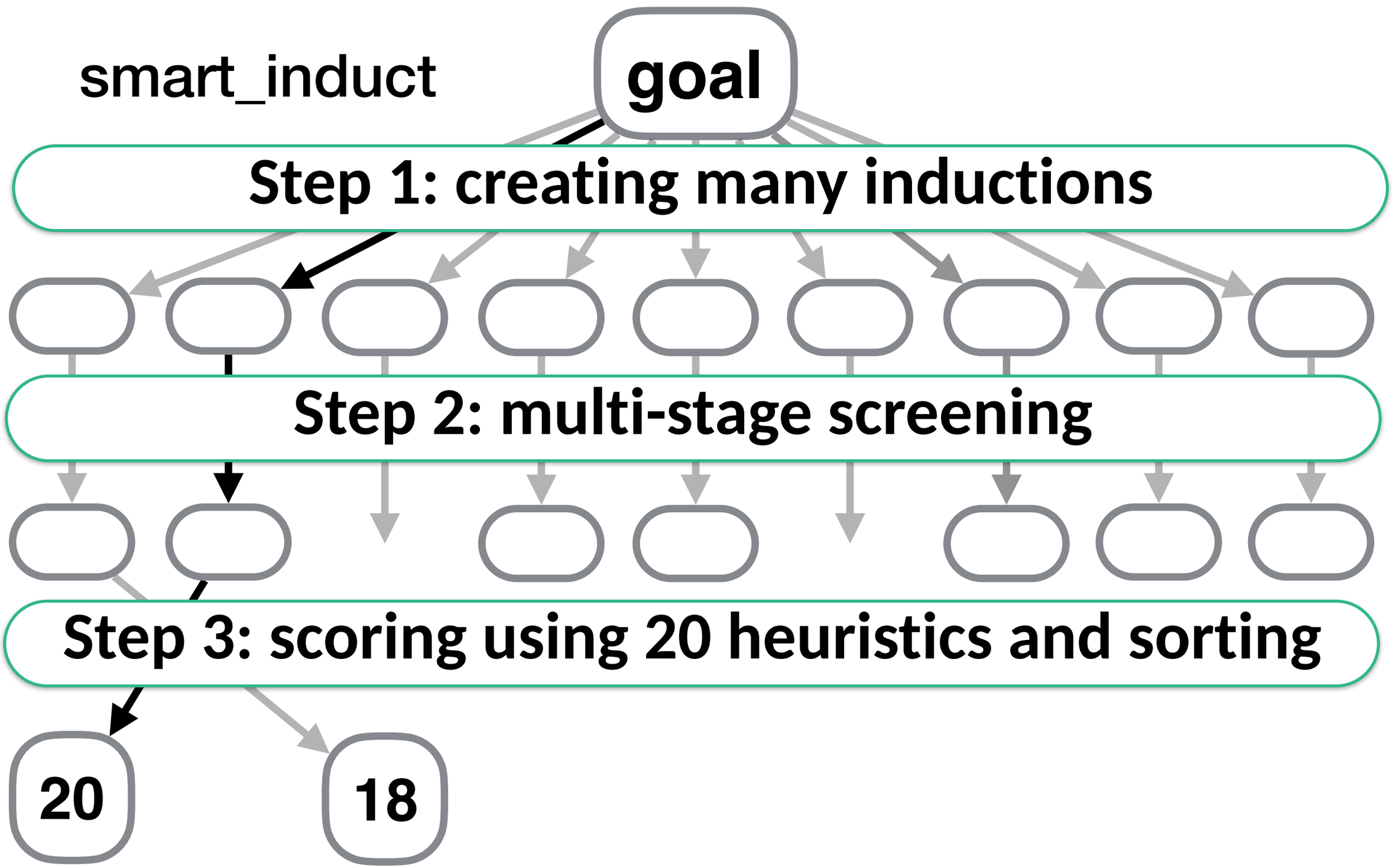
smart_induct

goal


smart_induct
goal
Step 1: creating many inductions


smart_induct
goal
Step 1: creating many inductions
Step 2: multi-stage screening


smart_induct
goal
Step 1: creating many inductions
Step 2: multi-stage screening
Step 3: scoring using 20 heuristics and sorting
18


smart_induct
goal
Step 1: creating many inductions
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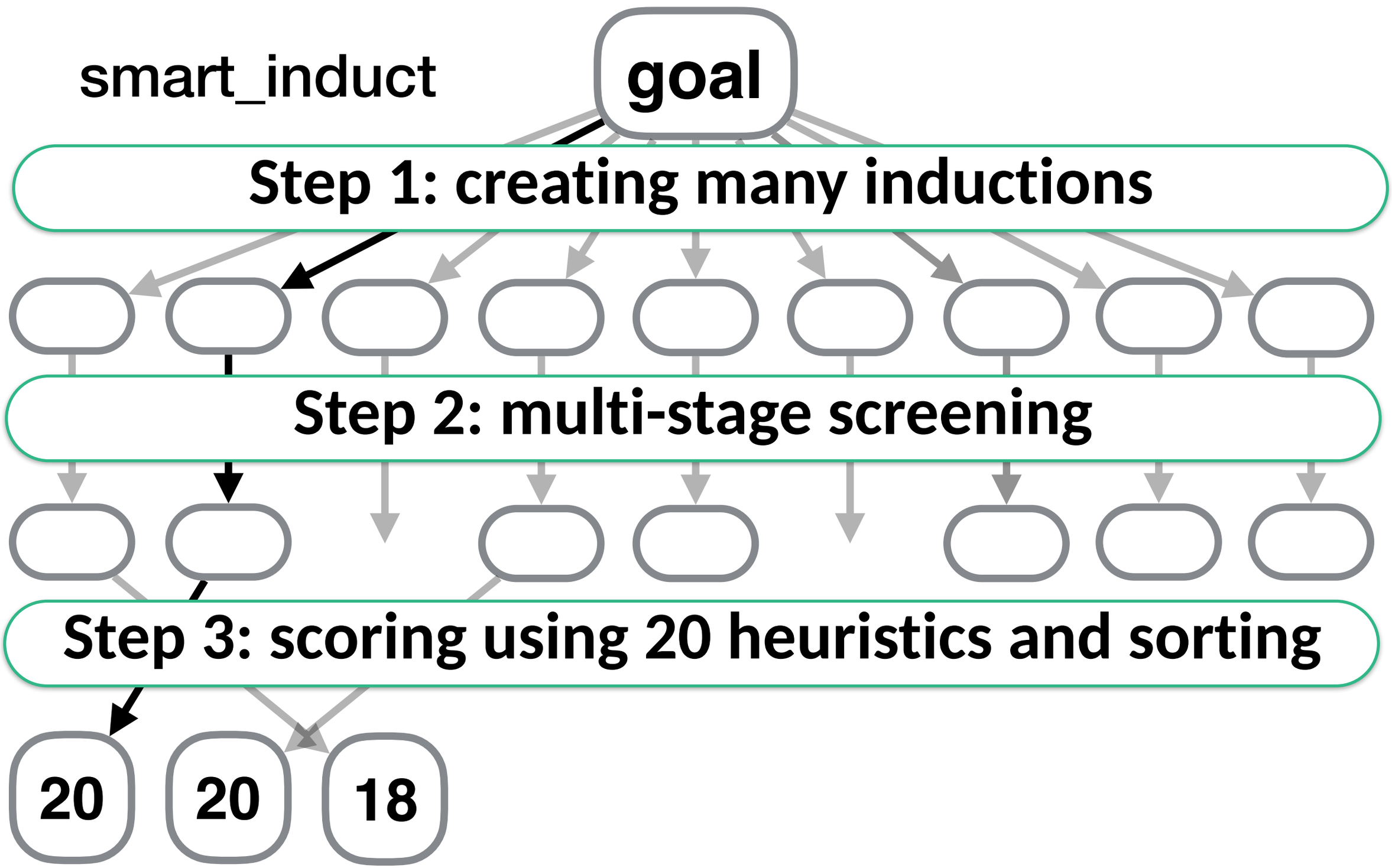
Step 3: scoring using 20 heuristics and sorting
20
18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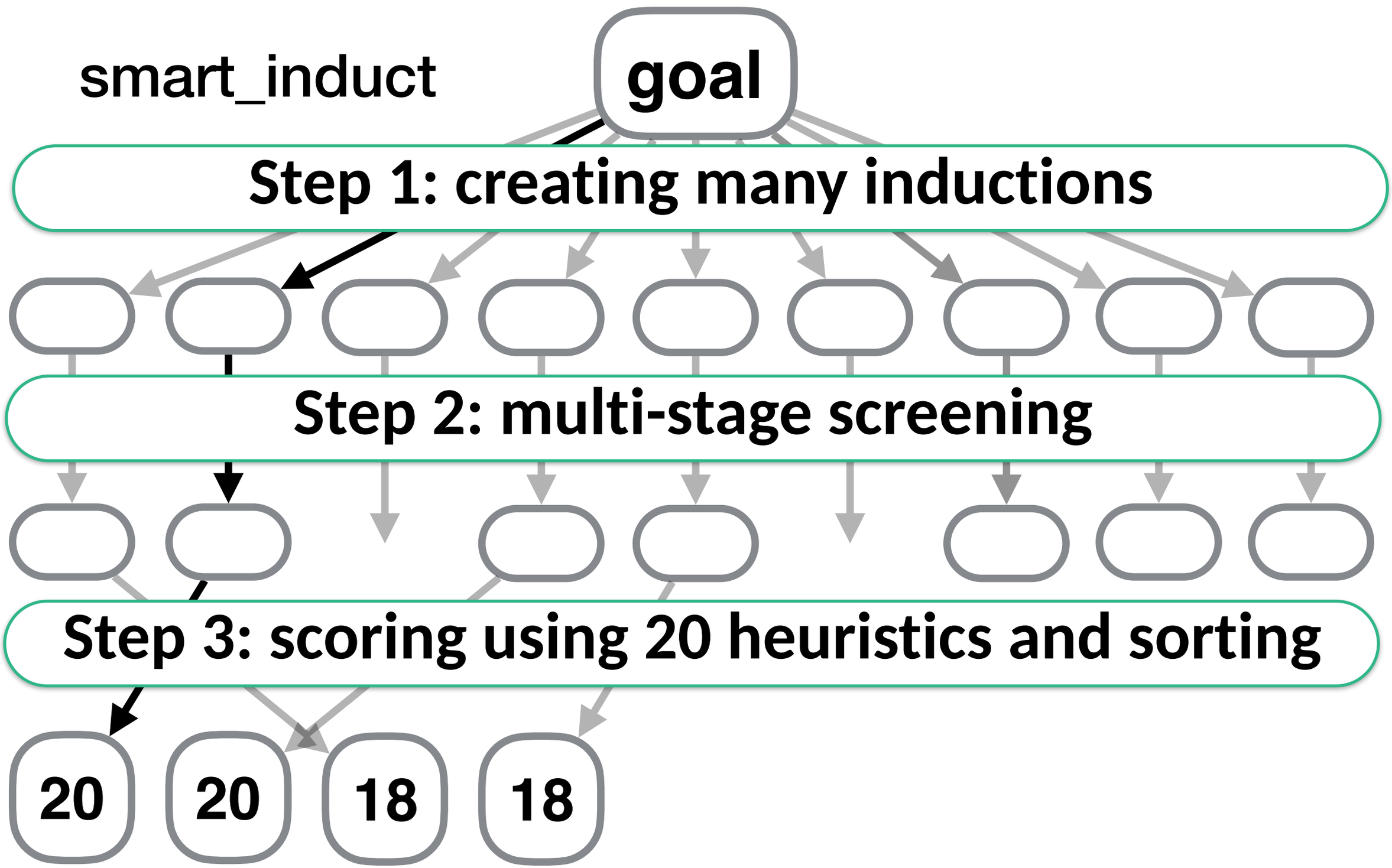
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20
20
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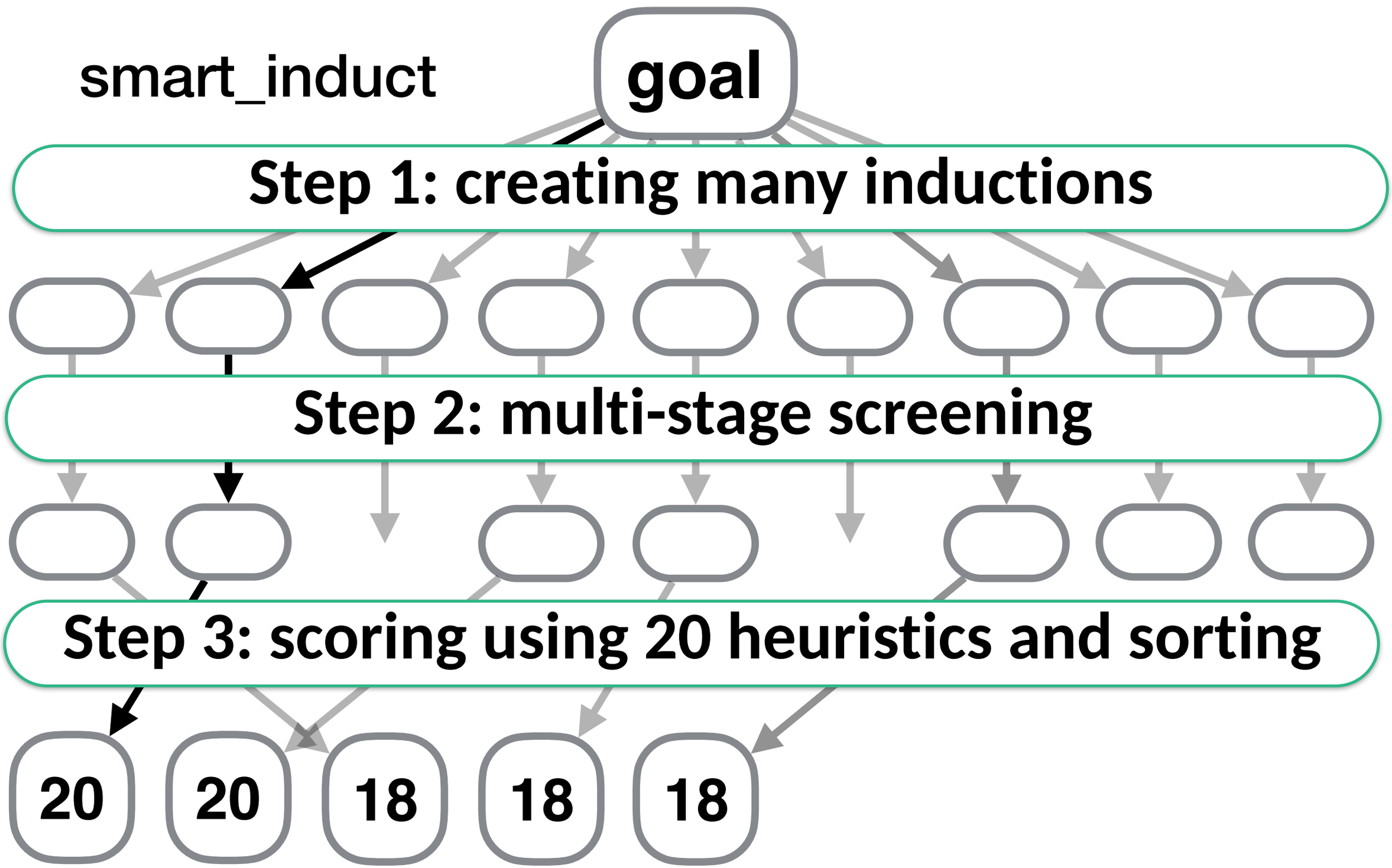
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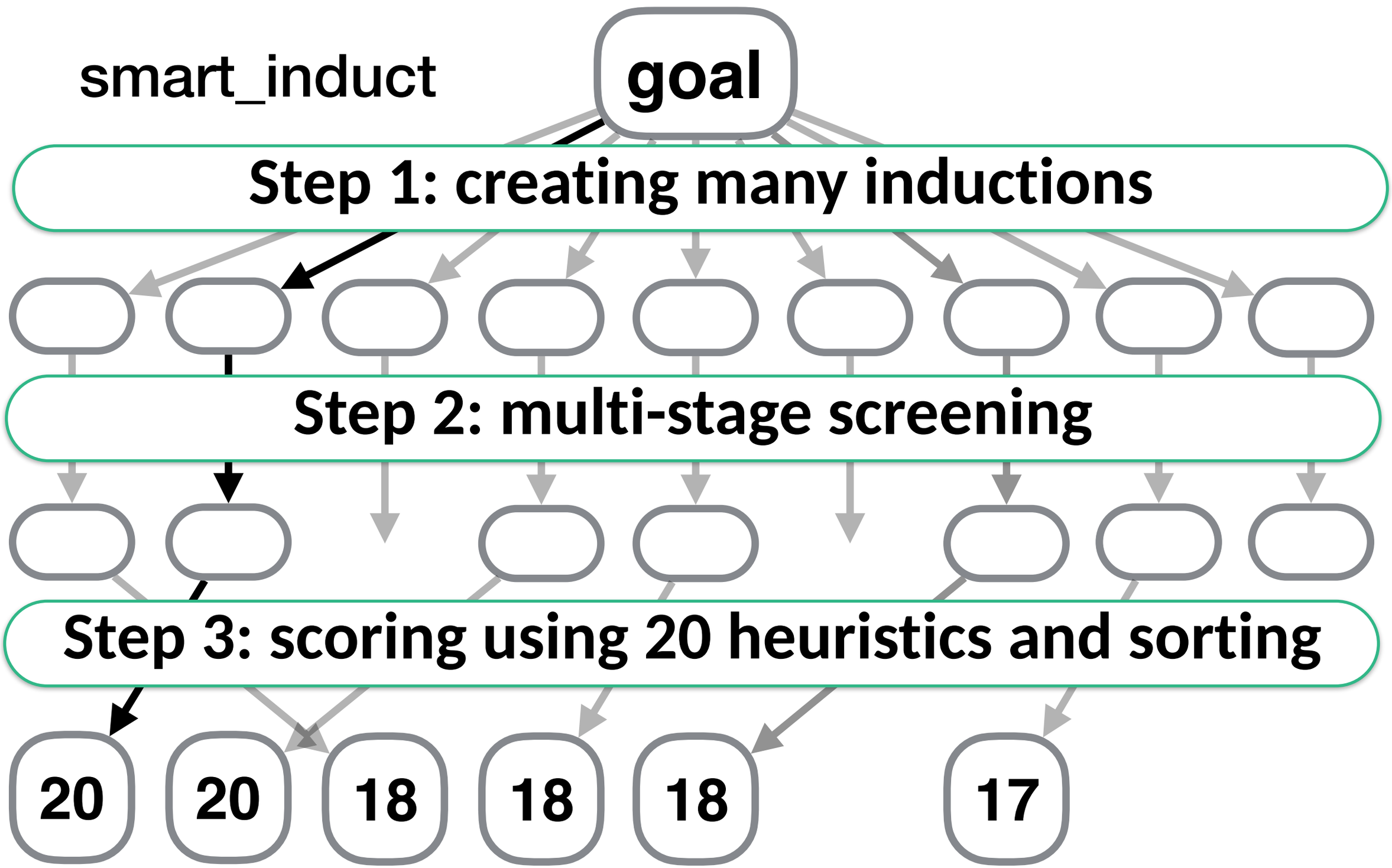
Step 3: scoring using 20 heuristics and sorting
20
20
18
18
18


smart_induct
goal
Step 1: creating many inductions
Step 2: multi-stage screening
Step 3: scoring using 20 heuristics and sorting
20
20
18
18
18
17

smart_induct

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

20 20 18 18 18 18 17

smart_induct

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

20 20 18 18 18 18 17

Step 4: short-listing

smart_induct

goal

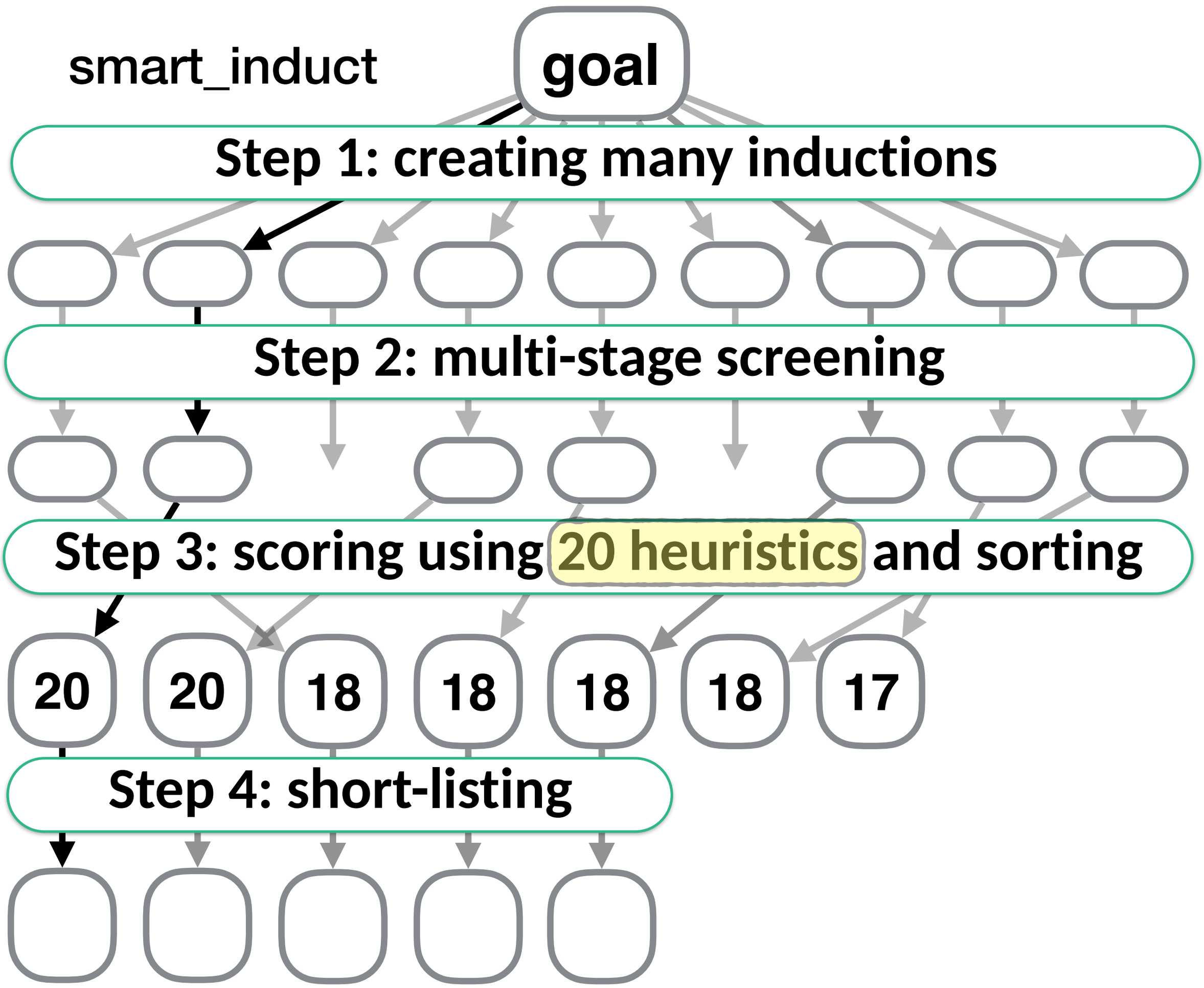
Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

20 20 18 18 18 18 17

Step 4: short-listing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Step 1: creating many inductions
Step 2: multi-stage screening
Step 3: scoring using 20 heuristics and sorting
20
20
18
18
18
18
17
Step 4: short-listing

smart_induct

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

?

20 20 18 18 18 18 17

Step 4: short-listing

smart_induct

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

heuristic : (proof goal * induction arguments) -> bool

20 20 18 18 18 18 17

Step 4: short-listing

It is difficult to write induction heuristics.

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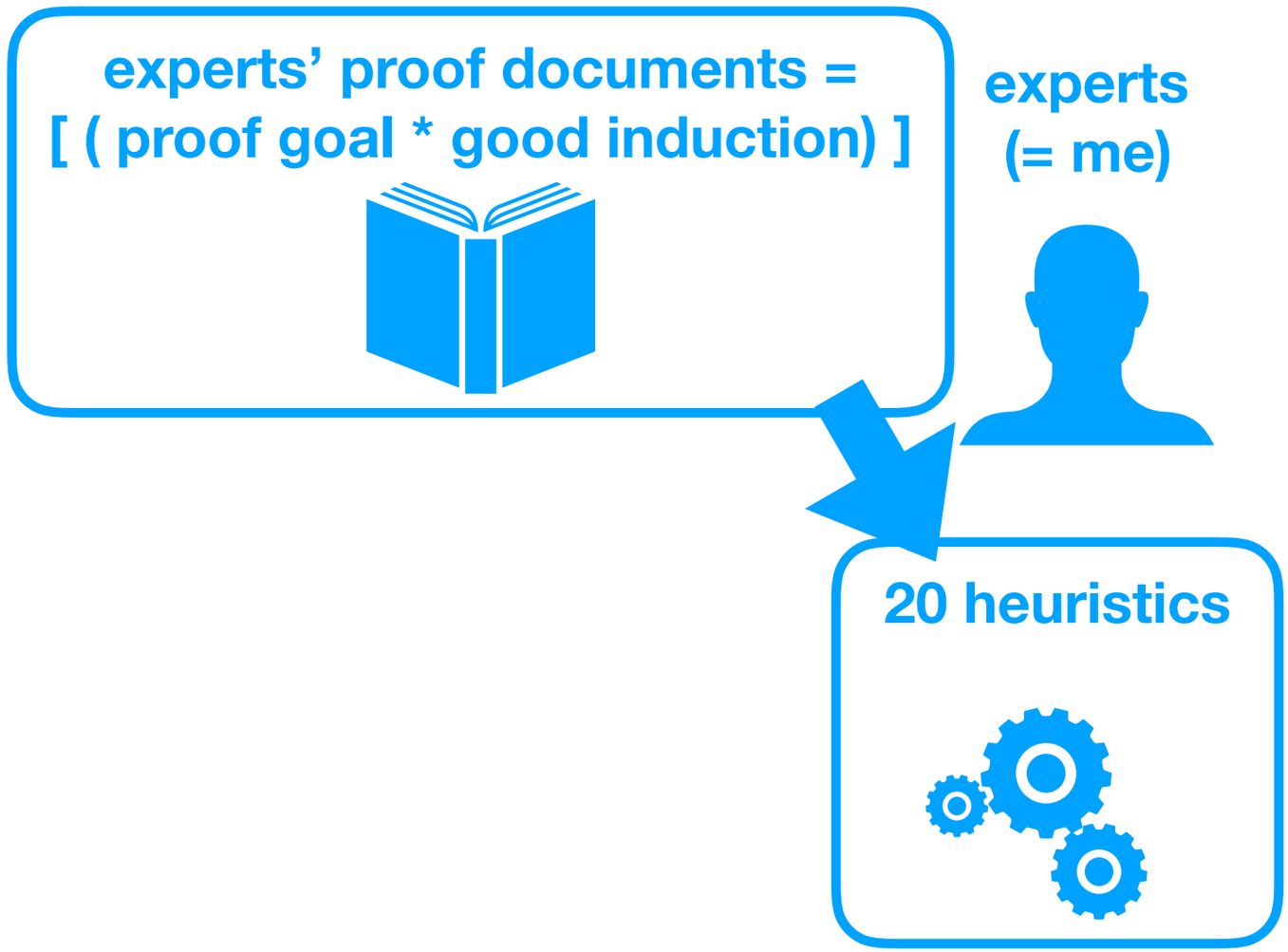

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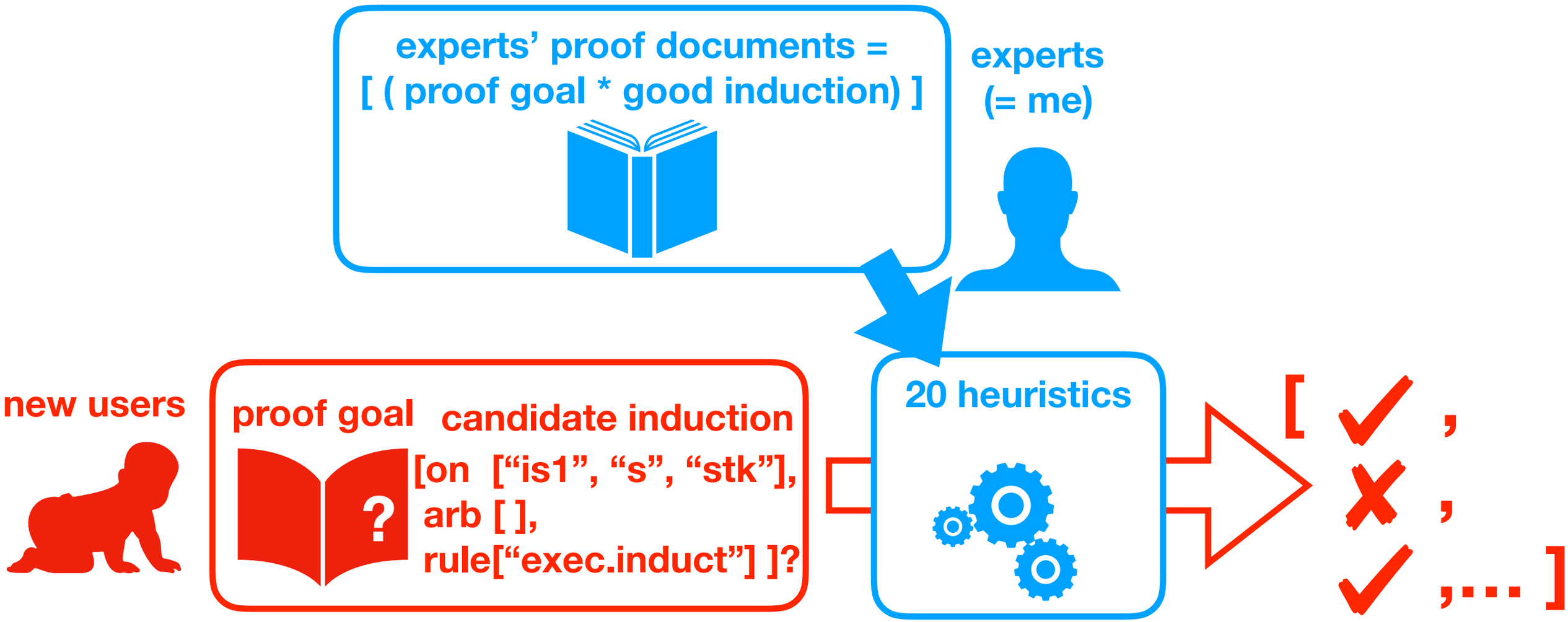


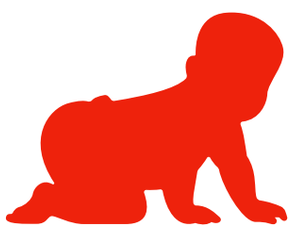
It is difficult to write induction heuristics.

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

experts' proof documents =
[(proof goal * good induction)]

experts
(= me)

new users

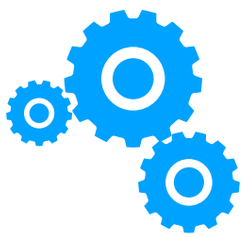


proof goal

candidate induction

[on [“is1”, “s”, “stk”],
arb [ ],
rule[“exec.induct”]]?

20 heuristics



[✓ ,
✗ ,
✓ ,...]

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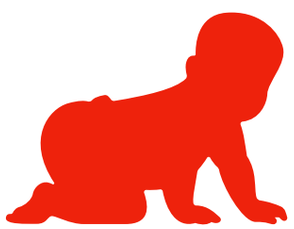

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[on ["is1", "s", "stk"],
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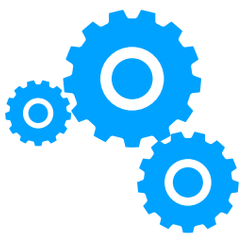

[✓ ,
✗ ,
✓ ,...]

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lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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① blue = what I can see before releasing smart_induct

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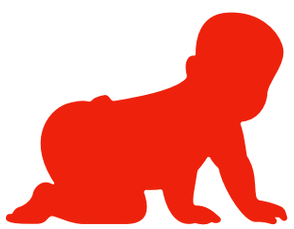

proof goal

candidate induction

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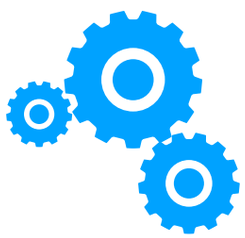

[✓ ,
✗ ,
✓ ,...]

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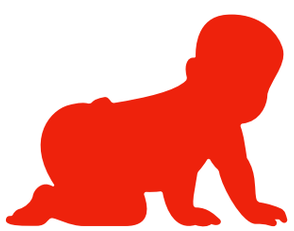


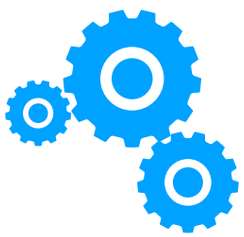
new users

proof goal

candidate induction

[on [“is1”, “s”, “stk”],
arb [ ],
rule[“exec.induct”]]?

20 heuristics



[✓ ,
✗ ,
✓ ,...]

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

② red = what will be developed after releasing smart_induct

It is difficult to write induction heuristics.

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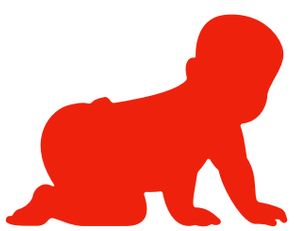

new users

proof goal

candidate induction

[on [“is1”, “s”, “stk”],
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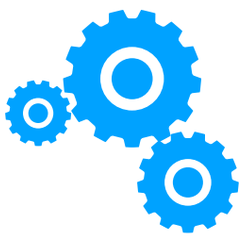
20 heuristics



[✓ ,
✗ ,
✓ ,...]

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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② red = what will be developed after releasing smart_induct

new proof goal consisting of new constants and variables of new types!

It is difficult to write induction heuristics.

① blue = what I can see before releasing smart_induct

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lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

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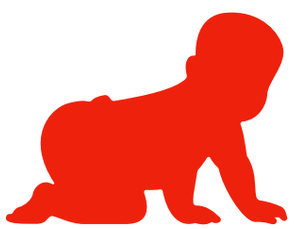
[(proof goal * good induction)]

experts
(= me)

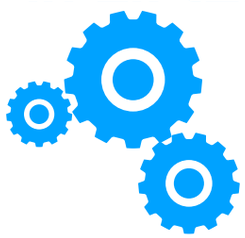
new users



proof goal candidate induction

[on ["is1", "s", "stk"],
arb [ ],
rule["exec.induct"]]?

20 heuristics
in LiFtEr



[✓ ,
✗ ,
✓ ,...]

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② red = what will be developed after releasing smart_induct

new proof goal consisting of new constants and variables of new types!

Abstract Syntax of LiFtEr

assertion := *atomic* | *connective* | *quantifier* | (*assertion*)
type := `term` | `term_occurrence` | `rule` | `number`
modifier_term := `induction_term` | `arbitrary_term`
quantifier := $\exists x$: *type* . *assertion*
| $\forall x$: *type* . *assertion*
| $\exists x$: `term` $\in$ *modifier_term* . *assertion*
| $\forall x$: `term` $\in$ *modifier_term* . *assertion*
| $\exists x$: `rule` . *assertion*
| $\exists x$: `term_occurrence` $\in y$: `term` . *assertion*
| $\forall x$: `term_occurrence` $\in y$: `term` . *assertion*
connective := `True` | `False` | *assertion* $\vee$ *assertion* | *assertion* $\wedge$ *assertion*
assertion $\rightarrow$ *assertion* | $\neg$ *assertion*
pattern := `all_only_var` | `all_constructor` | `mixed`
atomic :=
rule `is_rule_of` *term_occurrence*
| *term_occurrence* `term_occurrence_is_of_term` *term*
| `are_same_term` (*term_occurrence* , *term_occurrence*)
| *term_occurrence* `is_in_term_occurrence` *term_occurrence*
| `is_atomic` *term_occurrence*
| `is_constant` *term_occurrence*
| `is_recursive_constant` *term_occurrence*
| `is_variable` *term_occurrence*
| `is_free_variable` *term_occurrence*
| `is_bound_variable` *term_occurrence*
| `is_lambda` *term_occurrence*
| `is_application` *term_occurrence*
| *term_occurrence* `is_an_argument_of` *term_occurrence*
| *term_occurrence* `is_nth_argument_of` *term_occurrence*

Abstract Syntax of LiFtEr

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| $\forall x$: `term` $\in$ *modifier_term* . *a*
| $\exists x$: `rule` . *assertion*
| $\exists x$: `term_occurrence` $\in y$: `term` . *assertion*
| $\forall x$: `term_occurrence` $\in y$: `term` .
connective := `True` | `False` | *assertion* $\vee$ *asse* *assertion*
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| *term_occurrence* `term_occurrence` *erm*
| `are_same_term` (*term_occurrence* , *term_occ*
| *term_occurrence* `is_in_term_occurrence` *term*
| `is_atomic` *term_occurrence*
| `is_constant` *term_occurrence*
| `is_recursive_constant` *term_occurrence*
| `is_variable` *term_occurrence*
| `is_free_variable` *term_occurrence*
| `is_bound_variable` *term_occurrence*
| `is_lambda` *term_occurrence*
| `is_application` *term_occurrence*
| *term_occurrence* `is_an_argument_of` *occurrence*
| *term_occurrence* `is_nth_argument_of` *occurrence*

Example Assertion in LiFtEr (in Abstract Syntax)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧                                  conjunction
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Assertion in LiFtEr (in Abstract Syntax)

implication

variable for auxiliary lemmas

conjunction

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          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
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```

Example Assertion in LiFtEr (in Abstract Syntax)

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variable for auxiliary lemmas

variable for terms

conjunction

Example Assertion in LiFtEr (in Abstract Syntax)

implication

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              ∃ n : number.
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                ∧
                  t2 is_nth_induction_term n
```

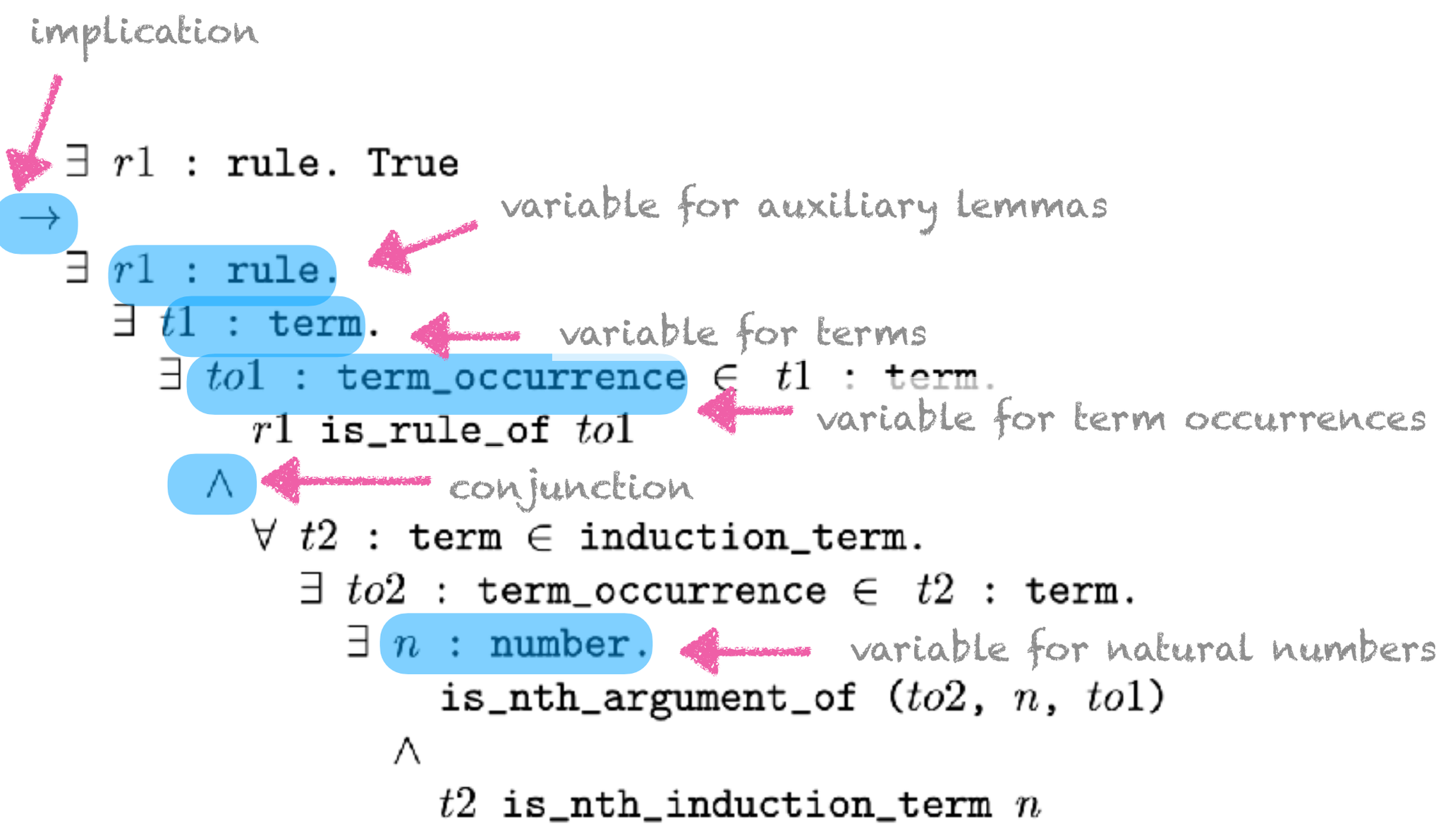
variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

Example Assertion in LiFtEr (in Abstract Syntax)



Example Assertion in LiFtEr (in Abstract Syntax)

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→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

implication

variable for auxiliary lemmas

variable for terms

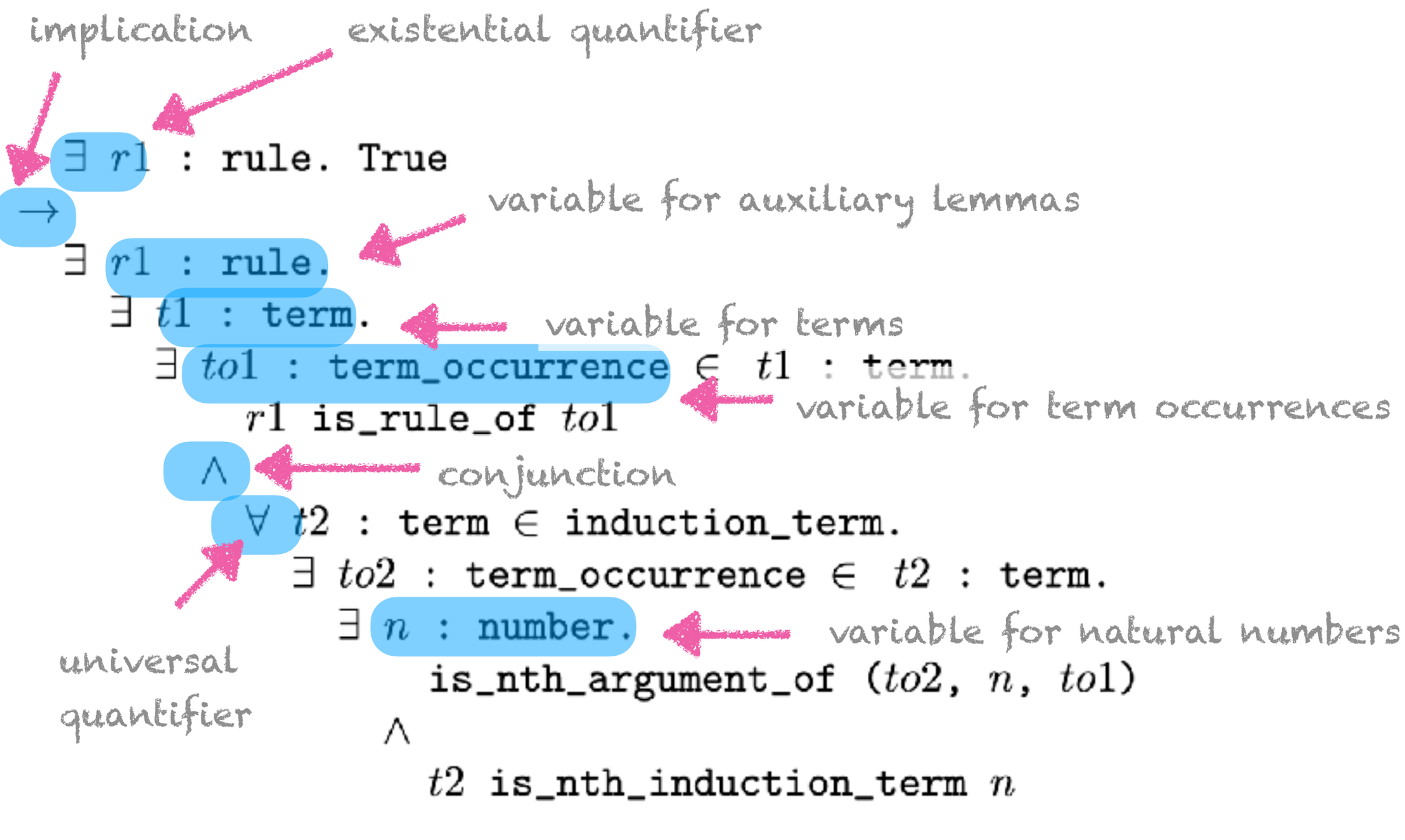
variable for term occurrences

conjunction

universal quantifier

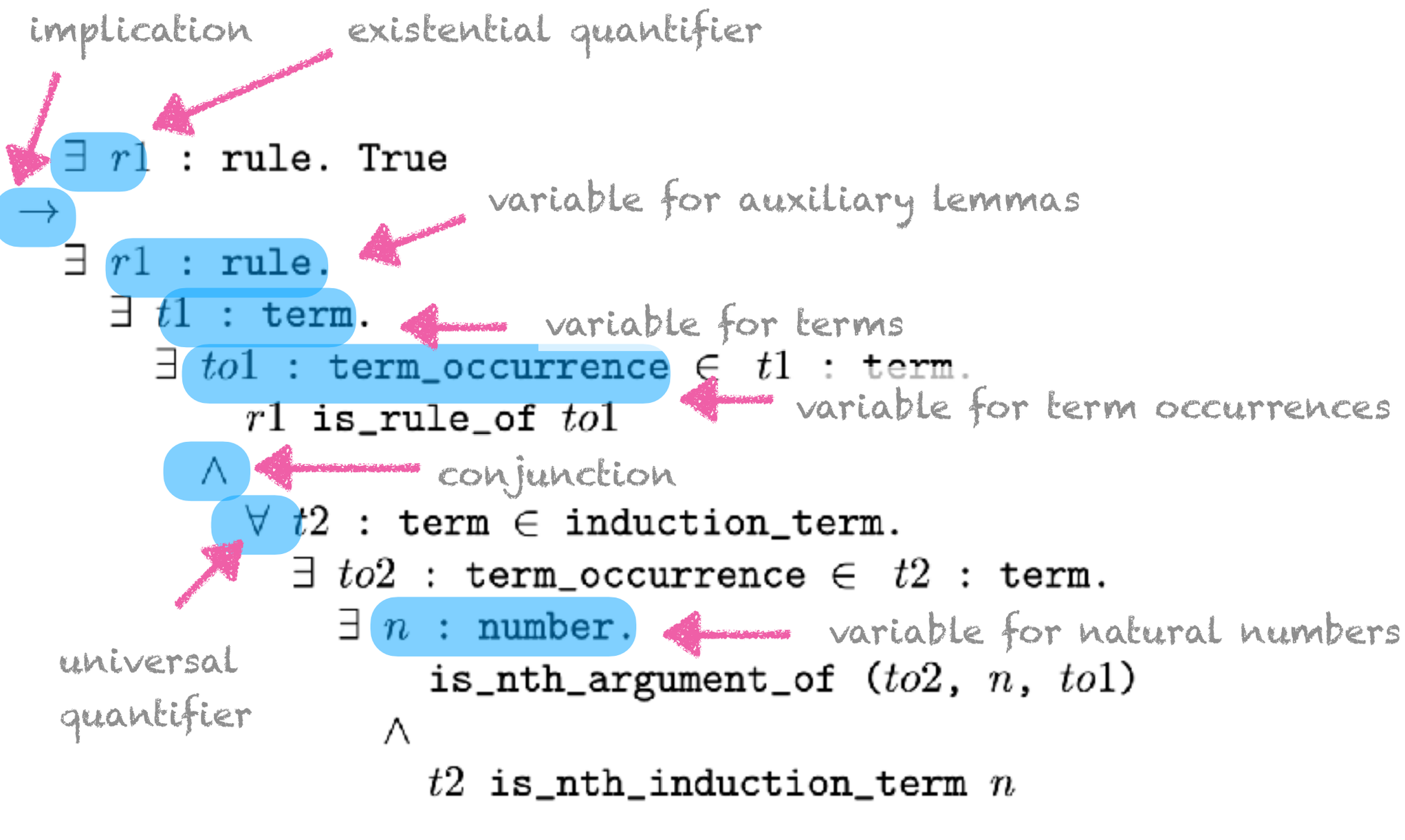
variable for natural numbers

Example Assertion in LiFtEr (in Abstract Syntax)



Example Assertion in LiFtEr (in Abstract Syntax)

LiFtEr assertion: (proof goal * induction arguments) -> bool



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primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
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fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
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r1

(r1 = itrev.induct)

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(r1 = itrev.induct)
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to1

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(r1 = itrev.induct)
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```
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→
  ∃ r1 : rule.                                    ( r1 = itrev.induct )
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        ∧
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True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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  ∃ r1 : rule.                                    ( r1 = itrev.induct )
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                is_nth_argument_of (to2, n, to1)
              ∧
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```

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

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to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
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```

r1

$\exists$ *r1* : rule. True

$\rightarrow$

$\exists$ *r1* : rule. (r1 = itrev.induct)
 $\exists$ *t1* : term. (t1 = itrev)
  $\exists$ *to1* : term_occurrence $\in$ *t1* : term. (to1 = itrev)
   r1 is_rule_of *to1* True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
  $\wedge$
   $\forall$ *t2* : term $\in$ induction_term.
    $\exists$ *to2* : term_occurrence $\in$ *t2* : term.
     $\exists$ *n* : number.
      is_nth_argument_of (*to2*, *n*, *to1*)
     $\wedge$
      t2 is_nth_induction_term *n*

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```

to1

t2

r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
  ∃ t1 : term. (t1 = itrev)
    ∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
      r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
    ∧
      ∀ t2 : term ∈ induction_term. (t2 = xs and ys)
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to1 to2

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t2 r1

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to1 to2 t2 r1

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to1 first to2 t2 first r1

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∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1) ?
∧
t2 is_nth_induction_term n

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to1 first second to2 t2 second first r1

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∃ t1 : term. (t1 = itrev)
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∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1) ?
∧
t2 is_nth_induction_term n when t2 is ys (n = 2) ?

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to1 first second to2 t2 second first r1

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            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

(t2 = xs and ys)
(to2 = xs and ys)

when t2 is xs (n = 1)

when t2 is ys (n = 2)

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to1 first second to2 t2 second first r1

∃ r1 : rule. True

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∃ r1 : rule. (r1 = itrev.induct)
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∃ n : number.
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∧
t2 is_nth_induction_term n when t2 is ys (n = 2)

$\exists$ $r1$: rule. True

$\rightarrow$

$\exists$ $r1$: rule.
$\exists$ $t1$: term.
$\exists$ $to1$: term_occurrence $\in$ $t1$: term.
$r1$ is_rule_of $to1$
$\wedge$
$\forall$ $t2$: term $\in$ induction_term.
$\exists$ $to2$: term_occurrence $\in$ $t2$: term.
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

the same LiFtEr assertion

$\exists\ r1$: rule. True

$\rightarrow$

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$\forall\ t2$: term $\in$ induction_term.
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$\exists\ n$: number.
is_nth_argument_of ($to2$, n, $to1$)
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$t2$ is_nth_induction_term n

new types ->

```
datatype      instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

the same LiFtEr assertion

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            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
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```

the same LiFtEr assertion

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
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new types ->

new constants ->

new lemma ->

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∃ r1 : rule. True

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            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
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new lemma ->

a model proof ->

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apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

r1

(r1 = exec.induct)

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
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new types ->

new constants ->

```
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 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

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              ∧
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r1

(r1 = exec.induct)

new types ->

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new lemma ->

a model proof ->

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       ∀ t2 : term ∈ induction_term.
         ∃ to2 : term_occurrence ∈ t2 : term.
           ∃ n : number.
               is_nth_argument_of (to2, n, to1)
             ∧
               t2 is_nth_induction_term n
```

r1

(r1 = exec.induct)

(t1 = exec)

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to1

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
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           ∃ n : number.
               is_nth_argument_of (to2, n, to1)
             ∧
               t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
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        ∀ t2 : term ∈ induction_term.
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new types ->

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 "exec (i#is) s stk = exec is s (exec1 i s stk)"
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new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
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              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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 "exec (i#is) s stk = exec is s (exec1 i s stk)"
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new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
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        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
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 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
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new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
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        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
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              ∧
                t2 is_nth_induction_term n
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(r1 = exec.induct)
(t1 = exec)
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True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

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 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

to1

r1

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

r1 is_rule_of to1 True! r1 (= exec.induct) is a lemma about to1 (= exec).

∧

∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)

∧

t2 is_nth_induction_term n

new types ->

new constants ->

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fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->
∃ r1 : rule. True

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 — is1, s, stk

to1 — exec is1 s stk

r1 — exec.induct

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
  ∧
   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
     ∧
      t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)

new types ->

new constants ->

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datatype     instr = LOADI val | LOAD vname | ADD
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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

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 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 to2

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

new types ->

new constants ->

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 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

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 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 r1

∃ r1 : rule. True

→

∃ r1 : rule.
(r1 = exec.induct)
∃ t1 : term.
(t1 = exec)
∃ to1 : term_occurrence ∈ t1 : term.
(to1 = exec)
r1 is_rule_of to1
True! r1 (= exec.induct) is a lemma about to1 (= exec).
∧
∀ t2 : term ∈ induction_term.
(t2 = is1, s, and stk)
∃ to2 : term_occurrence ∈ t2 : term.
(to2 = is1, s, and stk)
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

new types ->

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fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 r1

∃ r1 : rule. True

→

∃ r1 : rule.
(r1 = exec.induct)
∃ t1 : term.
(t1 = exec)
∃ to1 : term_occurrence ∈ t1 : term.
(to1 = exec)
r1 is_rule_of to1
True! r1 (= exec.induct) is a lemma about to1 (= exec).
∧
∀ t2 : term ∈ induction_term.
(t2 = is1, s, and stk)
∃ to2 : term_occurrence ∈ t2 : term.
(to2 = is1, s, and stk)
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
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fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first

to1

to2

t2

r1

first

∃ r1 : rule. True

→

∃ $r1$: rule.
 ∃ $t1$: term.
  ∃ $to1$: term_occurrence ∈ $t1$: term.
   $r1$ is_rule_of $to1$
  ∧
   ∀ $t2$: term ∈ induction_term.
    ∃ $to2$: term_occurrence ∈ $t2$: term.
     ∃ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
     ∧
      $t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

first

to1

t2

to2

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

r1

first

$\exists$ r1 : rule. True

$\rightarrow$

$\exists$ $r1$: rule.
$\exists$ $t1$: term.
$\exists$ $to1$: term_occurrence $\in$ $t1$: term.
r1 is_rule_of $to1$
$\wedge$
$\forall$ $t2$: term $\in$ induction_term.
$\exists$ $to2$: term_occurrence $\in$ $t2$: term.
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first second t2 to1 to2 r1

∃ r1 : rule. True

→

∃ $r1$: rule.
∃ $t1$: term.
∃ $to1$: term_occurrence ∈ $t1$: term.
$r1$ is_rule_of $to1$ True! r1 (= exec.induct) is a lemma about to1 (= exec).
∧
∀ $t2$: term ∈ induction_term. (t2 = is1, s, and stk) ✓
∃ $to2$: term_occurrence ∈ $t2$: term. (to2 = is1, s, and stk)
∃ n : number.
is_nth_argument_of ($to2$, n, $to1$) when t2 is is1 (n -> 1)
∧ when t2 is s (n -> 2) ?
$t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first second t2 to1 to2 r1

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1 ∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1) ∧
t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)
when t2 is s (n -> 2)

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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new constants ->

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```
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new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

first second third t2 s stk

first second third to1 s (exec is1 s stk)

to2

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?
when t2 is s (n -> 2) ?
when t2 is stk (n -> 3) ?

new types ->

new constants ->

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first second third

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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a model proof ->

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 apply auto done
```

t2 first second third

to1 r1

to2

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = exec.induct)
∃ t1 : term. (t1 = exec)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = exec)
r1 is_rule_of to1 True! r1 (= exec.induct) is a lemma about to1 (= exec).
∧
∀ t2 : term ∈ induction_term. (t2 = is1, s, and stk)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = is1, s, and stk)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is is1 (n -> 1)
∧ when t2 is s (n -> 2)
t2 is_nth_induction_term n when t2 is stk (n -> 3)

new types ->

```
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```
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```

a model proof ->

```
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
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∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
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∧
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first second third t2

first second third to1

r1

to2

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1)
when t2 is s (n -> 2)
when t2 is stk (n -> 3)

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

20 20 18 18 18 18 17

Step 4: short-listing

DEMO!

smart_induct… does it work?

Table 1: Coincidence Rates of `smart_induct`.

theory	total	top_1	top_3	top_5	top_10
`DFS`	10	6 (60%)	9 (90%)	9 (90%)	9 (90%)
`Nearest_Neighbors`	16	3 (19%)	4 (25%)	7 (44%)	12 (75%)
`RST_RBT`	24	24 (100%)	24 (100%)	24 (100%)	24 (100%)
sum	50	33 (65%)	37 (74%)	40 (80%)	45 (90%)

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Table 2: Coincidence Rates of `smart_induct` Based Only on Induction Terms.

theory	total	top_1	top_3	top_5	top_10
Nearest_Neighbors	16	5 (31%)	12 (75%)	15 (94%)	15 (94%)

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Overall, the resits are good.
But finding out what variables to generalise remains as a challenge!

smart_induct... does it work?

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theory	total	top_1	top_3	top_5	top_10
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Overall, the resits are good.
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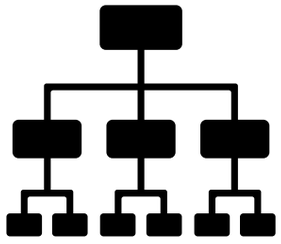
fin.

LiFtEr: (proof goal * induction arguments) -> bool

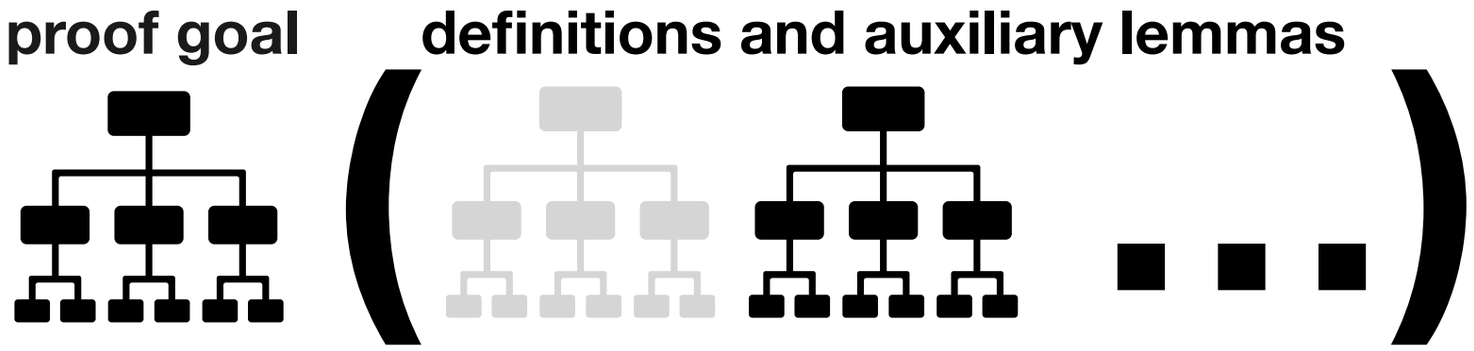
LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

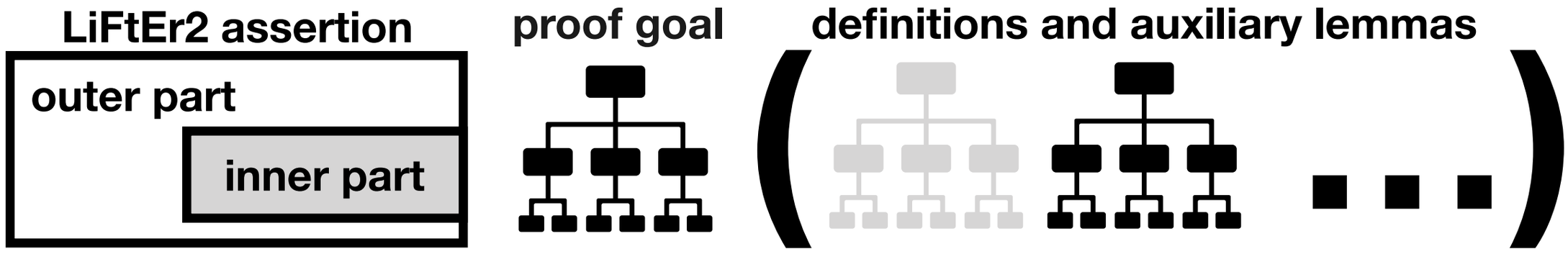
proof goal



LiFtEr$_2$ (proof goal * induction arguments) -> bool
*** relevant definitions**



LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions



LiFtEr$_2$ (proof goal * induction arguments) -> bool
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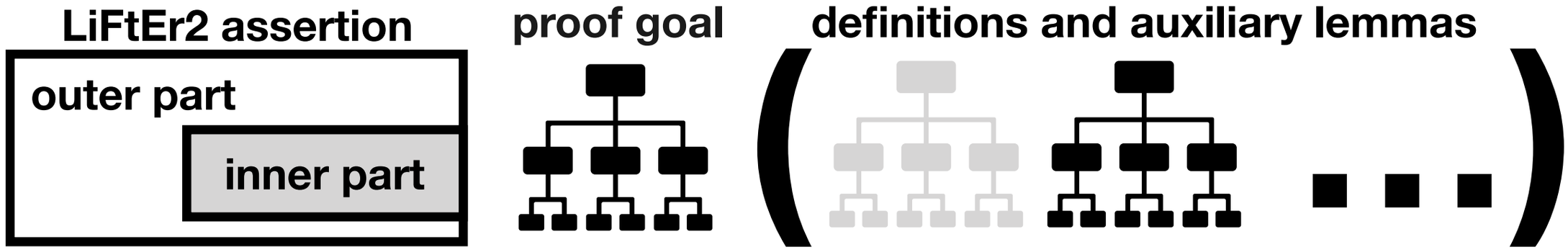
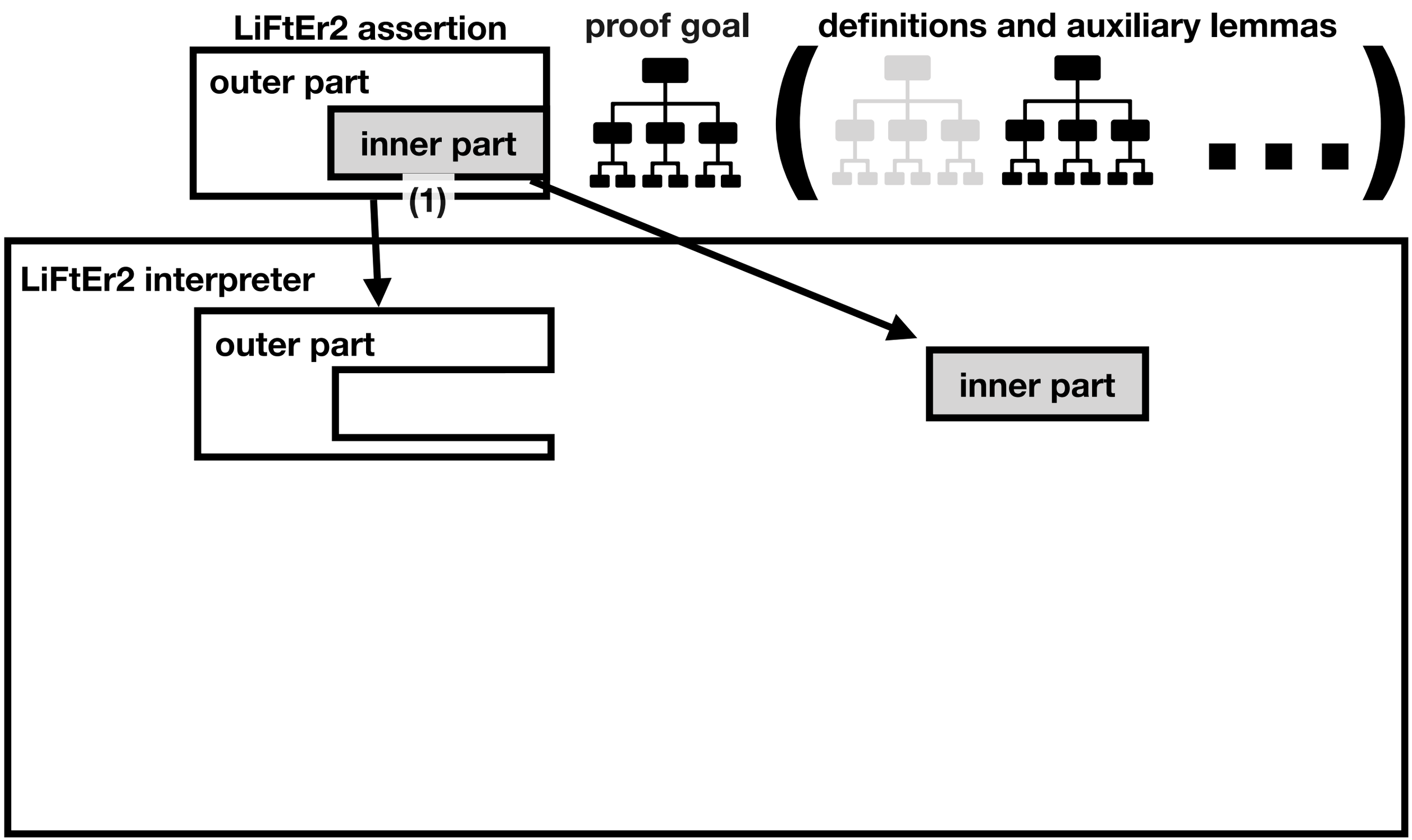
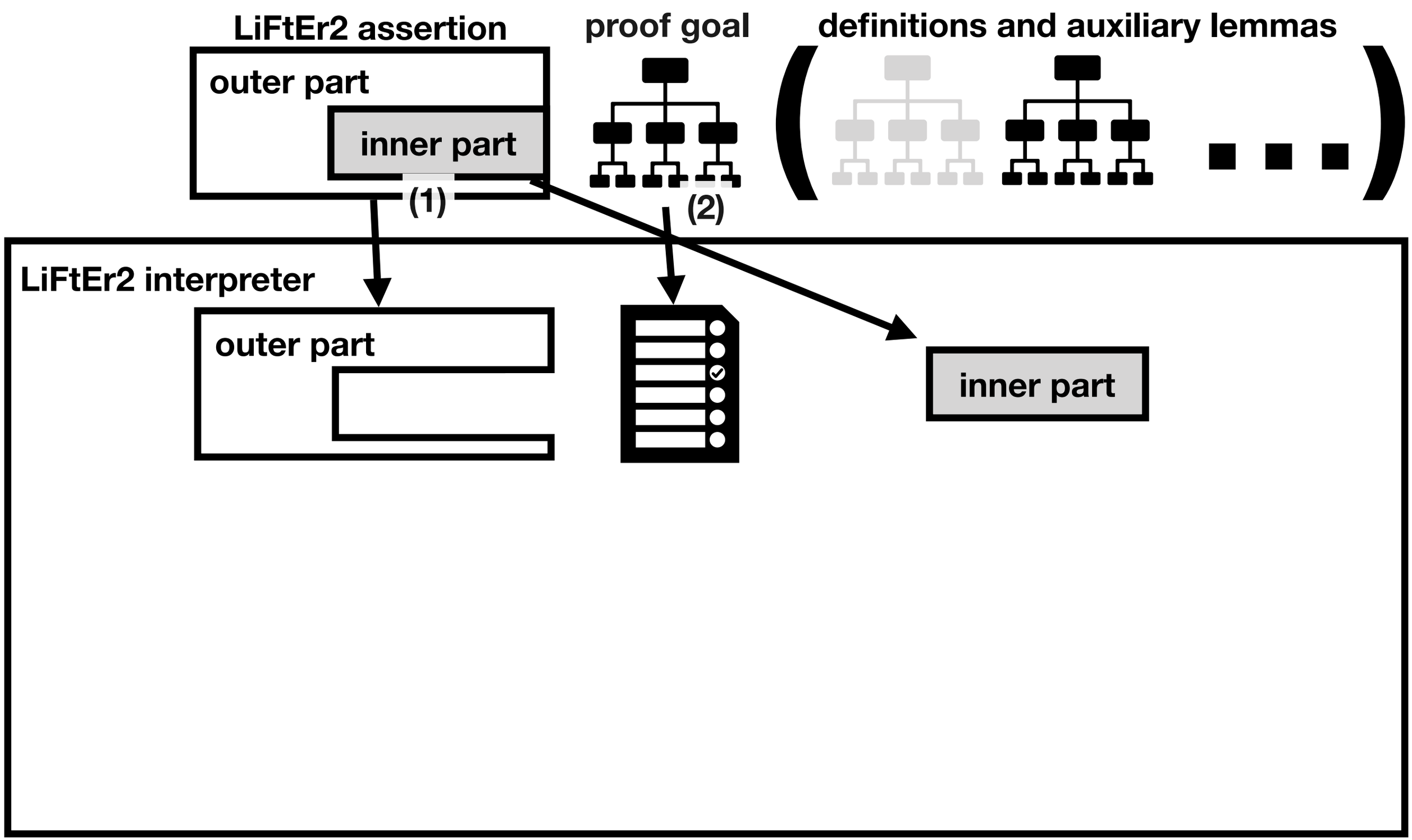


LiFtEr2 interpreter

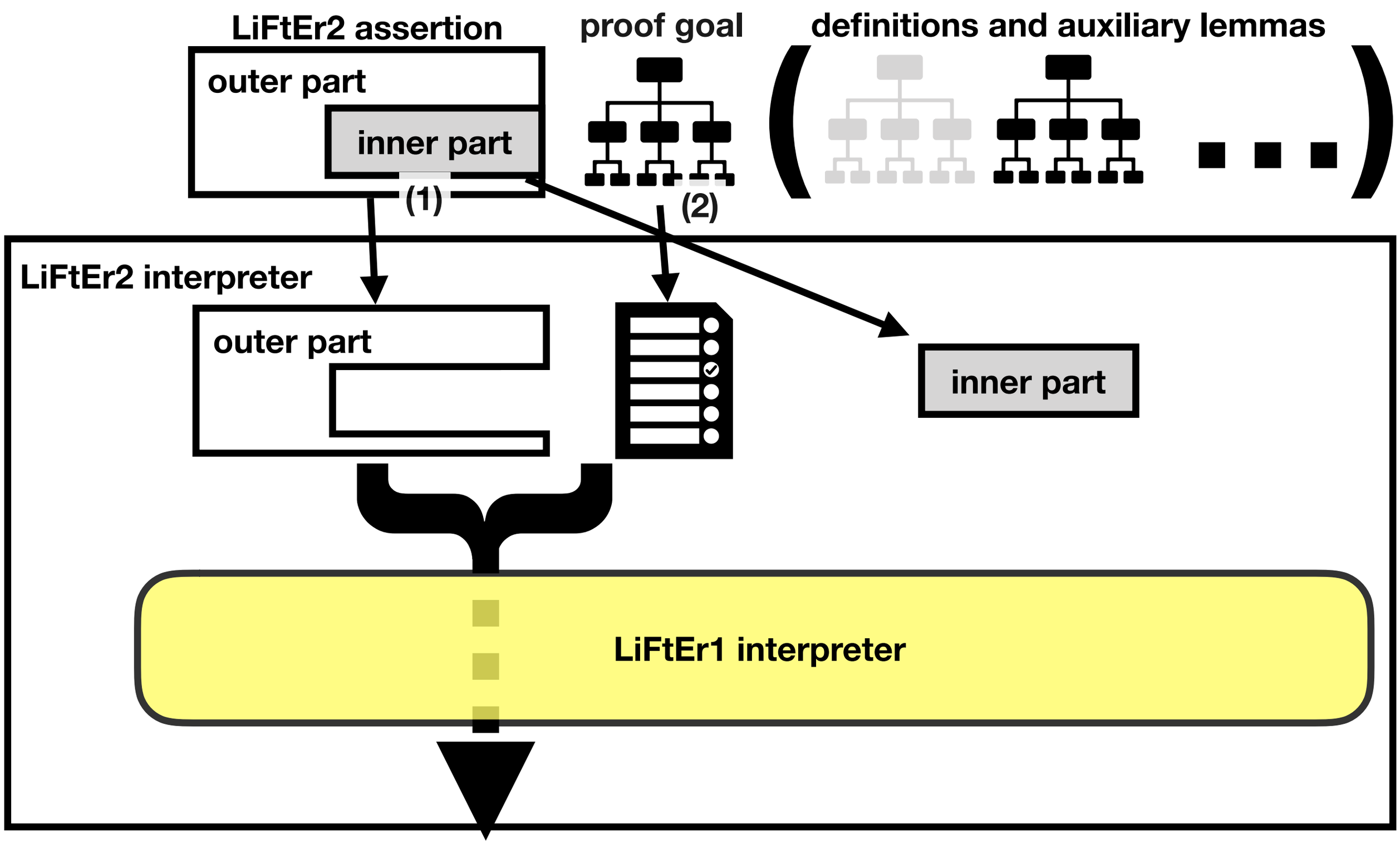
LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions



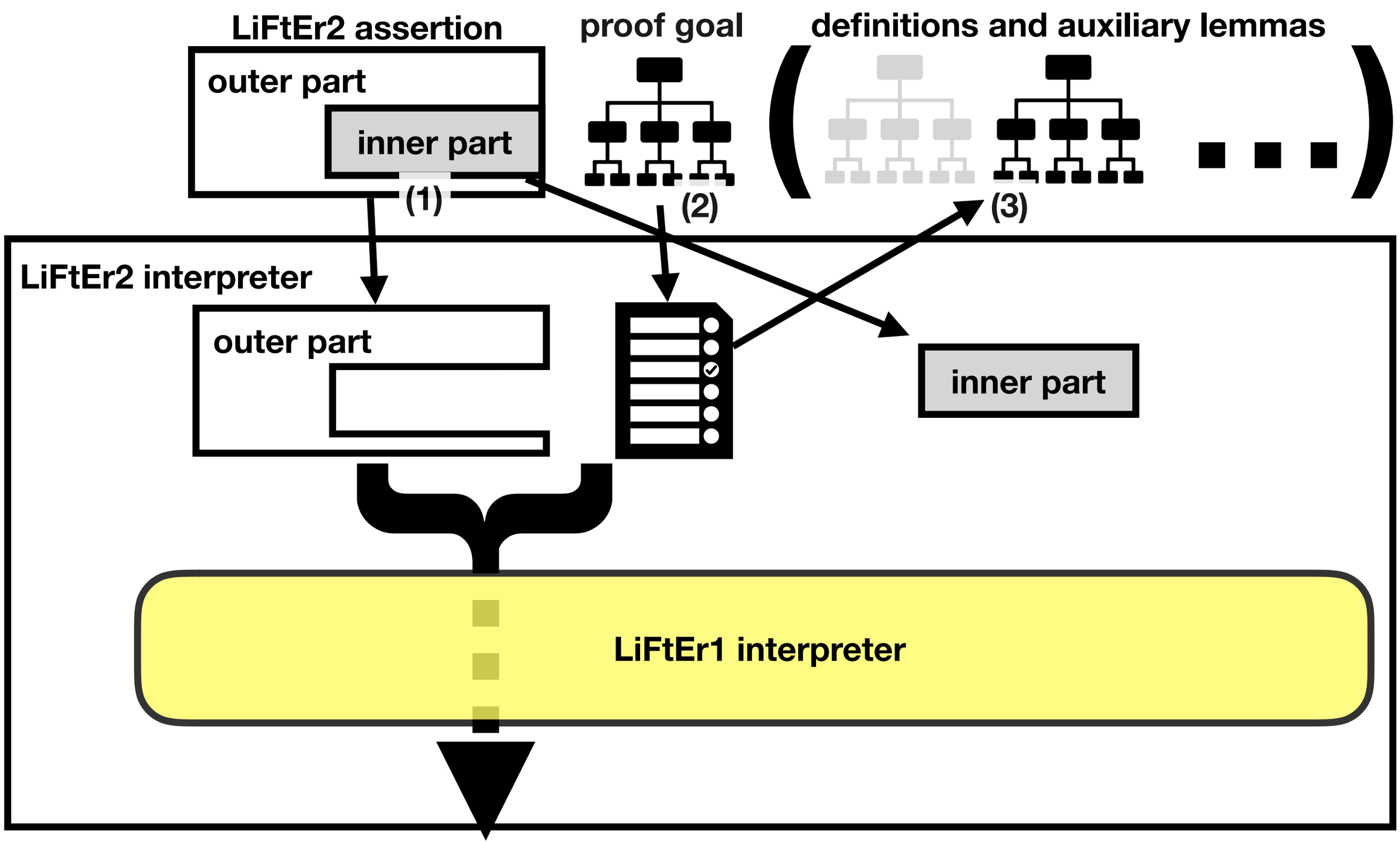
LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



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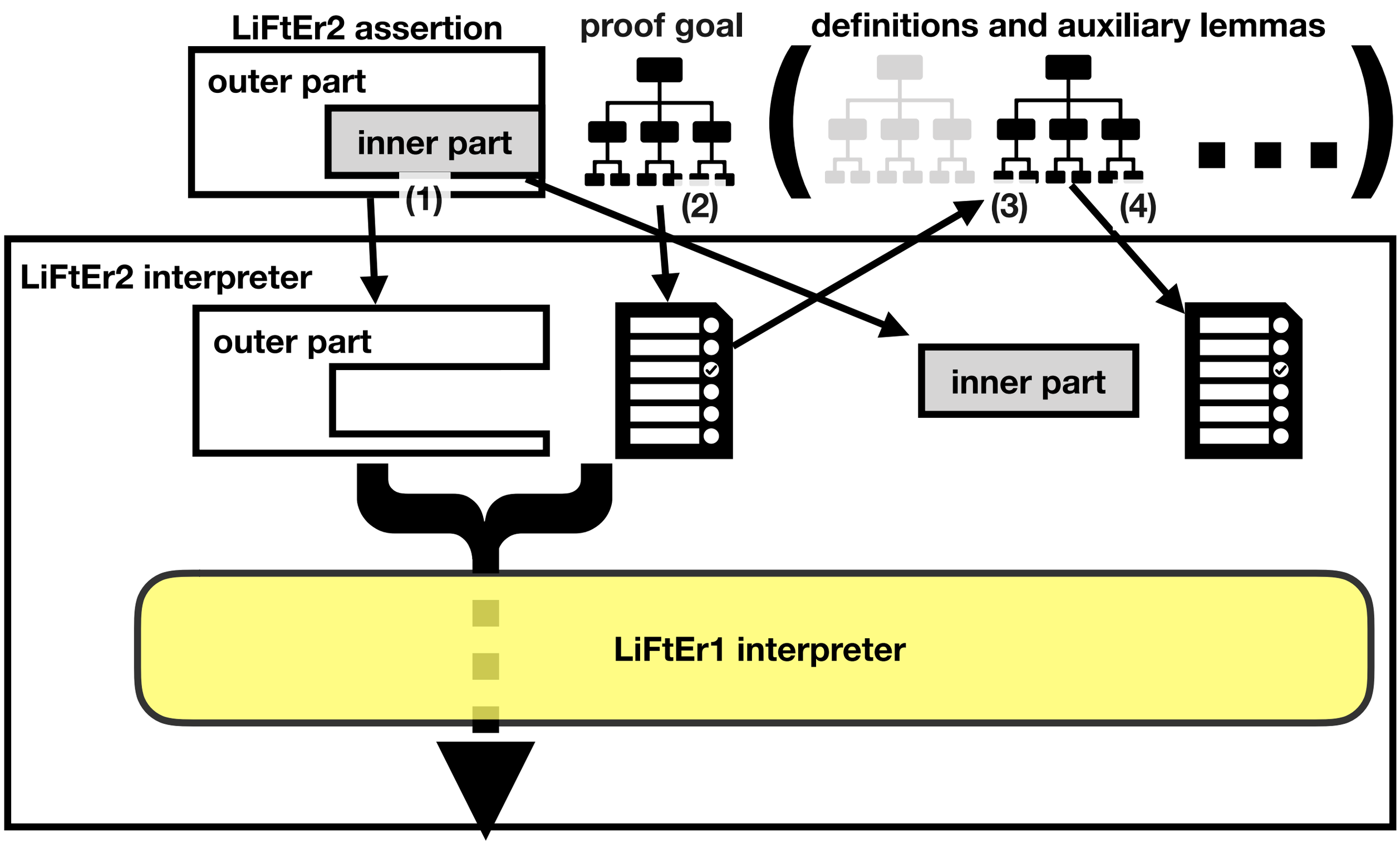

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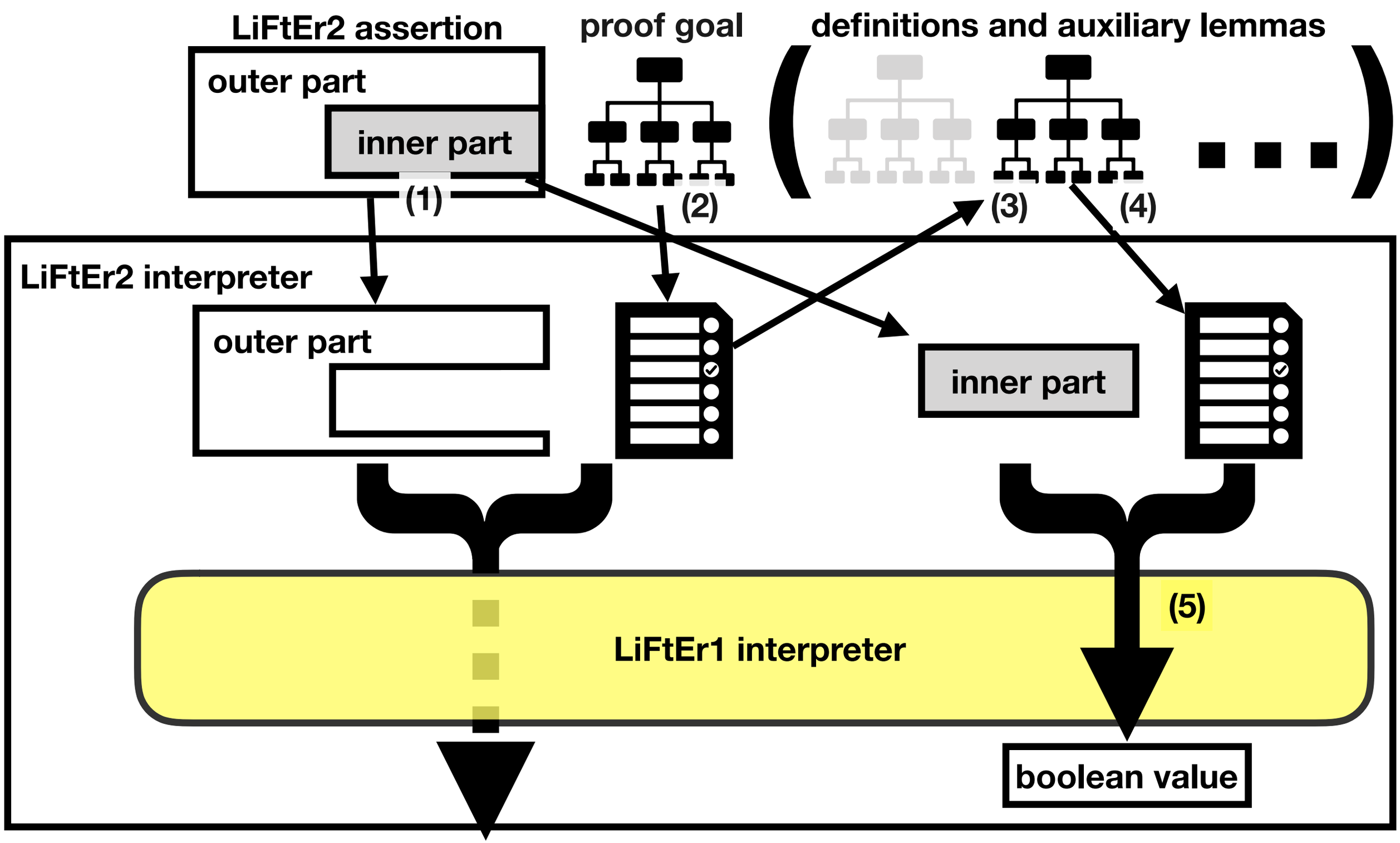

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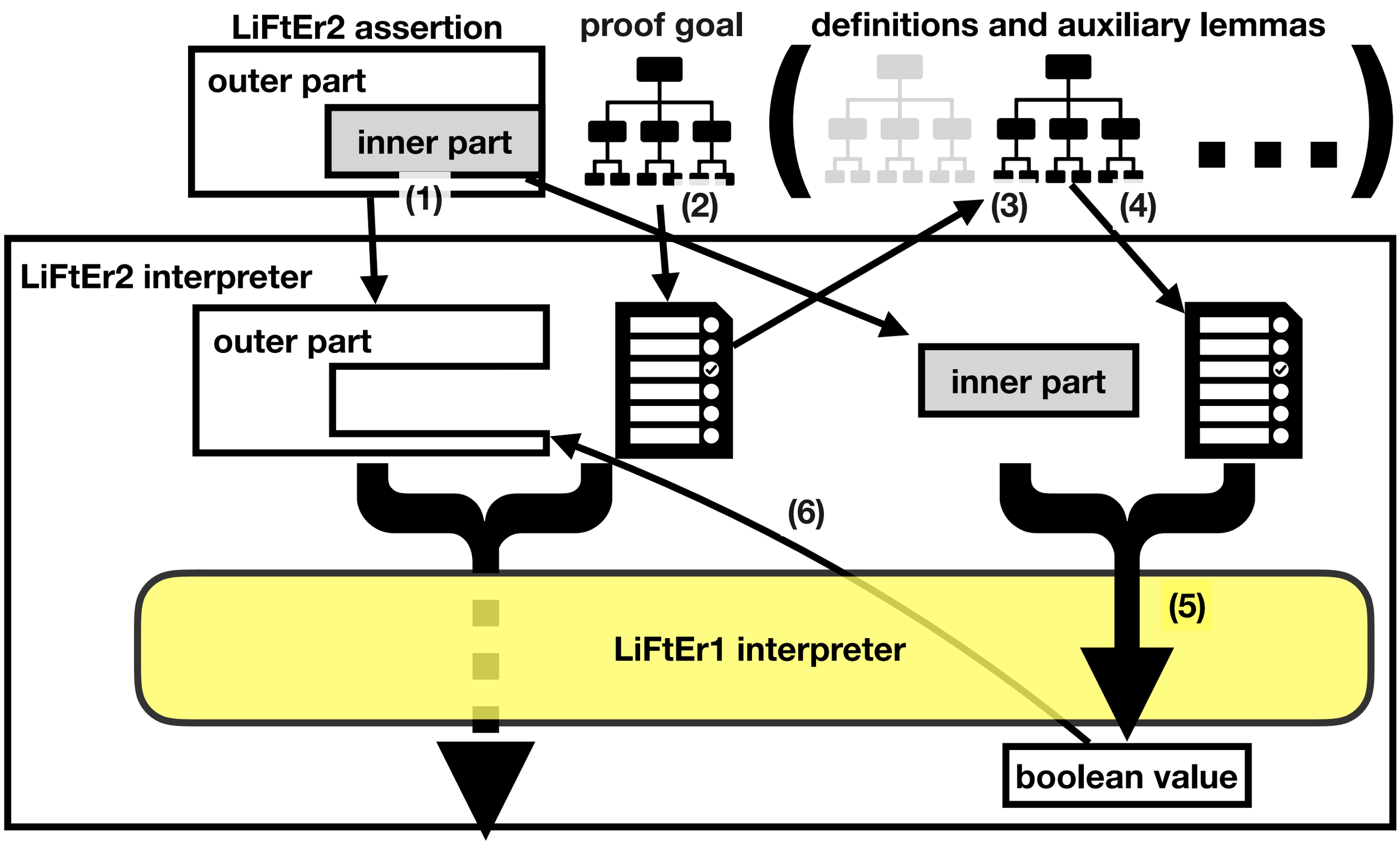
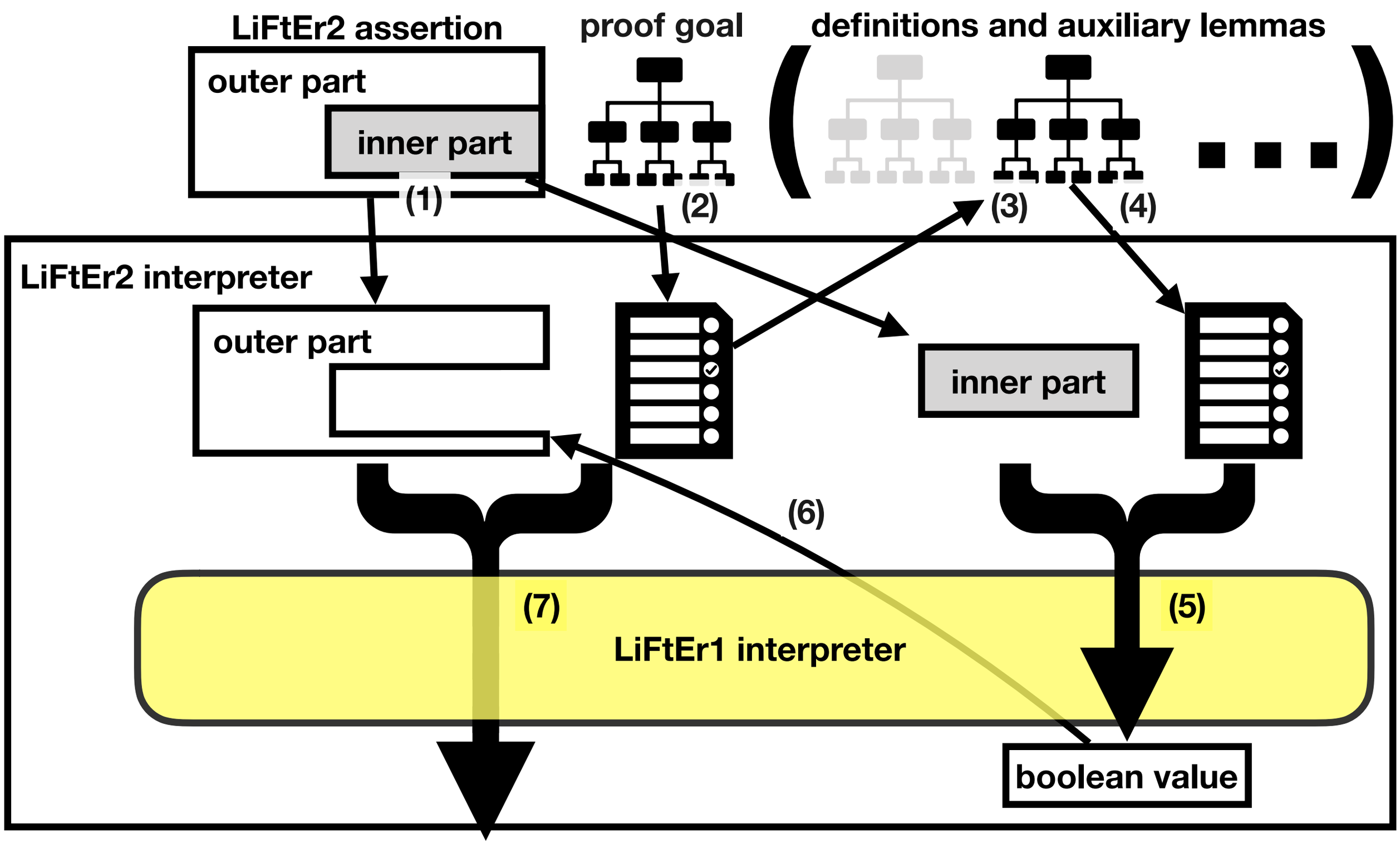
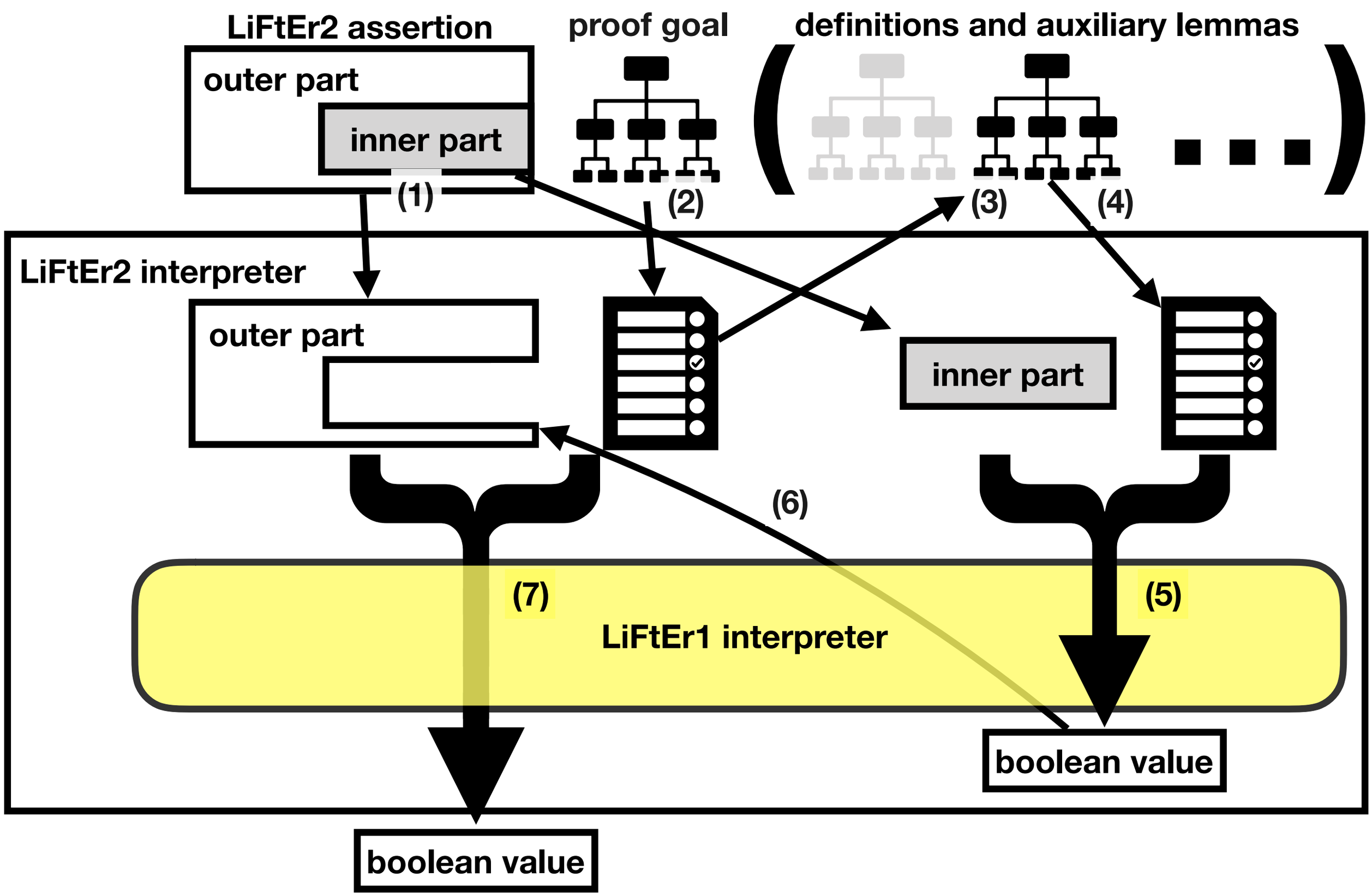


LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool

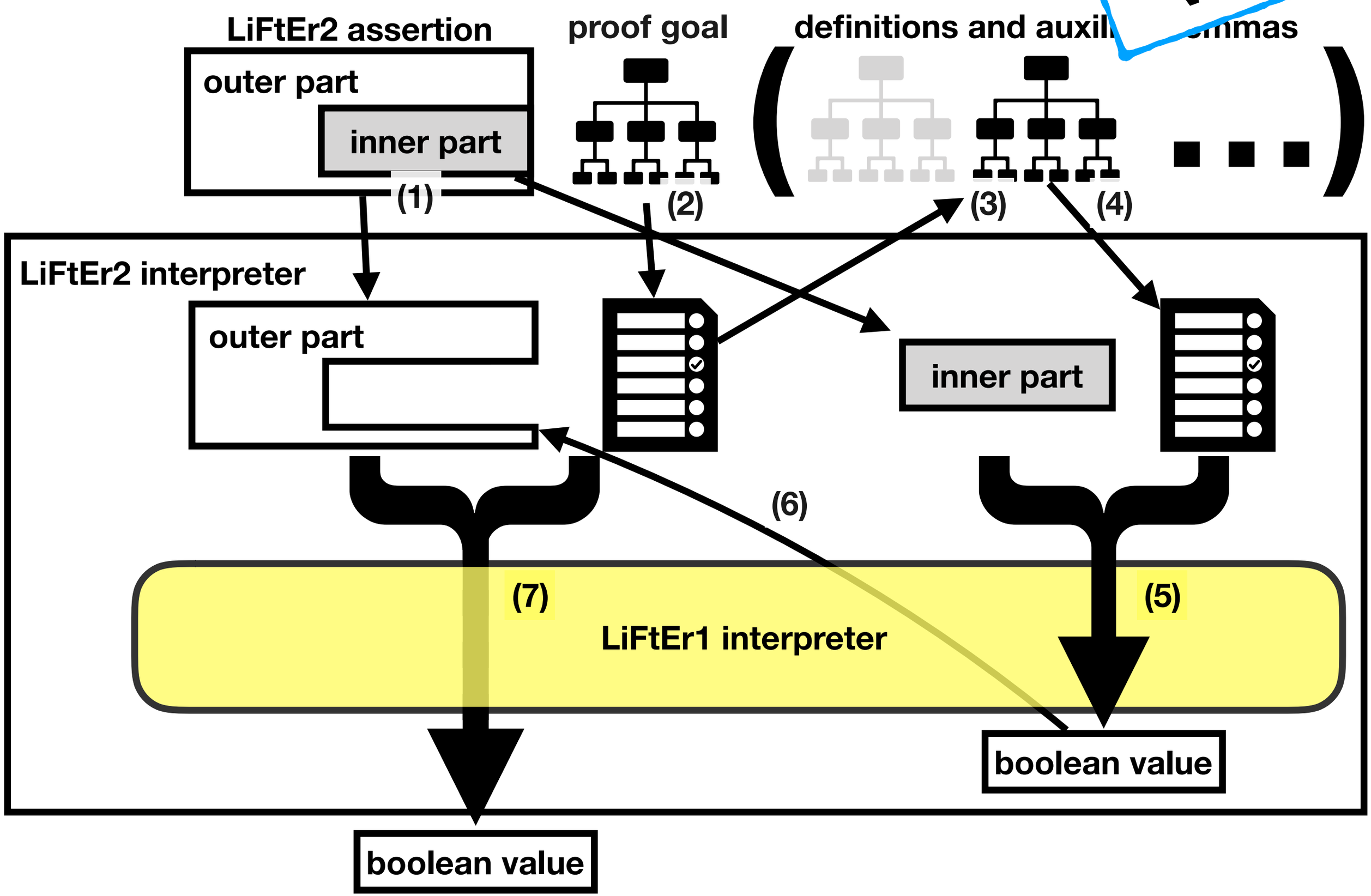


LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool

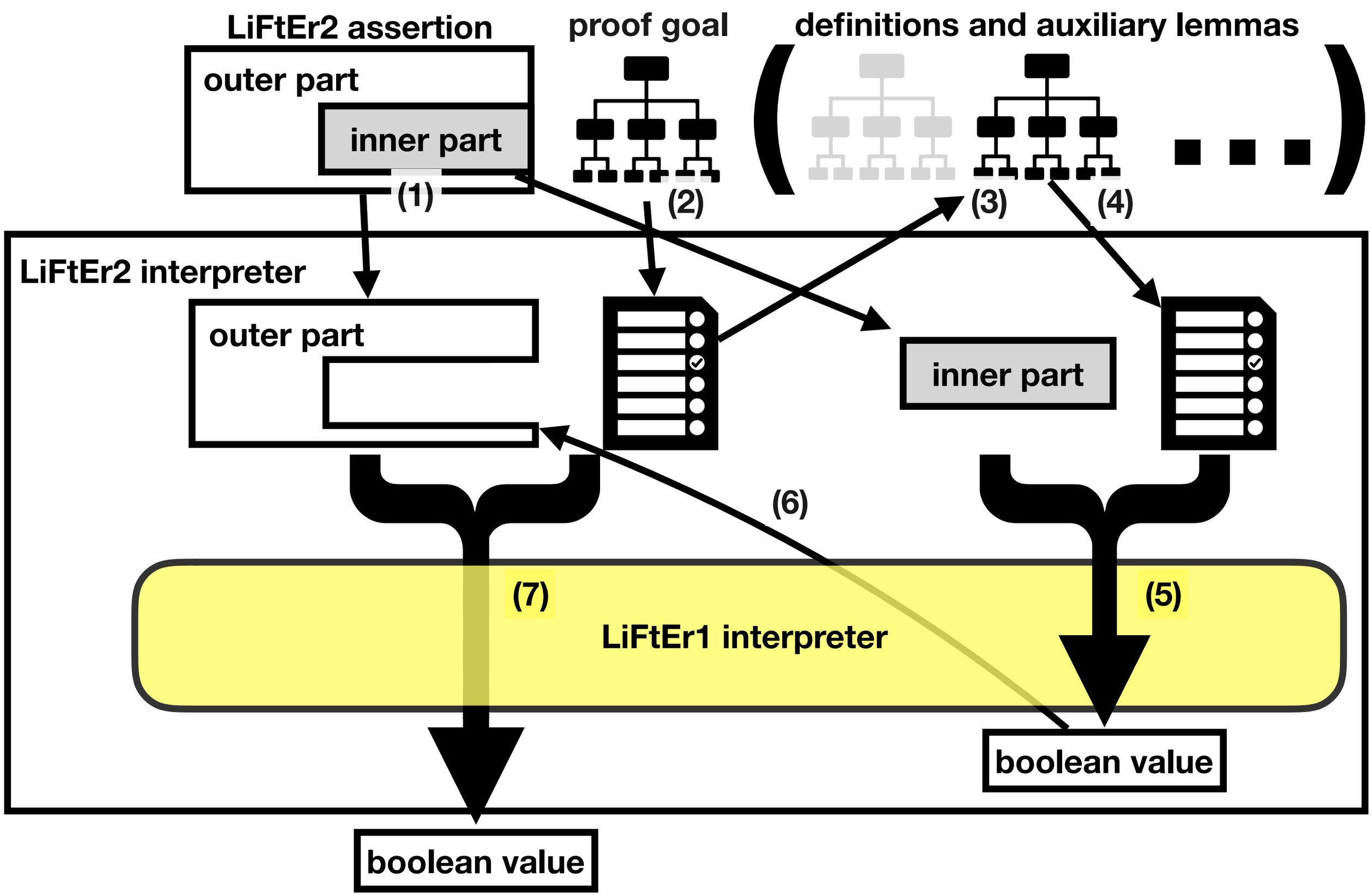


LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

WIP!



LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



<- simple representation


```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
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<- simple representation


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lemma "itrev xs ys = rev xs @ ys"
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```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev   []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

<- simple representation


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lemma "itrev xs ys = rev xs @ ys"
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 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
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<- relevant definitions

<- simple representation

LiFtEr

LiFtEr

LiFtEr

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr 2

LiFtEr 2

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
lemma "itrev xs ys = rev xs @
by(induct xs ys rule:"itrev.i                ) auto
```

<- relevant definitions

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev   []       = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

LiFtEr: (proof goal * induction arguments) -> bool

<- simple representation

LiFtEr LiFtEr LiFtEr

```
lemma "itrev xs ys = rev xs @ ys"
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```

LiFtEr 2 LiFtEr 2

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```

<- relevant definitions

```
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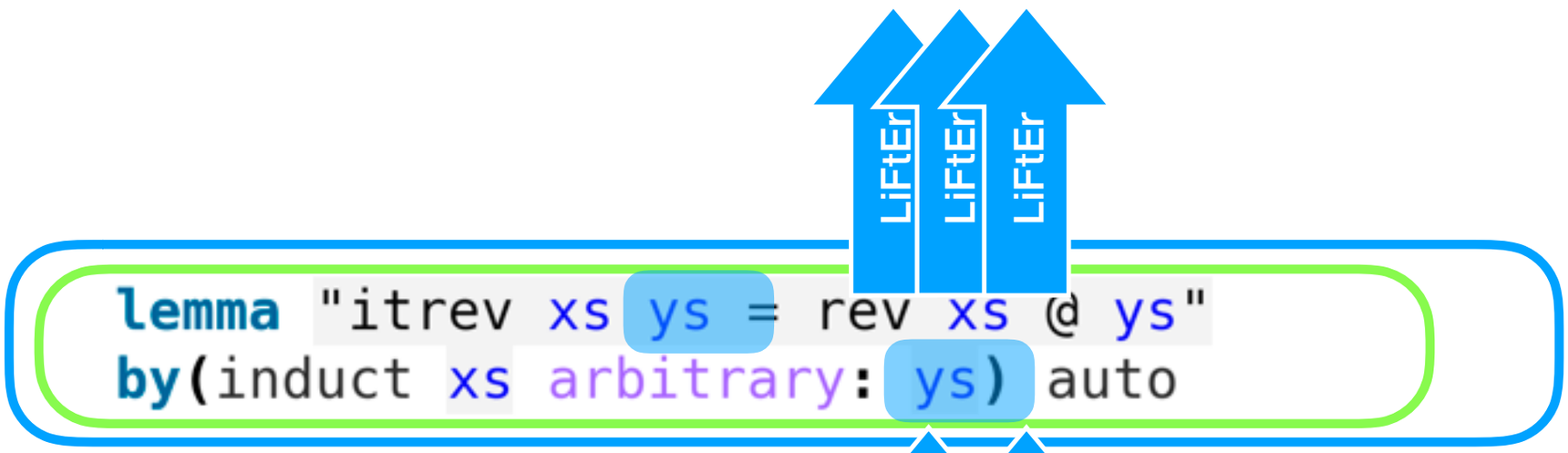
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

LiFtEr$_2$ (proof goal * induction arguments) -> bool
*** relevant definitions**

<- simple representation




```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev   []       = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

LiFtEr$_2$ (proof goal * induction arguments) -> bool
*** relevant definitions**

coming soon

LiFtEr LiFtEr LiFtEr

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr 2 LiFtEr 2

```
lemma "itrev xs ys = rev xs @
by(induct xs ys rule:"itrev.i                ) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev   []       = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions