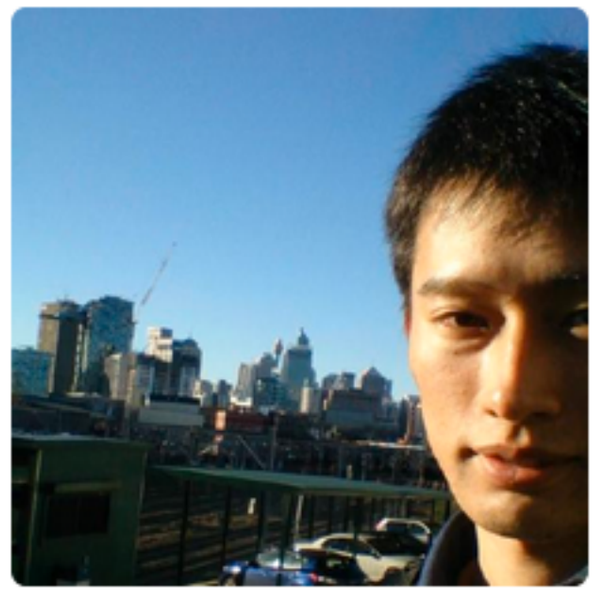
https://github.com/data61/PSL/blob/master/slide/2020_NUS.pdf

Automating proof by induction in Isabelle/HOL using DSLs

Yutaka Nagashima

universität innsbruck

CZECH INSTITUTE OF INFORMATICS ROBOTICS AND CYBERNETICS CTU IN PRAGUE



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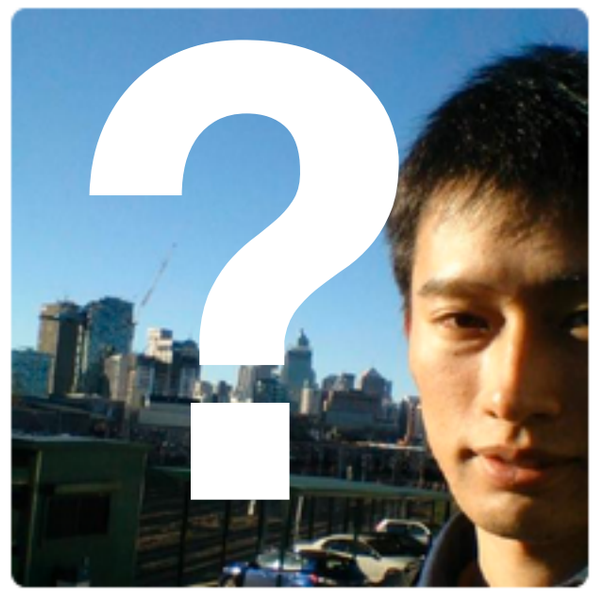

Yutaka Ng

https://github.com/data61/PSL/blob/master/slide/2020_NUS.pdf

Background

2013 ~ 2017

http://www.cse.unsw.edu.au/~kleing/

with Prof. Gerwin Klein

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Isabelle HOL

(seL4)

Security. Performance. Proof.

Cogent

with Prof. Gerwin Klein

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PSL

PaMpeR

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PSL

PaMpeR

2018 ~ 2020/21

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smart_induct

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smart_induct

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PSL: Proof Strategy Language

```
lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
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DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

Example.thy (~/Workplace/PSL/PGT/)

```
fun sep::"'a ⇒ 'a list ⇒ 'a list" where
  "sep a []       = []" |
  "sep a [x]      = [x]" |
  "sep a (x#y#zs) = x # a # sep a (y#zs)"
```

Proof state | Auto update | Update | Search: | 100%

```
consts
  sep :: "'a ⇒ 'a list ⇒ 'a list"
Found termination order: "(λp. length (snd p)) <*mlex*> {}"
```

Output | Query | Sledgehammer | Symbols

10,42 (221/935)

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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value "sep (1::int) [0,0,0]"
```

Proof state | Auto update | Update | Search: | 100%

```
"[0, 1, 0, 1, 0]"
  :: "int list"
```

Output | Query | Sledgehammer | Symbols

12,29 (251/963) (isabelle,isabelle,UTF-8-Isabelle) 1:17 AM

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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lemma
  "map f (sep x xs) = sep (f x) (map f xs)"
```

Proof state | Auto update | Update | Search: | 100%

```
proof (prove)
goal (1 subgoal):
 1. map f (sep x xs) = sep (f x) (map f xs)
```

Output | Query | Sledgehammer | Symbols

17,44 (304/1012) | Input/output complete | (isabelle,isabelle,UTF-8-Isabelle) | 241/535MB 1:17 AM

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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value "sep (1::int) [0,0,0]"

strategy DInd  = Thens [Dynamic (Induct), Auto, IsSolved]

lemma
  "map f (sep x xs) = sep (f x) (map f xs)"
```

Proof state | Auto update | Update | Search: | 100%

Output | Query | Sledgehammer | Symbols

14,58 (310/1067) (isabelle,isabelle,UTF-8-Isabelle) 1:20 AM

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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```
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value "sep (1::int) [0,0,0]"

strategy DInd  = Thens [Dynamic (Induct), Auto, IsSolved]

lemma
  "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd
```

Proof state | Auto update | Update | Search: | 100%

```
proof (prove)
goal (1 subgoal):
 1. map f (sep x xs) = sep (f x) (map f xs)
```

Output | Query | Sledgehammer | Symbols

17,44 (361/1084) (isabelle,isabelle,UTF-8-Isabelle) 242/535MB 1:05 AM

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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value "sep (1::int) [0,0,0]"

strategy DInd  = Thens [Dynamic (Induct), Auto, IsSolved]

lemma
  "map f (sep x xs) = sep (f x) (map f xs)"
  find_proof DInd
```

Proof state | Auto update | Update | Search: | 100%

```
Number of lines of commands: 3
apply (induct xs rule: Example.sep.induct)
apply auto
done
```

Output | Query | Sledgehammer | Symbols

18,18 (379/1084) (isabelle,isabelle,UTF-8-Isabelle) 251/535MB 1:05 AM

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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apply (induct xs rule: Example.sep.induct)
apply auto
done
```

Proof state | Auto update | Update | Search: | 100%

```
proof (prove)
goal:
No subgoals!
```

Output | Query | Sledgehammer | Symbols

20,11 (433/1147) (isabelle,isabelle,UTF-8-Isabelle) 255/535MB 1:05 AM

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

Example.thy (~/Workplace/PSL/PGT/)

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done
```

Proof state | Auto update | Update | Search: | 100%

```
proof (prove)
goal:
No subgoals!
```

What happened?

Output | Query | Sledgehammer | Symbols

20,11 (433/1147) (isabelle,isabelle,UTF-8-Isabelle) 535MB 1:05 AM

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

PSL: Proof Strategy Language

```
lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
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DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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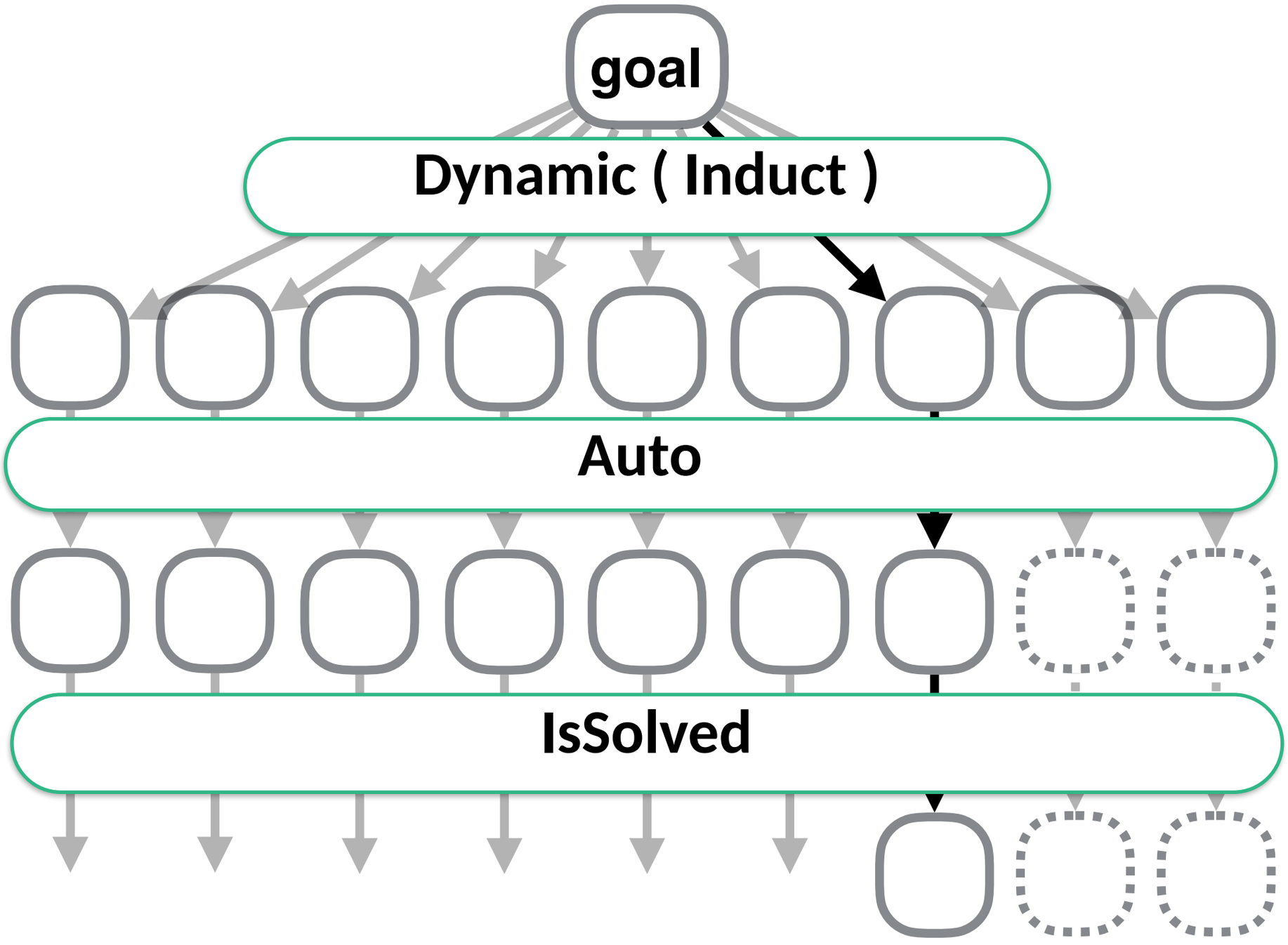

DEMO1

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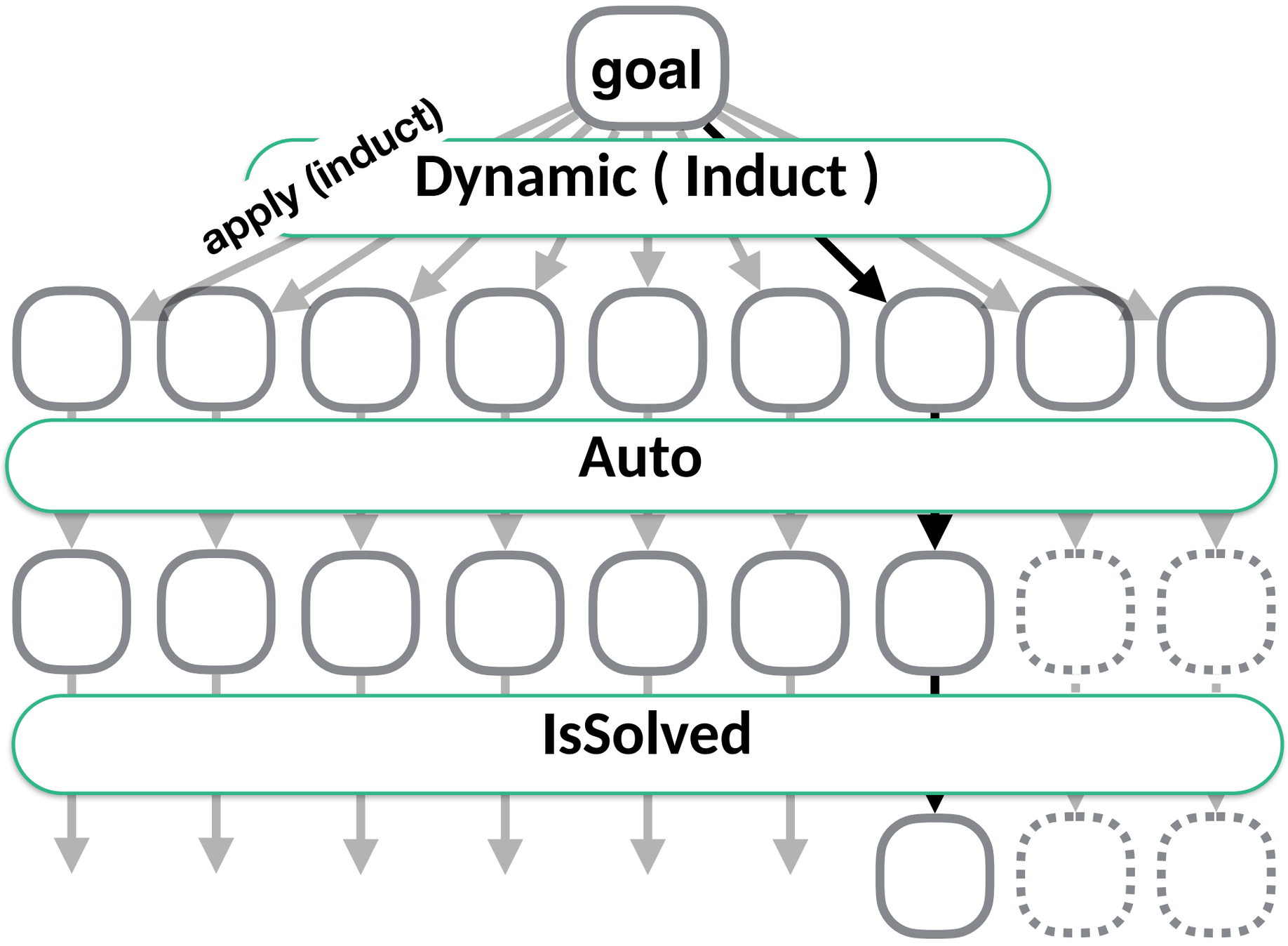

DEMO1

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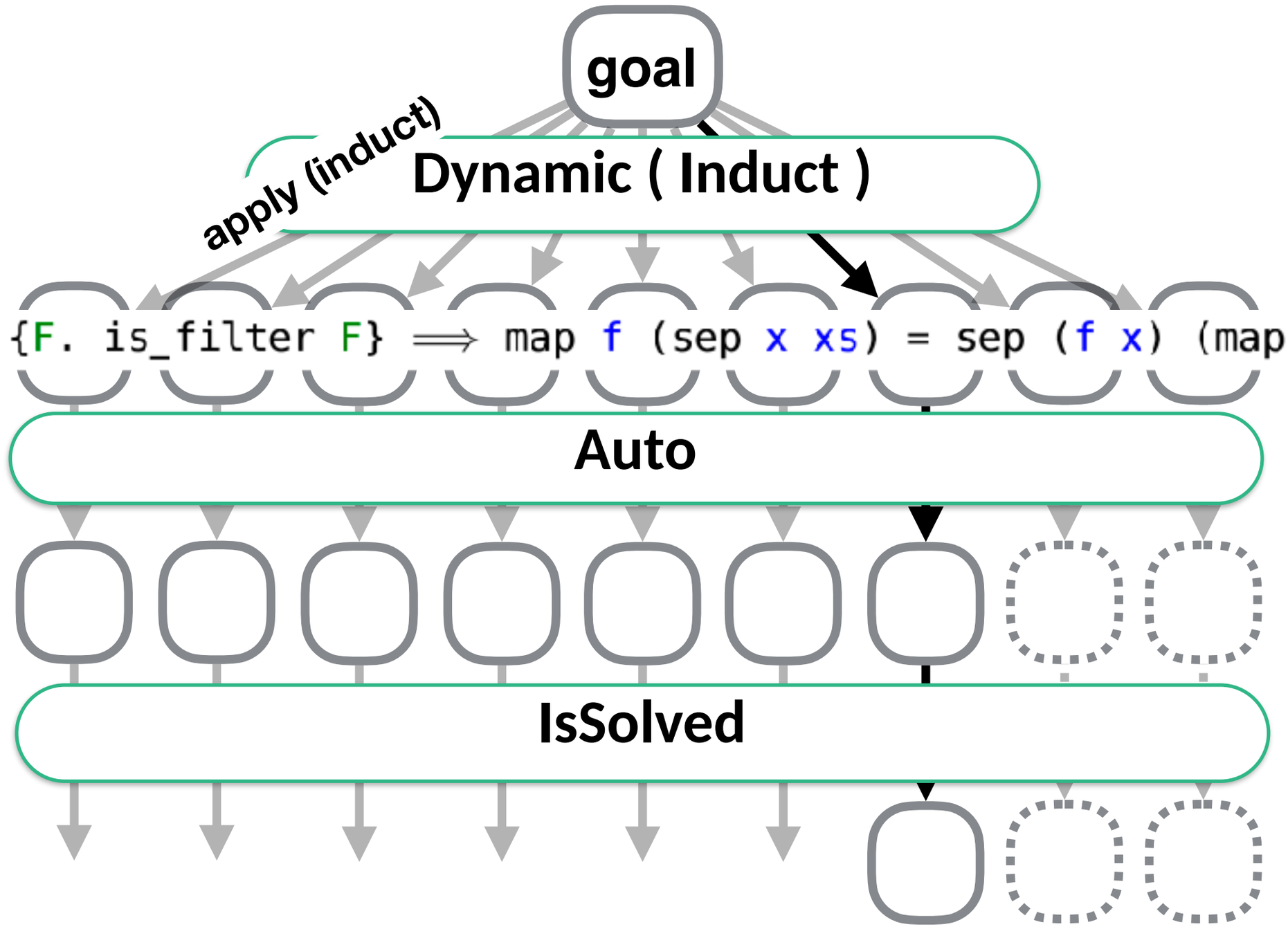


```
⋀y. y ∈ {F. is_filter F} ⟹ map f (sep x xs) = sep (f x) (map f xs)
```

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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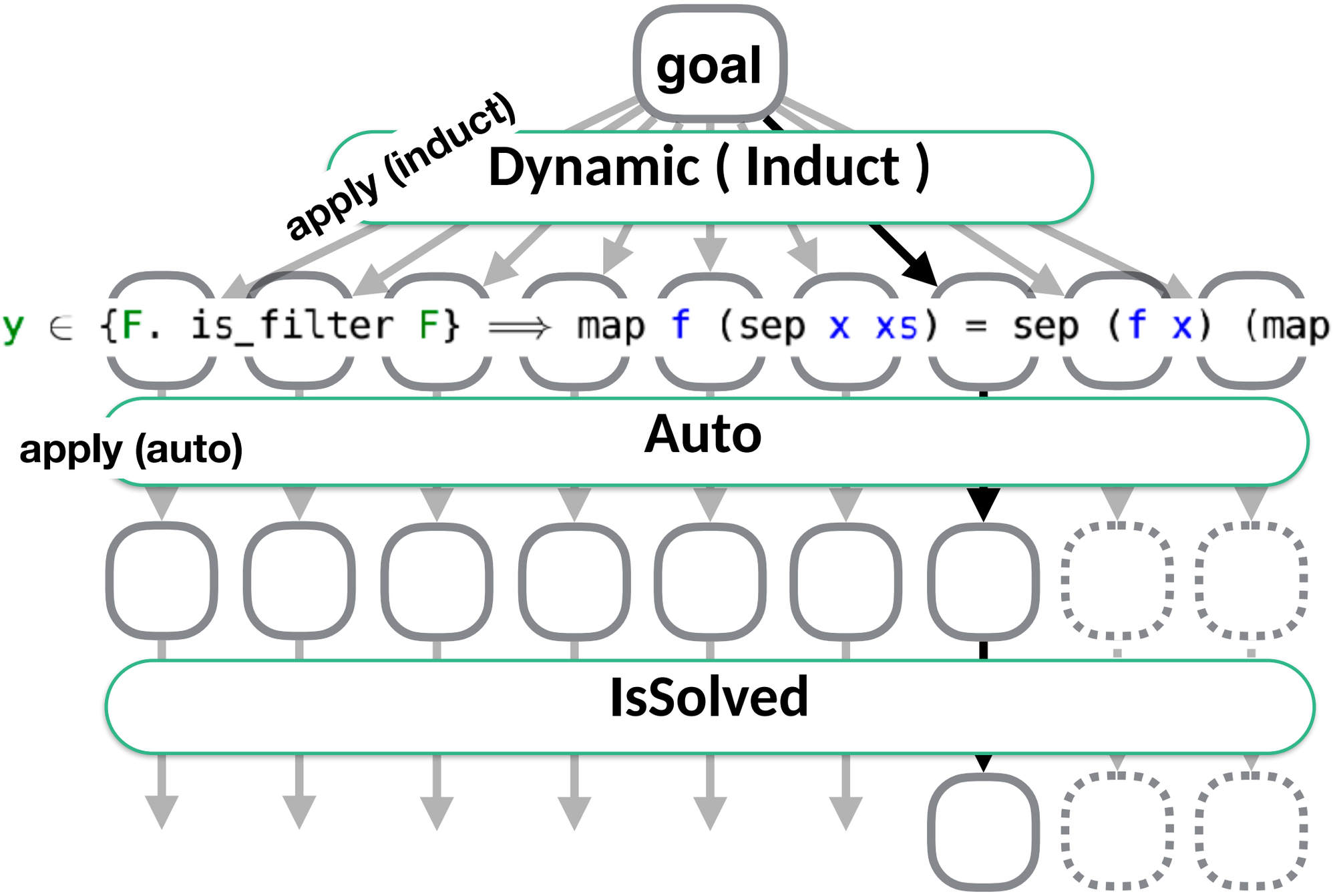

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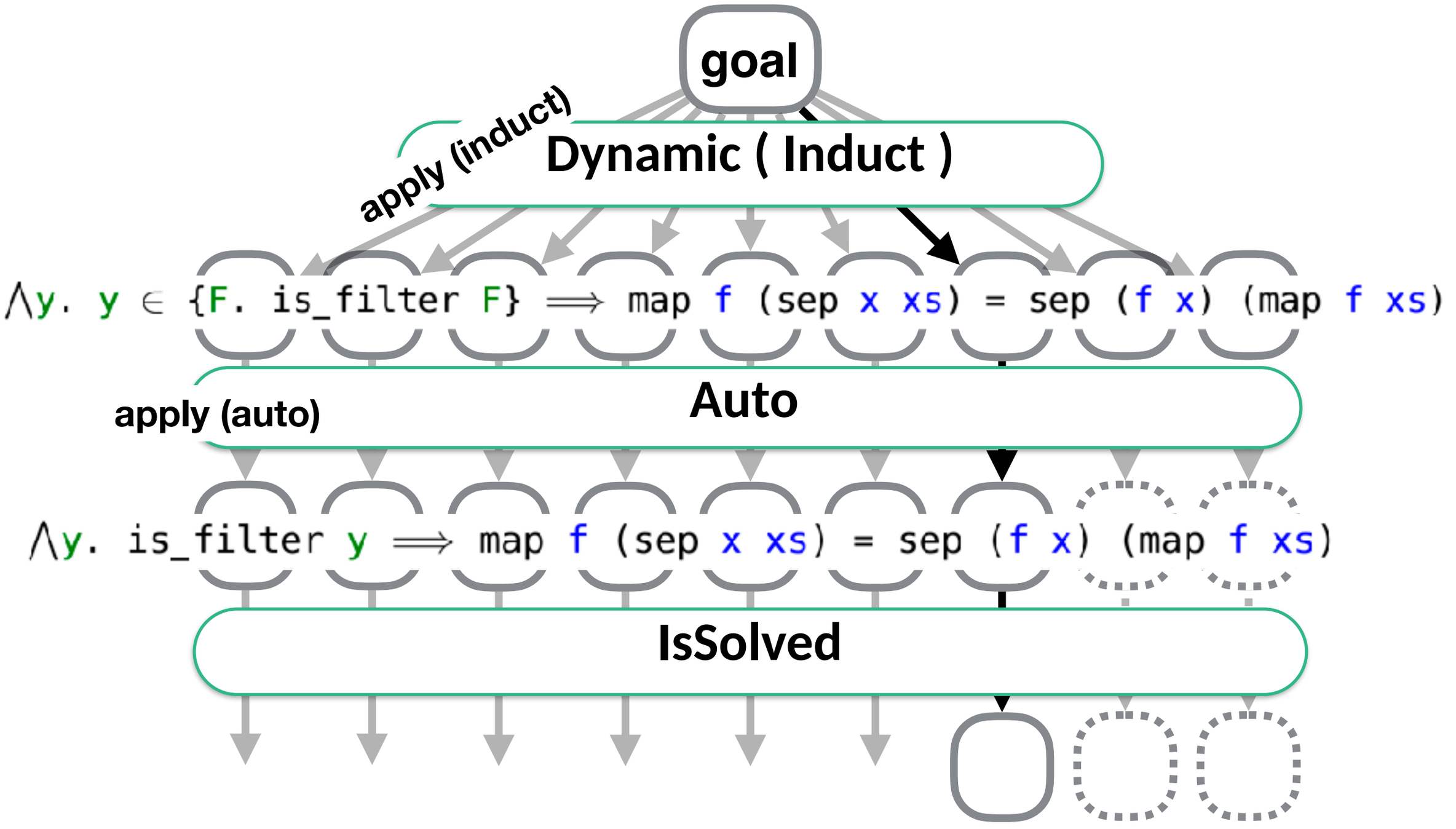

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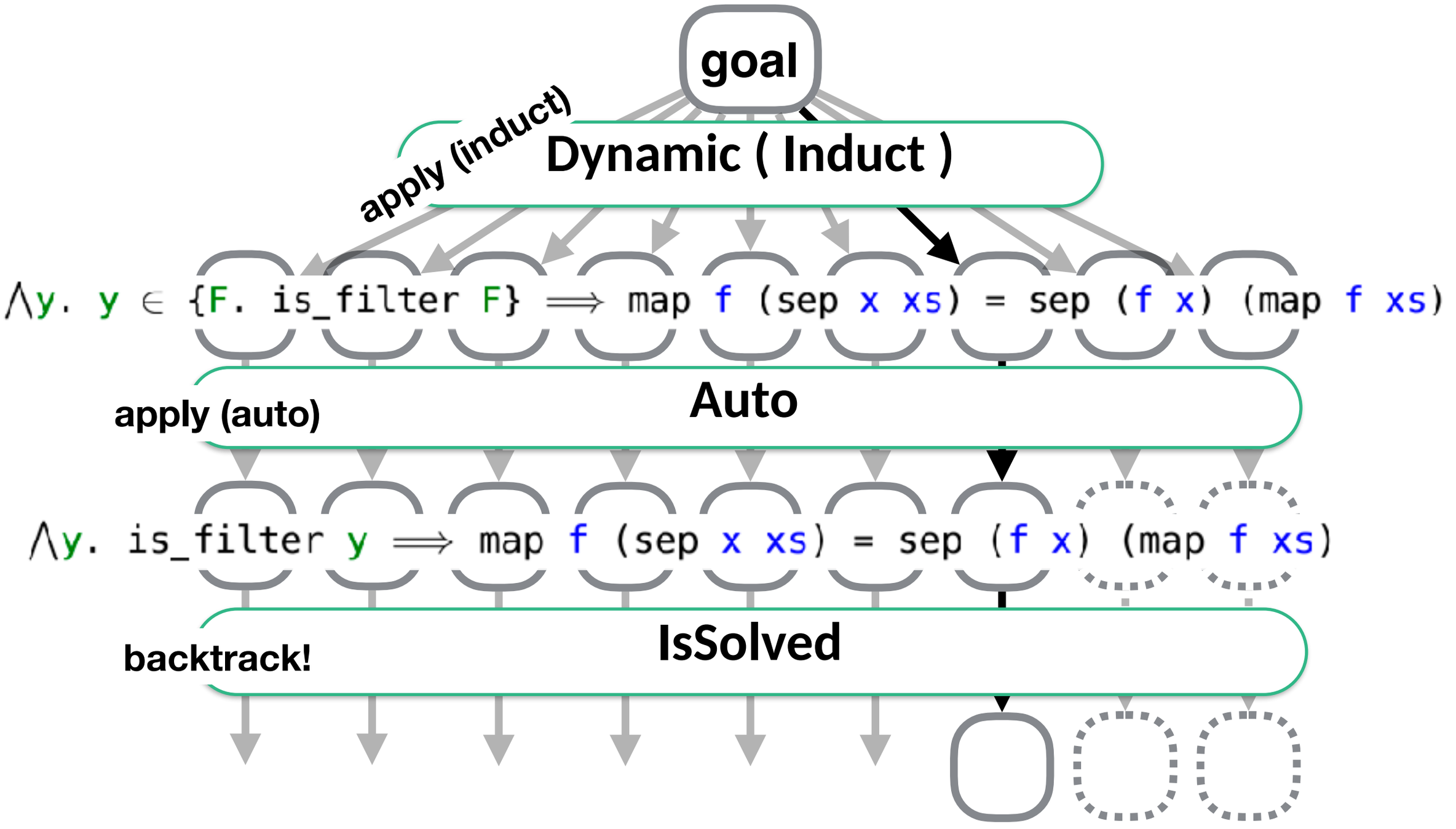

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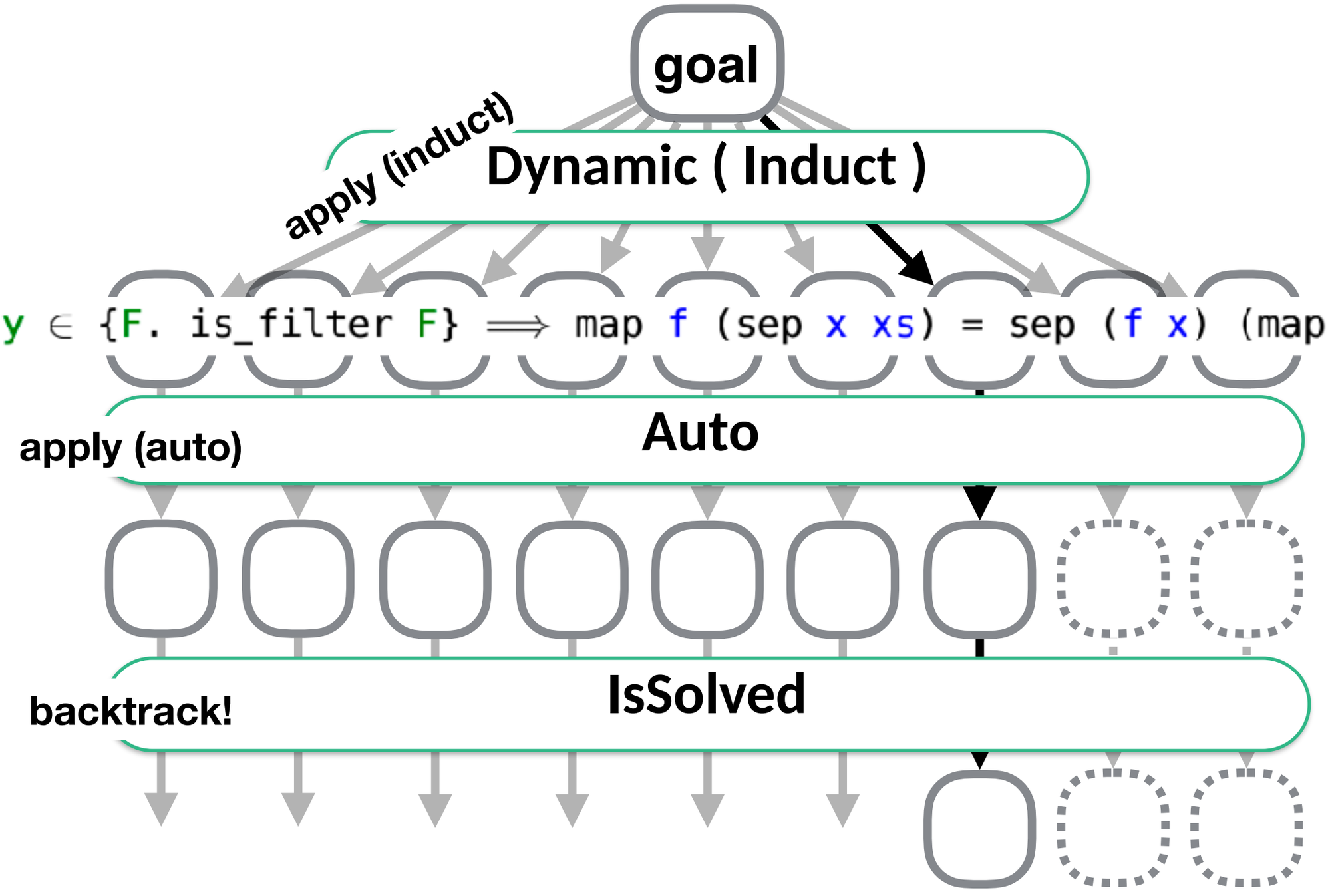

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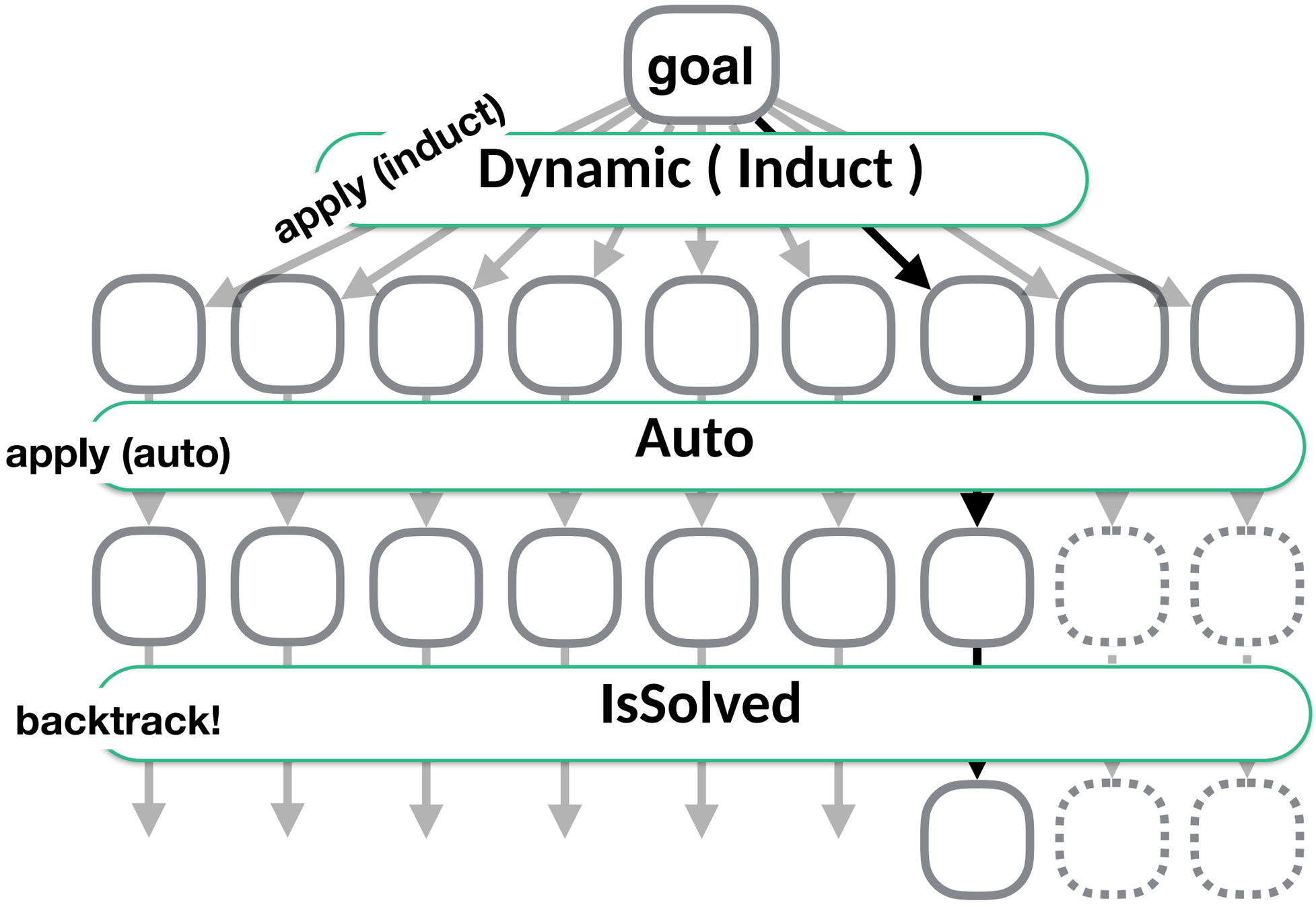

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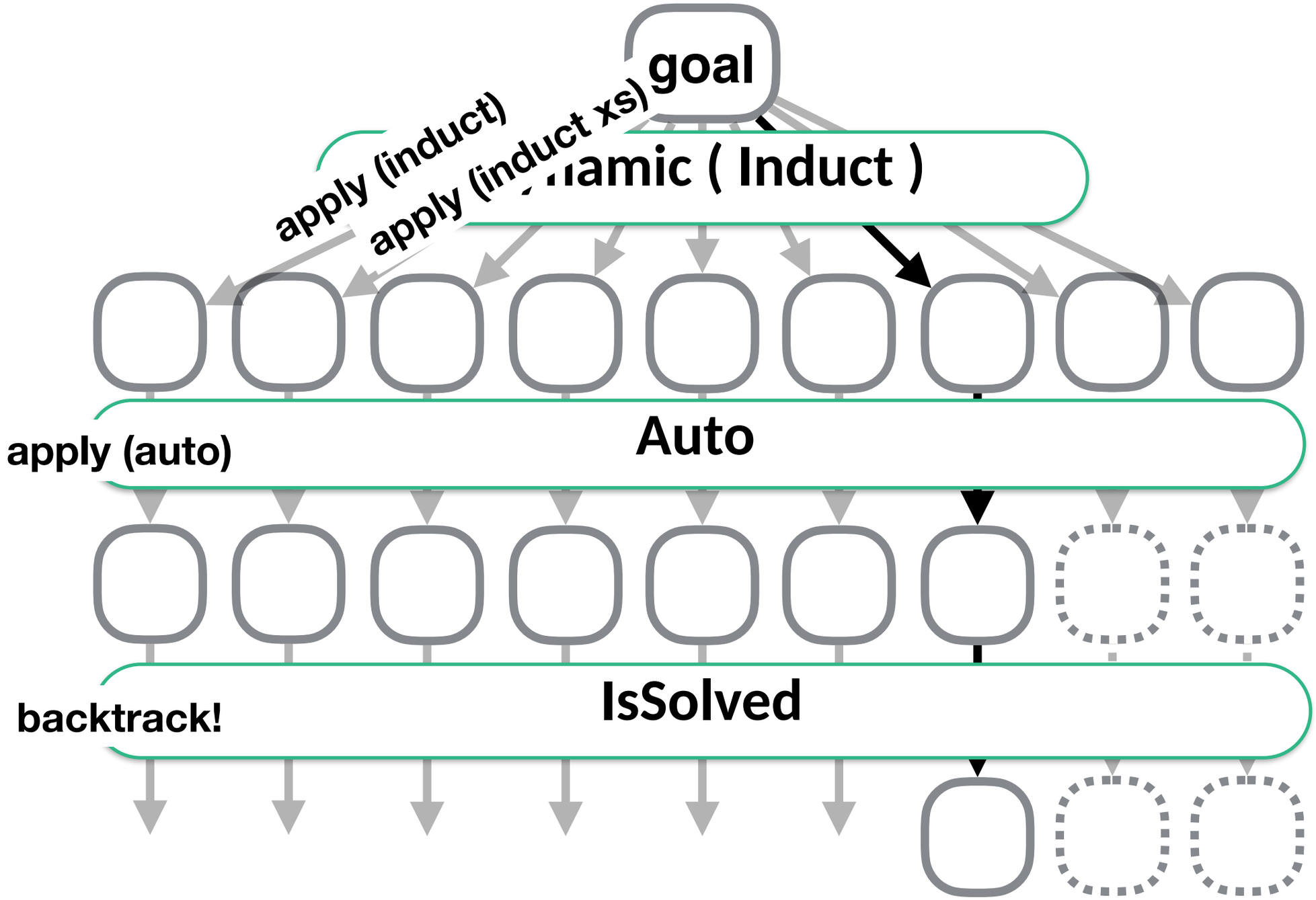

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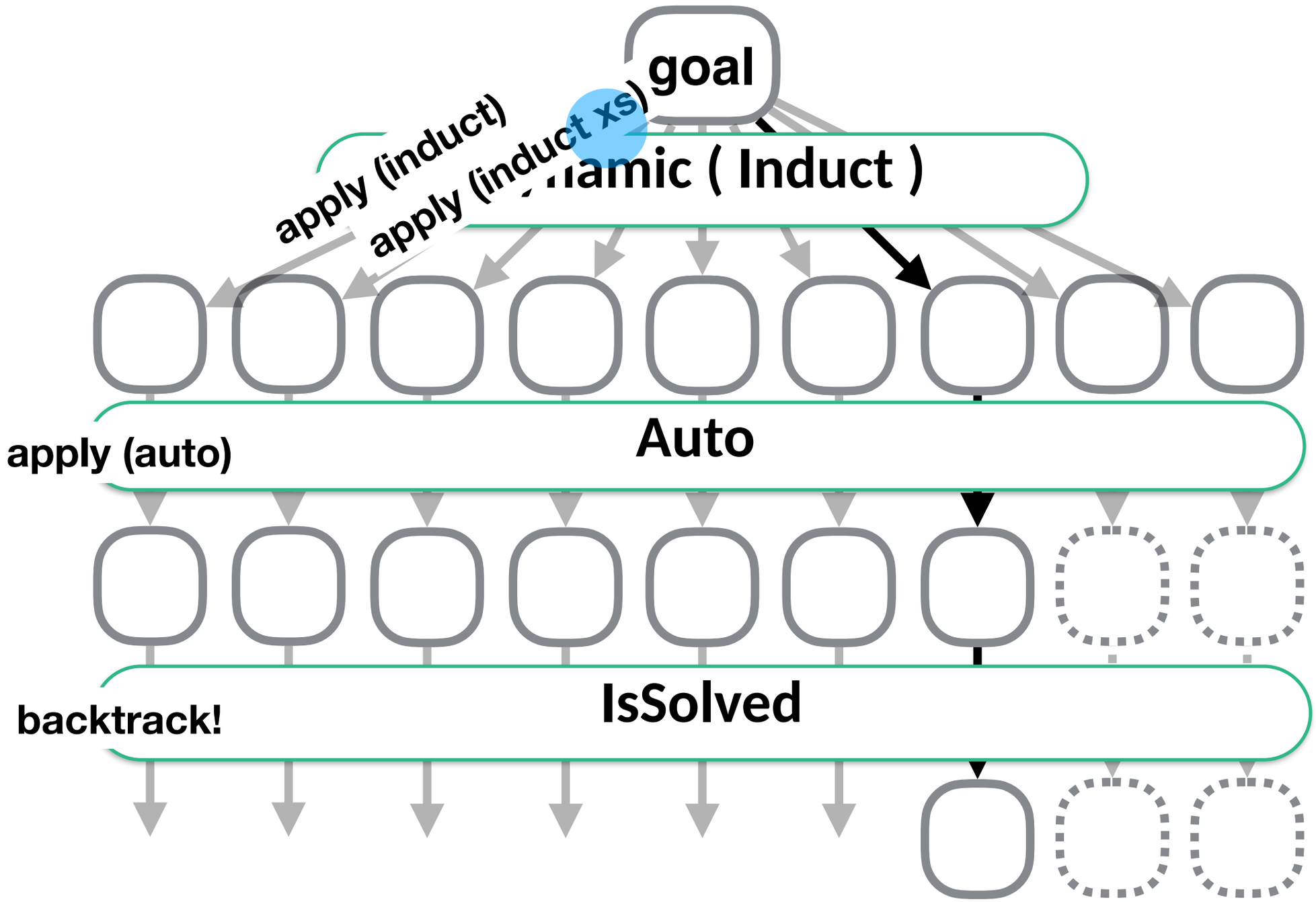

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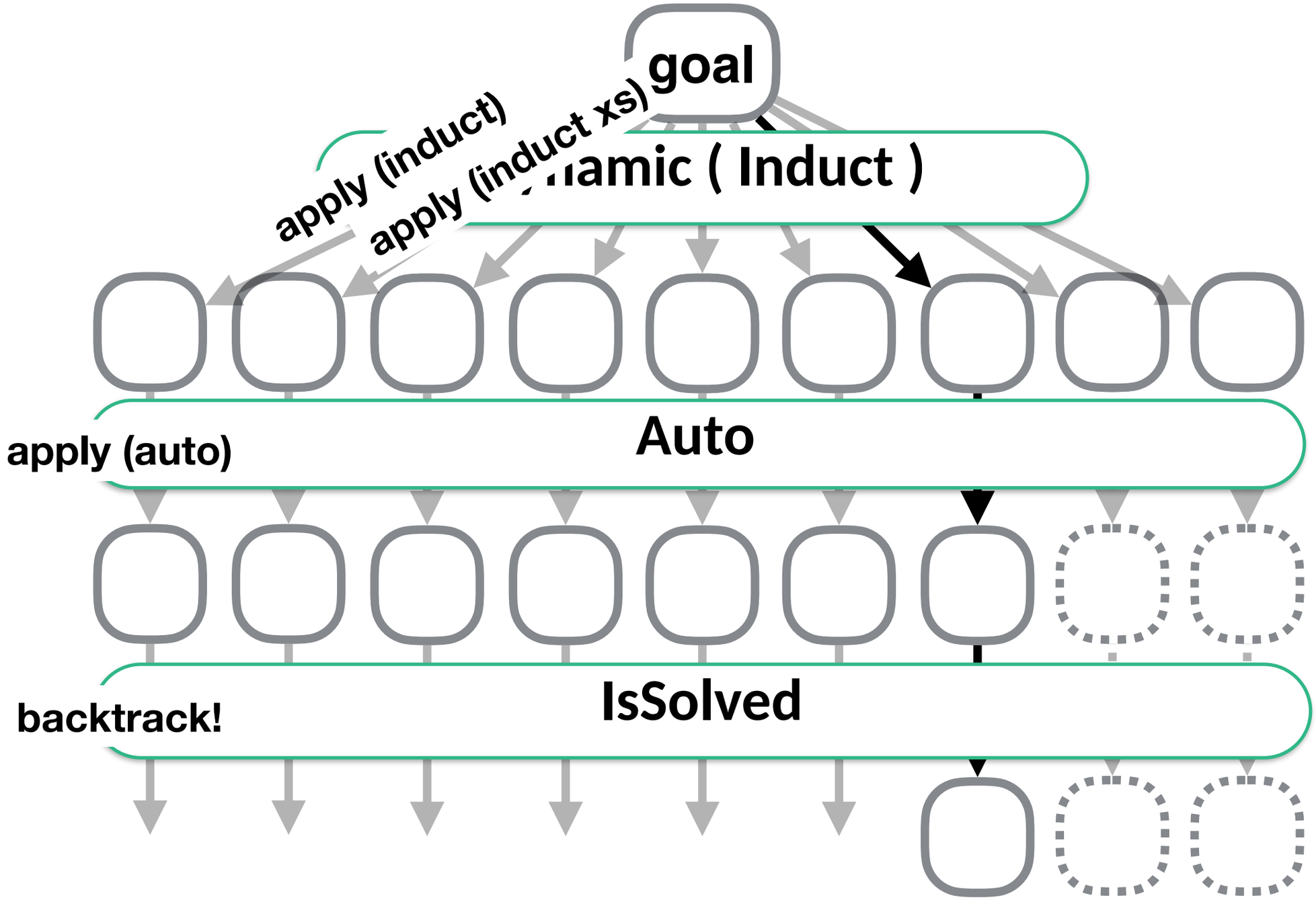

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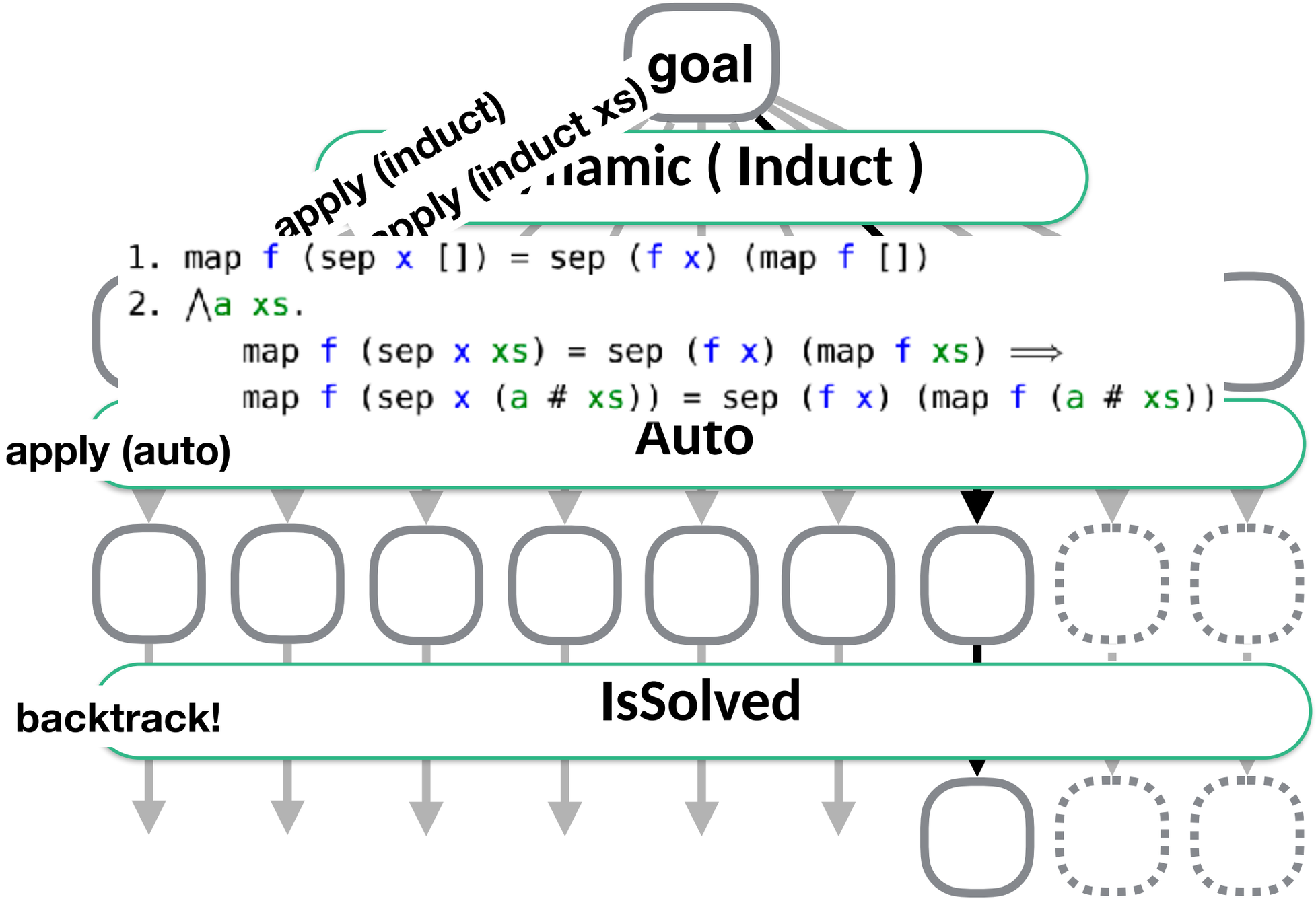

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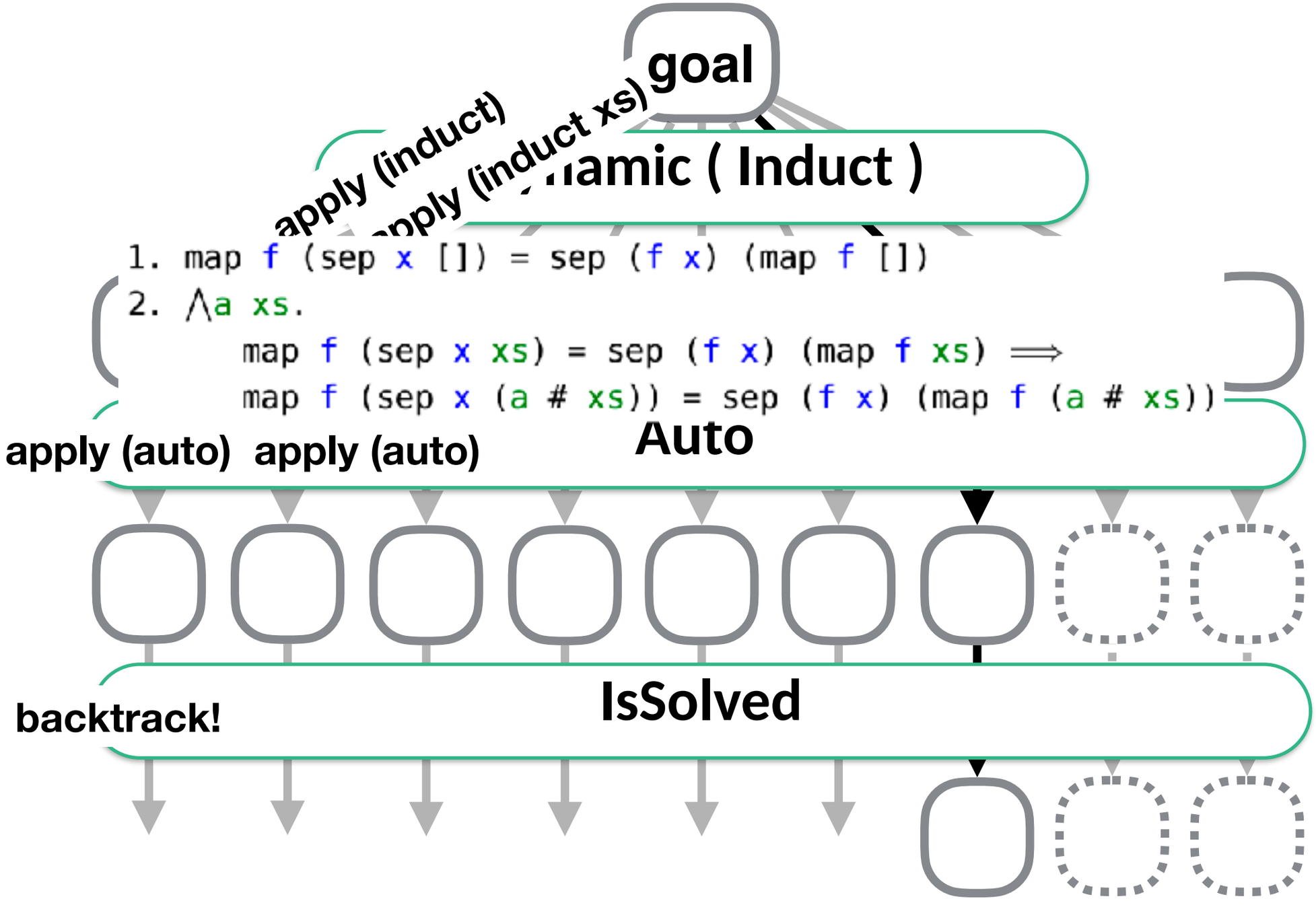

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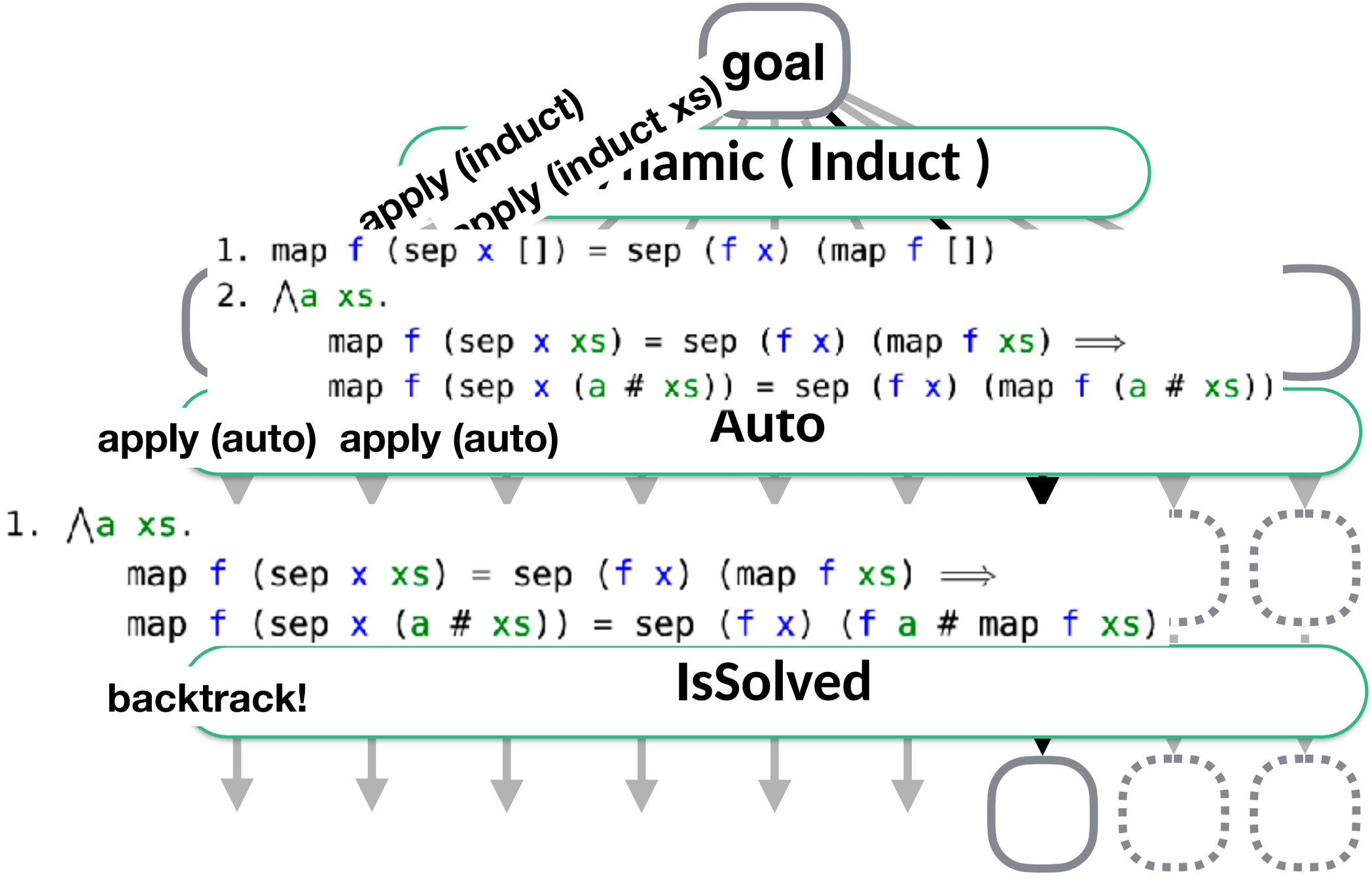

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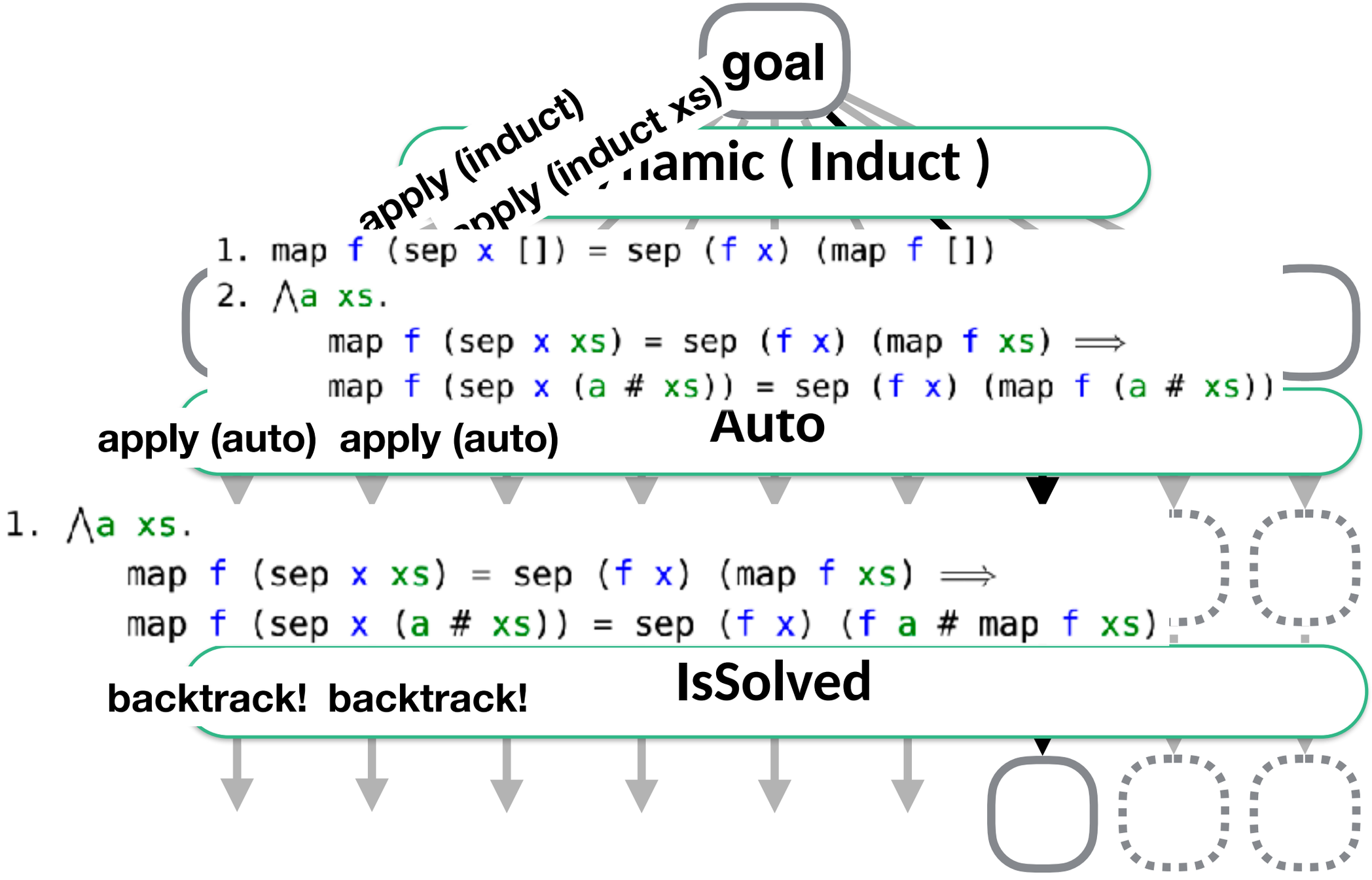

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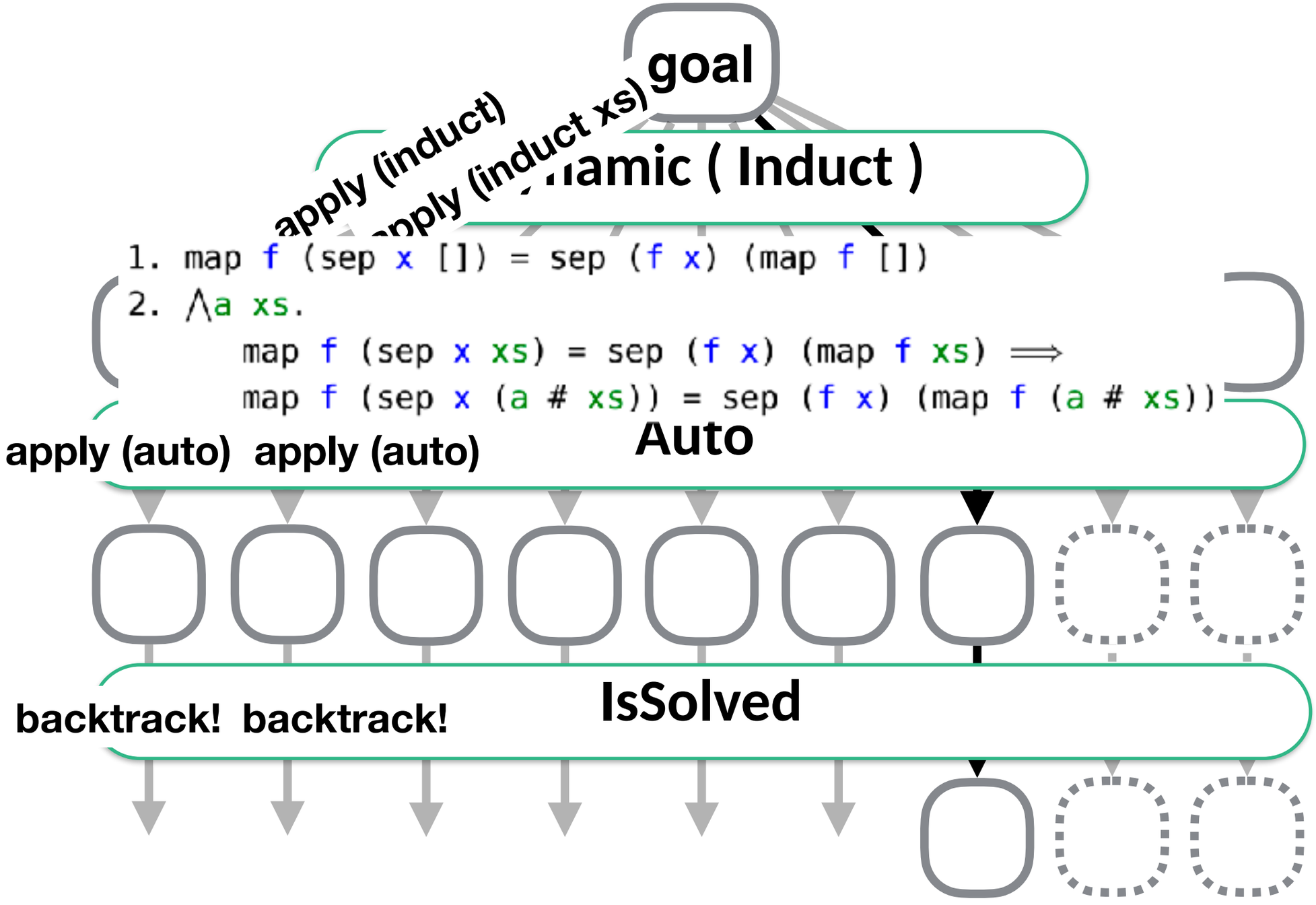

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PSL: Proof Strategy Language

```
lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
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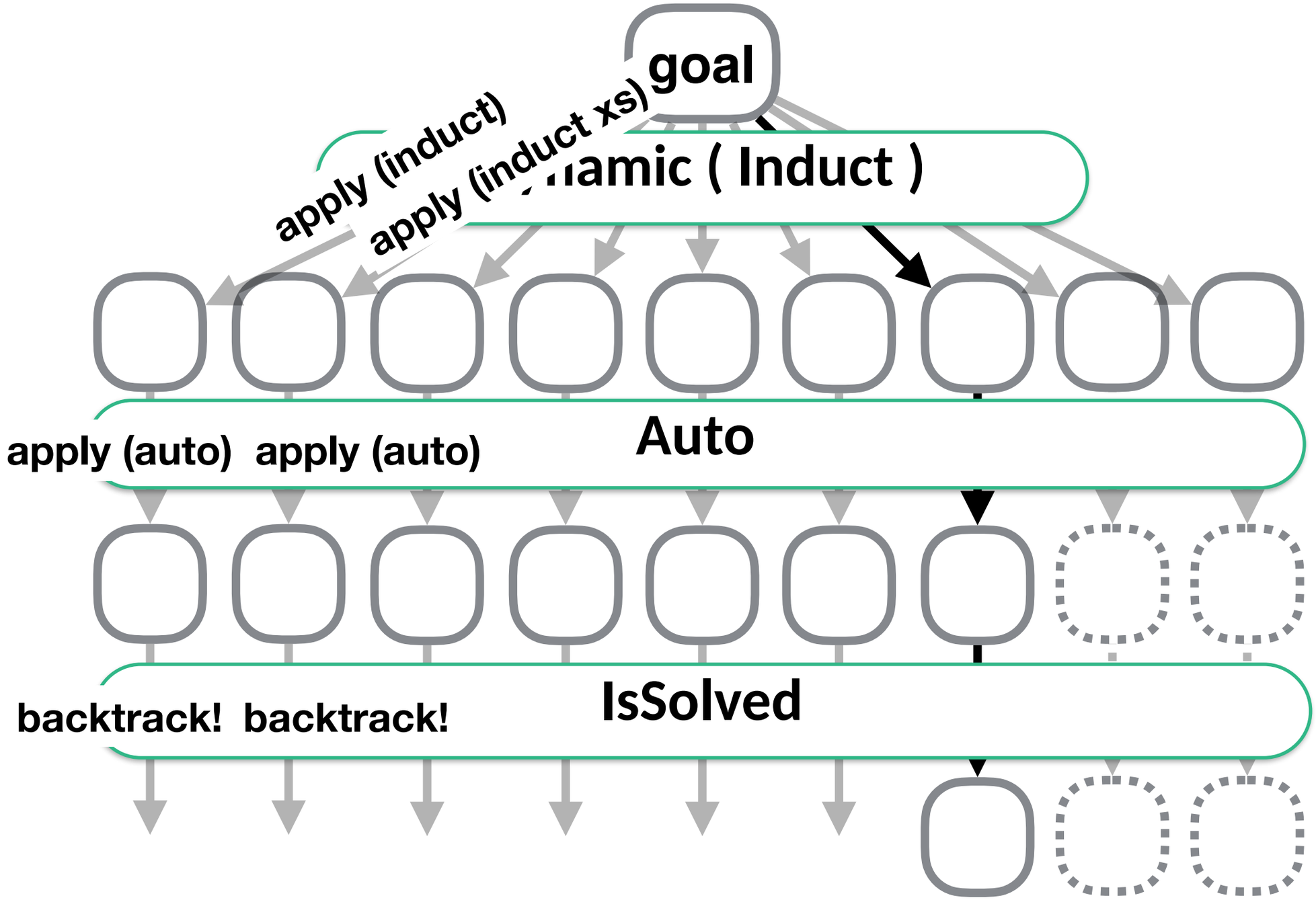

DEMO1

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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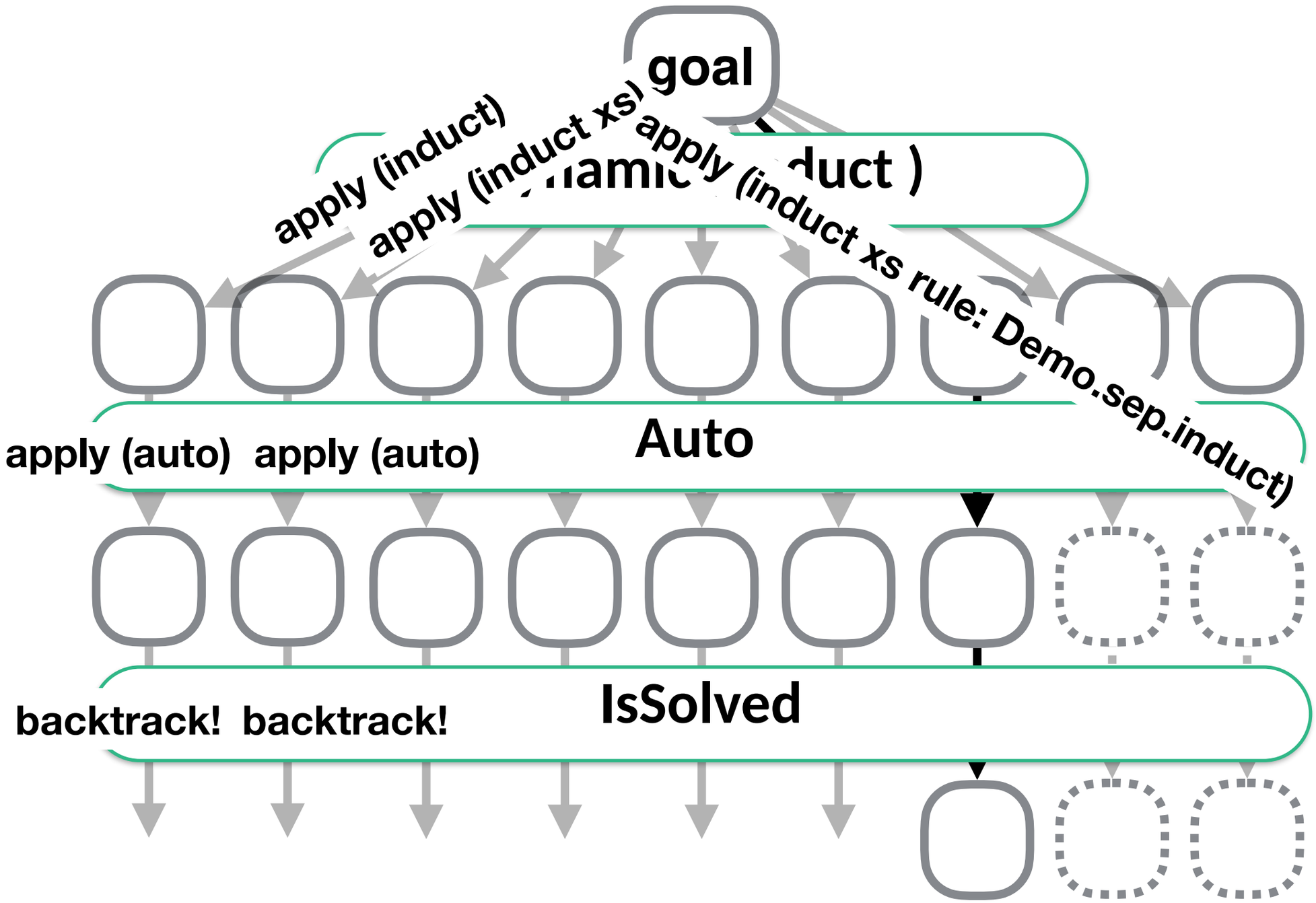

DEMO1

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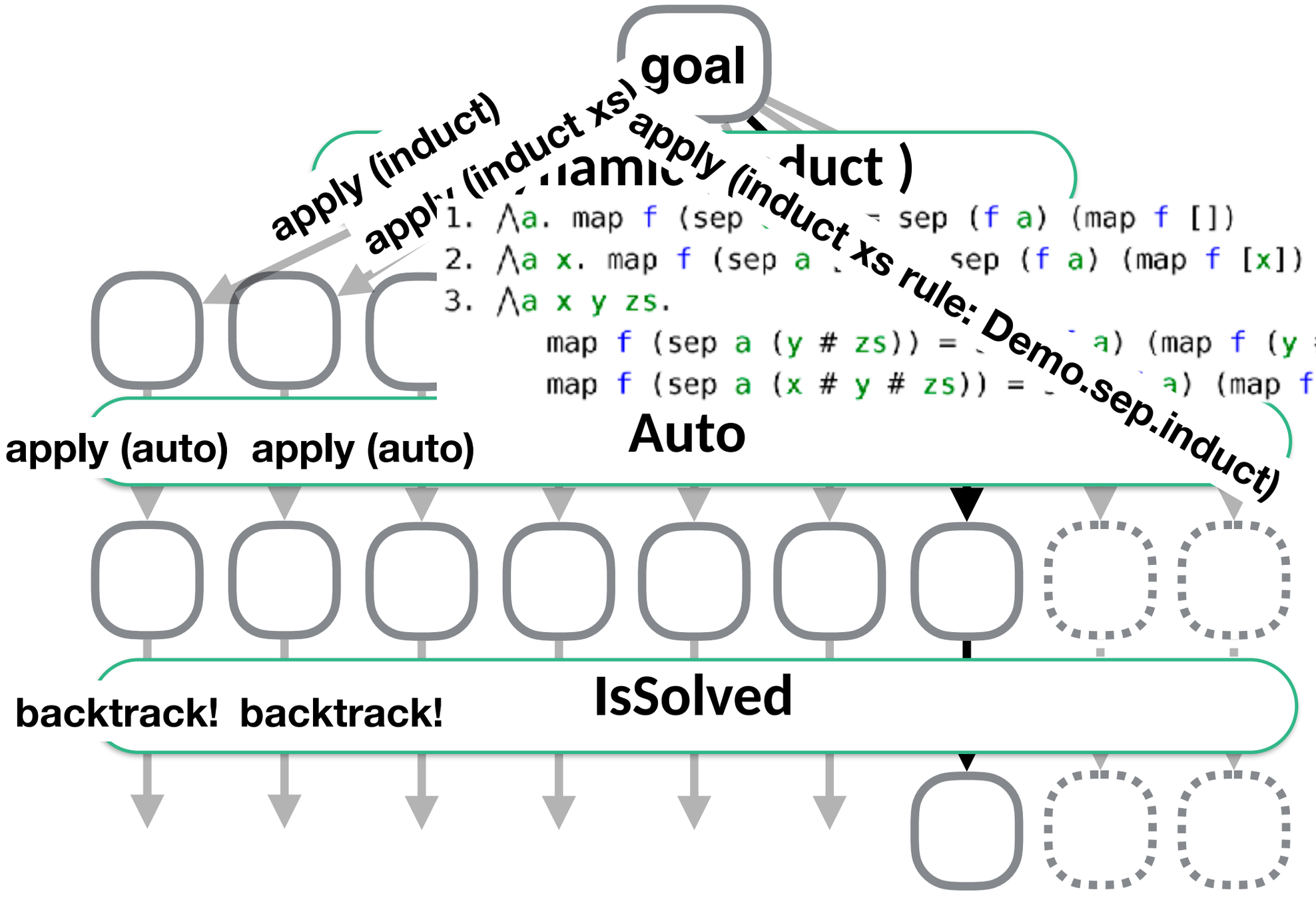

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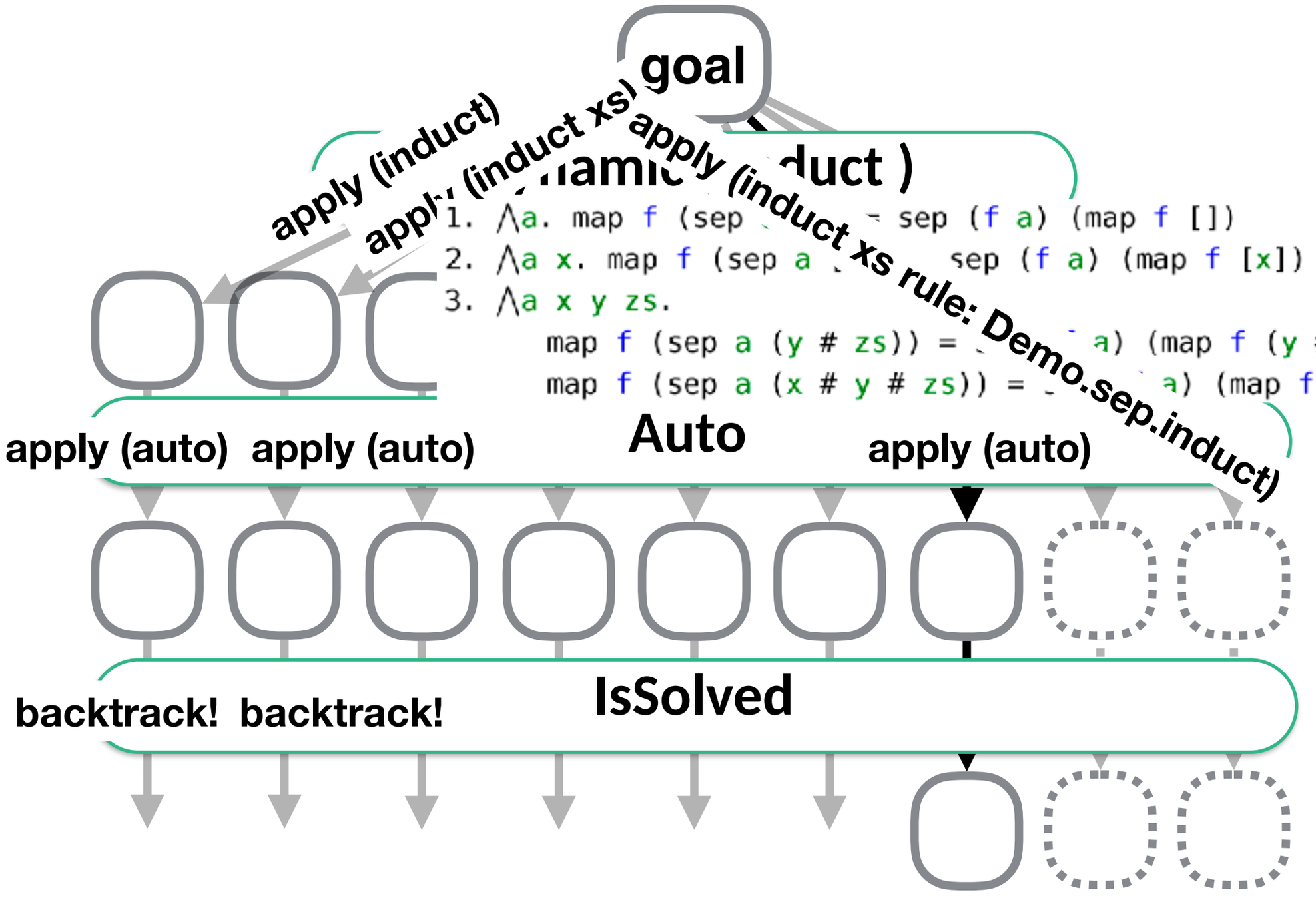

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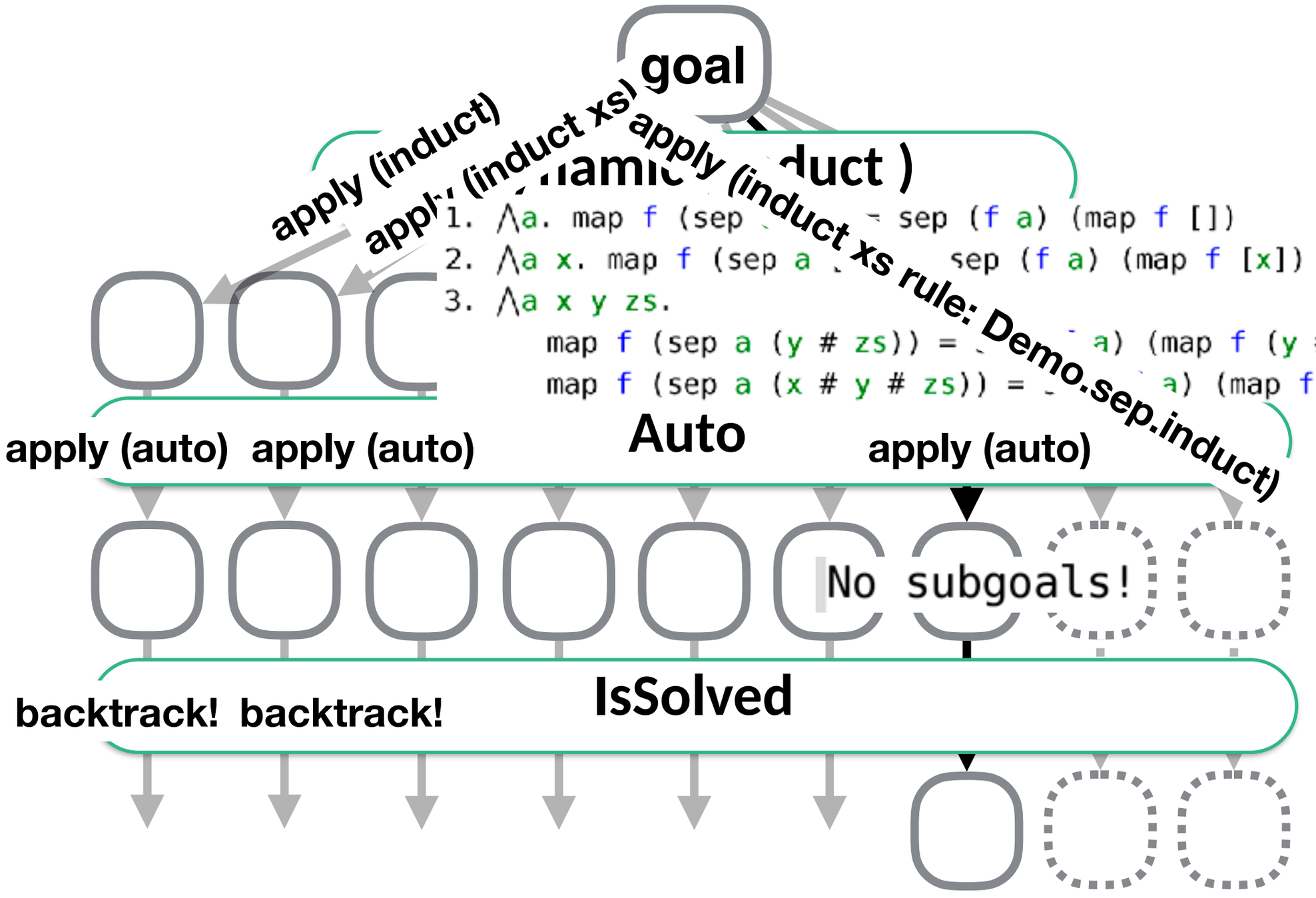

DEMO1

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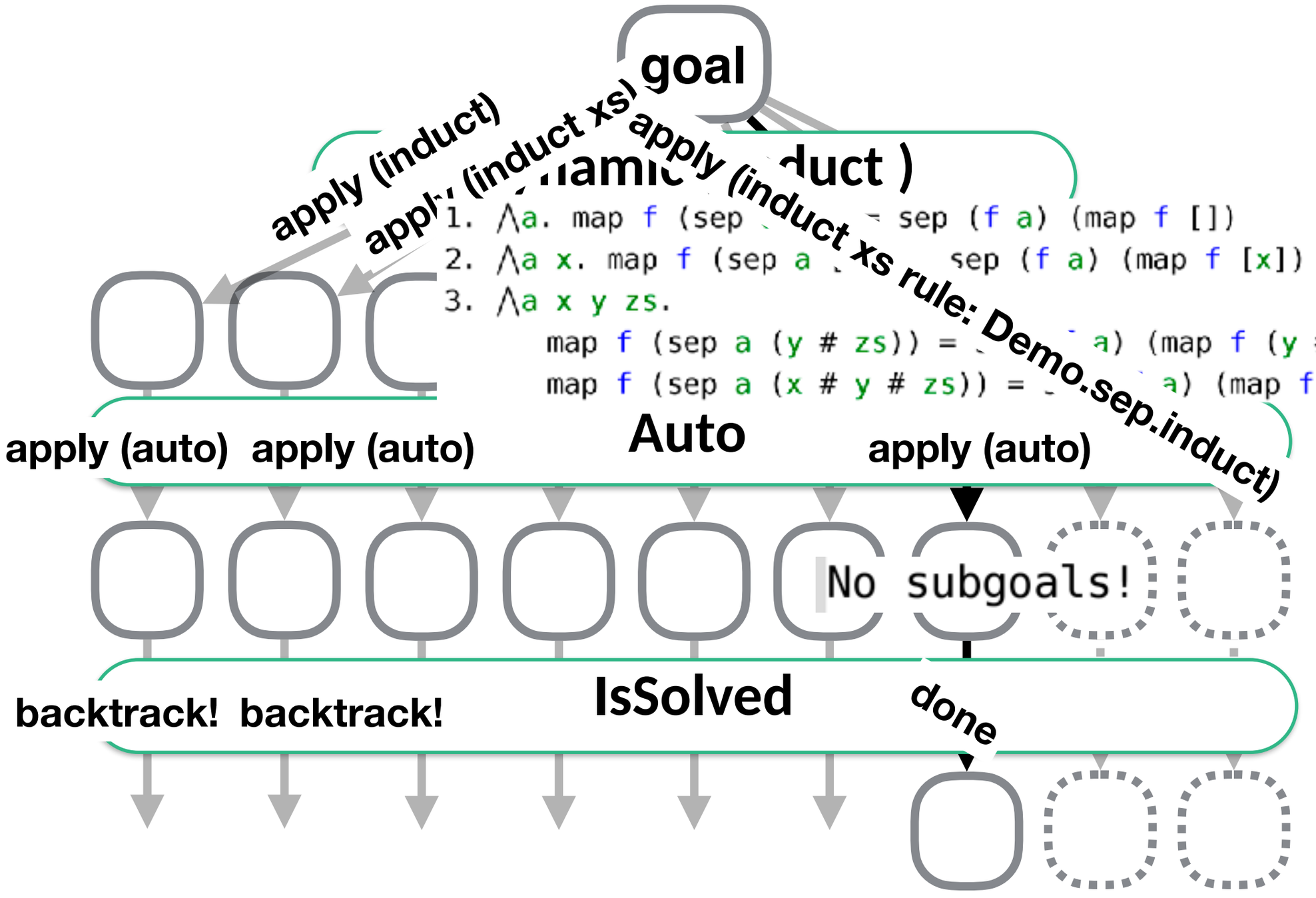

DEMO1

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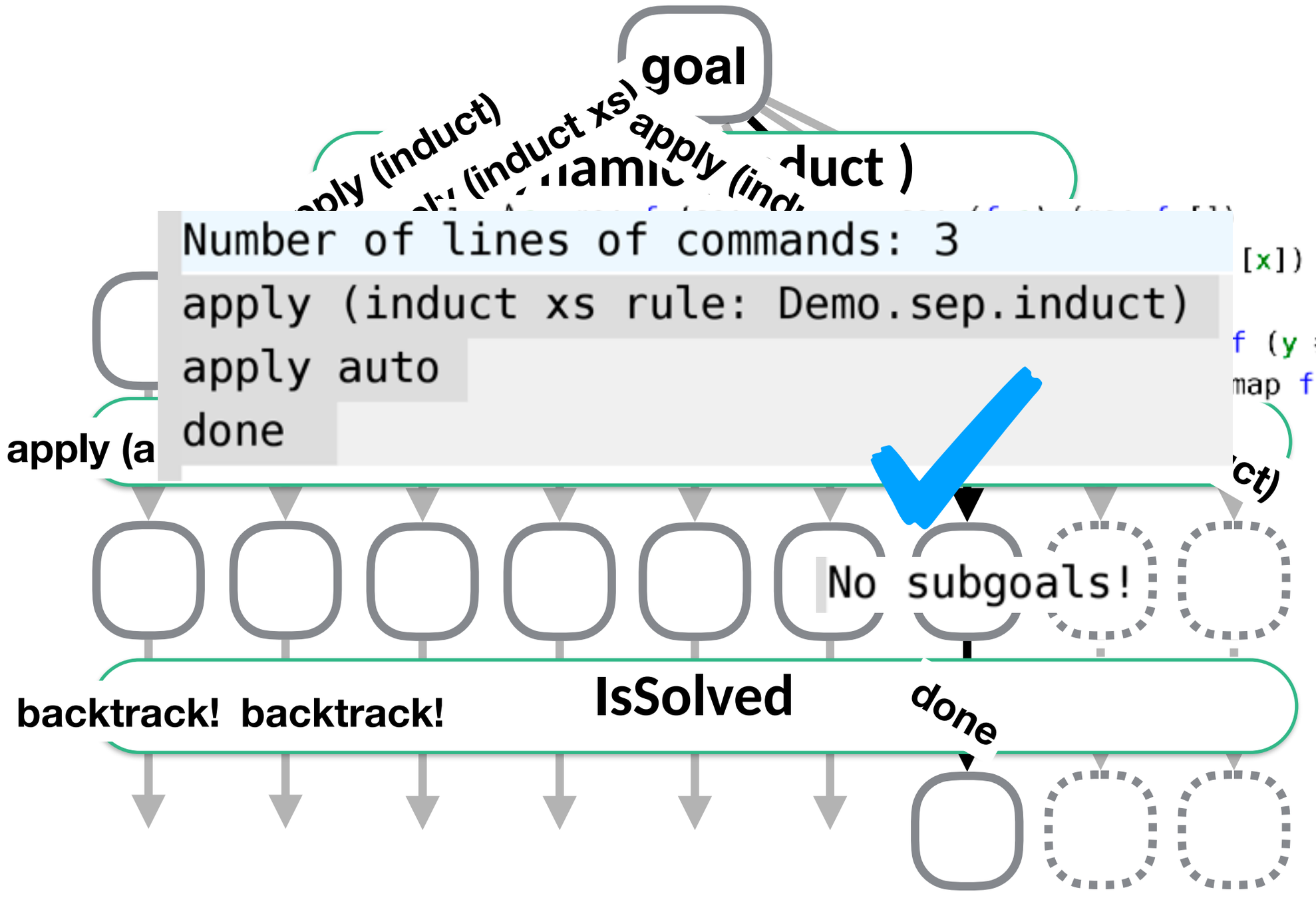

DEMO1

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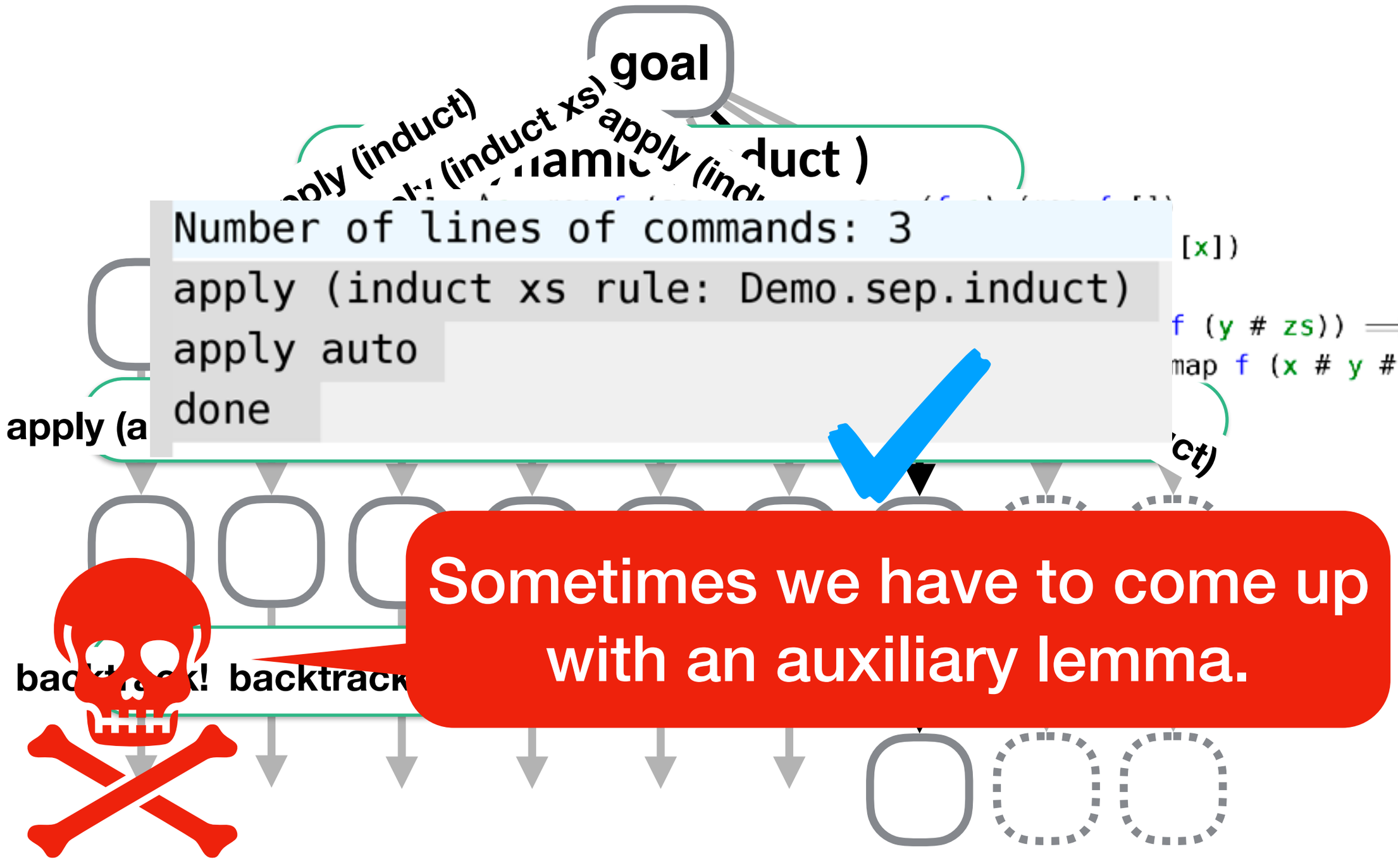

DEMO2

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

Example.thy (~/Workplace/PSL/PGT/)

```
primrec rev:: "'a list ⇒ 'a list" where
  "rev []       = []"
| "rev (x # xs) = rev xs @ [x]"
primrec itrev:: "'a list ⇒ 'a list ⇒ 'a list" where
  "itrev [] ys     = ys"
| "itrev (x#xs) ys = itrev xs (x#ys)"

strategy DInd  = Thens [Dynamic (Induct), Auto, IsSolved]


lemma "itrev xs [] = rev xs"
  find_proof DInd
```

Proof state | Auto update | Update | Search: | 100%

```
proof (prove)
goal (1 subgoal):
 1. itrev xs [] = Example.rev xs
```

Output | Query | Sledgehammer | Symbols

43,29 (837/1004)

DEMO2

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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lemma "itrev xs [] = rev xs"
  find_proof DInd
```

Proof state | Auto update | Update | Search: | 100%

```
empty sequence. no proof found.
```

Output | Query | Sledgehammer | Symbols

44,18 (855/1004)

DEMO2

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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lemma "itrev xs [] = rev xs"
  find_proof DInd
  find_proof DInd_Or_CDInd
```

Proof state | Auto update | Update | Search: | 100%

```
PGT creates 131 conjectures.
Testing conjecture with Quickcheck-exhaustive...
Testing conjecture with Quickcheck-exhaustive...
Testing conjecture with Quickcheck-exhaustive...
Testing conjecture with Quickcheck-exhaustive...
```

Output | Query | Sledgehammer | Symbols

45,27 (988/1000) (isabelle,isabelle,UTF-8-Isabelle) 371/535MB 6:17 PM

DEMO2

ithub.com/data61/PSL/blob/master/slide/2020_NUS.pdf

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Output | Query | Sledgehammer | Symbols

45,27 (988/1000)

DEMO2

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  find_proof DInd_Or_CDInd
```

Proof state | Auto update | Update | Search: | 100%

```
apply (subgoal_tac "∧Nil. itrev xs Nil = Example.rev xs @ Nil")
apply fastforce
apply (induct xs)
apply auto
done
```

Output | Query | Sledgehammer | Symbols

45,27 (988/1000)

DEMO2

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done
```

What happened?

Output | Query | Sledgehammer | Symbols

45,27 (988/1000)

```
goal (1 subgoal):
 1. itrev xs [] = rev xs
```

goal

DEMO2

```
strategy CDInd = Thens [Conjecture, Fastforce, Quickcheck, DInd]
```

```
goal (1 subgoal):
 1. itrev xs [] = rev xs
```

goal

Conjecture

Fastforce

```
strategy CDInd = Thens [Conjecture, Fastforce, Quickcheck, DInd]
```

Quickcheck

DInd

DInd

DEMO2

```
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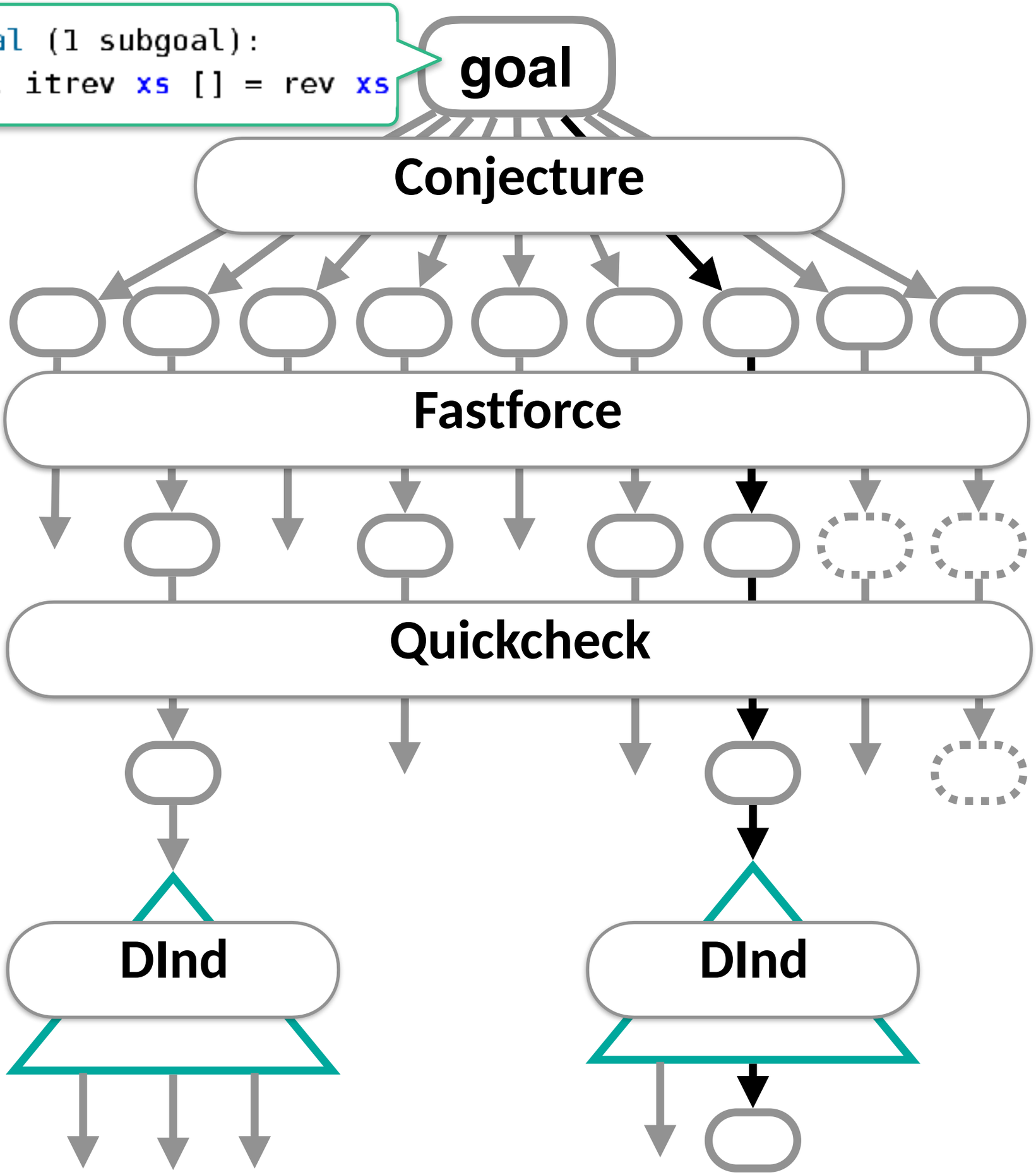

DEMO2

```
goal (1 subgoal):
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goal

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Conjecture

Fastforce

Quickcheck

DInd

DInd

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```
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```

```
goal (2 subgoals):
 1. (⋀Nil. itrev xs Nil = rev xs @ Nil) ⟹
    itrev xs [] = rev xs
 2. ⋀Nil. itrev xs Nil = rev xs @ Nil
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Conjecture

Fastforce

Quickcheck

DInd

DInd

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```

Conjecture

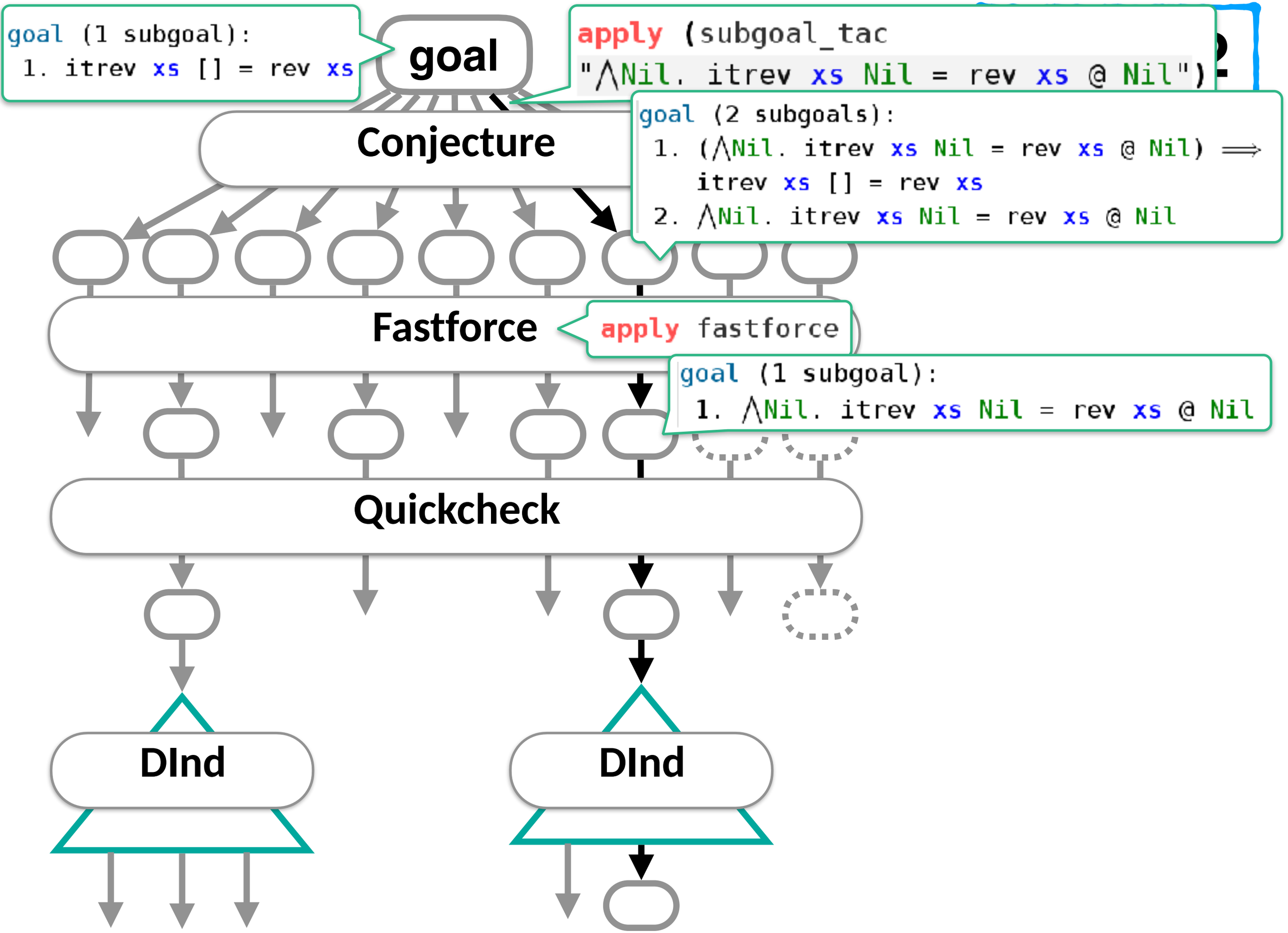
Fastforce

```
apply fastforce
```

Quickcheck

DInd

DInd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Conjecture

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DInd

DInd

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Fastforce

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Quickcheck

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DInd

DInd

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done
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theorem itrev ?xs [] = rev ?xs
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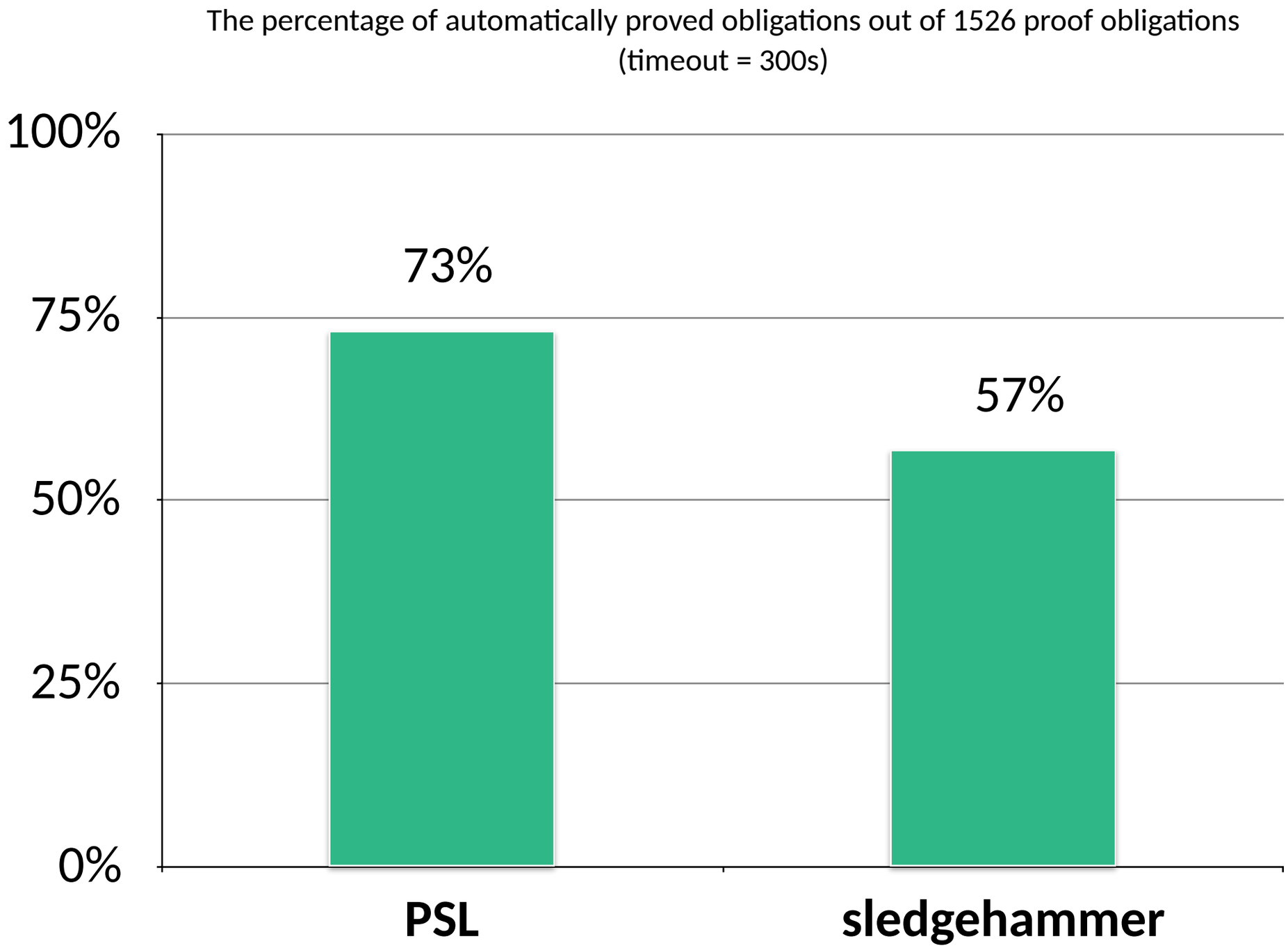
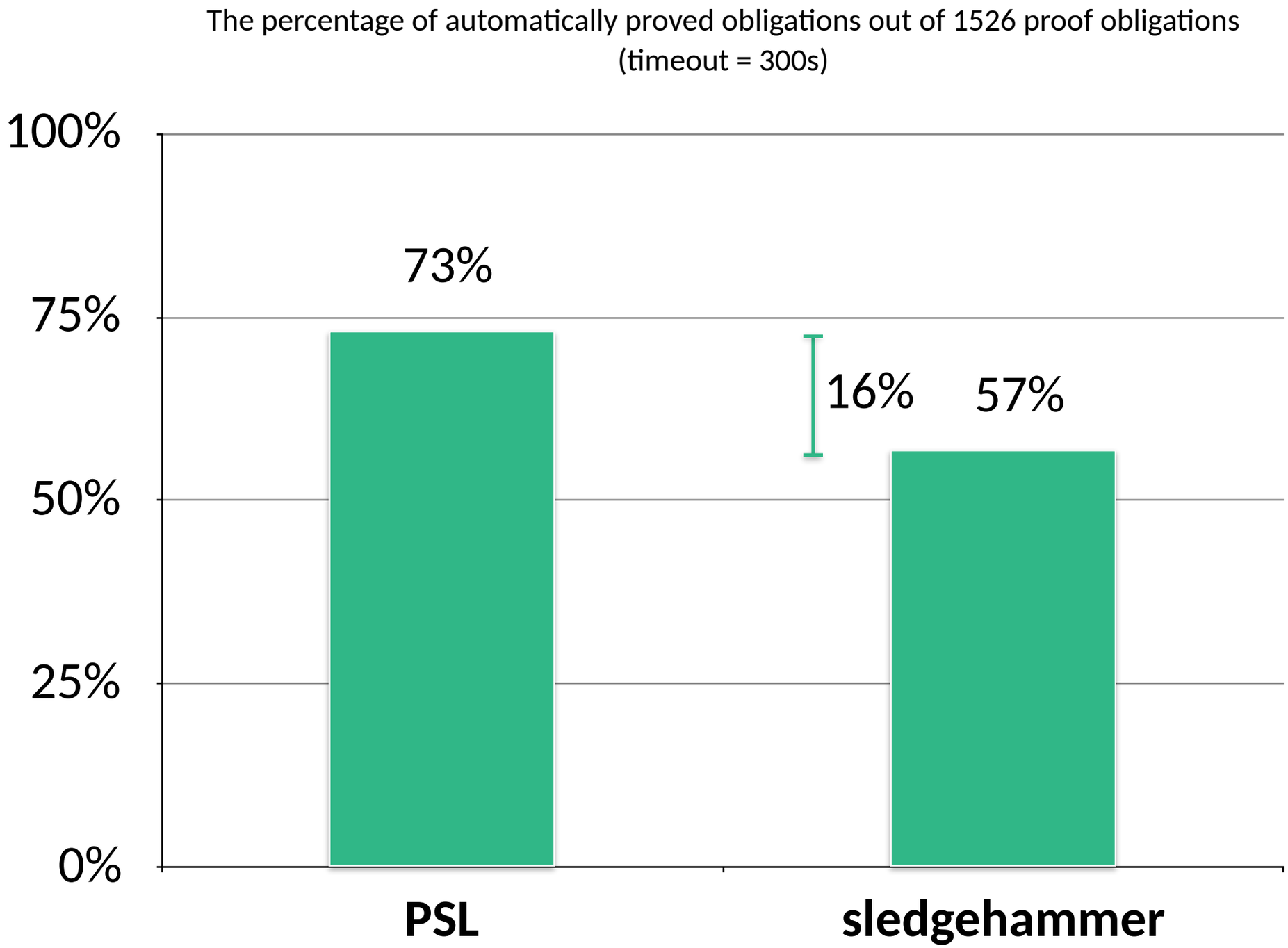
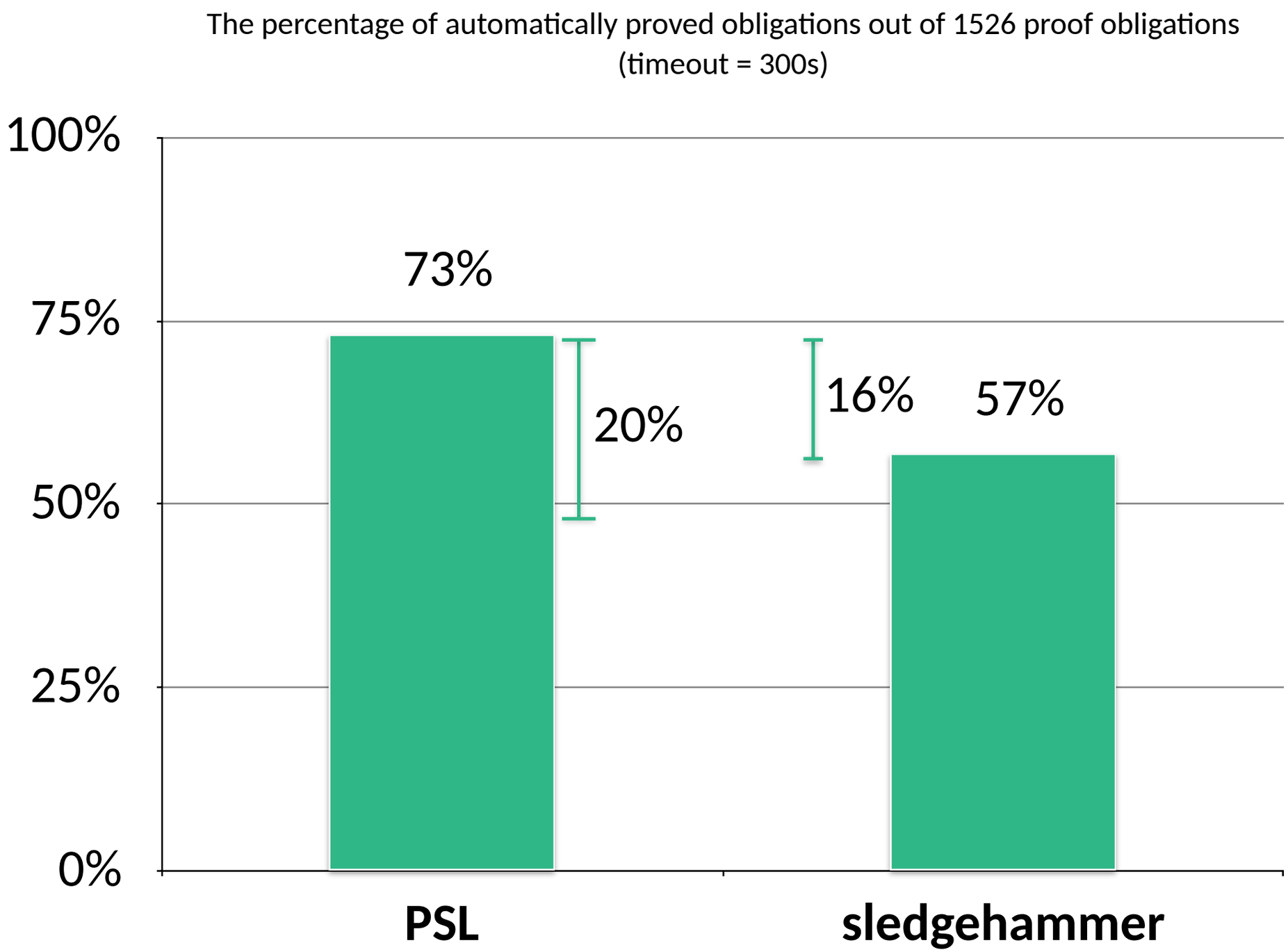
PSL vs sledgehammer



PSL vs sledgehammer

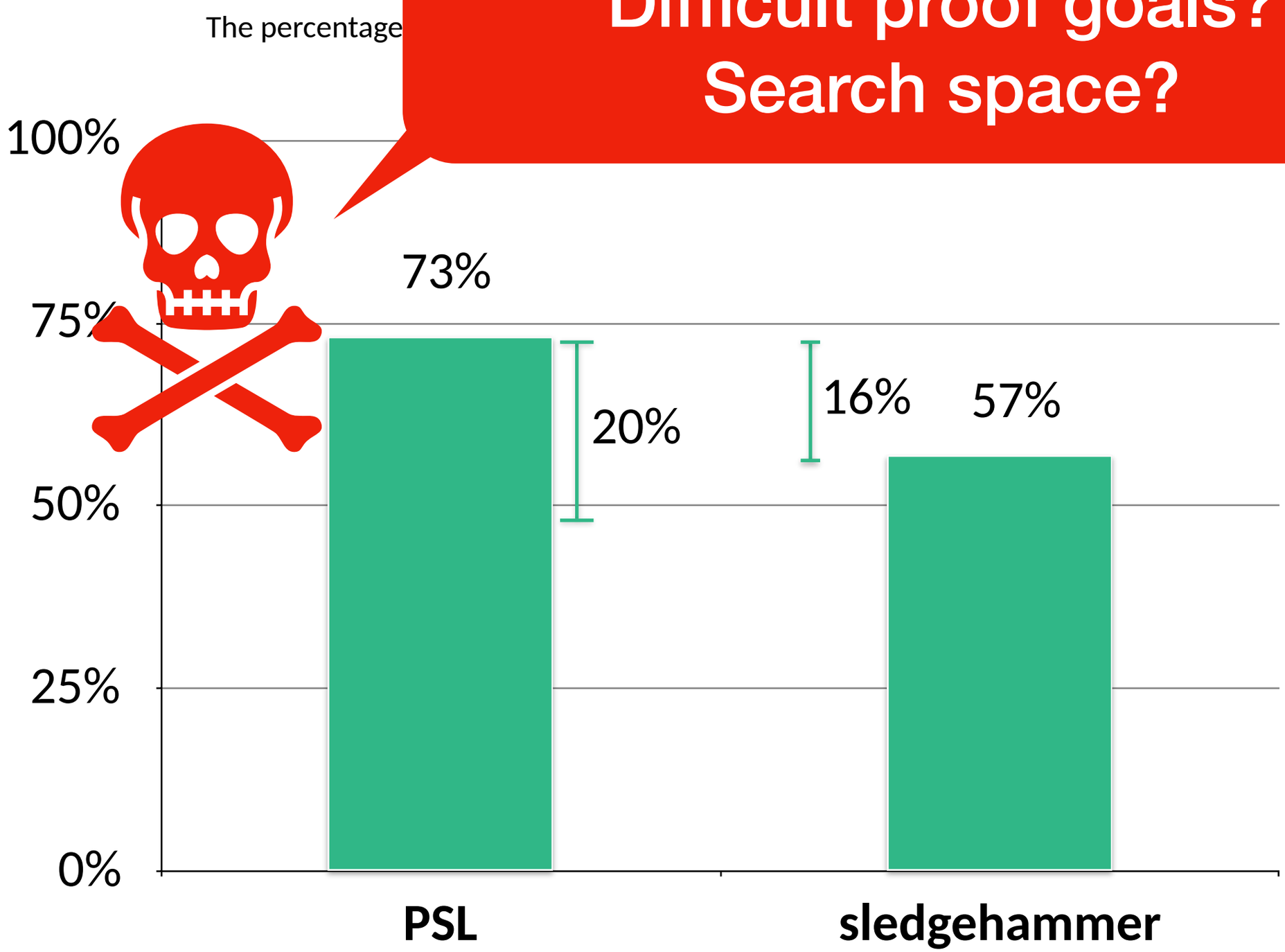


PSL vs sledgehammer



PSL vs sledgehammer

Difficult proof goals?
Search space?



PSL vs sledgehammer

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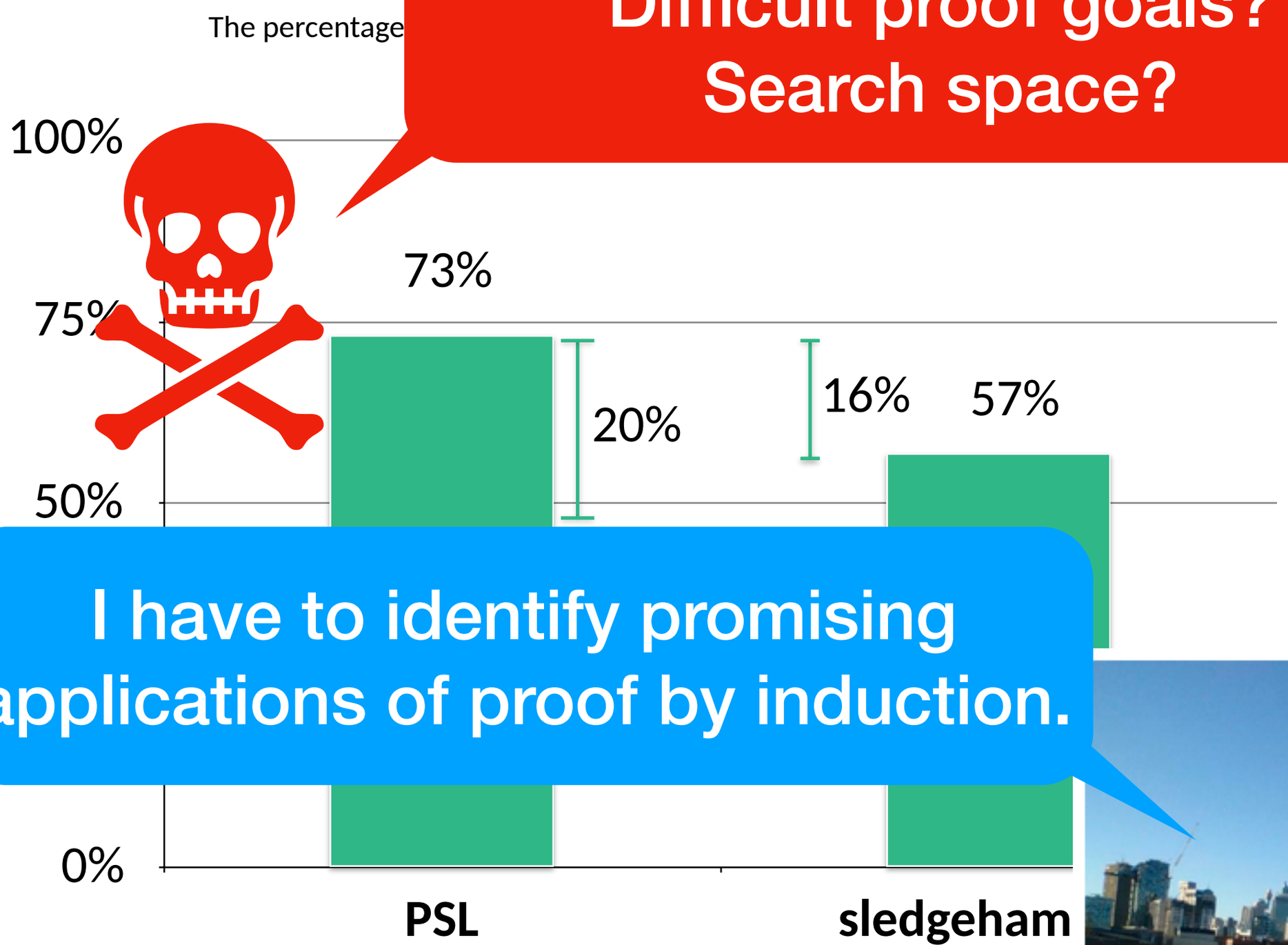
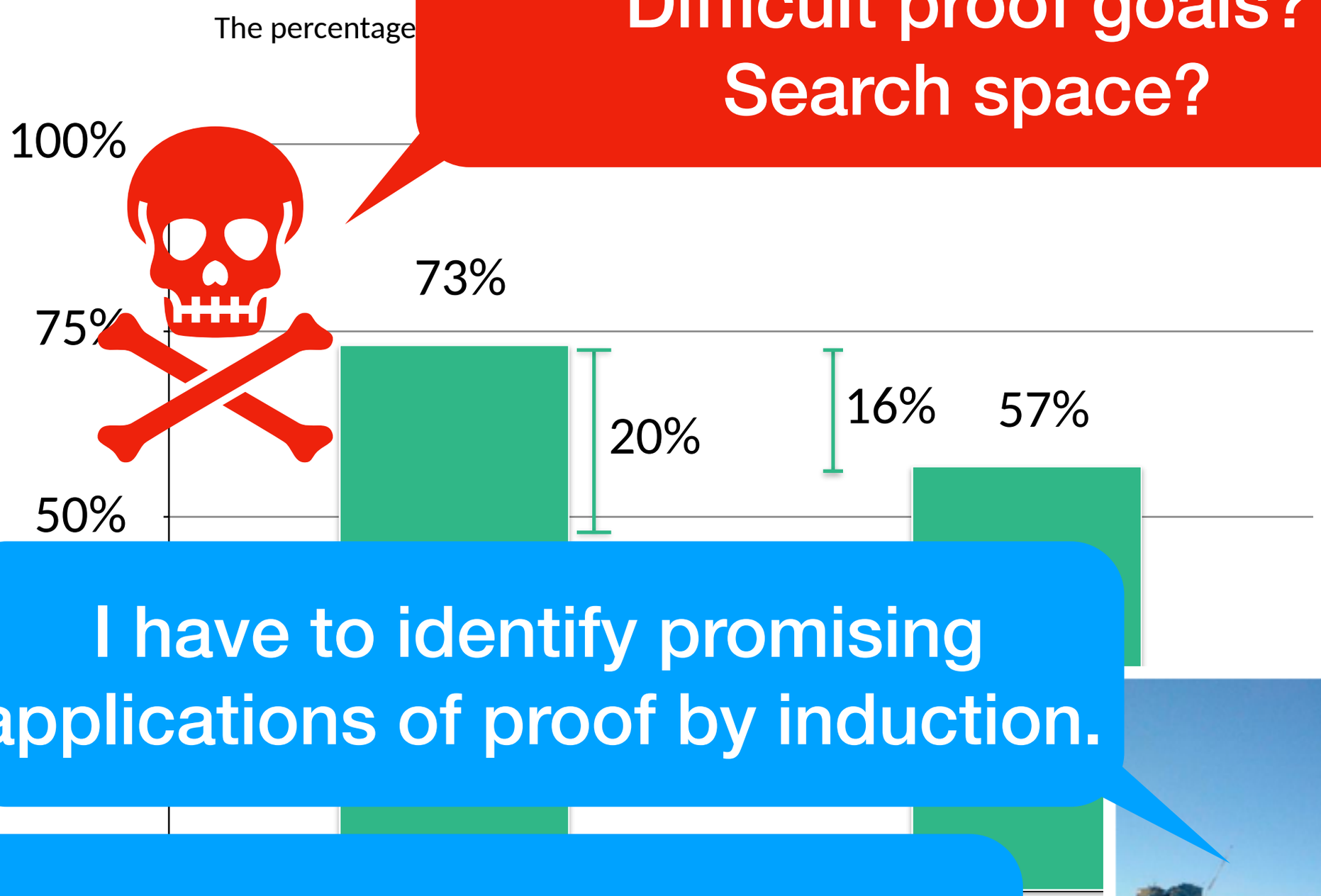


I have to identify promising applications of proof by induction.

PSL vs sledgehammer

Difficult proof goals?
Search space?



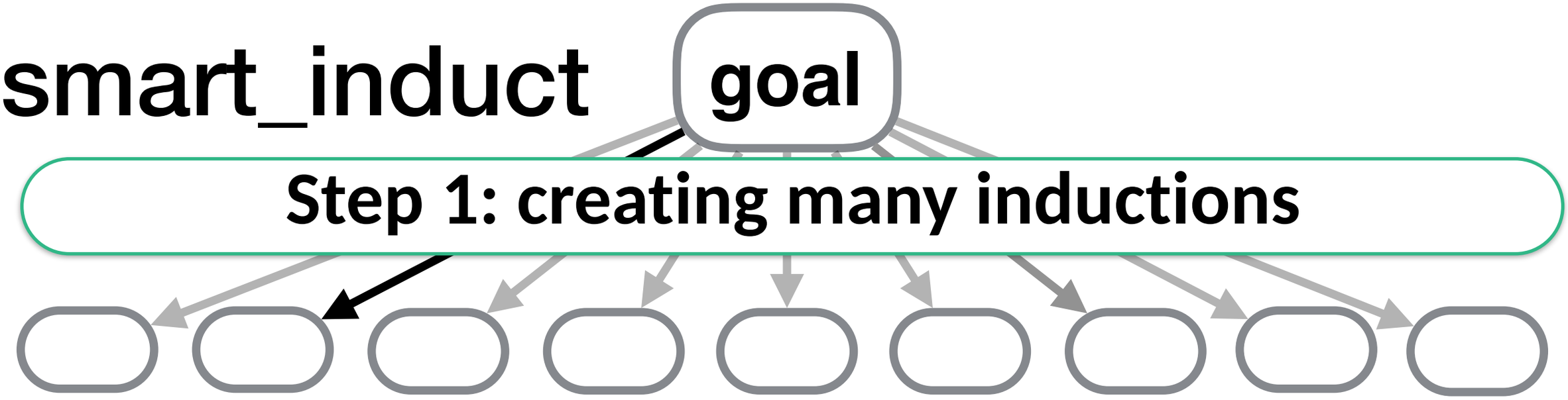
I have to identify promising applications of proof by induction.

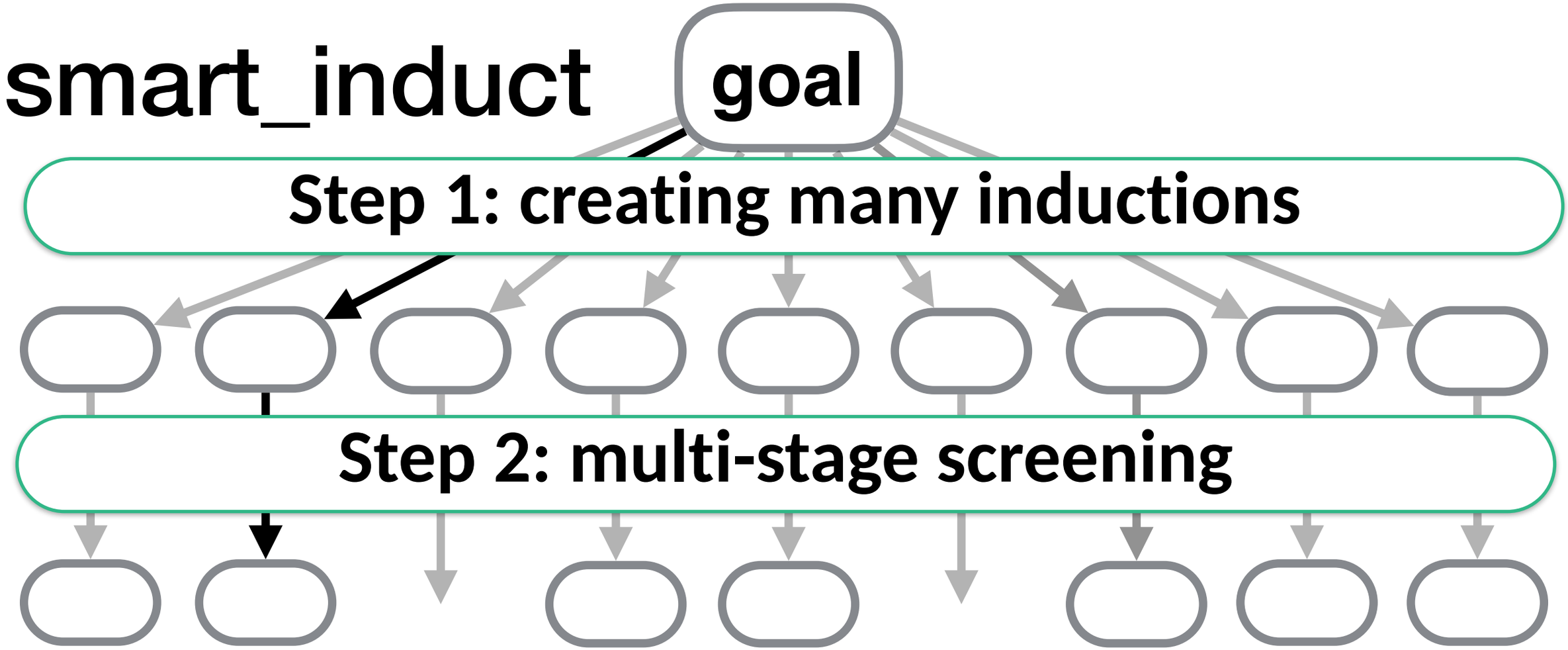
without completing a proof search

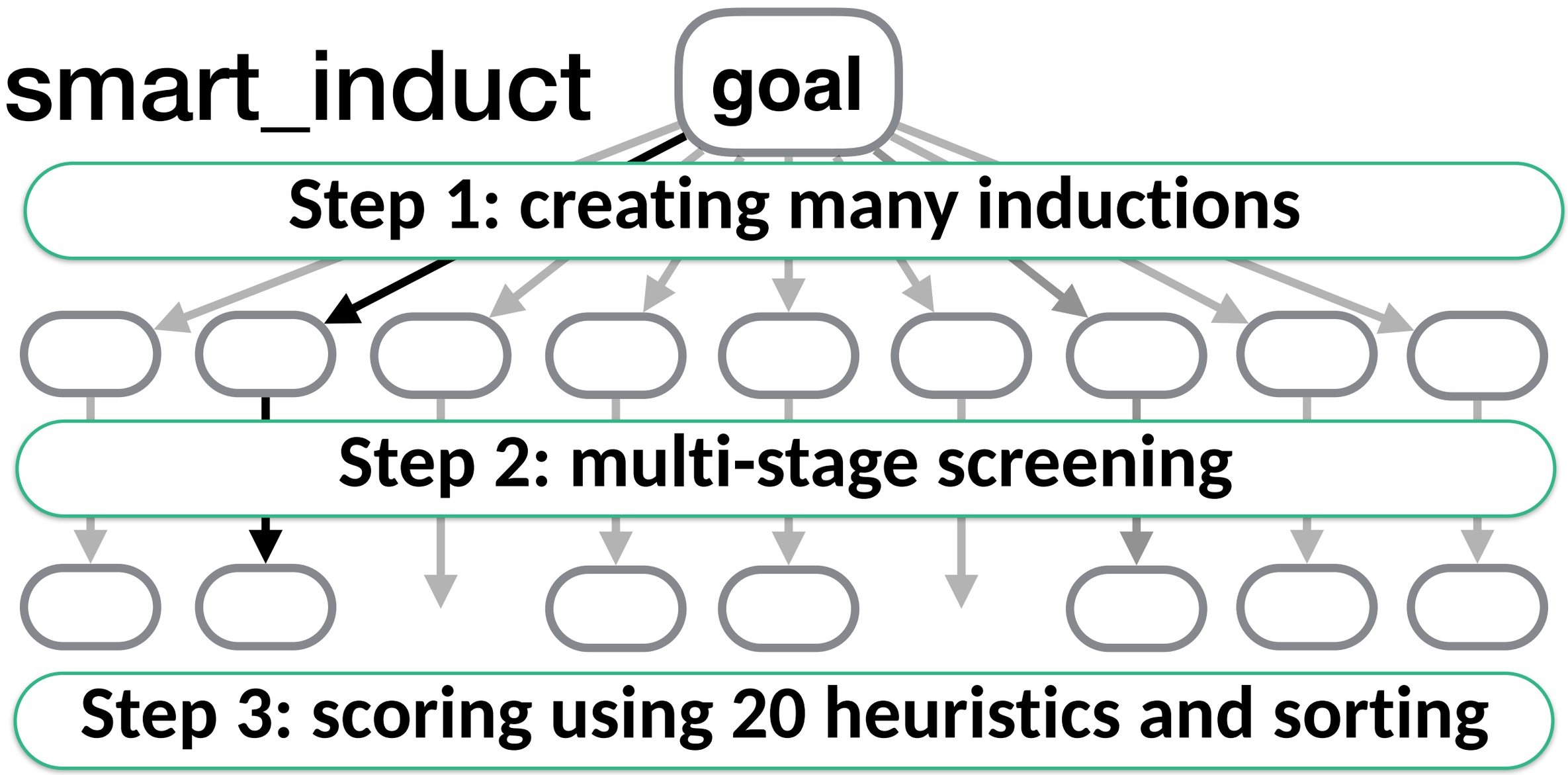
smart_induct

goal

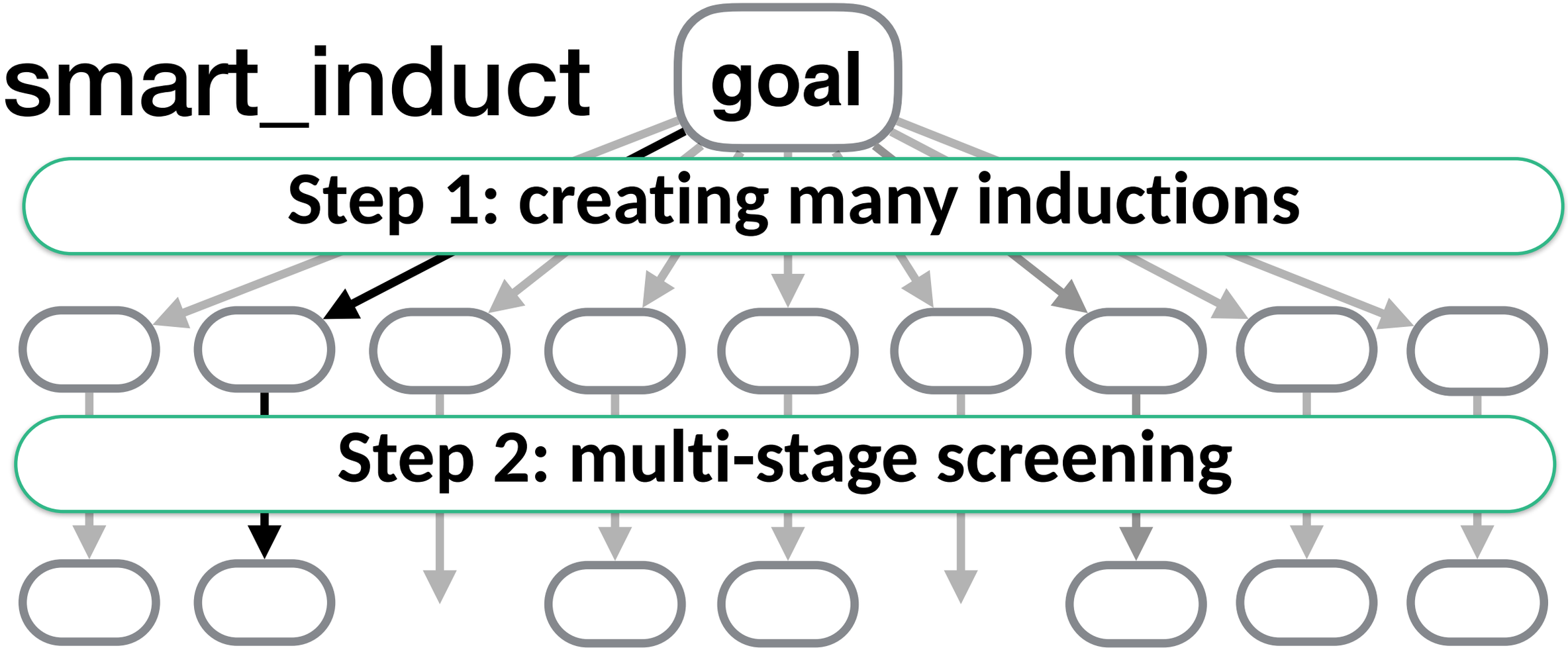
smart_induct




smart_induct
goal
Step 1: creating many inductions
Step 2: multi-stage screening


smart_induct
goal
Step 1: creating many inductions
Step 2: multi-stage screening
Step 3: scoring using 20 heuristics and sorting

smart_induct



Step 3: scoring using 20 heuristics and sorting

heuristic : (proof goal * induction arguments) -> bool

smart_induct

goal

Step 1: creating many inductions

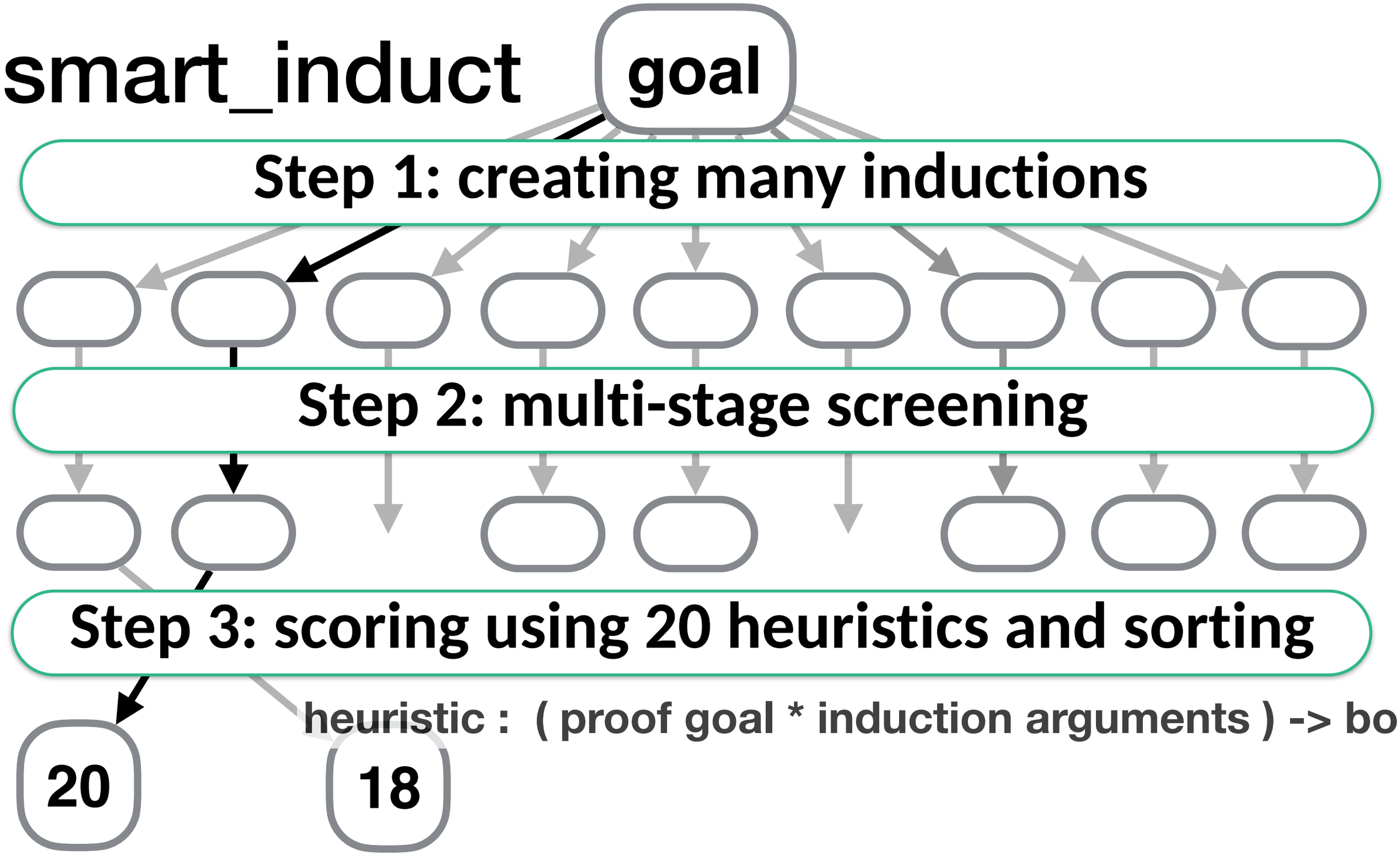
Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

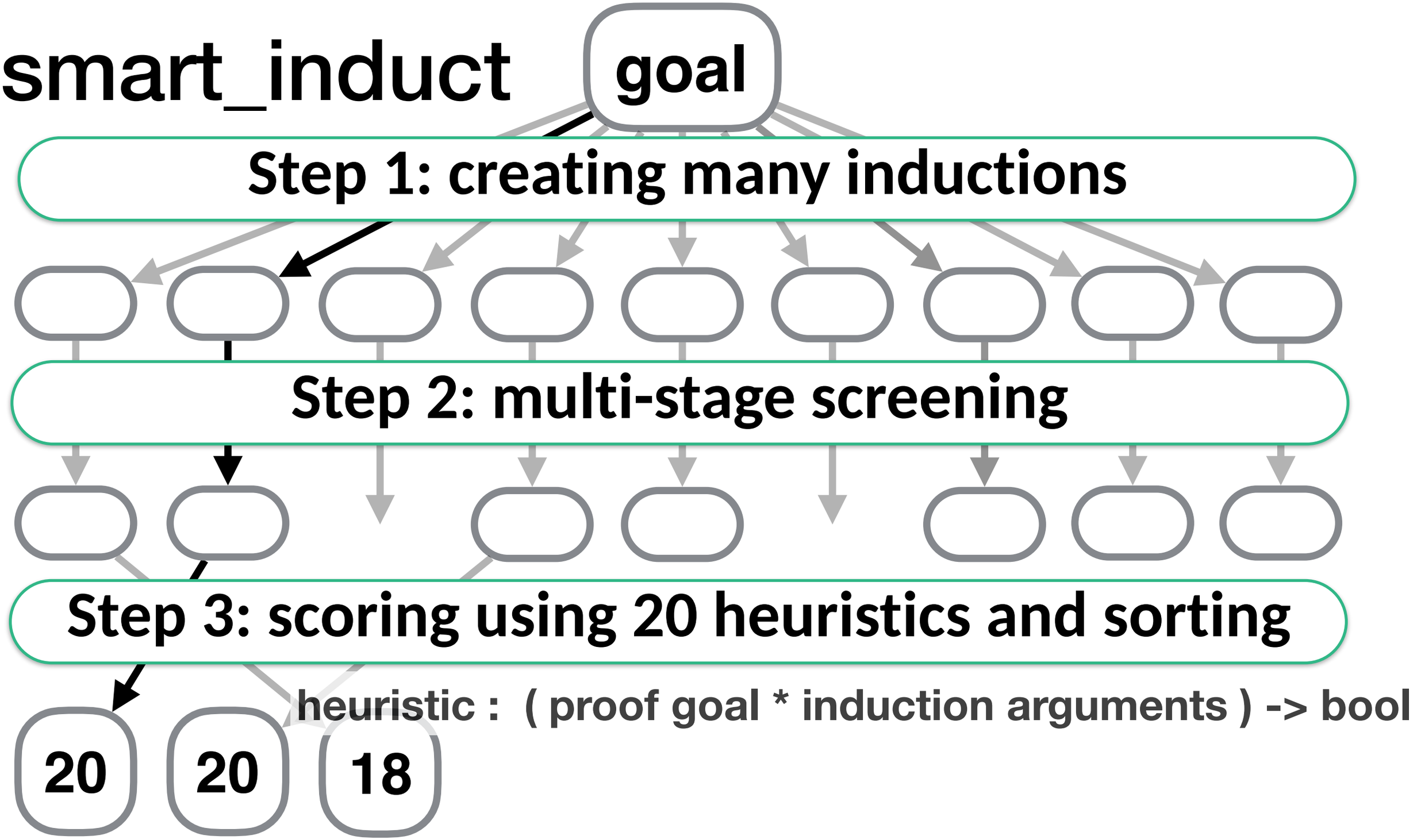
heuristic : (proof goal * induction arguments) -> bool

18

smart_induct



smart_induct



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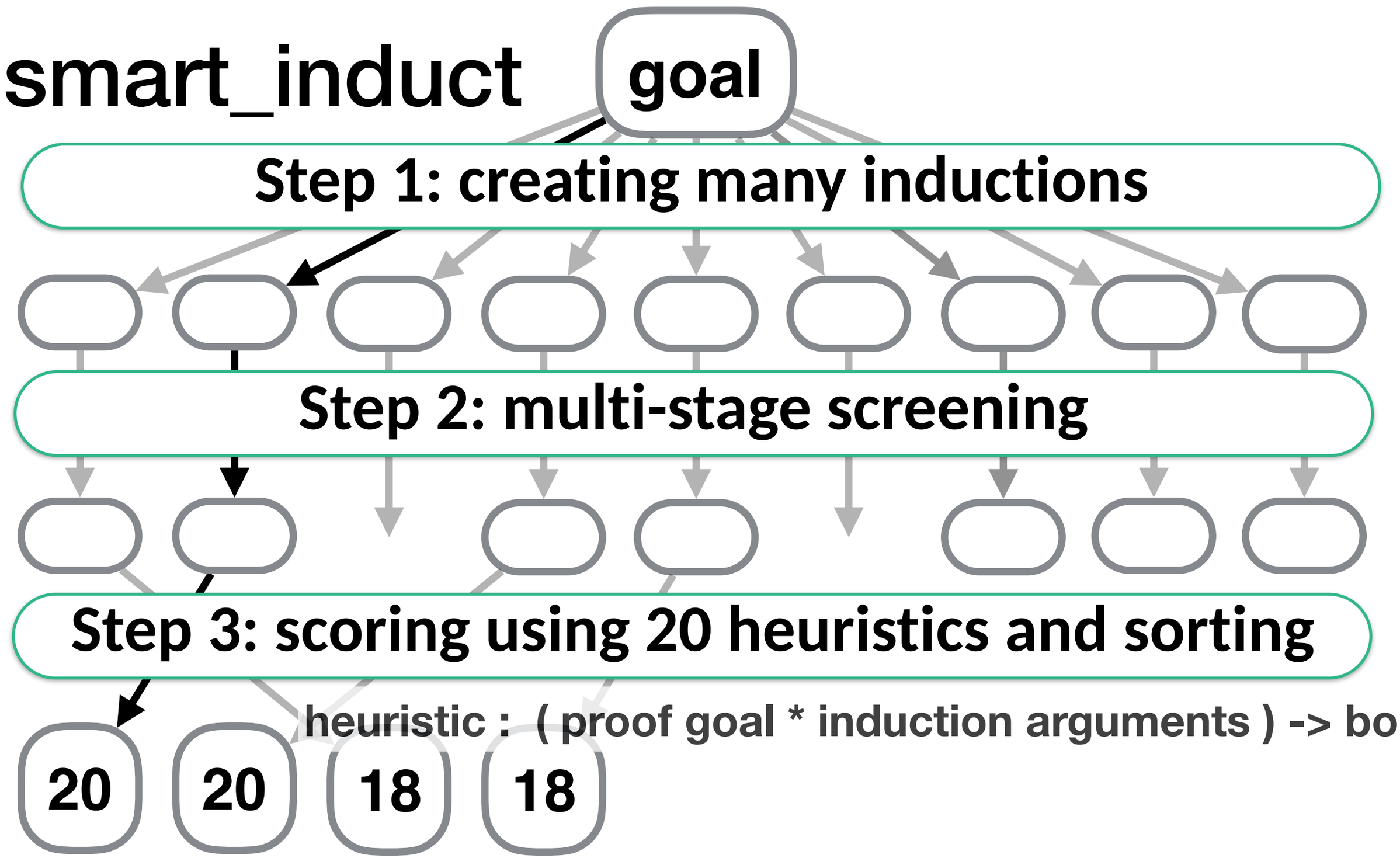

smart_induct

goal

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Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

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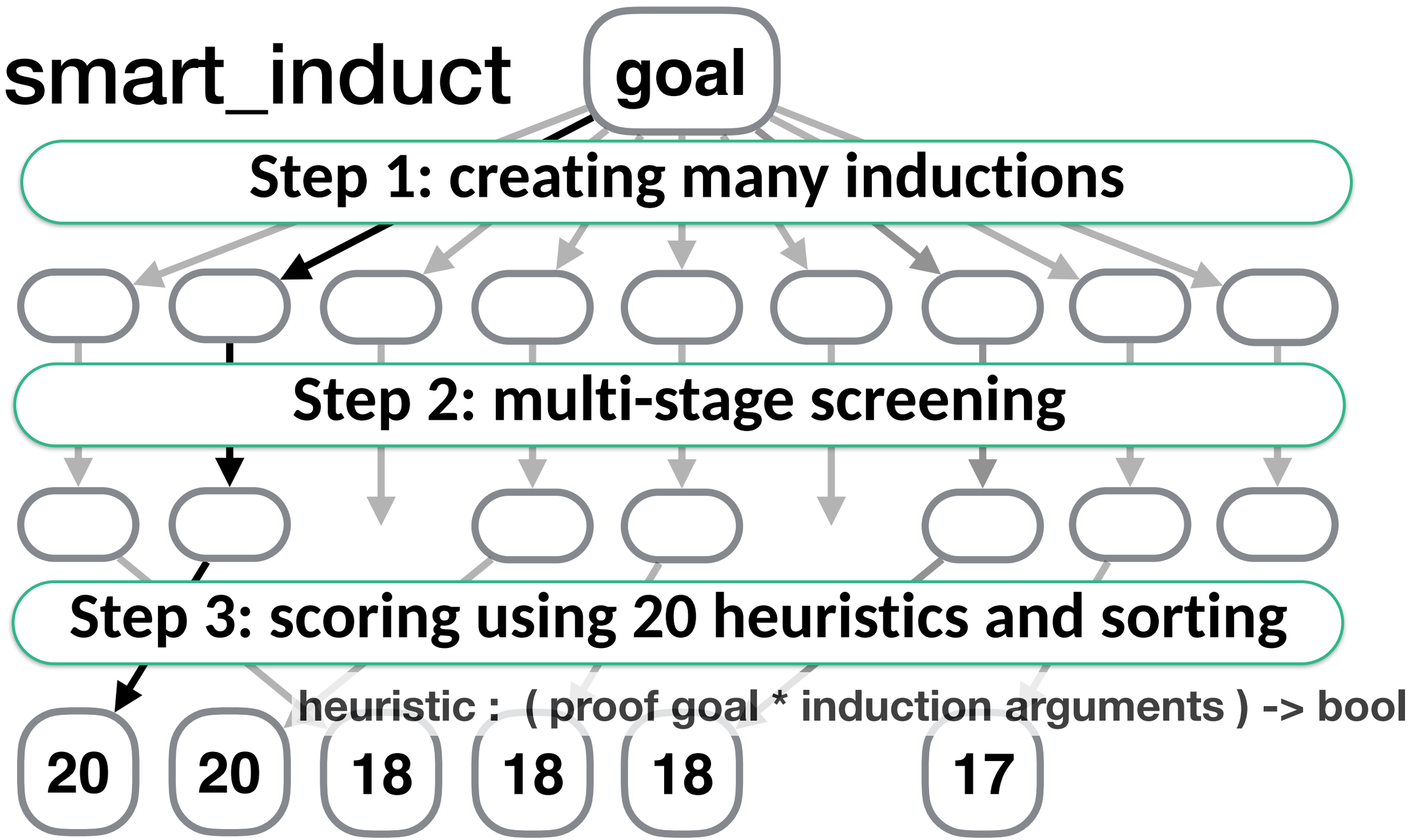
20 20 18 18 18

smart_induct



smart_induct

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

heuristic : (proof goal * induction arguments) -> bool

20 20 18 18 18 18 17

smart_induct

goal

Step 1: creating many inductions

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Step 3: scoring using 20 heuristics and sorting

heuristic : (proof goal * induction arguments) -> bool

20 20 18 18 18 18 17

Step 4: short-listing

smart_induct

goal

Step 1: creating many inductions

Step 2: multi-stage screening

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heuristic : (proof goal * induction arguments) -> bool

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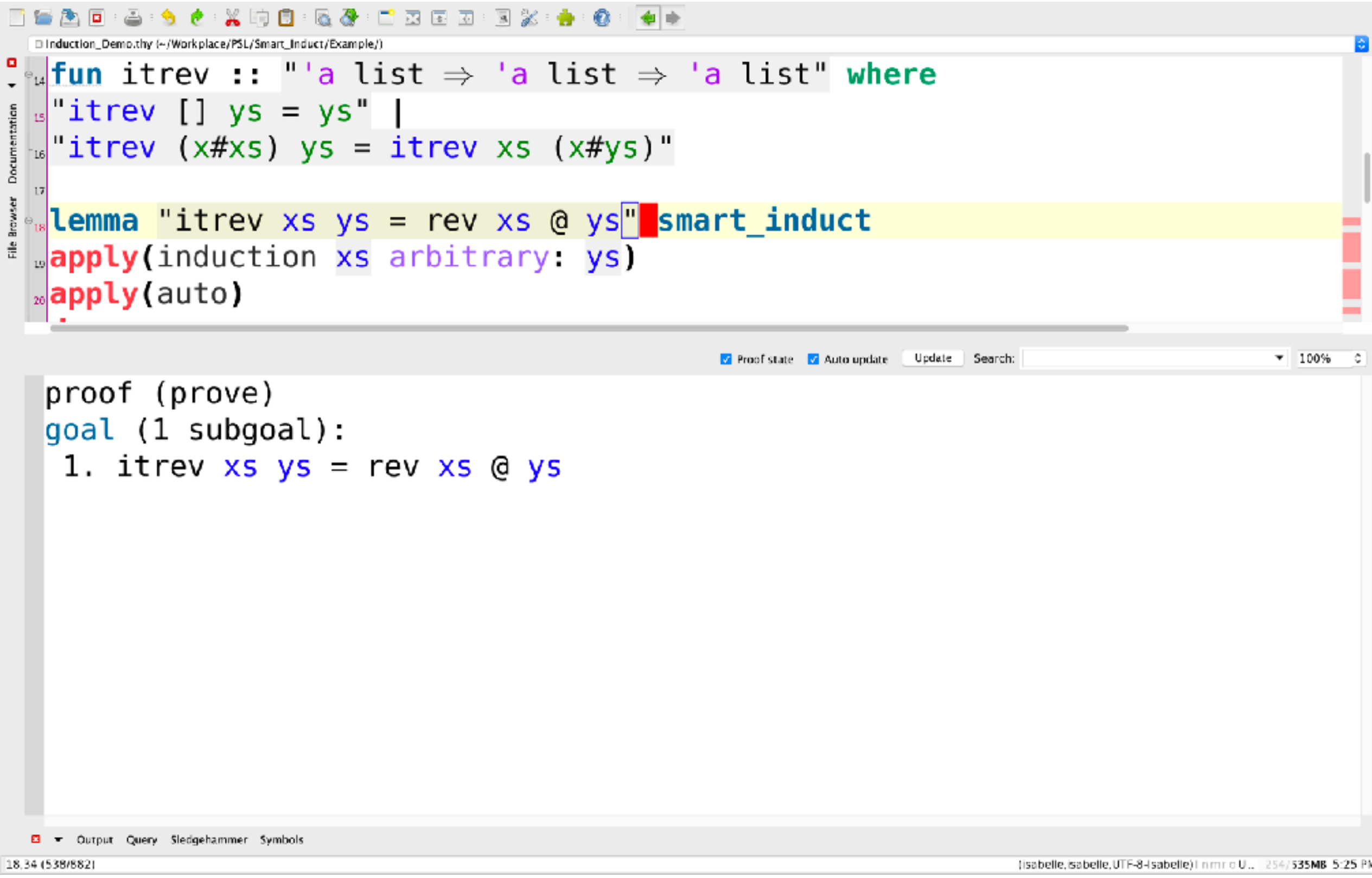
Step 4: short-listing

DEMO!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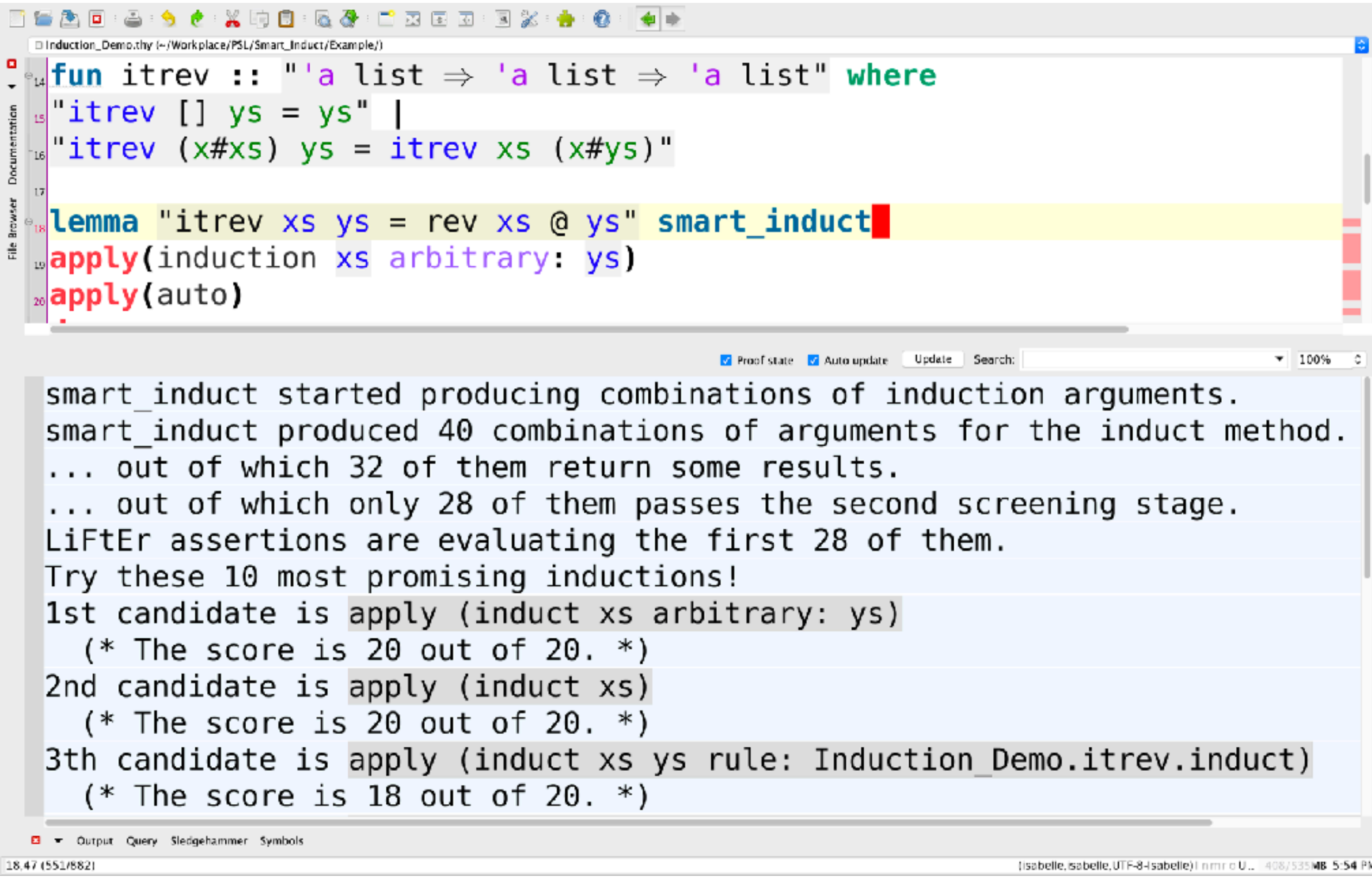
"itrev (x#xs) ys = itrev xs (x#ys)"
lemma "itrev xs ys = rev xs @ ys" smart_induct
apply(induction xs arbitrary: ys)
apply(auto)
Proof state
Auto update
Update
Search:
100%
proof (prove)
goal (1 subgoal):
1. itrev xs ys = rev xs @ ys
Output
Query
Sledgehammer
Symbols
18,34 (538/882)
Sidekick
State
Theories
Documentation
File Browser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... out of which 32 of them return some results.
... out of which only 28 of them passes the second screening stage.
LiFtEr assertions are evaluating the first 28 of them.
Try these 10 most promising inductions!
1st candidate is apply (induct xs arbitrary: ys)
(* The score is 20 out of 20. *)
2nd candidate is apply (induct xs)
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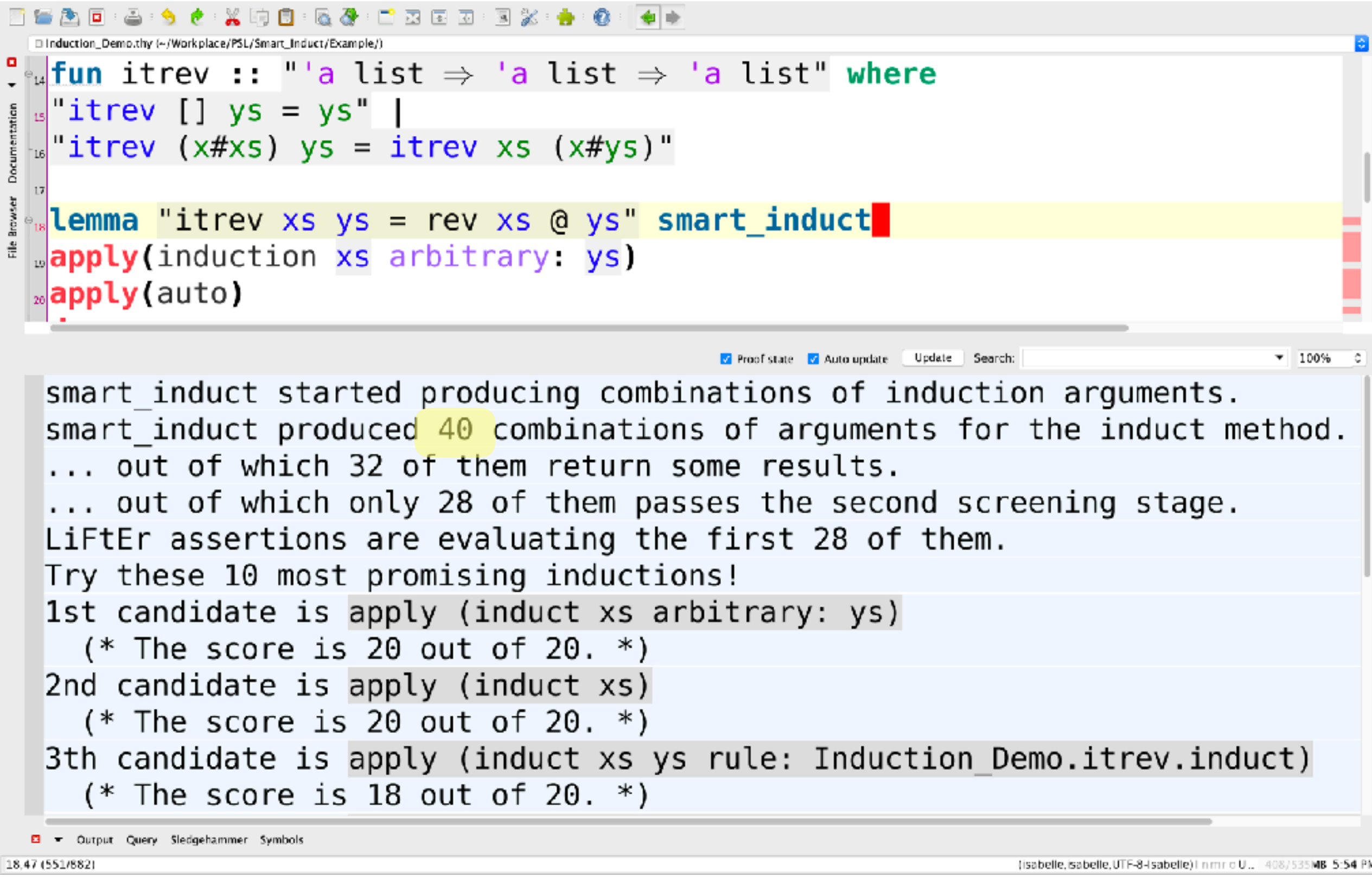
Output
Query
Sledgehammer
Symbols
18,47 (551/882)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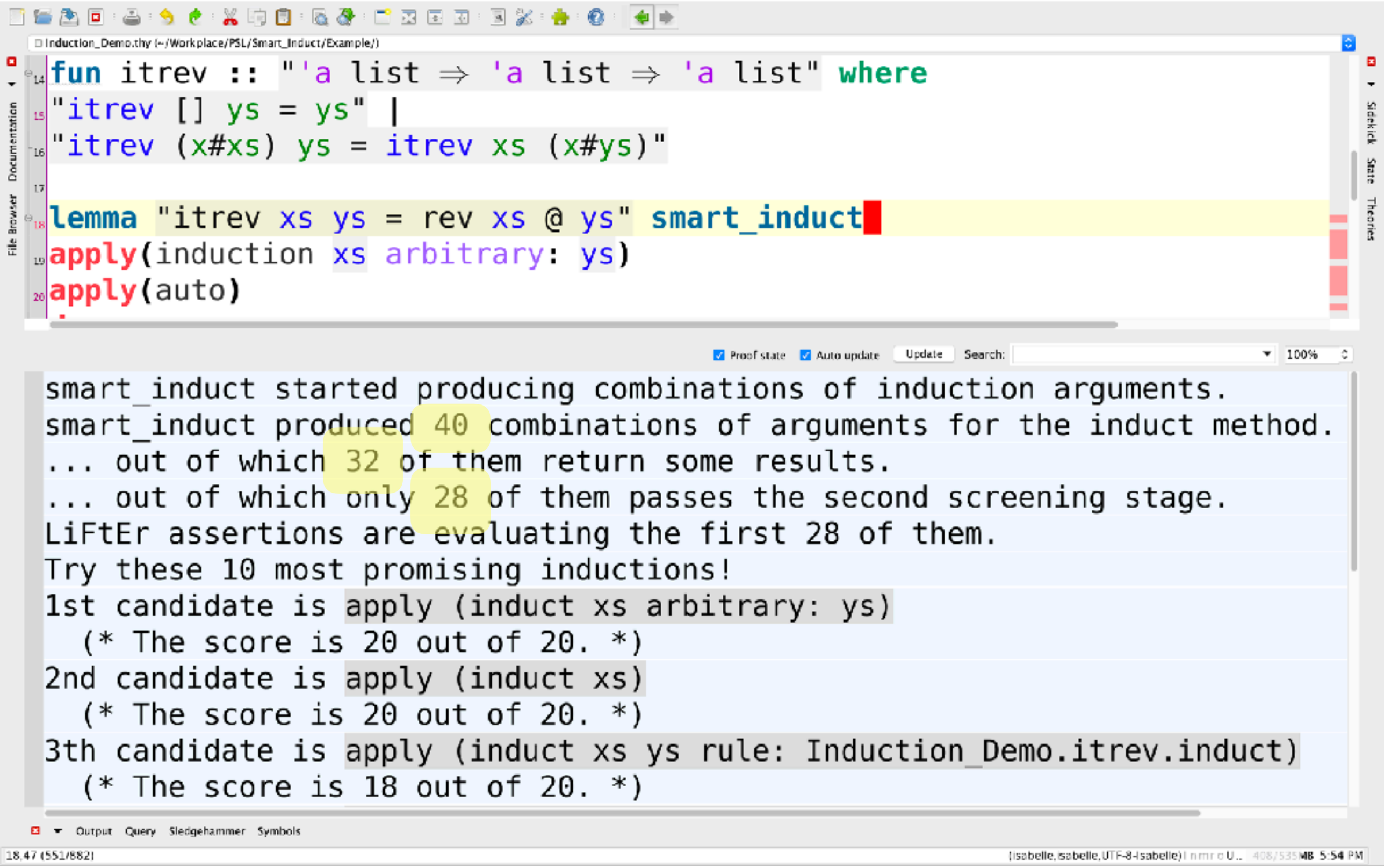
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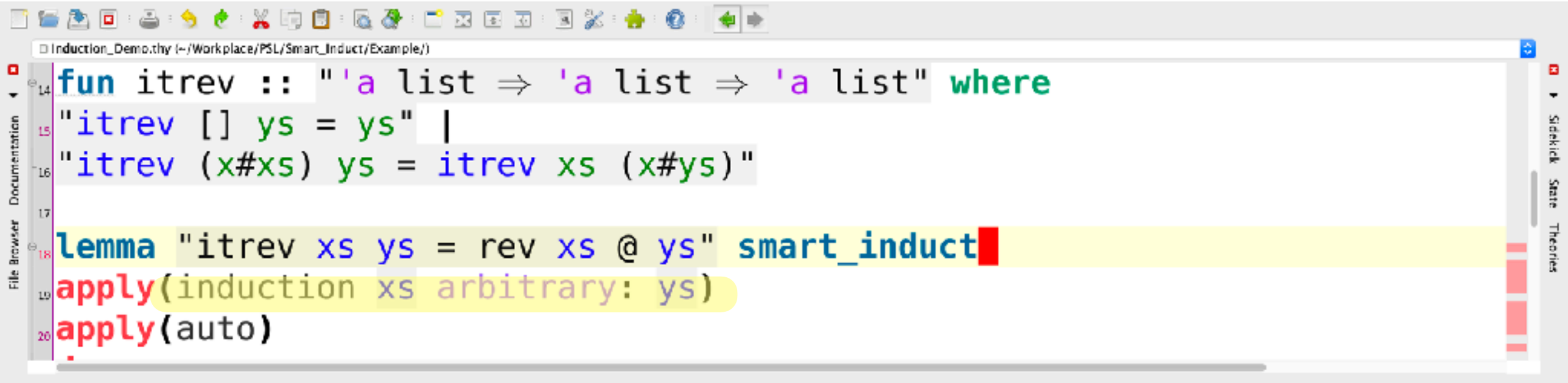
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18,47 (551/882)
5:54 PM

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

20 20 18 18 18 18 17

Step 4: short-listing

goal

Step 1: creating many inductions

Step 2: multi-stage screening

Step 3: scoring using 20 heuristics and sorting

?

20 20 18 18 18 18 17

Step 4: short-listing

It is difficult to write induction heuristics.

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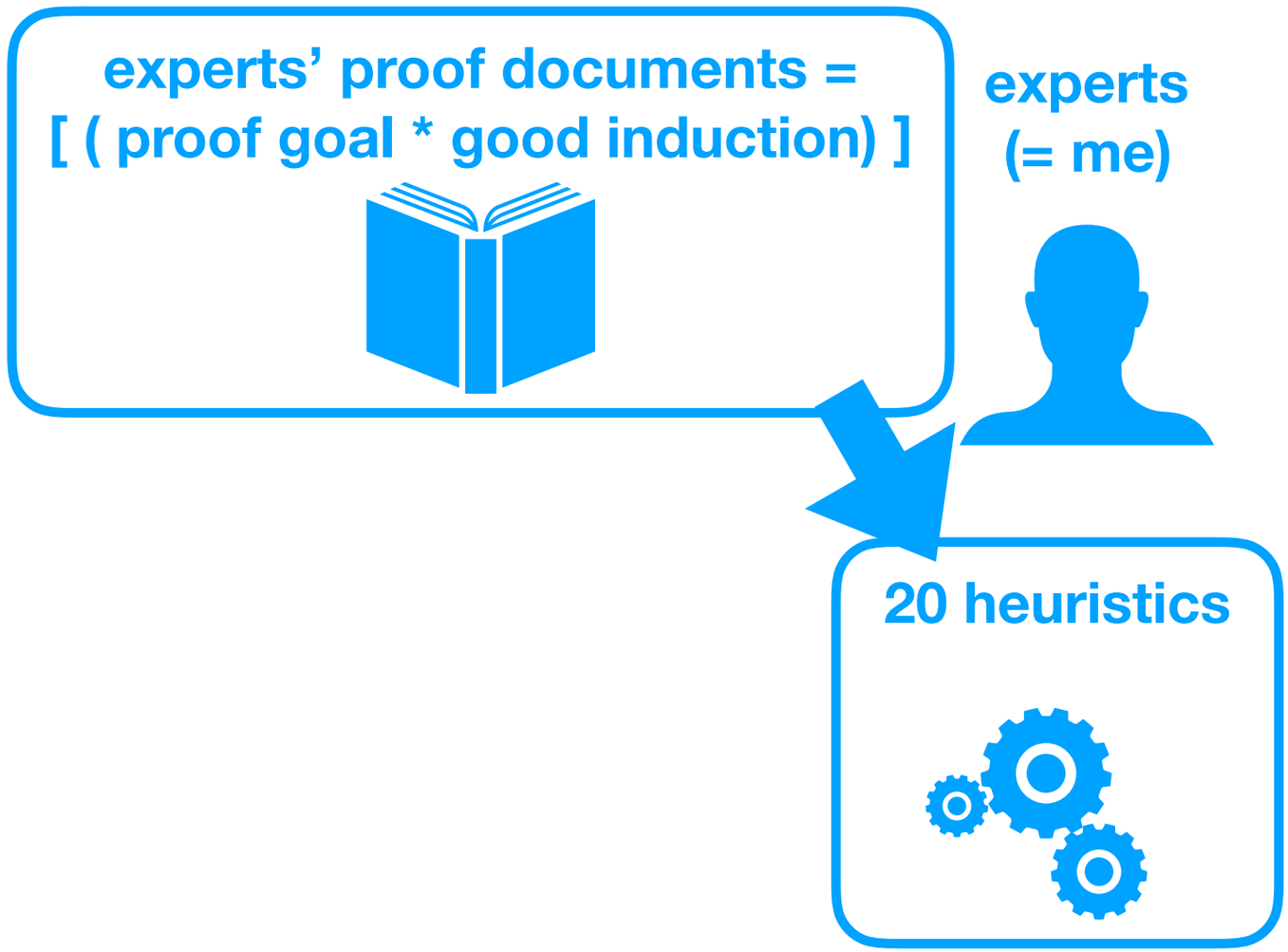

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lemma "itrev xs ys = rev xs @ ys"
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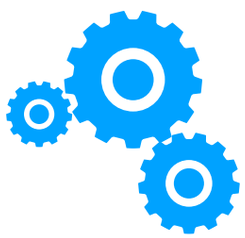
experts' proof documents =
[(proof goal * good induction)]

experts
(= me)

20 heuristics



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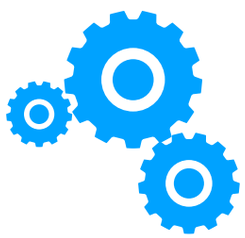
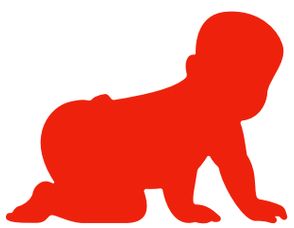


new users



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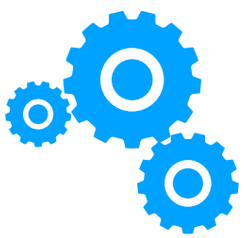

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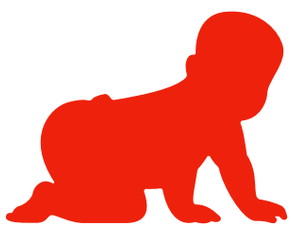

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lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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new proof goal consisting of new constants and variables of new types!

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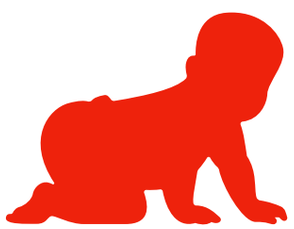
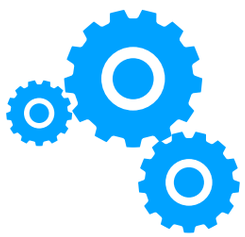


proof goal

candidate induction

apply (
induct is1 s stk
rule: exec.induct)

20 heuristics



[✓ ,
✗ ,
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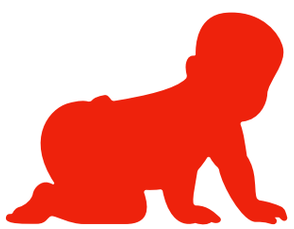

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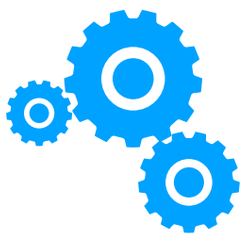

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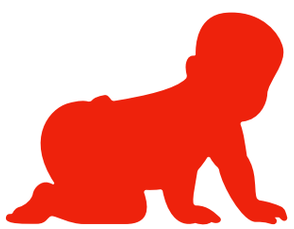

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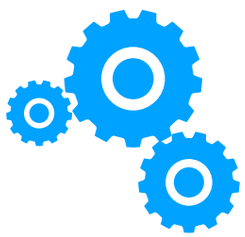

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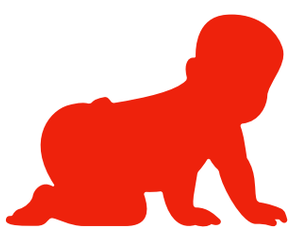

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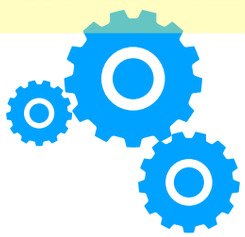
20 heuristics
in LiFtEr



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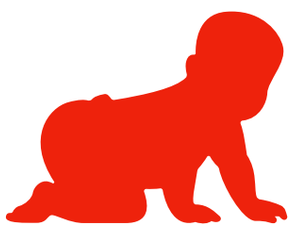
LiFtEr: Logical
Feature Extractor

new users



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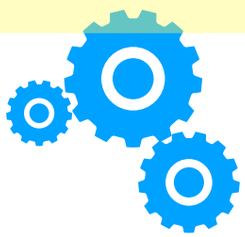

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Abstract Syntax of LiFtEr

assertion := *atomic* | *connective* | *quantifier* | (*assertion*)
type := `term` | `term_occurrence` | `rule` | `number`
modifier_term := `induction_term` | `arbitrary_term`
quantifier := $\exists x$: *type* . *assertion*
| $\forall x$: *type* . *assertion*
| $\exists x$: `term` $\in$ *modifier_term* . *assertion*
| $\forall x$: `term` $\in$ *modifier_term* . *assertion*
| $\exists x$: `rule` . *assertion*
| $\exists x$: `term_occurrence` $\in y$: `term` . *assertion*
| $\forall x$: `term_occurrence` $\in y$: `term` . *assertion*
connective := `True` | `False` | *assertion* $\vee$ *assertion* | *assertion* $\wedge$ *assertion*
assertion $\rightarrow$ *assertion* | $\neg$ *assertion*
pattern := `all_only_var` | `all_constructor` | `mixed`
atomic :=
rule `is_rule_of` *term_occurrence*
| *term_occurrence* `term_occurrence_is_of_term` *term*
| `are_same_term` (*term_occurrence* , *term_occurrence*)
| *term_occurrence* `is_in_term_occurrence` *term_occurrence*
| `is_atomic` *term_occurrence*
| `is_constant` *term_occurrence*
| `is_recursive_constant` *term_occurrence*
| `is_variable` *term_occurrence*
| `is_free_variable` *term_occurrence*
| `is_bound_variable` *term_occurrence*
| `is_lambda` *term_occurrence*
| `is_application` *term_occurrence*
| *term_occurrence* `is_an_argument_of` *term_occurrence*
| *term_occurrence* `is_nth_argument_of` *term_occurrence*

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| `is_atomic` *term_occurrence*
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| `is_recursive_constant` *term_occurrence*
| `is_variable` *term_occurrence*
| `is_free_variable` *term_occurrence*
| `is_bound_variable` *term_occurrence*
| `is_lambda` *term_occurrence*
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| *term_occurrence* `is_an_argument_of` *occurrence*
| *term_occurrence* `is_nth_argument_of` *occurrence*

Example Heuristic in LiFtEr (in Abstract Syntax)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Heuristic in LiFtEr (in Abstract Syntax)

implication

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∃ r1 : rule. True
→
∃ r1 : rule.
  ∃ t1 : term.
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        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

Example Heuristic in LiFtEr (in Abstract Syntax)

implication

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∃ r1 : rule. True
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      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧                          conjunction
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Heuristic in LiFtEr (in Abstract Syntax)

implication

variable for auxiliary lemmas

conjunction

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

Example Heuristic in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
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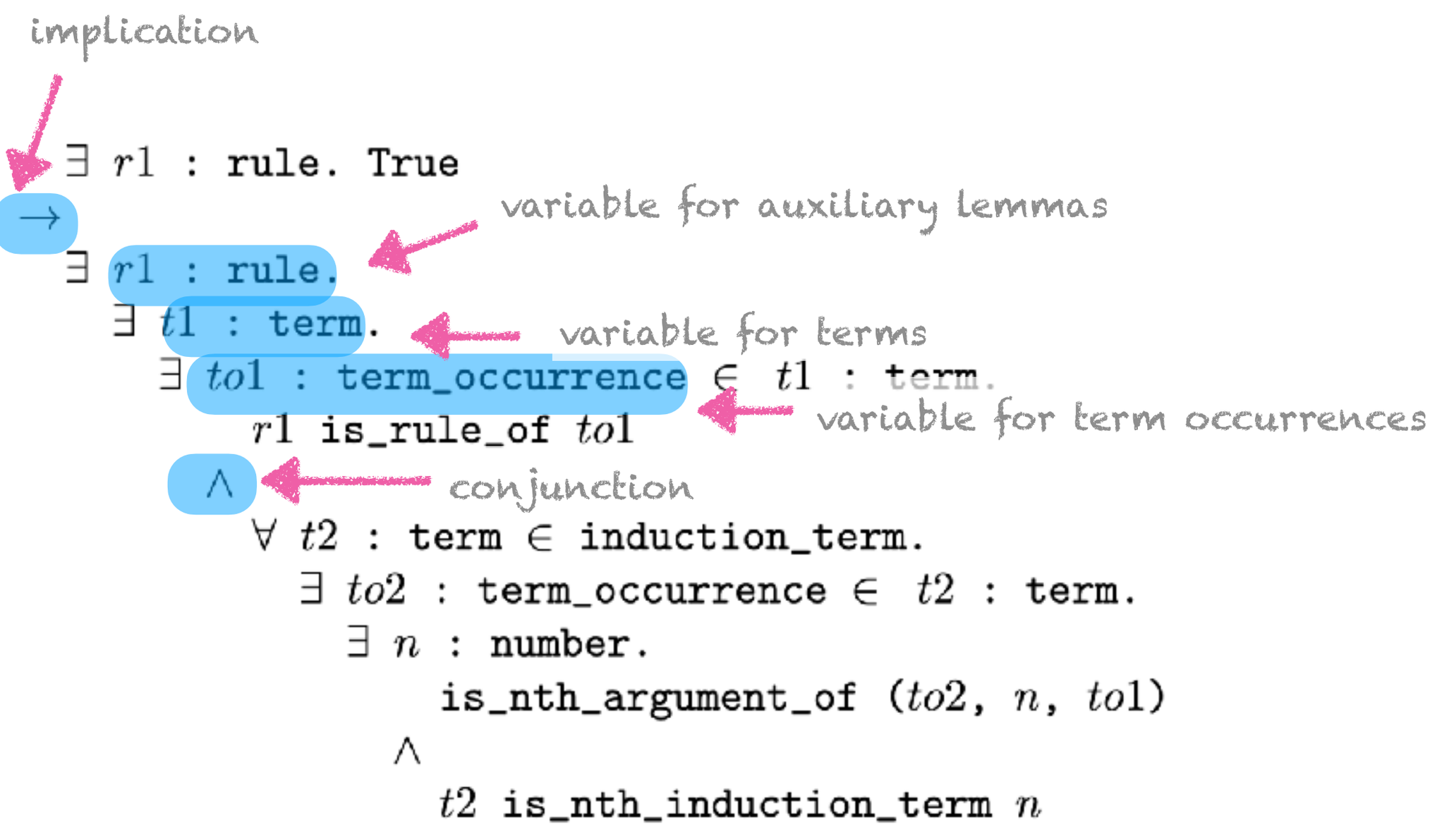
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

variable for auxiliary lemmas

variable for terms

conjunction

Example Heuristic in LiFtEr (in Abstract Syntax)



Example Heuristic in LiFtEr (in Abstract Syntax)

implication

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

variable for natural numbers

Example Heuristic in LiFtEr (in Abstract Syntax)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

implication

variable for auxiliary lemmas

variable for terms

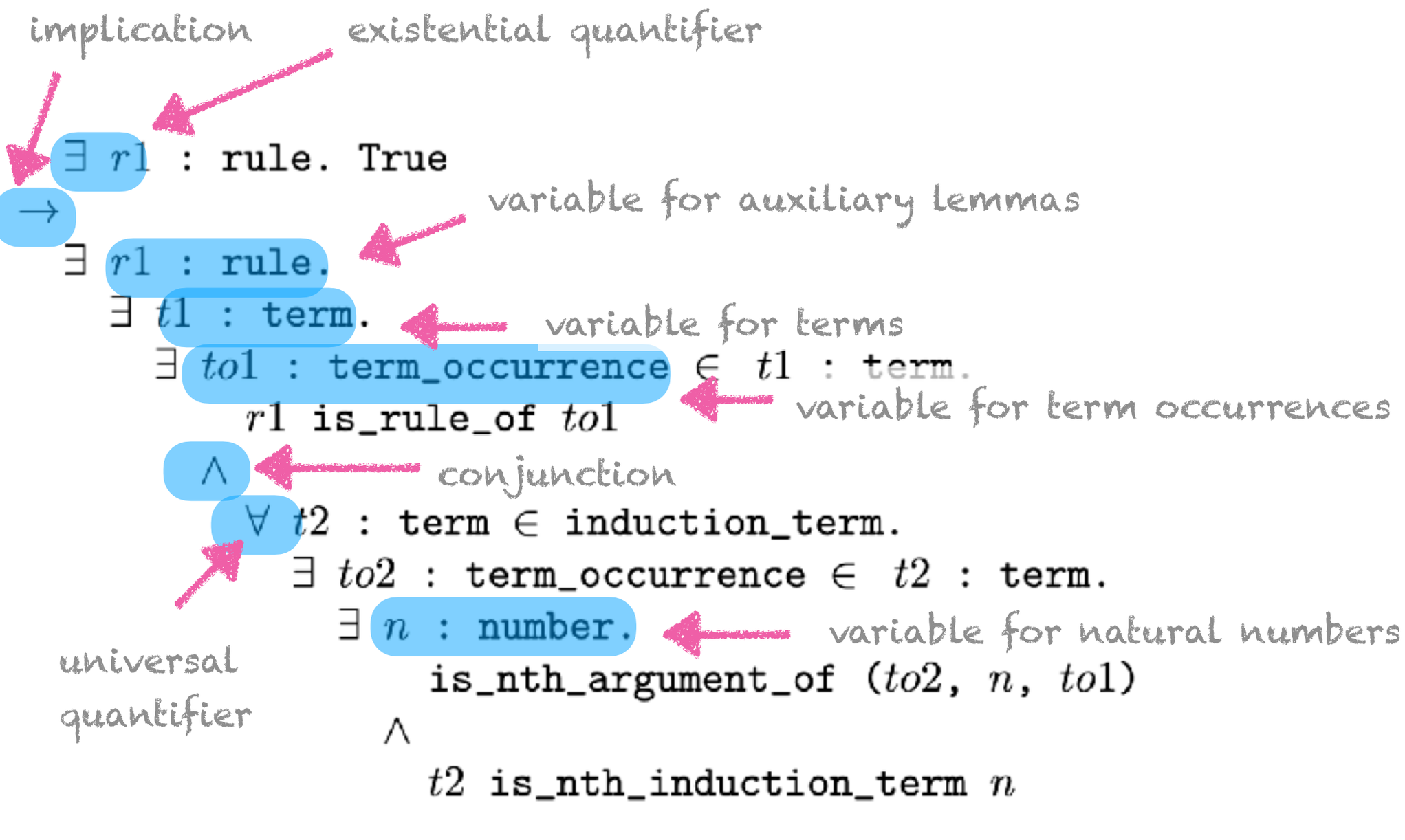
variable for term occurrences

conjunction

universal quantifier

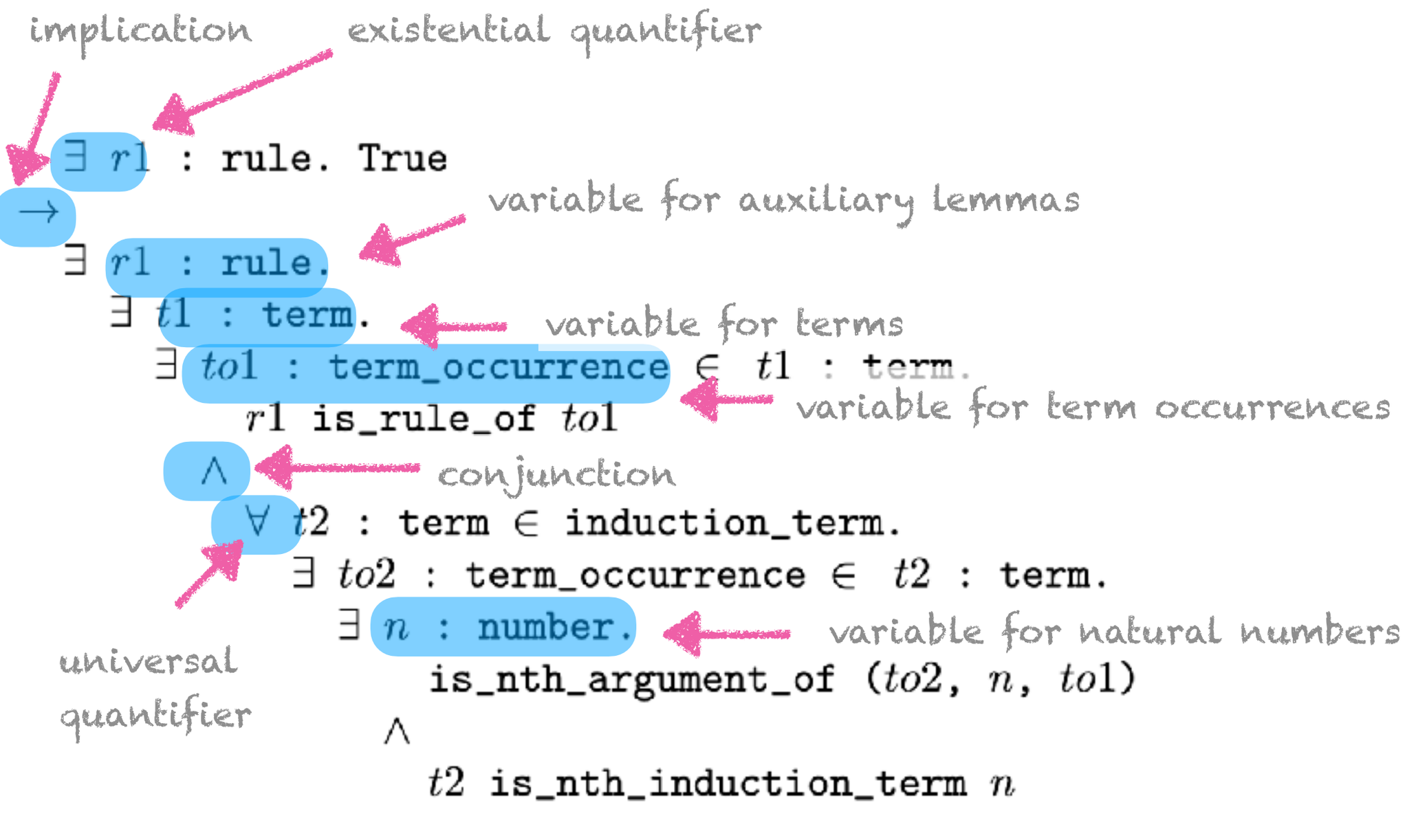
variable for natural numbers

Example Heuristic in LiFtEr (in Abstract Syntax)



Example Heuristic in LiFtEr (in Abstract Syntax)

LiFtEr heuristic: (proof goal * induction arguments) -> bool

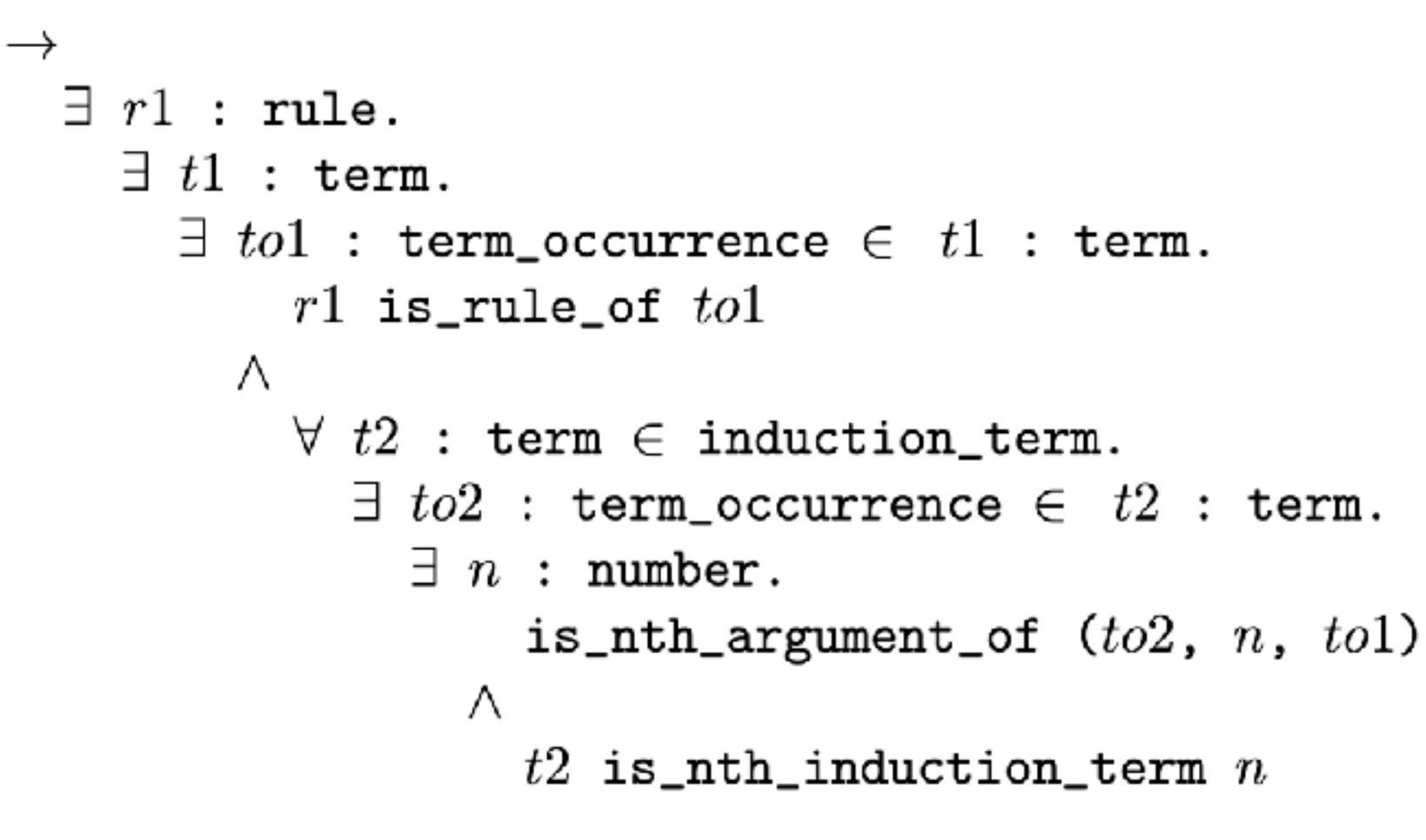


```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []       = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
∃ r1 : rule. True
```



```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

∃ r1 : rule. True

→ r1

∃ r1 : rule.

(r1 = itrev.induct)

∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

∃ $r1$: rule. True

→ r1

(r1 = itrev.induct)

∃ $r1$: rule.
 ∃ $t1$: term.
  ∃ $to1$: term_occurrence ∈ $t1$: term.
   $r1$ is_rule_of $to1$
  ∧
   ∀ $t2$: term ∈ induction_term.
    ∃ $to2$: term_occurrence ∈ $t2$: term.
     ∃ n : number.
      is_nth_argument_of ($to2$, n, $to1$)
      ∧
      $t2$ is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

∃ r1 : rule. True

→ r1

∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
      r1 is_rule_of to1
    ∧
      ∀ t2 : term ∈ induction_term.
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

(r1 = itrev.induct)
(t1 = itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

```
∃ r1 : rule. True
```

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
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fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

```
∃ r1 : rule. True
→
  ∃ r1 : rule.                                    ( r1 = itrev.induct )
    ∃ t1 : term.                                  ( t1  = itrev )
      ∃ to1 : term_occurrence ∈ t1 : term.        ( to1 = itrev )
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

∃ $r1$: rule. True

→

∃ $r1$: rule. (r1 = itrev.induct)
  ∃ $t1$: term. (t1 = itrev)
    ∃ $to1$: term_occurrence ∈ $t1$: term. (to1 = itrev)
      $r1$ is_rule_of $to1$ True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
      ∧
        ∀ $t2$: term ∈ induction_term.
          ∃ $to2$: term_occurrence ∈ $t2$: term.
            ∃ n : number.
              is_nth_argument_of ($to2$, n, $to1$)
              ∧
              $t2$ is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

∃ *r1* : rule. True

→

∃ *r1* : rule. (r1 = itrev.induct)
∃ *t1* : term. (t1 = itrev)
∃ *to1* : term_occurrence ∈ *t1* : term. (to1 = itrev)
r1 is_rule_of *to1*

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

∧
∀ *t2* : term ∈ induction_term.
∃ *to2* : term_occurrence ∈ *t2* : term.
∃ *n* : number.
is_nth_argument_of (*to2*, *n*, *to1*)
∧
t2 is_nth_induction_term *n*

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

$\exists$ *r1* : rule. True

$\rightarrow$

$\exists$ *r1* : rule.
  $\exists$ *t1* : term.
    $\exists$ *to1* : term_occurrence $\in$ *t1* : term.
      r1 is_rule_of *to1*
    $\wedge$
      $\forall$ *t2* : term $\in$ induction_term.
        $\exists$ *to2* : term_occurrence $\in$ *t2* : term.
          $\exists$ *n* : number.
            is_nth_argument_of (*to2*, *n*, *to1*)
          $\wedge$
            t2 is_nth_induction_term *n*

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
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 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

r1

∃ *r1* : rule. True

→

∃ *r1* : rule. (r1 = itrev.induct)
∃ *t1* : term. (t1 = itrev)
∃ *to1* : term_occurrence ∈ *t1* : term. (to1 = itrev)
r1 is_rule_of *to1*

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

∧
∀ *t2* : term ∈ induction_term.
∃ *to2* : term_occurrence ∈ *t2* : term.
∃ *n* : number.
is_nth_argument_of (*to2*, *n*, *to1*)
∧
t2 is_nth_induction_term *n*

```
primrec rev :: "'a list ⇒ 'a list" where
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 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1

t2

r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
  ∃ t1 : term. (t1 = itrev)
    ∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
      r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
    ∧
      ∀ t2 : term ∈ induction_term. (t2 = xs and ys)
        ∃ to2 : term_occurrence ∈ t2 : term.
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

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primrec rev :: "'a list ⇒ 'a list" where
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 "rev (x # xs) = rev xs @ [x]"

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 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 to2 t2 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

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primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 to2 t2 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
  ∃ t1 : term. (t1 = itrev)
    ∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
      r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
    ∧
      ∀ t2 : term ∈ induction_term. (t2 = xs and ys)
        ∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
          ∃ n : number.
            is_nth_argument_of (to2, n, to1)
          ∧
            t2 is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

to1 to2

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

t2 r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

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 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first to2 t2 first r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1) ?
∧
t2 is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first to2 t2 first r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1)
∧
t2 is_nth_induction_term n

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []       = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 first second to2 t2 second first r1

∃ r1 : rule. True

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
   ∧
   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
      ∧
      t2 is_nth_induction_term n

(r1 = itrev.induct)
(t1 = itrev)
(to1 = itrev)

True! r1 (= itrev.induct) is a lemma about to1 (= itrev).

(t2 = xs and ys)
(to2 = xs and ys)

when t2 is xs (n = 1) ?

when t2 is ys (n = 2) ?

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

to1 · first · second · to2 · t2 · second · first · r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
∃ t1 : term. (t1 = itrev)
∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
∧
∀ t2 : term ∈ induction_term. (t2 = xs and ys)
∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
∃ n : number.
is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1)
∧
t2 is_nth_induction_term n when t2 is ys (n = 2)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
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lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
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```

to1 · first · second · to2 · t2 · second · first · r1

∃ r1 : rule. True

→

∃ r1 : rule. (r1 = itrev.induct)
 ∃ t1 : term. (t1 = itrev)
  ∃ to1 : term_occurrence ∈ t1 : term. (to1 = itrev)
   r1 is_rule_of to1 True! r1 (= itrev.induct) is a lemma about to1 (= itrev).
  ∧
   ∀ t2 : term ∈ induction_term. (t2 = xs and ys ✓ ✓)
    ∃ to2 : term_occurrence ∈ t2 : term. (to2 = xs and ys)
     ∃ n : number.
      is_nth_argument_of (to2, n, to1) when t2 is xs (n = 1)
     ∧
      t2 is_nth_induction_term n when t2 is ys (n = 2)

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

```
datatype      instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

```
∃ r1 : rule. True
→
  ∃ r1 : rule.
    ∃ t1 : term.
      ∃ to1 : term_occurrence ∈ t1 : term.
          r1 is_rule_of to1
        ∧
          ∀ t2 : term ∈ induction_term.
            ∃ to2 : term_occurrence ∈ t2 : term.
              ∃ n : number.
                  is_nth_argument_of (to2, n, to1)
                ∧
                  t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

the same LiFtEr assertion

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

$\exists$ *r1* : rule. True

$\rightarrow$

$\exists$ $r1$: rule.
$\exists$ $t1$: term.
$\exists$ $to1$: term_occurrence $\in$ $t1$: term.
$r1$ is_rule_of $to1$
$\wedge$
$\forall$ $t2$: term $\in$ induction_term.
$\exists$ $to2$: term_occurrence $\in$ $t2$: term.
$\exists$ n : number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

r1

(r1 = exec.induct)

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

r1

(r1 = exec.induct)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

r1

(r1 = exec.induct)
(t1 = exec)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->
∃ r1 : rule. True

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

→

$$
\begin{array}{l}
\exists\, r1 : \mathtt{rule}.\\
\quad \exists\, t1 : \mathtt{term}.\\
\quad\quad \exists\, to1 : \mathtt{term\_occurrence} \in t1 : \mathtt{term}.\\
\qquad r1\ \mathtt{is\_rule\_of}\ to1\\
\qquad\quad \wedge\\
\qquad\quad \forall\, t2 : \mathtt{term} \in \mathtt{induction\_term}.\\
\qquad\qquad \exists\, to2 : \mathtt{term\_occurrence} \in t2 : \mathtt{term}.\\
\qquad\qquad\quad \exists\, n : \mathtt{number}.\\
\qquad\qquad\qquad \mathtt{is\_nth\_argument\_of}\ (to2,\ n,\ to1)\\
\qquad\qquad\qquad\quad \wedge\\
\qquad\qquad\qquad t2\ \mathtt{is\_nth\_induction\_term}\ n
\end{array}
$$

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

∃ r1 : rule. True

→

```
∃ r1 : rule.
  ∃ t1 : term.
    ∃ to1 : term_occurrence ∈ t1 : term.
        r1 is_rule_of to1
      ∧
        ∀ t2 : term ∈ induction_term.
          ∃ to2 : term_occurrence ∈ t2 : term.
            ∃ n : number.
                is_nth_argument_of (to2, n, to1)
              ∧
                t2 is_nth_induction_term n
```

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->
∃ r1 : rule. True

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->
∃ r1 : rule. True

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
apply(induct is1 s stk rule:exec.induct)
 apply auto done
```

to1

r1

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
  ∧
   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
     ∧
      t2 is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

r1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 to1 to2

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

∧

∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

new types ->

new constants ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n       # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->
a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.
r1 is_rule_of to1
∧
∀ t2 : term ∈ induction_term.
∃ to2 : term_occurrence ∈ t2 : term.
∃ n : number.
is_nth_argument_of (to2, n, to1)
∧
t2 is_nth_induction_term n

t2 = is1, s, and stk
to1 = exec
to2 = is1, s, and stk
r1 = exec.induct

True! r1 (= exec.induct) is a lemma about to1 (= exec).

new types ->

new constants ->

new lemma ->

a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
 "exec1 (LOAD x)  s   stk       =  s(x)     # stk" |
 "exec1  ADD      _ (j#i#stk)   =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

t2 t01 t02 r1

$\exists$ r1 : rule. True

$\rightarrow$

$\exists\ r1$: rule.
$\exists\ t1$: term.
$\exists\ to1$: term_occurrence $\in$ $t1$: term.
r1 is_rule_of $to1$
$\wedge$
$\forall\ t2$: term $\in$ induction_term.
$\exists\ to2$: term_occurrence $\in$ $t2$: term.
$\exists\ n$: number.
is_nth_argument_of ($to2$, n, $to1$)
$\wedge$
$t2$ is_nth_induction_term n

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk       =  n        # stk" |
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```
fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first

to1

to2

t2

r1

first

∃ r1 : rule. True

→

∃ $r1$: rule.
∃ $t1$: term.
∃ $to1$: term_occurrence ∈ $t1$: term.
$r1$ is_rule_of $to1$
∧
∀ $t2$: term ∈ induction_term.
∃ $to2$: term_occurrence ∈ $t2$: term.
∃ n : number.
is_nth_argument_of ($to2$, n, $to1$)
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$t2$ is_nth_induction_term n

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True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
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when t2 is is1 (n -> 1) ?

new types ->

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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []       _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"
```

first

t2 to1

new lemma ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

to2

r1

∃ r1 : rule. True

→

∃ r1 : rule.
∃ t1 : term.
∃ to1 : term_occurrence ∈ t1 : term.

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

r1 is_rule_of to1 True! r1 (= exec.induct) is a lemma about to1 (= exec).

∧

∀ t2 : term ∈ induction_term.

(t2 = is1, s, and stk)

∃ to2 : term_occurrence ∈ t2 : term.

(to2 = is1, s, and stk)

∃ n : number.

is_nth_argument_of (to2, n, to1) when t2 is is1 (n -> 1)

∧

t2 is_nth_induction_term n

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
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new lemma ->

```
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```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first, second, t2, to1, to2, r1

$\exists$ r1 : rule. True

$\rightarrow$

$\exists$ $r1$: rule.
$\exists$ $t1$: term.
$\exists$ $to1$: term_occurrence $\in$ $t1$: term.

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

$r1$ is_rule_of $to1$ — True! r1 (= exec.induct) is a lemma about to1 (= exec).

$\wedge$

$\forall$ $t2$: term $\in$ induction_term. (t2 = is1, s, and stk)

$\exists$ $to2$: term_occurrence $\in$ $t2$: term. (to2 = is1, s, and stk)

$\exists$ n : number.

is_nth_argument_of $(to2, n, to1)$ — when t2 is is1 (n -> 1) ?

$\wedge$

$t2$ is_nth_induction_term n — when t2 is s (n -> 2) ?

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

new constants ->

```
fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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```

a model proof ->

```
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

first (t2) second (to1) first (to2) second (r1)

∃ r1 : rule. True

→

∃ $r1$: rule.
∃ $t1$: term.
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∀ $t2$: term ∈ induction_term.
∃ $to2$: term_occurrence ∈ $t2$: term.
∃ n : number.
is_nth_argument_of ($to2$, n, $to1$) when t2 is is1 (n -> 1)
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(r1 = exec.induct)
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new lemma ->

a model proof ->

```
lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
 apply(induct is1 s stk rule:exec.induct)
  apply auto done
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∃ r1 : rule. True

→

∃ r1 : rule.
 ∃ t1 : term.
  ∃ to1 : term_occurrence ∈ t1 : term.
   r1 is_rule_of to1
  ∧
   ∀ t2 : term ∈ induction_term.
    ∃ to2 : term_occurrence ∈ t2 : term.
     ∃ n : number.
      is_nth_argument_of (to2, n, to1)
     ∧
      t2 is_nth_induction_term n

first second third t2

first second third to1

r1

to2

(r1 = exec.induct)
(t1 = exec)
(to1 = exec)

True! r1 (= exec.induct) is a lemma about to1 (= exec).

(t2 = is1, s, and stk)
(to2 = is1, s, and stk)

when t2 is is1 (n -> 1) ?
when t2 is s (n -> 2) ?
when t2 is stk (n -> 3) ?

new types ->

new constants ->

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first second third t2

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to2

new types ->

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first second third t2

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smart_induct… does it work?

Table 1: Coincidence Rates of `smart_induct`.

theory	total	top_1	top_3	top_5	top_10
`DFS`	10	6 (60%)	9 (90%)	9 (90%)	9 (90%)
`Nearest_Neighbors`	16	3 (19%)	4 (25%)	7 (44%)	12 (75%)
`RST_RBT`	24	24 (100%)	24 (100%)	24 (100%)	24 (100%)
sum	50	33 (65%)	37 (74%)	40 (80%)	45 (90%)

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Table 2: Coincidence Rates of `smart_induct` Based Only on Induction Terms.

theory	total	top_1	top_3	top_5	top_10
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smart_induct… does it work?

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Overall, the results are good!

But finding out what variables to generalise remains as a challenge!

smart_induct... does it work?

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But finding out what variables to generalise remains as a challenge!

WIP!

Today I talked about…

2013 ~ 2017

Intern & Engineer

http://www.cse.unsw.edu.au/~kleing/

with Prof. Gerwin Klein

Cogent

PhD in AI for theorem proving

2017 ~ 2018

http://cl-informatik.uibk.ac.at/users/cek/

with Prof. Cezary Kaliszyk

PSL

PaMpeR

2018 ~ 2020/21

http://ai4reason.org/members.html

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LiFtEr

smart_induct

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LiFtEr — **APLAS2019**

smart_indu — **under review at IJCAR2020**

Today I talked about… *fin.*

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backup slides for Q&A

<- simple representation


```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
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<- simple representation


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```

<- relevant definitions

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev   []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
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<- simple representation


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LiFtEr LiFtEr LiFtEr

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LiFtEr 2 LiFtEr 2

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<- nested assertions to examine the "semantics" of constants (rev and itrev)

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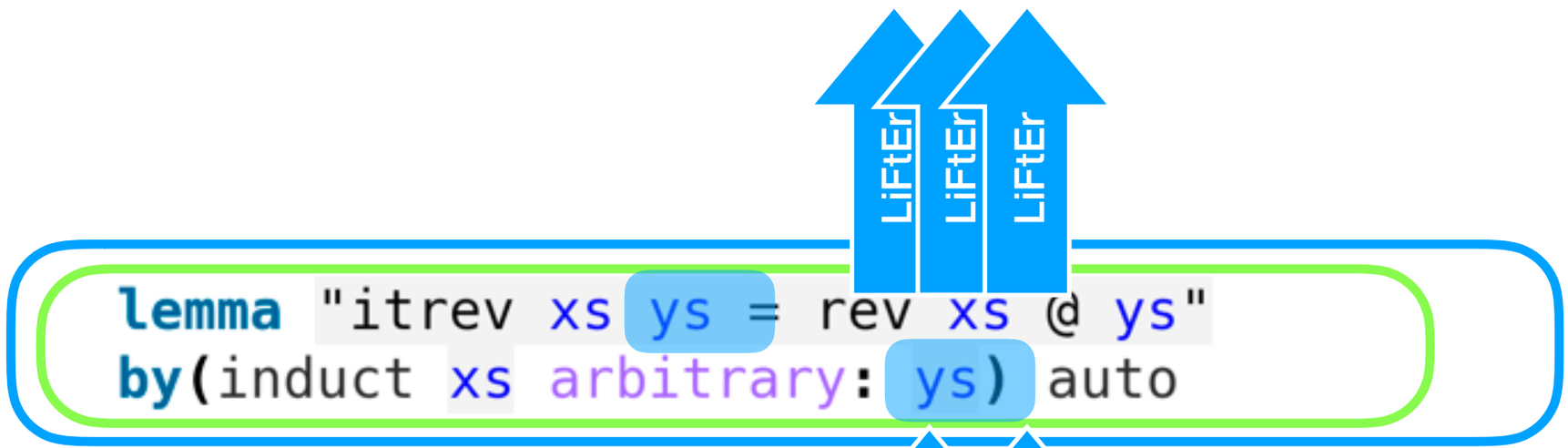
<- relevant definitions

LiFtEr: (proof goal * induction arguments) -> bool

<- simple representation




```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

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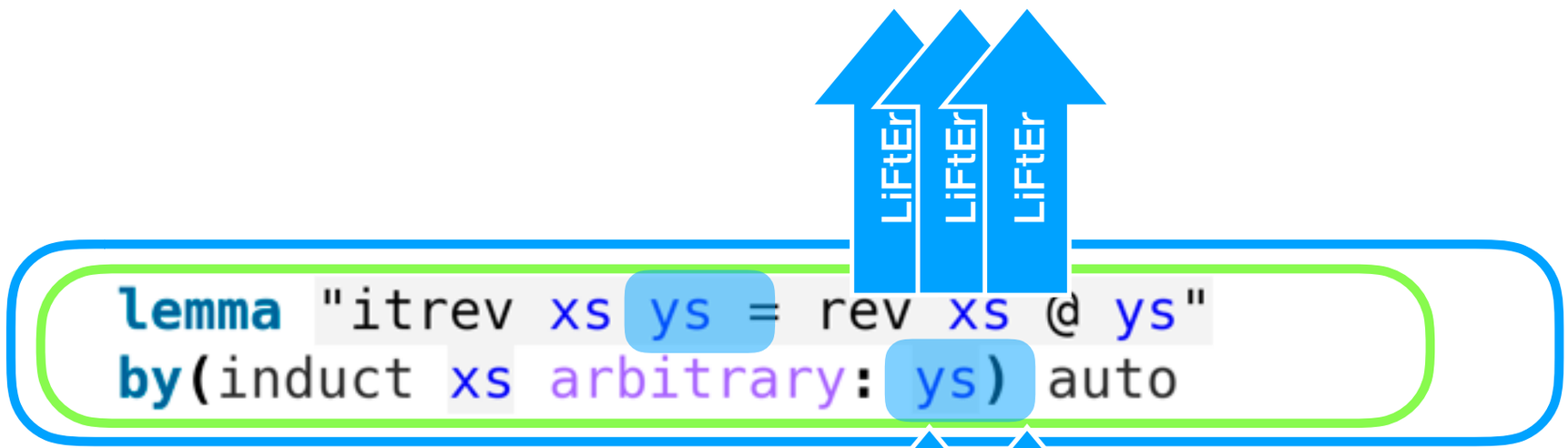
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

LiFtEr$_2$ (proof goal * induction arguments) -> bool
*** relevant definitions**

<- simple representation

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fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

<- relevant definitions

LiFtEr$_2$ (proof goal * induction arguments) -> bool
*** relevant definitions**

coming soon

LiFtEr LiFtEr LiFtEr

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs arbitrary: ys) auto
```

LiFtEr 2 LiFtEr 2

```
lemma "itrev xs ys = rev xs @
by(induct xs ys rule:"itrev.i                ) auto
```

<- nested assertions to examine the "semantics" of constants (rev and itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev   []       = []" |
 "rev (x # xs) = rev xs @ [x

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev   []     ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

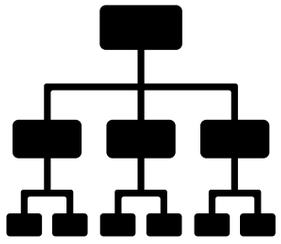
<- relevant definitions

LiFtEr: (proof goal * induction arguments) -> bool

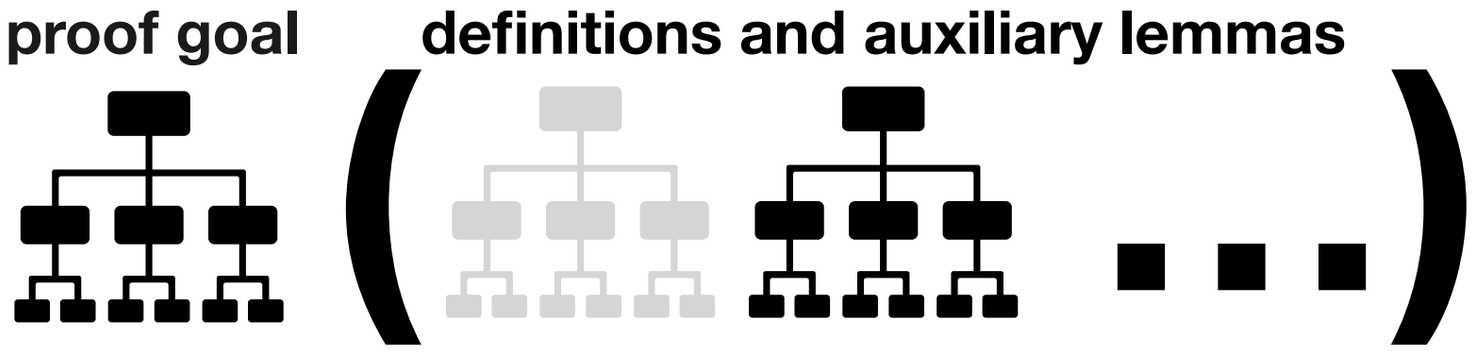
LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

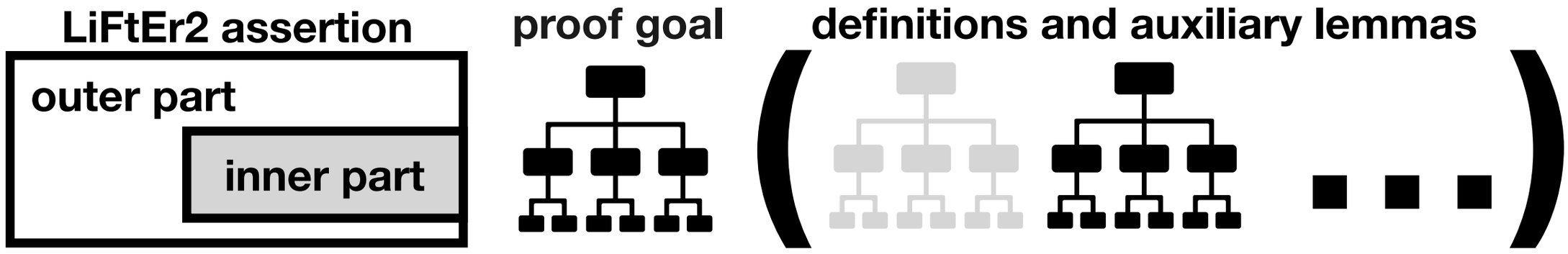
proof goal



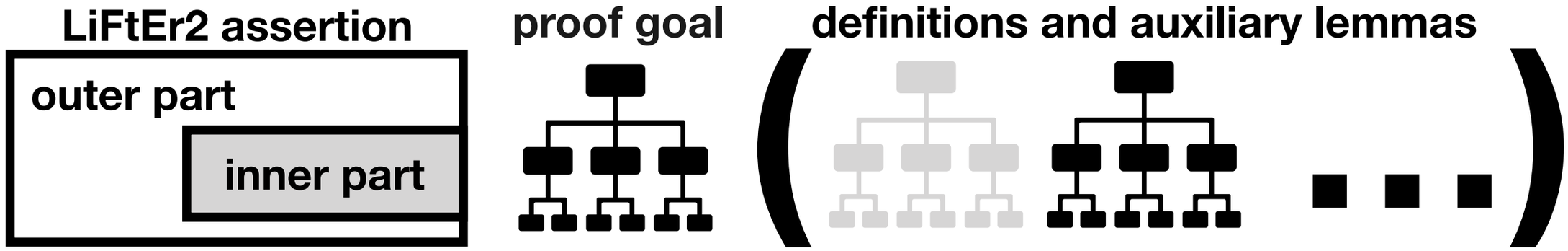
LiFtEr$_2$ (proof goal * induction arguments) -> bool
*** relevant definitions**



LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

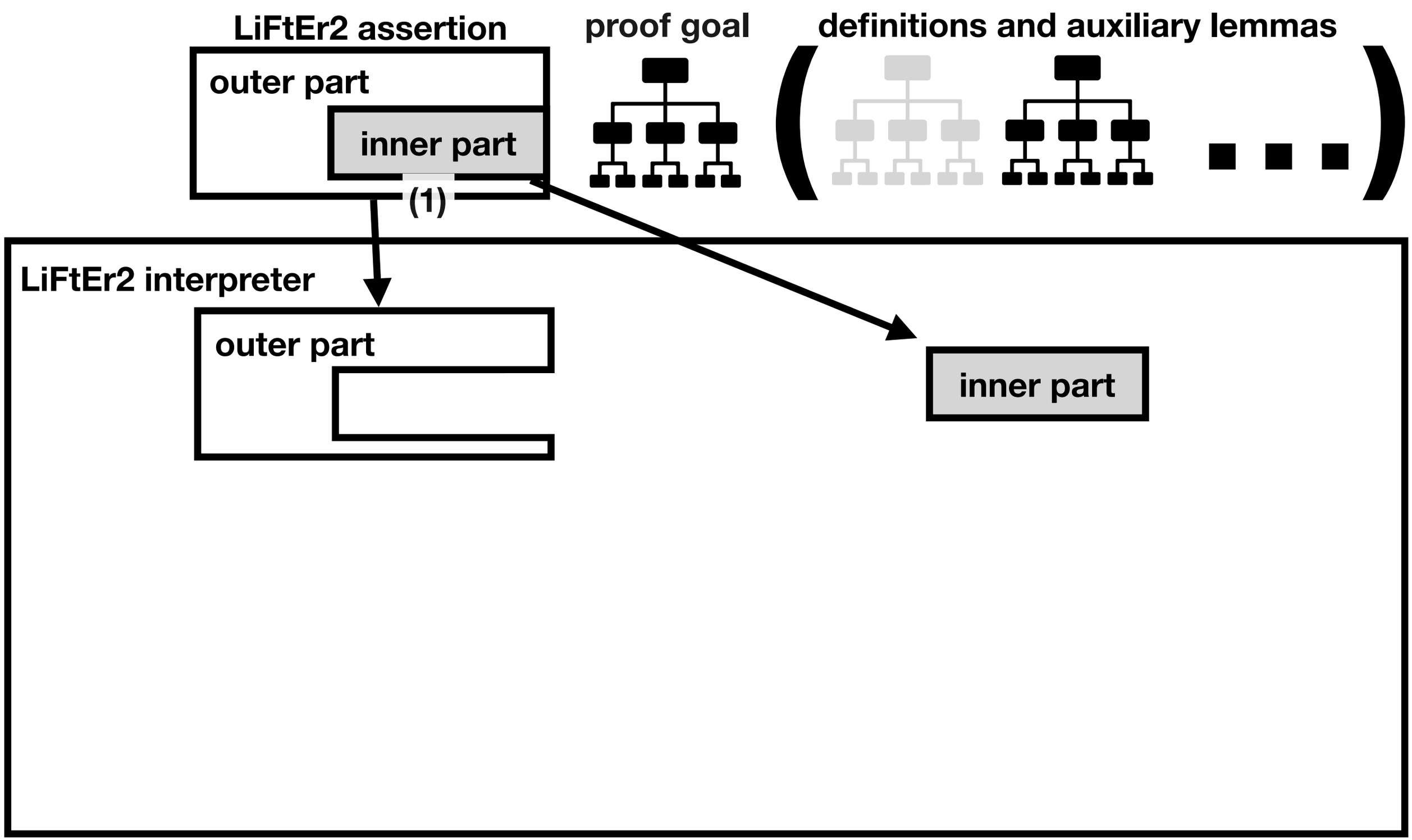


LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

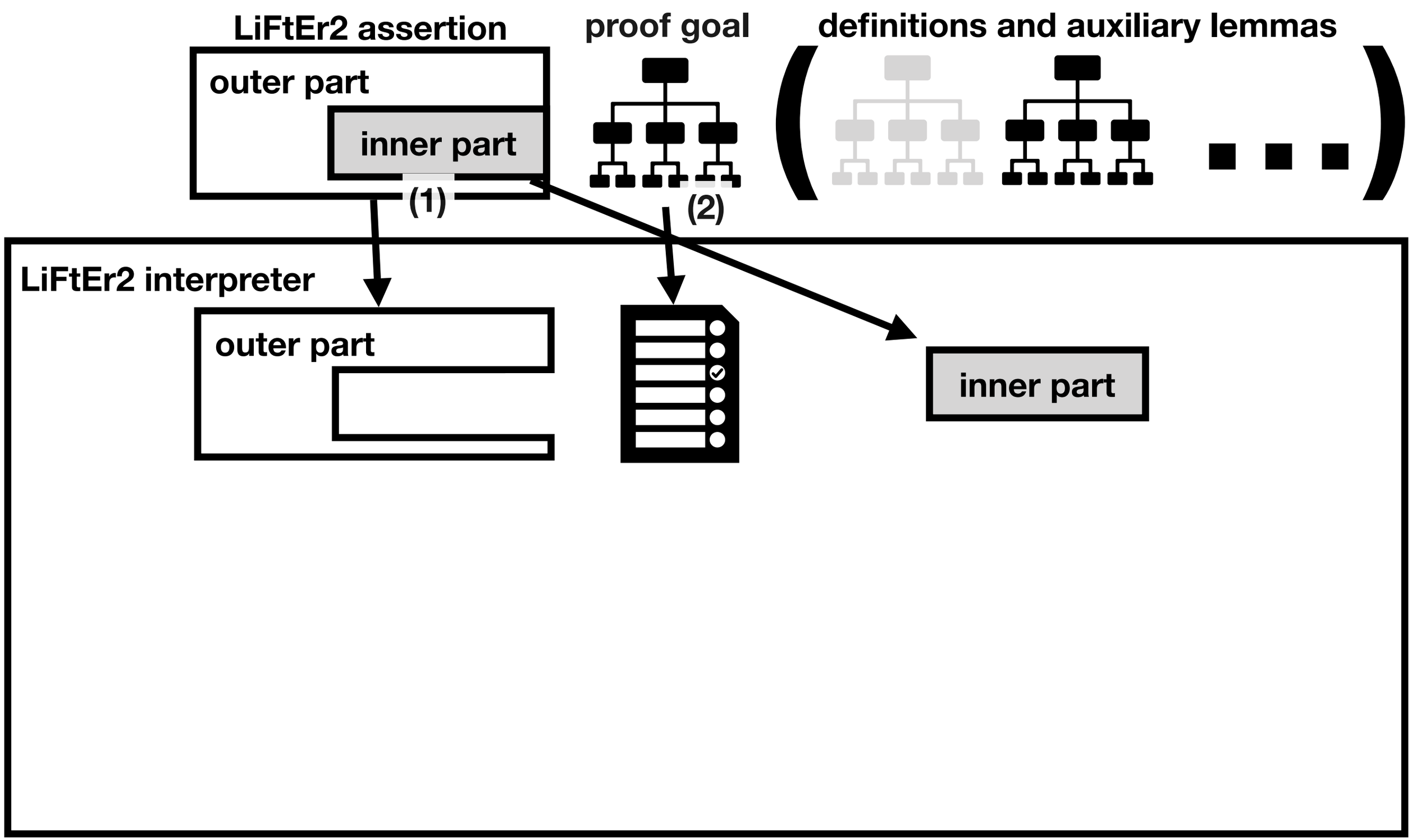


LiFtEr2 interpreter

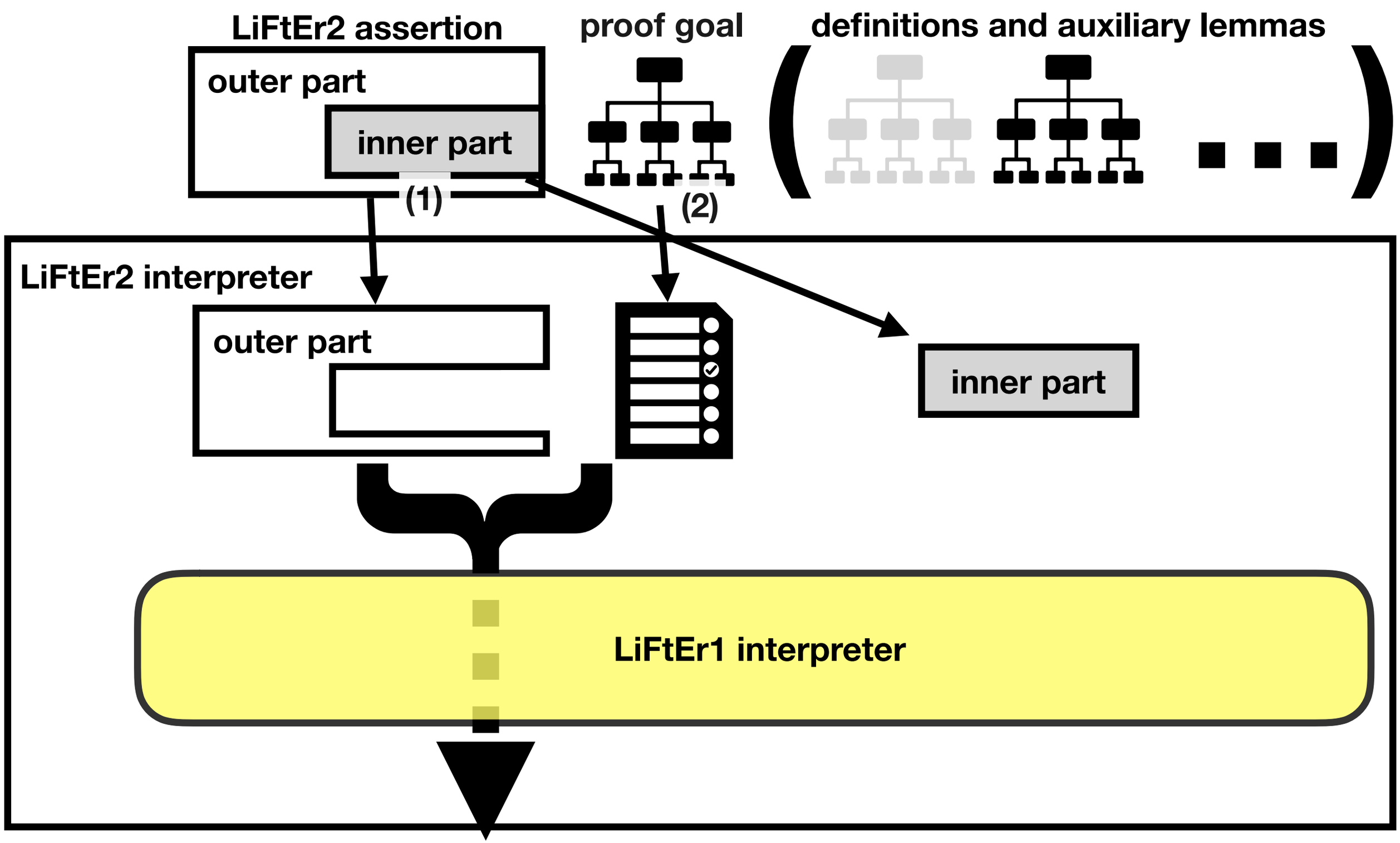
LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions



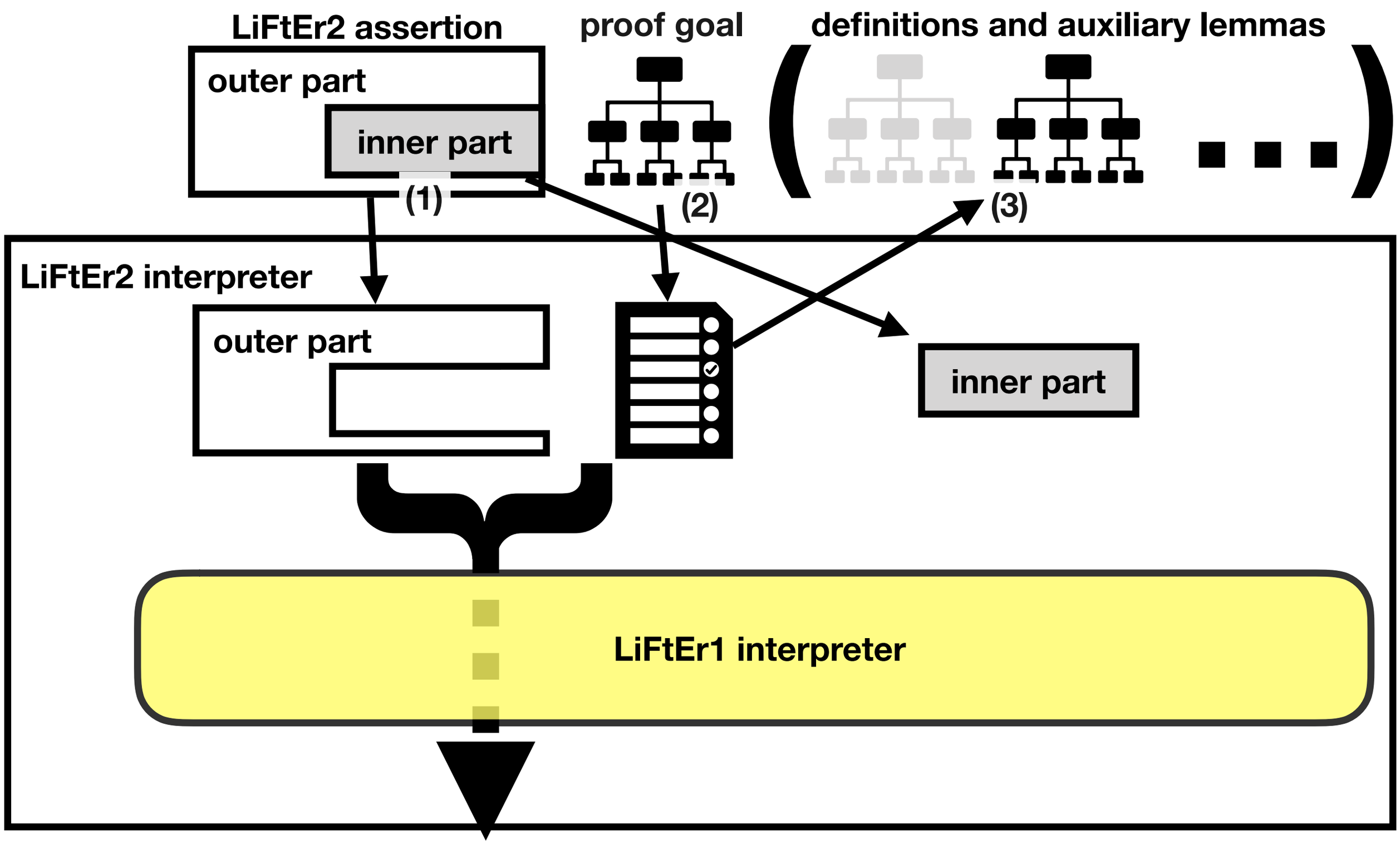
LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions



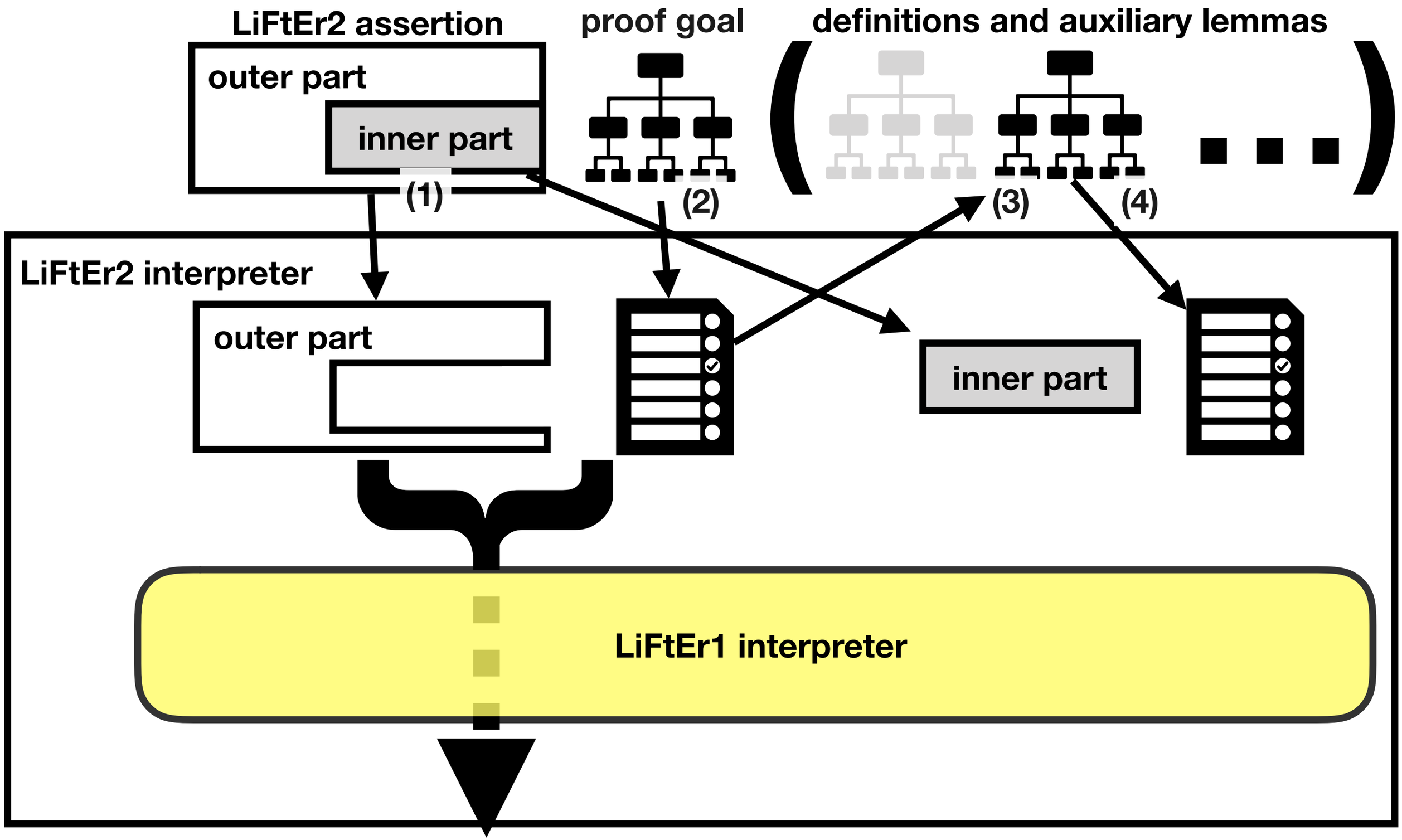
LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



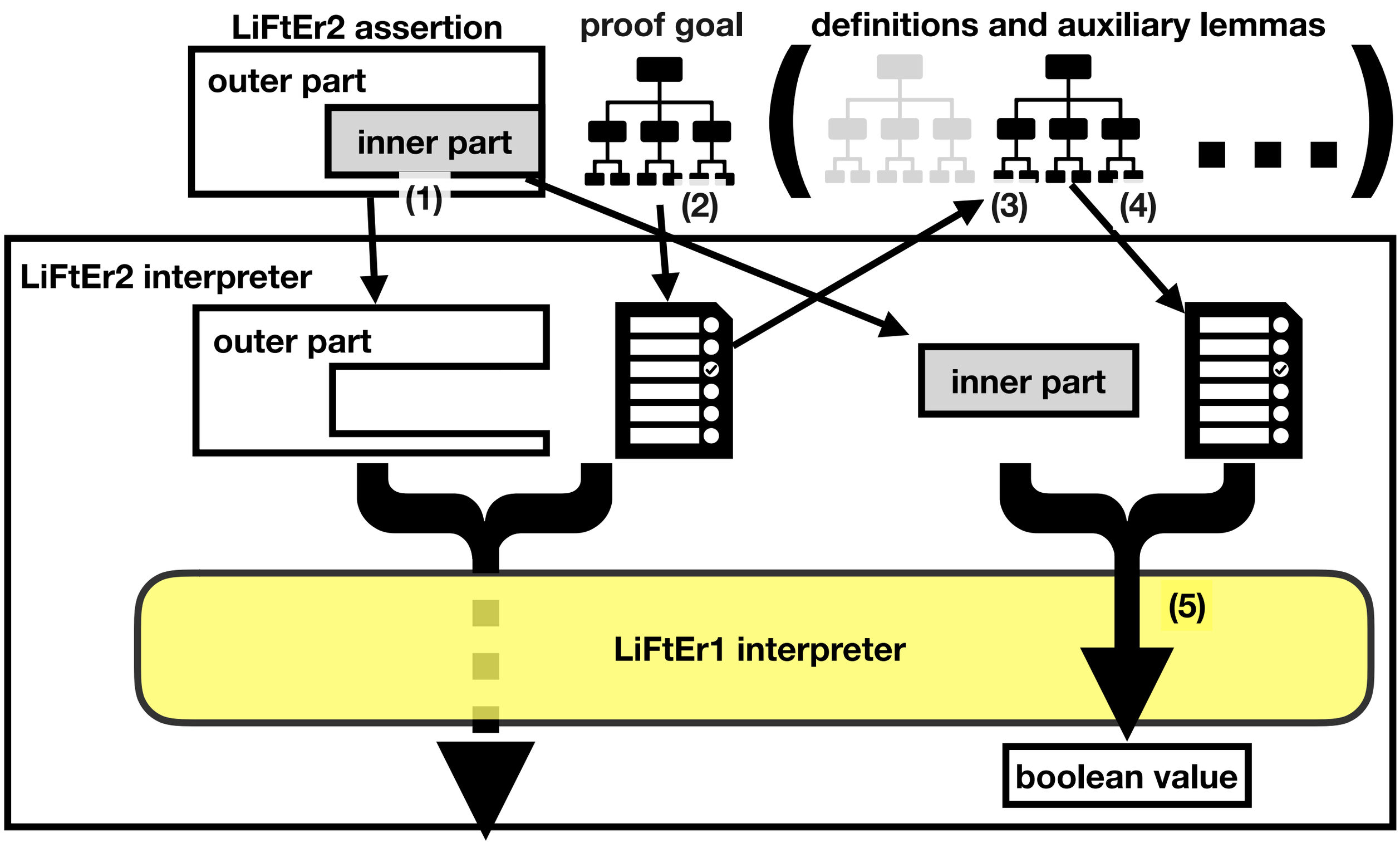
LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



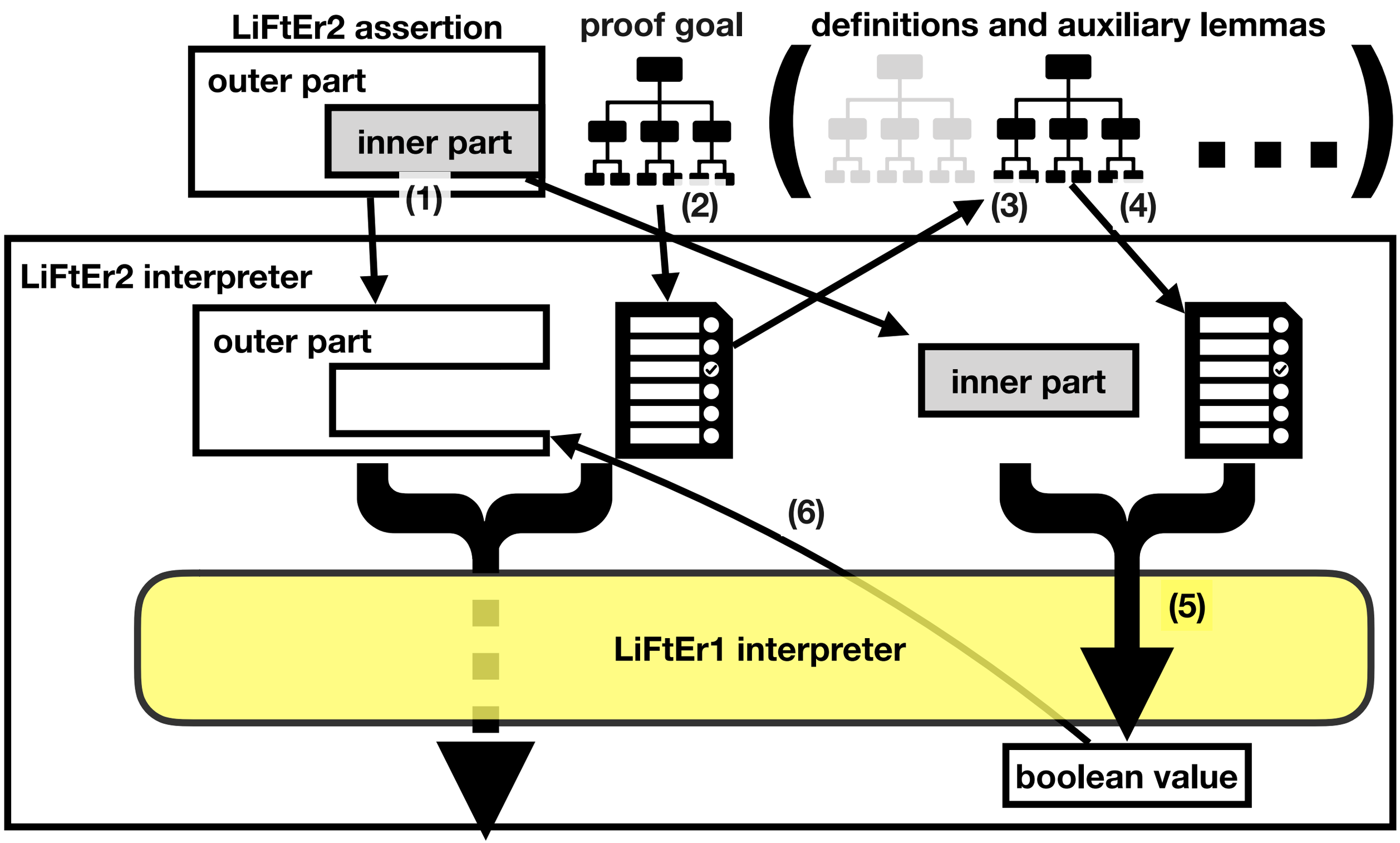
LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



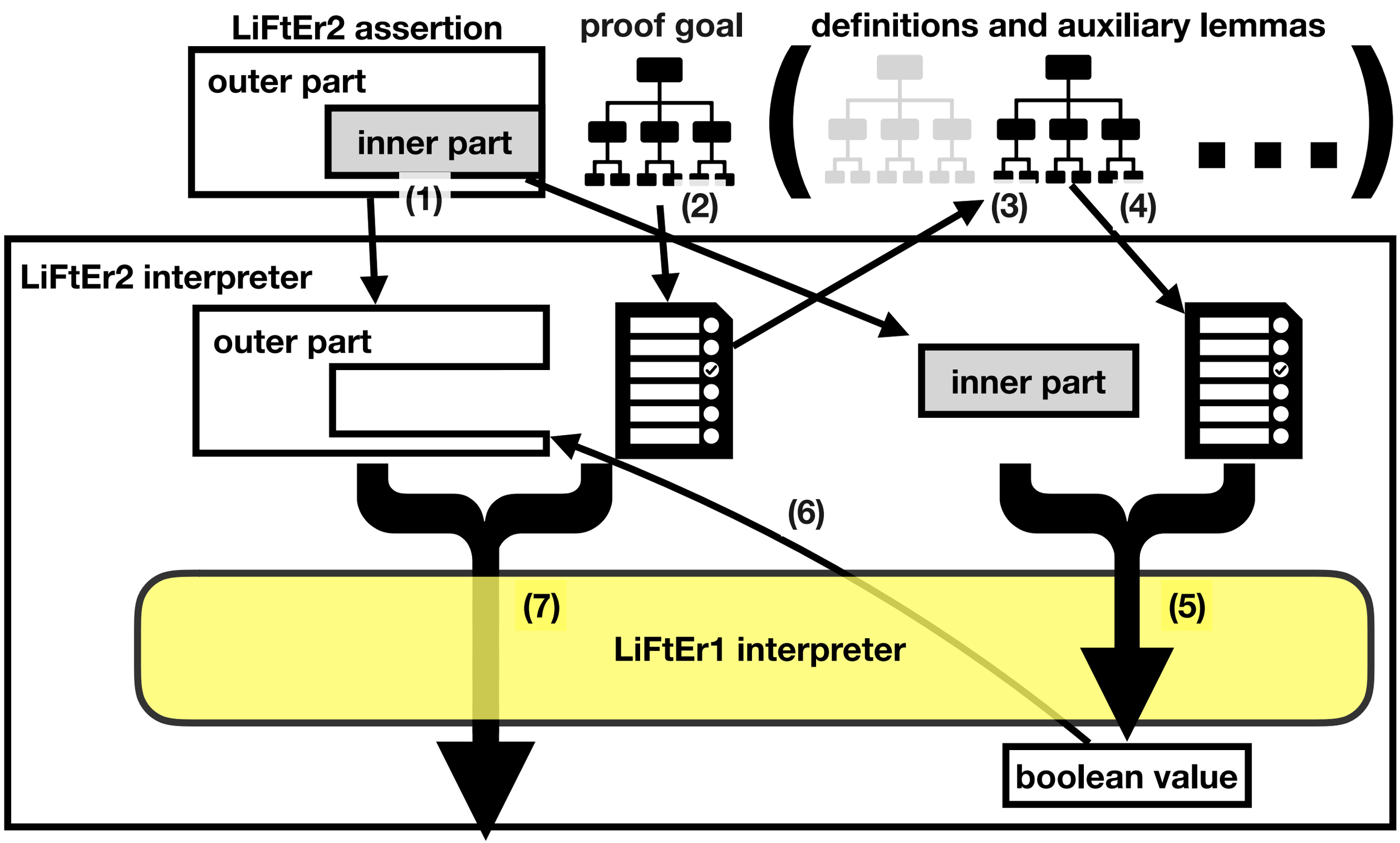
LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



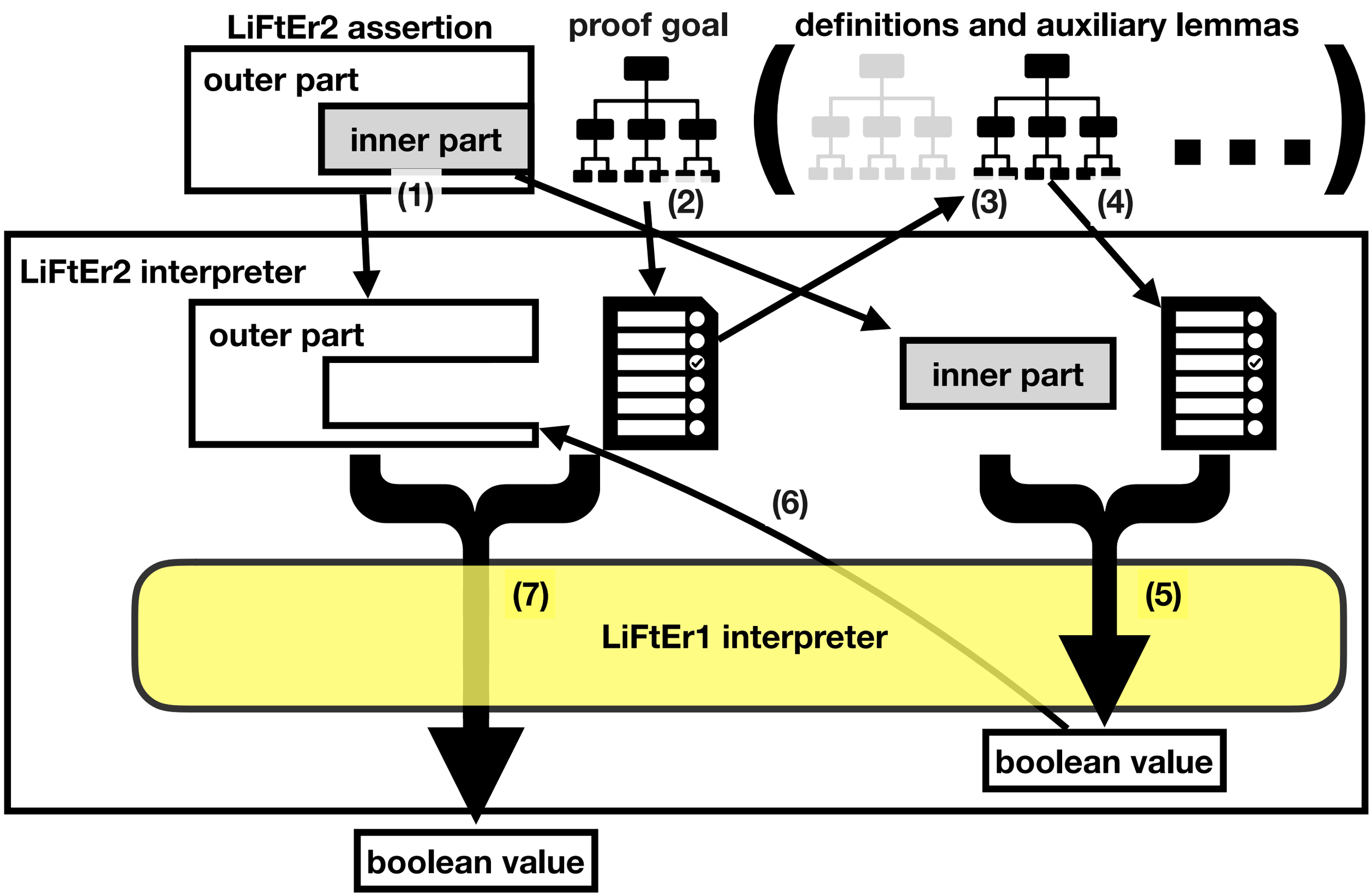
LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool

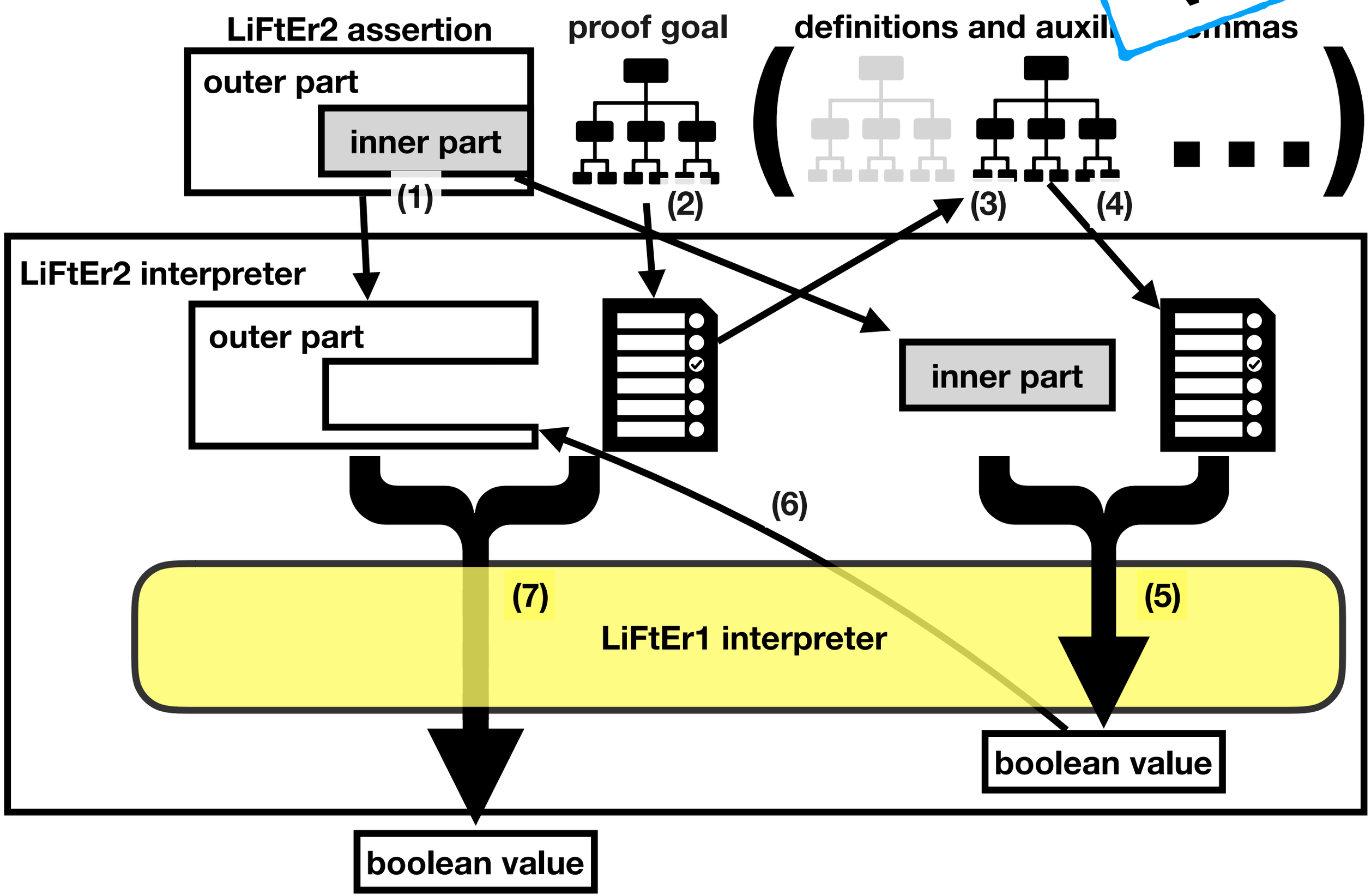


LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool

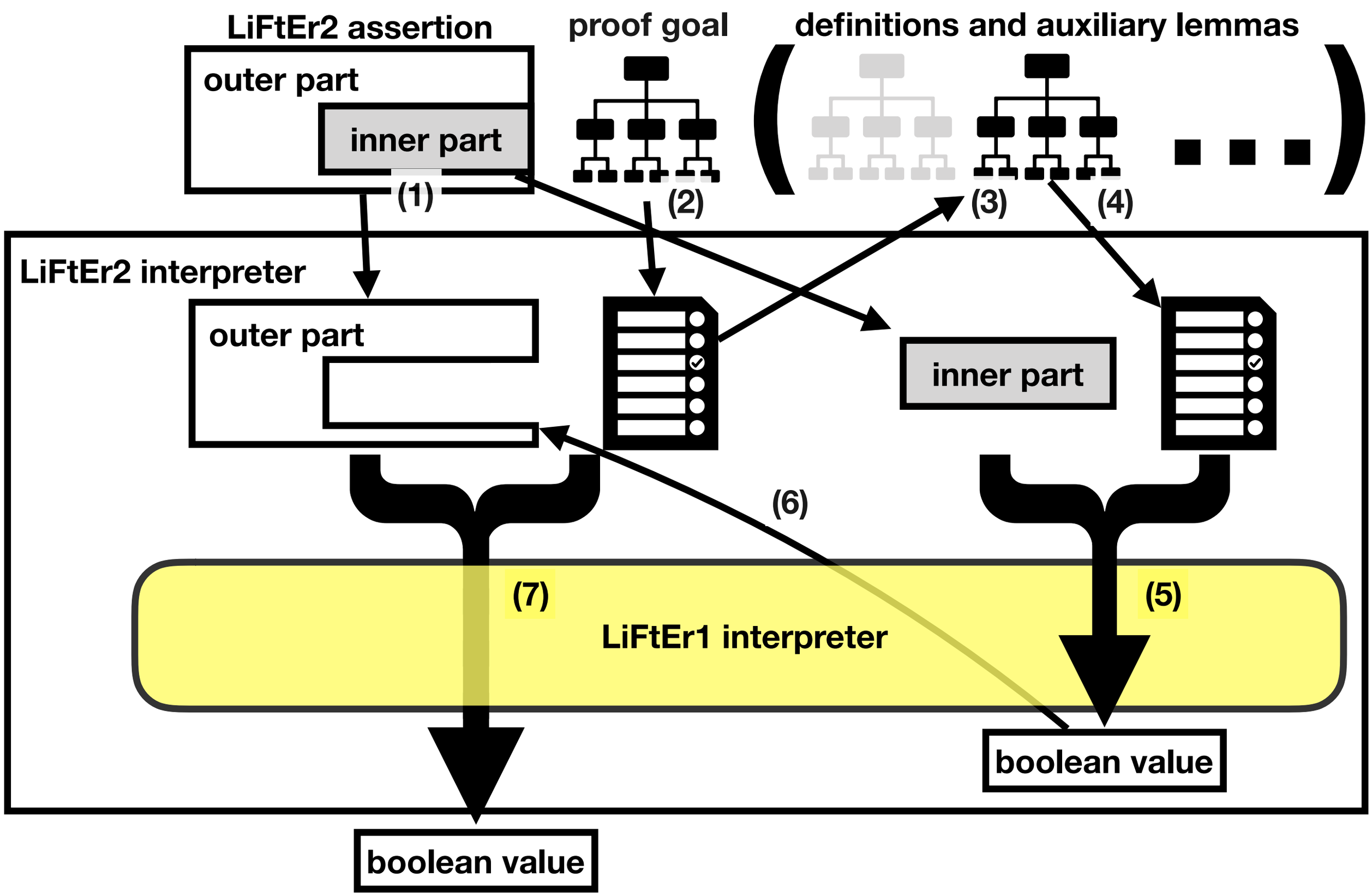


LiFtEr$_2$ (proof goal * induction arguments) -> bool
* relevant definitions

WIP!



LiFtEr$_2$ (proof goal * induction arguments * relevant definitions) -> bool



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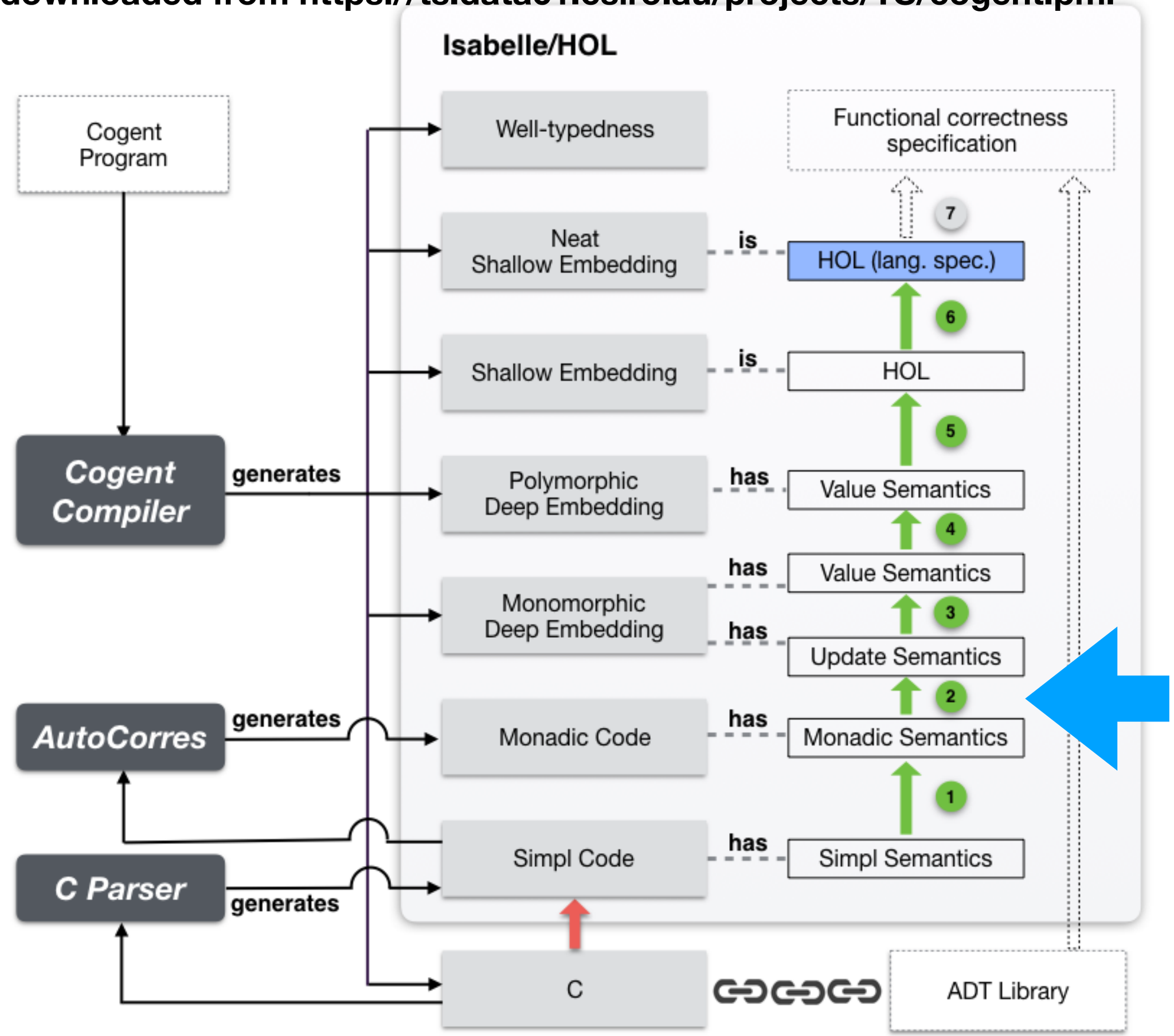
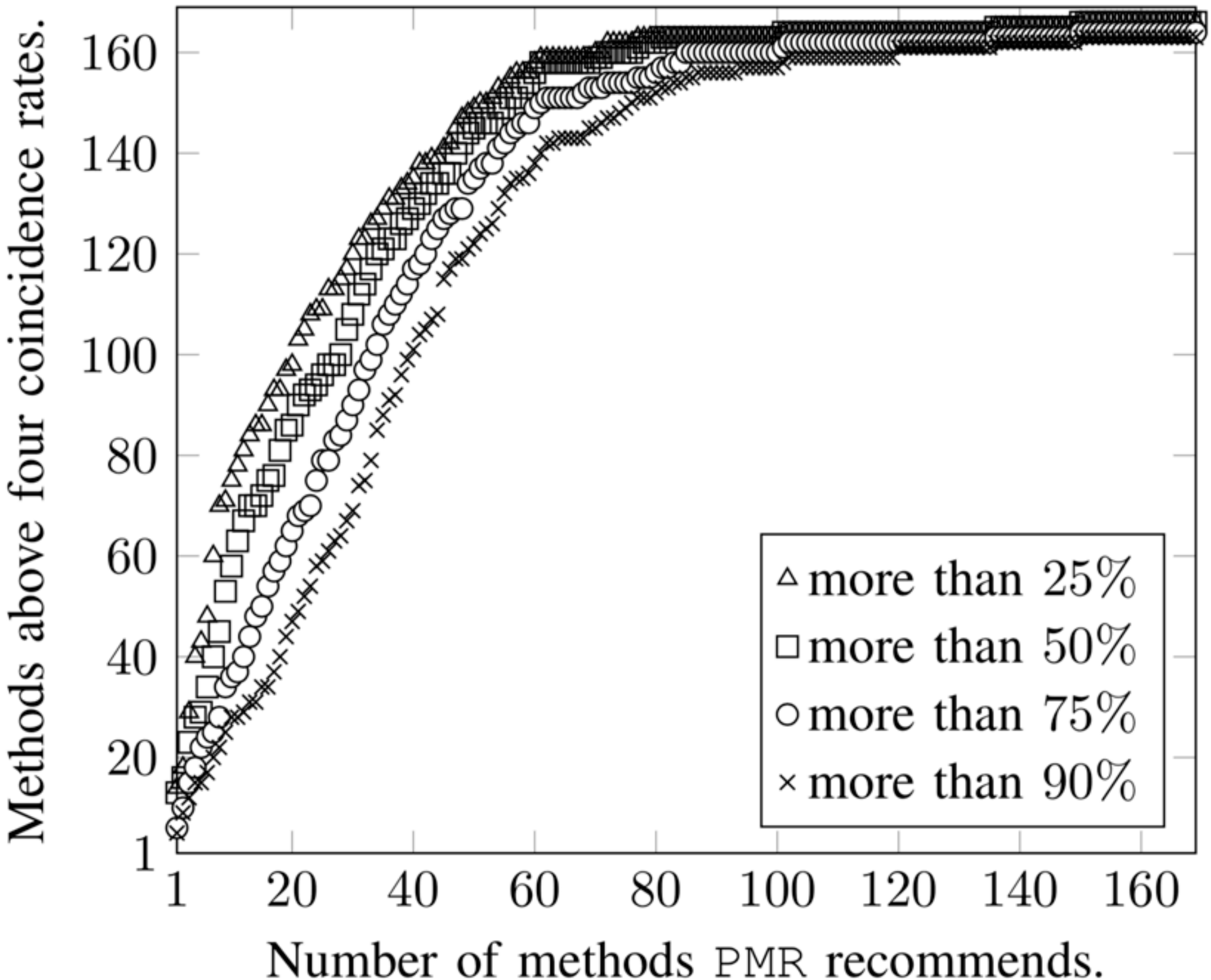
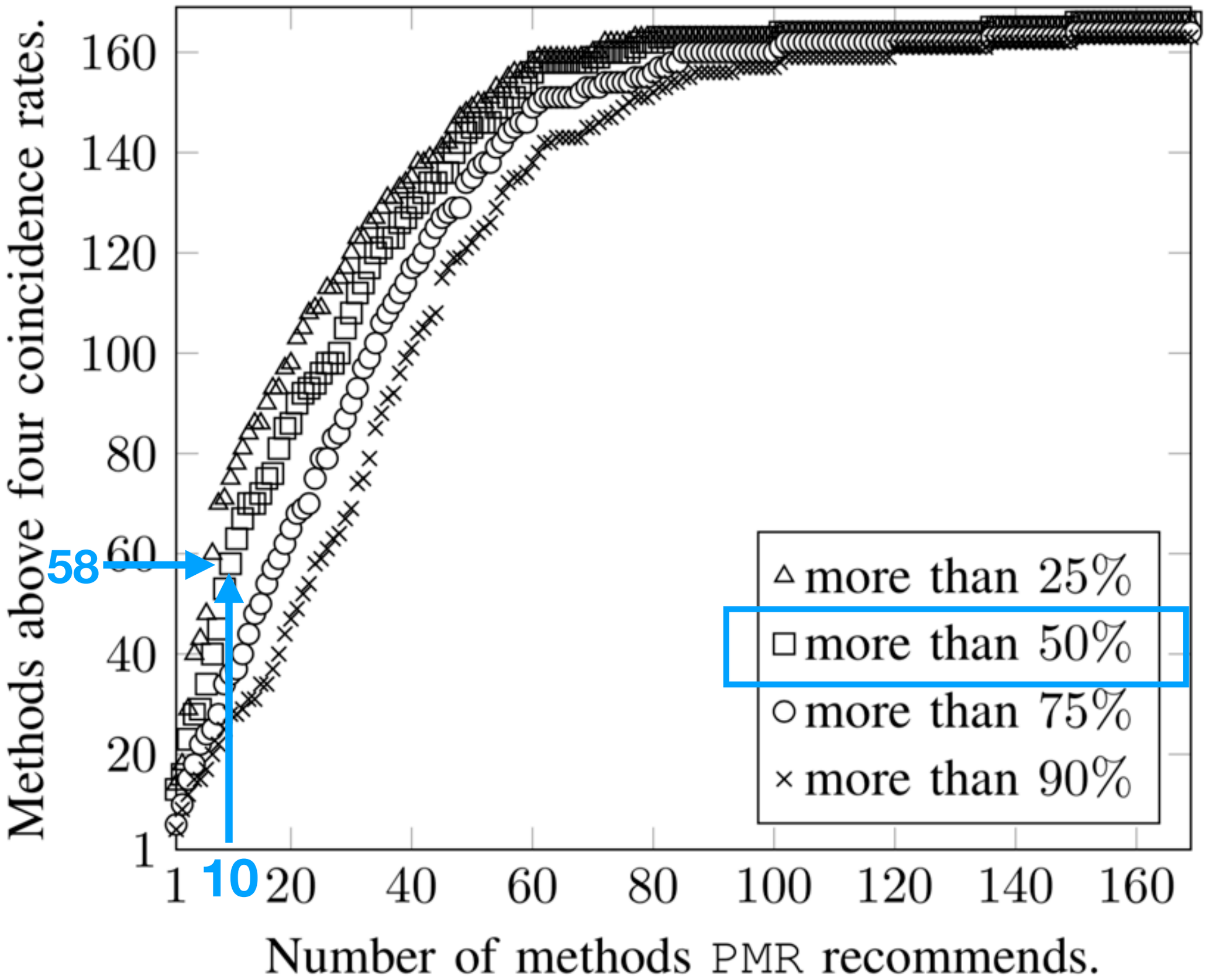


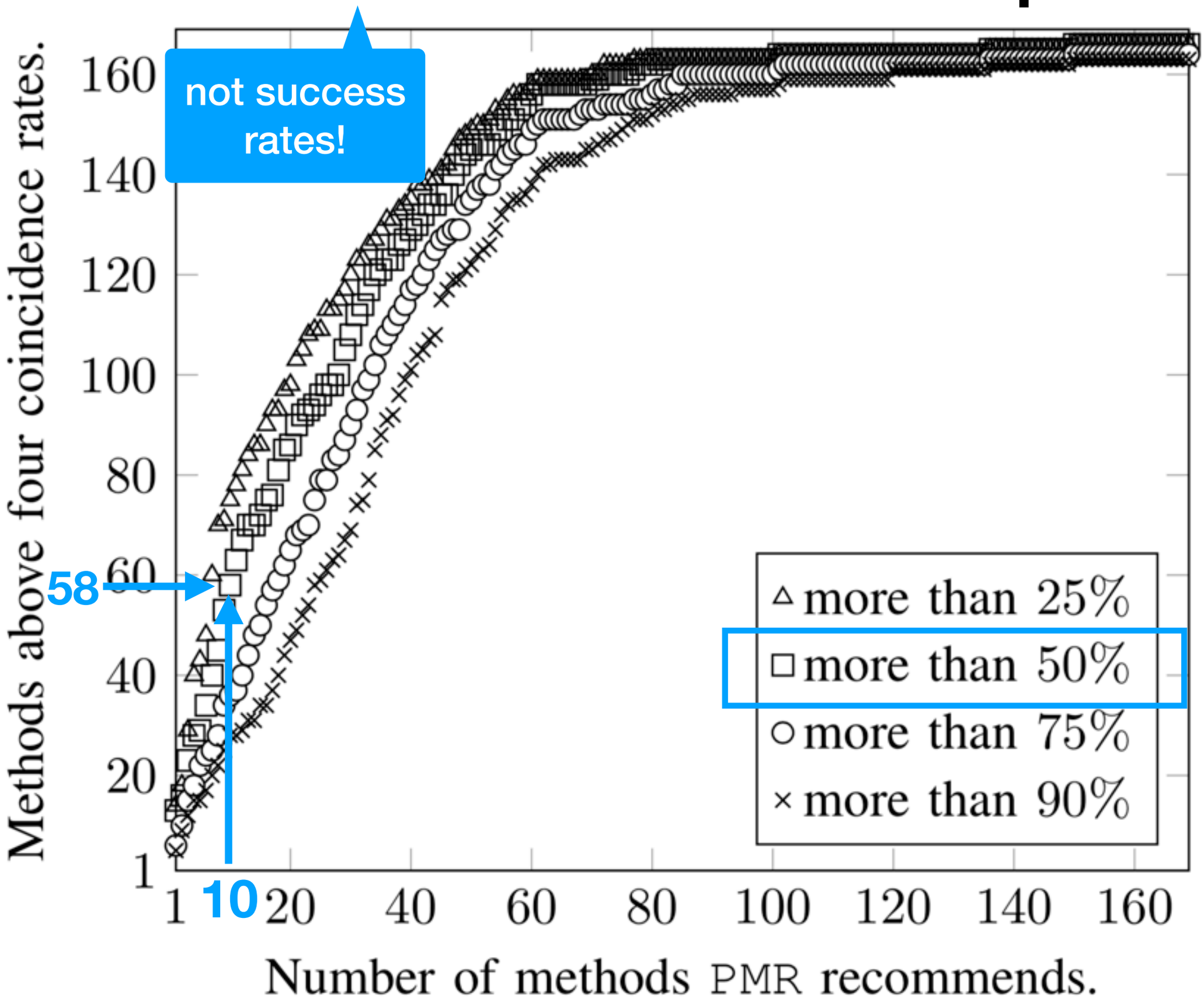
Coincidence rates of PaMpeR



Coincidence rates of PaMpeR



Coincidence rates of PaMpeR



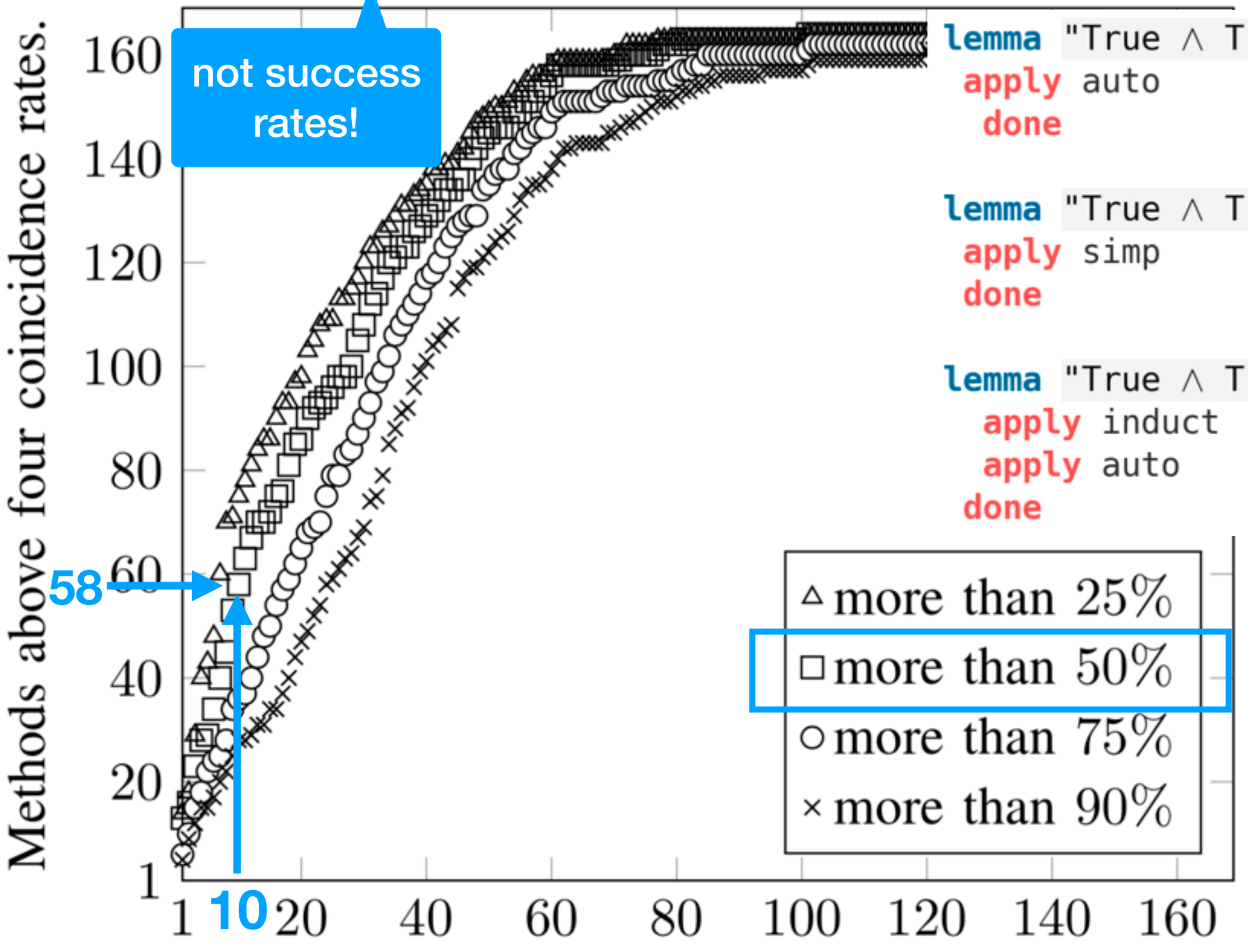
Coincidence rates of PaMpeR

not success rates!

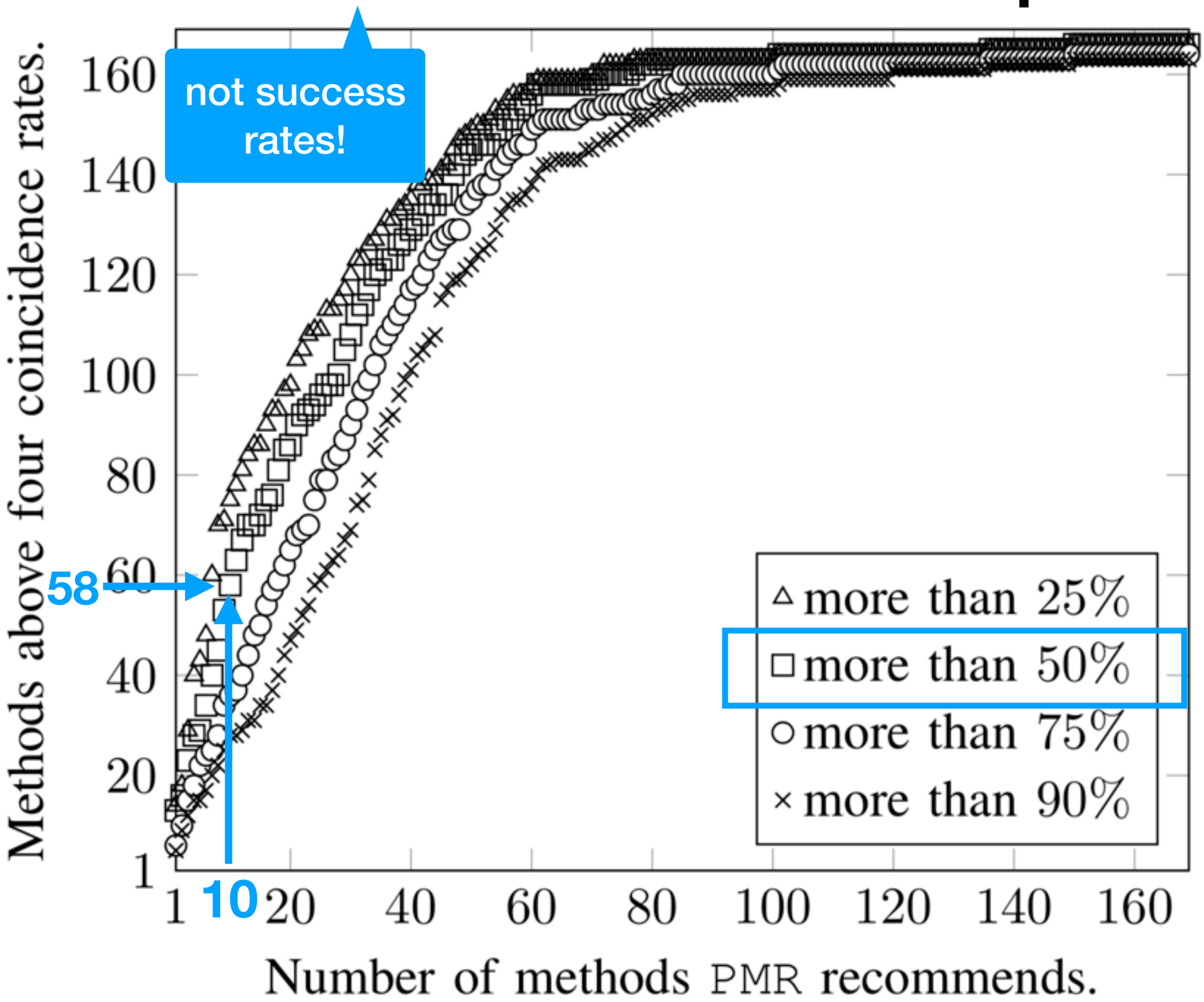
```
lemma "True ∧ True"
 apply auto
 done

lemma "True ∧ True"
 apply simp
 done

lemma "True ∧ True"
  apply induct
  apply auto
done
```



Coincidence rates of PaMpeR



Coincidence rates of PaMpeR