git clone https://github.com/data61/PSL

Domain-Specific Language to Encode Induction Heuristics

This work was supported by the project AI&Reasoning (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000466).

Yutaka Nagashima
University of Innsbruck
Czech Technical University

Yutaka Ng
yutakang

Block or report user

CVUT, CTU, CIIRC

CZECH INSTITUTE OF INFORMATICS ROBOTICS AND CYBERNETICS CTU IN PRAGUE

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Who is Isabelle?

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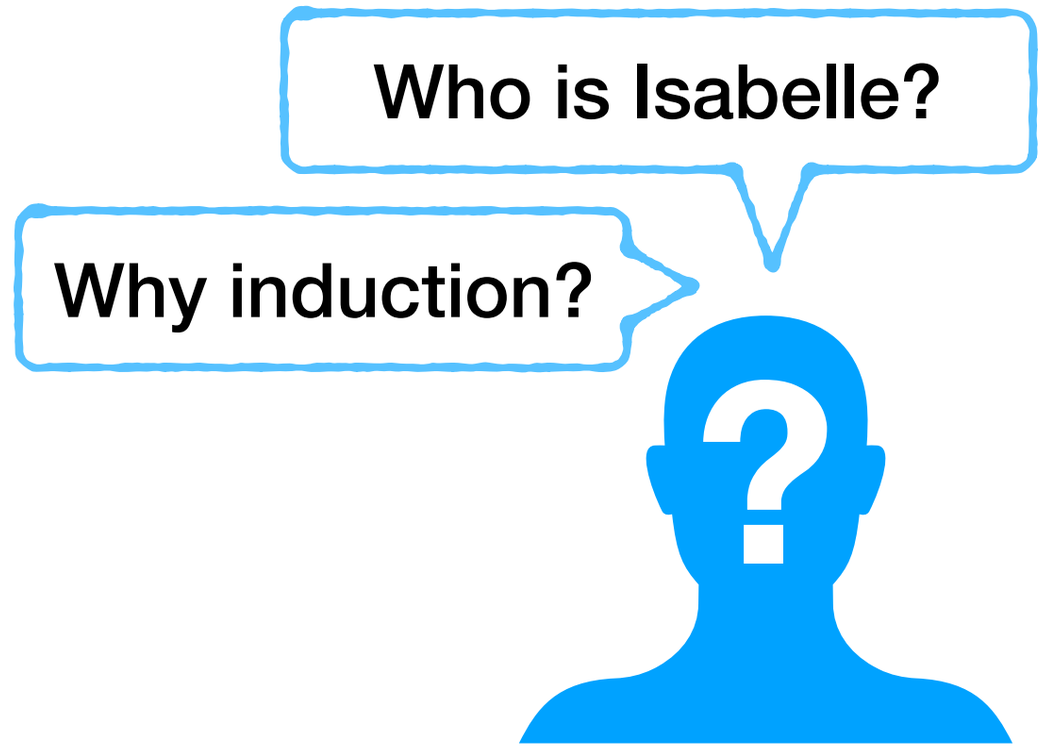

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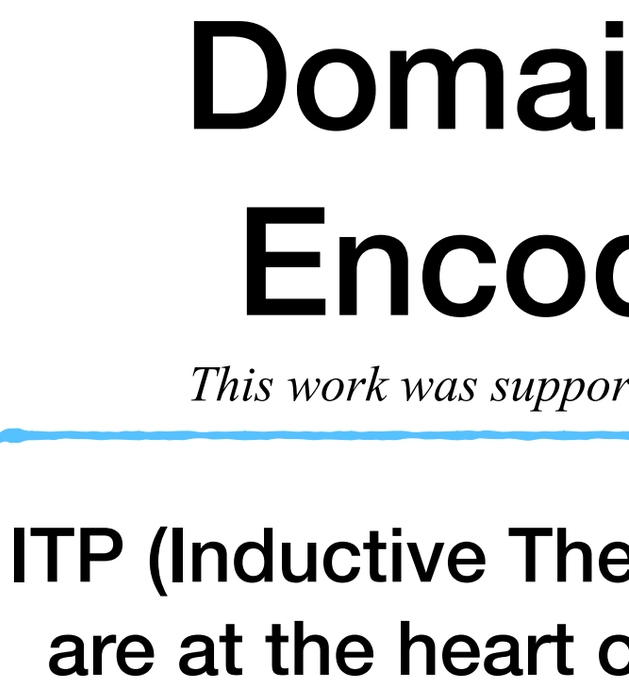
ITP (Inductive Theorem Proving) problems are at the heart of many verification and reasoning tasks in computer science.



Prof. Bernhard Gramlich
https://www.logic.at/staff/gramlich/

Who is Isabelle?

Why induction?

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we are convinced that substantial progress in ITP will take time.

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spectacular breakthroughs are unrealistic, in view of the enormous problems and the inherent difficulty of inductive theorem proving.

Strategic Issues, Problems and Challenges in Inductive Theorem Proving

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ITP (Inductive Theorem Pro
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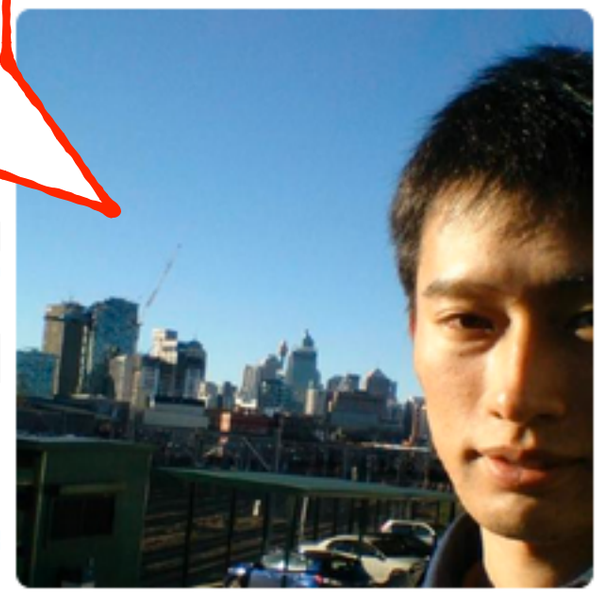
Challenge accepted!

?

Why in



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Prof. Bernhard Gramlich
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spectacular breakt **Yutaka Ng**
unrealistic, in view of yutakang
problems and the inhe Block or report user
inductive theorer CVUT, CTU, CIIRC

Strategic Issues, Problems and Challenges in Inductive Theorem Proving

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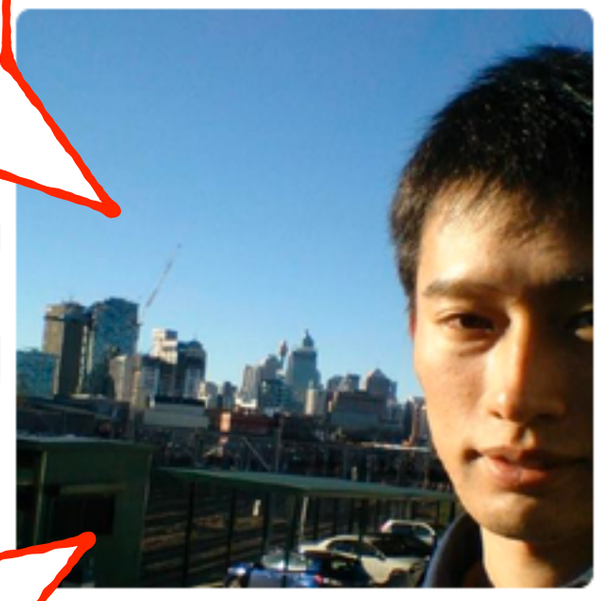

we are convinced that subst
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Let AI solve it!

Yutaka Ng
yutakang

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CVUT, CTU, CIIRC

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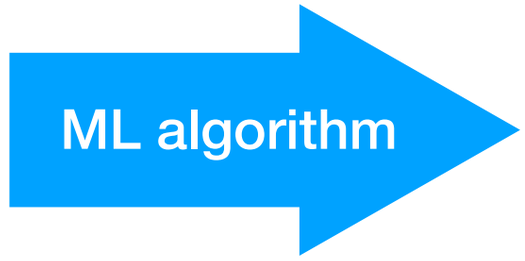
Strategic Issues, Problems and Challenges in Inductive Theorem Proving

git clone https://github.com/data61/PSL

Introduction to Machine Learning in 10 seconds



https://duckduckgo.com/?q=cat&t=ffab&iar=images&iax=images&ia=images

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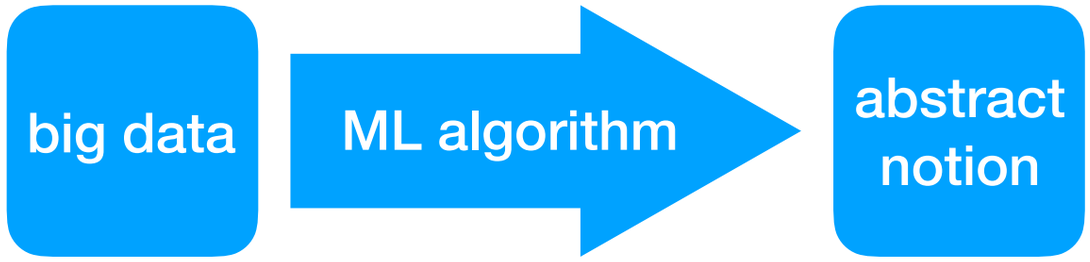

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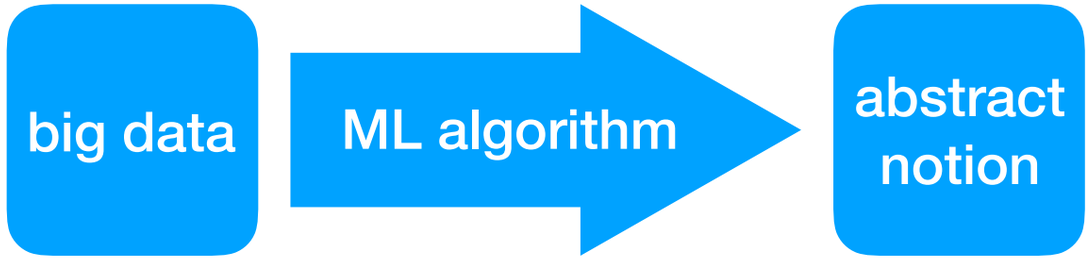

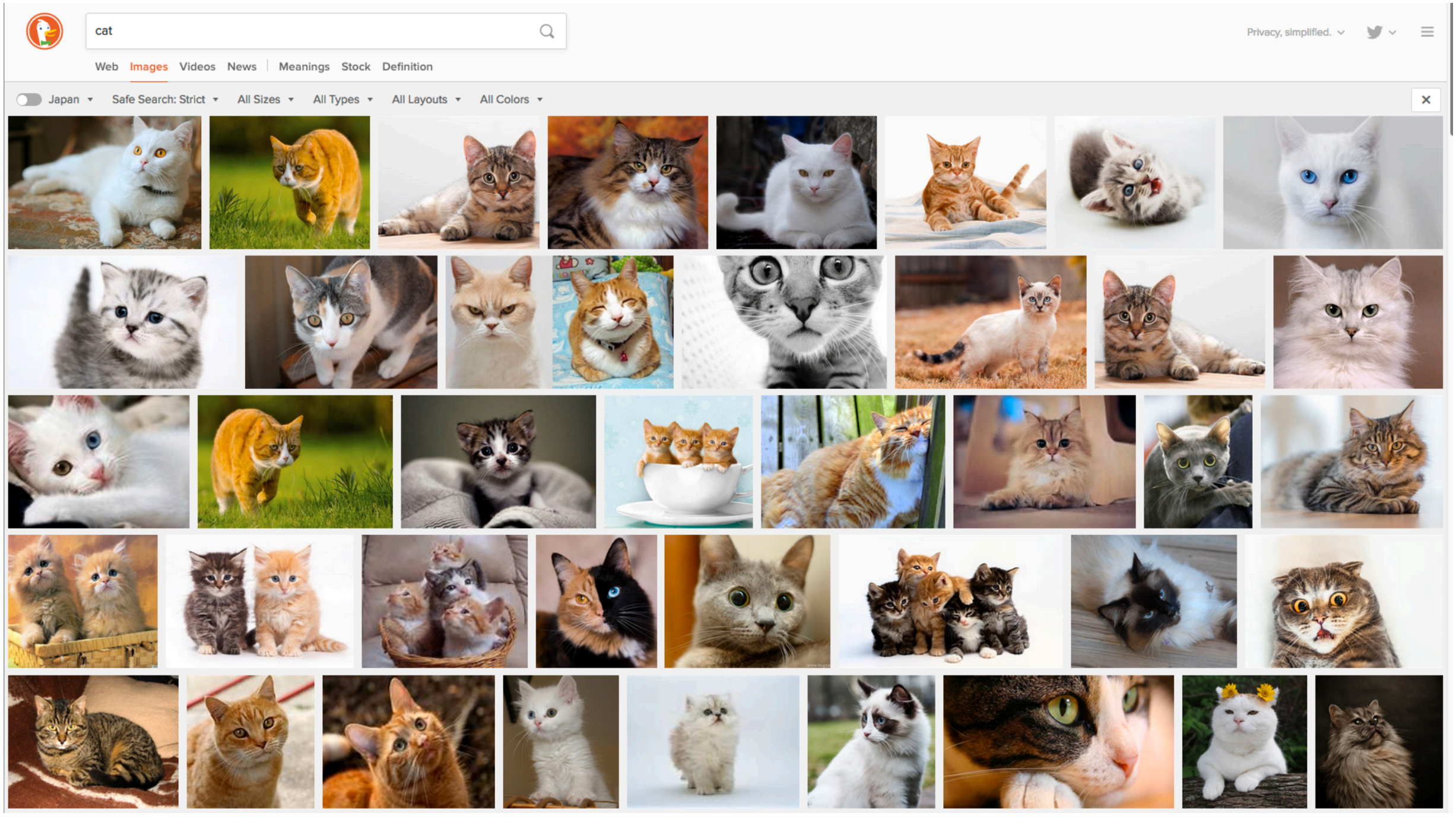

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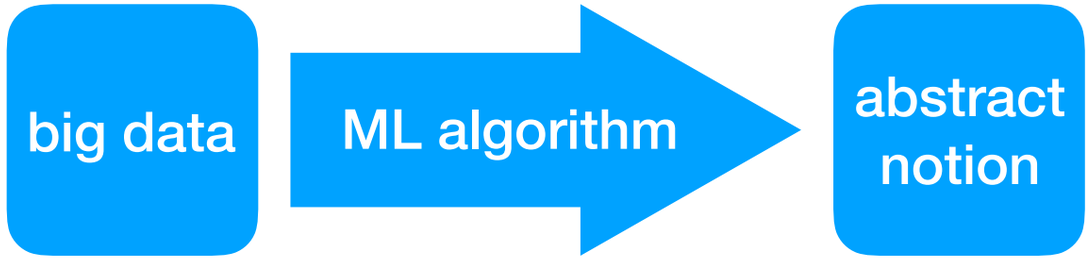

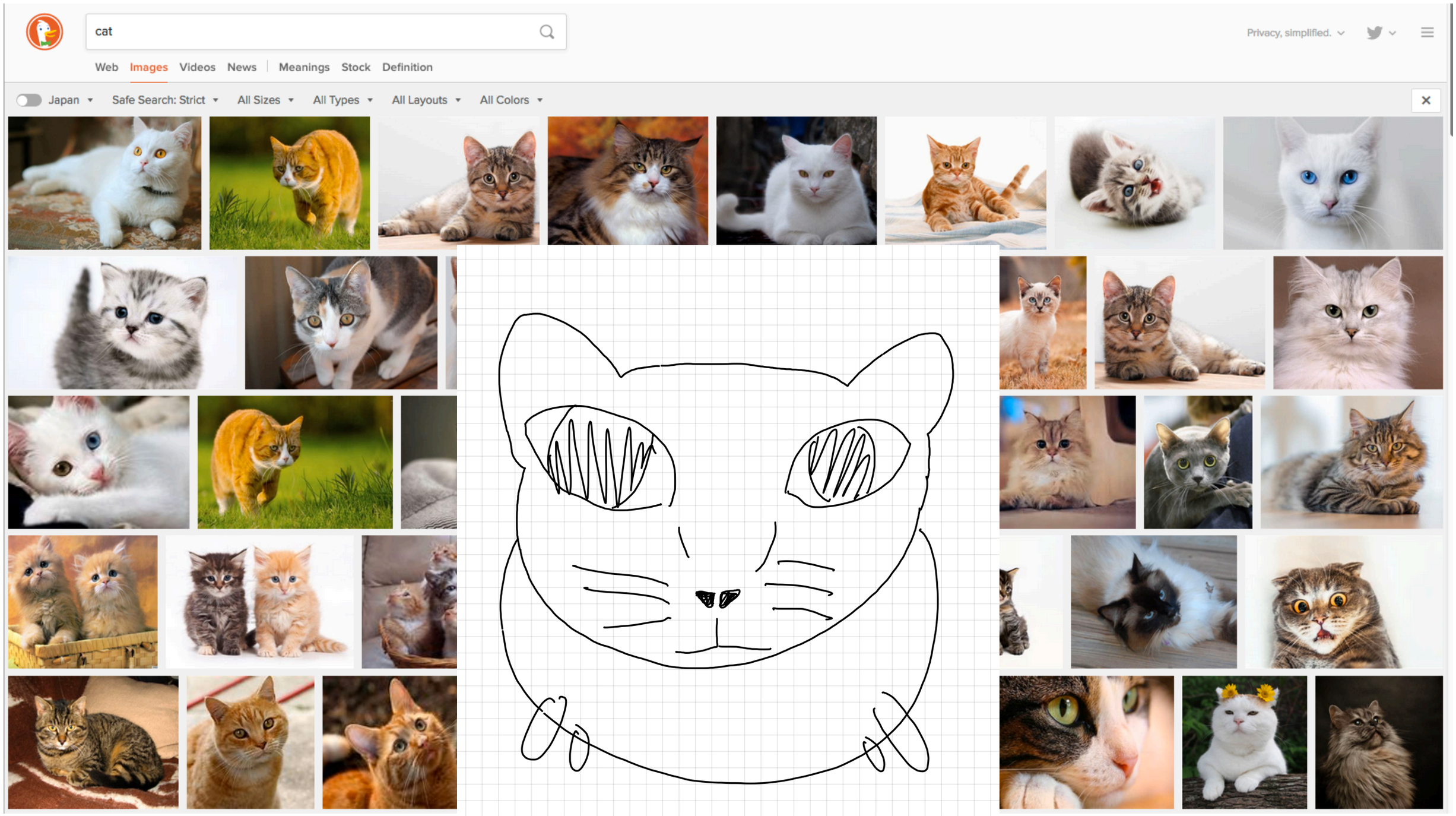

https://duckduckgo.com/?q=cat&t=ffab&iar=images&iax=images&ia=images

git clone https://github.com/data61/PSL

ML for Inductive Theorem Proving the BAD

```
lemma "itrev xs ys = rev xs @ ys"
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<- many concrete cases

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<- many concrete cases

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ML for Inductive Theorem Proving the BAD

```
lemma "itrev xs ys = rev xs @ ys"  by auto
by(induct xs ys rule:"itrev.induct") auto
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<- one abstract representation

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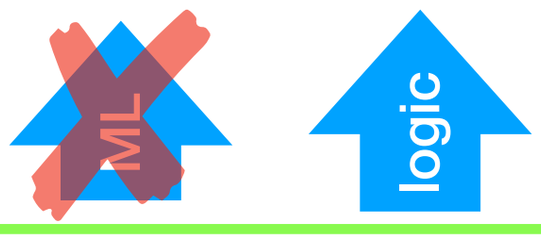

<- abstraction using expressive logic

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ML for Inductive Theorem Proving the BAD

polymorphism

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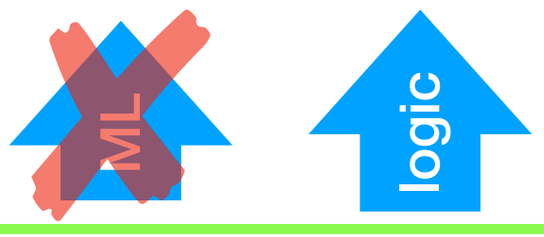

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ML for Inductive Theorem Proving the BAD

polymorphism **type class**

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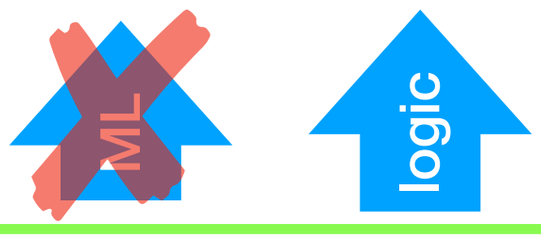

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<- many concrete cases

git clone https://github.com/data61/PSL

ML for Inductive Theorem Proving the BAD

polymorphism

type class

universal quantifier

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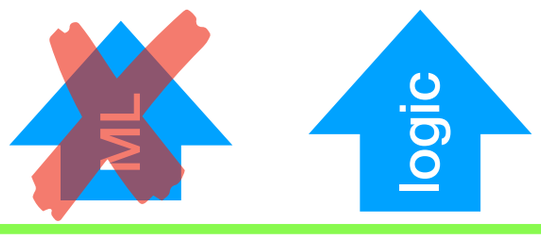

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<- many concrete cases

git clone https://github.com/data61/PSL

ML for Inductive Theorem Proving
the BAD

Higher-Order functions

polymorphism **type class**

universal quantifier

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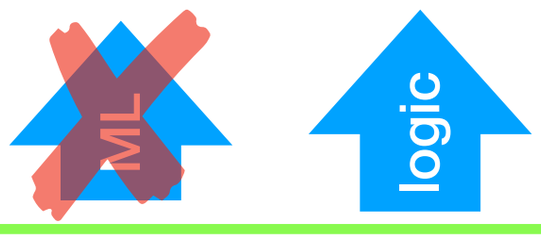

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<- many concrete cases

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ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism **type class**

universal quantifier

lambda abstraction

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```

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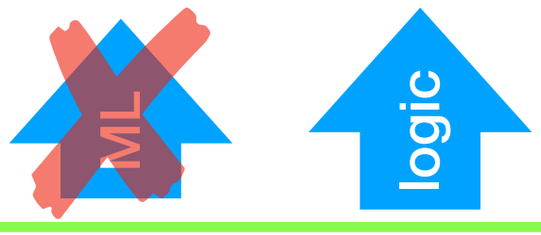

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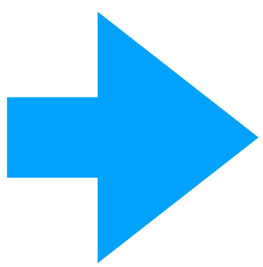
ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

universal quantifier



concise formula that can cover many concrete cases

lambda abstraction

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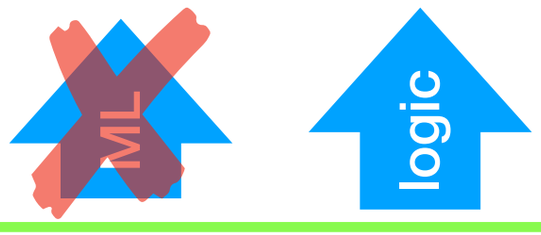
```

<- one abstract representation



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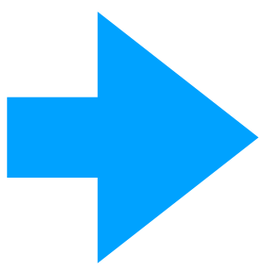
ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

universal quantifier



concise formula that can cover many concrete cases

different proof for general case

lambda abstraction

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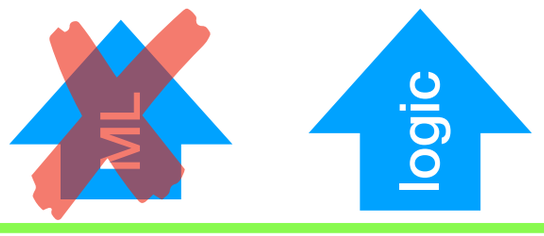
<- one abstract representation



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ML for Inductive Theorem Proving the BAD

Higher-Order functions

polymorphism

type class

universal quantifier

lambda abstraction

concise formula that can cover many concrete cases

different proof for general case

A small data set is not a failure but an achievement!

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lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
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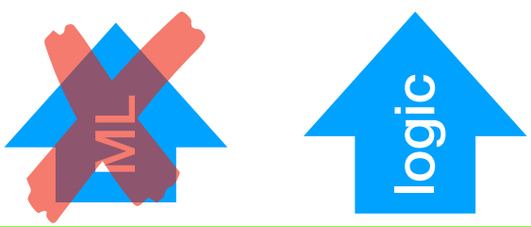
<- one abstract representation



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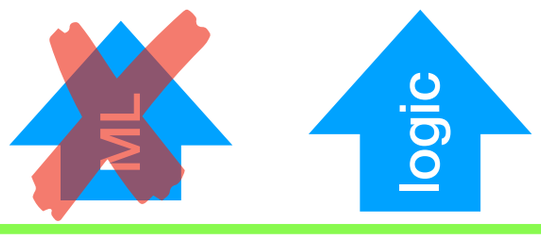

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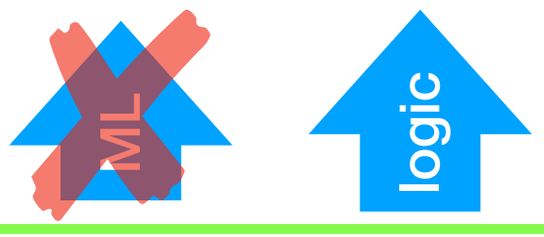

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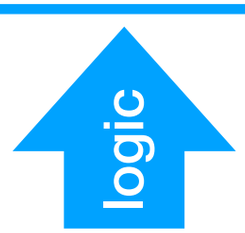

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Unsupervised Learning

Memory and one-shot learning

Imagination-based Planning with Generative Models

Learning Abstract Concepts

Transfer Learning

Language understanding

https://cbmm.mit.edu/video/power-self-learning-systems

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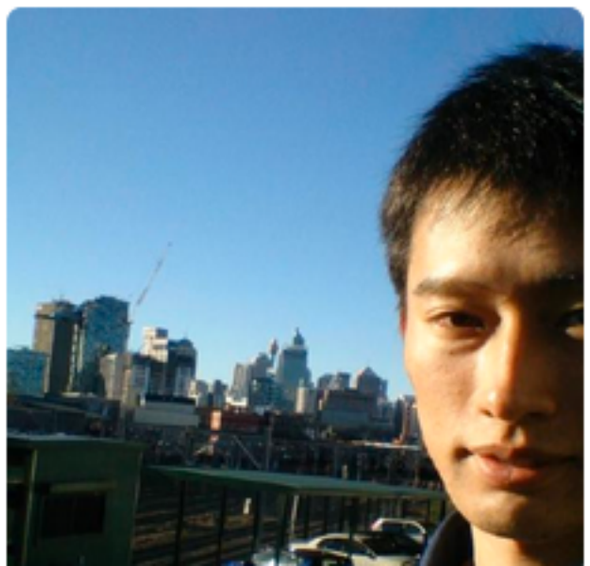
March 20, 2019

The Power of

Self



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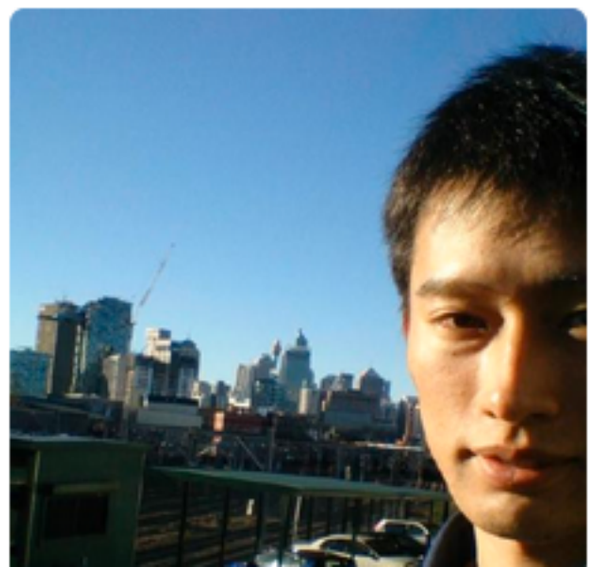

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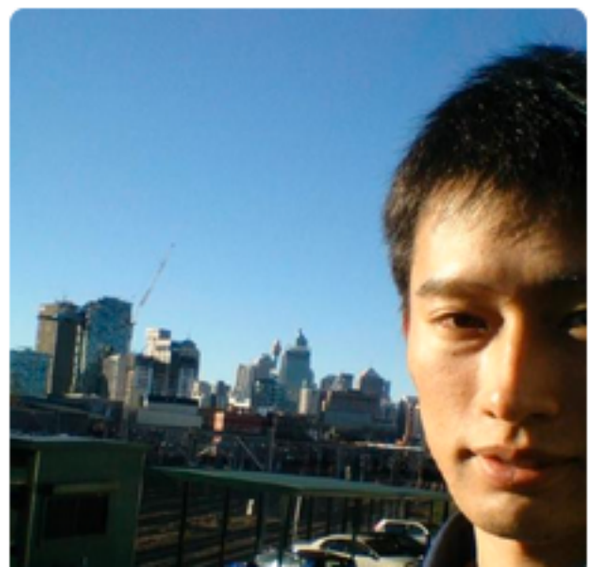

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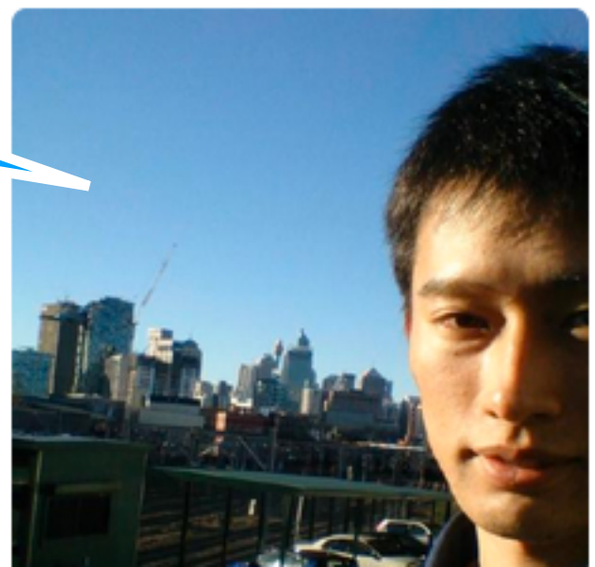

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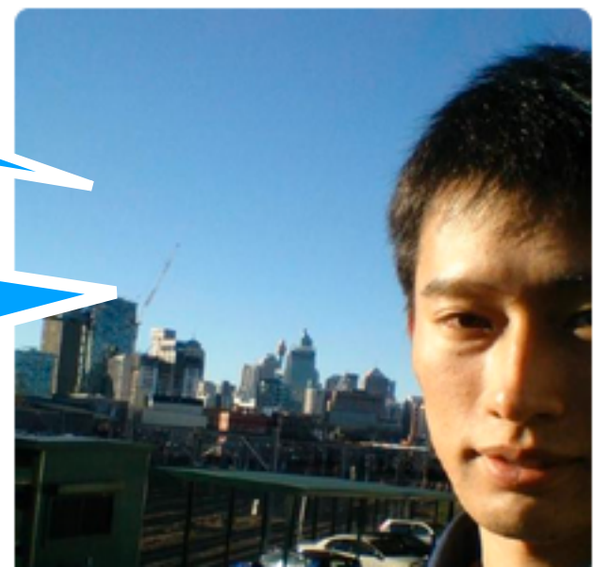
We already know a framework good for abstract concepts.

It is called logic!

The Power of Self



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Logic about Proofs to Abstract Abstraction

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

<- even more abstract

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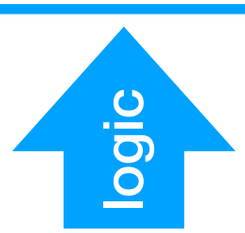

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lemma "star r x y ⟹ star r y z ⟹ star r x z"
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<- small dataset about different domains

<- one abstract representation

logic

<- abstraction using expressive logic

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lemma "itrev [1,2]         [] = rev [1,2]         @ []" by auto
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<- many concrete cases

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

https://github.com/data61/PSL

Big Picture

[[T,F,], [ ], [ ]]: bool list <- simple representation

LiFtEr

LiFtEr 2

abstraction using another logic (LiFtEr)

<- pros: good at rigorous abstraction

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LiFtEr
LiFtEr
LiFtEr 3

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LiFtEr 3 LiFtEr LiFtEr LiFtEr 1

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https://github.com/data61/PSL

Big Picture

[[T,F,T], [T,], [ ]]: bool list <- simple representation

LiFtEr 3 LiFtEr LiFtEr LiFtEr 1 LiFtEr 2

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<- abstraction using other logic (LiFtEr)

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lemma "star r ⟹ star r y z ⟹ star r x z"
by(induction r rule:star.induct)(auto simp: step)

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LiFtEr LiFtEr LiFtEr 3 LiFtEr 1 LiFtEr 2 LiFtEr 3 LiFtEr 1

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LiFtEr 1 LiFtEr 2 LiFtEr 3

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Big Picture

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Heuristics that are valid across problem domains.

ML

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr

LiFtEr 1 LiFtEr 2 LiFtEr 3

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this talk

```
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```

<- many concrete cases

Example Assertion in LiFtEr

```
val example_assertion =
  Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Example Assertion in LiFtEr

implication

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Example Assertion in LiFtEr

implication

conjunction

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Example Assertion in LiFtEr

implication

variable for auxiliary lemmas

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

conjunction

Example Assertion in LiFtEr

implication

variable for auxiliary lemmas

variable for terms

conjunction

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Example Assertion in LiFtEr

implication

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Example Assertion in LiFtEr

implication

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

variable for natural numbers

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Example Assertion in LiFtEr

implication

variable for auxiliary lemmas

variable for terms

variable for term occurrences

conjunction

variable for natural numbers

universal quantifier

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
         Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Example Assertion in LiFtEr

implication

existential quantifier

variable for auxiliary lemmas

```
val example_assertion =
 Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

variable for terms

variable for term occurrences

conjunction

universal quantifier

variable for natural numbers

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

```
val example_assertion =
  Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
          Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

(Rule 1 = itrev.induct)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,                          ( Rule 1      = itrev.induct )
  Some_Trm (Trm 1,                           ( Trm 1       = itrev )
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)

(Trm 1 = itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,                          ( Rule 1      = itrev.induct )
  Some_Trm (Trm 1,                           ( Trm 1       = itrev )
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,        ( Trm_Occ 1 = itrev )
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

Term_Occ 1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)
(Trm 1 = itrev)
(Trm_Occ 1 = itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,                          ( Rule 1      = itrev.induct )
  Some_Trm (Trm 1,                           ( Trm 1       = itrev )
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,        ( Trm_Occ 1 = itrev )
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"
```

Term_Occ 1

```
lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)
(Trm 1 = itrev)
(Trm_Occ 1 = itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)
(Trm 1 = itrev)
(Trm_Occ 1 = itrev)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1

Term 2

Term 1

Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,                                  ( Rule 1      = itrev.induct )
  Some_Trm (Trm 1,                                   ( Trm 1       = itrev )
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,                ( Trm_Occ 1 = itrev )
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,                                 ( Trm 2       = xs and ys )
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1 · Term_Occ 2 · Term 2 · Term 1 · Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,                                  ( Rule 1      = itrev.induct )
  Some_Trm (Trm 1,                                   ( Trm 1       = itrev )
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,                ( Trm_Occ 1 = itrev )
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,                                 ( Trm 2       = xs and ys )
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,             ( Trm_Occ 2 = xs and ys )
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1 — Term_Occ 2 — Term 2 — Term 1 — Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,                          ( Rule 1      = itrev.induct )
  Some_Trm (Trm 1,                           ( Trm 1       = itrev )
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,        ( Trm_Occ 1 = itrev )
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,                         ( Trm 2       = xs and ys )
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,     ( Trm_Occ 2 = xs and ys )
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1 · Term_Occ 2 · Term 2 · Term 1 · Rule 1

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,                          ( Rule 1      = itrev.induct )
  Some_Trm (Trm 1,                           ( Trm 1       = itrev )
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,        ( Trm_Occ 1 = itrev )
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,                                ( Trm 2       = xs and ys )
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,            ( Trm_Occ 2 = xs and ys )
       Some_Numb (Numb 1,
        Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
       And
       (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1 — first — Term_Occ 2

Term 2 — first — Term 1 — Rule 1

```
val example_assertion =
  Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

(Rule 1 = itrev.induct)
(Trm 1 = itrev)
(Trm_Occ 1 = itrev)
(Trm 2 = xs and ys)
(Trm_Occ 2 = xs and ys)

```
primrec rev :: "'a list ⇒ 'a list" where
 "rev  []     = []" |
 "rev (x # xs) = rev xs @ [x]"

fun itrev :: "'a list ⇒ 'a list ⇒ 'a list" where
 "itrev  []    ys = ys" |
 "itrev (x#xs) ys = itrev xs (x#ys)"

lemma "itrev xs ys = rev xs @ ys"
  apply(induct xs ys rule:"itrev.induct")
  apply auto done
```

Term_Occ 1 — first — second — Term_Occ 2

Term 2 — second — Term 1 — Rule 1

first

(Rule 1 = itrev.induct)
(Trm 1 = itrev)
(Trm_Occ 1 = itrev)
(Trm 2 = xs and ys)
(Trm_Occ 2 = xs and ys)

```
val example_assertion =
  Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
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     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
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         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
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```

```
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         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

```
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  Some_Rule (Rule 1, True)
 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
     (Rule 1 Is_Rule_Of Trm_Occ 1)
     And                                          <- the same LiFtEr assertion
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
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```

new types ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"
```

```
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new types ->

new constants ->

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 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
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  Some_Rule (Rule 1, True)
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         And
         (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

<- the same LiFtEr assertion

new types ->

new constants ->

new lemma ->
a model proof ->

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 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
  apply(induct is1 s stk rule:exec.induct)
  apply auto done
```

```
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<- the same LiFtEr assertion

new types ->

new constants ->

new lemma ->
a model proof ->

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new types ->

new constants ->

new lemma ->
a model proof ->

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Rule 1

(Rule 1 = exec.induct)

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
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        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

new types ->

new constants ->

new lemma ->
a model proof ->

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    (All_Ind (Trm 2,
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         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

new types ->

new constants ->

new lemma ->
a model proof ->

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```

Rule 1

(Rule 1 = exec.induct)

```
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 Some_Rule (Rule 1,
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    (All_Ind (Trm 2,
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        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

new types ->

new constants ->

new lemma ->
a model proof ->

```
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

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 "exec (i#is) s stk = exec is s (exec1 i s stk)"

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  apply(induct is1 s stk rule:exec.induct)
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```

Rule 1

(Rule 1 = exec.induct)

```
val example_assertion =
 Some_Rule (Rule 1, True)
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    (All_Ind (Trm 2,
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         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

new types ->

new constants ->

new lemma ->
a model proof ->

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datatype     instr = LOADI val | LOAD vname | ADD
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  apply(induct is1 s stk rule:exec.induct)
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```

Term 1 Rule 1

(Rule 1 = exec.induct)

(Trm 1 = exec)

```
val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

new types ->

new constants ->

new lemma ->
a model proof ->

```
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```
val example_assertion =
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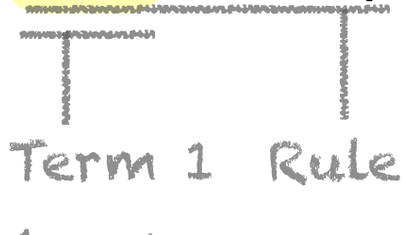
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```



Term 1 Rule 1

(Rule 1 = exec.induct)

(Trm 1 = exec)

new types ->

new constants ->

new lemma ->
a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
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fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
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lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
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```

Term_Occ 1

Term 1 Rule 1

(Rule 1   = exec.induct)

(Trm 1   = exec)

(Trm_Occ 1 = exec)

new types ->

new constants ->

new lemma ->
a model proof ->

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Term_Occ 1

Term 1 Rule 1

(Rule 1 = exec.induct)

(Trm 1 = exec)

(Trm_Occ 1 = exec)

new types ->

new constants ->

new lemma ->
a model proof ->

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Term_Occ 1

Term 1 Rule 1

(Rule 1 = exec.induct)

(Trm 1 = exec)

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new types ->

new constants ->

new lemma ->
a model proof ->

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```

Term_Occ 1

Term 1 Rule 1

(Rule 1 = exec.induct)

(Trm 1 = exec)

(Trm_Occ 1 = exec)

new types ->

new constants ->

new lemma ->
a model proof ->

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    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

Term_Occ 1

Term 1 Rule 1

(Rule 1   = exec.induct)

(Trm 1   = exec)

(Trm_Occ 1 = exec)

new types ->

new constants ->

new lemma ->
a model proof ->

```
datatype     instr = LOADI val | LOAD vname | ADD
type_synonym stack = "val list"

fun exec1 :: "instr ⇒ state ⇒ stack ⇒ stack" where
 "exec1 (LOADI n) _   stk      =  n       # stk" |
 "exec1 (LOAD x)  s   stk      =  s(x)    # stk" |
 "exec1  ADD      _ (j#i#stk) =  (i + j) # stk"

fun exec :: "instr list ⇒ state ⇒ stack ⇒ stack" where
 "exec []     _ stk = stk" |
 "exec (i#is) s stk = exec is s (exec1 i s stk)"

lemma "exec (is1 @ is2) s stk = exec is2 s (exec is1 s stk)"
  apply(induct is1 s stk rule:exec.induct)
  apply auto done

val example_assertion =
 Some_Rule (Rule 1, True)
Imply
 Some_Rule (Rule 1,
  Some_Trm (Trm 1,
   Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
    (Rule 1 Is_Rule_Of Trm_Occ 1)
    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

Term 2

Term_Occ 1

Term 1 Rule 1

(Rule 1 = exec.induct)

(Trm 1 = exec)

(Trm_Occ 1 = exec)

(Trm 2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->
a model proof ->

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    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Term 2
Term_Occ 1
Term_Occ 2

Term 1 Rule 1
(Rule 1   = exec.induct)
(Trm 1   = exec)
(Trm_Occ 1 = exec)
(Trm 2   = is1, s, and stk)
(Trm_Occ 2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->
a model proof ->

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        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

Term 2
Term_Occ 1
Term_Occ 2

Term 1 Rule 1
(Rule 1 = exec.induct)
(Trm 1 = exec)
(Trm_Occ 1 = exec)
(Trm 2 = is1, s, and stk)
(Trm_Occ 2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->
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 Some_Rule (Rule 1,
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    (All_Ind (Trm 2,
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       Some_Numb (Numb 1,
        Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
       And
       (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

Term 2
Term_Occ 1
Term_Occ 2

Term 1 Rule 1
(Rule 1   = exec.induct)
(Trm 1   = exec)
(Trm_Occ 1 = exec)

(Trm 2   = is1, s, and stk)
(Trm_Occ 2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->

a model proof ->

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    And
    (All_Ind (Trm 2,
     (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
       Some_Numb (Numb 1,
        Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
       And
       (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Term 2 — first

Term_Occ 1 — first

Term_Occ 2

Term 1 Rule 1

(Rule 1 = exec.induct)

(Trm 1 = exec)

(Trm_Occ 1 = exec)

(Trm 2 = is1, s, and stk)

(Trm_Occ 2 = is1, s, and stk)

new types ->

new constants ->

new lemma ->
a model proof ->

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datatype     instr = LOADI val | LOAD vname | ADD
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 "exec []     _ stk = stk" |
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     And
     (All_Ind (Trm 2,
      (Some_Trm_Occ_Of (Trm_Occ 2, Trm 2,
        Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1)))))))));
```

Term 2
first
second
Term_Occ 1
first
second
Term_Occ 2
Term 1 Rule 1
(Rule 1 = exec.induct)
(Trm 1 = exec)
(Trm_Occ 1 = exec)
(Trm 2 = is1, s, and stk)
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new types ->

new constants ->

new lemma ->
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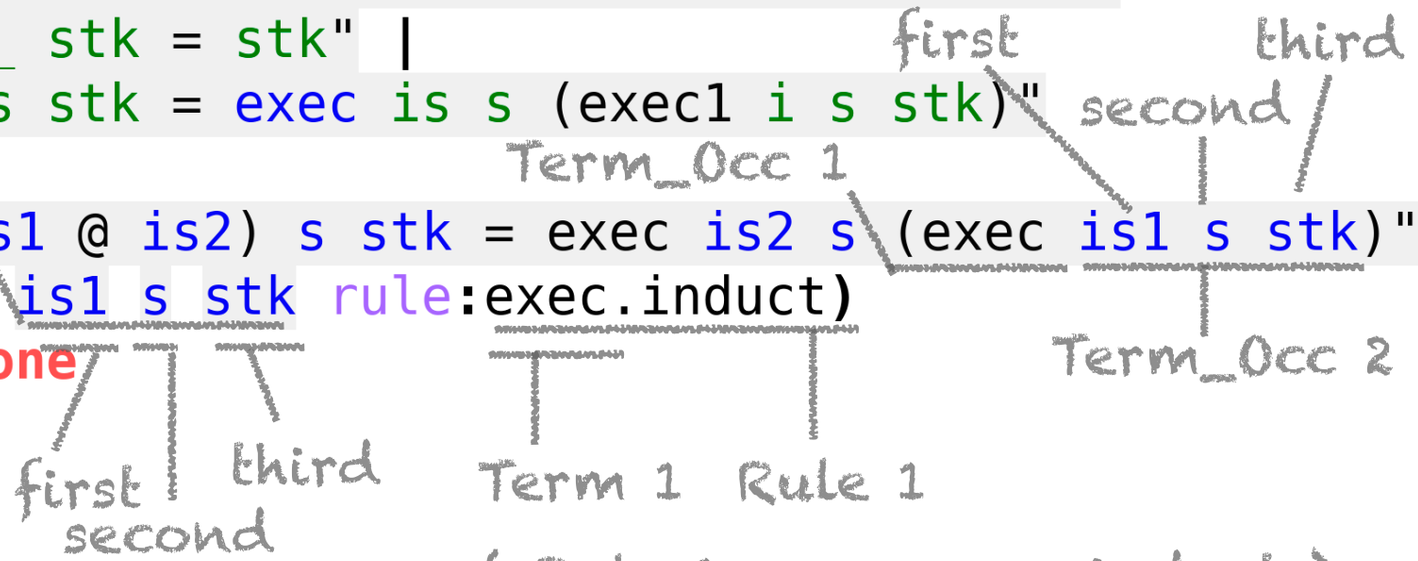


```
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 Imply
  Some_Rule (Rule 1,
   Some_Trm (Trm 1,
    Some_Trm_Occ_Of (Trm_Occ 1, Trm 1,
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        Some_Numb (Numb 1,
         Is_Nth_Arg_Of (Trm_Occ 2, Numb 1, Trm_Occ 1)
        And
        (Trm 2 Is_Nth_Ind Numb 1))))))))));
```

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

https://github.com/data61/PSL

Big Picture

ML

<- pros: good at ambiguity (heuristics)

[[T,F,T], [T,T,T], [F,T,T]]: bool list <- simple representation

LiFtEr LiFtEr LiFtEr 3

<- pros: good at rigorous abstraction

~~<- cons: bad at ambiguity (heuristics)~~

LiFtEr 1 LiFtEr 2 LiFtEr 3

```
lemma "star r                    star                ⟹ star r x z"
by(induction r                   ar.in               uto simp: step)

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                           )"
by(induct is1 s stk rule:e                           uct) auto
```

LiFtEr 1 LiFtEr 2 LiFtEr 3

<- small dataset about different domains

```
lemma "itrev xs ys = rev xs @ ys"
by(induct xs ys rule:"itrev.induct") auto
```

<- one abstract representation

logic

<- abstraction using expressive logic

```
lemma "itrev [1,2]          [] = rev [1,2]          @ []" by auto
lemma "itrev [1,2,3]        [] = rev [1,2,3]        @ []" by auto
lemma "itrev [''a'',''b''] [] = rev [''a'',''b''] @ []" by auto
lemma "itrev [x,y,z]        [] = rev [x,y,z]        @ []" by auto
```

<- many concrete cases

https://github.com/data61/PSL

Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

ML

[[T,F,T], [T,T,T], [F,T,T]]: bool list

<- pros: good at ambiguity (heuristics)

<- pros: good at rigoro… …tion

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?

LiFtEr

LiFtEr

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

```
lemma "star r …  star …  ⟹ star r x z"
by(induction r …ar.in…uto simp: step)

lemma "exec (is1 @ is2) s …
       exec is2 s (exec is…  …)"
by(induct is1 s stk rule:e…  …uct) auto

lemma "itrev xs ys = rev xs @ ys"
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Big Picture

Abstract notion of “good” application of induction.
Heuristics that are valid across problem domains.

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[[T,F,T], [T,T,T], [F,T,T]]: bool list

<- pros: good at ambiguity (heuristics)

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?

LiFtEr

LiFtEr

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

LiFtEr 1

LiFtEr 2

LiFtEr 3

17:40 @ ML workshop

```
lemma "star r                star
by(induction r                ar.in                ut

lemma "exec (is1 @ is2) s
       exec is2 s (exec is                )"
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<- many concrete cases

git clone https://github.com/data61/PSL