

Fix  $A$  as  $\{a, b, c\}$

$$\begin{aligned}\mathcal{L}_A = \{ & \epsilon, |a|, |b|, |c|, |a, b|, |b, c|, |a, c|, |a, b, c|, |a|b|, |a|c|, |b|a|, |b|c|, \\ & |c|a|, |c|b|, |a|b, c|, |b|a, c|, |c|a, b|, |a, b|c|, |a, c|b|, |b, c|a|, \\ & |a|b|c|, |a|c|b|, |b|a|c|, |b|c|a|, |c|b|a|, |c|a|b|\}\end{aligned}$$

Let  $L$  be  $\{|a|c|\}$ , and  $L'$  be  $\{|b|c|\}$

Thus,  $gap(L, L') = \{|a, b|, |a|b|, |a|c|\}$

$$\begin{aligned}\llbracket L \rrbracket_A &= \{|a|c|, |a|b|c|, |b|a|c|, |b, a|c|, |a|b, c|, |a|c|b|\} \\ \llbracket L' \rrbracket_A &= \{|a, b|c|, |b|a, c|, |a|b|c|, |b|c|, |b|c|a|, |b|a|c|\}\end{aligned}$$

$$\begin{aligned}res_A(L, L') &= \{|b, a, c|, |b, c|a|, |c|a|b|, |b, a|c|, |a|b|, |a|b, c|, |c|b|a|, |a|b|c|, \\ & |b|a|c|, |a|c|b|, |c|b|, |a, c|b|, |a|c|, |b, a|, |c|b, a|, |b, c|\}\end{aligned}$$

$$\begin{aligned}L \& L' &= \{|a, b|c|, |a|b|c|, |b|a|c|\} \\ (L \& L')_{\downarrow} &= \{\epsilon, |a|, |b|, |c|, |a, b|, |a|b|, |b|c|, |b|a|, |a|c|, |a, b|c|, |a|b|c|, |b|a|c|\}\end{aligned}$$

$$\begin{aligned}\llbracket (L \& L')_{\downarrow} \rrbracket_A &= \{\epsilon, |a|, |b|, |c|, |a, b|, |b, c|, |a, c|, |a, b, c|, |a|b|, |a|c|, |b|a|, |b|c|, \\ & |c|a|, |c|b|, |a|b, c|, |b|a, c|, |c|a, b|, |a, b|c|, |a, c|b|, |b, c|a|, \\ & |a|b|c|, |a|c|b|, |b|a|c|, |b|c|a|, |c|b|a|, |c|a|b|\}\end{aligned}$$

$$\therefore \llbracket (L \& L')_{\downarrow} \rrbracket_A = \mathcal{L}_A$$

$$\therefore \llbracket (L \& L')_{\downarrow} \rrbracket_A \cap res_A(L, L') = res_A(L, L')$$

$$\begin{aligned}\text{but } \llbracket gap(L, L') \rrbracket_A &= \{|a, b, c|, |a, b|, |a|b|, |a|c|, |a, b|c|, |a, c|b|, |a|b, c|, \\ & |c|a, b|, |a|b|c|, |a|c|b|, |b|a|c|, |c|a|b|\} \\ &\neq res_A(L, L')\end{aligned}$$

$$\therefore \llbracket gap(L, L') \rrbracket_A \neq \llbracket (L \& L')_{\downarrow} \rrbracket_A \cap res_A(L, L')$$

However,

$$(L \& L')_{\downarrow} \cap res_A(L, L') = \{|a, b|, |a|b|, |a|c|, |a, b|c|, |a|b|c|, |b|a|c|\}$$

$$\begin{aligned} \llbracket (L \& L')_{\downarrow} \cap res_A(L, L') \rrbracket_A &= \{|a, b, c|, |a, b|, |a|b|, |a|c|, |a, b|c|, |a, c|b|, |a|b, c|, \\ &\quad |c|a, b|, |a|b|c|, |a|c|b|, |b|a|c|, |c|a|b|\} \\ &= \llbracket gap(L, L') \rrbracket_A \end{aligned}$$