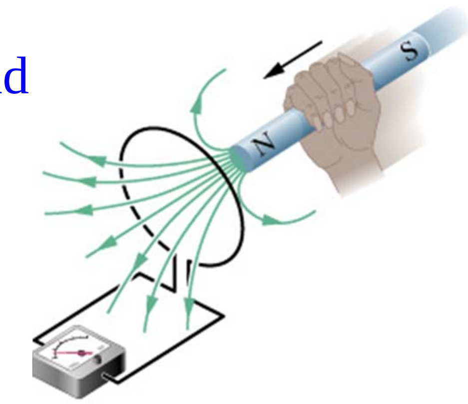
Tema 2:
Inducció Magnètica

2.1 Faraday's Experiments

In a series of experiments, Michael Faraday in England and Joseph Henry in the U.S. were able to generate electric currents without the use of batteries.

Below we describe some of these experiments that helped formulate what is known as "Faraday's law of induction."



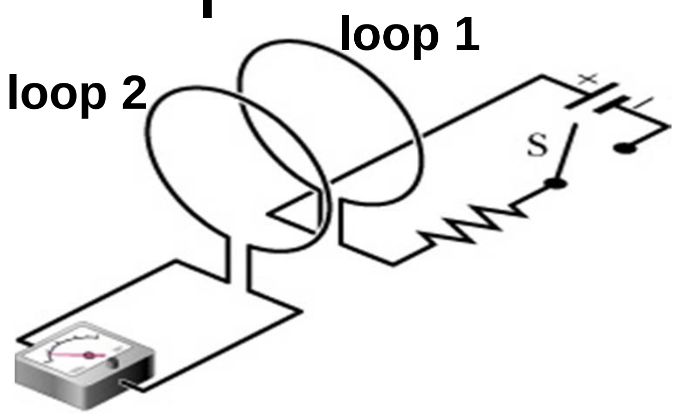
The circuit shown in the figure consists of a wire loop connected to a sensitive ammeter (known as a "galvanometer"). If we approach the loop with a permanent magnet we see a current being registered by the galvanometer. The results can be summarized as follows:

1. A current appears only if there is relative motion between the magnet and the loop.

2. Faster motion results in a larger current.

3. If we reverse the direction of motion or the polarity of the magnet, the current reverses sign and flows in the opposite direction.

The current generated is known as "***induced current***"; the emf that appears is known as "***induced emf***"; the whole effect is called "***induction.***"



In the figure we show a second type of experiment in which current is induced in loop 2 when the switch S in loop 1 is either closed or opened. When the current in loop 1 is constant no induced current is observed in loop 2. The conclusion is that the magnetic field in an induction experiment can be generated either by a permanent magnet or by an electric current in a coil.

Faraday summarized the results of his experiments in what is known as "**Faraday's law of induction.**"

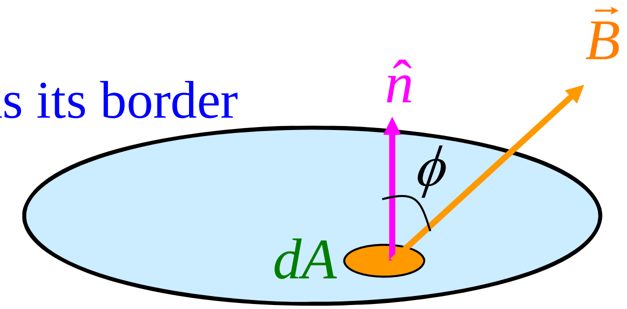
An emf is induced in a loop when the number of magnetic field lines that pass through the loop is changing.

Magnetic Flux Φ_B

$$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$$

The magnetic flux through a surface that borders a loop is determined as follows:

1. We divide the surface that has the loop as its border into elements of area *dA*.



2. For each element we calculate the magnetic flux through it: $d\Phi_B = BdA\cos\phi$.

Here ϕ is the angle between the normal $\hat{n}$ and the magnetic field $\vec{B}$ vectors at the position of the element.

3. We integrate all the terms. $\Phi_B = \int BdA\cos\phi = \int \vec{B} \cdot d\vec{A}$

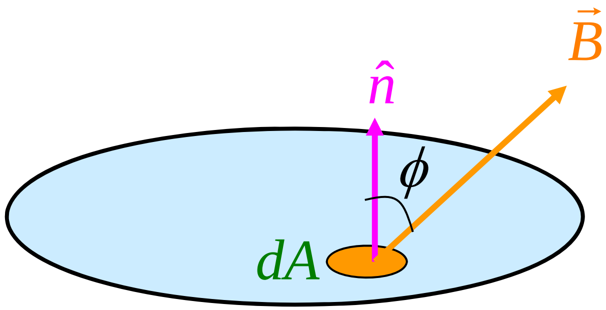
SI magnetic flux unit: $\mathrm{T} \cdot \mathrm{m}^2$ known as the Weber (symbol Wb).

We can express Faraday's law of induction in the following form:

The magnitude of the emf $\mathcal{E}$ induced in a conductive loop is equal to the rate at which the magnetic flux Φ_B through the loop changes with time.

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

$$\boxed{\Phi_B = \int B\,dA\cos\phi = \int \vec{B}\cdot d\vec{A}}$$

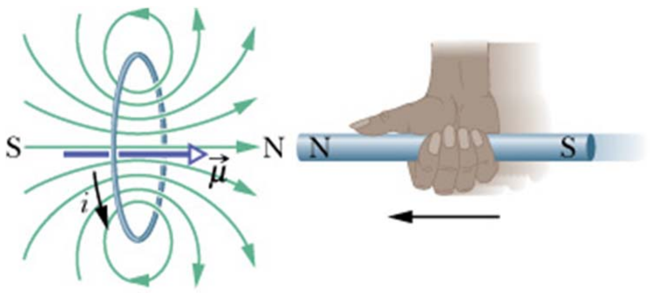


Methods for changing ϕ through a loop:

1. Change the magnitude of B within the loop.
2. Change either the total area of the coil or the portion of the area within the magnetic field.
3. Change the angle between **B** and **n**.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

2.2 Llei de Faraday-Lenz



Lenz's Rule

We now concentrate on the negative sign in the equation that expresses Faraday's law. The direction of the flow of induced current in a loop is accurately predicted by what is known as Lenz's rule.

An induced current has a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current.

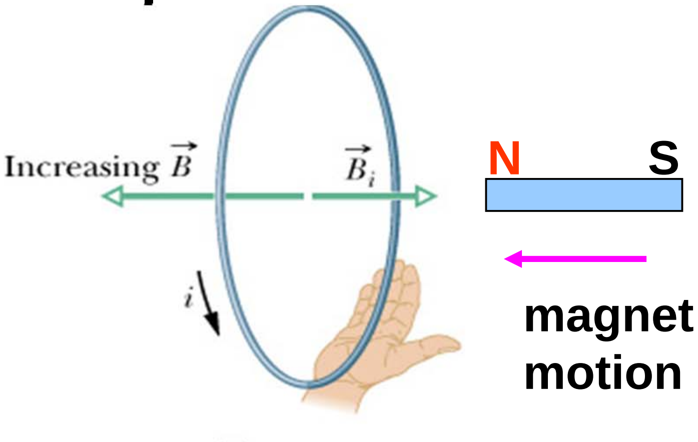
Lenz's rule can be implemented using one of two methods:

1. Opposition to pole movement

In the figure we show a bar magnet approaching a loop. The induced current flows in the direction indicated because this current generates an induced magnetic field that has the field lines pointing from left to right. The loop is equivalent to a magnet whose north pole faces the corresponding north pole of the bar magnet approaching the loop. The loop **repels** the approaching magnet and thus opposes the change in Φ_B that generated the induced current.

2. Opposition to flux change

Example *a* : Bar magnet approaches the loop with the north pole facing the loop.

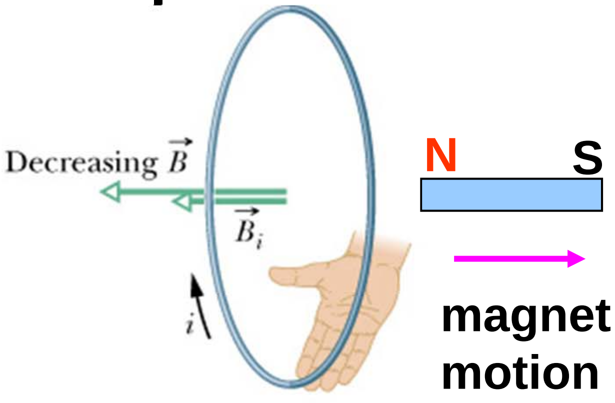


(*a*)

As the bar magnet approaches the loop, the magnetic field $\vec{B}$ points toward the left and its magnitude increases with time at the location of the loop. Thus the magnitude of the loop magnetic flux Φ_B also increases. The induced current flows in the **counterclockwise** (CCW) direction so that the induced magnetic field $\vec{B}_i$ opposes the magnetic field $\vec{B}$. The net field $\vec{B}_{\text{net}} = \vec{B} - \vec{B}_i$. The induced current is thus trying to **prevent** Φ_B from increasing. Remember that it was the increase in Φ_B that generated the induced current in the first place.

2. Opposition to flux change

Example *b* : Bar magnet moves away from the loop with north pole facing the loop.

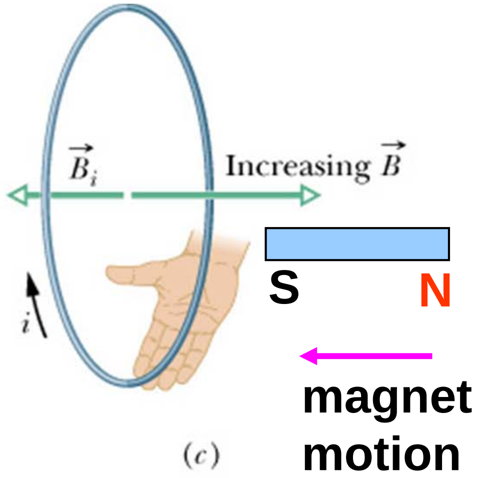


(*b*)

As the bar magnet moves away from the loop, the magnetic field $\vec{B}$ points toward the left and its magnitude decreases with time at the location of the loop. Thus the magnitude of the loop magnetic flux Φ_B also decreases. The induced current flows in the **clockwise** (CW) direction so that the induced magnetic field $\vec{B}_i$ adds to the magnetic field $\vec{B}$. The net field $\vec{B}_{\text{net}} = \vec{B} + \vec{B}_i$. The induced current is thus trying to **prevent** Φ_B from decreasing. Remember that it was the decrease in Φ_B that generated the induced current in the first place.

2. Opposition to flux change

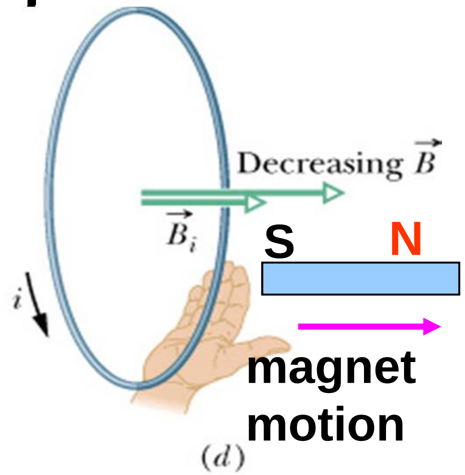
Example *c* : Bar magnet approaches the loop with south pole facing the loop.



As the bar magnet approaches the loop, the magnetic field $\vec{B}$ points toward the right and its magnitude increases with time at the location of the loop. Thus the magnitude of the loop magnetic flux Φ_B also increases. The induced current flows in the **clockwise** (CW) direction so that the induced magnetic field $\vec{B}_i$ opposes the magnetic field $\vec{B}$. The net field $\vec{B}_{\text{net}} = \vec{B} - \vec{B}_i$. The induced current is thus trying to **prevent** Φ_B from increasing. Remember that it was the increase in Φ_B that generated the induced current in the first place.

2. Opposition to flux change

Example *d* : Bar magnet moves away from the loop with south pole facing the loop.



As the bar magnet moves away from the loop, the magnetic field $\vec{B}$ points toward the right and its magnitude decreases with time at the location of the loop. Thus the magnitude of the loop magnetic flux Φ_B also decreases. The induced current flows in the **counterclockwise** (CCW) direction so that the induced magnetic field $\vec{B}_i$ adds to the magnetic field $\vec{B}$. The net field $\vec{B}_{\text{net}} = \vec{B} + \vec{B}_i$. The induced current is thus trying to **prevent** Φ_B from decreasing. Remember that it was the decrease in Φ_B that generated the induced current in the first place.

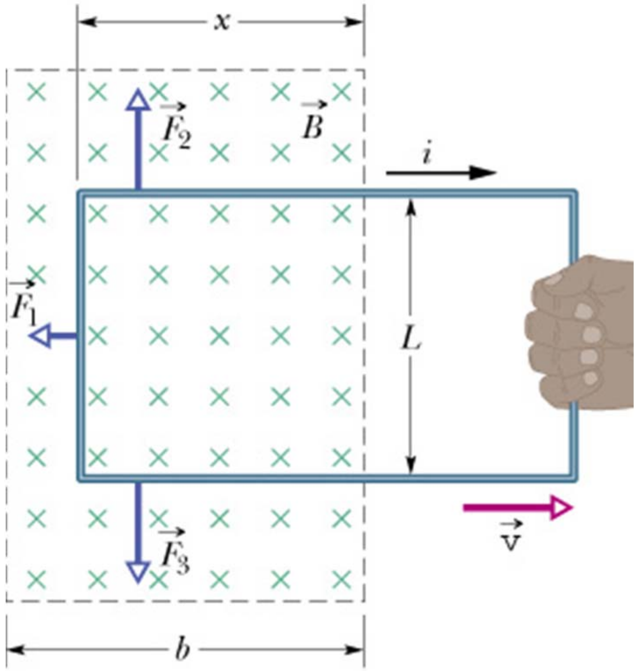
Induction and Energy Transfers

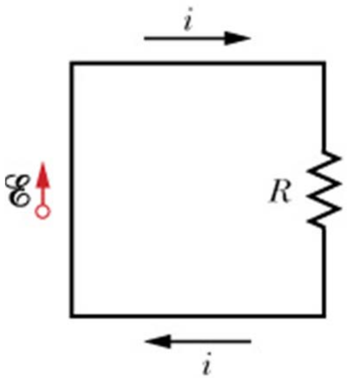
By Lenz's rule, the induced current always opposes the external agent that produced the induced current. Thus the external agent must always **do work** on the loop-magnetic field system. This work appears as thermal energy that gets dissipated on the resistance R of the loop wire.

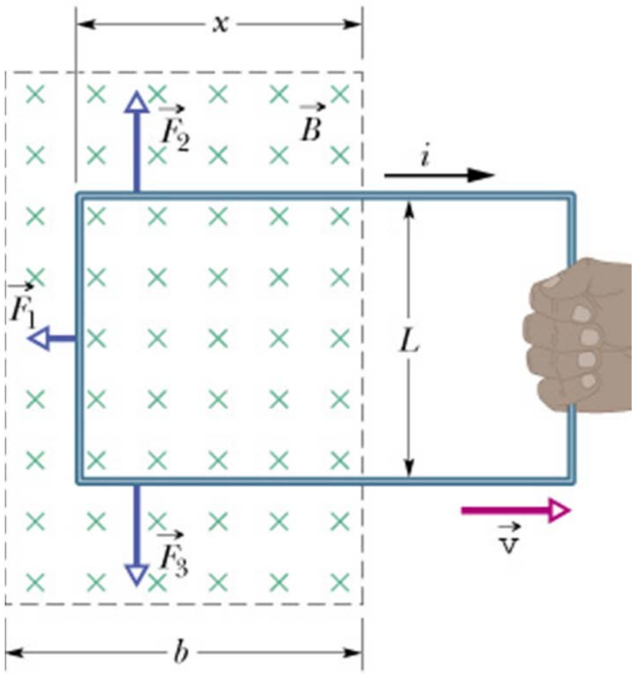
Lenz's rule is actually a different formulation of the principle of energy conservation.

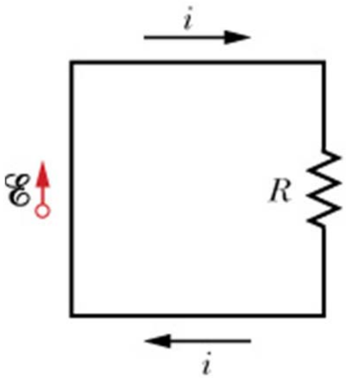
Consider the loop of width L shown in the figure. Part of the loop is located in a region where a uniform magnetic field B exists. The loop is being pulled outside the magnetic field region with constant speed v. The magnetic flux through the loop is $\Phi_B = BA = BLx$. The flux decreases with time:

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = BL\frac{dx}{dt} = BLv \qquad i = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$$









The rate at which thermal energy is dissipated on R is

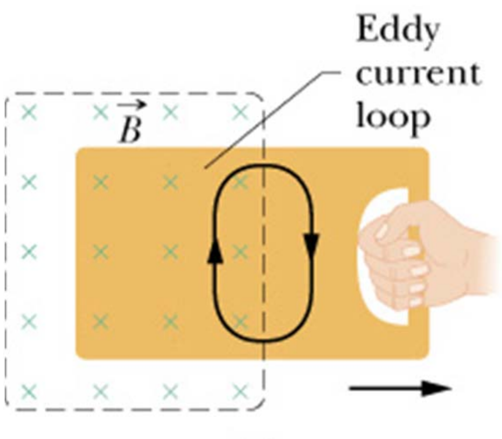
$$P_{\text{th}} = i^2 R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R} \quad \textbf{(eq. 1)}$$

The magnetic forces on the wire sides are shown in the figure. Forces $\vec{F}_2$ and $\vec{F}_3$ cancel each other:

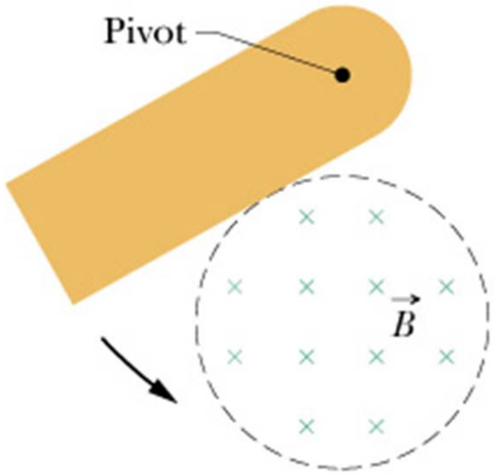
Force $\vec{F}_1 = i\vec{L} \times \vec{B}$, $\quad F_1 = iLB\sin 90° = iLB = \dfrac{BLv}{R}LB$,

$F_1 = \dfrac{B^2L^2v}{R}$. The rate at which the external agent is producing mechanical work is $P_{\text{ext}} = F_1 v = \dfrac{B^2L^2v^2}{R}$ **(eq. 2)**

If we compare equations 1 and 2 we see that indeed the mechanical work done by the external agent that moves the loop is converted into thermal energy that appears on the loop wires.



(a)

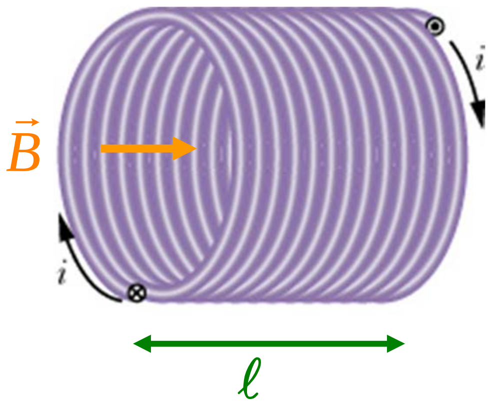


(b)

Eddy Currents

We replace the wire loop in the previous example with a solid conducting plate and move the plate out of the magnetic field as shown in the figure.

The motion between the plate and $\vec{B}$ induces a current in the conductor and we encounter an opposing force. With the plate, the free electrons do not follow one path as in the case of the loop. Instead, the electrons swirl around the plate. These currents are known as "**eddy currents.**" As in the case of the wire loop, the net result is that the mechanical energy that moves the plate is transformed into thermal energy that heats up the plate.



Inductance

Consider a solenoid of length ℓ that has N loops of area A each, and $n = \dfrac{N}{\ell}$ windings per unit length. A current i flows through the solenoid and generates a uniform magnetic field $B = \mu_0 n i$ inside the solenoid.

The solenoid magnetic flux is $\Phi_B = NBA$.

The total number of turns $N = n\ell \rightarrow \Phi_B = \left(\mu_0 n^2 \ell A\right) i$. The result we got for the special case of the solenoid is true for any inductor: $\Phi_B = Li$. Here L is a constant known as the ***inductance*** of the solenoid. The inductance depends on the geometry of the particular inductor.

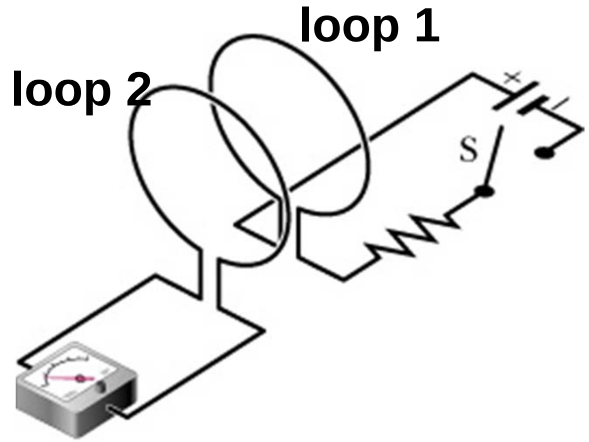
Inductance of the Solenoid

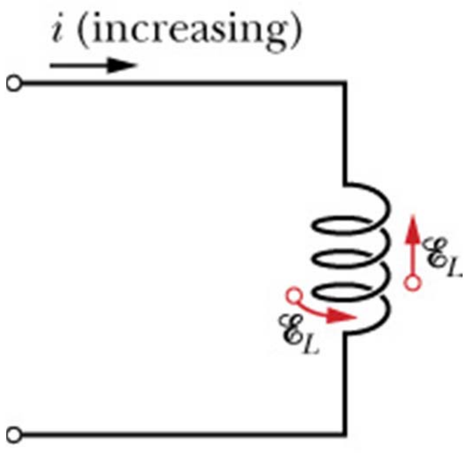
For the solenoid, $L = \dfrac{\Phi_B}{i} = \dfrac{\mu_0 n^2 \ell A i}{i} = \mu_0 n^2 \ell A.$

$$\boxed{L = \mu_0 n^2 \ell A}$$

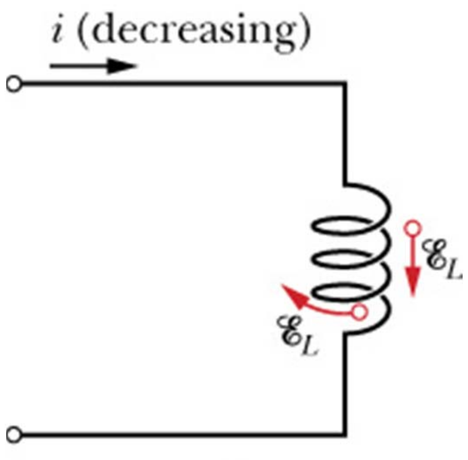
Self - Induction

In the picture to the right we already have seen how a change in the current of loop 1 results in a change in the flux through loop 2, and thus creates an induced emf in loop 2.





(a)



(b)

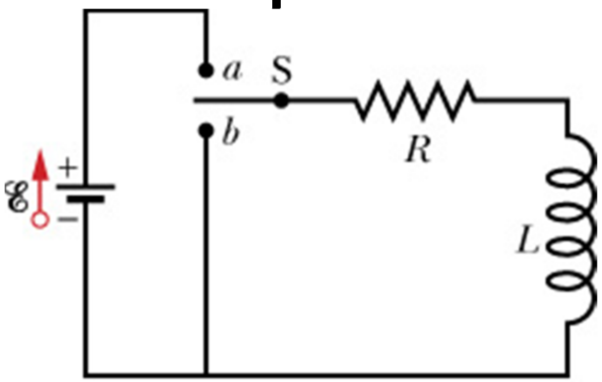
If we change the current through an inductor this causes a change in the magnetic flux $\Phi_B = Li$ through the inductor according to the equation $\dfrac{d\Phi_B}{dt} = L\dfrac{di}{dt}$. Using Faraday's law we can determine the resulting emf known as

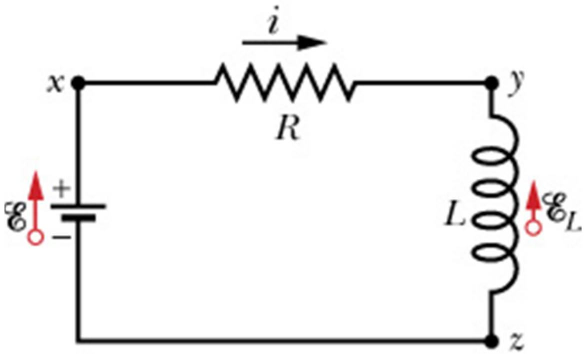
self - induced emf: $\mathrm{E} = -\dfrac{d\Phi_B}{dt} = -L\dfrac{di}{dt}$.

$$\boxed{\mathrm{E} = -L\frac{di}{dt}}$$

SI unit for *L*: the henry (symbol: H)

An inductor has inductance $L = 1\ \mathrm{H}$ if a current change of 1 A/s results in a self-induced emf of 1 V.





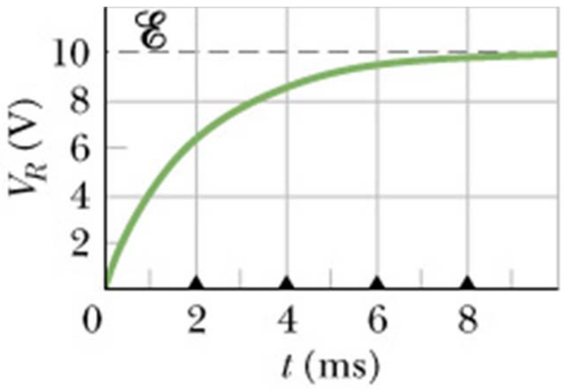
RL Circuits

Consider the circuit in the upper figure with the switch S in the middle position. At $t = 0$ the switch is thrown in position *a* and the equivalent circuit is shown in the lower figure. It contains a battery with emf E, connected in series to a resistor *R* and an inductor *L* (thus the name "*RL* circuit"). Our objective is to calculate the current *i* as a function of time *t*. We write Kirchhoff's loop rule starting at point *x* and moving around the loop in the clockwise direction:

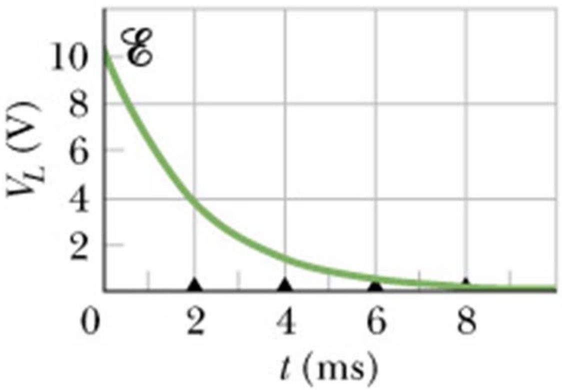
$$-iR - L\frac{di}{dt} + \mathrm{E} = 0 \rightarrow \quad L\frac{di}{dt} + iR = \mathrm{E}$$

The initial condition for this problem is $i(0) = 0$. The solution of the differential equation that satisfies the initial condition is

$i(t) = \dfrac{\mathrm{E}}{R}\left(1 - e^{-t/\tau}\right)$. The constant $\tau = \dfrac{L}{R}$ is known as the "***time constant***" of the *RL* circuit.



(a)



(b)

$$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right) \qquad \text{Here} \quad \tau = \frac{L}{R}.$$

The voltage across the resistor $V_R = iR = E\left(1 - e^{-t/\tau}\right)$.

The voltage across the inductor $V_L = L\dfrac{di}{dt} = Ee^{-t/\tau}$.

The solution gives $i = 0$ at $t = 0$ as required by the initial condition. The solution gives $i(\infty) = E / R$.

The circuit time constant $\tau = L / R$ tells us how fast the current approaches its terminal value:

$$i(t = \tau) = (0.632)(E / R)$$

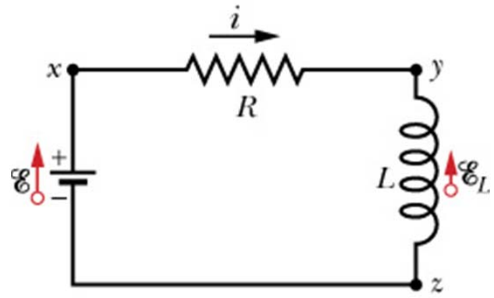
$$i(t = 3\tau) = (0.950)(E / R)$$

$$i(t = 5\tau) = (0.993)(E / R)$$

If we wait only **a few time constants** the current, for all practical purposes, has reached its terminal value (E / R).

Energy Stored in a Magnetic Field

We have seen that energy can be stored in the electric field of a capacitor. In a similar fashion, energy can be stored in the magnetic field of an inductor. Consider the circuit shown in the figure. Kirchhoff's loop rule gives



$\mathcal{E} = L\dfrac{di}{dt} + iR.$ If we multiply both sides of the equation we get: $\mathcal{E}i = Li\dfrac{di}{dt} + i^2 R.$

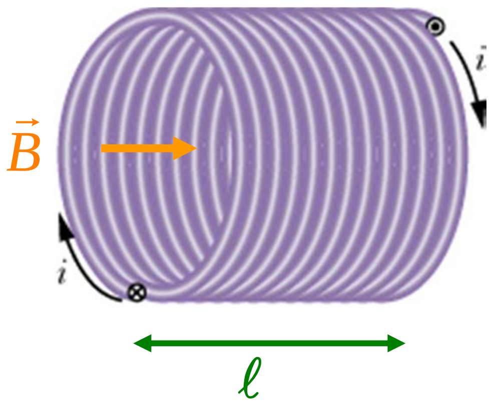
The term $\mathcal{E}i$ describes the rate at which the battery delivers energy to the circuit.

The term $i^2 R$ is the rate at which thermal energy is produced on the resistor.

Using energy conservation we conclude that the term $Li\dfrac{di}{dt}$ is the rate at which energy is stored in the inductor: $\dfrac{dU_B}{dt} = Li\dfrac{di}{dt} \to dU_B = Lidi.$ We integrate both sides of this equation: $U_B = \int_o^i Li'di' = \dfrac{L\left[i'^2\right]_o^i}{2} = \dfrac{Li^2}{2}.$

$$\boxed{U_B = \frac{Li^2}{2}}$$

Energy Density of a Magnetic Field



Consider the solenoid of length ℓ and loop area A that has n windings per unit length. The solenoid carries a current i that generates a uniform magnetic field $B = \mu_0 ni$ inside the solenoid. The magnetic field outside the solenoid is approximately zero.

The energy U_B stored by the inductor is equal to $\frac{1}{2}Li^2 = \frac{\mu_0 n^2 A\ell i^2}{2}$.

This energy is stored in the empty space where the magnetic field is present.

We define as energy density $u_B = \frac{U_B}{V}$ where V is the volume inside the solenoid. The density $u_B = \frac{\mu_0 n^2 A\ell i^2}{2A\ell} = \frac{\mu_0 n^2 i^2}{2} = \frac{\mu_0{}^2 n^2 i^2}{2\mu_0} = \frac{B^2}{2\mu_0}$.

This result, even though it was derived for the special case of a uniform magnetic field, holds true in general.

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$