Indeed this fact can be seen in the building of monuments and waging of wars, since they are given other names; for monuments are inscribed with the names of those with whose authority and reason they were ordained, and military triumphs are also similarly commemorated. But monuments and triumphs are not named or commemorated for the servitude and labor of those who carried these things to completion. Thus there are three kinds of people who are considered in relation to the musical art. The first type performs on instruments, the second composes songs, and the third type judges the instrumental performances and composed songs. But the type which buries itself in instruments is separated from the understanding of musical knowledge. Representatives of this type, for example kithara players and organists and other instrumentalists, devote their total effort to exhibiting their skill on instruments. Thus they act as slaves, as has been said; for they use no reason, but are totally lacking in thought.

The second type is that of the poets. But this type composes songs not so much by thought and reason as by a certain natural instinct. Thus this type is also separated from music.

The third type is that which has gained an ability of judging, whereby it can weigh rhythms and melodies and songs as a whole. Of course since this type is devoted totally to reason and thought, it can rightly be considered musical. And that man is a musician who has the faculty of judging the modes and rhythms, as well as the genera of songs and their mixtures, and the songs of the poets, and indeed all things which are to be explained subsequently; and this judgment is based on a thought and reason particularly suited to the art of music.

IV. CONCERNING THE DIVISION OF MATHEMATICS.[3]

Among all men of ancient authority who flourished through the purer reason of the mind under the leadership of Pythagoras, it was considered manifestly certain the no one was to go forth in the study of philosophy unless excellence and nobility were investigated by means of a certain four-way path (*quadrivium*) which led to such knowledge.[4] This four-way path will not hamper the skill of right reasoning. For wisdom, the comprehension of reality, is of things which partake in immutable substance. Moreover we say that these things are those which neither grow under tension nor shrink when reconsidered, things which are not susceptible to change through variations, but which always preserve themselves in their appropriate virtue *(vis),* supported through the aid of their own nature. These things are quantities, qualities, forms, magnitudes, minima, equalities, conditions, actualities *(actus),* dispositions, places, times, and whatever is found united in some way with bodies. Indeed these things themselves are incorporeal in nature and thrive by reason of their immutable substance, but they suffer radical change through participation in the corporeal, and through contact with variable things they change in veritable inconsistency. Therefore, since, as has been said, nature had allotted these things immutable substance and virtue *(vis),* they can truly and properly be said "to be." Therefore wisdom professes knowledge of these things which are in and of themselves, and which are called "essences."

There are two divisions of essences: the one is continuous and not divided by any limits, as in a tree, a stone, and all parts of this universe which are properly called magnitudes; the other division of essences is discontinuous with itself, and divided into parts, and just as a union is

[3]This section is from Boethius' The *Principles of Arithmetic,* Book I, Introduction.

[4]The Academy of Plato is said to have written over its gateway the motto "Let no one ignorant of geometry enter here," *cf Gorgias* 508a.

"Introduction to Arithmetic"

Trans: D'OOGE (different translation

reduced by heaps into one, so is the grass, the populace, a chorus, a mass of similar objects, and whatsoever thing whose parts are terminated by their own extremities and set apart from others by a limit. Multitude is the proper name for these things. Moreover, some multitudes are in and of themselves, such as 3 or 4, or a square, or any number which, as it were, is independent. Others are indeed ascertained through themselves, but are referred to something else, such as a duple, a half, a sesquialter, or a sesquitertian[5]; and anything is of such a nature that exists through its relation to something else and not in and of itself. On the other hand, some magnitudes are stationary and void of motion, while others are mobile, always turned in rotation and not still at any time. Therefore arithmetic explores that multitude which exists in and of itself, whereas the appropriate admixtures of musical modulation become fully acquainted with that multitude which is related to something. Geometry professes knowledge of immobile magnitude, whereas the skill of the astronomical discipline claims knowledge of mobile magnitude.

If the searcher should deprive himself of these four parts, he will not be able to find truth; and indeed, without this approach to knowledge he would not become properly wise concerning any truth. For wisdom, knowledge and complete comprehension, concerns itself with those things which truly are; and he who rejects these disciplines, these narrow ways to wisdom, I declare to him that he has no right to philosophize. For since philosophy is the love of wisdom, he would have despised philosophy when he rejected these disciplines.

I think it should be added that the entire potency of multitude, having progressed from one term, grows in an infinitely increasing progression. Magnitude, on the other hand, begins in a finite quantity, but admits no end to its division; for it submits to infinitesimal sectioning of its body. Thus philosophy willingly rejects this infinity and indeterminate potential of nature. For infinity can neither be unified through knowledge nor grasped by the mind. But hence reason takes unto itself those very things in which it can practice an adroit investigation of truth; for it assumes a term of finite quantity from a plurality of infinite multitude, and commits defined spaces unto itself for cognition, having rejected a section of interminable magnitude. It is therefore clear that if anyone neglect these disciplines, he would irrecoverably lose all the teaching of philosophy.

This then is that four-way path *(quadrivium)* by which one should come to those places where the more excellent mind, having been delivered from our senses, is led to the certainty of intelligence. Indeed there are degrees and fixed steps by which one is able to progress and ascend, so that the eye of that more worthy soul, as Plato said, might be saved and established free from corporeal bodies having many eyes. Then that eye might strive to investigate and see truth through the light of truth alone. I would say that these disciplines will illumine this eye cast down and made destitute by the corporeal senses.

Which then of these disciplines ought to be studied first unless it is that one which holds the first principle and position of a mother, as it were, to the others? This one is indeed arithmetic; for it is prior to all the others, not only because God the Creator of the great universe considered arithmetic first as the model of his reasoning and created all according to it, having rationally forged all things through numbers of assigned order to find concordance, but also because arithmetic is prior *by nature*. For if that which is prior by nature be abolished, the posterior things are also abolished; if the posterior things perish, no substantial change takes place in the prior. Animal, for example, is prior to man; for if you abolish animal, the nature of man is also

[5] 'Sesquialter' means relating to or denoting a ratio of 3:2 (from Latin, from *sesqui* (one and a half) + *alter 'second'*.) 'Sesquitertian' means having the ratio of one and one third to one (as 4:3).

abolished at the same time; but if you should abolish man, animal will not perish, Conversely those things are posterior which infer something else in themselves, whereas the prior, as has been said, imply nothing of the posterior. If you say "man," for example, you will also denote animal at the same time; for that which is man is animal. If you say "animal," you would not be implying the species of man at the same time; for animal is not the same thing as man.

The same is seen to occur in arithmetic and geometry; for if you abolish numbers, whence does the triangle, the quadrangle, or any other thing considered in geometry receive its name? But if you abolish quadrangle and triangle, and if all geometry be consumed, "three" and "four" and the names of other numbers will not perish. Likewise when I will have said any geometric form, the name of numbers is implied at the same time; when I will have said numbers, I in no way have named any geometric forms.

It can be easily proved that the power of numbers is prior to music, not only because things which exist in themselves are prior to those which are considered in relation to others, but also because musical modulation itself is discussed in terms of numerical names. The same can thus occur in the case of music that has already been discussed in terms of geometry; for diatessaron[6], diapente[7], and diapason take their names from those of antecedent numbers. Moreover, the proportion of these same sounds in relation to each other is found in none other than numbers; for the sound of the diapason consonance is drawn together according to a duple proportion in numbers [2:1]. The diatessaron of modulation is composed of the epitrita[8] relationship; that which is called diapente is joined together through the hemiolic mean; and the epogdous in numbers is the tone in music.[9] But let me not pursue only this point which the rest of this work will prove, namely that arithmetic without any doubt is a prior discipline.

Arithmetic is prior to the science of spheres and astronomy insofar as these other two disciplines by nature precede this third one. For in astronomy there are circles, the sphere, the center, parallel circles, and the middle axis, all of which are part of the geometric discipline. Why does this show the senior power of geometry? Because all motion is posterior to stillness, and motionlessness is by nature prior. Astronomy concerns itself with things in motion, whereas geometry considers things which are immobile. Moreover, the very motion of the stars is resounded in harmonic modulations. Wherefore it is certain that the power of music, which without doubt is naturally superseded by that of arithmetic, precedes the courses of the stars in authority. However, all the courses of the stars and all astronomical reasoning were properly established according to the very nature of numbers; for through arithmetical reason we collect risings and settings, we keep watch on the fast and slow velocities of the wandering stars, and we come to know the waning and multiple variations of the moon. Wherefore, since the power of arithmetic is clearly prior, we should begin our study with this discipline.[10]

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[6] The musical interval of a fourth.

[7] The musical interval of a fifth.

[8] Epitrita, hemiolic mean, and epogdous, mean respectively ratios of 4:3, 3:2 and 9:8

[9] Boethius' use of the term "tone" is confusing here. Each of the "two tones" referred to here are evidently separate pitches; so at first "a tone" seems to mean "a single pitch." When he goes on to say that these two tones (or pitches) "stand at the distance of a tone," he means that the interval between the pitches is a whole step (also called a "major second"), which he then consistently calls "a tone" throughout the rest of the work.

[10] This famous idea of the "music of the spheres" has its ultimate origin in the *Timaeus,* but from this and similar passages in Cicero found its way into Western literature. Perhaps the most famous passage in English is Shakespeare's *Merchant of Venice,* act 5, scene one, lines 54-65.

Therefore since high sounds are incited from the more frequent and faster motions, the low ones from the slower and less frequent, it is evident that a high sound grows from a low one by some increase in motion, whereas by a decrease in motion, a low sound descends from a high one. For a high sound consists of more motion than a low one. However, the plurality makes the difference in these matters; for the plurality necessarily consists of a certain numerical quantity, and every smaller quantity is considered to a larger quantity as a number compared to a number. Now of these things which are compared according to number, some are equal, others unequal. Thus some sounds are also equal; others indeed are different by virtue of an inequality. But in these sounds which do not harmonize by any inequality, there is no consonance at all. For a consonance is a concord which reduces mutually dissimilar voices into one.

V. WHAT SOUND IS, WHAT INTERVAL IS, AND WHAT CONSONANCE IS. (I,8)

Thus sound is the melodious inflection of the voice, that is, fitted to song in a single tune *(intensio)*. Indeed we do not wish to define sound generally now, but that which is called *phthongos* in Greek, called this from the similarity of speaking, that is *phthengesthai*. Interval is the distance of a high and low sound. Consonance is a mixture of high and low sound falling uniformly and pleasantly on the ears. Dissonance, on the other hand, is the harsh and unpleasant percussion of two sounds ill-mixed with each other coming to the ear. For while they are unwilling to mix, and both strive by some means to come together, since each interferes with the other, both are transmitted to the sense unpleasantly.

VIII. IN WHAT MANNER PYTHAGORAS INVESTIGATED THE PROPORTIONS OF CONSONANCES. (I,10)

Thus this was mainly the reason that Pythagoras, having forsaken aural judgment, turned to reason. He did not trust the human ears, which are subject to radical change, in part by nature, in part even by external accidents, and the perceptions of which are even affected by one's age. Nor did he rely on instruments, in conjunction with which much diversity and inconsistency often originate. Take strings, for example: either the more humid air deadens the vibrations, or more arid air excites them; either a larger string renders a low sound, or a smaller one returns a high sound; or by some other means, something would subject the stability of the prior pitch to radical change. And since the case would be the same in other instruments, Pythagoras placed a minimum of trust in these inconclusive things. And, being curious for some time, he sought a way to establish in his mind, by reason, firmly and consistently, the principles of consonances. In the meantime, by certain divine will, when he passed the workshops of blacksmiths, he overheard the beating hammers somehow resound one consonance from the diverse sounds. Thus in the presence of that which he had long sought, he approached the work amazed. And considering for a while, he thought the strength of the hammerers caused the diversity of sounds. Thus, in order to test this theory more clearly, he commanded the men to exchange hammers among themselves. But the property of sounds was not contingent on the muscles of the men, but rather it followed the exchanged hammers. Thus when he observed this, he examined the weight of the hammers. And since perchance there were five hammers, one was found to weigh twice as much as another [2:1], and these two resounded a diapason [octave] consonance. The one which had weighed twice as much as a second formed the sesquitertian [4:3] relation [with] a third, with which naturally it produced a diatessaron [fourth]. He found the one which weighed twice as much as a second to be the sesquialter [3:2] relation to a fourth, which was related to it by a diapente [fifth] consonance. Those two, to which the above one of double weight was proved to be sesquitertian and sesquialter relation, were discovered in turn to be related by the sesquioctave [9:8] proportion. The fifth hammer, which was dissonant with all, was rejected.

Therefore, since musical consonances before Pythagoras were called in part diapason, in part diapente, and in part diatessaron (which is the smallest consonance), Pythagoras first ascertained in this way by what proportion these consonances of sound were united. And in order that what was said might be clearer for the sake of discourse, the weights of the hammers were written underneath in numbers: 12, 9, 8, 6. Thus the hammers which weighed 12 and 6 pounds resounded, in the duple proportions, the diapason consonance. The hammer of 12 pounds with that of 9, and the hammer of 8 pounds with that of 6 were united by a diatessaron consonance according to the epitrita [sesquitertian or 4:3] proportion. Indeed the one of 9 pounds with that of 6, as well as those of 12 and 8 intermingled the diapente consonance. The one of 9 pounds with that of 8 resounded the tone according to the sesquioctave proportion.

XX. WHICH CONSONANCES PRECEDE OTHERS IN MERIT. (I,32)

But these consonances which we have discussed ought to be judged by the reason as well as by the ear. The reason must contemplate which of them is more harmonious. For as the ear is affected by a sound, and the eye by a view, in the same way the mind *(animus)* is affected by numbers or continuous quantity. Having supposed a line or a number, nothing is easier to contemplate, with either eye or the intellect, than its double. After this judgment of its double follows that of its half, and then its triple, and then a third part of it. Thus since the mental representation of the double is more easily accomplished, Nicomachus considered the diapason as the optimum consonance. After this followed the diapente, which is the half, and then the diapason and diapente, which is the triple. He ranked the other consonances in this same manner and form.

XXI. CONCERNING THE MERIT OR MANNER OF CONSONANCE ACCORDING TO NICOMACHUS. (II,18)

That consonance whose character is more easily grasped by the sense ought to be considered the first and pleasant consonance. For just as every single thing is in itself, so it is perceived by the sense. Therefore if that consonance which consists in duplicity is easier to know than the others, then there is no doubt that the diapason is the first consonance of all and that it excels the others in merit, since it precedes the others in being known. The remaining consonances, according to the Pythagoreans, necessarily stand in the order given by multiplications of multiple and diminutions of super particular proportions.[11] For it was demonstrated that, according to the ancients, multiple inequality transcends superparticular proportions in merit. Therefore let a natural numerical series be set out from 1 to 4: 1,2,3,4. Thus the 2 related to the 1 makes a duple proportion, and resounds the diapason consonance, which is the highest and most knowable because of its simplicity. If the 3 be compared with 1, it produces the diapason and diapente consonance. The 4 compared to 1 holds the quadruple proportion, naturally producing the bisdiapason consonance. When the 3 is compared with 2, it produces a diapente consonance, whereas the 4 to 3 produces a diatessaron. And now one comparison remains: If we compare 4 to 2 it ends in the duple proportion which the 2 compared to the 1 had held. Thus sounds are at their greatest distance in the bisdiapason, since the interval here is formed according to the quadruple proportion. The smallest interval of consonance is that which occurs when the higher

[11] In a section we have omitted Boethius defined "superparticular" proportions as those that differ in their terms by one part, as 4:3 or 3:2. The terms of "superpartient" proportions differ by more than one part; e.g. 5:3 is "superbipartient," 7:4 is "supertripartient," etc. This gives Boethius a way of classifying the complexity of ratios, beginning with pure multiples as the most intelligible and simplest (e.g. 2:1), then superparticular (e.g. 3:2), then superpartient (e.g. 256:243).

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sound transcends the lower by a third part of the lower (4:3). And thus the sizes of the consonances are established, and they can neither be extended beyond the quadruple nor compressed to less than a third part. And according to Nicomachus this is the order of consonances: first, the diapason; second, the diapason and diapente; third, the bisdiapason; fourth, the diapente; and fifth the diatessaron. Nicomachus held the diapason to be the principle of consonances in the manner of the following:

Diapason

Diapason and Diapente   Diapente

Bisdiapason   Diatessaron

But Nicomachus held that, although the whole scheme is related as above, nevertheless the multiple consonances precede in sweetness, and the superparticular proportions follow thereafter, just as we described a little earlier.

Therefore, since consonance is the fitting mixture of two voices, whereas sound is the event of an inflected voice produced at a single pitch *(intentio)*, and since sound is the smallest particle of modulation, whereas all sound consists in pulse, and all pulse is from motion, and since some motions are equal and others unequal, and of unequal motions some are more unequal, others less, others between: from all this it follows that equality of sound is born from equality, whereas inequalities of sound stem from inequality corresponding to the size of interval. Thus the clear, first, and simple proportions come into being, and they are naturally the consonances of the multiple and superparticular types: duple, triple, quadruple, sesquialter and sesquitertian. Dissonance is born from those inequalities which occur in the other types of proportions, or those sounds whose intervals are situated variously or vaguely, or at a considerable distance; and from these sounds no concord is produced.