Book 4

Against the Arithmeticians

1. Transition from *Against the Geometers* (1)

[1] Since one sort of quantity applies to continuous bodies, which is called magnitude, which is what geometry is especially about, while the other applies to discrete things, which is number, which is what arithmetic is concerned with,[1] let us move on from the principles and theories of geometry and look at those that are about number; for if this has been done away with, nor will there get to be the expertise that is assembled around it.

2. Pythagorean account of numbers and their importance (2–9)

[2] In general the Pythagoreans involved with mathematics[2] assign great power to numbers, treating the nature of the universe as being administered in accordance with them. Hence they were always proclaiming

[1] In Greek there is a close verbal connection between "number" (*arithmos*) and "arithmetic" (*arithmêtikê*). The Pythagorean and Platonic ideas that Sextus goes on to discuss may hardly look to us like arithmetic; and it has been suggested that he might better have called this book *Against Those who Teach that Numbers are Principles* (Brisson 2006). However, there were a variety of ways of studying numbers in antiquity, all of which could be called "arithmetic". In particular, besides the formal Euclidean approach that is the ancestor of our own conception of the subject, there was a more metaphysical approach associated with the Pythagorean and Platonic traditions, and it is the latter on which Sextus chooses to focus. See the Introduction, Section 4, also Corti 2015a, esp. section 2.

[2] This seems to refer to a subgroup within the Pythagorean tradition (although cf. n. 11). Elsewhere we hear of a division among Pythagoras' followers between *Mathêmatikoi*, who were concerned with mathematics (including the truths it supposedly afforded concerning the nature of the universe), and *Akousmatikoi*, who accumulated Pythagorean sayings (or *akousmata*, "things heard") and conducted themselves according to them, but were not theoretically inclined (Iamblichus, *Life of Pythagoras* 81–2). I take *apo tôn mathêmatôn* to refer to mathematics, not to "disciplines" in general; cf. 3.2 and accompanying note (n. 4).

Everything is like number,

swearing not only by number but also by the one who brought it to their attention, Pythagoras, treating him as a god because of the power in arithmetic, saying

> No, by the one who imparted to our soul the tetractys,
> Spring that holds the roots of everlasting nature.[3]

[3] Tetractys is what they called the "fourth number", composed out of the first four numbers.[4] For one and two and three and four add up to ten, which is the most perfect number, since when we have got to it, we return again to the unit and perform our counting from the beginning. And they have called it the "spring that holds the roots of everlasting nature" because according to them the rationale of everything's constitution lies in it—of the body and the soul, to take an obvious case; for it will be enough to mention these as an illustration. [4] Well then, the unit is set down as a sort of principle productive of the constitution of the other numbers, while the dyad[5] is productive of length. For just as in the case of geometrical principles we indicated first what the point is, and

[3] Sextus also quotes these anonymous Pythagorean verses at *M* 7.94; the previous "everything is like number" occurs in the same place and at *M* 7.109. Both also appear in other authors: "everything . . . " in Theon of Smyrna 99 and Simplicius, *Commentary on Aristotle's* Physics, *CAG* vol. 10, p. 1102, l. 22; the pair of lines in Theon of Smyrna 94, Aetius 282 Diels.

[4] With some hesitation I retain the mss. *tetartos*, "fourth", before "number", along with Mau. Bekker changed this to *deka*, "ten", giving the sense "the number ten". This is undeniably easier to understand. However, the phrase "fourth number" is clearly related to the term "tetractys" itself, which includes the Greek root for "four" but refers to the number ten (as being the sum of the first four numbers, as Sextus immediately goes on to explain). The term "fourth number" occurs in the mss. in [5] and is also a plausible correction in [9], so it would not be especially surprising to find it here. On the other hand, there is nothing in the manuscripts corresponding to "four" in the phrase "the first four numbers", and editors have added either *tessarôn*, "four", or (Mau's choice) the letter δ, which functions as the numeral 4. One might therefore take the copyist to have put the mention of four in the wrong place and either changed "four" to "fourth" or misunderstood δ as meaning "fourth" rather than "four"—it could stand for either depending on the context. (If I follow them correctly, this is the solution of Pellegrin et al.) In this case one would delete "fourth" and simply read "Tetractys is what they called the number composed out of the first four numbers".

[5] i.e. pair, group of two. The word "dyad" is perhaps unfamiliar, but I retain it because it figures regularly in the scholarship on Pythagoreanism. (I do not, however, extend this pattern to other numbered groups; cf. n. 25.)

then after it the line, which turns out to be a breadthless length,[6] in the same way in the present case, too, the unit occupies the role of the point, and the dyad that of the line and length; for when conceiving this, thought has gone from one place to another, and this is length. [5] And the triad was correlated with breadth and the surface; for the mind was carried from one place to another and again another. And when the dimension of breadth is added to the dimension of length, a surface is conceived. But if, over and above the triad, one considers a fourth unit, that is a fourth sign, a pyramid comes into being, a solid body and figure; for it has length and breadth and depth; so that the formula[7] of the body is contained in the fourth number. [6] Then again, so is that of the soul; for they say that as the whole world is organized in terms of harmony, so too the animal is endowed with soul. And the perfect harmony seems to obtain its subsistence in three concords, the fourth, the fifth, and the octave.[8] Well, the fourth lies in a one-third-again ratio [4:3], the fifth in a one-and-a-half ratio [3:2], and the octave in a double ratio [2:1].[9] [7] And a number is called one-third-again [4/3] which is constituted out of some whole number and a third part of that one, as eight stands in relation to six; for it includes the six itself and a third of it, that is the dyad. And a number is called one-and-a-half [3/2] when it includes a number and half of that one, as nine stands in relation to six; for it is constituted out of the six and half of it, that is, three. And the number equal to two equal numbers is called double [2/1], as four is in relation to two; for it includes the same one twice. [8] But since these things are so, and according to the original hypothesis there are four numbers, one and two and three and four, in which we said that the form of the soul is contained according to the harmonious ratio, four is double two and two is double the unit, in which the octave concord lies, [9] and three is one-and-a-half two (for it includes the two itself and half of this,

[6] Cf. 3.19ff.

[7] *Logos*, a very multifaceted word that is often hard to translate; in this case it seems to refer, roughly, to the rational principle that makes sense of the way something is (cf. "rationale" in [3], which is also *logos*, as is "ratio" in [6]).

[8] On the concords, cf. 6.45–6.

[9] "One-third-again" and "one-and-a-half" translate as literally as possible the Greek words *epitritos* and *hêmiolios*, which are used in various mathematical and musical contexts to express the 4:3 and 3:2 ratios respectively. For ease of comprehension I also insert the numerical expressions. For a brief account of the Pythagoreans' use of these ideas, and their connection with cosmic harmony, see Kahn 2001, chapter 3.

and hence furnishes the concord of a fifth), and four is one-third-again three, and in this the concord of a fourth is set down. So that it was reasonable for the fourth number to be called by the Pythagoreans "spring that holds the roots of everlasting nature".

3. Transition to counter-argument, to be centered around the unit (10)

[10] But it is clear from what has been said, more or less as an illustration, that they assigned a lot of power to numbers; for their discussion about numbers is extensive. But let's avoid lengthy treatment of this for now, and get to grips with the counter-argument, making the starting point of our arguments from the unit, which stands as the starting point of all number;[10] if this is done away with, there will not be number either.

4. Platonic conception of the one and the things that participate in it (11–13)

[11] So, Plato shapes for us the concept of the one in a somewhat Pythagorean manner;[11] he says "one is that without which nothing is called one" or "that by participation in which each thing is called one and many". For the vegetable, for example, and the animal and the stone are called one, but are not one on their own account,[12] but are conceived as one by participation in one, none of them having this status. [12] For neither a vegetable nor an animal nor a stone nor any other of the things counted is the really one. For if a vegetable or an animal is the one, what is not a vegetable or an animal will definitely not be called one; but a vegetable and an animal and millions of other things *are* called one; [13]

[10] On *archê*, "starting point", cf. 3.1 and accompanying note. This is another case where Sextus is having fun by using the same word for the *beginning* of his argument and for the claimed *principle* of arithmetic; were it not for this, I would translate *archê* in the latter case by "principle", as I did in [4].

[11] It would be more historically accurate to say that the ideas to follow originally derived from Plato and his immediate successors in the Academy, but quickly become associated with the Pythagorean tradition. On the interpenetration of Platonic and Pythagorean ideas, see Kahn 2001, chapters 4 and 5.

[12] i.e. (as the sequel makes clear), oneness does not belong to any of these things by definition and exclusively.