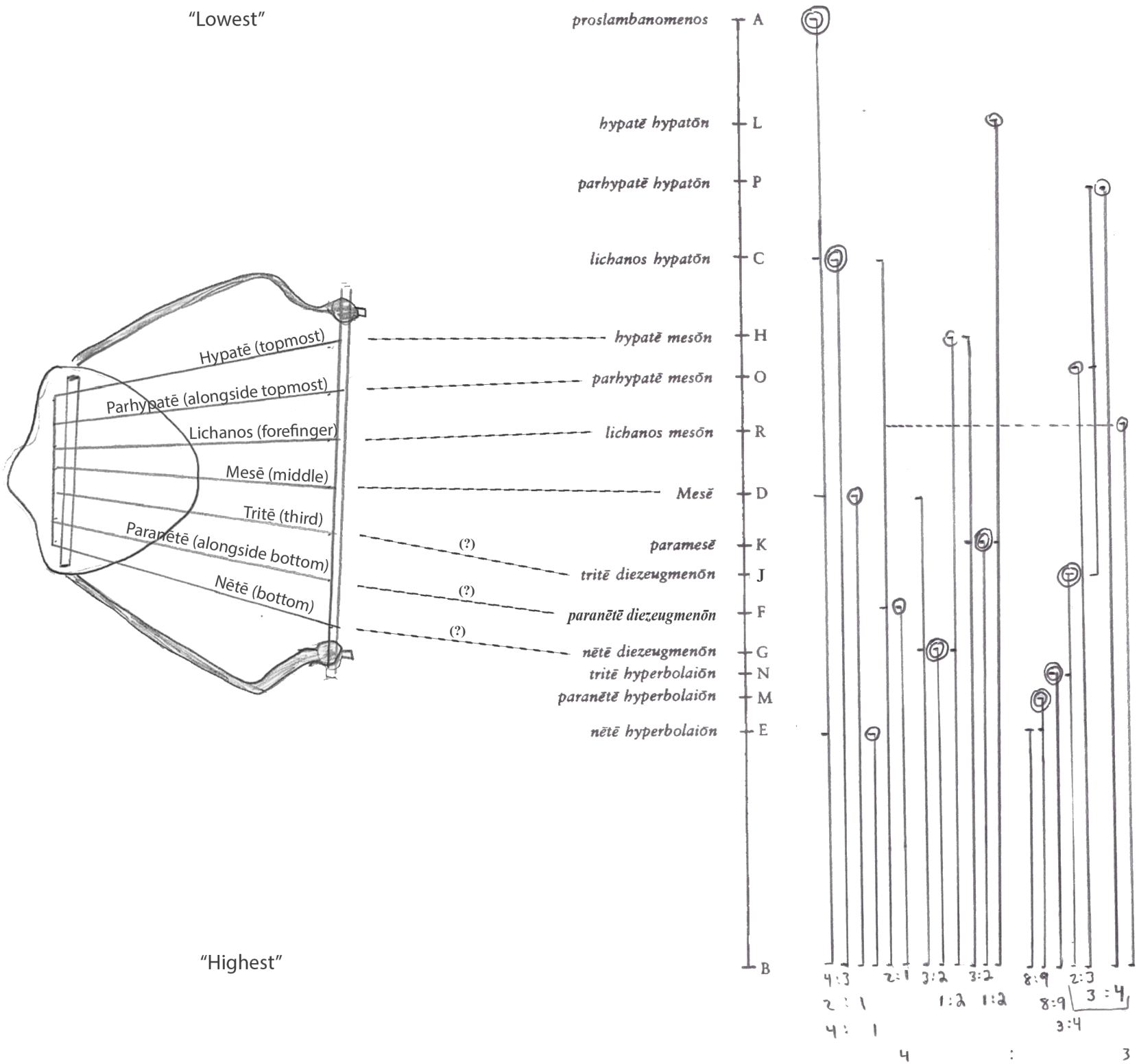
"Lowest"
Hypatē (topmost)
Parhypatē (alongside topmost)
Lichanos (forefinger)
Mesē (middle)
Tritē (third)
Paranētē (alongside bottom)
Nētē (bottom)
"Highest"
proslambanomenos A
hypatē hypatōn L
parhypatē hypatōn P
lichanos hypatōn C
hypatē mesōn H
parhypatē mesōn O
lichanos mesōn R
Mesē D
paramesē K
tritē diezeugmenōn J
paranētē diezeugmenōn F
nētē diezeugmenōn G
tritē hyperbolaiōn N
paranētē hyperbolaiōn M
nētē hyperbolaiōn E
B
(?)
(?)
(?)
4:3
2:1
4:1
2:1
3:2
1:2
3:2
1:2
8:9
8:9
2:3
3:4
3:4
4 : 3

Euclid - Division of a Monochord

Proposition 19

To mark the monochord according to the so-called immutable system.

Let there be the length of the monochord, which is also the length of the string, AB,

and let it be divided into four equal intervals by C, D, and E.

There will therefore be BA, which is the lowest note.

This AB is the sesquitertia (4:3) of CB, so CB will be consonant with AB a fourth higher.

AB is also the **proslambanomenos**.

Therefore CB will be the diatonic hypatōn (**lichanos hypatōn**).

Again, since AB is the double of BD, it will be consonant as an octave, and BD will be the **mesē**.

Again, since AB is the quadruple of EB, EB will be the **nētē hyperbolaiōn**.

I cut CB in two at F. And CB will be the duple of FB, so that CB is consonant as an octave with FB. So FB is the **paranētē diezeugmenōn** .

I take DG, the third part of DB. And DB will be the sesquialtera (3:2) of GB, so DB will be consonant with GB as a fifth. Therefore, GB will be the **nētē diezeugmenōn**.

I establish GH equal with GB, so HB will be consonant as an octave with GB, thus HB is the **hypatē mesōn**.

I take HK, the third part of HB. And HB will be the sesquialtera (3:2) of KB, so KB is the **paramesē**.

I take LK, equal with KB, and LB will be the low hypatē (**hypatē hypatōn**).

Therefore, every stationary note of the immutable system will be found in the monochord.

Proposition 20

It remains necessary to find the movable notes.

I cut EB in eight, and one equal part of these I establish EM, so MB becomes the sesquioctave (9:8) of EB.

Moreover, MB divides in eight, and one equal part of these I establish NM.

Therefore, by a tone, NB will be lower than BM, and MB than BE, so NB will be the trite hyperbolaion, and MB the diatonic hyperbolaiōn (**paranētē hyperbolaiōn**).

I take the third part of NB and establish NJ, so JB is the sesquitertia (4:3) of NB and is consonant as a fourth lower, and JB becomes the **tritē diezeugmenōn**.

Moreover, taking the half part of JB, I establish JO, so OB is consonant as a fifth with JB. Therefore, OB will be the **parhypatē mesōn**.

And equal to JO, I establish OP, so PB becomes the **parhypatē hypatōn**.

Now I take of BC a fourth part, CR, thus RB becomes the diatonic mesōn (**lichanos mesōn**).

[Therefore, the moveable notes will be found in the monochord.]

The components from which he made the soul and the way in which 35
he made it were as follows: In between the *Being* that is indivisible and always changeless, and the one that is divisible and comes to be in the corporeal realm, he mixed a third, intermediate form of being, derived from the other two. Similarly, he made a mixture of the *Same*, and then one of the *Different*, in between their indivisible and their corporeal, divisible counterparts. And he took the three mixtures and mixed them together to make a uniform mixture, forcing the Different, which was hard to mix, into conformity with the Same. Now when he had mixed these two together b
with Being, and from the three had made a single mixture, he redivided the whole mixture into as many parts as his task required,[14] each part remaining a mixture of the Same, the Different, and of Being. This is how he began the division: first he took one portion away from the whole, and then he took another, twice as large, followed by a third, one and a half times as large as the second and three times as large as the first. The fourth portion he took was twice as large as the second, the fifth three times as large as the third, the sixth eight times that of the first, and the seventh twenty-seven times that of the first.

After this he went on to fill the double and triple intervals by cutting 36
off still more portions from the mixture and placing these between them, in such a way that in each interval there were two middle terms, one exceeding the first extreme by the same fraction of the extremes by which it was exceeded by the second, and the other exceeding the first extreme by a number equal to that by which it was exceeded by the second. These connections produced intervals of 3/2, 4/3, and 9/8 within the previous intervals. He then proceeded to fill all the 4/3 intervals with the 9/8 b
interval, leaving a small portion over every time. The terms of this interval of the portion left over made a numerical ratio of 256/243. And so it was that the mixture, from which he had cut off these portions, was eventually completely used up.

14. In order to establish in the soul, through connected geometrical proportions, the source of the harmonious order it needs to impart to the three-dimensional body of the world, and in particular to the heaven and the bodies it contains.

Next, he sliced this entire compound in two along its length, joined the
c two halves together center to center like an X, and bent them back in a circle, attaching each half to itself end to end and to the ends of the other half at the point opposite to the one where they had been joined together. He then included them in that motion which revolves in the same place without variation, and began to make the one the outer, and the other the inner circle. And he decreed that the outer movement should be the movement of *the Same*, while the inner one should be that of *the Different*.[15] He made the movement of the Same revolve toward the right by way of the side, and that of the Different toward the left by way of the diagonal,
d and he made the revolution of the Same, i.e., the uniform, the dominant one in that he left this one alone undivided, while he divided the inner one six times, to make seven unequal circles.[16] His divisions corresponded to the several double and triple intervals, of which there were three each. He set the circles to go in contrary directions: three to go at the same speed, and the other four to go at speeds different from both each other's and that of the other three. Their speeds, however, were all proportionate to each other.

Once the whole soul had acquired a form that pleased him, he who
e formed it went on to fashion inside it all that is corporeal, and, joining center to center, he fitted the two together. The soul was woven together with the body from the center on out in every direction to the outermost limit of the universe, and covered it all around on the outside. And, revolving within itself, it initiated a divine beginning of unceasing, intelligent life for all time. Now while the body of the universe had come to be as a visible thing, the soul was invisible. But even so, because it shares in
37 reason and harmony, the soul came to be as the most excellent of all the things begotten by him who is himself most excellent of all that is intelligible and eternal.

Because the soul is a mixture of the Same, the Different and Being (the three components we've described), because it was divided up and bound together in various proportions, and because it circles round upon itself, then, whenever it comes into contact with something whose being is scatterable or else with something whose being is indivisible, it is stirred throughout its whole self. It then declares what exactly that thing is the same as,
b or what it is different from, and in what respect and in what manner, as well as when, it turns out that they are the same or different and are

15. The outer band is the circle responsible for the constant daily rotation of the fixed stars—hence for the "movement of *the Same*." The inner band is the circle responsible for contrary movements in the Zodiac of the seven "wandering" stars (moon and sun, plus the five planets known to the ancients)—hence for the "movements of *the Different*."

16. These circles or bands are the ones responsible for the individual movements in the Zodiac respectively of moon, sun, Mercury, Venus, Mars, Jupiter, and Saturn, the seven "wanderers" (see 38c–d). The sun, Venus, and Mercury are the three mentioned just below as going "at the same speed" (see 38d).

characterized as such. This applies both to the things that come to be, and to those that are always changeless. And when this contact gives rise to an account that is equally true whether it is about what is different or about what is the same, and is borne along without utterance or sound within the self-moved thing, then, whenever the account concerns anything that is perceptible, the circle of the Different goes straight and proclaims it throughout its whole soul. This is how firm and true opinions and convictions come about. Whenever, on the other hand, the account concerns c
any object of reasoning, and the circle of the Same runs well and reveals it, the necessary result is understanding and knowledge. And if anyone should ever call that in which these two arise, not soul but something else, what he says will be anything but true.

Now when the Father who had begotten the universe observed it set in motion and alive, a thing that had come to be as a shrine for the everlasting gods, he was well pleased, and in his delight he thought of making it more like its model still. So, as the model was itself an everlasting Living Thing, d
he set himself to bringing this universe to completion in such a way that it, too, would have that character to the extent that was possible. Now it was the Living Thing's nature to be eternal, but it isn't possible to bestow eternity fully upon anything that is begotten. And so he began to think of making a moving image of eternity: at the same time as he brought order to the universe, he would make an eternal image, moving according to number, of eternity remaining in unity. This number, of course, is what we now call "time."

Manual of Harmonics

by Nicomachus of Gerasa, the Pythagorean
dictated extempore in the
manner of the ancients

Chapter 1

The book is a manual outlining the doctrine of harmonics

EVEN though the study of intervals and their relations within the harmonic elements is in itself complex and hard to encompass in a single commentary, and even though I, being otherwise in a state of acute mental distraction because of the confusion and haste occasioned by travel, am unable to devote myself to your instruction on these subjects with the appropriate lucidity that demands above all one's leisurely and undivided attention—nevertheless I must exert myself with all zeal, since it is you who bid me, Your Noble Majesty, to set before you the main points in brief, undocumented though they may be, without proper foundation and detailed explication; so that, using this brief note as a manual, you may, with one glance at the chapter headings, recall what is discussed and explained at greater length in each chapter.

And if the gods are willing, as soon as I shall have leisure time and a rest from my journey, I shall compose a longer and more detailed Introduction for you on these same subjects, one that will be thoroughly articulated and crammed, so to speak, with close reasoning, and comprising several volumes. And at my first opportunity, I shall send it to you, wherever I hear that you and your family are residing. In order that it be easier for you to follow, I shall begin at roughly the same point as that at which I started your instruction when I was explaining the subject to you.

Chapter 2

On the two species of the voice, the intervallar and the continuous, and their regions

THE ADHERENTS of the Pythagorean school maintained that there are two species of the human voice subsumed under one genus; they called them the continuous and the intervallar, specifically, deriving their names from the attributes of each. The intervallar, which is the species of melody, stops on every note and renders the change in all the parts perceptible, they assumed to be free from confusion, discrete and graduated by the magnitudes of the intervals lying between each note, forming a progressive series, as it were, and not a blending of the parts of the voice lying adjacent to one another, these being well-defined, readily distinguished and in no instance dissolving into one another. For the melodic species is such that, to trained ears, it renders clear all the notes and the size of the interval in which each note participates. If, on that account, one were to deviate from this species of the voice, he would be said no longer to be singing but to be speaking. The other species is the continuous whereby we converse with one another and read, there being no need for us to make the pitches of the notes explicit and discrete from one another as we string together our discourse right up to the completion of our utterance. For if a person, whether in conversation or in recounting some event or in reading aloud, makes manifest the sizes of the intervals between each note by spacing and changing his voice from one note to another, such a one is said to be speaking or reading no longer, but to be singing. Now since the human voice has two parts, they thought it reasonable that there also be two regions which each of these parts passes through and occupies; that the region of the continuous species is by nature indeterminate in its compass, assuming its own limit from whatever point the speaker begins until he stops, that is, from the first utterance to the final silence, so that it is regulated for the most part by us.

The region of the intervallar species, however, is no longer

subject to our control, but to that of nature, and is itself limited by different operations. For its beginning is the first sound that can be perceived by the ear, while its final limit is the utmost sound that can be uttered by the voice. Our awareness and discrimination of the compasses of notes and their changes relative to one another begin at the point at which our sense of hearing receives its very first stimulus, it being possible that fainter sounds not yet perceptible to us are also produced in nature though they still elude our ears. As an example, in the case of weights, there are certain bodies which are almost impossible to express in terms of weight—chaff or bran or other such things; but when such items are added together and the balance-beam of the scale begins to move down, then we say that the first quantum of scientifically determined weight takes place. So too, when the faintness of the voice is intensified little by little to a greater point, we calculate the first degree that is perceptible to our ears to be the beginning of the region of the singing voice. However, it is not our sense of hearing that determines the end of the region, but the human voice. For we define the final limit of the region of this type of voice at that point which it reaches in its melodic progress as it advances musically.

Let it make no difference to us for the present whether we discuss the human voice or that of stringed, wind, and percussion instruments constructed in imitation of our voice; rather, let us pass over for the present the difference between them so that we do not dissipate our explanation at the very outset.

Chapter 3

Among objects of perception, the music of the planets is considered to be the prototype of our music according as we imitate it

IT IS probable that the names of the notes were derived from the seven stars which traverse the heavens and travel around the earth. For they say that all swiftly whirling bodies necessarily produce sounds when something gives way to them and is very easily vibrated; and that these sounds differ from one another in magnitude and in region of the voice either because of the weights of the bodies or their particular speeds, or because of the position in which the motion of each is accomplished, these positions being more subject to fluctuation or, conversely, more resistant. These three differences are clearly observed in the case of the planets, which differ from one another in size and speed and position as they whir continuously and without pause through the ethereal expanse. Hence the word ἀστήρ (star) was formed, inasmuch as each is deprived of στάσις (*status quo*) and is ἀεὶ θέων (constantly running), whereby the words θεός (god) and αἰθήρ (ether) were also formed. However, from the fact that the movement of Kronos is the farthest up from us, the lowest pitched note in the octave was called *hypate*, for *hypaton* means the highest up. On the other hand, from the fact that the course of the Moon is the lowest of all and is situated nearer to the earth, the name *neate* was derived, for *neaton* signifies the lowest. The term *parhypate* was derived from the position of Zeus, being below Kronos on one side, while at the other side, the position of Aphrodite, being above the Moon, occasioned the name, *paraneate*. The term *mese* was derived from the Sun's most central position, this being the fourth from either end, since the *mese* in fact stood at a distance of a fourth in the ancient heptachord. The term *hypermese*, also called *lichanos*, was derived from one of the positions on either side of the Sun, that of Ares, which was assigned the sphere between Zeus and the Sun. On the other side of the Sun between Aphrodite and the Sun, the position occupied by Hermes provided the name *paramese*.

We shall provide more detailed substantiation respecting these identifications, complete with linear and numerical examples, in the commentaries we promised you earlier, Your Most Serene Highness, and we shall explain why we ourselves do not hear this cosmic symphony with its deep complementation of sound and its all-embracing attunement, as tradition describes. But now, because of the pressure of time, we must run over the material that follows.

Chapter 4

The properties in musical notes are regulated by number

WE SAY that sound in general is a percussion of air that is unbroken in its progress to the ear; a note, however, is a pitch without breadth of a melodic voice, and that pitch is a kind of persistence and a sameness of a note that is without intervals in its extent. An interval is a kind of passage from low to high or the reverse. A system is a synthesis of more than one interval.

When a considerable blow or breath falls upon the surrounding air and strikes it in many parts, a loud sound is produced; when a small blow or breath falls upon the surrounding air, a soft sound is produced; when a regular blow falls, a smooth sound is produced; when an irregular blow falls, a harsh sound results; when the blow is applied slowly, a low sound is produced; when quickly, a high sound results.

Wind instruments, such as auloi, trumpets, panpipes and hydraulic organs, necessarily have inverse properties and the same things hold true for the stringed instruments, such as the cithara, lyre, spadix and similar instruments. There are instruments that appear to be intermediate between these in the sense that they share common properties and are subject to the same affects; these are the monochords, commonly called pandouroi but which the Pythagoreans call canons. Classed also among the stringed instruments are the triangular harps; and there are also the transverse auloi along with the photinxes, as the coming discourse will show.

Among stringed instruments, the greater and more powerful the tension, the greater and higher pitched are the notes produced; while the lesser tensions produce more slow moving and lower pitched notes. For when the plectrum displaces the strings, some, when shifted from their proper position, return very quickly and with considerable vibration, striking the surrounding air many times, as if impelled by the very intensity of the tension; on the other hand, others return to their position along the instrument maker's plumbline slowly and without vibration.

Again, in wind instruments the greater bores and greater lengths produce a slow moving and relaxed note. For if the breath, extenuated by a long air-column, issues forth into the surrounding air and strikes and agitates it without much force, the resulting note is low pitched.

We must observe here that the greater and the lesser depend upon the quantity we obtain by our own tightening or slackening of strings, or by our own lengthening or shortening of the parts of air-columns. Wherefore, it is abundantly clear that all these factors are governed by number. For quantity is held to be the property of number and number alone.

Chapter 5

Pythagoras, by adding the eighth string to the seven-stringed lyre, instituted the attunement of the octave

PYTHAGORAS is the first one who—in order that the middle note, when combined itself with both extremes through conjunction, might not produce the consonance of a fourth only, a fourth with hypate at one extreme, and with nete at the other; and in order that we might be able to envisage a more varied scheme, the extremes themselves producing with one another the most satisfying consonance, that is, the octave in a double ratio, which could not result from the two tetrachords—intercalated an eighth note, which he fitted between mese and paramese and separated it from mese by a whole-tone and from paramese by a semi-tone. The result was that the string which was formerly paramese in the heptachord, still being the third string calculating from nete, is called trite and is situated in just this position; the intercalated string, on the other hand, is the fourth string calculating from nete and forms with it the consonance of a fourth, which is the consonance mese formed originally with hypate. The whole-tone between both these two strings, mese and the intercalated string which is named after the former paramese, to whichever tetrachord it is added, whether to the one that is contingent on hypate, in which case it is relatively high in pitch, or to the one contingent on nete, in which case it is relatively deep in pitch, will produce the consonance of a fifth, which is a system of both the tetrachord itself and the added whole tone. Thus the hemiolic ratio [3:2] of the fifth is found to be a system composed of the epitritic ratio [4:3] and of the sesquioctave ratio [9:8]. The whole-tone then is in a sesquioctave ratio.

Chapter 6

How the numerical proportions of the notes were discovered

THE INTERVAL of strings comprising a fourth, that of a fifth, and that formed by the union of both, which is called an octave, as well as that of the whole-tone lying between the two tetrachords, was confirmed by Pythagoras to have this numerical quantity by means of a certain method which he discovered. One day he was deep in thought and seriously considering whether it could be possible to devise some kind of instrumental aid for the ears which would be firm and unerring, such as vision obtains through the compass and the ruler or the *dioptra*; or touch obtains with the balance-beam or the system of measures. While thus engaged, he walked by a smithy and, by divine chance, heard the hammers beating out iron on the anvil and giving off in combination sounds which were most harmonious with one another, except for one combination. He recognized in these sounds the consonance of the octave, the fifth and the fourth. But he perceived that the interval between the fourth and the fifth was dissonant in itself but was otherwise complementary to the greater of these two consonances. Elated, therefore, since it was as if his purpose was being divinely accomplished, he ran into the smithy and found by various experiments that the difference of sound arose from the weight of the hammers, but not from the force of the blows, nor from the shapes of the hammers, nor from the alteration of the iron being forged. After carefully examining the weights of the hammers and their impacts, which were identical, he went home.

He planted a single stake diagonally in the walls in order that no difference might arise from this procedure or, in short, that no variation might be detected from the use of several stakes, each with its own peculiar properties. From this stake he suspended four strings of the same material and made of an equal number of strands, equal in thickness and of equal torsion. He then attached a weight to the bottom of each string, having suspended each by

each in succession. When he arranged that the lengths of the strings should be exactly equal, he alternately struck two strings simultaneously and found the aforementioned consonances, a different consonance being produced by a different pair of strings.

He found that the string stretched by the greatest weight produced, when compared with that stretched by the smallest, an octave. The weight on one string was twelve pounds, while that on the other was six pounds. Being therefore in a double ratio [2:1], it produced the octave, the ratio being evidenced by the weights themselves. Again, he found that the string under the greatest tension compared with that next to the string under the least tension (the string stretched by a weight of eight pounds), produced a fifth. Hence he discovered that this string was in a hemiolic ratio [3:2] with the string under the greatest tension, the ratio in which the weights also stood to one another. Then he found that the string stretched by the greatest weight, when compared with that which was next to it in weight, being under a tension of nine pounds, produced a fourth, analogous to the weights. He concluded, therefore, that this string was undoubtedly in an epitritic ratio [4:3] with the string under the greatest tension and that this same string was by nature in a hemiolic ratio with the string under the least tension (for this is the case with the ratio of 9 to 6). In a similar way, the string neighboring on that under the least tension, that is, the string stretched by a weight of eight pounds, compared with that stretched by a weight of six pounds, was in an epitritic ratio, but it was in a hemiolic ratio with the string stretched by a weight of twelve pounds.

Then that interval which is between the fifth and the fourth, that is, the interval by which the fifth is greater than the fourth, was confirmed to be in a sesquioctave ratio [9:8], which is as 9 is to 8. And either way it was proved that the octave is a system consisting of the fifth and the fourth in conjunction, just as the double ratio consists of the hemiolic ratio and the epitritic, as for example, 12, 8, and 6; or conversely, it consists of the fourth and the fifth, just as the double ratio consists of the epitritic ratio and the hemiolic, as for example, 12, 9, and 6, in such order.

And having inured his hand and his hearing to the suspended

weights and having established on their basis that ratio of their relations, he ingeniously transferred the bond, which fastened all the strings, from the diagonal stake to the bridge of the instrument, which he called *chordotonon* or string-stretcher, and he transferred the amount of tension on the strings analogous to the weights, to the commensurate turning of the tuning pegs set in the upper part of the instrument. Using this as a standard and as it were an infallible pointer, he extended the test henceforward to various instruments, namely, to the percussion on plates, to auloi and panpipes, to monochords and triangular harps, and the like. And in all of these he found consistent and unchanging, the determination by number.

He called the note partaking of the number 6, hypate, that of the number 8, mese, this number being in an epitritic proportion with the number 6; that of the number 9, he called paramese, which is higher than mese by a whole tone and what is more, stands in a sesquioctave proportion with it; that of 12, he called nete. Filling out the intervening intervals in the diatonic genus with analogous notes, he thus subordinated the octachord to the consonant numbers, the double ratio [2:1], the hemiolic [3:2], the epitritic [4:3], and the difference between them, the sesquioctave [9:8].

Chapter 7

On the division of the octave in the diatonic genus

THUS HE discovered, on the basis of a certain natural necessity, the progression in this diatonic genus from the lowest note to the highest. (For from this procedure he also revealed the structure of the chromatic and enharmonic genera, as it will be possible for me to show you some time later.) This diatonic genus appears, however, to comprehend by nature the degrees and progressions such as follow: semi-tone, a whole-tone, and then a whole-tone. And this is the system of a fourth, consisting of two whole-tones and the semi-tone, so-called. Then, by the addition of another whole-tone, namely, the intercalated whole-tone, the fifth results, being a system of three whole-tones and a semi-tone. Then next in order to this come a semi-tone, a whole-tone, and a whole-tone, being another system of a fourth, that is, another epitritic proportion. So that in the more ancient heptachord all the notes four removed from one another, starting from the lowest, were consonant throughout with each other by a fourth, the semi-tone occupying by transference the first, the middle, and the third place in the tetrachord.

In the Pythagorean octachord, however, whether it be composed of a tetrachord and a pentachord by conjunction, or of two tetrachords separated by a whole-tone from one another by disjunction, the progression by ascent will result in all the notes five removed from one another forming the consonance of a fifth with each other, the semitone as one advances, shifting into the four places, first, second, third, and fourth.

Chapter 8

Explanation of the references to harmonics in the *Timaeus*

IT IS USEFUL, now that we have reached this point, to open up at this opportune moment the passage in the *Psychogony* in which Plato expressed himself as follows: "so that within each interval there are two means, the one superior and inferior to the extremes by the same fraction, the other by the same number. He (the Demiurge) filled up the distance between the hemiolic interval [3:2] and the epitritic [4:3] with the remaining interval of the sesquioctave [9:8]."

For the double interval is as 12 is to 6, but there are two means, 9 and 8. The number 8, however, in the harmonic proportion is midway between 6 and 12, being greater than 6 by one third of 6 (that is, 2), and being less than 12 by one third of 12 (that is, 4). That is why Plato said that the mean, 8, inasmuch as it is of the harmonic proportion, is greater and lesser than the extremes by the same fraction. For the greatest term compared with the smallest is thus in a double proportion; and so it follows that the difference between the greatest term and the middle is 4, compared with the difference between the middle and the smallest, which is 2; for these differences are in a double proportion, 4 to 2. The peculiar property of such a mean is that, when the extremes are added to one another and multiplied by the middle term, a product is yielded which is the double of the product of the extremes; for 8 multiplied by the sum of the extremes, that is, 18, gives 144, which is double the product of the extremes, that is, 72.

The other mean, 9, which is fixed at the paramese degree, is observed to be at the arithmetic mean between the extremes, being less than 12 and greater than 6 by the same number (3). And the peculiar property of this mean is that the sum of the extremes is the double of the middle term itself, and the square of the middle term (which is 81) is greater than the product of the extremes (that is, 72) by the whole square of the differences, that is, by 3 times 3, or 9, for

this is the difference.

One can also point out the third mean, more properly called "proportion," in both the middle terms, 9 and 8. For 12 is in the same proportion to 8 as 9 is to 6; for both are in a hemiolic proportion. And the product of the extremes is equal to the product of the middle terms, 12 times 6 being equal to 9 times 8.

Chapter 9

The evidence of Philolaus concerning our statements

THAT the ancients also offered explanations that are in agreement with what has been demonstrated by us—on the one hand, calling the octave *harmonia*, and the fourth *syllaba* (for it is the first "taking together" of consonant notes), and the fifth *dioxeian* (for the fifth, progressing towards the treble, is adjacent to the primitive consonance, the fourth), and, on the other hand, asserting that the system formed of both intervals, the *syllaba* and the *dioxeian*, is the octave (called *harmonia* from the very fact that it is the first consonance "fitted together" from consonant intervals)—Philolaus, the successor of Pythagoras, makes evident by speaking to this effect in the first book of his *On Nature*. Because of haste, we shall content ourselves with a single witness even though many others have often made similar statements on the same subject. The text of Philolaus reads as follows:

> The size of a *harmonia* is a *syllaba* and a *dioxeian*. The *dioxeian* is greater than the *syllaba* by a sesquioctave. For from *hypata* to *mesa* is a *syllaba*, and from *mesa* to *neata*, a *dioxeian*; from *neata* to *trita* is a *syllaba*, and from *trita* to *hypata*, a *dioxeian*. The interval between *trita* and *mesa* is a sesquioctave, the *syllaba* is epitritic, the *dioxeian* is *hamiolic* and the *dia pasan* is a double proportion. Thus a *harmonia* consists of five sesquioctaves and two *dieseis*. A *dioxeian* is three sesquioctaves and a *diesis*, a *syllaba* is two sesquioctaves and a *diesis*.

We must remember that Philolaus means by trite, the note in the heptachord, now called paramese, but called trite before the insertion of the disjunctive whole-tone in the octachord. For this note was distant from paraneate by an incomposite trihemitone, from which interval the intercalated string subtracted a whole-tone, and the remaining semi-tone between trite and paramese was left in the disjunction. Consequently, the ancient trite was distant from the nete by a fourth, which interval the paramese has now assumed in

its stead. But there are those who do not understand this and claim that it is impossible for trite to be distant from nete by an interval in the epitritic proportion. Others say not unpersuasively that the intercalated note was not inserted between mese and trite, but between trite and paraneate; but that the ancient trite became paramese in the disjunction; and that Philolaus calls the paramese by the former name, trite, although it was distant from the nete by a fourth.

Chapter 10

On the tuning of the notes by means of numerical proportions

TAKING up our earlier discussion again, let me add the following to it by saying that measurements based on the lengths and thicknesses of strings and on those of the air-columns of auloi are seen to be inverse to measurements that are based on tension, in which case the smaller the term, the lower the pitch, and the greater the term, the higher the pitch. For in the former case, there is an inverse proportion in that the smaller the term, the higher is the pitch, while the greater the term, the lower is the pitch. If, therefore, one takes a long string that is kept under one and the same tension and that lies over a ruler, but is raised far enough above it so as not to touch it, and if one compares the note produced by plucking the entire string with that produced by plucking half the string—the string having been stopped by a bridge or some such contrivance at its very center so that the vibration caused by the plucking of the string may not progress beyond the half-way point—he will find the interval of an octave, the sound of half the string compared with that of the whole string being in a greater proportion, that is, in a duple proportion, a result exactly inverse to the reciprocal data of the length. And if one keeps the vibration down to a third of the string, this part having been measured off exactly, the sound from two thirds of the string will necessarily be in a hemiolic [3:2] relation to that of the sound from the whole string, or inversely proportional to the length of the string. And if one sections off a fourth part of the string, preventing the vibration from reaching any further than that, the sound from three parts of the string will be in an epitritic [4:3] relation to that of the whole string, or inversely proportional to the length of the string. So too in the case of the aulos that has three holes distributed over four equal lengths; if the holes are first stopped by the fingers and if we compared the note of the whole aulos with that produced by the middle hole, the finger being lifted from the hole, a duple proportion would be found and the note produced by the middle hole

would form an octave with that of the whole aulos. The same note compared with that produced by the hole below it, namely, the hole lying next to the bottom extremity of the pipe, would stand in a hemiolic [3:2] proportion with it. This latter note, however, compared with that of the whole pipe stands in an epitritic [4:3] proportion with it. But the note produced by the hole next to the mouthpiece compared with that produced by the middle hole, stands in a duple [2:1] relation and, compared with that produced by the whole aulos, in a quadruple [4:1] relation, or inverse to the proportion of the lengths. In the case of syringes (pan-pipes), the lengths of the pipes and the widths of their bores produce a result similar to the thicknesses of the strings; for those made of two strands emit a sound in duple proportion to those made of four strands.

Chapter 11

On the double octave in the diatonic genus

THIS, then, is the diagrammatic structure of the scale in the diatonic genus, consisting of a double octave of quadruple breadth. For this is all that the voice at full capacity encompasses without any insecurity or vacillation, it being difficult for the voice to place itself at either extreme. On the one hand, the voice has a tendency to hoot like a cuckoo in the treble register, while on the other hand, it tends to buzz in the deeper register of the bass notes.

Accordingly, to the old-fashioned lyre, that is, to the heptachord composed of two tetrachords in conjunction—the mese itself delimiting both of the consonant intervals, the lower one contingent on hypate as one ascends, the higher one contingent on nete as one descends—to these they attached two other tetrachords, one at each extremity. Adjoining the original nete the hyperbolaion tetrachord was situated, called "hyperbolaion" because it was composed of a higher and "transcending" vocal register, beginning from the ancient nete, again by conjunction with it. Consequently, the extended tetrachord found its limit after three notes only had been added which were suitably named as follows: trite hyperbolaion, then paranete hyperbolaion, then nete of the same tetrachord.

In order that the tetrachord preceding it [hyperbolaion] and formed by conjunction with the mese, might have its notes so designated as to distinguish it from the hyperbolaion, its notes were given these names: after mese comes trite synemmenon, then paranete synemmenon, then nete synemmenon. And the entire high-pitched range reckoning from the mese itself necessarily completes a heptachord also. To the original hypate they added at the bass the other of the tetrachords mentioned, again by conjunction, the ancient hypate being also comprehended in it since it is the higher of the notes in the tetrachord. In a similar way, for the sake of distinguishing it from the disposition of the prior tetrachord, this one also acquired more distinctive names. For to each name was added the word, "hypaton," such as: hypate hypaton, parhypate

hypaton, diatonos hypaton or lichanos hypaton, for it makes no difference which of the two names we use.

And this entire system from the mese to the hypate of the hypaton tetrachord turns out to be a heptachord composed of two conjunct tetrachords, these using one common note, the ancient hypate. Consequently, from hypate hypaton to nete hyperbolaion there are four conjunct tetrachords. A thirteen-stringed system is found, the seventh string being fixed diatonically from either end. Then, as we said previously, those intending to vary the attunement intercalated the eighth note at a distance of a whole-tone between mese and the ancient trite (or, as some say, between trite and paranete) and clearly emphasized the interval of a fifth. And the mese was found to be no longer truly a middle note. For in the case of strings that are fixed to an equal number of degrees, there cannot be one middle note, but necessarily two, the seventh and eighth strings.

Again, they added one outermost note below the hypate, the lowest of the existing notes, which they called because of this "addition," proslambanomenos, it too being at a distance of a whole-tone below the hypate hypaton as one descends basswards. The purpose of this addition was that the systems on either side of the mese might be octachords, and that the mese might truly be a middle note by being situated among fifteen notes as the eighth string from either end; and that the double octave, the total compass of the scale, might be a doubly duple proportion, that is, a quadruple proportion; and that the order of names, moving upward in succession, might be such as follow:

proslambanomenos
then after an interval of a whole-tone, hypate hypaton
then after a semi-tone, parhypate hypaton
then after a whole-tone, lichanos hypaton, named after
the finger of the left hand, the finger next to the thumb,
which is called forefinger and is
always applied to this string
then after another whole-tone, hypate meson

following in succession after a semi-tone, parhypate meson
and after a whole-tone, lichanos meson, which they
also call "diatonos," naming it after the
diatonic genus itself
then after another whole-tone, mese
then paramesos, after a whole-tone
then trite diezeugmenon, after a semi-tone
then after a whole-tone, paranete diezeugmenon
and after another whole-tone, nete diezeugmenon
succeeding this, after a semi-tone, trite hyperbolaion
then after a whole-tone, paranete hyperbolaion
and following upon all, after a whole-tone, nete hyperbolaion

In reminiscence of the original conjunction in the heptachord, another tetrachord called synemmenon was inserted between the meson tetrachord and the diezeugmenon, complete with its own trite, separated by a semi-tone from the mese; then after a whole-tone, its own paranete, then after another whole-tone, the conjunctive nete of exactly the same pitch and the same sound as the disjunctive paranete. Consequently, there are five tetrachords in all, hypaton, meson, synemmenon, diezeugmenon, hyperbolaion—among which there are two disjunctions and three conjunctions. The disjunctions occur between the synemmenon and the hyperbolaion tetrachords and between the meson and the diezeugmenon tetrachords, each comprising the interval of a whole-tone—and the three conjunctions, one conjoining hypaton with meson, one conjoining meson itself to synemmenon, and the last one conjoining diezeugmenon to hyperbolaion.

The discoveries of all these elements note by note, their causes and developments, how they came about, by whom they were discovered and when, and from what type of origin, I shall explain to you in my extensive treatment, beginning with the tetrachord and ending with the complete *katapyknosis* of the octave, not only in this diatonic genus, but also in the chromatic and the enharmonic, together with the testimonies of ancient witnesses who are the most reliable and who are held in the highest esteem. And in

addition we shall set forth the division of the so-called Pythagorean canon up to the twenty-seventh multiple, in rigorous conformity with the intention of this master, not as Eratosthenes or Thrasyllus misrepresented it, but as the Locrian Timaeus rendered it, whom even Plato followed closely.

Chapter 12

On the progression and division of the notes in the three genera

IN ORDER that you may have the progression in the three genera from proslambanomenos up to hyperbolaion nete in an orderly extension, it is suitable that I recapitulate for the sake of clarity, beginning with the statements I made a little earlier.

A note is an indivisible vocal utterance, an auditory monad, as it were. But as the moderns say, it is the incidence of the voice upon a single and simple point of pitch. While, according to some, it is a sound without breadth and without intervals in its region.

An interval is the intervening space between two notes. Relationship, however, is the ratio which measures the distance in each interval; and difference is the excess or deficiency of the notes compared with one another. Those who believe that difference and relationship are the same thing are wrong in their thinking. For consider: 2 to 1 comprises the same difference as 1 does to 2 but not the same relationship. For 2 is twice that of 1, but 1 is half of 2. And further, among all the terms of an arithmetic mean, whether they be three or even more, the difference is the same among all, but the relationship is of one sort or another. But you will be provided with more copious information on this question in my extensive treatment of the subject.

A system is a combination of two or more intervals. But no note among the intervals is consonant with the one immediately following it, but is completely dissonant with it. Among the systems, however, some are consonant, others dissonant. Systems are consonant when the notes comprising them, though they be different in compass, commingle with one another when played together or are somehow sounded simultaneously, in such a way that the sound produced from them is of a oneness like a single voice. Notes are dissonant, however, when the sound emanating from both of them is heard to be disparate in some way and unblended.

Since the first and most elementary consonance is the fourth in a continuous tetrachord in epitritic [4:3] proportion, it is reasonable

to find in this tetrachord the variations from one to another of the three genera of melody. For the diatonic, about which we spoke earlier, progresses in this way: semi-tone, then a whole-tone, then a whole-tone—three intervals within four numbers, that is to say, within four notes. For this reason it is called "diatonic," from the fact that it is the only one of the rest of the genera that progresses "by tones." The chromatic progresses in this way: semi-tone, then another semi-tone, then after these, an incomposite trihemitone. And this genus is disposed in such a way that even if it is not composed outright of two whole-tones and a semi-tone, it may be observed, nevertheless, to comprise intervals that are equivalent to two whole-tones and a semi-tone. The enharmonic comprehends by nature such a progression as the following: quarter-tone, which is half of a semi-tone, another quarter-tone, the sum of both being equal to a semi-tone, and the remaining interval of the tetrachord, an entire incomposite ditone. And this is distributed in such a way that it is also equivalent to two whole-tones and a semi-tone; for within these intervals it is impossible for one note to be consonant with another.

It is evident, therefore, that the variations of the genera do not assume their difference within the four notes of the tetrachord, but within the two middle notes only. In the chromatic, therefore, the third note was altered in relation to the diatonic, but the second remained the same as in the diatonic and at the same pitch as the third note in the enharmonic. In the enharmonic the two middle notes were altered in relation to the diatonic in such a way that the enharmonic is opposite to the diatonic, and the chromatic is between these two, for it deviates minimally from the diatonic, by one semi-tone only. From whence we say that men who are changeable have "color."

The extreme notes of the tetrachord, then, are called "fixed notes," for they do not deviate in any of the genera; while the middle notes are "movables," at least in the enharmonic. In the chromatic, however, the second note is both movable and immovable, for relative to the diatonic it does not vary, but relative to the enharmonic it does vary.

The octave, being a system distributed over eight strings, whether

one reckons downwards from mese to proslambanomenos or upwards from mese to nete hyperbolaion—amounting to a fourth which consists of two whole-tones and a semi-tone and a fifth which consists of three whole-tones and a semi-tone—does not consist of six whole-tones exactly, as the moderns believe, but of five whole-tones and two semi-tones so-called. If these semi-tones were really halves of whole-tones, what would prevent a whole-tone from being composed of them and the octave from consisting of six whole-tones? We shall offer a clear and very detailed explanation of this problem in our extensive treatment of the subject. Indeed, Philolaus agrees with us in the preceding citation in which he says, "A *harmonia* consists of five sesquioctaves and two *dieseis*," that is, two semi-tones, which would make one whole-tone if they were truly halves of whole-tones.

Accordingly, the names of the three genera amalgamated with one another in the same diagram will be these:

proslambanomenos
hypate hypaton
parhypate hypaton enharmonic
parhypate hypaton chromatic and diatonic
enharmonic hypaton
chromatic hypaton
diatonos hypaton
hypate meson
parhypate meson enharmonic
parhypate meson chromatic and diatonic
meson enharmonic
meson chromatic
meson diatonic
mese
trite synemmenon enharmonic
trite synemmenon chromatic and diatonic

synemmenon enharmonic
synemmenon chromatic
synemmenon diatonic
nete synemmenon
paramese
trite diezeugmenon enharmonic
trite diezeugmenon chromatic and diatonic
enharmonic diezeugmenon
chromatic diezeugmenon
diatonic diezeugmenon
nete diezeugmenon
trite hyperbolaion enharmonic
trite hyperbolaion chromatic and diatonic
enharmonic hyperbolaion
chromatic hyperbolaion
diatonic hyperbolaion
nete hyperbolaion

Please forgive the haste of such writing as this—for you are aware that you asked me while I was completely unsettled in transit—and, consistent with your most gentle and altogether most intelligent nature, please accept this as first fruits of a sort and friendship's offering. And if the gods be willing, please await my treatise which will be a very copious and complete discourse on the subject discussed here. I shall send it to you directly, as the earliest opportunity presents itself.