

```

> restart:
> Digits := 25:
> with(Statistics):
> assume(m11, real, m12, real, m13, real, m14, real, m21, real,
m22, real, m23, real, m24, real, m31, real, m32, real, m33,
real, m34, real);
> valnum := m11 = 1, m12 = -0.05, m13 = 1, m14 = -0.5, m21 = 0.5,
m22 = 1, m23 = -0.05, m24 = 0.3, m31 = -0.5, m32 = -0.1, m33 =
1.2, m34 = -0.8;

```

```

valnum:= m11~ = 1, m12~ = -0.05, m13~ = 1, m14~ = -0.5, m21~ = 0.5,
m22~ = 1, m23~ = -0.05, m24~ = 0.3, m31~ = -0.5, m32~ = -0.1, m33~
= 1.2, m34~ = -0.8

```

(1)

```

> p1 := PDF(RandomVariable(Normal(0, 1)), x1);

```

$$p1 := \frac{1}{2} \frac{\sqrt{2} e^{-\frac{1}{2} x1^2}}{\sqrt{\pi}}$$

(2)

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> p2 := PDF(RandomVariable(Normal(2, 1)), x2)/2+PDF
(RandomVariable(Normal(-2, 1)), x2)/2;

```

$$p2 := \frac{1}{4} \frac{\sqrt{2} e^{-\frac{1}{2} (x2-2)^2}}{\sqrt{\pi}} + \frac{1}{4} \frac{\sqrt{2} e^{-\frac{1}{2} (x2+2)^2}}{\sqrt{\pi}}$$

(3)

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> p3 := PDF(RandomVariable(Uniform(0, 1)), x3);
p4 := PDF(RandomVariable(Uniform(0, 1)), x4);

```

$$p3 := \begin{cases} 0 & x3 < 0 \\ 1 & x3 < 1 \\ 0 & otherwise \end{cases}$$

$$p4 := \begin{cases} 0 & x4 < 0 \\ 1 & x4 < 1 \\ 0 & otherwise \end{cases}$$

(4)

```

> N := Matrix(4, 4, [m11, m12, m13, m14, m21, m22, m23, m24, m31,
m32, m33, m34, 0, 0, 0, 1]);

```

$$N := \begin{bmatrix} m11~ & m12~ & m13~ & m14~ \\ m21~ & m22~ & m23~ & m24~ \\ m31~ & m32~ & m33~ & m34~ \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(5)

```

> with(LinearAlgebra):
> R := MatrixInverse(N):
> x := R . Vector([z1, z2, z3, z4]):

```

$$expr :=$$
$$e^{-5.000000000000000000000000 \cdot 10^{-53} (7.139656460044809559372666 \cdot 10^{25} z_1 - 2.389843166542195668409261 \cdot 10^{24} z_2 - 5.959671396564600448095594 \cdot 10^{25} z_3 - 1.126213592233009708737864 \cdot 10^{25} z_4)}$$
$$+ 2.135922330097087378640777 \cdot 10^{24} z_4 - 2.000000000000000000000000 \cdot 10^{25})^2) \left(\begin{cases} 0. & 0.268857356235 \\ 1. & 0.268857356235 \\ 0. & 1. \leq 0.268857356 \end{cases} \right)$$

0.2467776322519220390482646

$$\begin{aligned} & e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z_1^2} \\ & - 0.7385888813721198615145846 z_1 + 0.4616180508575749134466154 z_2 \\ & + 0.6347248199291655059890962 z_3 - 0.3367240424027403214796267 z_2 z_3 \\ & + 0.3918243402504614649944747 z_1 z_2 + 0.5387584678443845143674027 z_1 z_3 \\ & - 0.1224451063282692078107733 z_2^2 - 0.2314977791518839710172434 z_3^2 \operatorname{erf}(\\ & -1.250969618762476439094141 + 0.02058761868341532275350798 z_1 \\ & + 0.6274127753223006280722546 z_2 \\ & + 0.008985850068916423875087319 z_3) \left\{ \begin{array}{ll} 0. & z_3 + 0.121951219512 \\ 1. & \end{array} \right. \end{aligned}$$

[illegible]

$$\begin{aligned}
& + 0.008985850068916423875087319 \, z^3) \left(\begin{cases} 0. & z^3 + 0.121951219512195121951219512195 \, z^2 + 0.439024390243902439024390243902 \, z1 < 0 \\ 1. & \text{otherwise} \end{cases} \right) \\
&) \\
& - 0.2467776322519220390482646 \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 \, z1^2} \\
& + 0.7385888813721198615145846 \, z1 - 0.4616180508575749134466154 \, z2 \\
& - 0.6347248199291655059890962 \, z3 - 0.3367240424027403214796267 \, z2 \, z3 \\
& + 0.3918243402504614649944747 \, z1 \, z2 + 0.5387584678443845143674027 \, z1 \, z3 \\
& - 0.1224451063282692078107733 \, z2^2 \\
& - 0.2314977791518839710172434 \, z3^2 \operatorname{erf}(1.250969618762476439094141 \\
& + 0.02058761868341532275350798 \, z1 \\
& + 0.6274127753223006280722546 \, z2 \\
& + 0.008985850068916423875087319 \, z^3) \left(\begin{cases} 0. & z^3 + 0.121951219512195121951219512195 \, z^2 + 0.439024390243902439024390243902 \, z1 < 0 \\ 1. & \text{otherwise} \end{cases} \right) \\
&) \\
& - 0.2467776322519220390482646 \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 \, z1^2} \\
& - 0.7385888813721198615145846 \, z1 + 0.4616180508575749134466154 \, z2 \\
& + 0.6347248199291655059890962 \, z3 - 0.3367240424027403214796267 \, z2 \, z3 \\
& + 0.3918243402504614649944747 \, z1 \, z2 + 0.5387584678443845143674027 \, z1 \, z3 \\
& - 0.1224451063282692078107733 \, z2^2 \\
& - 0.2314977791518839710172434 \, z3^2 \operatorname{erf}(0.09684926080741753076857857 \, z1 \\
& + 0.6485965648011901302986631 \, z2 \\
& + 0.1826929237958103421316370 \, z3 - 1.250969618762476439094141) \\
& \left(\begin{cases} 0. & z^3 + 0.121951219512195121951219512195 \, z^2 + 0.439024390243902439024390243902 \, z1 < 0 \\ 1. & \text{otherwise} \end{cases} \right) \\
&) \\
& - 0.2467776322519220390482646 \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 \, z1^2}
\end{aligned}$$

$$\begin{aligned}
& + 0.7385888813721198615145846 z1 - 0.4616180508575749134466154 z2 \\
& - 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\
& + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\
& - 0.1224451063282692078107733 z2^2 \\
& - 0.2314977791518839710172434 z3^2 \operatorname{erf}(0.09684926080741753076857857 z1 \\
& + 0.6485965648011901302986631 z2 \\
& + 0.1826929237958103421316370 z3 + 1.250969618762476439094141) \\
& \left(\left\{ \begin{array}{ll} 0. & z3 + 0.1219512195121951219512195 z2 + 0.43902439024390243902 z1 < 0 \\ 1. & \text{otherwise} \end{array} \right. \right. \\
& \left. \right) + 0.2467776322519220390482646 \left(\left\{ \begin{array}{ll} 0. & z3 + 0.1219512195121951219512195 z2 + 0.43902439024390243902 z1 < 0 \\ 1. & \text{otherwise} \end{array} \right. \right. \\
& \left. \right) \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\
& - 0.7385888813721198615145846 z1 + 0.4616180508575749134466154 z2 \\
& + 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\
& + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\
& - 0.1224451063282692078107733 z2^2 \\
& - 0.2314977791518839710172434 z3^2 \operatorname{erf}(0.09684926080741753076857857 z1 \\
& + 0.6485965648011901302986631 z2 \\
& + 0.1826929237958103421316370 z3 - 1.534620559884806873905750) \\
& + 0.2467776322519220390482646 \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\
& - 0.7385888813721198615145846 z1 + 0.4616180508575749134466154 z2 \\
& + 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\
& + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\
& - 0.1224451063282692078107733 z2^2 - 0.2314977791518839710172434 z3^2 \operatorname{erf}(\\
& - 1.421710961962325827038993 + 0.02058761868341532275350798 z1 \\
& + 0.6274127753223006280722546 z2 \\
& + 0.008985850068916423875087319 z3) \left(\left\{ \begin{array}{ll} 0. & z3 + 0.1219512195121951219512195 z2 + 0.43902439024390243902 z1 < 0 \\ 1. & \text{otherwise} \end{array} \right. \right. \\
& \left. \right) - 0.2467776322519220390482646 \left(\left\{ \begin{array}{ll} 0. & z3 + 0.1219512195121951219512195 z2 + 0.43902439024390243902 z1 < 0 \\ 1. & \text{otherwise} \end{array} \right. \right. \\
& \left. \right)
\end{aligned}$$

)

$$e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\ + 0.7385888813721198615145846 z1 - 0.4616180508575749134466154 z2 \\ - 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\ + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\ - 0.1224451063282692078107733 z2^2 \\ - 0.2314977791518839710172434 z3^2 \operatorname{erf}(0.09684926080741753076857857 z1 \\ + 0.6485965648011901302986631 z2 \\ + 0.1826929237958103421316370 z3 + 0.9673186776401460042825308) \\ - 0.2467776322519220390482646$$

$$e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\ - 0.7385888813721198615145846 z1 + 0.4616180508575749134466154 z2 \\ + 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\ + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\ - 0.1224451063282692078107733 z2^2 - 0.2314977791518839710172434 z3^2 \operatorname{erf}(\\ -1.421710961962325827038993 + 0.02058761868341532275350798 z1 \\ + 0.6274127753223006280722546 z2$$

$$+ 0.008985850068916423875087319 z3) \left(\left\{ \begin{array}{ll} 0. & z3 + 0.12195121951219512195 z1 \\ 1. & \end{array} \right. \right.$$

)

$$+ 0.2467776322519220390482646 \\ e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\ + 0.7385888813721198615145846 z1 - 0.4616180508575749134466154 z2 \\ - 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\ + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\ - 0.1224451063282692078107733 z2^2 \\ - 0.2314977791518839710172434 z3^2 \operatorname{erf}(1.080228275562627051149288 \\ + 0.02058761868341532275350798 z1 \\ + 0.6274127753223006280722546 z2$$

$$+ 0.008985850068916423875087319 z3) \left(\left\{ \begin{array}{ll} 0. & z3 + 0.12195121951219512195 z1 \\ 1. & \end{array} \right. \right.$$

$$\begin{aligned}
& \left. \begin{aligned}
& -0.2467776322519220390482646 \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\
& + 0.7385888813721198615145846 z1 - 0.4616180508575749134466154 z2 \\
& - 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\
& + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\
& - 0.1224451063282692078107733 z2^2 \\
& - 0.2314977791518839710172434 z3^2 \operatorname{erf}(1.080228275562627051149288 \\
& + 0.02058761868341532275350798 z1 \\
& + 0.6274127753223006280722546 z2 \\
& + 0.008985850068916423875087319 z3) \left(\begin{cases} 0. & z3 + 0.12195121951219512195 z2 + 0.43902439024390243902 z1 < 0. \\ 1. & \text{otherwise} \end{cases} \right)
\end{aligned} \right) \\
& \left. \begin{aligned}
& + 0.2467776322519220390482646 \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\
& - 0.7385888813721198615145846 z1 + 0.4616180508575749134466154 z2 \\
& + 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\
& + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\
& - 0.1224451063282692078107733 z2^2 \\
& - 0.2314977791518839710172434 z3^2 \operatorname{erf}(0.09684926080741753076857857 z1 \\
& + 0.6485965648011901302986631 z2 \\
& + 0.1826929237958103421316370 z3 - 1.250969618762476439094141) \\
& \left(\begin{cases} 0. & z3 + 0.12195121951219512195 z2 + 0.43902439024390243902 z1 < 0. \\ 1. & \text{otherwise} \end{cases} \right)
\end{aligned} \right) \\
& \left. \begin{aligned}
& + 0.2467776322519220390482646 \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\
& + 0.7385888813721198615145846 z1 - 0.4616180508575749134466154 z2 \\
& - 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\
& + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3
\end{aligned} \right)
\end{aligned}$$

$$\begin{aligned}
& -0.1224451063282692078107733 z^2 \\
& -0.2314977791518839710172434 z^3 \operatorname{erf}(0.09684926080741753076857857 z1 \\
& + 0.6485965648011901302986631 z2 \\
& + 0.1826929237958103421316370 z3 + 1.250969618762476439094141) \\
& \left(\begin{cases} 0. & z3 + 0.1219512195121951219512195 z2 + 0.43902439024390243902 z1 < 0 \\ 1. & \text{otherwise} \end{cases} \right) \\
& -0.2467776322519220390482646 \left(\begin{cases} 0. & z3 + 0.1219512195121951219512195 z2 + 0.43902439024390243902 z1 < 0 \\ 1. & \text{otherwise} \end{cases} \right) \\
& e^{-0.4350750129332643559234349 - 0.3134594722003691719955797 z1^2} \\
& -0.7385888813721198615145846 z1 + 0.4616180508575749134466154 z2 \\
& + 0.6347248199291655059890962 z3 - 0.3367240424027403214796267 z2 z3 \\
& + 0.3918243402504614649944747 z1 z2 + 0.5387584678443845143674027 z1 z3 \\
& -0.1224451063282692078107733 z^2 \\
& -0.2314977791518839710172434 z^3 \operatorname{erf}(0.09684926080741753076857857 z1 \\
& + 0.6485965648011901302986631 z2 \\
& + 0.1826929237958103421316370 z3 - 1.534620559884806873905750)
\end{aligned}$$

```

> with(plots):
npts:=21:
p1:=implicitplot3d(pdfZ=0.02,z1=-4..4,z2=-6..6,z3=-3..3,grid=
[npts,npts,npts],transparency=0,axes=boxed):
display3d(p1);
p2:=implicitplot3d(pdfZ=0.001,z1=-4..4,z2=-6..6,z3=-3..3,grid=
[npts,npts,npts],transparency=0.5,axes=boxed):
display3d(p2);
display3d({p1,p2});

```






