

SVEUČILIŠTE U ZAGREBU  
FAKULTET ELEKTROTEHNIKE I RAČUNARSTVA

NEIZRAZITO, EVOLUCIJSKO I NEURORAČUNARSTVO

## **6. Domaća zadaća - Sustav ANFIS** **(engl. *Adaptive Neuro-Fuzzy*** ***Inference System*)**

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# 1. Zadatak 1

Funkcija pogreške za  $k$ -ti uzorak:

$$E_k = \frac{1}{2}(y_k - o_k)^2,$$

gdje  $y_k$  predstavlja željeni izlaz za  $k$ -ti uzorak, dok  $o_k$  predstavlja stvarni izlaz neuro-fuzzy sustava.

Ažuriranje parametara u skladu s algoritmom gradijentnog spusta:

$$\psi(t+1) = \psi(t) - \eta \cdot \frac{\partial E_k}{\partial \psi}.$$

Promatrat ćemo ANFIS koji koristi zaključivanje tipa 3 (metodu Takagi-Sugeno-Kang). Neuro-fuzzy sustav će imati dva ulaza i jedan izlaz čime će obavljati preslikavanje  $\mathbb{R}^2 \rightarrow \mathbb{R}$ . U općem slučaju, neuro-fuzzy sustav će raspolagati s  $m$  pravila:

$$\mathbb{R}_1 : \text{Ako } x \text{ je } A_1 \text{ i } y \text{ je } B_1 \text{ tada } z = p_1x + q_1y + r_1$$

$$\mathbb{R}_2 : \text{Ako } x \text{ je } A_2 \text{ i } y \text{ je } B_2 \text{ tada } z = p_2x + q_2y + r_2$$

...

$$\mathbb{R}_m : \text{Ako } x \text{ je } A_m \text{ i } y \text{ je } B_m \text{ tada } z = p_mx + q_my + r_m$$

Uz zadani skup pravila od  $m$  pravila, temeljem primjera za učenje sustav treba naučiti ukupno  $2 \cdot m$  neizrazitih skupova koji se nalaze u antecedent dijelu pravila i  $3 \cdot m$  parametara koji određuju  $m$  funkcija u konzekvens dijelovima pravila.

Neizraziti skupovi  $A$  i  $B$  su definirani sljedećim funkcijama pripadnosti:

$$\alpha_i = A_i(x) = \frac{1}{1 + e^{b_i(x-a_i)}},$$

$$\beta_i = B_i(y) = \frac{1}{1 + e^{d_i(y-c_i)}}.$$

Iz navedenog se može zaključiti da je potrebno naučiti sveukupno  $7 \cdot m$  parametara sustava.

Stvarni izlaz neuro-fuzzy sustava je definiran na sljedeći način:

$$o_k = \frac{\sum_{i=1}^m \pi_i z_i}{\sum_{i=1}^m \pi_i},$$

gdje je  $\pi_i = \alpha_i \cdot \beta_i$ . Sada kada znamo kako je sve definirano, možemo izvesti formule za ažuriranje svih parametara tijekom učenja algoritmom gradijenti spust, pa krenimo:

1.

$$\begin{aligned}
\frac{\partial E_k}{\partial a_i} &= \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial \pi_i} \frac{\partial \pi_i}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial a_i} \\
\frac{\partial E_k}{\partial o_k} &= \frac{\partial}{\partial o_k} \left( \frac{1}{2} (y_k - o_k)^2 \right) = 2 \cdot \frac{1}{2} (y_k - o_k) \cdot (-1) = -(y_k - o_k) \\
\frac{\partial o_k}{\partial \pi_i} &= \frac{\partial}{\partial \pi_i} \left( \frac{\sum_{j=1}^m \pi_j z_j}{\sum_{j=1}^m \pi_j} \right) = \frac{z_i \sum_{j=1, j \neq i}^m \pi_j - \sum_{j=1, j \neq i}^m \pi_j z_j \cdot 1}{(\sum_{j=1}^m \pi_j)^2} = \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \\
\frac{\partial \pi_i}{\partial \alpha_i} &= \frac{\partial}{\partial \alpha_i} (\alpha_i \cdot \beta_i) = \beta_i \\
\frac{\partial \alpha_i}{\partial a_i} &= \frac{\partial}{\partial a_i} \left( \frac{1}{1 + e^{b_i(x-a_i)}} \right) = (-1) \cdot \frac{1}{(1 + e^{b_i(x-a_i)})^2} \cdot e^{b_i(x-a_i)} \cdot (-b_i) = \alpha_i(1-\alpha_i) \cdot b_i \\
\frac{\partial E_k}{\partial a_i} &= -(y_k - o_k) \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \beta_i \alpha_i (1 - \alpha_i) b_i \\
\implies a_i(t+1) &= a_i(t) + \eta \cdot (y_k - o_k) \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \beta_i \alpha_i (1 - \alpha_i) b_i
\end{aligned}$$

2.

$$\begin{aligned}
\frac{\partial E_k}{\partial b_i} &= \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial \pi_i} \frac{\partial \pi_i}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial b_i} \\
\frac{\partial E_k}{\partial o_k} &= -(y_k - o_k) \\
\frac{\partial o_k}{\partial \pi_i} &= \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \\
\frac{\partial \pi_i}{\partial \alpha_i} &= \beta_i \\
\frac{\partial \alpha_i}{\partial b_i} &= \frac{\partial}{\partial b_i} \left( \frac{1}{1 + e^{b_i(x-a_i)}} \right) = (-1) \cdot \frac{1}{(1 + e^{b_i(x-a_i)})^2} \cdot e^{b_i(x-a_i)} \cdot (x-a_i) = \alpha_i(1-\alpha_i) \cdot (a_i - x) \\
\frac{\partial E_k}{\partial b_i} &= -(y_k - o_k) \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \beta_i \alpha_i (1 - \alpha_i) (a_i - x) \\
\implies b_i(t+1) &= b_i(t) + \eta \cdot (y_k - o_k) \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \beta_i \alpha_i (1 - \alpha_i) (a_i - x)
\end{aligned}$$

3. Po uzoru na ažuriranje parametra  $a$  uz  $\frac{\partial \pi_i}{\partial \beta_i} = \alpha_i$ :

$$\implies c_i(t+1) = c_i(t) + \eta \cdot (y_k - o_k) \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \alpha_i \beta_i (1 - \beta_i) d_i$$

4. Po uzoru na ažuriranje parametra  $b$  uz  $\frac{\partial \pi_i}{\partial \beta_i} = \alpha_i$ :

$$\implies d_i(t+1) = d_i(t) + \eta \cdot (y_k - o_k) \frac{\sum_{j=1, j \neq i}^m \pi_j (z_i - z_j)}{(\sum_{j=1}^m \pi_j)^2} \alpha_i \beta_i (1 - \beta_i) (c_i - y)$$

5.

$$\begin{aligned} \frac{\partial E_k}{\partial p_i} &= \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial p_i} \\ \frac{\partial E_k}{\partial o_k} &= -(y_k - o_k) \\ \frac{\partial o_k}{\partial p_i} &= \frac{\partial}{\partial p_i} \left( \frac{\sum_{j=1}^m \pi_j z_j}{\sum_{j=1}^m \pi_j} \right) = \frac{\partial}{\partial p_i} \left( \frac{\sum_{j=1}^m \pi_j (p_j x + q_j y + r_j)}{\sum_{j=1}^m \pi_j} \right) = \frac{\pi_i}{\sum_{j=1}^m \pi_j} x \\ \frac{\partial E_k}{\partial p_i} &= -(y_k - o_k) \frac{\pi_i}{\sum_{j=1}^m \pi_j} x \\ \implies p_i(t+1) &= p_i(t) + \eta \cdot (y_k - o_k) \frac{\pi_i}{\sum_{j=1}^m \pi_j} x \end{aligned}$$

6.

$$\begin{aligned} \frac{\partial E_k}{\partial q_i} &= \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial q_i} \\ \frac{\partial E_k}{\partial o_k} &= -(y_k - o_k) \\ \frac{\partial o_k}{\partial q_i} &= \frac{\partial}{\partial q_i} \left( \frac{\sum_{j=1}^m \pi_j z_j}{\sum_{j=1}^m \pi_j} \right) = \frac{\partial}{\partial q_i} \left( \frac{\sum_{j=1}^m \pi_j (p_j x + q_j y + r_j)}{\sum_{j=1}^m \pi_j} \right) = \frac{\pi_i}{\sum_{j=1}^m \pi_j} y \\ \frac{\partial E_k}{\partial q_i} &= -(y_k - o_k) \frac{\pi_i}{\sum_{j=1}^m \pi_j} y \\ \implies q_i(t+1) &= q_i(t) + \eta \cdot (y_k - o_k) \frac{\pi_i}{\sum_{j=1}^m \pi_j} y \end{aligned}$$

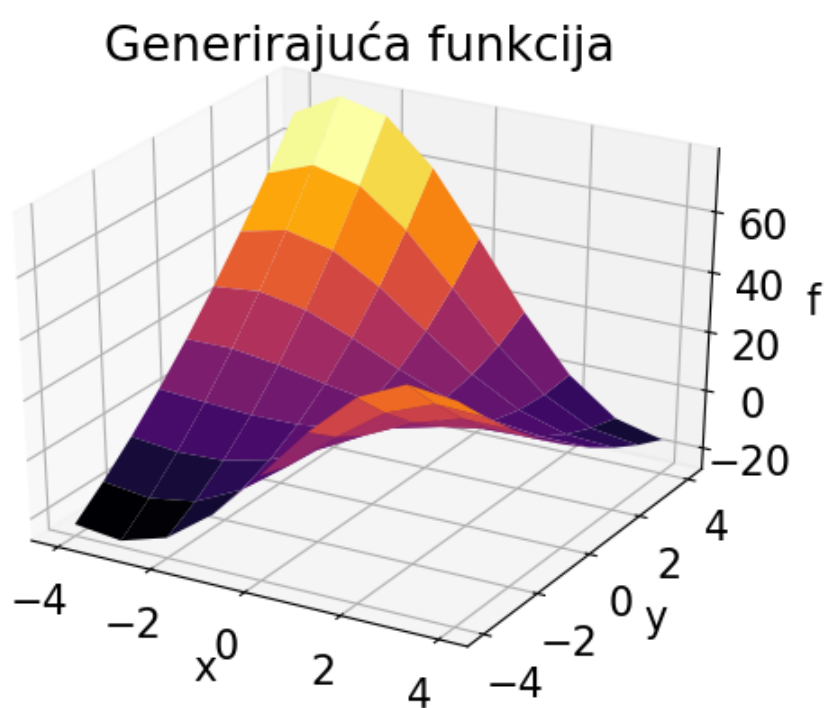
7.

$$\begin{aligned} \frac{\partial E_k}{\partial r_i} &= \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial r_i} \\ \frac{\partial E_k}{\partial o_k} &= -(y_k - o_k) \\ \frac{\partial o_k}{\partial r_i} &= \frac{\partial}{\partial r_i} \left( \frac{\sum_{j=1}^m \pi_j z_j}{\sum_{j=1}^m \pi_j} \right) = \frac{\partial}{\partial r_i} \left( \frac{\sum_{j=1}^m \pi_j (p_j x + q_j y + r_j)}{\sum_{j=1}^m \pi_j} \right) = \frac{\pi_i}{\sum_{j=1}^m \pi_j} \\ \frac{\partial E_k}{\partial r_i} &= -(y_k - o_k) \frac{\pi_i}{\sum_{j=1}^m \pi_j} \\ \implies r_i(t+1) &= r_i(t) + \eta \cdot (y_k - o_k) \frac{\pi_i}{\sum_{j=1}^m \pi_j} \end{aligned}$$

**Napomena:** Ažuriranje parametara je izvedeno za *online* tip učenja, dok bi se za pravi gradijent morala uzimati suma pogrešaka svih uzoraka za učenje tijekom pronalaska parcijalnih derivacija, tj.

$$E = \sum_{k=1}^N E_k.$$

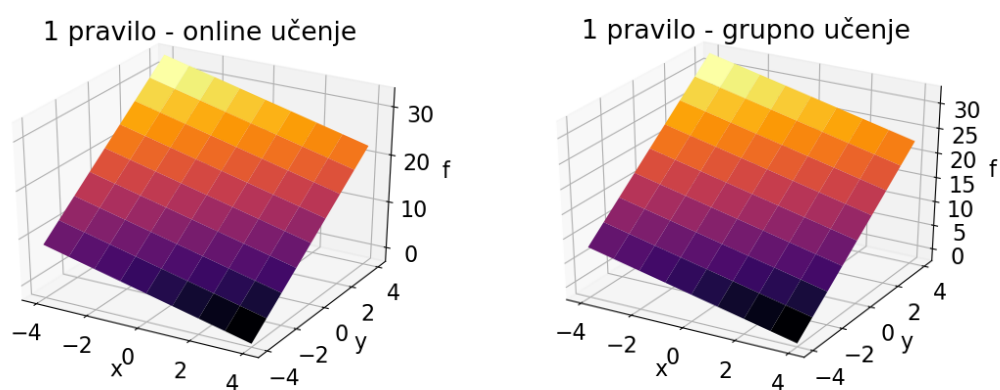
## 2. Zadatak 3



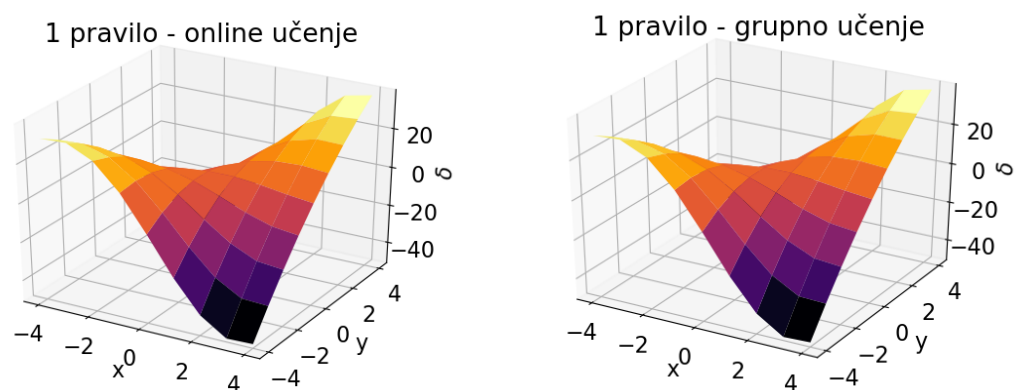
Slika 2.1: Graf generirajuće funkcije

## 3. Zadatak 4

### 3.1. Jedno pravilo



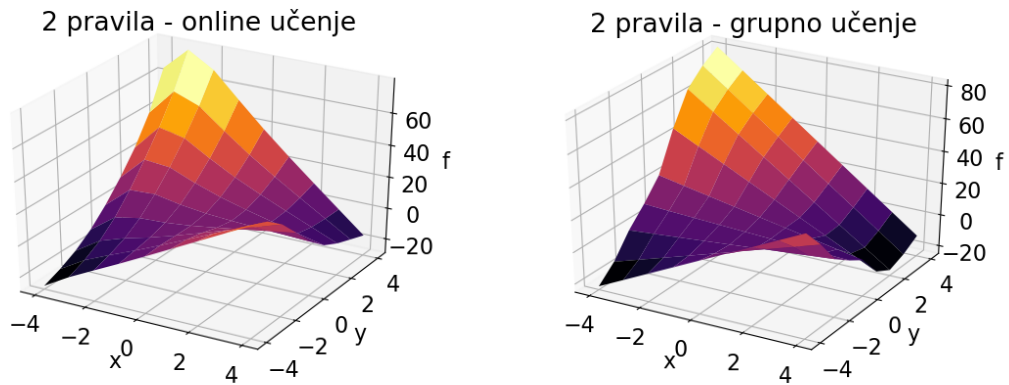
Slika 3.1: Grafovi aproksimirane generirajuće funkcije



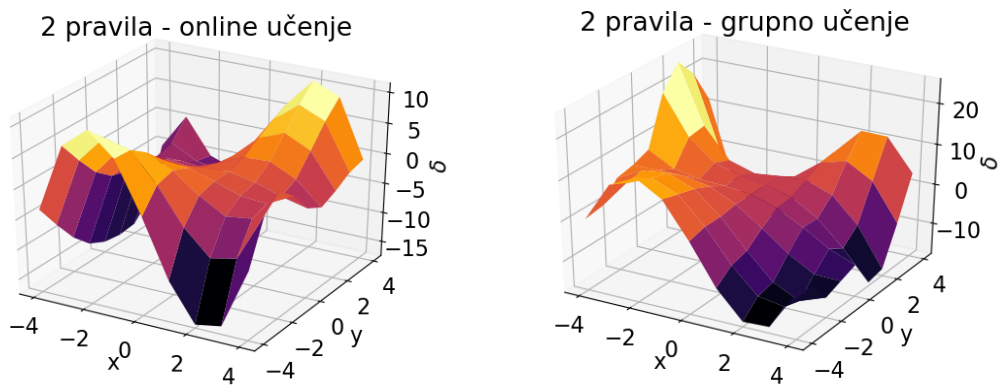
Slika 3.2: Grafovi pogrešaka



## 3.2. Dva pravila

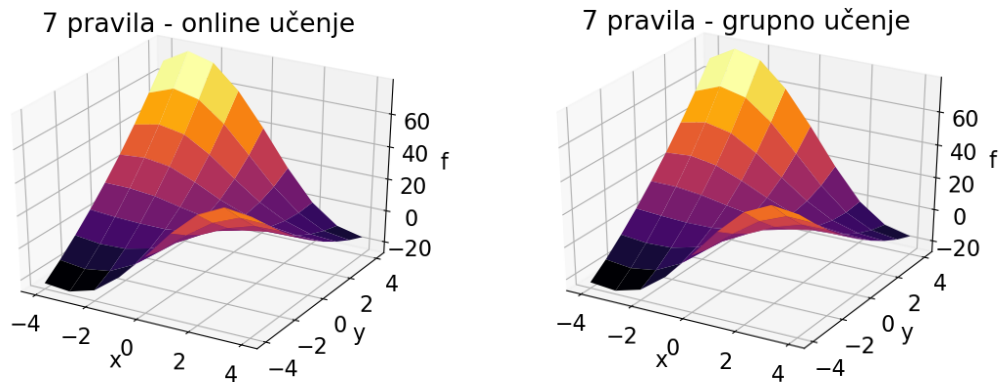


**Slika 3.3:** Grafovi aproksimirane generirajuće funkcije

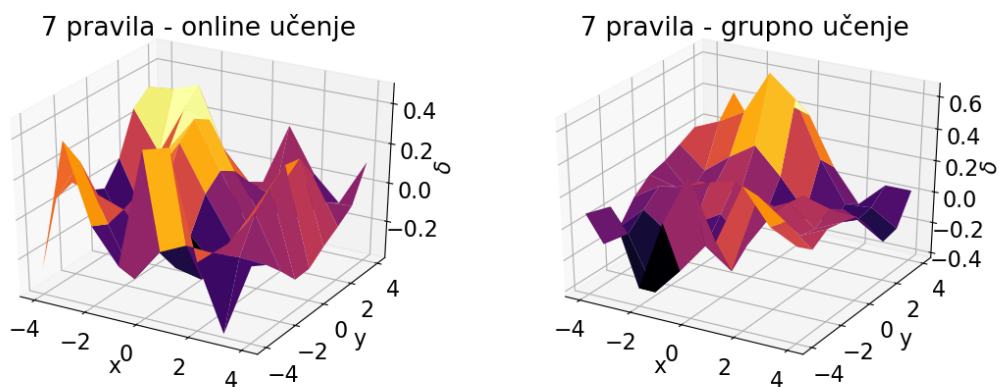


**Slika 3.4:** Grafovi pogrešaka

### 3.3. Prikladan broj pravila - sedam

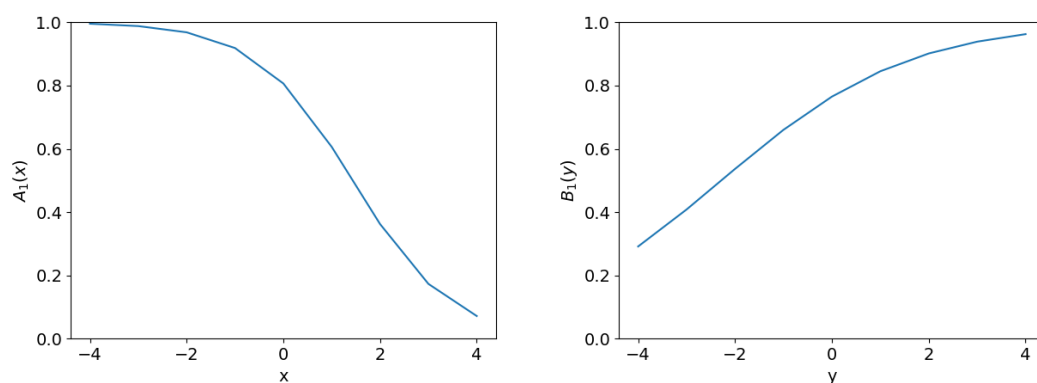


Slika 3.5: Grafovi aproksimirane generirajuće funkcije

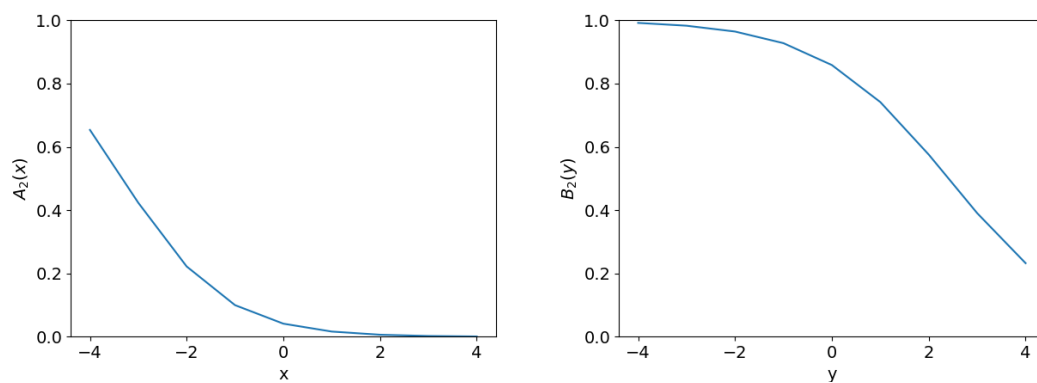


Slika 3.6: Grafovi pogrešaka

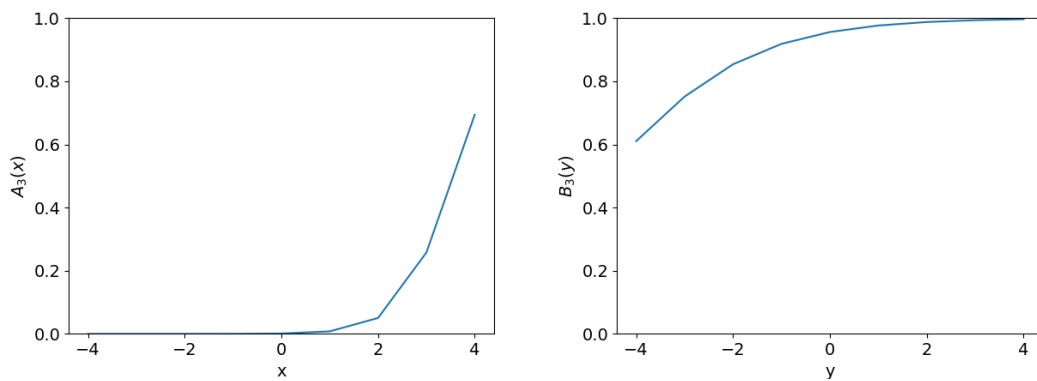
## 4. Zadatak 5



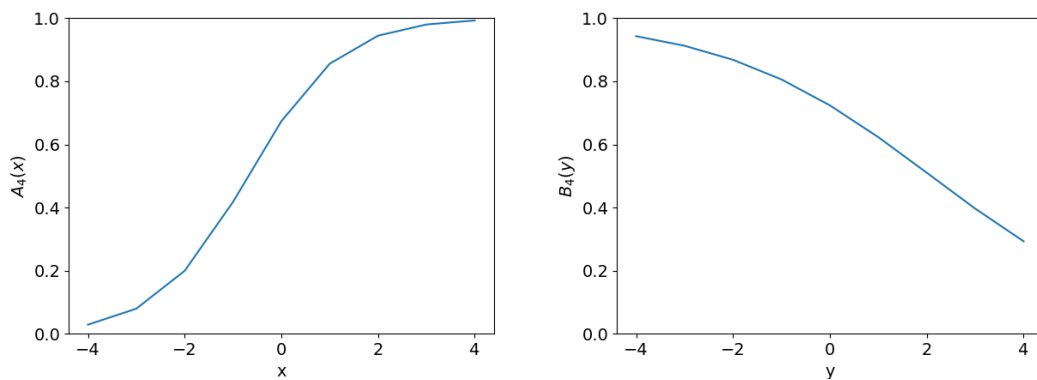
Slika 4.1: Funkcije pripadnosti za prvo pravilo



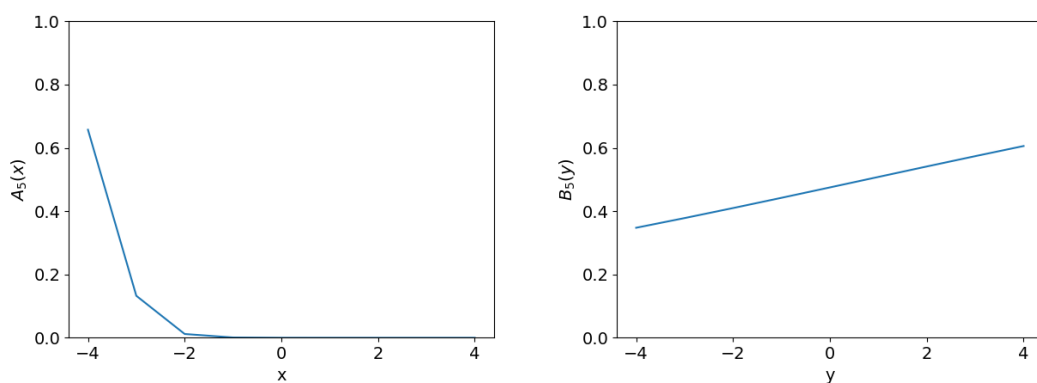
Slika 4.2: Funkcije pripadnosti za drugo pravilo



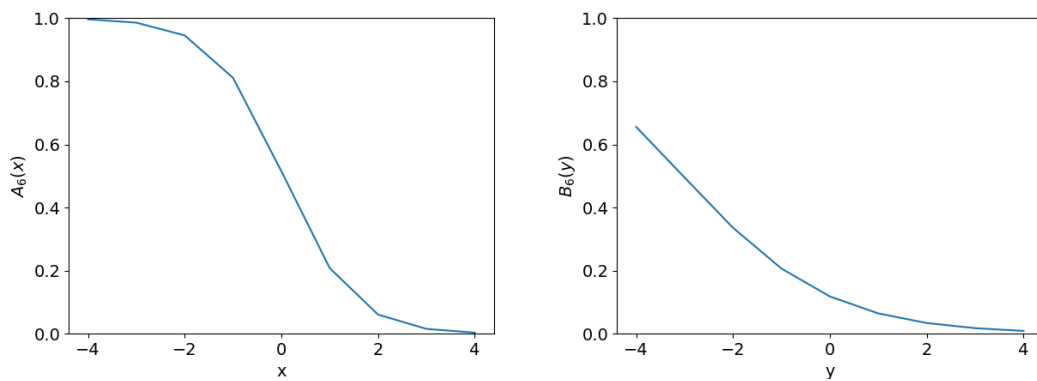
**Slika 4.3:** Funkcije pripadnosti za treće pravilo



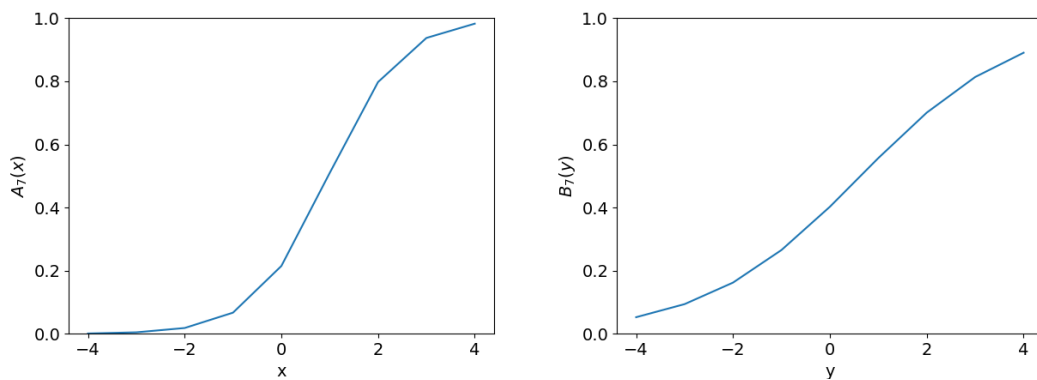
**Slika 4.4:** Funkcije pripadnosti za četvrto pravilo



**Slika 4.5:** Funkcije pripadnosti za peto pravilo

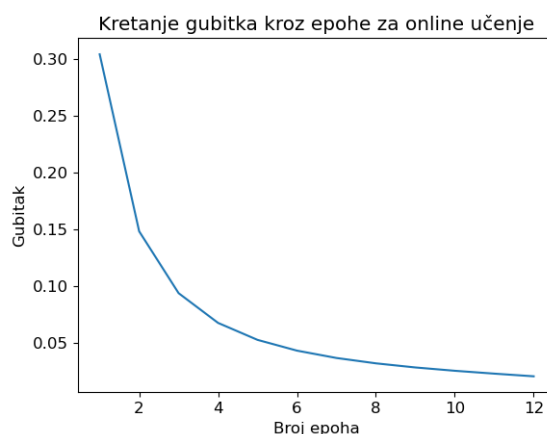


**Slika 4.6:** Funkcije pripadnosti za šesto pravilo

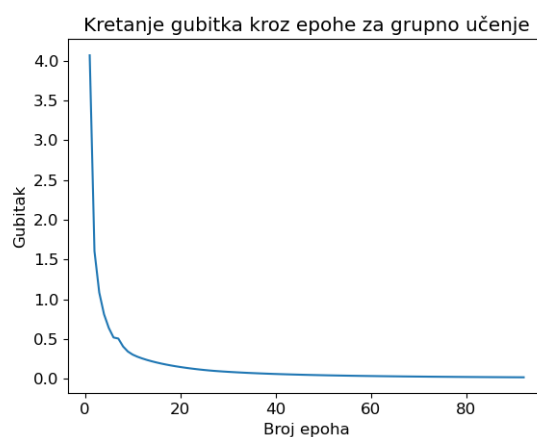


**Slika 4.7:** Funkcije pripadnosti za sedmo pravilo

## 5. Zadatak 7



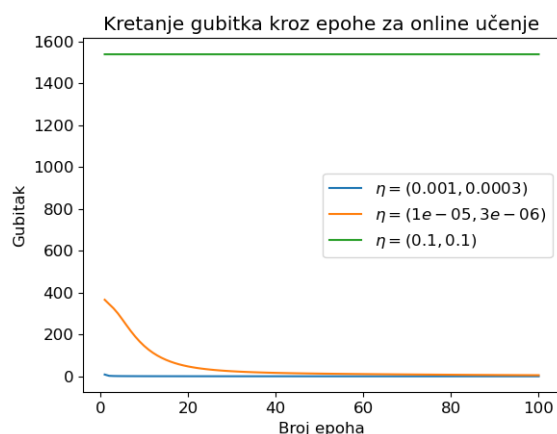
Slika 5.1



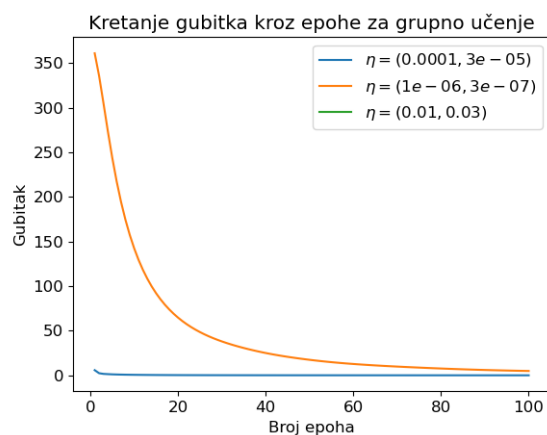
Slika 5.2

Možemo vidjeti da *online* učenje puno brže konvergira nego li grupno, no valja uzeti u obzir i stope učenja. U ovom eksperimentu za *online* učenje su korištene stope učenja  $10^{-3}$  i  $3 \cdot 10^{-4}$ , dok za grupno  $10^{-4}$  i  $3 \cdot 10^{-5}$ . Dakle, razlika u redu veličina je 10.

## 6. Zadatak 8



Slika 6.1



Slika 6.2

Što je stopa učenja manja, to algoritam sporije konvergira, ali konvergira, dok što je stopa učenja veća, algoritam divergira. Također, optimalni iznosi stopa učenje za *online* i grupno učenje razlikuju se za 10 puta.

## 7. Literatura

M. Čupić, B. Dalbelo Bašić, i M. Golub. *Neizrazito, evolucijsko i neuroračunarstvo*, kolovoz 2013. URL <http://java.zemris.fer.hr/nastava/nenr/knjiga-0.1.2013-08-12.pdf>. [Pristupljeno 15-Prosinac-2020.].