Q5 (b) J: closest representation of I

Both J and I can be written as linear combination of mean vector and first 4 eigen vectors.

For any matrix A, Frobenius norm

$$\|A_{frob}\| = \sqrt{\sum_i \sum_j A_{ij}^2}$$

On reshaping J into column vector, we obtain vector 2 norm of row vector because change in basis does not change the norm of the vector and so we change the eigenbasis of the covariance matrix.

The basis changed to:-

$$\bar{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{19200 \times 1} \quad \bar{v}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}_{19200 \times 1} \quad \cdots \quad v_{19200} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}_{19200 \times 1}$$

Now $J = a_1\bar{u} + a_2\bar{v}_1 + a_3\bar{v}_2 + a_4\bar{v}_3 + a_5\bar{v}_4$

As $\bar{u}$ is in the same domain,

$u = \sum u_i \vec{v_i}$ $\quad u_i =$ a real number

& $u_i = \bar{u}.\bar{v}_i$

$J = \sum_i J_i \vec{v_i}$

Frobenius norm on matrix of difference b/w I & J is used.

$$\Delta = I - J$$

$$\|\Delta_{Frob}\| = (j_1 - u_1 a_1 - a_2)^2 + (j_2 - u_2 a_1 - a_3)^2 + (j_3 - u_3 a_1 - a_4)^2 \ldots$$

$$+ (j_4 - u_4 a_1 - a_5)^2$$

$$+ \sum_{i=5}^{19200} (j_1 - u_i a_1)^2.$$

To make $\|\Delta_{Frob}\|_{min}$, first 4 terms can be made 0 irrespective of choice of a_1

i.e

$$a_2 = j_1 - u_1 a_1$$
$$a_3 = j_2 - u_2 a_1$$
$$\vdots$$
$$a_5 = j_4 - u_4 a_1$$

Differentiate remaining expression:-

$$\frac{d}{da_1} \sum_{i=5}^{19200} (j_i - u_i a_i)^2 = 0.$$

$$\sum 2 (j_i - u_i a_i)(-u_i) = 0$$

$$a_1 = \frac{\bar{J}.\bar{u} - \sum_{i=1}^{4} j_i u_i}{\bar{u}.\bar{u} - \sum_{i=1}^{4} u_i^2} = \frac{\sum_{i=5}^{19200} u_i j_i}{\sum_{i=1}^{5} u_i^2}$$

Hence we get closed representation of I i.e

J will be

$$J = a_1 \vec{u} + (j_1 - u_1 a_1)\vec{v_1} + (j_2 - u_2 a_1)\vec{v_2} + (j_3 - u_3 a_1)\vec{v_3} + (j_4 - u_4 a_1)\vec{v_4}$$

$\cancel{a = J}$ a as derived previously

(c) To get random images from MVG to generate new images of fruit, we follow the steps:-

MVG

$\cancel{X = Aw + u}$ $X = u + Aw$ where w is a vector consisting of independent and identically distributed variables from univariate Gaussian.

$AA^T = C$ (Covariance matrix)

u = mean

Now we can get U = matrix whose columns are corresponding eigenvectors for respective eigenvalues

As used in 2nd question

$A = C$^0.5

Now w = randn(19200, 1);

get X.

By reshaping X, we can obtain reqd image we do this process 3 times to obtain 3 images that are distinct from given images & are representative of the data set

But since $C = 19200 \times 19200$

code is taking a lot of time

so I used SVD.

$$U S U^T = C.$$

U ↓ matrix with columns as corresponding e.vectors for respective eigen values of C

$[U, D] = \mathrm{eig}(C, 10);$

U: orthogonal. (2 columns of U; 2 eigen vectors are mutually $\perp^r$)

$D_1 = 4 \times 4$ diag matrix such that

$D_1 = S(1:4, 1:4)$

$S_1 = D_1 \hat{} 0.5 \qquad S_1 * S_1 = D$

$U_1 = U(:, 1:4)$ i.e matrix of first 4 columns of U.

Hence we observe that

$$A = U_1 S_1 U_1^T$$

$$AA^T = U_1 S_1 \underbrace{U_1^T U_1}_{I} S_1^T U_1^T$$

$$= U_1 S_1 S_1^T U_1^T = U_1 D_1 U_1^T$$

Hence we get A, U and we proceed similarly as in previous page