LELEC2880
Modem design Project:
Coded OFDM transmission on a frequency selective channel

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1 Introduction

OFDM (Orthogonal Frequency Division Multiplexing) is the modulation of choice for most nowadays systems. It makes equalization very easy with the use of the cyclic prefix, and converts the transmission through a wideband frequency selective channel into multiple parallel and independent frequency flat subchannels. This offers a lot of flexibility for per-subcarrier processing, allowing for instance bit and power allocation, coding across the frequencies to obtain diversity, or simplified MIMO schemes.

One of the main drawbacks associated with OFDM is the issue of synchronization. It is quite sensitive to errors in the carrier frequency (also called CFO for carrier frequency offset), and requires frame (or block) synchronization to ensure that the FFT operation is applied at the right instants. Another important drawback of OFDM is the increased PAPR (peak to average power ratio) and its impact on the non-linear behaviour of the power amplifiers.

The objective of this project is to study several parts of an OFDM system. The study is based on mathematical derivations, computer calculations and simulations (using Matlab for instance). It focuses on the following points:

- Implementation of an OFDM chain on a frequency selective channel
- Adaptive modulation with bit and power allocation
- Channel estimation
- Coding across the frequencies and optimal decoding.

SNR → p°13

$\sigma_E^2 = \sigma_R$

2 Context: OFDM transmission

For all the steps of this project, we consider a common OFDM transmission with the following characteristics. The number of subcarriers is equal to $N = 128$. The subcarriers are assumed to be modulated around a carrier frequency of 2 GHz, and with subcarrier spacing 15 kHz. The cyclic prefix is fixed at $L = 16$ samples. The assumptions on the channel will be different for each part but in all cases, the cyclic prefix is assumed to be long enough (and the frame synchronization sufficiently accurate) so that the transmission is performed without inter-carrier and inter-symbol interference.

#sub → N ; f_c ; CP

2.1 Step 1: Basic OFDM chain

As an initial step in the project, you are required to simulate a basic OFDM chain on an ideal AWGN channel, using computer simulations. The simulations should generate random input symbols using 4-QAM constellation, implement all the *digital* operations of the OFDM communication chain, and compute the bit error rate. In this first step, the channel is a simple AWGN channel. Pay particular attention to the computation of the E_s/N_0 ratio. The simulation should be able to be run for different values of this parameter and should thus be able to recover the standard BER vs. E_s/N_0 curve.

• Gray Mapping
• ⚠ normalisation FFT
S10
Lien SNR, $\frac{E_s}{N_b}$
calcule E_s/N_0 ↓ pareil ∀ canaux à cette étape

goal: BER-SNR

3 Transceiver optimization

3.1 Step 2: Resource allocation

The second step aims at implementing the resource allocation procedure that can take place in an adaptive OFDM transmission. When channel knowledge is available at the transmitter, the powers p_k and the number of bits b_k (adaptive modulation) associated with each subcarrier $k = 0, \ldots, N-1$ can be optimized. We consider QAM modulation and a target symbol error rate of 10^{-5} on each subcarrier. For this part, one particular channel realization (normalized to an energy of 1) is assumed (it is provided on the Moodle), but several levels of noise will be considered. The channel can be assumed perfectly known both at the transmitter and receiver.

- Provide a model of the parallel channels obtained with OFDM: establish the relationship between the OFDM transmission over the frequency selective multipath channel on one hand, and the AWGN parallel subchannels with different SNRs as used in the lecture about "Adaptive modulation" on the other hand.
- Establish the general procedure for maximizing the available bit rate by optimizing the allocated powers p_k and the number of bits b_k for

S10

a given channel impulse response (and hence frequency response) and based on the constraints described above. In order to simplify the procedure, it is assumed that b_k is real and is not restricted to an integer.

- Implement this optimization procedure for the particular realization of the channel impulse response provided to you for 3 different values $E_s/N_0 = 0, 10$ and 20 dB, and evaluate the gain (in terms of bit rate) with respect to uniform power allocation and/or uniform bit allocation.

3.1.1 Bonus: Power allocation only

In some situations, it may be too costly to implement adaptive modulation. In that case, the power allocation can still be optimized in order to improve the performance of the transmission. Assume that a fixed 4-QAM modulation is used on all subcarriers. The bit rate is now fixed, so the optimization has to be based on another criterion. Several criterions can be considered. For this part, consider the minimization of the sum MSE (mean square error) over the subcarriers at the ouptut of the OFDM receiver, across all subcarriers.

- Based on the parallel channel model obtained above, provide the model of the mean square error at the output of each subcarrier of the OFDM receiver as a function of the powers p_k.
- Establish the general procedure for minimizing the sum mean square error (sum MSE) by optimizing the allocated powers p_k. Compare to the previous procedure and discuss the differences.

3.2 Step 3: Channel estimation

The third and fourth steps build on the basic OFDM chain designed earlier and extend it to include two particular operations of a communication chain: the channel estimation, and the decoding of the error correcting code.
For channel estimation, it is assumed that a training sequence (also called preamble) is sent by the transmitter. The preamble considered here is made of two identical OFDM symbols in which the symbol I_k sent on subcarrier k ($k - 0, \ldots, N-1$) is given by

$$I_k = (-1)^k. \tag{1}$$

Hence the transmitted power is equal on all subcarriers.

In this part, the channel is assumed to be an 8-tap channel with 8 independent Rayleigh taps with uniform power delay profile. In all your simulations, and for a given E_s/N_0 the results should be averaged over a sufficient number of realizations of the randomly generated channel.

S11

- Choose and derive a channel estimation method for this training sequence.
- Implement the channel estimation in your OFDM chain by simulating the transmission of the training sequence and performing the computation of the channel estimation. Evaluate and plot the performance of your estimator (in terms of mean square error) as a function of E_s/N_0 ranging from 0 dB to 20 dB.

3.3 Step 4: Optimal Viterbi decoding

If the channel is not known at the transmitter, convolutional coding is usually used across the frequencies in order to counteract the possibility of deep fades on some of the subcarriers. For simplicity we assume a code of rate 1/2 working on a frame of $L_f = 128$ bits. The convolutional code is given by $G(D) = [1 + D \quad 1 + D + D^2]$. The 256 coded bits are multiplexed on the 128 subcarriers modulated in 4-QAM, and forming one OFDM symbol. The receiver is either assuming perfect channel knowledge or using the channel estimation obtained in step 3. We consider Viterbi decoding, but the objective is to revisit the algorithm by taking into account the channel knowledge and the specific gain in each subcarrier.

S12

- Establish a model for the transmission of the 256 coded bits, taking into account the channel frequency selectivity (remember that ideal OFDM transmission can be seen as independent parallel channels).
- Assuming that the channel is perfectly known at the receiver (decoder) side, derive the expression of the maximum likelihood sequence decoder.
- Adapt the Viterbi algorithm accordingly.
- Implement the Viterbi algorithm and show the performance improvement that can be obtained by correctly taking into account the knowledge of the channel (at the receiver). Provide a BER vs. E_s/N_0 curve for the two versions of the algorithm by averaging over many channel realizations. Compare the performance obtained with perfect channel knowledge and with the estimation obtained from your estimator implemented in step 3.

4 Organization and Schedule

The project is designed for groups of 2 people. Students who cannot find a classmate to form a group may contact the assistant in order to find one, or can be allowed to work on the projet alone with some reduced requirements. Groups of 3 people are not allowed.

You will be asked to report periodically just to show your progress. No written report is required, but you are asked to come to the normal schedule of the exercise session and show your simulation/computation results on your computer. The schedule of milestones to be presented is summarized in the table below.

Date	Milestone
Tue Apr 20st	Step 1 (basic OFDM chain) & Step 2 (Resource allocation)
Tue Apr 27th	Step 2 (Resource allocation & Step 3 (Channel estimation)
Tue May 6th	Step 4 (Viterbi decoding)
S13 ← May 13th - 16th	Presentation of the project + written report

The final presentations of the project will take place during the final week of the semester. It should be made of approximately 15-20 min presentation (with slides) of your main results, followed by 10-15 min of discussion and questions. The questions may naturally cover all aspects of the course related to the project. The schedule of the different groups will be arranged the week before. The final written report should be provided at the end of the semester (last Friday of the semester). The grade of the project counts for around one third of the overall grade of the course.

33%

STEP 1

Single C.

SNR

$\sigma_n^2 \triangleq 2N_0$ (Base band)

$$x \longrightarrow \boxed{\alpha} \longrightarrow \overset{n \downarrow}{\oplus} \longrightarrow \boxed{1/\alpha} \longrightarrow z = x + \frac{n}{\alpha} \;\; (v)$$

$$\boxed{SNR \triangleq \frac{E[|x|^2]}{E[|v|^2]}} = \frac{\sigma_x^2 |\alpha|^2}{2N_0}$$

$$\sigma_v^2 \triangleq E\left[\frac{n}{\alpha}\left(\frac{n}{\alpha}\right)^*\right] = \frac{2N_0}{|\alpha|^2}$$

On doit pouvoir comparer SNR ∀ chaînes

⇒ On calcul $\frac{E_s}{N_0}$ — reçu ≡ Entrée récepteur

$$E_s \triangleq \frac{E[|\alpha x|^2]}{2} = \frac{\sigma_x^2 |\alpha|^2}{2}$$

$\int \cos^2 = 1/2$

⇒ ici $SNR = E_s/N_0$

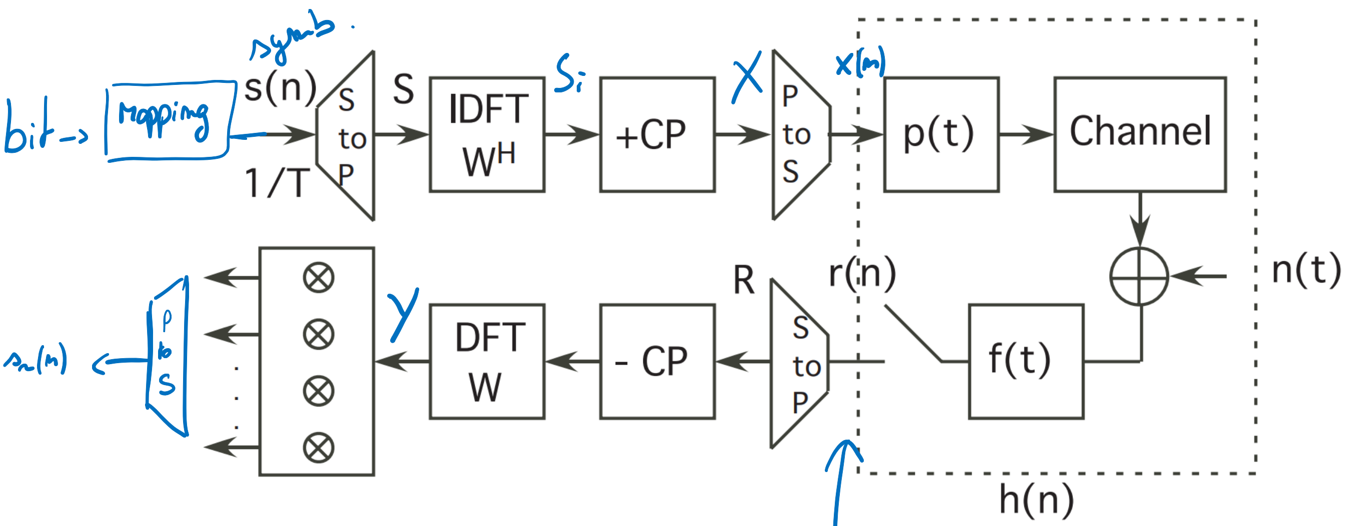
Pour faire BER(SNR) ⇝ $\frac{E_s}{N_0} \rightarrow$ SNR ⇝ BER

⚠ const. normalisée

Pour l'instant, on a m̂ SNR sur chaque channel.

⟶ calcul sur un canal

$X = [S_i, C_p]$



Pour E_s/N_0

$$E_s = \frac{E[s[m]\, s^*[m]]}{2}$$

$r(m) = x(m) + n(m)$ car AWGN

$\rightarrow E_s = \sigma_x^2$

$$= \frac{1}{2} E[x[n]\, x^*[n]]$$

$$= \frac{1}{2} E\left[\left(\sum_{m=0}^{N-1} \frac{s[m]\, e^{j\frac{2\pi m}{N}(n-L_c)}}{\sqrt{N}}\right)\left(\sum_{m'=0}^{N-1} \frac{s^*[m']\, e^{-2\pi j \frac{m'(n-L_c)}{N}}}{\sqrt{N}}\right)\right]$$

$$= \frac{1}{2N} \sum_{m} \sum_{m'} \overbrace{E[s[m]\, s^*[m']]}^{\sigma^2 \delta(m-m')} e^{j 2\pi \frac{(n-L_c)}{N}(m-m')}$$

$\frac{\sigma_{OFDM}^2}{N}$? ⟵ $\sigma^2 \delta(m-m')$

bloc de N_t symboles = # porteuses $- L_c$

$\sigma_N^2 = 2N_0$

$$R = H_c X + N$$

$$= W^H \Lambda \underbrace{W W^H}_{I} S + N$$

$$= W^H \Lambda S + N$$

$$W \begin{pmatrix} h[0] \\ \vdots \\ h[N-1] \end{pmatrix}$$

λ_0

$Y = WR$

$= \Lambda S + \underbrace{WN}_{\nu} \rightarrow \sigma_\nu^2 \stackrel{?}{=} E[WNN^H W^H] = I_N \frac{\sigma_n^2}{2N_0}$

$$= \begin{pmatrix} \lambda_0 & & 0 \\ & \ddots & \\ 0 & & \lambda_{N-1} \end{pmatrix} \begin{pmatrix} S_0 \\ \vdots \\ S_{N-1} \end{pmatrix} + \begin{pmatrix} \nu_0 \\ \vdots \\ \nu_{N-1} \end{pmatrix}$$

Chaque sous-porteuse est un narrowband 'fréq. dispersive' ? = channel

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Si AWGN $h[n] = \delta[n]$

$$W \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}_{\Lambda_{\text{centré}}} \Leftrightarrow \forall \lambda_i = 1$$

$\rightarrow y_i = s_i + \nu_i$

p° 13

$$\boxed{SNR_i = \frac{\sigma_{s_i}}{2N_0/|\lambda_i|} \underset{\text{AWGN}}{=} \frac{\sigma_{s_i}}{2N_0}}$$

$= \frac{1}{2N} \sum_m \sum_{m'} \sigma^2 \delta(m-m') \frac{e^{j2\pi\frac{(m-L_c)}{N}(m-m')}}{1}$

$= \sum_m \frac{\sigma^2}{2N}$

$= \frac{\sigma^2}{2} \stackrel{?}{=} \sigma_s \frac{(L_c+N)}{2N}$

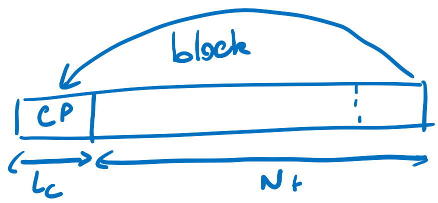
$$\Rightarrow \boxed{\frac{E_s}{N_0} = \frac{\sigma_x^2}{N_0} = \frac{\sigma_s^2}{2N_0} \frac{L_c+N}{N}}$$

Lien $|e|$ σ_{OFDM}^2 et σ_s^2 ($E_{\text{symbol QAM}}$)

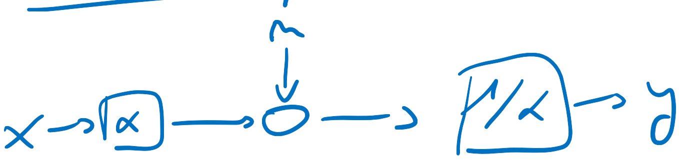
On a N_t symbole / x

$\rightarrow \underbrace{\frac{\sigma_{OFDM}^2}{N}}_{\text{E à mettre dans chq QAM}} = \frac{(L_c+N_t)}{N} \underbrace{\sigma_s^2}_{\text{E qui va réellement dedans}}$ Si même P sur chaque subcarrier

$\rightarrow \sigma_{OFDM}^2 = \sum_{i=0}^{L_c} 2\sigma_{si}^2 + \sum_{i=L_c+1}^{N_t} \sigma_{si}^2$ (Slide 39)



BER théorique ?



$P_{e,s} \stackrel{?}{=} \text{erfc}\left(\sqrt{\frac{SNR}{2}}\right)$

lien avec BER ?

Qst°

1 → SNR; su base de // AWGN ⊗α

2 → $\frac{1}{\sqrt{N}}$ ds E_b/N_0 ?

3 → BER Théorique / Pe

Slide 2795 :

on envoie N_t symb en $\neq$ tps

$\forall N_t$ paquet

$$\circ\ x(n) = \mathbb{R}\left[\sum_{i=-\infty}^{+\infty} \sum_{p=0}^{N_t-1} s_p(i)\, \underbrace{u_p(nT - iN_tT)}_{\frac{1}{\sqrt{N_t}} \sum_{m=0}^{N_t-1} e^{2\pi j m p / N_t}\, \underbrace{u(nT - mT)}}\, e^{j\omega_c t}\right]$$

RF bande

$= \mu$ si $n = m$, 0 ailleurs

Si synchro

$$= \mathbb{R}\left[\sum_i \sum_p s_p(i) \frac{1}{\sqrt{N_t}} e \ldots\right]$$

$$\circ\ r(n) = x(n) + n(n)$$

Livre Telecom

$$\text{QAM} \leftarrow x(n) = \frac{\sum_{m=0}^{N-1} s[m]\, e^{j\frac{2\pi m}{N}(n - L_c)}}{\sqrt{N}} \qquad n = 0, \ldots, N + L_c - 1$$

$\sigma_s^2 = E$ symb QAM

$N + L_c$

$x(n) = x(n+N)$ si $n \in [0, L_c - 1]$

$$r(n) = \sum_{\ell=0}^{L_h-1} h(\ell)\, x(n - \ell) + n(n)$$

Soit $\bar{r}(n) = r(n + L_c) \rightarrow$ on supprime CP

$$= \sum_{\ell=0}^{L_h-1} h(\ell)\, x(n + L_c - \ell) \qquad n \in [0, N-1]$$

$$= \frac{1}{\sqrt{N}} \sum_{0}^{L_h-1} h(\ell) \sum_{m=0}^{N-1} s(m)\, e^{j2\pi \frac{m(n + \cancel{L_c} - \cancel{L_c} - l)}{N}}$$

$$= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \left(\sum_{\ell=0}^{L_h-1} h(\ell)\, e^{-2\pi j \frac{m l}{N}}\right) s(m)\, e^{2\pi j \frac{m n}{N}}$$

$$\boxed{\frac{L_c + N}{N}}$$

On prend $\underbrace{\text{DFT}}_{\text{ac } \frac{1}{\sqrt{N}}}$ de $\bar{r}(n)$

$\hookrightarrow y[k] = W[k]\, s[k] + V[k]$

ac $W[k] = \frac{1}{\sqrt{N}} \sum_{l=0}^{L_h - 1} h[l]\, e^{-2\pi j \frac{kl}{N}}$

revenir sur équation matricielle

3.1 Step 2: Resource allocation

The second step aims at implementing the resource allocation procedure that can take place in an adaptive OFDM transmission. When channel knowledge is available at the transmitter, the powers p_k and the number of bits b_k (adaptive modulation) associated with each subcarrier $k = 0, \ldots, N-1$ can be optimized. We consider QAM modulation and a target symbol error rate of 10^{-5} on each subcarrier. For this part, one particular channel realization (normalized to an energy of 1) is assumed (it is provided on the Moodle), but several levels of noise will be considered. The channel can be assumed perfectly known both at the transmitter and receiver.

$y = \Delta s + \underbrace{WN}_{\nu}$

lin. equa

$\hat{y} = \Delta^{-1} y = s + \Delta^{-1}\nu = s + \underbrace{\Delta^{-1} W N}_{\mathcal{CN}}$

?

$SNR_i = \dfrac{\sigma_{si}^2}{\underbrace{\boxed{2N_0/|\lambda_i|^2}}_{\sigma_{\nu_i}^2}}$

- Provide a model of the parallel channels obtained with OFDM: establish the relationship between the OFDM transmission over the frequency selective multipath channel on one hand, and the AWGN parallel subchannels with different SNRs as used in the lecture about "Adaptive modulation" on the other hand.
- Establish the general procedure for maximizing the available bit rate by optimizing the allocated powers p_k and the number of bits b_k for (A)

Pour chaque

Channel i : $\dfrac{E_{si}}{N_0} = \dfrac{|\lambda_i| \sigma_{si}^2}{2N_0} \stackrel{?}{=} SNR_i$

$|\lambda_i| \searrow \Rightarrow SNR_i \searrow$

$\Rightarrow BER \nearrow$

a given channel impulse response (and hence frequency response) and based on the constraints described above. In order to simplify the procedure, it is assumed that b_k is real and is not restricted to an integer.

- Implement this optimization procedure for the particular realization of the channel impulse response provided to you for 3 different values $E_s/N_0 = 0, 10$ and 20 dB, and evaluate the gain (in terms of bit rate) with respect to uniform power allocation and/or uniform bit allocation.

(A) on veut max bit rate → b_k, p_k ?

avec $P_{e,symb}$ fixé

points $= 2^{2b}$

M-QAM

$\hookrightarrow P_{e,symb_k} \leq 2 \underbrace{\dfrac{\sqrt{M}-1}{\sqrt{M}}}_{\simeq 1} \operatorname{erfc}\left(\sqrt{\dfrac{3\, SNR_k}{2(M-1)}}\right)$

pire cas ($\simeq$)

HYPO

$\Leftrightarrow b_k \simeq \dfrac{1}{2} \log_2\left(1 + \underbrace{\boxed{\dfrac{3\, SNR_k}{2\left[\operatorname{erfc}^{-1}\left(\frac{P_{e,symb,k}}{2}\right)\right]^2}}}_{1/\Gamma \to \text{SNR gap}}\right)$

on veut

$\max_{p_k} \sum_k b_k$

De base tjs

$\sigma_s^2 = 1$

$$\max_{p_k} \frac{1}{2}\log_2\left(1+\frac{SNR_k}{\Gamma_k}\right)$$

$$\boxed{\max_{p_k} \frac{1}{2}\log_2\left(1+\frac{|\lambda_k|^2\,\overset{p_k}{\sigma_{si}^2}}{2N_0\Gamma_k}\right)}$$

$$p_k = \begin{cases} 0 & \text{si } \sigma_{w,k}^2 > \dfrac{1}{2\lambda\ln(2)} \\ \dfrac{1}{2\lambda\ln(2)} - \underbrace{\sigma_{w,k}^2}_{\frac{2N_0}{|\lambda_k|^2}} & \text{otherwise} \end{cases}$$

waterfilling $\leftarrow$

3.1.1 Bonus: Power allocation only

In some situations, it may be too costly to implement adaptive modulation. In that case, the power allocation can still be optimized in order to improve the performance of the transmission. Assume that a fixed 4-QAM modulation is used on all subcarriers. The bit rate is now fixed, so the optimization has to be based on another criterion. Several criterions can be considered. For this part, consider the minimization of the sum MSE (mean square error) over the subcarriers at the ouptut of the OFDM receiver, across all subcarriers.

- Based on the parallel channel model obtained above, provide the model of the mean square error at the output of each subcarrier of the OFDM receiver as a function of the powers p_k.
- Establish the general procedure for minimizing the sum mean square error (sum MSE) by optimizing the allocated powers p_k. Compare to the previous procedure and discuss the differences.

3.2 Step 3: Channel estimation

④ Lg training sig ? L_c+N

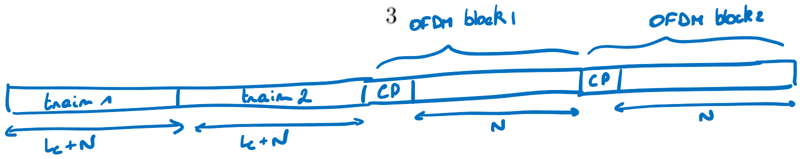
The third and fourth steps build on the basic OFDM chain designed earlier and extend it to include two particular operations of a communication chain: the channel estimation, and the decoding of the error correcting code.
For channel estimation, it is assumed that a training sequence (also called preamble) is sent by the transmitter. The preamble considered here is made of two identical OFDM symbols in which the symbol I_k sent on subcarrier k $(k-0,\dots,N-1)$ is given by

$$I_k = (-1)^k. \tag{1}$$

Hence the transmitted power is equal on all subcarriers.

In this part, the channel is assumed to be an 8-tap channel with 8 independent Rayleigh taps with uniform power delay profile. In all your simulations, and for a given E_s/N_0 the results should be averaged over a sufficient number of realizations of the randomly generated channel.

3



On envoie S_T = training sequence

$$\underline{Y}_T = \Lambda S_T + \underbrace{WN}_{\nu}$$

↳ $y_i = \lambda_i s_i + \nu_i$

$\Longleftrightarrow \lambda_i = \dfrac{y_i}{s_i}$ — symbole reçu porteuse i / symbole envoyé porteuse i

Zero Forcing

$= \lambda_i + \dfrac{\nu_i}{s_i}$

- Choose and derive a channel estimation method for this training sequence.
- Implement the channel estimation in your OFDM chain by simulating the transmission of the training sequence and performing the computation of the channel estimation. Evaluate and plot the performance of your estimator (in terms of mean square error) as a function of E_s/N_0 ranging from 0 dB to 20 dB.

$$\hat{\underline{\lambda}} = \underline{\lambda} + \underline{\underline{S}}^{-1}\underline{\underline{W}}\,\underline{N} = \underline{\underline{S}}^{-1}\underline{Y}$$

$$\underline{\underline{S}}^{-1} = \begin{pmatrix} 1/s_0 & & 0 \\ & \ddots & \\ 0 & & 1/s_N \end{pmatrix}$$

$\boxed{\text{Ensuite moyenne sur 2 essais}}$

Mean square Error of $\hat{\lambda}$

$$MSE \triangleq E\left[\|\hat{\lambda} - \lambda\|^2\right]$$

$$= E\left[\|\underline{\underline{S}}^{-1}\underline{\underline{W}}\,\underline{N}\|^2\right]$$

$$= E\left[\left\|\sum_{i=0}^{N} \frac{\nu_i}{s_i}\right\|^2\right]$$

$L_c = 18$ dans notre cas

⟹ on peut egualize h_i pour $i=[0,\dots,18]$

⟺ 19 coefs.

Sur $h(n)$ décroît

et $h[n] = W^H \lambda[k]$

sans $\sqrt{N}$

On considère $\underline{h}$ lqr 8 ms 8 éléments $\neq 0$

$$\underline{h} = \begin{pmatrix} h_0 \\ \vdots \\ h_7 \\ 0 \\ \vdots \\ 0 \end{pmatrix} N = \begin{pmatrix} \tilde{h} \\ 0 \end{pmatrix} \begin{matrix} 8 \\ N-8 \end{matrix}$$

avec $\underline{\lambda} = \sqrt{N}\, \underline{\underline{W}}\, \underline{h}$

$$\underline{Y}_T = \underbrace{\underline{\underline{\Delta}}}_{\begin{pmatrix} \lambda_0 & & 0 \\ & \ddots & \\ 0 & & \lambda_{N-1} \end{pmatrix}} \underline{S}_T + \underline{\underline{W}}\,\underline{N}$$

$$= \underbrace{\underline{\underline{\Sigma}}}_{\begin{pmatrix} S_0 & & 0 \\ & \ddots & \\ 0 & & S_{N-1} \end{pmatrix}} \cdot \underline{\lambda} + \underline{\underline{W}}\,\underline{N}$$

$$\underline{Y}_T = \underbrace{\sqrt{N}\,\underline{\underline{\Sigma}}_T\,\underline{\underline{W}}}_{M} \begin{pmatrix} \tilde{h} \\ 0 \end{pmatrix} + \underbrace{\underline{\underline{W}}\,\underline{N}}_{\underline{\nu}}$$

$$\underline{Y}_T = \underbrace{\begin{pmatrix} \tilde{M} & \square \end{pmatrix}}_{M} \begin{pmatrix} \tilde{h} \\ 0 \end{pmatrix} + \begin{pmatrix} \nu \end{pmatrix}$$

régr. linéaire

$$\Longrightarrow \boxed{\underset{N\times 1}{\underline{Y}_T} = \underset{N\times 8}{\underline{\underline{\tilde{M}}}}\ \underset{8\times 1}{\underline{\tilde{h}}} + \underset{N\times 1}{\underline{\nu}}}$$

$\boxed{\hat{h}\ ?}$

MLE pour gaussien

$\Updownarrow$

Min de dist. euclidienne

Given set of data $(\underline{S}_T, \underline{Y}_{T1}), (\underline{S}_T, \underline{Y}_{T2})$

$$\hat{h} = \min_{\tilde{h}} \sum_{i=1}^{2} \left(\underline{Y}_{T,i} - \underline{\underline{\tilde{M}}}_i \tilde{h}\right)^H \left(\underline{Y}_{T,i} - \underline{\underline{\tilde{M}}}_i \tilde{h}\right)$$

$$\frac{\partial}{\partial h}\left(\sum_i (Y_i - M_i h)^H (Y_i - M_i h)\right) = 0$$

$$\Leftrightarrow \sum_{i=1}^{2} -(Y - Mh)^H M = 0$$

$$\Leftrightarrow -(Y_1 - M_1 h)^H M_1 - (Y_2 - M_2 h)^H M_2 = 0$$

$$\Leftrightarrow -Y_1^H M_1 + h^H M_1^H M_1 - Y_2^H M_2 + h^H M_2^H M_2 = 0$$

$$\Leftrightarrow h^H (M_1^H M_1 + M_2^H M_2) = Y_1^H M_1 + Y_2^H M_2$$

$$\Leftrightarrow \hat{h} = \left((Y_1^H M_1 + Y_2^H M_2)(M_1^H M_1 + M_2^H M_2)^{-1}\right)^H$$

$$\boxed{\hat{h} = (M_1 M_1^H + M_2 M_2^H)^{-1} (Y_1 M_1^H + Y_2 M_2^H)}$$

avec

$M_i = \underline{\underline{\tilde{M}}}_i$

$Y_i = \underline{Y}_{T,i}$

3.3 Step 4: Optimal Viterbi decoding

If the channel is not known at the transmitter, convolutional coding is usually used across the frequencies in order to counteract the possibility of deep fades on some of the subcarriers. For simplicity we assume a code of rate 1/2 working on a frame of $L_f = 128$ bits. The convolutional code is given by $G(D) = [1 + D \quad 1 + D + D^2]$. The 256 coded bits are multiplexed on the 128 subcarriers modulated in 4-QAM, and forming one OFDM symbol. The receiver is either assuming perfect channel knowledge or using the channel estimation obtained in step 3. We consider Viterbi decoding, but the objective is to revisit the algorithm by taking into account the channel knowledge and the specific gain in each subcarrier.

- Establish a model for the transmission of the 256 coded bits, taking into account the channel frequency selectivity (remember that ideal OFDM transmission can be seen as independent parallel channels).
- Assuming that the channel is perfectly known at the receiver (decoder) side, derive the expression of the maximum likelihood sequence decoder.
- Adapt the Viterbi algorithm accordingly.
- Implement the Viterbi algorithm and show the performance improvement that can be obtained by correctly taking into account the knowledge of the channel (at the receiver). Provide a BER vs. E_s/N_0 curve for the two versions of the algorithm by averaging over many channel realizations. Compare the performance obtained with perfect channel knowledge and with the estimation obtained from your estimator implemented in step 3.