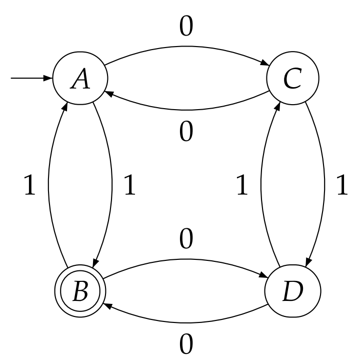
CS 310, Assignment 2

Answers

1. Let $\Sigma = \{0, 1\}$ and consider the following state-transition diagram:



   (a) Give three examples of strings that are accepted by the transition diagram and three examples of strings that are not accepted by the transition diagram.

   ANSWER: Strings that are accepted by the automaton include: 1, 111, 010, 01110, 00001. Strings that are not accepted include: 0, 00, 01, 011, 0111.

   (b) Write explicitly the transition function δ that defines the transitions of the diagram.

   ANSWER: $\delta(A, 0) = C$ $\delta(A, 1) = B$ $\delta(C, 0) = A$ $\delta(C, 1) = D$
   $\delta(B, 0) = D$ $\delta(B, 1) = A$ $\delta(D, 0) = B$ $\delta(D, 1) = C$

   (c) Is the transition diagram deterministic or nondeterministic? Explain briefly.

   ANSWER: The automaton is deterministic as there are no ε-transitions and each state has only one transition enabled for every input symbol.

   (d) What is the language recognized by the state transition diagram? Describe (in English) conditions that characterize exactly all the strings in the language.

ANSWER: The language contains exactly all the binary strings that have an even number of 0s and an odd number of 1s.

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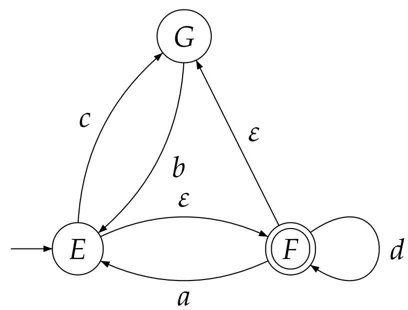
2. Let $\Sigma = \{a, b, c, d\}$ and consider the following nondeterministic state transition diagram with ε-transitions:

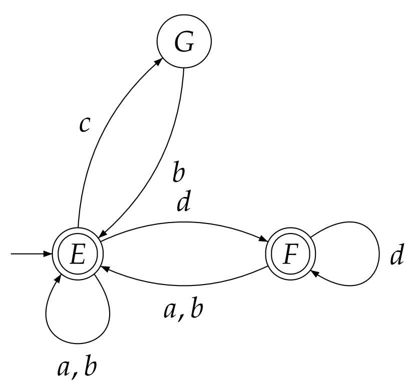


Using the systematic method described in class (and in the text), convert the transition diagram into an equivalent (non)deterministic transition diagram without ε-transitions. Do not modify or simplify the resulting diagram any further.

ANSWER: The following paths will need to be replaced by single transitions:

$$\begin{array}{lll}
E \xrightarrow{\varepsilon} F \xrightarrow{d} F & \text{becomes} & E \xrightarrow{d} F \\
E \xrightarrow{\varepsilon} F \xrightarrow{a} E & \text{becomes} & E \xrightarrow{a} E \\
F \xrightarrow{\varepsilon} G \xrightarrow{b} E & \text{becomes} & F \xrightarrow{b} E \\
E \xrightarrow{\varepsilon} F \xrightarrow{\varepsilon} G \xrightarrow{b} E & \text{becomes} & E \xrightarrow{b} E
\end{array}$$

The two ε-transitions $E \xrightarrow{\varepsilon} F$ and $F \xrightarrow{\varepsilon} G$ are then removed. State E becomes accepting since there is an ε-path from it to an accepting state. The following is the resulting state transition diagram:



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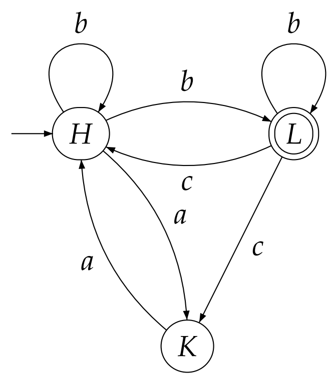
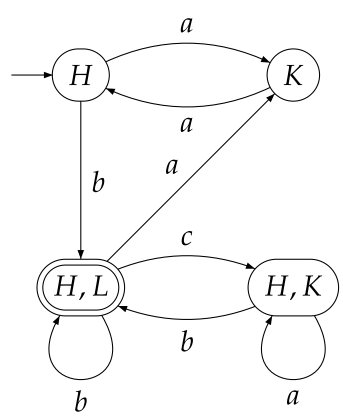
3. Let $\Sigma = \{a, b, c\}$. Using the systematic method described in class and textbook convert the following nondeterministic state transition diagram into a deterministic transition diagram:



Describe how the deterministic transition diagram is obtained from the nondeterministic one as follows: label the states of the deterministic diagram by sets of states of the nondeterministic diagram (as we did in class).

ANSWER:



For people needing yet another example here is a walk-through:

(a) We start from the initial state H (technically $\{H\}$). With an a we can only go to K; with a b we can reach H or L; we do not go anywhere with a c. The new states that need to be considered are $\{K\}$ and $\{H, L\}$.

(b) From $\{K\}$: with an a we reach H and we do not go anywhere with anything else. The state $\{H\}$ has already been considered; the only new state left is $\{H, L\}$.

(c) From $\{H, L\}$: with an a we reach $\{K\}$; with a b we reach $\{H, L\}$; and with a c we reach $\{H, K\}$. The latter is the only new state thus introduced.

(d) From $\{H, K\}$: with an a we reach $\{H, K\}$; with a b we reach $\{H, L\}$; and with a c we do not go anywhere. There are no new states needing consideration so we stop here.

The only accepting state in the original automaton is L and so the only accepting state in the deterministic automaton is $\{H, L\}$ since this is the only set that contains the accepting state L.
