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HW 4

Problem 1

(a)

We use the notation for treap analysis in the lecture notes.

We know that x_j is a proper ancestor of node x_i iff x_j is the minimal element in $X(i,j)$. Also x_j is a proper ancestor of x_k iff x_j is the minimal element in $X(j,k)$. Thus, x_j is a common ancestor of x_i and x_k iff x_j is the minimal element in $X(i,k)$. The probability is $\frac{1}{k-i+1}$.

(b)

Let x_j be the deepest common ancestor of x_i and x_k, $i \leq j \leq k$. Then the length of the unique path from x_i to x_k is

path length$= depth(x_i) + depth(x_k) - 2depth(x_j)$

We take the expectation value according to the formula from the lecture nodes, i.e. $E[depth(x_i)] = H_i + H_{n-i+1} - 2$, then

$$
\begin{aligned}
E[\text{path length between node i and k}] &= E[depth(x_i) + depth(x_k) - 2depth(x_j)] \\
&= E[depth(x_i)] + E[depth(x_k)] - 2E[depth(x_j)] \\
&= H_i + H_{n-i+1} - 2 + H_k + H_{n-k+1} - 2 \\
&+ \sum_{j=i}^{k} \frac{1}{i-k+1}(H_j + H_{n-j+1} - 2)
\end{aligned}
$$