Yongzuan Wu
wu68
cs573
HW 5
Problem 1
reference: http://en.wikipedia.org/wiki/Maximum$_{f}low_{p}roblem$
We reduce the problem to a bipartite directed graph matching.

Let n be the number of boxes. Let $G = (V \cup V', E)$ be a bipartite graph, where $V = \{v_1, v_2 ..., v_n\}$, $V' = \{v'_1, v'_2 ..., v'_n\}$, $E = \{(v_i, v'_j) \,|\, \text{box j can be placed inside box i}\}$. Let M be a matching of G. Every edge in M corresponds to nesting two boxes. Thus a vertex $v'_j \in V'$ that is not covered by M indicates that box j is not inside another box, i.e. it is a visible box. Thus the number of visible boxes is $n - |M|$. Therefore the smallest number of visible boxes is given by ($n-$the maximum matching of G). Thus the algorithm follows

boxNexsting(box[1..n])
 construct a bipartite graph G as described above
 compute the maximum matching M of G
 return $n - |M|$

Analysis: We could use the Ford-Fulkerson algorithm to compute the maximun matching. The algortihm runs in $O(VE)$. By construction, $|V| = O(n)$, $|E| = O(n^2)$, thus the total running time is $O(n^3)$.