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HW 5
Problem 5
(a)
Proof: Here's Sally's stratrgy

GameSally(G,r)
 run Paul's algorithm GamePaul(G,r), returning path P
 return any edge in P

Sally will always win since the edge she returns is in P, thus $P \cap S \neq \emptyset$

(b)
Here's Sally's stratrgy

GameSally(G,r)
 compute a min (s,t)-cut C in G
 return C

Sally will always win since C is a cut. Any path from s to t that Paul computes must include at least one edge in C. Thus $P \cap S \neq \emptyset$.

(c)
Proof:
Here's Paul's strategy

GamePaul(G,r)
 compute Sally's edge set S by GameSally(G,r)
 remove S from G to get G'
 return any path P in G'

The algorithm is correct iff after removing S, the remaining graph still has a path from s to t. Since $|S| \leq r < M$, and M is the size of the minimum (s,t)-cut, S can't be an (s,t)-cut. Therefore, after removing S, s and t are still connected. Thus such path does exist.

(d)
Here's Sally's stratrgy

GameSally(G,r)
compute a min (s,t)-cut C in G
if $(r \geq M)$
return C
else
uniformly randomly choose r edges in c
return that subset C'

Proof:

When $r \geq M$, Sally is gauranteed to win as proved in part (b).

When $r < M$, since C is a minimum (s,t)-cut in G, then any path Paul choose will have at least one edge in C. When we uniformly randomly choose r edges from C, the probability that we hit that edge is $\frac{r}{|C|} = \frac{r}{M}$. Thus the probalility that Sally wins is at least $\frac{r}{M}$ in this case.

Therefore the probability that Sally wins is $min(\frac{r}{M}, 1)$.

(e)

Here's Paul's strategy

GamePaul(G,r)
compute a min (s,t)-cut C in G
uniformly randomly choose an edge $e \in C$
remove $(C - \{e\})$ from G to get G'
compute all paths from s to t in G'
uniformly randomly choose one path and return

I don't know of a proof for the winning probability.