

MITx: 6.041x Introduction to Probability - The Science of Uncertainty

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Problem 4 Vertical: A three-sided die

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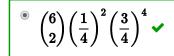
Problem 4: A three-sided die

2.0/4.0 points (graded)

The newest invention of the 6.041x staff is a three-sided die. On any roll of this die, the result is 1 with probability 1/2, 2 with probability 1/4, and 3 with probability 1/4.

Consider a sequence of six independent rolls of this die.

1. Find the probability that exactly two of the rolls results in a 3.





$$^{\circ} \quad \binom{6}{2} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^2$$

2. Given that exactly two of the six rolls resulted in a 1, find the probability that the first roll resulted in a 1. Note: Your answer should be a number. Do not enter '!' or combinations in your answer.



3. We are told that exactly three of the rolls resulted in a 1 and exactly three rolls resulted in a 2. Given this information, find the probability that the six rolls resulted in the sequence (1, 2, 1, 2, 1, 2). **Note:** Your answer should be a number. Do not enter '!' or combinations in your answer.



4. The conditional probability that exactly k rolls resulted in a 3, given that at least one roll resulted in a 3, is of the form:

$$rac{1}{1-(c_1/c_2)^{c_3}}inom{c_3}{k}igg(rac{1}{c_2}igg)^kigg(rac{c_1}{c_2}igg)^{c_3-k},\quad ext{for } k=1,2,\ldots,6.$$

Find the values of the constants c_1 , c_2 , and c_3 :





Answer:

1. Each roll is an independent trial with probability 1/4 of resulting in a 3 (a "success"). The probability of exactly 2 successes in 6 trials is given by the binomial probabilities with n=6, k=2, and p=1/4:

$$\binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4$$

- 2. The probability of obtaining a 1 on a single roll is 1/2, and the probability of obtaining a 2 or 3 on a single roll is also 1/2. For the purposes of solving this problem, we treat obtaining a 2 or a 3 as an equivalent result. We know that there are $\binom{6}{2}$ ways of rolling exactly two 1's. Of these $\binom{6}{2}$ ways, exactly $\binom{5}{1} = 5$ ways result in a 1 on the first roll, since we can place the other 1 in any of the five remaining rolls. The rest of the rolls must be either 2 or 3. Thus the probability that the first roll is a 1 given exactly two rolls resulted in a 1 is $\frac{5}{\binom{6}{2}} = \frac{1}{3}$.
- 3. We want to find

$$\mathbf{P}(121212 \mid ext{exactly three 1's and three 2's}) = \frac{\mathbf{P}(121212)}{\mathbf{P}(ext{exactly three 1's and three 2'}}$$

Any particular sequence of three 1's and three 2's will have the same probability: $\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)^3$. There are $\binom{6}{3}$ possible sequences with exactly three 1's and three 2's, of which exactly one sequence is 121212. Therefore,

$$\mathbf{P}(121212 \mid \text{exactly three 1's and three 2's}) = \frac{\left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right)^3}{\left(\frac{6}{3}\right) \left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right)^3} = \frac{1}{20}.$$

4. Let $m{A}$ be the event that at least one roll results in a 3. Then,

$$\mathbf{P}(A) = 1 - \mathbf{P}(\text{no rolls resulted in a 3}) = 1 - \left(\frac{3}{4}\right)^6$$
.

Let B be the event that there were exactly k rolls that resulted in a 3, where $k\in\{1,2,\ldots,6\}$. Note that $\mathbf{P}(B)=\binom{6}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{6-k}$.

Note also that $B \subset A$. Thus, the desired probability is:

$$egin{align} \mathbf{P}(B \mid A) &= rac{\mathbf{P}(A \cap B)}{\mathbf{P}(A)} \ &= rac{\mathbf{P}(B)}{\mathbf{P}(A)} \ &= rac{1}{1 - (3/4)^6} inom{6}{k} inom{1}{4}^k inom{3}{4}^{6-k} ext{ for } k = 1, 2, \dots, 6. \end{split}$$

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You have used 2 of 2 attempts

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Discussion

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