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Problem 5: Probabilities on a continuous sample space

3/6 points (graded)

Alice and Bob each choose at random a real number between zero and one. We assume that the pair of numbers is chosen according to the uniform probability law on the unit square, so that the probability of an event is equal to its area.

We define the following events:

$$\begin{aligned} A &= \{\text{The magnitude of the difference of the two numbers is greater than } 1/3\} \\ B &= \{\text{At least one of the numbers is greater than } 1/4\} \\ C &= \{\text{The sum of the two numbers is } 1\} \\ D &= \{\text{Alice's number is greater than } 1/4\} \end{aligned}$$

Find the following probabilities:

1. $\mathbf{P}(A) =$ 0.49 ✗ **Answer:** 0.44444

2. $\mathbf{P}(B) =$ 0.9375 ✓ **Answer:** 0.9375

3. $\mathbf{P}(A \cap B) =$ ✗ **Answer:** 0.44444

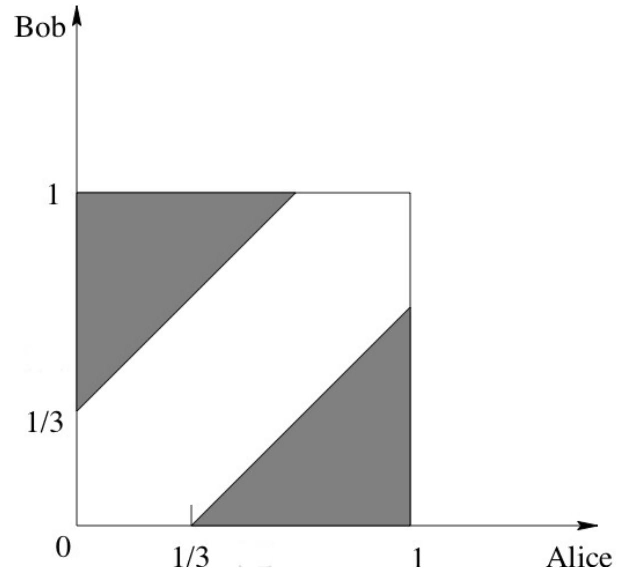
4. $\mathbf{P}(C) =$ 0 ✓ **Answer:** 0

5. $\mathbf{P}(D) =$ 0.75 ✓ **Answer:** 0.75

6. $\mathbf{P}(A \cap D) =$ 0.71625 ✗ **Answer:** 0.30903

Answer:

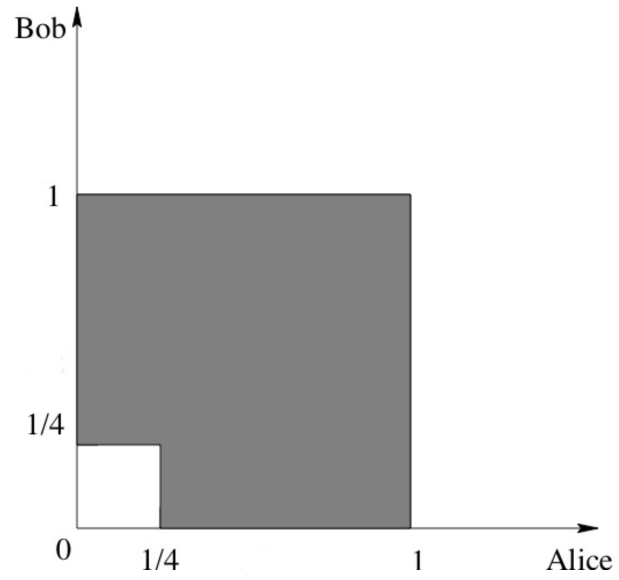
1. We have the following figure, where the axes represent Alice and Bob's choices, and the shaded areas (the two triangles) represent points where Alice's and Bob's choices differ by more than $1/3$.



Under the uniform probability law, the probability of the event is its area. Using the formula for the area of a triangle, we find

$$\mathbf{P}(A) = 2 \cdot \frac{(2/3)^2}{2} = \boxed{4/9}.$$

2. The set of points for which at least one of the numbers is greater than $1/4$ is the shaded region in the following figure:



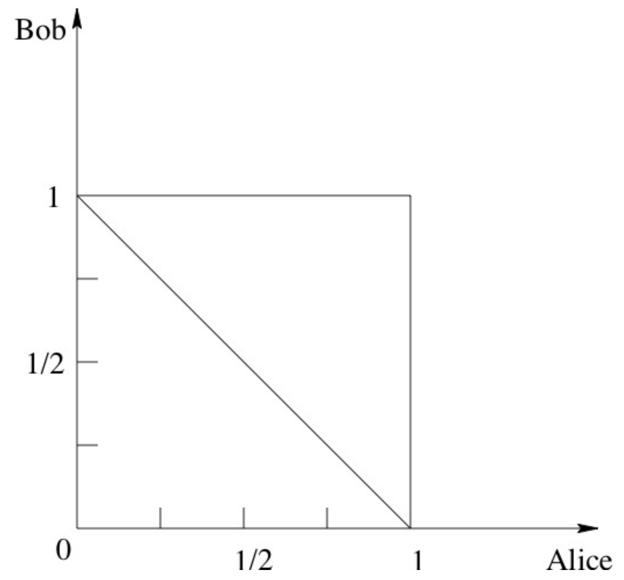
Its probability is:

$$\begin{aligned}\mathbf{P}(B) &= 1 - \mathbf{P}(\text{both numbers are less than or equal to } 1/4)\\ &= 1 - \text{Area of unshaded square}\\ &= 1 - 1/4 \cdot 1/4\\ &= 1 - 1/16\end{aligned}$$

$$= \boxed{15/16}.$$

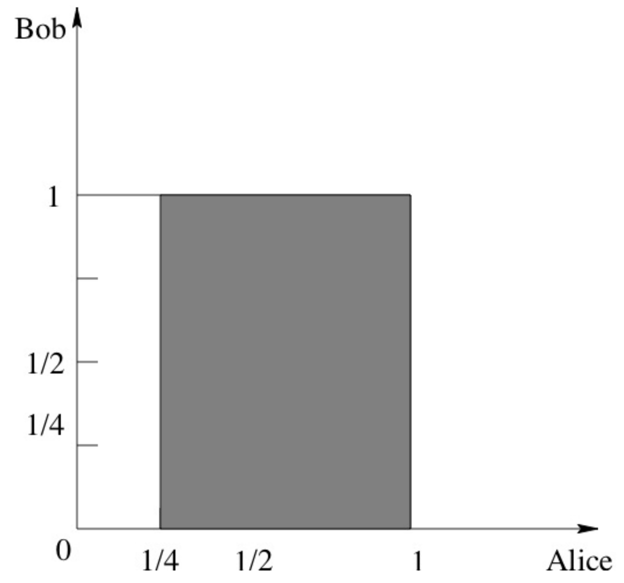
3. Event A is a subset of event B, so that $A \cap B = A$. Thus, $\mathbf{P}(A \cap B) = \mathbf{P}(A) = \boxed{4/9}$.

4. The set of points where the sum of the two numbers is 1 is the diagonal of slope -1 shown in the next figure.



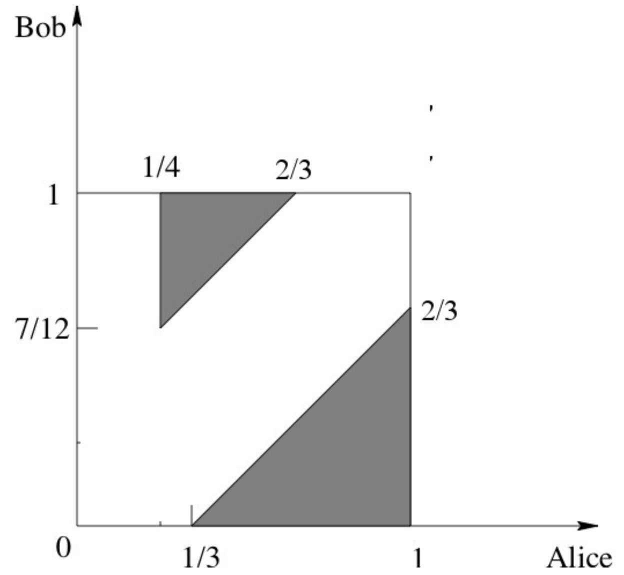
Since it is a line segment, it has zero area and $\mathbf{P}(C) = \boxed{0}$.

5. Event D is the shaded region in the figure below.



Thus, $\mathbf{P}(D) = \textbf{area of shaded region} = 1 \cdot (3/4) = \boxed{3/4}$.

6. By intersecting the shaded areas associated with events A and D, we obtain the shaded region shown below.



Then,

$$\begin{aligned} \mathbf{P}(A \cap D) &= \text{area of shaded region} \\ &= 2/3 \cdot 2/3 \cdot 1/2 + 5/12 \cdot 5/12 \cdot 1/2 \\ &= 2/9 + 25/288 \\ &= \boxed{89/288}. \end{aligned}$$

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