Exercise: Counting committees

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2/2 points (graded)

We start with a pool of n people. A chaired committee consists of $k \geq 1$ members, out of whom one member is designated as the chairperson. The expression $k\binom{n}{k}$ can be interpreted as the number of possible chaired committees with k members. This is because we have $\binom{n}{k}$ choices for the k members, and once the members are chosen, there are then k choices for the chairperson. Thus,

$$c = \sum_{k=1}^{n} k\binom{n}{k}$$

is the total number of possible chaired committees of any size.

Find the value of c (as a function of n) by thinking about a different way of forming a chaired committee: first choose the chairperson, then choose the other members of the committee. The answer is of the form

$$c = (\alpha + n^{\beta})2^{\gamma n + \delta}.$$

What are the values of α, β, γ, and δ?

$\alpha =$	0	✔ Answer: 0
$\beta =$	1	✔ Answer: 1
$\gamma =$	1	✔ Answer: 1
$\delta =$	-1	✔ Answer: -1

Answer:
We first choose the chairperson, for which there are n choices, and then choose an arbitrary subset of the remaining $n-1$ people, who will be the remaining committee members. For example, this arbitrary subset could be the empty set, which would mean that the committee is of size 1: only the chairperson. There are 2^{n-1} possible subsets of a set with $n-1$

elements, and so there are 2^{n-1} ways of choosing the remaining committee members. Thus, an alternative expression for the number of possible chaired committees of any size is $n2^{n-1}$, from which we can extract the values of α, β, γ, and δ.

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