Bookmarks

- Unit 0: Overview
- Entrance Survey
- Unit 1: Probability models and axioms
- Unit 2: Conditioning and independence
  - Unit overview
  - Lec. 2: Conditioning and Bayes' rule
    Exercises 2 due Feb 2, 2017 20:59 ART
  - Lec. 3: Independence
    Exercises 3 due Feb 2, 2017 20:59 ART
  - Solved problems
  - Problem Set 2
    Problem Set 2 due Feb 2, 2017 20:59 ART
- Unit 3: Counting



# Exercise: Bayes' rule and the false-positive puzzle

Bookmark this page

## Exercise: Bayes' rule and the false-positive puzzle

4.0/4.0 points (graded)

A test for a certain rare disease is assumed to be correct $95\%$ of the time: if a person has the disease, the test result is positive with probability $0.95$, and if the person does not have the disease, the test result is negative with probability $0.95$. A person drawn at random from a certain population has probability $0.001$ of having the disease.

1. Find the probability that a random person tests positive.

   0.0509 ✔ **Answer:** 0.0509

2. Given that the person just tested positive, what is the probability he actually has the disease?

   0.0186 ✔ **Answer:** 0.01866

Answer:

Let $A$ be the event that the person has the disease, and $B$ the event that the test result is positive.

1. The desired probability is

$$\mathbf{P}(B) = \mathbf{P}(A)\mathbf{P}(B \mid A) + \mathbf{P}(A^c)\mathbf{P}(B \mid A^c) = 0.001 \cdot 0.95 + 0.999 \cdot 0.05 = 0.05$$

2. The desired probability is

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A)\mathbf{P}(B \mid A)}{\mathbf{P}(B)} = \frac{0.001 \cdot 0.95}{0.0509} \approx 0.01866.$$

   Note that even though the test was assumed to be fairly accurate, a person who has tested positive is still very unlikely (probability less than $2\%$) to have the disease. The explanation is that when testing 1000 people, we expect about 1 person to have the disease (and most likely test positive), but also expect about $1000 \cdot 0.999 \cdot 0.05 \approx 50$ people to test positive without having the disease. Hence, when we see a positive test, it is about 50 times more likely to correspond to one of the 50 false positives.

Submit You have used 1 of 10 attempts