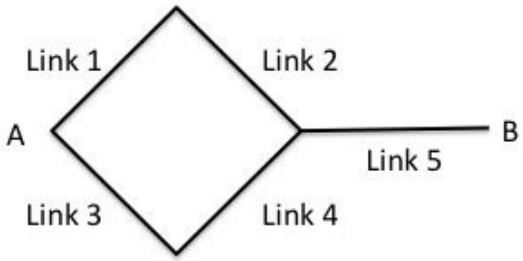
Problem 2 Vertical: A reliability problem

Bookmark this page

Problem 2: A reliability problem

4.0/4.0 points (graded)

Consider the communication network shown in the figure below and suppose that each link can fail with probability p. Assume that failures of different links are independent.



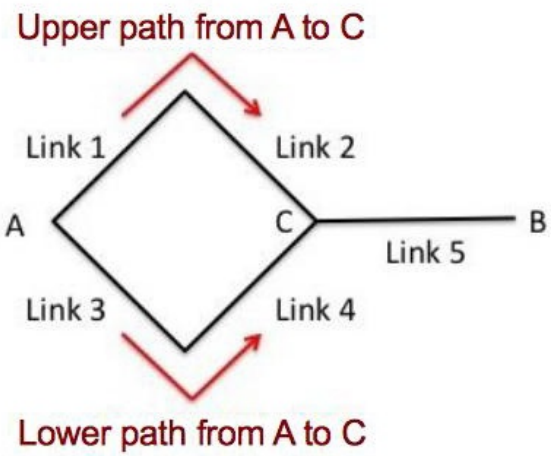
1. Assume that $p = 1/3$. Find the probability that there exists a path from A to B along which no link has failed. (Give a numerical answer.)

0.4609 ✔ **Answer:** 0.46091

2. Given that exactly one link in the network has failed, find the probability that there exists a path from A to B along which no link has failed. (Give a numerical answer.)

0.8 ✔ **Answer:** 0.8

Answer:



Let E be the event that there exists an operational path from A to B. Note that the probability that the Upper path from A to C is operational is $(1-p)^2$. So the probability that the Upper path fails is $1-(1-p)^2$. Similarly, the Lower path fails with probability $1-(1-p)^2$ as well.

1. We can break the problem down into two parts. In order for there to be an operational path from A to B, there must be an operational path from A to C, and Link 5 must be operational.

$$\begin{aligned}\mathbf{P}(E) &= \mathbf{P}(\text{there exists a path from } A \text{ to } C \text{ and Link 5 is operational}) \\ &= (1 - \mathbf{P}(\text{Upper path fails and Lower path fails})) \cdot \mathbf{P}(\text{Link 5 is operationa} \\ &= \left\{1 - \left[1 - (1-p)^2\right]^2\right\} \cdot (1-p).\end{aligned}$$

When $p = 1/3$, this gives us $\mathbf{P}(E) \approx 0.46091$.

2. Since all links are equally likely to fail and since exactly one link has failed, each link has the same probability $1/5$ of being the one that failed. There will be no path from A to B only in the case where the link that failed is Link 5, which happens with probability $1/5$. Therefore, the desired probability is $1 - 1/5 = 4/5$.

Submit You have used 1 of 2 attempts

DISCUSSION

Click “Show Discussion" below to see discussions on this problem.

Discussion
Topic: Unit 2/Problem Set 2 / A reliability problem

Show Discussion