Problem 4 Vertical: Victor and his umbrella

Bookmark this page

Problem 4: Victor and his umbrella

4.0/6.0 points (graded)

Before leaving for work, Victor checks the weather report in order to decide whether to carry an umbrella. On any given day, with probability 0.2 the forecast is “rain" and with probability 0.8 the forecast is “no rain". If the forecast is “rain", the probability of actually having rain on that day is 0.8. On the other hand, if the forecast is “no rain", the probability of actually raining is 0.1.

1. One day, Victor missed the forecast and it rained. What is the probability that the forecast was “rain"?

   0.666 ✔ **Answer:** 0.66667

2. Victor misses the morning forecast with probability 0.2 on any day in the year. If he misses the forecast, Victor will flip a fair coin to decide whether to carry an umbrella. (We assume that the result of the coin flip is independent from the forecast and the weather.) On any day he sees the forecast, if it says “rain" he will always carry an umbrella, and if it says “no rain" he will not carry an umbrella. Let U be the event that “Victor is carrying an umbrella", and let N be the event that the forecast is “no rain". Are events U and N independent?

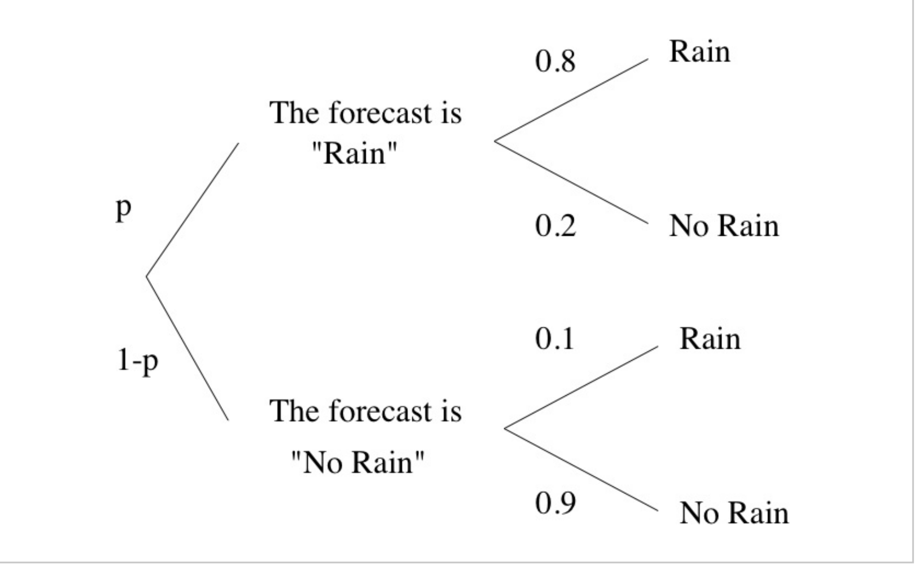
   No ✔ **Answer:** No

3. Victor is carrying an umbrella and it is not raining. What is the probability that he saw the forecast?

   ✘ **Answer:** 0.29630

Answer:

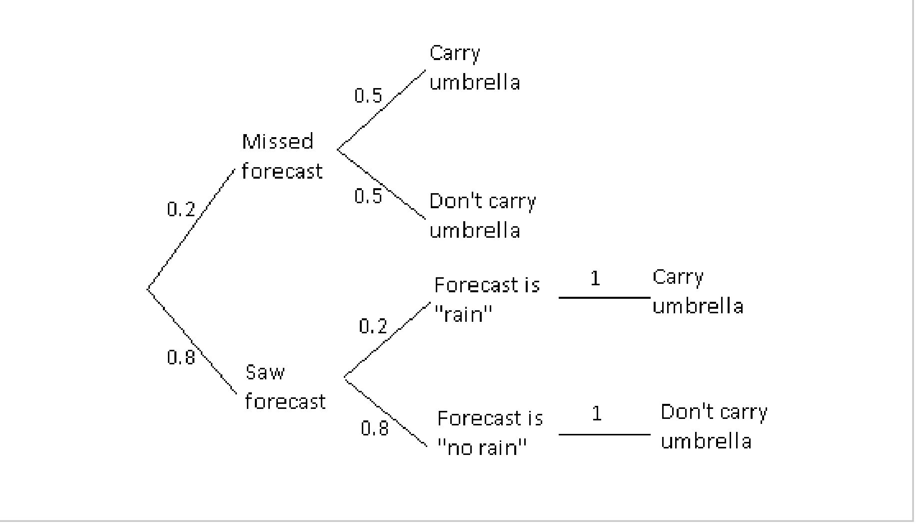
1. We can represent the scenario using the following tree diagram, where $p = 0.2$ is the probability that the forecast is “rain".



Let A be the event that the forecast was “rain", and let B be the event that it rained. Then,

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A)\mathbf{P}(B \mid A)}{\mathbf{P}(B)} = \frac{(0.2)(0.8)}{(0.2)(0.8) + (0.8)(0.1)} = \frac{2}{3}.$$

2. The tree diagram in this case is the following:



$$\begin{aligned}
\mathbf{P}(U) &= (0.2)(0.5) + (0.8)(1)(0.2) = 0.26 \\
\mathbf{P}(N) &= 0.8 \\
\mathbf{P}(U \cap N) &= \mathbf{P}(U \cap N \mid \text{Missed forecast})\mathbf{P}(\text{Missed forecast}) \\
&\quad + \mathbf{P}(U \cap N \mid \text{Saw forecast})\mathbf{P}(\text{Saw forecast}) \\
&= (0.5 \cdot 0.8)(0.2) + (0)(0.8) = 0.08
\end{aligned}$$

Since $\mathbf{P}(U \cap N) \neq \mathbf{P}(U)\mathbf{P}(N)$, the two events are not independent.

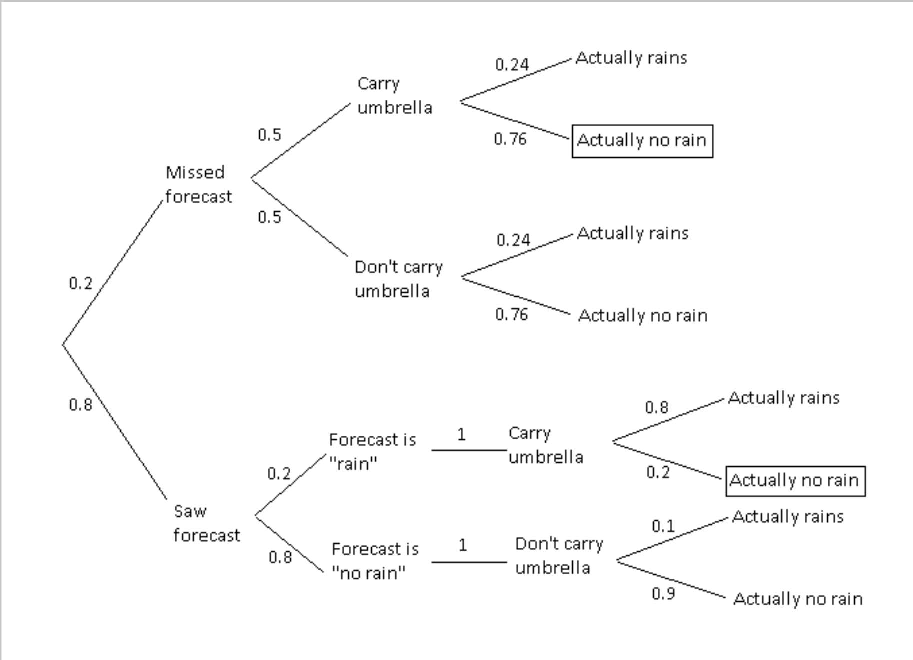
3. Let us first find the probability that it actually rains:

$$\mathbf{P}(\text{Actually rains}) = (0.8)(0.2) + (0.1)(0.8) = 0.24.$$

Thus,

$$\mathbf{P}(\text{Actually no rain}) = 1 - 0.24 = 0.76.$$

We can then extend the tree from part 2 as follows:



The event that Victor is carrying an umbrella and it is not raining corresponds to the two boxed cases in the figure above, of which the lower one is the one where Victor saw the forecast. Therefore, the desired probability is

$$\begin{aligned}&\mathbf{P}(\text{Saw forecast} \mid \text{Umbrella and actually no rain})\\ &= \frac{(0.8)(0.2)(1)(0.2)}{(0.8)(0.2)(1)(0.2) + (0.2)(0.5)(0.76)}\\ &= \frac{8}{27}.\end{aligned}$$

Submit You have used 2 of 2 attempts

DISCUSSION

Click “Show Discussion" below to see discussions on this problem.

Discussion

Topic: Unit 2/Problem Set 2 / Victor and his umbrella

Show Discussion