The multinomial probabilities

- balls of different colors: $i = 1, \ldots, r$
- probability of picking a ball of color i is p_i
- draw n balls, independently
- given nonnegative numbers n_i, with $n_1 + \cdots + n_r = n$
- Find: $\mathbf{P}\big(n_1$ balls of color 1, n_2 balls of color $2, \ldots, n_r$ balls of color $r\big)$

p $\quad$ $1-p$

- Special case $r = 2$; colors: "heads", "tails"

$n_1 \longleftrightarrow k$ heads $\qquad p_1 = p \qquad \binom{n}{k} p^k (1-p)^{n-k} \qquad \dfrac{n!}{n_1!\, n_2!}\, p_1^{n_1} p_2^{n_2}$.

$n_2 \longleftrightarrow n-k$ tails $\qquad p_2 = 1-p$

The multinomial probabilities

$r = 3$

$n = 7$

$1\ 1\ 3\ 1\ 2\ 2\ 1$

type: $(4, 2, 1)$

Prob $= P_1 P_1 P_3 P_1 P_2 P_2 P_1 = P_1^4 P_2^2 P_3$

$$n_1 + \cdots + n_r = n$$

$$\mathbf{P}\Big(\text{particular sequence of "type" } (n_1, n_2, \ldots, n_r)\Big) = p_1^{n_1} p_2^{n_2} \cdots p_r^{n_r}$$

sequence of type $(n_1, n_2, \ldots, n_r)$ $\longleftrightarrow$ partition of $\{1, \ldots, n\}$ into subsets of sizes $n_1, n_2, \ldots, n_r$

$$\boxed{\mathbf{P}\Big(\text{get type } (n_1, n_2, \ldots, n_r)\Big) = \frac{n!}{n_1!\, n_2! \cdots n_r!} p_1^{n_1} p_2^{n_2} \cdots p_r^{n_r}}$$