Problem Set - 0

Introduction to Mathematical Logic and Proofs

OpenCourseWare Discord

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Fundamentals of Mathematical Logic

1. Complete the following truth table:

P	Q	$\neg Q$	$P \wedge (\neg Q)$
T	T	-	-
T	F	-	-
F	T	-	-
F	F	-	-

2. A college student makes the following statement:

"If I don't see my advisor today, then I'll see her tomorrow."

Determine whether for each of the following the above statement is true or false:

- The student doesn't see his advisor either day.
- The student sees his advisor both days.
- The student sees his advisor on one of the two days.
- The student doesn't see his advisor today and waits until next week to see her.

3. Let P and Q be statements. Which of the following implies that $P \vee Q$ is false?

- $(\neg P) \vee (\neg Q)$ is false.
- $(\neg P) \vee Q$ is true.
- $(\neg P) \wedge (\neg Q)$ is true.
- $Q \implies P$ is true.
- $P \wedge Q$ is false.

4. "Translate" the following logical statements into statements in English[1]:

- $(\neg Q) \implies (P \wedge (\neg P))$
- $(P \vee Q) \implies R$
- $(P \implies R) \wedge (Q \implies R)$

Are the last two statements logically equivalent? Check and explain why they are or aren't the same.

Proofs

5. Let $x \in \mathbb{Z}$ Prove that if 2^{2x} is an odd integer, then 2^{-2x} is an even integer.

6. Prove that $2n^2 + n$ is odd iff $\cos\left(\dfrac{n\pi}{2}\right)$ is even.

7. Find a counterexample for the following statements:

- If a and b are any two real numbers, then $\log(ab) = \log(a) + \log(b)$.
- Let $a, b \in \mathbb{Z}$. If ab and $(a+b)^2$ are of opposite parity, then a^2b^2 and $a + ab + b$ are of opposite parity.

8. Prove that there exist no positive integers m and n for which $m^2+m+1 = n^2$.

9. Prove $\sqrt{2} + \sqrt{3}$ is an irrational number.

10. Show that $7^n - 1$ is divisible by 6, for all positive integers n.

11. [**Optional Bonus**] Let x be a real number such that $x + x^{-1}$ is an integer. Prove that $x^n + x^{-n}$ is an integer, for all positive integers n.

[1]That is, if your logical statement is $P \implies Q$ then you "translate" it to : "P implies Q" or anything equivalent to that. You're also encouraged to use examples as you like, such as "If A is an apple, then B is a ball.