

Homework #1

Due on February 11th, 2020
Knowledge Discovery & Data Mining
CS513A—Spring 2020
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Problem 1

Jerry and Susan have a joint bank account. Jerry goes to the bank 20% of the days. Susan goes there 30% of the days. Together they are at the bank 8% of the days.

- Susan was at the bank last Monday. What is the probability that Jerry was there too?
- Last Friday, Susan was not at the bank. What's the probability that Jerry was there?
- Last Wednesday at least one of them was at the bank. What is the probability that both of them were there?

Solution

	Susan at Bank	Susan not at Bank	
Jerry at Bank	8	12	20
Jerry not at Bank	22	58	80
	30	70	100

- $P(\text{Jerry at Bank} / \text{Susan at Bank}) = 8\% / 30\% = 26.67\%$
- $P(\text{Jerry at Bank} / \text{Susan not at Bank}) = 12\% / 70\% = 17.14\%$
- $P(\text{Jerry} \cap \text{Susan at Bank} / \text{Jerry} \cup \text{Susan at Bank}) = 8\% / (1 - 58\%) = 19.05\%$

Problem 2

Harold and Sharon are studying for a test. Harold's chances of getting a "B" are 80%. Sharon's chances of getting a "B" are 90%. The probability of at least one of them getting a "B" is 91%.

- What is the probability that only Harold gets a "B"?
- What is the probability that only Sharon gets a "B"?
- What is the probability that both won't get a "B"?

Solution

	Sharon "B"	Sharon not "B"	
Harold "B"	79	1	80
Harold not "B"	11	9	20
	90	10	100

$$\begin{aligned}
 P(\text{Harold and Sharon}) &= P(\text{Harold}) + P(\text{Sharon}) - P(\text{Harold} \cup \text{Sharon}) \\
 &= 80\% + 90\% - 91\% = 79\%
 \end{aligned}$$

- $P(\text{Only Harold "B"}) = P(\text{Harold "B"}) - P(\text{Harold "B"} \cap \text{Sharon not "B"}) = 80\% - 79\% = 1\%$
- $P(\text{Only Sharon "B"}) = P(\text{Sharon "B"}) - P(\text{Sharon "B"} \cap \text{Harold not "B"}) = 90\% - 79\% = 11\%$
- $P(\text{Harold} \cap \text{Sharon not "B"}) = 1 - 91\% = 9\%$

Problem 3

Jerry and Susan have a joint bank account. Jerry goes to the bank 20% of the days. Susan goes there 30% of the days. Together they are at the bank 8% of the days. Are the events "Jerry is at the bank" and "Susan is at the bank" independent?

Solution

By definition, two events are independent if and only if:

$$P(A \text{ and } B) = P(A) \times P(B) \tag{1}$$

In the situation described within the problem statement:

$$P(A \text{ and } B) = 8\% \neq 20\% \times 30\% = 6\%$$

Therefore, the statements "Jerry is at the bank" and "Susan is at the bank" are dependent.

Problem 4

You roll 2 dice.

- a. Are the events “the sum is 6” and “the second die shows 5” independent?
- b. Are the events “the sum is 7” and “the first die shows 5” independent?

Solution

- a. The events “the sum is 6” and “the second die shows 5” are dependent.

$$P(\text{Sum is 6}) = n(\{(3, 3), (2, 4), (4, 2), (1, 5), (5, 1)\})/36 = 5/36$$

$$P(\text{Second die is 5}) = n(\{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)\})/36 = 6/36$$

$$P(\text{Sum is 6} \cap \text{second die is 5}) = n(\{(1, 5)\})/36 = 1/36$$

$$1/36 \neq 5/36 \times 6/36 = 30/1296 = 5/216$$

- b. The events “the sum is 7” and “the first die shows 5” are dependent.

$$P(\text{Sum is 7}) = n(\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}) = 6/36$$

$$P(\text{First die is 5}) = 6/36$$

$$P(\text{Sum is 7} \cap \text{first die is 5}) = 1/36$$

$$1/36 \neq 6/36 \times 1/36 = 1/216$$

Problem 5

An oil company is considering drilling in either TX, AK and NJ. The company may operate in only one state. There is 60% chance the company will choose TX and 10% chance - NJ. There is 30% chance of finding oil in TX, 20% - in AK, and 10% - in NJ.

1. What's the probability of finding oil?
2. The company decided to drill and found oil. What is the probability that they drilled in TX?

Solution

	TX	AK	NJ	
Oil	18	6	1	25
No Oil	42	24	9	75
	60	30	10	100

$$P(\text{Oil}|\text{TX}) = 30\% = P(\text{Oil} \cap \text{TX})/P(\text{TX}) = P(\text{Oil} \cap \text{TX})/60\%$$

$$P(\text{Oil} \cap \text{TX}) = 30\% \times 60\% = 18\%$$

$$P(\text{Oil}|\text{NJ}) = 10\% = P(\text{Oil} \cap \text{NJ})/P(\text{NJ}) = P(\text{Oil} \cap \text{NJ})/10\%$$

$$P(\text{Oil} \cap \text{NJ}) = 10\% \times 10\% = 1\%$$

$$P(\text{Oil}|\text{AK}) = 30\% = P(\text{Oil} \cap \text{AK})/P(\text{AK}) = P(\text{Oil} \cap \text{AK})/20\%$$

$$P(\text{Oil} \cap \text{AK}) = 30\% \times 20\% = 6\%$$

1. $P(\text{Oil}) = 25\%$
2. $P(\text{TX}|\text{Oil}) = 18\%/25\% = 0.72\%$