CS559: Linear Algebra Practice Problems.

1. Let $A = \begin{bmatrix} 3 & -1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ -1 & -3 & 2 & -4 \end{bmatrix}$ and $b = \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$. Denote the columns of A by $\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}, \boldsymbol{a_4}$. Let $W = Span\{\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}, \boldsymbol{a_4}\}$.

A. Is b is W?

B. If **b** is in W, then express **b** as a linear combination of the vectors $\mathbf{a_1}$, $\mathbf{a_2}$, $\mathbf{a_3}$, and $\mathbf{a_4}$.

2. Suppose $\{\mathbf{v_1}, \mathbf{v_2}\}$ is a linearly independent set in R^n. Show that $\{\mathbf{v_1}, \mathbf{v_1}+\mathbf{v_2}\}$ is also linearly independent.

3. Define the linear transformation $T: R^2 \to R^2$ and $S: R^2 \to R^2$ so that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ 3x_1 - 2x_2 \end{bmatrix} \text{ and } S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + 3x_2 \\ -x_1 + x_2 \end{bmatrix}$$

A. Is $S \cdot T$ an invertible linear transformation? Explain.

B. If $T \cdot S$ is invertible, find the formula for $(T \cdot S)^{-1}$ (Hint: use $(AB)^{-1} = B^{-1}A^{-1}$).

4. Use **the invertible matrix theorem** to determine the value(s) of λ for which the matrix

$$A = \begin{bmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

is **NOT** invertible.

5. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and assume that det(A)=2. Find $\det(-2\mathrm{A})$, $\det\left(\frac{1}{2}A^{-1}\right)$, $\det((2A)^3)$, and $\det(3A^T)^{-1})$.