Octupus

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1.

The general form for hyperbolic conservation laws is

$$\frac{\partial}{\partial t}\mathbf{w} + \nabla \cdot \mathbf{f}\left(\mathbf{w}\right) = \mathbf{s}, \tag{1}$$

where the matrix $\frac{\partial \mathbf{f}(\mathbf{w})}{\partial \mathbf{w}}$ has real eigenvalues. For Euler's equations of incompressible ideal gas flow,

$$\mathbf{w} = \begin{pmatrix} \rho \\ \mathbf{s} \\ \mathrm{E} \end{pmatrix}, \tag{2}$$

and

$$\mathbf{f}\left(\mathbf{w}\right) = \begin{pmatrix} \rho\mathbf{v} \\ \mathbf{s}\mathbf{v} + \mathrm{p} \\ \left(\mathrm{E} + \mathrm{p}\right)\mathbf{v} \end{pmatrix}, \tag{3}$$

where ρ is the mass density, $\mathbf{s}$ is the momentum density, E is the energy density,

$$\mathbf{v} = \frac{\mathbf{s}}{\rho} \tag{4}$$

is the velocity, and

$$p := \left(\gamma - 1\right)\left(\mathrm{E} - \frac{1}{2}\rho u^2\right) \tag{5}$$

is the gas pressure for a given ratio of specific heats, γ. For flow in conservative scalar potentials, such as classical gravity,

$$\mathbf{s} := \begin{pmatrix} 0 \\ -\rho\nabla\phi \\ -\rho\mathbf{u}\cdot\nabla\phi \end{pmatrix}, \tag{6}$$

where ϕ is the potential.

We use the positivty limiter of Zhang & Shu (2010).

REFERENCES

Zhang, X., & Shu, C.-W. 2010, Journal of Computational Physics, 229, 8918

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