changing the vote assignment ⇒ [a) all possible quorums overlap in one correct replica
b) even with up to f failed replicas, there is always some quorum available in the system

requirements

GOALS

a) rely primarily on the fastest replicas in the system
b) preserve original safety and liveness properties

WHEAT VOTE ASSIGNMENT

A) CFT ASSIGNMENT

$m = 2f + 1 + \Delta$ → extra replicas available in the system

$N_V = \sum V_i = 2F_V + 1$

F_V → max # votes that can be dismissed in the system

let λ be # replicas we assign V_{max} to ⇒ $N_V = \lambda V_{max} + (m-\lambda) V_{min}$

Obs: votes need to be distributed such that $Q_V = F_V + 1$ gathered, quorums always overlap by at least one correct replica.

$F_V = (\Delta + f) V_{min} = f V_{max}$ → observed from quorum formation schemes

⇓

$$\begin{cases} V_{max} = 1 + \frac{\Delta}{f} \\ V_{min} = 1 \end{cases} \quad \text{and } \lambda = f$$

Proof of correctness

Safe minimality (there exists at least one minimal quorum in the system)

↳ S_{max} subset of f replicas that hold V_{max} ⇒ they add up to F_V votes

⇒ S_{max} + one additional replica holding $V_{min} = 1$ ⇒ $Q_V = F_V + 1$, the necessary votes

Availability (there is always a quorum in the system that holds Q_V votes)

↳ S_{min} subset of $(m-f)$ replicas holding V_{min} | ⇒ quorum is reached even in the case of all V_{max} replicas failing

$(m-f) = f+\Delta+1$

Consistency (all quorums that hold Qv votes intersect by at least one CORRECT replica)

↳ minimal quorum contains Qv votes => at least one V_{min} replica

B) BFT ASSIGNMENT

$n = 3f + 1 + \Delta$

$N_v = \sum V_i = 2F_v + 1 \Rightarrow \left[\begin{array}{l} V_{max} = 1 + \frac{\Delta}{f} \\ V_{min} = 1 \end{array}\right. \Rightarrow d = 2f$

$\Rightarrow Q_v = 2f + 1$

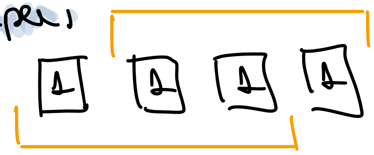
MULTIPLE WEIGHTS CHALLENGE

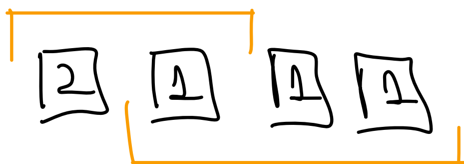
A) CFT MODE

$n = 2f + 1 + \Delta$

$Q_v = f + 1$

example:



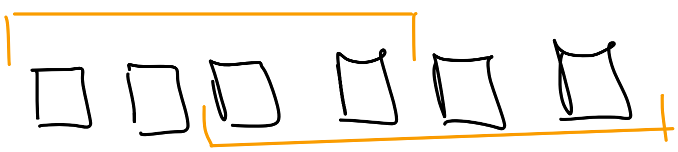


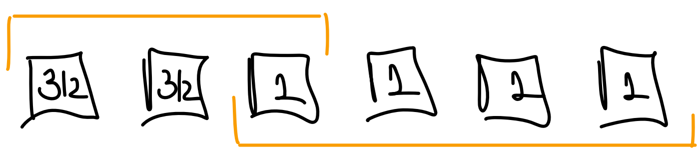
$n = 4,\ f = 1,\ \Delta = 1 \rightarrow \text{majority} = \left\lceil \frac{4+1}{2} \right\rceil = 3$

weighted voting → $N_v = 5$ → 1 V_{max}, 3 V_{min}

$F_v = f V_{max} = (\underset{1}{\Delta} + \underset{1}{f}) V_{min} = 1 \cdot 2 = 2$

$Q_v = 3$



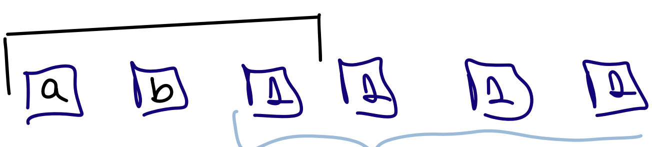


$n = 6,\ f = 2,\ \Delta = 1 \Rightarrow \text{majority} = \left\lceil \frac{6+1}{2} \right\rceil = 4$

weighted voting → $V_{max} = 1 + \frac{1}{2} = 1.5$

$V_{min} = 1$

$Q_v = 4$



↓ need to remain 1 to keep availability

$a + b + 1 = 4 \Leftrightarrow a + b = 3$

↳ weighted voting → each gets half

i) FIRST IDEA → add one more weight

$$N_v = \alpha_1 V_1 + \alpha_2 V_2 + (n - \alpha_1 - \alpha_2) V_0 \xrightarrow{} 1$$

From weighted voting

$$FV = (\Delta + f) V_{min} = f V_{max}$$

$(\Delta + f) V_0$ $\alpha_1 V_1 + \alpha_2 V_2 = \Delta + f$

$\alpha_1 + \alpha_2 = f$, $\alpha_1 = \alpha_2 = f/2$ $\Big|$ $\Rightarrow f/2 (V_1 + V_2) = \Delta + f \Rightarrow$

$$\underbrace{V_1 + V_2}_{\text{make them almost equal}} = 2 \cdot \frac{\Delta}{f} + 1$$

make them almost equal $V_1 = \Delta / f + 1/3$

$V_2 = \Delta / f + 2/3$

ii) SECOND IDEA → multiple weights + some kind of ranking

$$N_v = \underbrace{\alpha_K V_K + \dots + \alpha_1 V_1}_{K \text{ weights to account for } f V_{max}\ (K \leq f)} + (n - f) V_0 \xrightarrow{} V_{min} = 1$$

K weights to account for $f V_{max}$ $(K \leq f)$

$FV = (\Delta + f) V_{min} = f V_{max}$

$(\Delta + f) V_0 = \Delta + f$ $\Big|$ $\Rightarrow \Delta + f = \alpha_K V_K + \dots + \alpha_1 V_1$

$\alpha_K + \dots + \alpha_1 = f \Rightarrow \alpha_{K-1} = \dots = \alpha_1 = \lceil f/K \rceil$

$\alpha_K = f - \lceil f/K \rceil (K - 1)$

i) f weights $\Rightarrow \Delta + f = V_f + \dots + V_1$, since $\forall\, i = \overline{1, f}$, $V_i \geq 1$ $\Big|$ $\Rightarrow$ f will be spread one to each V_i $\Rightarrow$ remains the Δ to be spread to all f replicas

1) Δ / f to each replica (equal) $\Rightarrow V_{max}$ normal weighted voting

2) $V_1 \quad V_2 \dots \quad V_f$

feasible idea to be tested

$$\frac{\Delta}{m} \quad \frac{2\Delta}{m} \quad \dots \quad \frac{f\Delta}{m} \Rightarrow \sum_{1}^{f} V_i = \Delta \Leftrightarrow \frac{\Delta}{m} + \frac{2\Delta}{m} + \dots + \frac{f\Delta}{m} = \Delta \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{m}(1 + 2 + \dots + f) = 1 \Leftrightarrow m = \frac{f(f+1)}{2}$$

Thus, we can do weighting by assigning each replica $V_i = \frac{i \cdot \Delta}{m}$ weight,

→ generalization of f weights case

ii) less than f weights

↳ $d_{k-1} = \dots = d_1 = \lceil f/k \rceil$
(*) $d_k = f - \lceil f/k \rceil (k-1)$ → this way we spread the f component to each of the $V_1, V_2 \dots V_k$ equally

$$\begin{array}{cccc} V_1 & V_2 & \dots & V_k \\ \downarrow & \downarrow & & \downarrow \\ \frac{\Delta}{m} & \frac{2\Delta}{m} & \dots & \frac{k\Delta}{m} \end{array} \Rightarrow d_1 \frac{\Delta}{m} + d_2 \cdot \frac{2\Delta}{m} + \dots + d_k \cdot \frac{k\Delta}{m} = \Delta \Leftrightarrow$$

$$\Leftrightarrow m = (d_1 + 2d_2 + \dots + (k-1)d_{k-1} + k d_k)$$

$$\overset{(*)}{=} \lceil f/k \rceil \underbrace{(1+2+\dots+(k-1))}_{\frac{(k-1)\cdot k}{2}} + (f - \lceil f/k \rceil (k-1)) \cdot k$$

$$= \lceil f/k \rceil \frac{k(k-1)}{2} + f - k(k-1) \cdot \lceil f/k \rceil$$

$$= f - \lceil f/k \rceil \frac{k(k-1)}{2}$$

Hence, we can do weighted voting by assigning to d_i replicas weight $V_i = \frac{i \cdot \Delta}{m}, i = \overline{1,k}$.

B) BFT MODE

↳ we apply the same ideas but $m = 3f + 1 + \Delta$, $Q_v = 2f + 1$ and hence we have $f' = 2f$ instead of f in everything we presented above.