CSC 565 2020 Fall Homework 1

name1, unityID1 name2, unityID2

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1 k-connectivity

Let G be a 4-connected graph and let $v_1, \ldots, v_4$ be four vertices of G. Let H_0 be formed from G by adding a new vertex u and four edges $(u, v_1), \ldots, (u, v_4)$.

1.1. Prove that H_0 is 4-connected.

Answer:

Let H_1 be formed from G by adding a new vertex u and the $|V_G|$ edges $\{(u, v) \mid v \in V_G\}$.

1.2. Prove that H_1 is 5-connected.

Answer:

2 Hamiltonian Paths

Recall that an Eulerian path in G is (the image of) a homomorphism $f : P_m \to G$ such that f_E is bijective. Similarly, a Hamiltonian path in G is (the image of) a homomorphism $f : P_m \to G$ such that f_V is bijective. Let G be a graph with a vertex u such that $G \setminus u$ has three connected components. **Prove that G does not have a Hamiltonian path.**

Answer:

3 Graph Coloring

Let G, H be simple graphs and $f : G \to H$ be a graph homomorphism. **Prove that G is $|V_H|$-colorable.**

Answer: