CSC 565 2020 Fall Homework 2

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You can create a latex project of this homework through this link.

Cartesian Products

Let G, H be graphs. The Cartesian product between G and H can be defined as:

$$\begin{aligned} G\square H := (&V_G \times V_H, \\ &\{(ab, cd) \mid (a = c \text{ and } (b, d) \in E_H) \\ &\qquad \text{or} \\ &\qquad (b = d \text{ and } (a, c) \in E_G), \\ &\qquad a, c \in V_G, \\ &\qquad b, d, \in V_H\}) \end{aligned}$$

1. Give a function $f_V : V_{G\square H} \to V_{H\square G}$ that induces an isomorphism. No need to prove it is an isomorphism.

Answer:

2. Prove that for all $i, j \geq 3$, $C_i\square P_j$ is planar.
Hint: You can try to do it by providing an embedding for the general case.

Answer:

3. Prove $C_3\square C_3$ is <u>NOT</u> planar by completing the following steps. It is a fact that $C_3\square C_3$ is 3-connected. Think about this and convince yourself of this first.

3.1 Suppose it is planar. How many faces should it have according to Euler's formula?

Answer:

3.2 How many non-separating induced cycles are there? You may assume without proving it that all such cycles have length 3 or 4.

Answer:

3.3 Complete the proof.

Answer:

Outerplanar Graphs

4. Let G be an outerplanar graph with n vertices and as many edges as possible but subject to the constraint that G is not 2-connected. How many edges does G have? Prove it.

Answer:

5. Let G be the graph:

$$G = ([n], \{(i,j) \mid (j-i) \ \% \ n = 1 \vee (j-i) \ \% \ n = 2\}), n \geq 4$$

$\%$ is the modulo operator. Prove that G is <u>NOT</u> outerplanar.

Answer:

Extra Questions

These questions are for your interest and practice. It's recommended to think about them. They will not be graded.

6. Prove that every outerplanar graph is 3-colorable.

7. Prove that no outerplanar graph is 3-connected.