# CSC 565 2020 Fall Homework 2

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You can create a latex project of this homework through this link.

## Cartesian Products

Let $G$, $H$ be graphs. The Cartesian product between $G$ and $H$ can be defined as:

$$\begin{aligned} G\square H := (V_G \times V_H, \\ \{(ab, cd) \mid (a = c \text{ and } (b, d) \in E_H) \\ \text{or} \\ (b = d \text{ and } (a, c) \in E_G), \\ a, c \in V_G, \\ b, d, \in V_H\}) \end{aligned}$$

**1. Give a function $f_V : V_{G\square H} \to V_{H\square G}$ that induces an isomorphism. No need to prove it is an isomorphism.**

*Answer:*

**2. Prove that for all $i, j \geq 3$, $C_j\square P_j$ is planar.**
**Hint: You can try to do it by providing an embedding for the general case.**

*Answer:*

**3. Prove $C_3\square C_3$ is <u>NOT</u> planar by completing the following steps. It is a fact that $C_3\square C_3$ is 3-connected. Think about this and convince yourself of this first.**

**3.1 Suppose it is planar. How many faces should it have according to Euler's formula?**

*Answer:*

**3.2 How many non-separating induced cycles are there? You may assume without proving it that all such cycles have length 3 or 4.**

*Answer:*

**3.3 Complete the proof.**

*Answer:*

## Outerplanar Graphs

**4. Let $G$ be an outerplanar graph with $n$ vertices and as many edges as possible but subject to the constraint that $G$ is not 2-connected. How many edges does $G$ have? Prove it.**

*Answer:*

**5. Let $G$ be the graph:**

$$G = ([n], \{(i,j) \mid (j-i) \ \% \ n = 1 \vee (j-i) \ \% \ n = 2\}), n \geq 4$$

**$\%$ is the modulo operator. Prove that $G$ is <u>NOT</u> outerplanar.**

*Answer:*

## Extra Questions

These questions are for your interest and practice. It's recommended to think about them. They will not be graded.

**6. Prove that every outerplanar graph is 3-colorable.**

**7. Prove that no outerplanar graph is 3-connected.**