

CSC 565 2020 Fall Homework 5

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You can create a latex project of this homework through this [link](#).

Boundary Matrices

1. Write out the boundary matrix for the following graphs:

1. K_3
2. P_3
3. $K_2 \times K_2$

Answer:

2. Let T be a tree with 5 vertices. Let ∂ be the boundary matrix for T . Let A be the 4×4 matrix formed by deleting one row of ∂ . What are the possible values of $\det(A)$?

Hint: Try to find an ordering on vertices such that A is an upper-triangular matrix.

Answer:

Challenge to think about: What if T were not a tree?

The Adjacency Matrix

Remember $M^k = \overbrace{M * \dots * M}^k$, where M is a square matrix and $*$ is matrix multiplication.

Here is an interesting fact about the adjacency matrix A_G of a graph G . For any k , the entries $[A_G^k]_{ij}$ are the number of distinct walks from i to j of length k in G . If you haven't seen this before, you might want to work it out by expanding the matrix multiplication and applying induction. This fact has the following interesting consequences for you to prove.

3. Prove that the trace of A_G^3 is equal to six times the number of K_3 subgraphs in G .

Answer:

4. Let $M = A_G^2 - L_G$, where L_G is the Laplacian of G . Prove that M_{ij} is the number of paths (not walks) from v_i to v_j of length 1 or 2.

Answer: