Cybersecurity in quantum era

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- So before we continue, we provide some necessary definitions.
- *Cybersecurity* is the protection of computer systems and networks from information disclosure, theft or damage from malicious attack (cyberattacks).
- Also, we provide some definitions concerning cryptography.

ALICE, BOB AND EVE

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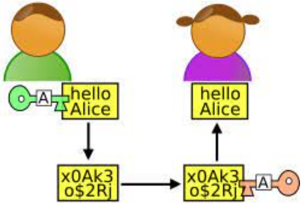

FIGURE: Bob encrypts the message *Hello Alice* and he sends it to Alice.

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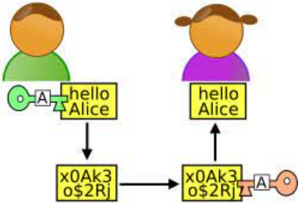

Figure: Bob encrypts the message *Hello Alice* and he sends it to Alice.

- Although, their friend Eve , sometimes, tries to intercept their communication and if she is lucky or smart enough she may decrypt the message. Usually, Eve has enough resources, sophisticated algorithms, very powerful computers and she is very skilled in social engineering.

Three cryptographic primitives

- There two basic schemes. Symmetric cryptosystems (Scs) and Public key Cryptosystems (PkCs). We use both of them to build security protocols, such as SSL/TLS or IPsec or ssh. Also, sometimes instead of a PkC we use a key agreement protocol, such as Diffie-Hellman.

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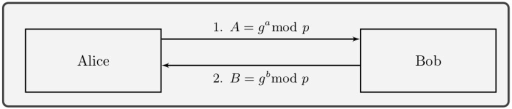
  - 

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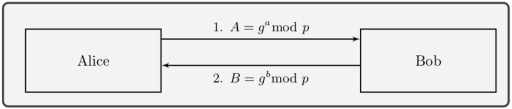

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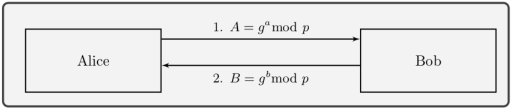
- 

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- Here Eve, if somehow discovers a or b she will compromise all the communication.

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- Public Key Cryptography, as well as Key agreement protocols, as we use them today, are based on the following number theoretic problems : Factorization and Discrete Logarithm Problem.

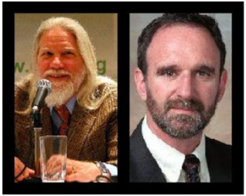

FIGURE: Whit Diffie and Martin Hellman. ACM Turing Medal 2015.

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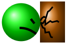
- *disclaimer* : Since I am not a physicist I can't say much for Quantum Computers, because I don't understand how they work! 

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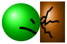
- *disclaimer* : Since I am not a physicist I can't say much for Quantum Computers, because I don't understand how they work! 
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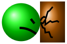
- *disclaimer* : Since I am not a physicist I can't say much for Quantum Computers, because I don't understand how they work! 
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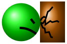
- *disclaimer* : Since I am not a physicist I can't say much for Quantum Computers, because I don't understand how they work! 
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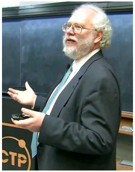
- It runs only in quantum computers (since it exploits quantum properties) and solves the problem of factorization and DL problem in polynomial time.

Peter Shor's quantum algorithm

- 

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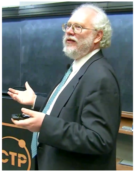
- 

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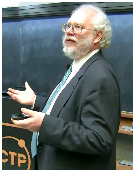
- 
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- If a quantum computer with large memory ever constructed, then the most well known public key cryptosystem, RSA (and also Diffie-Hellman), shall break and all the current security in Internet will be compromised.

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- *Bristlecone*, the google's quantum computer, it has 72 qubits

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- The current quantum computers are noisy
- How large is a 2048-bit integer?

How large is a 2048 or 4096 bits integer?

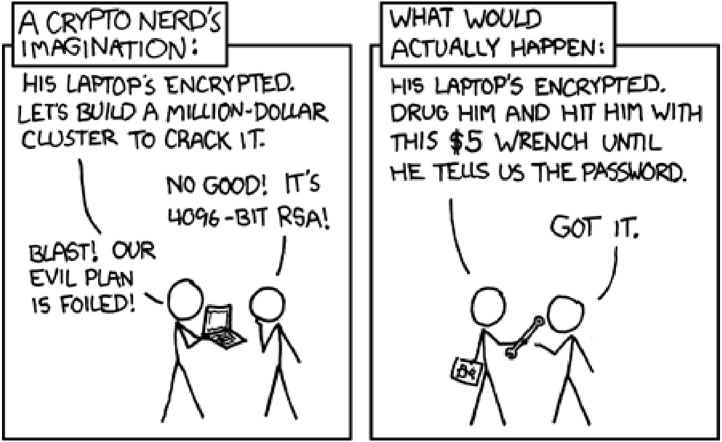

FIGURE: Licence:xkcd.com, CC BY-NC 2.

How large is a 2048 bits integer?

- Here is the number you have to factor in order to break the security of `https://www.sfhmmy.gr/`

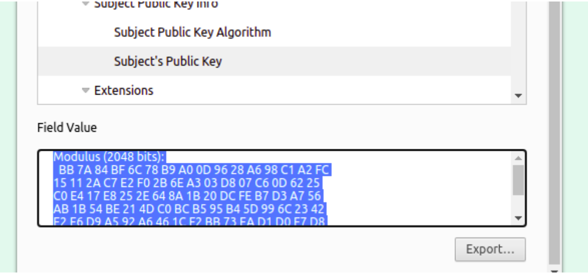

Figure: RSA public key

How large is a 2048 bits integer?

- ...and if we represent it as an integer

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- A quantum computer capable of implementing Shor's algorithms will return the two prime factors.

How large is a 2048 bits integer?

- The current record for factorization RSA modulus with 829 bits and it was factored in Feb. 2020 using CADO-NFS software.

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- The (heuristic) time complexity of GNFS is $2^{O(\sqrt[3]{\log_2(N)})}$.

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- Although, we don't know or predict when a large memory quantum computer has been built. But we have such computers with low memory.

Maybe it's time to panic

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- When we are talking about PQC we (usually) mean cryptography which is **not** based on Factorization and Discrete Logarithm Problem (DLP).

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- For more information `https://csrc.nist.gov/projects/post-quantum-cryptography`

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- We must have good evidences that they are indeed quantum resistant. So, years of research need to be done.

Lattice based PQC

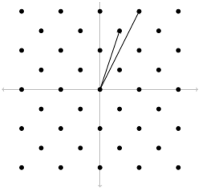
- 

FIGURE: The 2 dimensional lattice $\mathbb{Z}^2$

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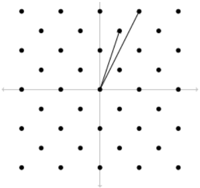
- 

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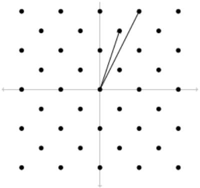
- 

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- The problem of finding a shortest vector (which always exists) is called Shortest Vector Problem (SVP).

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- A famous example in this class of problems is the Ring Learning with Errors RLWE.
- Another example is the Short Integer Problem (SIS).

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- It seems easy, but it was proved to be NP-hard.

Thank you!