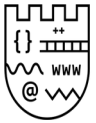
ARISTOTLE UNIVERSITY OF THESSALONIKI



SCHOOL OF INFORMATICS

Written with Libreoffice + TexMaths

K. A. Draziotis




# What is CAS?

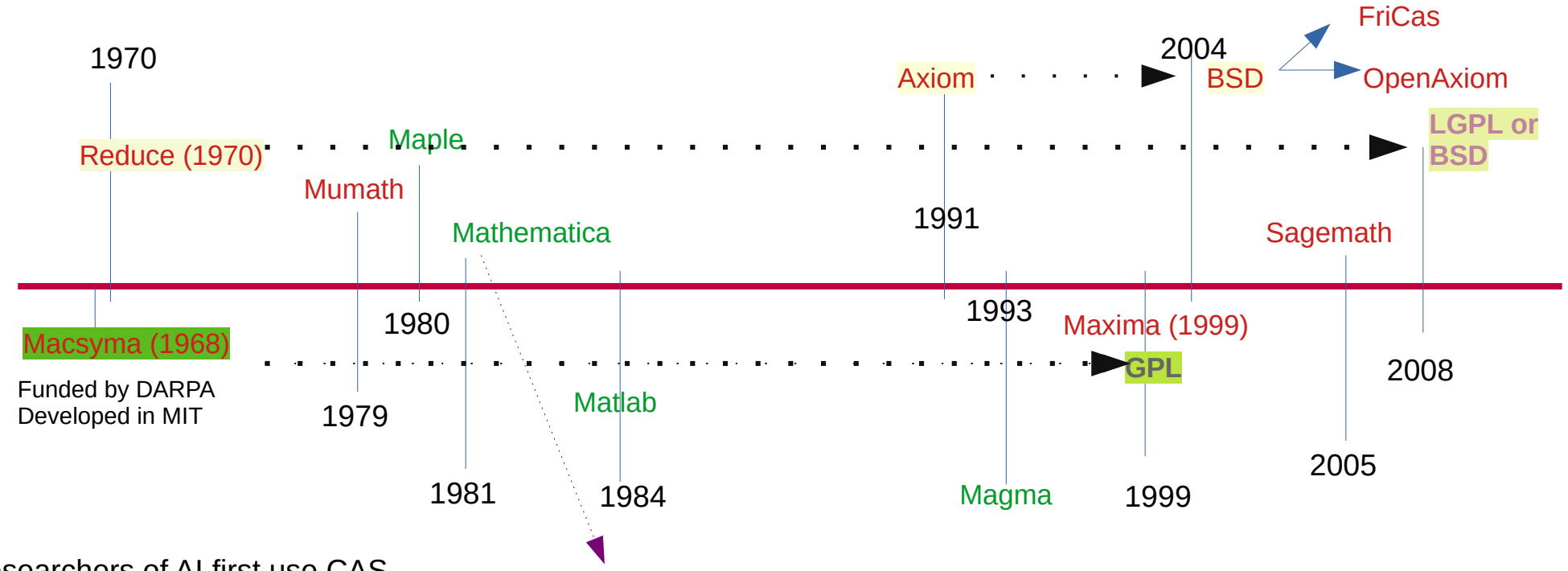
Wikipedia definition : CAS is any mathematical software with the ability to manipulate mathematical expressions  in a way similar to the traditional manual computations of mathematicians and scientists.

There is a language to implement more complex functions. Sometimes a CAS includes an interface.
Almost always there is an interpreter, a memory manager to manipulate the Itermediate data, which sometimes are huge. Also, there is an arbitrary precision library for manipulating large numbers.

# What is CAS?


VS CAS

Calculator does Numerical calculations but CAS does also symbolic calculations can br programmed etc




Researchers of AI first use CAS

Maple, Mathematica and Matlab are well known commercial CAS

# What is CAS?

Sagemath is free Open Source licensed under the GPL
Computer Algebra System (CAS). First version appeared in 24 Feb. 2005.

Wikipedia definition : CAS is any mathematical software with the ability to manipulate mathematical expressions in a way similar to the traditional manual computations of mathematicians and scientists.

The first Computer Algebra Systems were muMATH, Reduce, Derive and Macsyma

Macsyma : Project MAC's SYmbolic Manipulator it is developed from 1968 to 1982 in MIT. Later, in 1999, it was released under GPL with the name MAXIMA.

Other modern CAS, except MAXIMA, are Axiom, Magma, Maple, Mathematica, and SageMath, FriCas.

# What is CAS?

Also there are some specialized open source CAS such as
Pari gp (number theory)
Singular (polynomial computations)
Gap (group theory)
CoCoA (commutative algebra)
Cadabra (Quantum mechanics)
Macaulay2 (Commutative algebra and algebraic geometry)

See, comparison between CAS'

# Sagemath

- It is based on Python/Cython
- It also co-exists with other open source computer algebra systems such as Maxima, Singular, Pari gp, Gap, Flint, R and there are some optional choices, for Octave (open source), Magma (closed source).

# Sagemath

- It is developed for education and research
- There are implementations that covers many large branches of mathematics, such as, calculus, algebra, statistics, combinatorics, number theory, cryptography.

# Sagemath and cython

Cython is a derivative of pyrex of Greg Ewing. We had two forks of Pyrex, SageX and the other of Stefan Behnel. These two forks made Cython

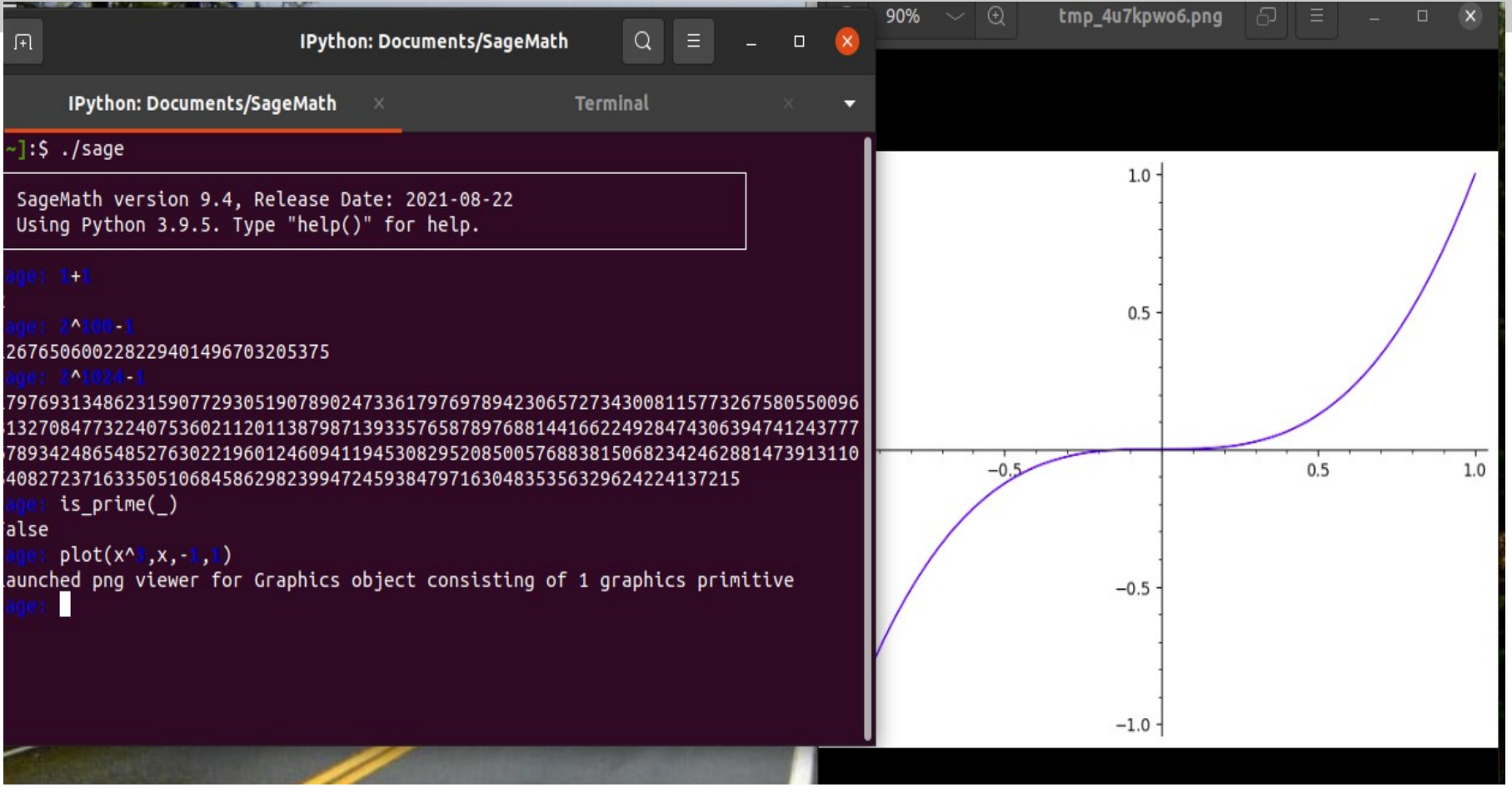
Some examples in colab : here

# Sagemath

- There two modes of operation, command line mode and notebook mode (jupyter)
- There are implementations that covers many large branches of mathematics, such as,

  calculus, algebra, statistic, combinatorics, number theory, cryptography.

# User mode


IPython: Documents/SageMath
IPython: Documents/SageMath
Terminal
~]:$ ./sage
SageMath version 9.4, Release Date: 2021-08-22
Using Python 3.9.5. Type "help()" for help.
age: 1+1
age: 2^100-1
2676506002282294014967032053 75
age: 2^1024-1
7976931348623159077293051907890247336179769789423065727343008115773267580550096
1327084773224075360211201138798713933576587897688144166224928474306394741243777
7893424865485276302219601246094119453082952085005768838150682342462881473913110
4082723716335051068458629823994724593847971630483535632962422413721 5
age: is_prime(_)
alse
age: plot(x^3,x,-1,1)
aunched png viewer for Graphics object consisting of 1 graphics primitive
age:
90%
tmp_4u7kpwo6.png
1.0
0.5
−0.5
0.5
1.0
−0.5
−1.0

# Notebook mode

jupyter Untitled2 Last Checkpoint: a few seconds ago (unsaved changes)

Logout

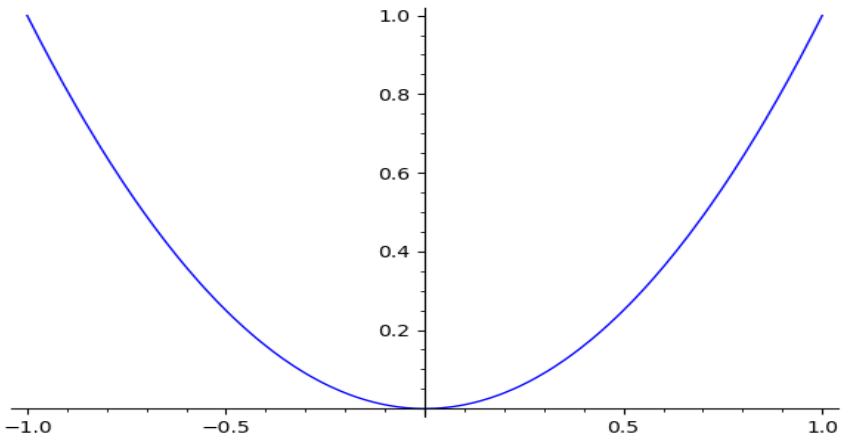
File Edit View Insert Cell Kernel Widgets Help

Trusted | SageMath 9.4

Run Code

In [1]:
```
f = x^2
plot(f)
```

Out[1]:




In [ ]:

# Other useful features

Also notebook and user mode support Tab completion or help for some commands for instance sudoku? Of course we can just right in python.
If you want to see the source code of a function we just type : sudoku??
Of course you can visit https://github.com/sagemath/sage/

```
Source:
def sudoku(m):
    r"""
    Solves Sudoku puzzles described by matrices.

    INPUT:

    - ``m`` - a square Sage matrix over `\ZZ`, where zeros are blank entries

    OUTPUT:

    A Sage matrix over `\ZZ` containing the first solution found,
    otherwise ``None``.

    This function matches the behavior of the prior Sudoku solver
    and is included only to replicate that behavior.  It could be
    safely deprecated, since all of its functionality is included in the :class:`~sage.games.sudoku.Sudoku` class.
```

# Notebook mode

Also cooperates well with latex. For instance, if we define a matrix `M=matrix([ [2,3],[4,5] ])` with the command `print(latex(M))` we get the latex code of the matrix.

In [142]:
```
1 M=matrix([ [2,3],[4,5] ])
2 print(latex(M))
3 show(M)
```

```
\left(\begin{array}{rr}
2 & 3 \\
4 & 5
\end{array}\right)
```

$$\left(\begin{array}{rr} 2 & 3 \\ 4 & 5 \end{array}\right)$$

# Notebook mode

We can search the well known integer sequence database of N. Sloane

```
search = oeis([1,2,3,5,8,13]) ; search
```

```
In [147]: 1 search = oeis([1,2,3,5,8,13,21]) ; search

Out[147]: 0: A000045: Fibonacci numbers: F(n) = F(n-1) + F(n-2) with F(0) = 0 and F(1) = 1.
          1: A290689: Number of transitive rooted trees with n nodes.
          2: A027926: Triangular array T read by rows: T(n,0) = T(n,2n) = 1 for n >= 0; T(n,1) = 1 for n >= 1; T(n,k) = T(n-1,k-2
```

# Education and Sagemath

Combining Sagemath with jupyter we can prepare our course.

- Tip. We can use latex and Markdown for the text and Sagemath commands to explain math notions.

# Education and Sagemath

- It is suitable both for secondary education and for college.
- The student does not have to download anything, they can use the online calculator : https://sagecell.sagemath.org/

# Education and Sagemath (in greek)

[1] Βρείτε το πεδίο ορισμού της συνάρτησης

$$f(x) = \frac{x+2}{x^2 - 3x + 2}$$

Αυτή η άσκηση υπάρχει στο βιβλίο Γ' Λυκείου του οργανισμού.

# Education and Sagemath

Είναι αρκετό να λύσουμε την εξίσωση,

$$x^2 - 3x + 2$$

```
In [3]:  1  solve(x^2-3*x+2==0,x)
Out[3]: [x == 1, x == 2]
```

Το σύνολο $\mathbb{R} - \{1, 2\}$ ειναι το πεδίο ορισμού της f.

# Education and Sagemath (in greek)

[2] Μια άλλη άσκηση από το βιβλίο του οργανισμού.

Βρείτε το α ώστε f(g(x))=g(f(x)), όπου

$$f = x + 1, g = ax + 2$$

# Education and Sagemath

In [1]:
```
1 var('a')
2 f = x+1;g=a*x+2;
3 f1=f.subs(x=g) #f(g(x))
4 f2=g.subs(x=f).expand() #g(f(x))
5 f1,f2
```

Out[1]: (a*x + 3, a*x + a + 2)

In [5]:
```
1 solve(f1==f2,a)
```

Out[5]: [a == 1]

# Education and Sagemath

[3] Limits in sagemath. Say we want to study if there exists the

$$\lim_{x \to \infty} \sin x$$

We shall compute some values of sin(x) for “large” values of x

# Education and Sagemath

```
def f(x):
    return sin(x)

def table():
    print('|   x   |   f(x)    |')
    print('|-------------------|')
    for x in [50000..50015]:
        print('|%6i | %+f |'%(x, f(x)))

table()
```

```
|   x   |   f(x)    |
|-------------------|
| 50000 | -0.999840 |
| 50001 | -0.555259 |
| 50002 | +0.399825 |
| 50003 | +0.987311 |
| 50004 | +0.667069 |
| 50005 | -0.266474 |
| 50006 | -0.955022 |
| 50007 | -0.765527 |
| 50008 | +0.127790 |
| 50009 | +0.903617 |
| 50010 | +0.848663 |
| 50011 | +0.013452 |
| 50012 | -0.834127 |
| 50013 | -0.914813 |
| 50014 | -0.154425 |
| 50015 | +0.747941 |
```

# Education and Sagemath

[4] For symbolic integration, we can use FriCas.

```
def integrate_fricas(f):
    return pretty_print(fricas(integrate(f,x)))
```

In [124]:
```
1 f = 1/sqrt((x^2+1)*(x+1))
2 integrate_fricas(f)
```

$$2\,weierstrassPInverse\left(-\frac{8}{3},\ -\frac{80}{27},\ \frac{3\,x+1}{3}\right)$$

In [125]:
```
1 show(integral(f,x)) #native function of sagemath
```

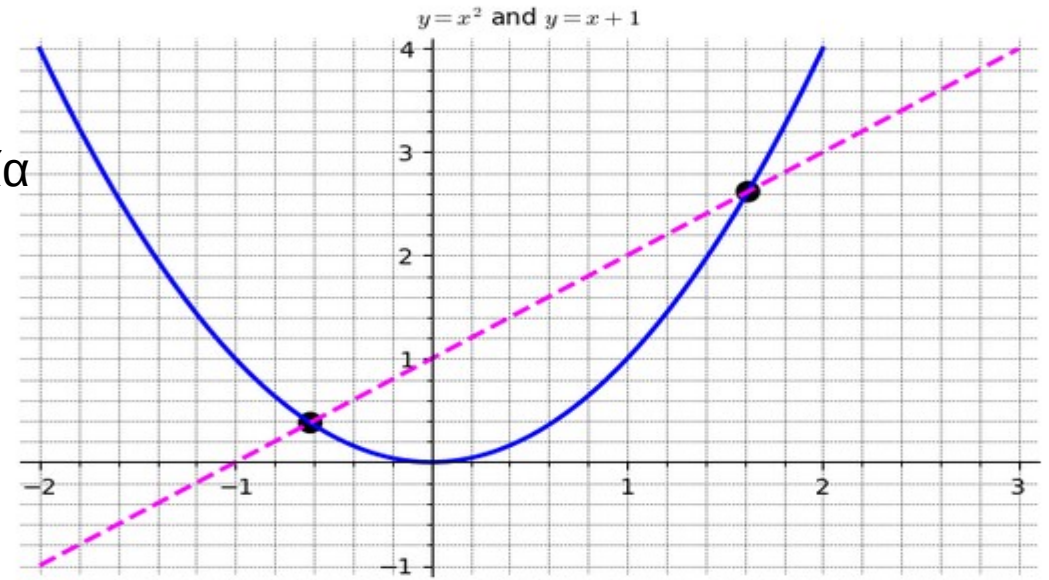
$$\int \frac{1}{\sqrt{(x^2+1)(x+1)}}\,dx$$

# Education and Sagemath

For ploting function is sagemath we can use the awesome library matplatlib which is independent from sagemath. Also we can use the command plot of sagemath.

```
sage:f = x^2;g=x+1;
sage:root1 =  -1/2*sqrt(5) + 1/2;
sage:root2 =   1/2*sqrt(5) + 1/2;
sage:plot(f,x,-2,2,thickness=2,title="$y=x^2$ and $y=x+1$",gridlines='minor') + \
sage:plot(g,x,-2,3,thickness=2,color='magenta',linestyle="--") + \
sage:list_plot([(root1,f.subs(x=root1)), (root2,f.subs(x=root2))],color='black',pointsize=100)
```

Out[68]:




Σχεδιάζουμε τα δύο σημεία root1,root2

# Research and Sagemath

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Language

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## Publications Citing SageMath

Below is a list of publications that cite SageMath and/or the SageMath cluster. This list is also available in BibTeX format. The publications listed in each section are sorted in chronological order. Where two or more items are published in the same year, these items are sorted alphabetically by the authors' last names. See the section Citing SageMath for information on how to cite SageMath and/or the SageMath cluster in your publications.

See also the list of publications citing SageMath-Combinat.

### Articles

1. William Stein and David Joyner. {SAGE}: System for Algebra and Geometry Experimentation. ACM SIGSAM Bulletin, volume 39, number 2, pages 61–64, 2005.
2. Timothy Brock. Linear Feedback Shift Registers and Cyclic Codes in SAGE. Rose-Hulman Undergraduate Mathematics Journal, volume 7, number 2, 2006.
3. John Cremona. The Elliptic Curve Database for Conductors to 130000. In Florian Hess, Sebastian Pauli, and Michael Pohst (ed.). ANTS VII: Proceedings of the 7th International Symposium on Algorithmic Number Theory. Springer, Lecture Notes in Computer Science, volume 4076, pages 11–29, 2006.
4. David Joyner and Amy Ksir. Automorphism Groups of Some AG codes. IEEE Transactions on Information Theory, volume 52, pages 3325–3329, 2006.
5. Barry Mazur. Controlling Our Errors. Nature, volume 443, number 7, pages 38–40, 2006.
6. Jaap Spies. Dancing School Problems. Nieuw Archief voor Wiskunde, volume 5/7, number 4, pages 283–285, 2006.
7. Baur Bektemirov, Barry Mazur, William Stein, and Mark Watkins. Average Ranks of Elliptic Curves: Tension Between Data and Conjecture. Bulletin of the AMS, volume 44, number 2, pages 233–254, 2007.

pages 729–745, 2012.

198. Martin Albrecht, Pooya Farshim, Jean-Charles Faugère, Gottfried Herold, and Ludovic Perret. Polly Cracker, Revisited. Electronic Colloquium on Computational Complexity, volume 19, pages 165, 2012.
199. Jennifer S. Balakrishnan and Amnon Besser. Computing Local $p$-adic Height Pairings on Hyperelliptic Curves. International Mathematics Research Notices, volume 2012, number 11, pages 2405–2444, 2012.
200. Pilar Benito, Daniel de-la-Concepción, and others. {Sage computations of $sl_2(k)$-Levi extensions}. Tbilisi Mathematical Journal, volume 5, number 2, pages 3–16, 2012.
201. Chris Berg, Nantel Bergeron, Steven Pon, and Mike Zabrocki. Expansions of $k$-Schur Functions in the Affine nilCoxeter Algebra. Electronic Journal of Combinatorics, volume 19, number 2, pages P55, 2012.
202. Chris Berg, Nantel Bergeron, Hugh Thomas, and Mike Zabrocki. Expansion of $k$-Schur functions for maximal $k$-rectangles within the affine nilCoxeter algebra. Journal of Combinatorics, volume 3, number 3, pages 563–589, 2012.
203. Corentin Boissy and Erwan Lanneau. Pseudo-Anosov Homeomorphisms on Translation Surfaces in Hyperelliptic Components Have Large Entropy. Geometric and Functional Analysis, volume 22, number 1, pages 74–106, 2012.
204. Felix Breuer. Ehrhart $f^*$-Coefficients of Polytopal Complexes are Non-

406 articles
41 theses
45 books
61 preprints

# Research and Sagemath

Many researchers use Sagemath.

They use sagemath for doing
[1] Combinatorics
[2] Algebra
[3] Numerical analysis
[4] Linear Algebra
[5] Number Theory
[6] Algebraic Geometry
[7] Diophantine Equations
[8] Calculus
[9] Graph Theory

# How to install Sagemath locally

- We suppose that we have GNU/linux in our PC.

  If for some reason the online calculator does not satisfy us, then we have the

- Easy way :

  [1] Go to https://repology.org/project/sagemath/versions
  and check if your linux distribution supports Sagemath. All major distributions Arch/Debian/Fedora/Manjaro/Ubuntu support sagemath.

- Not so easy way :

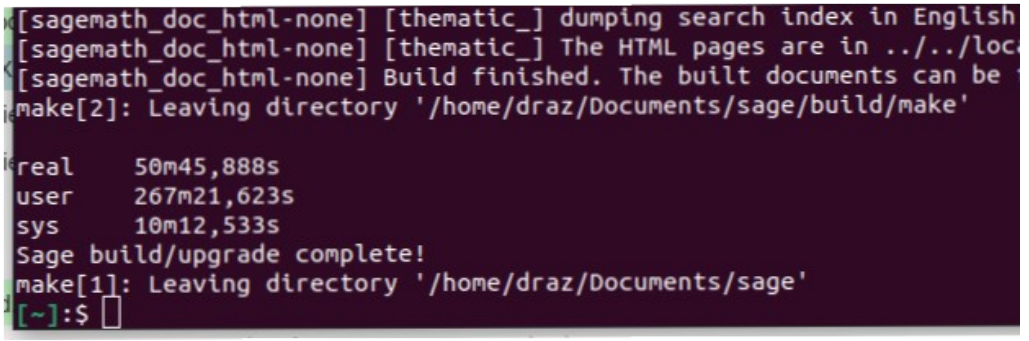
  [2] Compile from source

# How to install Sagemath locally

- To compile from source :
- Clone the latest github dev repo

```
$ORIG=https://github.com/sagemath/sage.git
$git clone -c core.symlinks=true --branch develop --tags $ORIG
$cd sage
$make configure
$./ configure
Install necessary packages and then
$./config.status --recheck && ./config.status
$make -j4
```

...drink a coffee...and maybe a second one...



# How to Contribute

- Translate the official Sage tutorial to Greek
- Pay attention to https://trac.sagemath.org/
- join the sage-devel group

# References


[1] J. R. Culham, Computer Alegbra Systems, online
[2] Reduce system, Dep. Of Math. - Univ. of Utah
[3] W. Stein's talk about sagemath, https://wstein.org/talks/2016-06-sage-bp/bp.pdf

# Thank you