Computing for Mathematics: Handout 2

This handout contains a summary of the topics covered and an activity to carry out prior or during your lab session.

At the end of the handout is a specific coursework like exercise.

For further practice you can do the exercises available at the algebra chapter of Python for Mathematics.

1 Summary

The purpose of this handout is to cover Algebra which corresponds to the Algebra chapter of Python for Mathematics.

The topics covered are:

- Creating symbolic numeric values
- Getting numerical value of a symbolic expression
- Factorising an expression
- Expanding an expression
- Simplifying an expression
- Solving an equation
- Substituting values in to expressions

2 Activity

We will be tackling the problem from the tutorial of the Algebra chapter of Python for Mathematics.

1. Rationalise the denominator of $\frac{1}{\sqrt{2}+1}$
2. Consider the quadratic: $f(x) = 2x^2 + x + 1$:
   (a) Calculate the discriminant of the quadratic equation $2x^2+x+1 = 0$. What does this tell us about the solutions to the equation? What does this tell us about the graph of $f(x)$?
   (b) By completing the square, show that the minimum point of $f(x)$ is $\left(-\frac{1}{4}, \frac{7}{8}\right)$

There are instructions for how to do all of this is in the Algebra chapter of Python for Mathematics.

1. Create the variable `expression` which has value $\frac{1}{\sqrt{2}} + 1$.
2. Use the `sympy.simplify` command to rationalise the denominator.
3. Create the variable `expression` which has value the quadratic from the second part of the question: $f(x) = 2x^2+x+1$.
4. Use the `sympy.equation` and `sympy.sovleset` command to find the roots of f.
5. Create the variable `expression` which has value the expression $a(x-b)^2 + c$.
6. Solve the various equations that give the correct values of a, b and c to be able to complete the square for $f(x)$.

3 Coursework like exercise

Consider the equation: $x^2 + 4 - y = \frac{1}{y}$:

1. Create a variable `general_solution` which has value the set of solutions to the equation for x (as a function of y).
2. Create a variable `specific_solution` which has value the set of solutions when $y = 5$.

4 Summary examples

Create the symbolic value $1/3$

```
import sympy
value = 1 / sympy.S(3)
```

Get the numeric value of a symbolic variable $1/3$

```
import sympy
float(value)
```

Factor $x^2 - 81$

```
import sympy
x = sympy.Symbol("x")
sympy.factor(x ** 2 - 81)
```

Expand $(x - 1)(x + 1)$

```
import sympy
x = sympy.Symbol("x")
sympy.expand((x - 1) * (x + 1))
```

Simplify $(x - 3)(x - 3)$

```
import sympy
x = sympy.Symbol("x")
sympy.simplify((x - 3) * (x - 3))
```

Solve $x + 4 = x^2$

```
import sympy
x = sympy.Symbol("x")
equation = sym.Eq(x + 4, x ** 2)
sympy.solveset(equation, x)
```

Substitute $x = -2$ in to $x^2 - 4$

```
import sympy
x = sympy.Symbol("x")
expression = x ** 2 - 4
expression.subs({x: -2})
```