Computing for Mathematics: Handout 4

This handout contains a summary of the topics covered and an activity to carry out prior or during your lab session.

At the end of the handout is a specific coursework like exercise.

For further practice you can do the exercises available at the combinatorics chapter of Python for Mathematics.

1 Summary

The purpose of this handout is to cover combinatorics which corresponds to the combinatorics chapter of Python for Mathematics.

The topics covered are:

- Generating and counting permutations and combinations of elements.
- · Directly computing binomial coefficients.

2 Activity

We will be tackling the problem from the tutorial of the combinatorics chapter of Python for Mathematics.

The digits 1, 2, 3, 4 and 5 are arranged in random order, to form a five-digit number.

- 1. How many different five-digit numbers can be formed?
- 2. How many different five-digit numbers are:
 - (a) Odd
 - (b) Less than 23000

There are instructions for how to do all of this is in the combinatorics chapter of Python for Mathematics.

- 1. Create a variable digits which has value the collection of integers 1, 2, 3, 4 and 5.
- 2. Use the itertools permutations tool to create the variable permutations which has value the permutations of the integers 1, 2, 3, 4 and 5.
- 3. Convert the permutations variable to a tuple and use the len command to count the number of different permutations that exist.
- 4. Use the sum command to calculate:

$$\sum_{\pi \in \Pi} \pi_5 \mod 2$$

where Π is the collection of all permutations.

5. Use the sum command to calculate:

$$\sum_{\pi \in \Pi \text{ if } \pi_1 10^4 + \pi_2 10^3 + \pi_3 10^2 + \pi_4 10 + \pi_5 < 23000} 1$$

where Π is the collection of all permutations.

3 Summary examples

Create the collection: (A, A, B).

```
collection = ("A", "A", "B")
```

Obtain the 2nd element in the collection (A, A, B).

```
collection = ("A", "A", "B")
collection[1]
```

Check a boolean condition like if "C" $\in (A, A, B)$.

```
collection = ("A", "A", "B")
"C" in collection
```

Create the collection of integers from 1 to 11:

```
range(1, 12)
```

Create the permutations of the collection (A, A, B) of size 2.

```
import itertools
collection = ("A", "A", "B")
itertools.permutations(collection, r=2)
```

Create the combinations of the collection (A, A, B) of size 2.

```
import itertools
collection = ("A", "A", "B")
itertools.combinations(collection, r=2)
```

Calculate $\sum_{i=1}^{20} \operatorname{if} i \operatorname{even} i^2$

```
sum(i ** 2 for i in range(1, 21) if (i % 2 == 0))
```

Obtain 5!:

```
import math
math.factorial(5)
```

Obtain $\binom{5}{2}$:

```
import scipy.special
scipy.special.comb(5, 2)
```

Obtain 5P_2 :

```
import scipy.special
scipy.special.perm(5, 2)
```