Computing for Mathematics: Handout 3

This handout contains a summary of the topics covered and an activity to carry out prior or during your lab session.

At the end of the handout is a specific coursework like exercise.

For further practice you can do the exercises available at the calculus chapter of Python for Mathematics.

1 Summary

The purpose of this handout is to cover Calculus which corresponds to the Calculus chapter of Python for Mathematics.

The topics covered are:

- Getting the derivative of a symbolic expression.
- Getting the indefinite integral of a symbolic expression.
- Getting the definite integral of a symbolic expression.
- Getting the limit of a symbolic expression.

2 Activity

We will be tackling the problem from the tutorial of the Calculus chapter of Python for Mathematics.

Consider the function $f(x) = \frac{24x(a-4x)+2(a-8x)(b-4x)}{(b-4x)^4}$

1. Given that $\frac{df}{dx}|_{x=0} = 0$, $\frac{d^2f}{dx^2}|_{x=0} = -1$ and that $b > 0$ find the values of a and b.
2. For the specific values of a and b find:
   (a) $\lim_{x\to 0} f(x)$;
   (b) $\lim_{x\to \infty} f(x)$;
   (c) $\int f(x)dx$;
   (d) $\int_5^{20} f(x)dx$.

There are instructions for how to do all of this is in the Calculus chapter of Python for Mathematics.

1. Create the variable `expression` which has value $f(x) = \frac{24x(a-4x)+2(a-8x)(b-4x)}{(b-4x)^4}$.
2. Use the `sympy.diff` command to obtain the derivative.
3. Create the variable `first_equation` which has value the equation that comes from the first condition of the question: $\frac{df}{dx}|_{x=0} = 0$.
4. Create the variable `second_equation` which has value the equation that comes from the second condition of the question: $\frac{d^2f}{dx^2}|_{x=0} = -1$.
5. Solve both equations (use substitution if you helpful) and recalling that $b > 0$ substitute the correct values of a and b in to `expression`.
6. Obtain the required limits.

3 Coursework like exercise

Consider the second derivative $f''(x) = 4x + \cos(x)$.

1. Create a variable `derivative` which has value $f'(x)$ (use the variables `x` and `c1` if necessary):
2. Create a variable `equation` that has value the equation $f'(0) = 0$.
3. Using the solution to that equation, output the value of $\int_0^{5\pi} f(x)dx$.

4 Summary examples

Calculate the second derivative of $\cos(x^2)$:

```
import sympy as sym
x = sym.Symbol("x")
expression = sym.cos(x ** 2)
sym.diff(expression, x, 2)
```

Calculate the definite integral $\int_0^5 1/x$

```
import sympy as sym
x = sym.Symbol("x")
expression = 1 / x
sym.integrate(expression, (x, 0, 5))
```

Calculate the indefinite integral of $e^(x)$

```
import sympy as sym
x = sym.Symbol("x")
expression = sym.exp(x)
sym.integrate(expression, x)
```

Obtain the limit $\lim_{h\to\infty} \frac{1}{cos^2(x)}$

```
import sympy
x = sympy.Symbol("x")
expression = 1 / (sym.cos(x) ** 2)
sym.limit(expression, x, sym.oo)
```