MATHEMATICS

Time: Two Hours **Max.Marks:60**

(8.30 AM – 10.30 AM)

NOTE:-

1. *Attempt all questions.*
2. *Answer to each question should begin on a fresh page, and rough work must be enclosed with answer book.*
3. *There is no negative marking.*
4. *Answer all parts of a question at one place.*
5. *Use of calculators, slide rule, graph paper and logarithmic, trigonometric and statistical tables is not permitted.*

Note:- All answers to questions in **Section-A, Section-B, Section-C** must be supported by mathematical arguments. In each of these sections order of the parts must be maintained.

SECTION-A

I. **This section has Five questions. Each question is provided with five alternative answers. Only one of them is the correct answer. Indicate the correct answer by A,B,C,D,E.** (5× 2=10 MARKS)

1. Let m be the l.c.m. of $3^{2002}-1$ and $3^{2002}+1$. Then the last digit of m is

A) 0 B) 4 C) 5 D) 8 E) none

Hint: Read off g.c.d. and hence determine the l.c.m.

2. In every cyclic quadrilateral ABCD with AB parallel to CD

A) AD = BC, AC $\neq$ BD B) AD $\neq$ BC, AC $\neq$ BD
C) AD $\neq$ BC, AC = BD D) AD = BC, AC = BD
E) none of these

3. f,g,h:R $\rightarrow$ R are defined by f(x) = x-1, g(x) = $\dfrac{x^2-1}{x+1}$ if x $\neq$ -1 h(x) = $\dfrac{x^3+2x^2-x-2}{x^2+3x+2}$ if x $\neq$ -1, x $\neq$ -2 -2 if x = -1 -2 if x = -1 -3 if x = -2 Then f(x) + g(x) – 2.h(x) is

A) not a polynomial B) a polynomial whose degree is undefined
C) a polynomial of degree 0 D) a polynomial of degree 1
E) none of these

4. A, B are two distinct points on a circle with center C_1 and radius r_1. $\overline{AB}$ also is a chord of a different circle with center C_2 and radius r_2. S denotes the statement: The arc AB divides the first circle into two

parts of equal area.

A) S happens if $C_1C_2 > r_2$

B) If S happens then $C_1C_2 > r_2$

C) If S happens then it is possible that $C_1C_2 = r_2$

D) If S happens then it is necessary that $C_1C_2 < r_2$

E) none of these

5. Triangle T_1 has vertices at (-3,0), (2,0) and (0,4). Triangle T_2 has vertices at (-3,3), (3,5) and (1,-2). Plot all points $l \cap m$ where the line segment l is a side of T_1 and the line segment m is a side of T_2, whenever $l \cap m$ is a unique point. These points of intersection determine a convex polygon, which is a

A) triangle B) quadrilateral C) pentagon D) hexagon E) septagon

SECTION-B

II. This section has Five questions. In each question a blank is left. Fill in the blank. (5× 2=10 MARKS)

1. The number of integral solutions of the equation $x^4 - y^4 = 2002$ is _____

2. A,B,C,D are distinct and concyclic. $\overleftrightarrow{DL}$ is perpendicular to $\overleftrightarrow{BC}$, $\overleftrightarrow{DM}$ is perpendicular to $\overleftrightarrow{AC}$. $\overleftrightarrow{LM}$ intersects $\overleftrightarrow{AB}$ in N. The nonobtuse angle enclosed between $\overleftrightarrow{AB}$ and $\overleftrightarrow{DN}$ is _____ in degrees

3. Let m be the 2002-digit number each digit of which is 6. The remainder obtained when m is divided by 2002 is ______

4. If x is a real number Int.x denotes the greatest integer less than or equal to x.

Int. $\sqrt{5 - (Int. - 2002.2002)}$ = __________

5. O is the origin and P_i is a point on the curve $x^2 + y^2 = i^2$ for any natural number i.

$\sum_{i=1}^{2002} OP_i = OP_j$, where j = _________

SECTION-C

III. This section has Five questions. Each question has a short answer. Elegant solutions will be rewarded. (5× 2=10 MARKS)

1. If m is the right most nonzero digit of $(n!)^4$ where n is a positive integer greater than 1, determine the possible values of m.

2. Sketch the graph of the function f(x) = $\sqrt{x^2 - 4x + 4}$, $1 \le x \le 2$.

3. ΔABC is right angled at A. AB=60, AC=80, BC=100 units. D is a point between B and C such that the triangles ADB and ADC have equal perimeters. Determine the length of AD.

4. Do there exist positive integers p, q, r such that $\frac{p}{q} + \frac{q}{r} + \frac{r}{p} = 1$?

5. k is a constant, $(x_i,0)$ is the mid point of (i-k, 0) and (i+k, 0) for i=1,2,…,2002 then find $\prod_{1 \le i \le j \le 2002} x_i x_j$.

SECTION-D

IV. This section has Five questions. These questions are long answer questions. Elegant solutions will be rewarded. (5× 6=30 MARKS)

1. If m and n are natural numbers such that

$$\frac{m}{n} = \frac{1}{3} + \frac{1}{5} + \frac{1}{17} + \frac{1}{19} + \frac{1}{23} + \frac{1}{1979} + \frac{1}{1983} + \frac{1}{1985} + \frac{1}{1997} + \frac{1}{1999}.$$

Prove that 2002 divides m.

2. ΔABC is right angled at A. Prove that the three mid points of the sides and the foot of the altitude through A are concyclic.

3. Determine all possible finite series in GP with first term 1, common ratio an integer greater than 1 and sum 2002.

4. A convex pentagon ABCDE has the property that the area of each of the five triangles ABC, BCD, CDE, DEA, and EAB is unity. Prove that the area of the pentagon is $\frac{5+\sqrt{5}}{2}$.

5. If a,b,c, are real numbers such that the sum of any two of them is greater than the third and $\sum\{c(a^2+b^2-c^2)\}$ =3abc, then prove that a=b=c.

6. Explain how you will construct a triangle of area equal to the area of a given convex quadrilateral. Use only rough sketches and give proof.