

# Lecture slides

Philip Murray

# Lecture 1

- ▶ Introduction to nonlinear difference equations
- ▶ The Malthusian model
- ▶ The Ricker model

Why difference equations?

## A general model

Consider the first order difference equation

$$N_{t+1} = N_t f(N_t) = H(N_t), \quad (1)$$

where  $f(N_t)$  is a function that defines the per capita growth rate.  
The function  $H(N_t)$  describes the total (net) growth rate.

# The Malthusian model

The population size at time  $t + 1$  is

$$N_{t+1} = N_t + bN_t - dN_t = rN_t,$$

Exercise: solve the Malthusian model and classify qualitative behaviours

# Nonlinear models

- ▶ Beverton-Holt

$$N_{t+1} = \frac{rN_t}{1 + \frac{N_t}{K}},$$

- ▶ Hassell model

$$N_{t+1} = \frac{rN_t}{(1 + \frac{N_t}{K})^b},$$

- ▶ Ricker model

$$N_{t+1} = N_t e^{r(1 - \frac{N_t}{K})}.$$

# Numerical simulation of the Ricker model

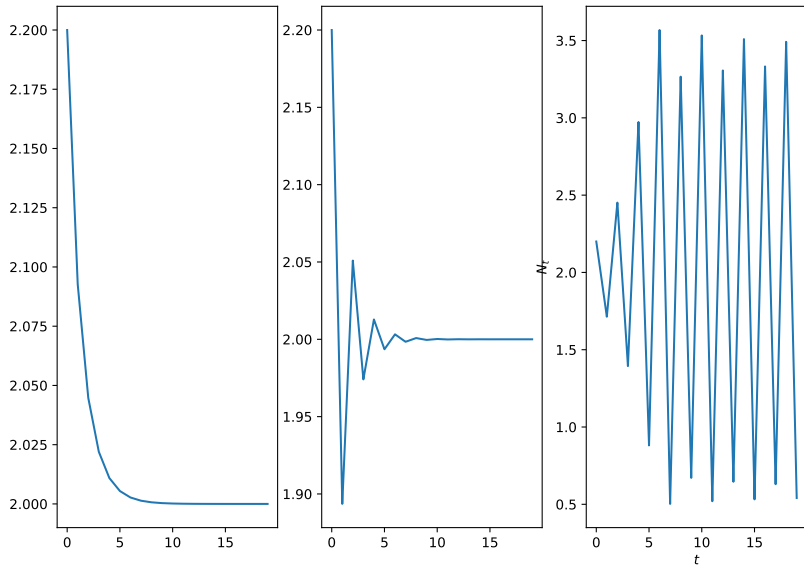


Figure 1: A plot of numerical solutions of the Ricker model. (a)  $r=0.5$ .



# Summary

- ▶ Motivated use of difference equation models
- ▶ Introduced general model for one population
- ▶ Solved the Malthusian model
- ▶ Introduced nonlinear models