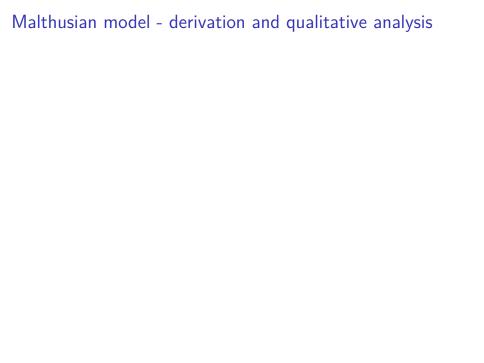
#### MA32009 Lecture slides

Philip Murray

#### Lecture 1



#### A general model

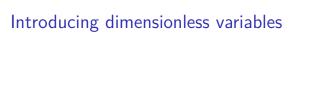
$$\frac{dN}{dt} = f(N)N = H(N),$$

# Numerical solution

#### Dimensions and nondimensionalisation

As an example, consider the linear ODE

$$\frac{dN}{dt} = rN.$$

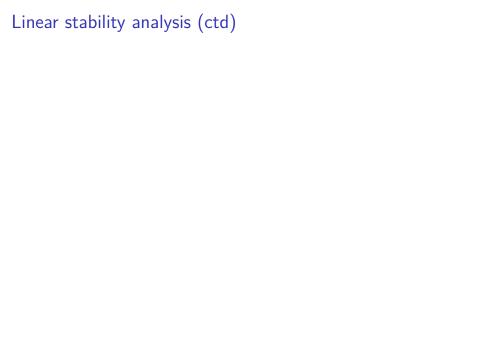


# Steady-state analysis

#### Lecture 10

- Recap
- ► Techniques for single first order ODE (ctd)
- Example model 1: Logistic growth
- Example model 2: Spruce budworm





# Graphical solution

# Bifurcation diagrams

# Example model 1: the logistic growth equation

$$\frac{dN}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right).$$

#### Numerical solution

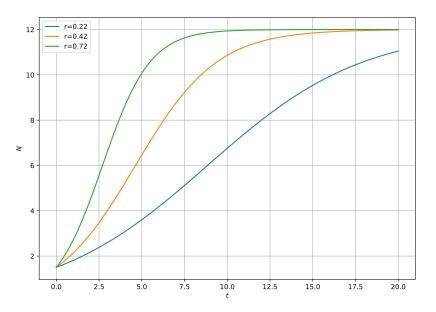
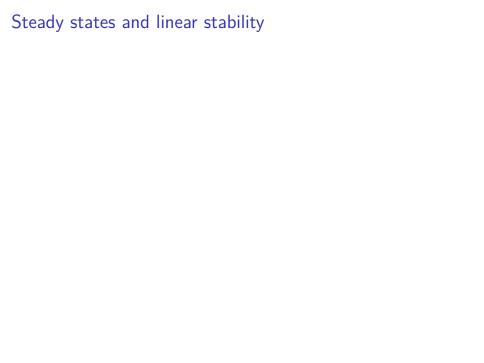
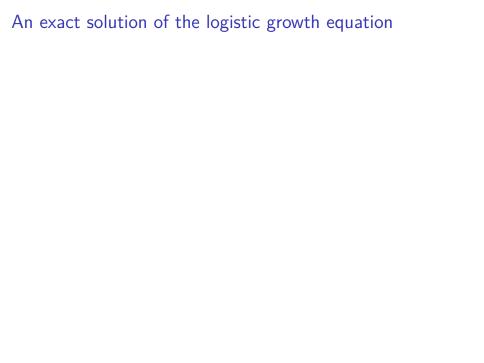


Figure 1: Numerical solution of the logistic growth model



# Graphical analysis



#### Example model 2: the spruce budworm model

$$\frac{dN}{dt} = r_B N \left( 1 - \frac{N}{K_B} \right) - \frac{BN^2}{A^2 + N^2},\tag{1}$$

#### Nondimensionalisation

$$\frac{dn}{d\tau} = rn\left(1 - \frac{n}{q}\right) - \frac{n^2}{1 + n^2} = H(n),$$

## Plotting the RHS

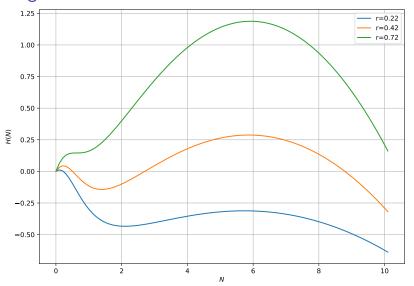
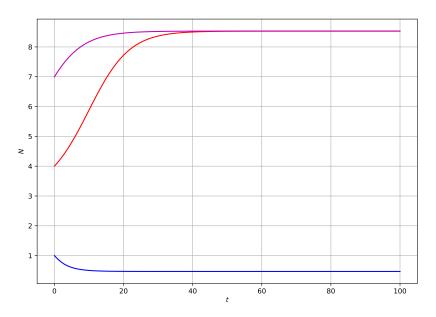


Figure 2: RHS of spr. budworm model

#### Numerical solution



## Steady state analysis

$$rn^*(1 - \frac{n^*}{q}) - \frac{n^{*2}}{1 + n^{*2}} = 0.$$



#### Lecture 12

Recap - Spruce budworm model

$$\frac{dn}{d\tau} = rn\left(1 - \frac{n}{q}\right) - \frac{n^2}{1 + n^2} = H(n),$$

Steady states:  $n^* = 0$  or

$$rn^*(1-\frac{n^*}{q})-\frac{n^{*2}}{1+n^{*2}}=0.$$

- ightharpoonup r small one stable steady state
- ightharpoonup r large one stable steady state (outbreak)
- ightharpoonup r intermediate bistability (two stable steady states and one unstable)

Today: bifurcation analysis, hysteresis, harvesting

# Tangent bifurcations in $\emph{rq}$ space

# Plotting stability regions in the rq plane

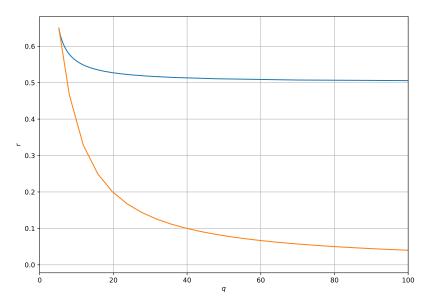
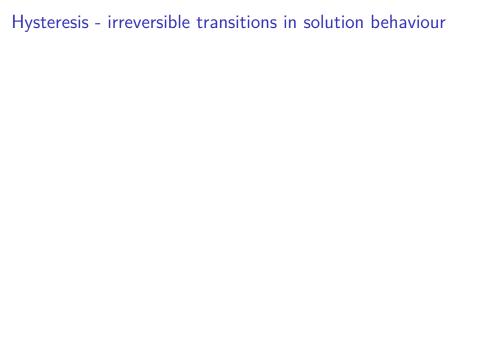


Figure 3: Bifurcations in the rq plane



#### Hysteresis

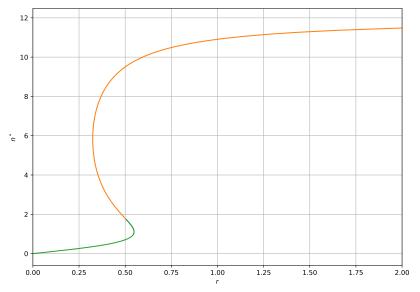


Figure 4: Bifurcations in the rq plane

#### Harvesting

- use models to simulate how much resource can be extracted?
- approach: take model without harvesting and add in harvesting terms

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - EN.$$

where E is the harvesting rate.

Question: what value of  ${\cal E}$  maximises the long term yield?

#### Delay differential equation models

$$\frac{dN}{dt} = H(N(t), N(t-T)),$$