0.1 Takagi-Sugeno Fuzzy Model

The design procedure begins with representing a given non-linear plant by the Takagi-Sugeno fuzzy model. This model is characterized by fuzzy IF-THEN rules which describe local linear input-output relations of a non-linear system. The TS Fuzzy model expresses the local dynamics of each fuzzy rule using a linear system model, while the global model is achieved by combining these linear system models

The i-th fuzzy rules for Continuous Fuzzy Systems (CFS) are of the following forms:

Model Rule i:

IF $z_1(t)$ isM_{i1} and... and$z_p(t)$ isM_{ip}

$$\textbf{THEN} \begin{cases} \dot{x} = A_i x(t) + B_i u(t) \\ y = C_i x, \end{cases} \quad , \quad i = 1, 2, \ldots, r.$$

Here, M_{ij} is the fuzzy set and r is the number of model rules; $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(y) \in \mathbb{R}^q$ is the output vector, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ and $C_i \in \mathbb{R}^{q\times n}$; $z_i(t), \ldots, z_p(t)$ are known premise variables which may be functions of the state variables, external disturbances, and/or time.

Given a pair of $(x(t), u(t))$, the final outputs of the CFS are inferred as follows:

$$\dot{x} = \frac{\sum\limits_{i=1}^{r} w_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum\limits_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\} \tag{1}$$

$$y(t) = \frac{\sum\limits_{i=1}^{r} w_i(z(t)) C_i x(t)}{\sum\limits_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \tag{2}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_p(t)]$,

$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \qquad \text{and} \qquad h_i(z(t)) = \frac{w_i(z(t))}{\sum\limits_{j=1}^{r} w_i(z(t))}, \tag{3}$$

for all time t. The term $M_{ij}(z(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. Since

$$\sum_{i=1}^{r} w_i(z(t)) > 0, \quad w_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r, \tag{4}$$

we have

$$\sum_{i=1}^{r} h_i(z(t)) > 0, \quad h_i(z(t)) \geq 0, \quad i = 1, 2, \ldots, r. \tag{5}$$

Chapter 1

DC-DC Converters Modeling

1.1 Buck Converter

The buck converter circuit is illustrated in Figure 1.1. In this circuit, $v_{\text{in}}(t)$ represents the input voltage, and D denotes the diode. The output voltage filter consists of an inductor with winding resistance R_L and inductance L, and a capacitor with capacitance C. Finally, two types of loads are supplied: a Constant Resistance Load with resistance $R(t)$ and a Constant Power Load (CPL) with power $P_\ell(t)$, represented by a controlled current source.

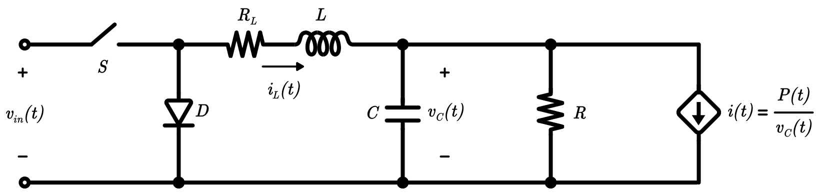

Figure 1.1: Buck converter circuit.

1.1.1 Non-linear Buck Converter Model

When the switch S is in the off state fot the duration t_{off}, the dynamic equations describing the circuit's behavior at this moment, derived from Kirchhoff's laws, are as follows:

$$\begin{cases} \dot{i}_L = -\dfrac{R_L}{L} i(t) - \dfrac{1}{L} v_C(t) \\ \dot{v}_C = \dfrac{1}{C} i_L(t) - \dfrac{1}{CR} v_C(t) - \dfrac{1}{C v_C(t)} P_\ell(t) \end{cases} \tag{1.1}$$

And, when the switch S is in the on state for the duration t_{on},

$$\begin{cases} \dot{i}_L = -\dfrac{R_L}{L} i(t) - \dfrac{1}{L} v_C(t) + \dfrac{1}{L} v_{\text{in}}(t) \\ \dot{v}_C = \dfrac{1}{C} i_L(t) - \dfrac{1}{CR} v_C(t) - \dfrac{1}{C v_C(t)} P_\ell(t) \end{cases} \tag{1.2}$$

Building on the equations (1.1) and (1.2), the average dynamic model that represents the behavior of the buck converter throughout its operation is:

$$\begin{cases} \dot{i}_L = \left[-\dfrac{R_L}{L}i(t) - \dfrac{1}{L}v_C(t)\right]\dfrac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} + \left[-\dfrac{R_L}{L}i(t) - \dfrac{1}{L}v_C(t) + \dfrac{1}{L}v_{\text{in}}(t)\right]\dfrac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} \\ \dot{v}_C = \left[-\dfrac{1}{C}i_L(t) - \dfrac{1}{CR}v_C(t) - \dfrac{1}{Cv_C(t)}P_\ell(t)\right]\dfrac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} + \left[-\dfrac{1}{C}i_L(t) - \dfrac{1}{CR}v_C(t) - \dfrac{1}{Cv_C(t)}P_\ell(t)\right]\dfrac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} \end{cases} \tag{1.3}$$

Defining the following expression:

$$d = \frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}}, \tag{1.4}$$

yields,

$$\frac{t_{\text{off}}+t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} = 1 \Rightarrow \frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} + \frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} = 1 \Rightarrow \frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} = 1 - \frac{t_{\text{on}}}{t_{\text{off}}+t_{\text{on}}} \Rightarrow \frac{t_{\text{off}}}{t_{\text{off}}+t_{\text{on}}} = 1 - d \tag{1.5}$$

Therefore, the equation (1.3) can be rewritten as follows:

$$\begin{cases} \dot{i}_L = \left[-\dfrac{R_L}{L}i(t) - \dfrac{1}{L}v_C(t)\right](1-d) + \left[-\dfrac{R_L}{L}i(t) - \dfrac{1}{L}v_C(t) + \dfrac{1}{L}v_{\text{in}}(t)\right]d \\ \dot{v}_C = \left[-\dfrac{1}{C}i_L(t) - \dfrac{1}{CR}v_C(t) - \dfrac{1}{Cv_C(t)}P_\ell(t)\right](1-d) + \left[-\dfrac{1}{C}i_L(t) - \dfrac{1}{CR}v_C(t) - \dfrac{1}{Cv_C(t)}P_\ell(t)\right]d \end{cases}$$

Then,

$$\begin{cases} \dot{i}_L = -\dfrac{R_L}{L}i(t) - \dfrac{1}{L}v_C(t) + \dfrac{1}{L}v_{\text{in}}(t)d \\ \dot{v}_C = -\dfrac{1}{C}i_L(t) - \dfrac{1}{CR}v_C(t) - \dfrac{1}{Cv_C(t)}P_\ell(t) \end{cases} \tag{1.6}$$

The variable d is commonly known as the switching duty cycle, which plays a crucial role in controlling switch states. Its value can be determined based on the control input $u_d(t)$, defined by:

$$d(u_d(t)) = \max\left\{\min\{u_d(t), 1\}, 0\right\}. \tag{1.7}$$

Choosing the operation point $P^{\text{o}} = (i_L^{\text{o}}, v_C^{\text{o}}, u_d^{\text{o}}, v_{\text{in}}^{\text{o}}, P_\ell^{\text{o}})$, where $u_d^{\text{o}} \in [0,1]$, the following coordinate change can be performed:

$$\delta i_L(t) = i_L(t) - i_L^{\text{o}}, \qquad \delta v_C(t) = v_C(t) - v_C^{\text{o}}, \qquad \delta u_d(t) = u_d(t) - u_d^{\text{o}} \tag{1.8}$$

$$\delta v_{\text{in}}(t) = v_{\text{in}}(t) - v_{\text{in}}^{\text{o}} \qquad \delta P_\ell(t) = P_\ell(t) - P_\ell^{\text{o}} \tag{1.9}$$

Moreover, let the control input saturation be modeled by means of the function $\text{sat} : \mathbb{R} \to [-\upsilon, \upsilon]$ such that:

$$\delta d = \text{sat}(\delta u_d(t)) = \max\left\{\min\left\{\delta u_d(t), \upsilon\right\}, -\upsilon\right\} \tag{1.10}$$

$$\upsilon = \min\left\{1 - u_d^{\text{o}}, u_d^{\text{o}}\right\} \tag{1.11}$$

Thus, the following non-linear model is obtained:

$$\begin{cases} \delta\dot{i}_L = -\dfrac{R_L}{L}\delta i(t) - \dfrac{1}{L}\delta v_C(t) + \dfrac{v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t)}{L}\text{sat}(\delta u_d(t)) + \dfrac{u_d^{\text{o}}}{L}\delta v_{\text{in}}(t) \\ \delta\dot{v}_C = \dfrac{1}{C}\delta i_L(t) + \left[-\dfrac{1}{CR} + \dfrac{P_\ell^o}{Cv_C^{\text{o}}\left[v_C^{\text{o}} + \delta v_C(t)\right]}\right]\delta v_C(t) - \dfrac{1}{C\left[v_C^{\text{o}} + \delta v_C(t)\right]}\delta P_\ell(t) \end{cases} \tag{1.12}$$

where,

$$i_L^{\text{o}} = \frac{1}{R}v_C^{\text{o}} + \frac{1}{v_C^{\text{o}}}P_\ell^{\text{o}}, \qquad u_d^{\text{o}} = \frac{R_L}{v_{\text{in}}^{\text{o}}}i_L^{\text{o}} + \frac{v_C^{\text{o}}}{v_{\text{in}}^{\text{o}}}.$$

Finally, the model (1.12) can be rewritten as:

$$\dot{x} = A(x)x(t) + B(w)\text{sat}(u(t)) + E(x,u)w(t) \tag{1.13}$$

where $x(t) = \begin{bmatrix} i_L(t) & v_C(t) \end{bmatrix}^{\text{T}}$, $u(t) = \delta u_d(t)$, $w(t) = \begin{bmatrix} \delta v_{\text{in}}(t) & \delta P_\ell(t) \end{bmatrix}^{\text{T}}$, and

$$A(x) = \begin{bmatrix} -\dfrac{R_L}{L} & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{CR} + \dfrac{P_\ell^o}{Cv_C^{\text{o}}\left[v_C^{\text{o}} + \delta v_C(t)\right]} \end{bmatrix} \qquad B(w) = \begin{bmatrix} \dfrac{v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t)}{L} \\ 0 \end{bmatrix}$$

$$E(x,u) = \begin{bmatrix} \dfrac{u_d^{\text{o}}}{L} & 0 \\ 0 & -\dfrac{1}{C\left[v_C^{\text{o}} + \delta v_C(t)\right]} \end{bmatrix} \tag{1.13}$$

1.1.2 Buck Converter Fuzzy Model

In order to address the nonlinearity introduced by saturation, an approach based on substituting sat($\circ$) with a dead-zone type nonlinearity is used. The dead-zone nonlinearity is defined as:

$$\psi(u(t)) \triangleq u(t) - \text{sat}(u(t)) \tag{1.14}$$

From the matrices presented in equation (1.13), it follows $\dfrac{1}{v_C^{\text{o}} + \delta v_C^{\text{o}}}$ and $v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t)$ are non-linear terms. For the non-linear terms, are defined

$$z_0(t) \equiv \frac{1}{v_C^{\text{o}} + \delta v_C^{\text{o}}} \qquad \text{and} \qquad z_1(t) \equiv v_{\text{in}}^{\text{o}} + \delta v_{\text{in}}(t). \tag{1.15}$$

Thus, the equation (1.13) can be rewritten as:

$$\dot{x} = A(z(t))x(t) + B(z(t))u(t) - B(w)\psi(u(t)) + E(z(t))w(t), \tag{1.16}$$

where $z(t) = \begin{bmatrix} z_0(t) & z_1(t) \end{bmatrix}$, $A(z(t)) = \begin{bmatrix} -\dfrac{R_L}{L} & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{CR} + \dfrac{P_\ell^o}{Cv_C^o} z_0(t) \end{bmatrix}$, $B(z(t)) = \begin{bmatrix} \dfrac{1}{L} z_1(t) \\ 0 \end{bmatrix}$ and $E(z(t)) = \begin{bmatrix} \dfrac{u_d^o}{L} & 0 \\ 0 & -\dfrac{1}{C} z_0(t) \end{bmatrix}$.

Next, the minimum and maximum values of $z_0(t)$ e $z_1(t)$ under $v_C(t) \in \left[v_C^{\min}, v_C^{\max}\right]$ and $v_{\text{in}}(t) \in \left[v_{\text{in}}^{\min}, v_{\text{in}}^{\max}\right]$, are obtained as follows:

$$\begin{cases} z_i^0 & = \min\limits_{v_C(t), v_{\text{in}}(t)} z_i(t) \\ z_i^1 & = \max\limits_{v_C(t), v_{\text{in}}(t)} z_i(t) \end{cases}, \quad i = 1, 2 \tag{1.17}$$

From the maximum and minimum values of $z_0(t)$ and $z_1(t)$, the membership functions can be calculated as:

$$z_i(t) = \sum_{j=0}^{1} M_i^j(z_i(t)) z_i^j, \qquad \text{where} \quad M_i^1 = \frac{z_i(t) - z_i^0}{z_i^1 - z_i^0} \quad \text{and} \quad M_i^0 = 1 - M_i^1, \qquad \text{for} i = \{1, 2\}. \tag{1.18}$$

Therefore, the Takagi-Sugeno fuzzy model for the buck converter is:

$$\dot{x} = \sum_{i=0}^{1} \sum_{j=0}^{1} \prod_{k=1}^{2} M_k^i(z_k(t)) \left[A\left(z^{\{i,j\}}\right) x(t) + B\left(z^{\{i,j\}}\right) u(t) - B\left(z^{\{i,j\}}\right) \psi(u(t)) + E\left(z^{\{i,j\}}\right) w(t) \right] \tag{1.19}$$

where $z^{\{i,j\}}(t)$ is a shorthand for $\begin{bmatrix} z_1^i(t) & z_2^j(t) \end{bmatrix}$.

The summations in (1.19) can be aggregated as one summations:

$$\dot{x} = \sum_{p=\{0,0\}}^{\{1,1\}} h_p(z(t)) \left\{ A_p x(t) + B_p u(t) - B_p \psi(u(t)) + E_p w(t) \right\} \tag{1.20}$$

where $p = \{b_1, b_2\} \in \mathbb{B}^2$, $\mathbb{B} = \{0, 1\}$,

$$h_p(z(t)) = \prod_{k=1}^{2} M_k^{b_k}(z_k(t)) \tag{1.21}$$

$$A_p = A(z^p) \qquad\qquad B_p = B(z^p) \qquad\qquad E_p = E(z^p) \tag{1.22}$$

Chapter 2

Event-based Control for DC-DC Converters

2.1 Problem formulation

Dynamic system:

$$\dot{x} = \sum_{p \in \mathbb{B}^n} h_p(z(t)) \left\{ A_p x(t) + B_p u(t) - B_p \psi(u(t)) + E_p w(t) \right\} \tag{2.1}$$

$$h_p(z(t)) = \prod_{k=1}^{n} M_k^{b_k}(z_k(t)) \tag{2.2}$$

$$A_p = A(z^p) \qquad B_p = B(z^p) \qquad E_p = E(z^p) \tag{2.3}$$

Control law:

$$u(t) = K(\hat{x})\hat{x}(t) + L(\hat{x})\hat{w}(t) \Rightarrow u(t) = \sum_{p \in \mathbb{B}^n} h_p(z(t)) \left\{ K_p \hat{x}(t) + L_p \hat{w}(t) \right\} \tag{2.4}$$

Consider a vector $v \in \mathbb{R}^{n_u}$ such that the following set $\mathcal{S}$ can be defined:

$$\mathcal{S} = \{ u, v \in \mathbb{R}^{n_u} : |u_i - v_i| \leq u_{0i},\ i = 1, \ldots, n_u \} \tag{2.5}$$

If u and $v \in \mathcal{S}$, then the relation

$$\psi(u)^{\mathrm{T}} T \left[\psi(u) - v \right] \leq 0 \tag{2.6}$$

is satisfied for any diagonal and positive definite matrix $T \in \mathbb{R}^{n_u \times n_u}$.

$$v = G(\hat{x})x(t) = \sum_{p \in \mathbb{B}^n} h_p(z(t)) \left\{ G_p \hat{x}(t) \right\} \tag{2.7}$$

If $x \in \mathcal{S}$, such that

$$\mathcal{S} = \{x \in \mathbb{R}^n : |K(\hat{x})\hat{x}(t) + L(\hat{x})\hat{w}(t) - G(\hat{x})\hat{x}(t)| \leq u_{0i},\ i = 1, \ldots, n_u\} \tag{2.8}$$

$$\mathcal{S} = \{x \in \mathbb{R}^n : |\left[K(\hat{x}) - G(\hat{x})\right]\hat{x}(t) + L(\hat{x})\hat{w}(t)| \leq u_{0i},\ i = 1, \ldots, n_u\} \tag{2.9}$$

$$\mathcal{S} = \left\{x \in \mathbb{R}^n : \left|\sum_{p \in \mathbb{B}^n} h_p(z(t))\left\{(K_p - G_p)\hat{x}(t) + L_p\hat{w}(t)\right\}\right| \leq u_{0i},\ i = 1, \ldots, n_u\right\} \tag{2.10}$$

then,

$$\psi^{\mathrm{T}}(K(\hat{x})x(t) + L(\hat{x})\hat{w})T\left[\psi(K(\hat{x})\hat{x} + L(\hat{x})\hat{w}) - G(\hat{x})\hat{x}\right] \leq 0 \tag{2.11}$$

$$\psi^{\mathrm{T}}(u(t))T\left[\psi(u(t)) - G(\hat{x})\hat{x}\right] \leq 0 \tag{2.12}$$

$$\psi^{\mathrm{T}}(u(t))T\psi(u(t)) - \psi^{\mathrm{T}}(u(t))TG(x(t) + e_x(t))\left[x(t) + e_x(t)\right] \leq 0 \tag{2.13}$$

$$\begin{aligned}\psi^{\mathrm{T}}(u(t))T\psi(u(t)) - \psi^{\mathrm{T}}(u(t))TG(x(t))x(t) - \psi^{\mathrm{T}}(u(t))TG(x(t))e_x(t)&\\ - \psi^{\mathrm{T}}(u(t))TG(e_x(t))x(t) - \psi^{\mathrm{T}}(u(t))TG(e_x(t))e_x(t) \leq 0&\end{aligned} \tag{2.14}$$

$$\begin{aligned}\psi^{\mathrm{T}}(u(t))T\psi(u(t)) - \psi^{\mathrm{T}}(u(t))TG(x(t))x(t) - \psi^{\mathrm{T}}(u(t))TG(x(t))e_x(t)&\\ - \psi^{\mathrm{T}}(u(t))T\left[G(x(t) + e_x(t)) - G(x)\right]\left[x(t) + e_x(t)\right] \leq 0&\end{aligned} \tag{2.15}$$

$$\psi^{\mathrm{T}}(u(t))T\psi(u(t)) - \psi^{\mathrm{T}}(u(t))TG(x(t))x(t) - \psi^{\mathrm{T}}(u(t))TG(x(t))e_x(t) - \psi^{\mathrm{T}}(u(t))TG(e_x(t))\left[x(t) + e_x(t)\right] \leq 0 \tag{2.16}$$

2.1.1 Sistema

Seja $e_x(t) = \hat{x}(t) - x(t)$ and $e_w(t) = \hat{w}(t) - w(t)$. Têm-se,

$$\dot{x} = A(x)x(t) + B(x)u(t) - B\psi(u(t)) + E(x)w(t) \tag{2.17}$$

$$\dot{x} = A(x)x(t) + B(x)\left[K(\hat{x})\hat{x}(t) + L(\hat{x})\hat{w}(t)\right] - B\psi(u(t)) + E(x)w(t) \tag{2.18}$$

$$\dot{x} = A(x)x(t) + B(x)K(\hat{x})\hat{x}(t) + B(x)L(\hat{x})\hat{w}(t) - B\psi(u(t)) + E(x)w(t) \tag{2.19}$$

$$\dot{x} = A(x)x(t) + B(x)K(\hat{x}(t))\left[e_x(t) + x(t)\right] \\ + B(x)L(\hat{x}(t))\left[e_w(t) + w(t)\right] - B\psi(u(t)) + E(x)w(t) \quad (2.20)$$

$$\dot{x} = \left[A(x) + B(x)K(x(t))\right]x(t) + B(x)K(x(t))e_x(t) + B(x)K(e_x(t))\left[e_x(t) + x(t)\right] \\ + \left[E(x) + B(x)L(x(t))\right]w(t) + B(x)L(x(t))e_w(t) + B(x)L(e_x(t))\left[e_w(t) + w(t)\right] \\ - B\psi(u(t)) \quad (2.21)$$

ETM dinâmico:

$$t_0 = 0, t_{k+1} = \inf\{t > t_k : \eta(t) + \theta\Gamma(x(t), e_x(t), e_w(t)) < 0\},\ \forall k \in \mathbb{N}. \quad (2.22)$$

Função de ativação:

$$\Gamma(x, e_x) = x^T(t)\Psi x(t) - e_x^T \Xi_x e_x - \zeta(x, e_x) \quad (2.23)$$

onde,

$$\zeta(x, e_x) = 2x^T P\left[B(x)\left(K(x + e_x) - x\right)(x + e_x)\right] \quad (2.24)$$

LMI de restrição,

$$\begin{bmatrix} \mathbf{He}(A_iX + B_i\tilde{K}_j) + (1-\beta)I & E_iX + B_i\tilde{L}_j & B_i\tilde{K}_j & B_i\tilde{L}_j & -B_iX - \frac{1}{2}XTG_iX & X \\ \star & -(\mu-\alpha)I & 0 & 0 & 0 & 0 \\ \star & \star & -\tilde{\Xi} - \beta & 0 & -\frac{1}{2}XTG_iX & 0 \\ \star & \star & \star & \alpha I & 0 & 0 \\ \star & \star & \star & \star & \tilde{T} & 0 \\ \star & \star & \star & \star & \star & -\tilde{\Psi} \end{bmatrix} < 0 \quad (2.25)$$

Multiplicando por $diag(X^{-1}, X^{-1}, X^{-1}, X^{-1}, I)$, e fazendo as mudança de variáveis: $K_j = \tilde{K}_jX^{-1}$, $L_j = \tilde{L}_jX^{-1}$, $\Xi = X^{-1}\tilde{\Xi}X^{-1}$, $T = X^{-1}\tilde{T}X^{-1}$, $P = X^{-1}$

$$\begin{bmatrix} \Omega & P\left[E(x) + B(x)L(x)\right] & PB(x)K(x) & PB(x)L(x) & -PB(x) - \frac{1}{2}TG(x) & I \\ \star & -(\mu-\alpha)I & 0 & 0 & 0 & 0 \\ \star & \star & -\Xi - \beta & 0 & -\frac{1}{2}TG(x) & 0 \\ \star & \star & \star & \alpha I & 0 & 0 \\ \star & \star & \star & \star & T & 0 \\ \star & \star & \star & \star & \star & -\tilde{\Psi} \end{bmatrix} < 0 \quad (2.26)$$

onde $\Omega = P\,\mathbf{He}(A(x) + B(x)K(x)) + (1-\beta)I$. Pelo complemento de Schur,

$$\begin{bmatrix} \Omega+\Psi & P\left[E(x)+B(x)L(x)\right] & PB(x)K(x) & PB(x)L(x) & -PB(x)-\frac{1}{2}TG(x) \\ \star & -(\mu-\alpha)I & 0 & 0 & 0 \\ \star & \star & -\Xi-\beta & 0 & -\frac{1}{2}TG(x) \\ \star & \star & \star & \alpha I & 0 \\ \star & \star & \star & \star & T \end{bmatrix} < 0 \tag{2.27}$$

Considerando $v = \begin{bmatrix} x^T(t) & w^T(t) & e_x^T(t) & e_w^T(t) & \psi(u)^T \end{bmatrix}$, pré e pós-multiplicando por v e v^T, respectivamente, obtêm-se:

$$\begin{aligned} &2x^T(t)P\left\{\left[A(x)+B(x)K(x(t))\right]x(t)+\left[E(x)+B(x)L(x(t))\right]w(t)+B(x)K(x(t))e_x(t)+B(x)L(x(t))e_w(t)\right. \\ &\left.-B(x)\psi(u(t))\right\}-\psi^T(u(t))TG(x)x(t)+(1-\beta)x^T(t)x(t)-(\mu-\alpha)w^T(t)w(t)+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t) \\ &-\beta e_x^T(t)e_x(t)+\alpha e_w^T(t)e_w(t)+\psi^T(u(t))T\psi(u(t))-\psi^T(u(t))TG(x)e_x(t)<0 \end{aligned} \tag{2.28}$$

Onde $\alpha \in \mathbb{R}_{>0}$ é tal que:

$$2x^T PB(x)L(e_x(t))\left[w(t)+e_w(t)\right] \le \alpha^{-1}\|x^T(t)PB(x)L(e_x)\|^2+\alpha\|w(t)+e_w(t)\|^2 \tag{2.29}$$

$$2x^T PB(x)L(e_x(t))\left[w(t)+e_w(t)\right] \le \alpha^{-1}\|x^T(t)PB(x)L(e_x)\|^2+\alpha\|w(t)\|^2+\alpha\|w(t)\|\|e_w(t)\|+\alpha\|e_w(t)\|^2 \tag{2.30}$$

$$-\alpha^{-1}\|x^T(t)PB(x)L(e_x)\|^2-\alpha\|w(t)\|^2-\alpha\|w(t)\|\|e_w(t)\|-\alpha\|e_w(t)\|^2 \le -2x^T PB(x)L(e_x(t))\left[w(t)+e_w(t)\right] \tag{2.31}$$

Assim,

$$\begin{aligned} &2x^T(t)P\left\{\left[A(x)+B(x)K(x(t))\right]x(t)+\left[E(x)+B(x)L(x(t))\right]w(t)+B(x)K(x(t))e_x(t)+B(x)L(x(t))e_w(t)\right. \\ &\left.-B(x)\psi(u(t))\right\}-\psi^T(u(t))TG(x)x(t)+(1-\beta)x^T(t)x(t)-\mu w^T(t)w(t)+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)-\beta e_x^T(t)e_x(t) \\ &+\psi^T(u(t))T\psi(u(t))-\psi^T(u(t))TG(x)e_x(t)<-\alpha w^T(t)w(t)-\alpha e_w^T(t)e_w(t) \end{aligned} \tag{2.32}$$

$$\begin{aligned} &2x^T(t)P\left\{\left[A(x)+B(x)K(x(t))\right]x(t)+\left[E(x)+B(x)L(x(t))\right]w(t)+B(x)K(x(t))e_x(t)+B(x)L(x(t))e_w(t)\right. \\ &\left.-B(x)\psi(u(t))\right\}-\psi^T(u(t))TG(x)x(t)+(1-\beta)x^T(t)x(t)-\mu w^T(t)w(t)+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)-\beta e_x^T(t)e_x(t) \\ &+\psi^T(u(t))T\psi(u(t))-\psi^T(u(t))TG(x)e_x(t)-\alpha\|w(t)\|\|e_w(t)\|-\alpha^{-1}\|x^T(t)PB(x)L(e_x)\|^2 \\ &<-\alpha w^T(t)w(t)-\alpha e_w^T(t)e_w(t)-\alpha\|w(t)\|\|e_w(t)\|-\alpha^{-1}\|x^T(t)PB(x)L(e_x)\|^2 \end{aligned} \tag{2.33}$$

$$
\begin{aligned}
&2x^T(t)P\left\{[A(x)+B(x)K(x(t))]\,x(t)+[E(x)+B(x)L(x(t))]\,w(t)+B(x)K(x(t))e_x(t)+B(x)L(x(t))e_w(t)\right.\\
&\left.-B(x)\psi(u(t))\right\}-\psi^T(u(t))TG(x)x(t)+(1-\beta)x^T(t)x(t)-\mu w^T(t)w(t)+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)-\beta e_x^T(t)e_x(t)\\
&+\psi^T(u(t))T\psi(u(t))-\psi^T(u(t))TG(x)e_x(t)-\alpha\|w(t)\|\|e_w(t)\|-\alpha^{-1}\|x^T(t)PB(x)L(e_x)\|^2\\
&\qquad< -2x^TPB(x)L(e_x(t))\,[w(t)+e_w(t)]
\end{aligned}
\tag{2.34}
$$

$$
\begin{aligned}
&2x^T(t)P\left\{[A(x)+B(x)K(x(t))]\,x(t)+[E(x)+B(x)L(x(t))]\,w(t)+B(x)K(x(t))e_x(t)+B(x)L(x(t))e_w(t)\right.\\
&\left.+B(x)L(e_x(t))\,[w(t)+e_w(t)]-B(x)\psi(u(t))\right\}-\psi^T(u(t))TG(x)x(t)+(1-\beta)x^T(t)x(t)-\mu w^T(t)w(t)\\
&+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)-\beta e_x^T(t)e_x(t)+\psi^T(u(t))T\psi(u(t))-\psi^T(u(t))TG(x)e_x(t)-\alpha\|w(t)\|\|e_w(t)\|\\
&-\alpha^{-1}\|x^T(t)PB(x)L(e_x)\|^2<0
\end{aligned}
\tag{2.35}
$$

$$
\begin{aligned}
&2x^T(t)P\left\{[A(x)+B(x)K(x(t))]\,x(t)+[E(x)+B(x)L(x(t))]\,w(t)+B(x)K(x(t))e_x(t)+B(x)L(x(t))e_w(t)\right.\\
&\left.+B(x)L(e_x(t))\,[w(t)+e_w(t)]-B(x)\psi(u(t))\right\}-\psi^T(u(t))TG(x)x(t)+(1-\beta)x^T(t)x(t)-\mu w^T(t)w(t)\\
&+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)-\beta e_x^T(t)e_x(t)+\psi^T(u(t))T\psi(u(t))-\psi^T(u(t))TG(x)e_x(t)<0
\end{aligned}
\tag{2.36}
$$

Somando $\zeta(x(t),e_x(t))$, têm-se:

$$
\begin{aligned}
&2x^T(t)P\dot{x}(t)-\psi^T(u(t))TG(x)x(t)+(1-\beta)x^T(t)x(t)-\mu w^T(t)w(t)\\
&+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)-\beta e_x^T(t)e_x(t)+\psi^T(u(t))T\psi(u(t))-\psi^T(u(t))TG(x)e_x(t)<0
\end{aligned}
\tag{2.37}
$$

De forma semelhante, $\beta\in\mathbb{R}_{>0}$ é tal que,

$$
\psi^T(u(t))G(e_x(t))\,[x(t)+e_x(t)]\le\beta^{-1}\|\psi(u(t))G(e_x(t))\|^2+\beta\|x(t)+e_x(t)\|^2
\tag{2.38}
$$

$$
\psi^T(u(t))G(e_x(t))\,[x(t)+e_x(t)]-\beta^{-1}\|\psi(u(t))G(e_x(t))\|^2-\beta\|x(t)\|^2-\beta\|x(t)\|\|e_x(t)\|-\beta\|e_x(t)\|^2\le 0
\tag{2.39}
$$

Logo,

$$
\begin{aligned}
&2x^T(t)P\dot{x}(t)+x^T(t)x(t)-\mu w^T(t)w(t)+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)-\beta x^T(t)x(t)+\psi^T(u(t))T\psi(u(t))\\
&-\psi^T(u(t))TG(x)x(t)-\psi^T(u(t))TG(x)e_x(t)-\psi^T(u(t))G(e_x(t))\,[x(t)+e_x(t)]\\
&+\psi^T(u(t))G(e_x(t))\,[x(t)+e_x(t)]-\beta e_x^T(t)e_x(t)<0
\end{aligned}
\tag{2.40}
$$

De, ,

$$
\begin{aligned}
&2x^T(t)P\dot{x}(t)+x^T(t)x(t)-\mu w^T(t)w(t)+x^T(t)\Psi x(t)-e_x^T(t)\Xi e_x(t)\\
&+\psi^T(u(t))G(e_x(t))\,[x(t)+e_x(t)]-\beta x^T(t)x(t)-\beta e_x^T(t)e_x(t)<0
\end{aligned}
\tag{2.41}
$$

$$\begin{aligned}2x^T(t)P\dot{x}(t) + x^T(t)x(t) - \mu w^T(t)w(t) + x^T(t)\Psi x(t) - e_x^T(t)\Xi e_x(t) + \psi^T(u(t))G(e_x(t))\,[x(t) + e_x(t)]\\ - \beta x^T(t)x(t) - \beta e_x^T(t)e_x(t) - \beta\|x(t)\|\|e_x(t)\| - \beta^{-1}\|\psi(u(t))G(e_x(t))\|^2 < 0\end{aligned} \tag{2.42}$$

$$2x^T(t)P\dot{x}(t) + x^T(t)x(t) - \mu w^T(t)w(t) + x^T(t)\Psi x(t) - e_x^T(t)\Xi e_x(t) < 0 \tag{2.43}$$

Como $\lambda\eta(t) > 0$, então:

$$2x^T(t)P\dot{x}(t) + x^T(t)x(t) - \mu w^T(t)w(t) + x^T(t)\Psi x(t) - e_x^T(t)\Xi e_x(t) - \lambda\eta(t) < 0 \tag{2.44}$$

Logo,

$$\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \dot{\eta}(t) + x^T(t)x(t) - \mu w^T(t)w(t) < 0 \tag{2.45}$$

$$\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \dot{\eta}(t) < 0 \tag{2.46}$$

$$\dot{W}(x,\eta) < 0 \tag{2.47}$$

Portanto, a origem do sistema em malha fechada sob o ETM dinâmico é estável.