Argumentation Theory
Nonmonotonic Reasoning

BACHELOR'S THESIS

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Statutory Declaration

Hereby I assure that this thesis is a result of my personal work and that no other than the indicated aids have been used for its completion.

Furthermore I assure that all the quotations and statements, that have been inferred literally or in a general manner from published or unpublished writings, are marked as such.

Beyond this I assure that the work has not been used, neither completely nor in parts, to pass any previous examination.

Vienna, April 28, 2015

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Christoph Groß

Acknowledgements

Abstract

Throughout this thesis, I am going to explore the applications of argumentation theory in the context of nonmonotonic reasoning.

First off, I will introduce the necessary formal definitions, give a few easy examples as clarification and draw the theoretical conclusions.

In the next step, the reader is given an overview over the real-life applications of argumentation theory. Finally I will append the documentation of an application specifically written during the course of this thesis, which purpose is to graphically help understanding the subject.

Chapter 1

Introduction

1.1 Nonmonotonic Reasoning

When talking about nonmonotonic reasoning, one is referring to the process of deducing knowledge within a formal logic, whose consequence relation does not fulfill the requirements of monotonicity.

Elaboration 1.1 (Monotonicity)
If for a given set of formulas ψ and a formula A the logical consequence $\psi \vdash A$ holds, then $\psi, B \vdash A$ still holds.

As one can see, nonmonotonic sets of formulas might change their deducable knowledge as new facts are added.

1.2 Argumentation Theory

In a human context, we (hopefully) argue to reach a sound conclusion. We do that by claiming statements based on premises, and usually, due to different perception and other factors, these statements tend to vary in their degree of believability.

Argumentation Theory, from a logical point of view, tries to formalize these strategies and introduces rules to that purpose. Therefore arguments are modeled as formulas which might or might not attack each other. For instance, the most believeable argument out of two, so to say the "winner", would be the one, that attacks the other argument, and at the same time, does not get attacked by said argument.
In the following chapter, I will provide all the necessary definitions to achieve the above mentioned aim.

1.2.1 Argumentation Frameworks

Definition 1.1 (Argumentation Framework)
$$AF := (AR, att) \blacksquare$$

An argumentation framework AF is defined as a tuple, consisting of a set of arguments and a set of attack relations. The following example should help illustrating this construct a little better.

Example 1.1
$s =$"Superman is the best superhero!", $b =$"Batman is way cooler.."
$AR = \{s, b\}$, $att = \{(b, s)\}$ $\blacksquare$

Since the only tuple in att is the one that represents b attacking s, the (frankly obvious) outcome is that b is the winning argument.

1.2.2 Properties of (Sets of) Arguments

Definition 1.2 (Conflict-freeness)
A set S of arguments is said to be conflict-free, if for any argument $a \in S$, there is no other argument $b \in S$ so that $(a, b) \in att$. $\blacksquare$

Definition 1.3 (Acceptability)
An argument $a \in AR$ is acceptable with respect to a set S, iff S attacks any argument $b \in AR$ that attacks a. $\blacksquare$

Remark
If for said argument a is not being attacked by any $b \in AR$, then it is acceptable with even the empty set. $\blacksquare$

Definition 1.4 (Admissibility)
A conflict-free Set S is said to be admissible, iff each argument $a \in S$ is acceptable with respect to S. Often, S is also called an admissible extension. $\blacksquare$

These are the basic properties that can be fulfilled by arguments or sets of arguments. Based on them, we will now introduce a few more interesting definitions. For instance, argumentation theory knows a number of different extensions, which can be used to model the degree of "credulity" of an agent.

Before we discuss those extensions and their implications on argumentation theory, I would like to make a detour and discuss the conventional meaning of extensions in predicate logic.

Elaboration 1.2 (Extensions)
For any n-ary predicate Φ, its extension can be written as $\{(x_1, ..., x_n)|\Phi(x_1, ..., x_n)\}$.
So, the extension of a predicate is nothing other than the maximum set of tuples
fulfilling said predicate. ∎

The aforementioned extensions, adapted fo fit the needs of argumentation theory, use this idea in combination with certain selective criteria to model the desired behaviour. The first extension takes a rather gullible approach to things; the so called *preferred extension* is defined as follows:

Remark
From here on, we will use E_i to denote an arbitrary but fixed extension of type i, while E_I denotes the set of all extensions of type i.
Also, if we want to talk about the set of all extensions of one specific type, we will use $i(AF)$. ∎

Definition 1.5 (Preferred Extensions)
E_p is a preferred extension of AF, iff E_p is a maximum admissible set of AF. ∎

As you can see, the preferred extension is made up of the biggest, conflict-free set of arguments, that can be defended against all other arguments not in the set.
The following lemma will be useful later on.

Lemma 1.1
For every AF, there exists at least one preferred extension. ∎

Proof
The proof will be depending on two different properties of admissible sets:

1. *The empty set is always admissible.*

2. *Given an admissible Set S and an argument a, which is acceptable with respect to S, the unification $S \cup a$ is also admissible.*

(1) *directly follows from the definition 1.4*
(2) *We just have to show, that $S \cup a$ is conflict-free. To achieve that, we assume the contrary:*
There exists an argument $b \in S$ so that either (a, b) or (b, a) holds. There has to be an argument $c \in S$, so that either (c, a) or (c, b). We know, that S is conflict-free, so $(c, b) \implies \lightning$
On the other hand, c and a are both acceptable with respect to S; $(c, a) \implies \lightning$. ∎

Another approach would be the *stable extension*, which is defined as follows:

Definition 1.6 (Stable Extensions)
The conflict-free set E_s is a stable extension of AF, iff E_s attacks each argument $a \notin E_s$; which is equivalent to $\{a | a$ is not attacked by $E_s\}$. ∎

Apparently, the stable extension contains all arguments that it does not attack itself. Therefore it has to be a conflict-free, admissible set.
We will explore the differences between those extensions in a later chapter.

For now, there are two more types of extensions, which have to be introduced.

Definition 1.7 (Complete Extensions)
The admissible set E_c is a complete extension of AF, iff every argument which is acceptable with respect to E_c, is included in E_c. ∎

It is easy to see that many different complete extensions can exist, depending on E_c. That leads directly to the *grounded extension*.

Definition 1.8 (Grounded Extensions)
The intersection of all complete extensions is called the grounded extension.
$$E_g := \bigcap comp(AF)$$ ∎

All those types of extensions can be originally found in [**Dung**]. Next, we will take a look at two more extensions, one originally introduced in [**CaCaDu**], the other found in [**DuMaTo**].

Definition 1.9 (Semi-stable Extensions)
A set E_{ss} is a semi-stable extensions, if it is a complete extension and the set
$E_{ss} \cup \{a \in AR | E_{ss}$ *attacks* $a\}$ *is maximal (with respect to set inclusion).* ∎

Definition 1.10 (Ideal Extensions)
A set E_i is an ideal extension, if it is the maximal admissible set, that is contained in every preferred extension. ∎

Since all the necessary, basic definitions of argumentation frameworks have been laid out, I will go on and provide the promised examples.

1.2.3 Examples of different Extensions

By expanding example 1.1 with a few more arguments, we now have to consider the following example.

Example 1.2
$s_1 =$ ”Superman is the best superhero!”, $b_1 =$ ”Batman is way cooler..”,
$s_2 =$ ”But Superman is stronger.”, $b_2 =$ ”Batman possesses genius-level intellect.”
$AR = \{s_1, s_2, b_1, b_2\}$, $att = \{(b_1, s_1), (s_2, b_1), (b_2, s_2)\}$ ∎

This argumentation framework could also be depicted by following graph.
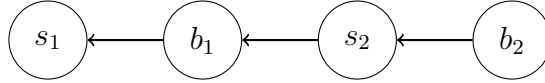
Figure 1.1: graph of AF (example 1.2)

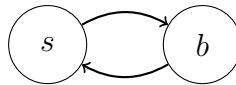The extensions of this example would be

- $E_P = E_S = E_C = E_G = \{\{b_1, b_2\}\}$

The next example is going to be a little more interesting in its outcome.

Example 1.3
$s =$ ”Superman was born a hero.”, $b =$ ”Batman trained hard to become a hero.”
$AR = \{s, b\}$, $att = \{(s, b), (b, s)\}$ ∎

This examples graph looks like this.

Figure 1.2: graph of AF (example 1.3)

Following extensions can be calculated

- $E_P = E_S = \{\{s\}, \{b\}\}$
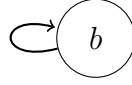- $E_C = \{\{\emptyset\}, \{s\}, \{b\}\}$
- $E_G = \{\{\emptyset\}\}$

The following example (on the contrary to the above) has different preferred and stable extensions.

Example 1.4
$b =$ "Batman is Clark Kent."
$AR = \{b\}$, $att = \{(b, b)\}$ ∎

The corresponding graph looks like follows.

Figure 1.3: graph of AF (example 1.4)

For this example, the extensions would look as follows.

- $E_P = E_C = E_G = \{\{\emptyset\}\}$

- $E_S = \{\emptyset\}$

Example 1.5
Another good example, specifically showing that not every semi-stable extension is also stable, was provided in [$\boldsymbol{CaCaDu}$].
In the following graph, the set $\boldsymbol{E_{ss}} = \{\boldsymbol{b, d}\}$ is semi-stable but not stable. ∎
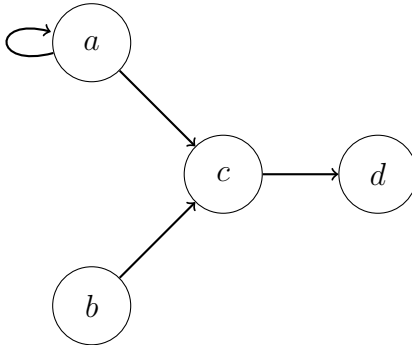
Figure 1.4: semi-stable but not stable (example 1.5)

Example 1.6

The following example illustrates ideal extensions (and how they differ from e.g. grounded extensions).

In the following graph, the set $E_i = \{b\}$ is an ideal extension. This example can also be found in [DuMaTo]. ∎
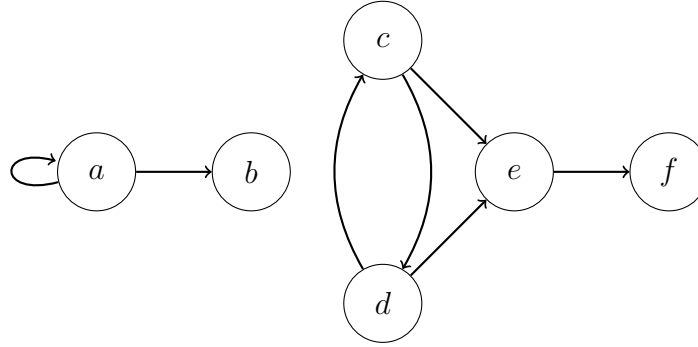
Figure 1.5: ideal extension (example 1.6)

As we can see, the ideal extension $E_i = \{b\}$ is a subset of every preferred extension $(\{b, cf\}, \{b, d, f\})$.

Chapter 2

Conclusions and Implications
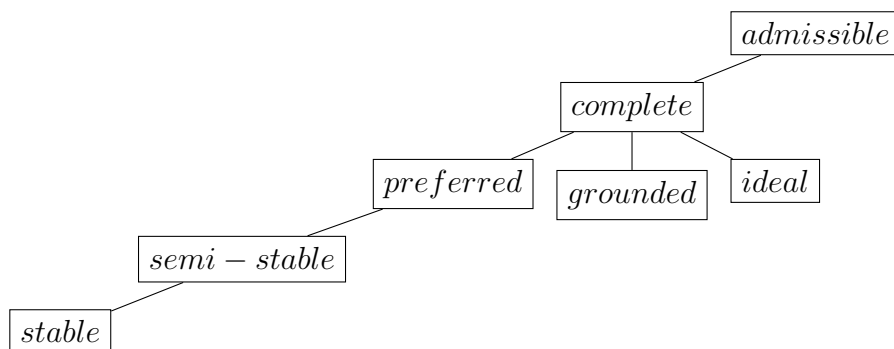
Now that we have established the basic definitions, it is time to travel further down the rabbit hole and explore the meaning of what we just defined.

2.1 Relations between different Extensions

The following, simple tree graph (borrowed from [**HuGo**]) shows the hierarchy of extension types better than words ever could describe it.

Figure 2.1: hierarchy of extension types

Next, I want to clarify a few relations between those aforementioned extensions, since they can be reused in later proofs.

Lemma 2.1

1. *Every preferred extension is a complete extension, but not vice versa.*

2. *The ground extension is the least (with respect to set inclusion) complete extension*

■

Proof (1)
It is obvious that every preferred extension (maximum admissible set) is a complete extension. So we only need to show that the other way round is not true.

Assumption: Every complete extension is also a preferred extension.
Let $\boldsymbol{AR} = \{\boldsymbol{a}, \boldsymbol{b}\}$ and $\boldsymbol{att} = \{(\boldsymbol{a}, \boldsymbol{b}), (\boldsymbol{b}, \boldsymbol{a})\}$.
Then it is easy to see, that $\boldsymbol{E_P} = \{\{\boldsymbol{a}\}, \{\boldsymbol{b}\}\}$ but $\boldsymbol{E_C} = \{\emptyset, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}\}$
$\implies \exists \boldsymbol{E_c} : \boldsymbol{E_c} \in \boldsymbol{E_C} \wedge \boldsymbol{E_c} \notin \boldsymbol{E_P} \implies \lightning$ ■

Proof (2)
Proof for this lemma follows directly from definition 1.8. ■

2.2 Properties of Argumentation Frameworks

Up to now, we have only talked about sets of arguments and their properties. Now we will focus on the whole argumentation framework and make a connection between its properties and extensions.

2.2.1 Well-Founded Argumentation Frameworks

The first new property establishes a coherence between preferred, grounded and stable extensions.

Definition 2.1 (Well-Founded Argumentation Frameworks)
An argumentation framework is considered well-founded, iff there exists no infinit sequence of arguments $A_0, A_1, ..., A_n, ...$ where each A_{i+1} attacks A_i. ■

Remark
This infinite sequence of arguments can be formed by a finite set of arguments (for instance $AR = \{a, b\}, att = (a, b), (b, a)$). ■

Although it might not be the most intuitive observation, following lemma can be extracted from this definition.

Lemma 2.2
Every well-founded AF has exactly one complete extension, which is also a preferred, grounded and stable extension. ■

Proof
Assumption: $\exists \boldsymbol{AF} : AF$ is well-founded and $\boldsymbol{E_p} = \boldsymbol{E_c} = \boldsymbol{E_g} \notin \boldsymbol{E_S}$. We further assume, that $\exists \boldsymbol{a} \in \boldsymbol{AR} \backslash \boldsymbol{E_g}$ and $\nexists \boldsymbol{b} \in \boldsymbol{E_g} : (\boldsymbol{b}, \boldsymbol{a}) \in \boldsymbol{att}$.

If $\boldsymbol{a}$ is not acceptable with respect to $\boldsymbol{E_g}$, this means that there has to be another argument $\boldsymbol{c} \in \boldsymbol{AR} \backslash \boldsymbol{E_g}$, which is also not attacked by $\boldsymbol{E_g}$.
Therefore an infinite sequence of arguments like $\boldsymbol{a}$ and $\boldsymbol{c}$ has to exist $\implies \notmid$. ■

2.2.2   Coherent Argumentation Frameworks

If an AF has a preferred extension, which is not a stable one too, that indicates some sort of "anomaly" in the framework (see example 1.4).
So what we want to do next, is to find out what criteria has to be met so that preferred and stable extension coincide.

Definition 2.2 (Coherent Argumentation Frameworks)
An AF is called coherent, if $\boldsymbol{E_P} = \boldsymbol{E_S}$. ■

Remark
It follows directly from the definition, that there always exists at least one stable extension in a coherent AF. ■

Chapter 3

Argumentation Theory and Nonmonotonic Reasoning

3.1 Default Logic

Chapter 4

Further Topics

Chapter 5

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