1)

a)

1) $\lim\limits_{n\to\infty} \frac{T_1(n)}{T_2(n)} = \lim\limits_{n\to\infty} \frac{3\log n+3}{4\log(\log n)} \xrightarrow{\text{L' Hopital}} = \lim\limits_{n\to\infty} \frac{\frac{3}{n}}{\frac{4}{n.\ln n}} = \lim\limits_{n\to\infty} \frac{3}{\cancel{n}} \cdot \frac{\cancel{n}\ln n}{4} = \infty$

So, $T_2(n) \in O(T_1(n))$

2) $\lim\limits_{n\to\infty} \frac{T_1(n)}{T_3(n)} = \lim\limits_{n\to\infty} \frac{3\log n+3}{n^5+8n^4} \xrightarrow{\text{L' Hopital}} = \lim\limits_{n\to\infty} \frac{\frac{3}{n}}{5n^4+32n^3} = \lim\limits_{n\to\infty} \frac{3}{n} \cdot \frac{1}{5n^4+32n^3} = 0$

So, $T_1(n) \in O(T_3(n))$

3) $\lim\limits_{n\to\infty} \frac{T_1(n)}{T_4(n)} = \lim\limits_{n\to\infty} \frac{3\log n+3}{2000n+1} \xrightarrow{\text{L' Hopital}} = \lim\limits_{n\to\infty} \frac{3}{n} \cdot \frac{1}{2000} = 0$

So, $T_1(n) \in O(T_4(n))$

4) $\lim\limits_{n\to\infty} \frac{T_1(n)}{T_5(n)} = \lim\limits_{n\to\infty} \frac{3\log n+3}{\left(\frac{n}{6}\right)^2} \xrightarrow{\text{L' Hopital}} = \lim\limits_{n\to\infty} \frac{3}{n} \cdot \frac{1}{\left(2\cdot\frac{n}{6}\cdot\frac{1}{6}\right)} = 0$

So, $T_1(n) \in O(T_5(n))$

5) $\lim\limits_{n\to\infty} \frac{T_1(n)}{T_6(n)} = \lim\limits_{n\to\infty} \frac{3\log n+3}{3^n+n^2} \xrightarrow{\text{L' Hopital}} = \lim\limits_{n\to\infty} \frac{3}{n} \cdot \frac{1}{3^n\cdot\ln 3+2n} = 0$

So, $T_1(n) \in O(T_6(n))$

6) $\lim\limits_{n\to\infty} \frac{T_1(n)}{T_7(n)} = \lim\limits_{n\to\infty} \frac{3\log n+3}{n^n+1000n} \xrightarrow{\text{L' Hopital}} = \lim\limits_{n\to\infty} \frac{3}{n} \cdot \frac{1}{n^n\cdot(\ln n+1)+1000} = 0$

So, $T_1(n) \in O(T_7(n))$

7) $\lim\limits_{n\to\infty} \frac{T_1(n)}{T_8(n)} = \lim\limits_{n\to\infty} \frac{3\log n+3}{2^n+n^3} \xrightarrow{\text{L' Hopital}} = \lim\limits_{n\to\infty} \frac{3}{n} \cdot \frac{1}{2^n\cdot\ln 2+3n^2} = 0$

So, $T_1(n) \in O(T_8(n))$

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b)

1) $\lim_{n\to\infty} \frac{T_2(n)}{T_3(n)} = \lim_{n\to\infty} \frac{4\log(\log n)}{n^5+8n^4} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{4}{n.\ln n} \cdot \frac{1}{5n^4+32n^3} = 0$

So, $T_2(n) \in O(T_3(n))$

2) $\lim_{n\to\infty} \frac{T_2(n)}{T_4(n)} = \lim_{n\to\infty} \frac{4\log(\log n)}{2000n+1} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{4}{n.\ln n} \cdot \frac{1}{2000} = 0$

So, $T_2(n) \in O(T_4(n))$

3) $\lim_{n\to\infty} \frac{T_2(n)}{T_5(n)} = \lim_{n\to\infty} \frac{4\log(\log n)}{\left(\frac{n}{6}\right)^2} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{4}{n.\ln n} \cdot \frac{1}{\left(2\cdot\frac{n}{6}\cdot\frac{1}{6}\right)} = 0$

So, $T_2(n) \in O(T_5(n))$

4) $\lim_{n\to\infty} \frac{T_2(n)}{T_6(n)} = \lim_{n\to\infty} \frac{4\log(\log n)}{3^n+n^2} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{4}{n.\ln n} \cdot \frac{1}{3^n.\ln 3+2n} = 0$

So, $T_2(n) \in O(T_6(n))$

5) $\lim_{n\to\infty} \frac{T_2(n)}{T_7(n)} = \lim_{n\to\infty} \frac{4\log(\log n)}{n^n+1000n} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{4}{n.\ln n} \cdot \frac{1}{n^n(\ln n+1)+1000} = 0$

So, $T_2(n) \in O(T_7(n))$

6) $\lim_{n\to\infty} \frac{T_2(n)}{T_8(n)} = \lim_{n\to\infty} \frac{4\log(\log n)}{2^n+n^3} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{4}{n.\ln n} \cdot \frac{1}{2^n.\ln 2+3n^2} = 0$

So, $T_2(n) \in O(T_8(n))$

c)

1) $\lim_{n\to\infty} \frac{T_3(n)}{T_4(n)} = \lim_{n\to\infty} \frac{n^5+8n^4}{2000n+1} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{5n^4+32n^3}{2000} = \infty$

So, $T_4(n) \in O(T_3(n))$

2) $\lim_{n\to\infty} \frac{T_3(n)}{T_5(n)} = \lim_{n\to\infty} \frac{n^5+8n^4}{\left(\frac{n}{6}\right)^2} \xrightarrow{L'Hopital} = \lim_{n\to\infty} \frac{5n^4+32n^3}{2\,\frac{n}{6}\cdot\frac{1}{6}} = \lim_{n\to\infty} \frac{5n^3+32n^2}{\frac{1}{6}} = \infty$

So, $T_5(n) \in O(T_3(n))$

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3) $\lim_{n\to\infty} \frac{T_3(n)}{T_6(n)} = \lim_{n\to\infty} \frac{n^5+8n^4}{3^n+n^2} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{5n^4+32n^3}{3^n.\ln 3+2n} \xrightarrow{\text{After 4 L'Hopital}} \lim_{n\to\infty} \frac{120}{3^n.(\ln 3)^4} = 0$

So, $T_3(n) \in O(T_6(n))$

4) $\lim_{n\to\infty} \frac{T_3(n)}{T_7(n)} = \lim_{n\to\infty} \frac{n^5+8n^4}{n^n+1000n} \xrightarrow{\text{L'Hopital}} \frac{5n^4+32n^3}{n^n.(\ln n+1)+1000} \xrightarrow{\text{After 4 L'Hopital}} \frac{120}{n^n.(\ln n+1)^4} = 0$

So, $T_7(n) \in O(T_3(n))$

5) $\lim_{n\to\infty} \frac{T_3(n)}{T_8(n)} = \lim_{n\to\infty} \frac{n^5+8n^4}{2^n+n^3} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{5n^4+32n^3}{2^n.\ln 2+3n^2} \xrightarrow{\text{After 4 L'Hopital}} \lim_{n\to\infty} \frac{120}{2^n.(\ln 2)^4} = 0$

So, $T_3(n) \in O(T_8(n))$

d)

1) $\lim_{n\to\infty} \frac{T_4(n)}{T_5(n)} = \lim_{n\to\infty} \frac{2000n+1}{\left(\frac{n}{6}\right)^2} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{2000}{2.\frac{n}{6}.\frac{1}{6}} = 0$

So, $T_4(n) \in O(T_5(n))$

2) $\lim_{n\to\infty} \frac{T_4(n)}{T_6(n)} = \lim_{n\to\infty} \frac{2000n+1}{3^n+n^2} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{2000}{3^n.\ln 3+2n} = 0$

So, $T_4(n) \in O(T_6(n))$

3) $\lim_{n\to\infty} \frac{T_4(n)}{T_7(n)} = \lim_{n\to\infty} \frac{2000n+1}{n^n+1000n} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{2000}{n^n.(\ln n+1)+1000} = 0$

So, $T_4(n) \in O(T_7(n))$

4) $\lim_{n\to\infty} \frac{T_4(n)}{T_8(n)} = \lim_{n\to\infty} \frac{2000n+1}{2^n+n^3} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{2000}{2^n.\ln 2+3n^2} = 0$

So, $T_4(n) \in O(T_8(n))$

e)

1) $\lim_{n\to\infty} \frac{T_5(n)}{T_6(n)} = \lim_{n\to\infty} \frac{\left(\frac{n}{6}\right)^2}{3^n+n^2} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{2.\frac{n}{6}.\frac{1}{6}}{3^n.\ln 3+2n} \xrightarrow{\text{L'Hopital}} \lim_{n\to\infty} \frac{\frac{1}{18}}{3^n(\ln 3)^2+2} = 0$

So, $T_5(n) \in O(T_6(n))$

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2) $\lim_{n\to\infty} \frac{T_5(n)}{T_7(n)} = \lim_{n\to\infty} \frac{\left(\frac{n}{6}\right)^2}{n^n + 1000n} \xrightarrow{\text{L'Hopital}} = \lim_{n\to\infty} \frac{2 \cdot \frac{n}{6} \cdot \frac{1}{6}}{n^n \cdot (\ln n + 1) + 1000} = 0$

So, $T_5(n) \in O(T_7(n))$

3) $\lim_{n\to\infty} \frac{T_5(n)}{T_8(n)} = \lim_{n\to\infty} \frac{\left(\frac{n}{6}\right)^2}{2^n + n^3} \xrightarrow{\text{L'Hopital}} = \lim_{n\to\infty} \frac{2 \cdot \frac{n}{6} \cdot \frac{1}{6}}{2^n \cdot \ln 2 + 3n^2} \xrightarrow{\text{L'Hopital}} = \lim_{n\to\infty} \frac{\frac{2}{36}}{2^n \cdot (\ln 2)^2 + 6n} = 0$

So, $T_5(n) \in O(T_8(n))$

f)

1) $\lim_{n\to\infty} \frac{T_6(n)}{T_7(n)} = \lim_{n\to\infty} \frac{3^n + n^2}{n^n + 1000n}$ $\quad 3^{2n} > 3^n + n^2$, $n^n < n^n + 1000n$ $\Rightarrow \lim_{n\to\infty} \frac{3^{2n}}{n^n} = \lim_{n\to\infty} \left(\frac{9}{n}\right)^n = 1$

So, $T_6(n) \in O(T_7(n))$

2) $\lim_{n\to\infty} \frac{T_6(n)}{T_8(n)} = \lim_{n\to\infty} \frac{3^n + n^2}{2^n + n^3}$ $\quad 3^n < 3^n + n^2$, $2^{2n} > 2^n + n^3$ $\Rightarrow \lim_{n\to\infty} \frac{3^n}{2^{2n}} = \lim \left(\frac{3}{4}\right)^n = \infty$

So, $T_8(n) \in O(T_6(n))$

g)

1) $\lim_{n\to\infty} \frac{T_7(n)}{T_8(n)} = \lim_{n\to\infty} \frac{n^n + 1000n}{2^n + n^3}$ $\quad n^n < n^n + 1000n$, $2^{2n} > 2^n + n^3$ $\Rightarrow \lim_{n\to\infty} \frac{n^n}{2^n} = \lim \left(\frac{n}{2}\right)^n = \infty$

So, $T_8(n) \in O(T_7(n))$

Therefore, $O(T_2(n)) \subset O(T_1(n)) \subset O(T_4(n)) \subset O(T_5(n)) \subset O(T_3(n)) \subset O(T_8(n)) \subset O(T_6(n)) \subset O(T_7(n))$

2)

a) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{99n}{n} = 99 \Rightarrow f(n) \in \Theta(g(n))$

b) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2n^4 + n^2}{(\log n)^6} \Rightarrow \lim_{n\to\infty} \frac{2n^4 + n^2}{n^3} \xrightarrow{\text{L'Hopital 3 times}} \Rightarrow \lim_{n\to\infty} \frac{48n}{6} = \infty$

$\Rightarrow f(n) \in \Omega(g(n))$

* $\sqrt{n} > \log n$

c) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\sum_{x=1}^{n} x}{4n + \log n} = \lim_{n\to\infty} \frac{n \cdot \frac{n+1}{2}}{4n + \log n} = \lim_{n\to\infty} \frac{n^2 + n}{8n + 2\log n} = \lim_{n\to\infty} \frac{2n+1}{8 + \frac{2}{n}} = \infty$

$\Rightarrow f(n) \in \Omega(g(n))$

d) It is not found.

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3) a) Gets an array and size of the array as parameters. Counts how many same elements there are. If the number of an element is bigger than the half of the size of the array, returns the element. If there are not identical elements as many as the half of the size of the array, returns -1.

b) Worst case occurs when the number of identical elements is less than the half of the size of the array. The time complexity of worst case is $O(n^2)$. Best case occurs when there are identical elements as many as the half of the size of the array to first element of array. The time complexity is $O(n)$.

4)
a) Gets an array and size of the array as parameters and finds the max number in the nums array. Creates a new array with size of one more of max number in nums array. After that, counts how many same elements are in nums array. For example, nums[0] is 2, then increases map[2] by 1. Finally, check every elements in map array, if one of the elements is bigger than half of size of the nums array, returns the repeating value. Otherwise, returns -1.

b) The best and worst case time complexities of this function is $O(n)$. Because first and second loop iterate n times regardless of best case and worst case. So, the general time complexity is $\Theta(n)$.

5) Time complexity of second function is better than first function. Because in worst case, second function has $O(n)$ time complexity, whereas first function has $O(n^2)$. But their best cases are almost equal. In best case, second function takes $\Theta(n)$ times but first function takes $\Theta(n)$ at most. In terms of space complexity, first function better than second function. First function uses $O(n)$ memory whereas second function's space complexity can be anything according to the max number in array.

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6)

a)

```
maxA = A[0]
maxB = B[0]
For i=1  i<n
    If  maxA < A[i]
          maxA = A[i]
For  i=1  i<m
    If  maxB < B[i]
          maxB = B[i]
Return  maxA * maxB
```

Note: Works when there is no negative element.

Best and worst case time complexity is depends on max of n and m. So time complexity is $\Theta(max(n,m))$.

b)

```
Declare an array X size of n+m integers
For  i=0   i<n+m
      max= A[0]
      For  J=1  J<n
           If  max < A[J]
                max = A[J]
                index = J
                First = true
      For  k=0   k<m
           If  max < B[k]
                max = B[k]
                index = k
                first = false
      If  first == true
           A[index] = -1
      else
           B[index] = -1
      X[i] = max
```

Best and worst case time complexity is depends on (m+n). max of m and n. So time complexity is $\Theta((m+n).max(m,n))$

c)

```
For  i=n   i>position
      A[i] = A[i-1]
A[position] = element
```

Best case occurs when the element is added to the end of the array. It takes $O(1)$. Worst case occurs when the element is added to the beginning of the array. It takes $O(n)$. Avarage is $O(n)$.

d)

```
For  i=position  i<n
      A[i] = A[i+1]
A[position] = null
```

Best case occurs when the element is deleted from the end of the array. It takes $O(1)$. Worst case occurs when the element is deleted from the beginning of the array. It takes $O(n)$. Avarage is $\Theta(n)$.

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