

AGEC 317

Problem Set 2

Show all your work for this assignment in a new document and submit in-class on **February 4th, 2020**. This assignment is worth 5 points. No late work will be accepted.

Solve the following systems of equations for x and y :

1.

$$\begin{aligned}x + y &= 2 \\2x + 3y &= 10\end{aligned}$$

2.

$$\begin{aligned}\frac{1}{2}x + \frac{3}{4}y &= 1 \\4x + 10y &= 15\end{aligned}$$

3.

$$\begin{aligned}4\mu + 3\sigma &= 12 \\8\mu - 2\sigma &= 10\end{aligned}$$

Take the first derivative of the following functions with respect to x ($\frac{\partial y}{\partial x}$):

4. $y = 2 + 3x$

5. $y = 12 - 2x + 4x^2 - 12x^3$

6. $y = e^{2\psi + \beta - 0.5x}$

7. $y = \beta \ln(x)$

Perform the following linear algebra operations:

8.

$$\begin{bmatrix} 1 & 3 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ -12 & -1 \end{bmatrix}$$

9.

$$\begin{bmatrix} 0 & 12 \\ -2 & 12 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

10. Let A be represented by the following matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 10 \end{bmatrix}$$

Find A^{-1}

Solve the following optimization problems:

11. Suppose Toni drives for Uber. Her daily profit (*Profit*) is a function of the number of routes (*Routes*) she makes in a night. Let her profit function be:

$$\text{Profit} = 250\text{Routes} - 3\text{Routes}^2$$

What is the number of routes that will maximize her profit?

12. Suppose you own a local power utility, and can produce electricity from one of two plants: a natural gas generator (output in MW = x) and a coal-fired plant (output in MW = y). Suppose you need to produce 40 MW in the next hour. Suppose further that your cost of production is:

$$C(x, y) = x^2 + 2y^2 - xy$$

What is the optimal output of MW from the natural gas and coal-fired generators? (*Hint: don't forget about the 40MW constraint. This is a cost-minimization problem.*)

13. Suppose *Starbucks* produces high-quality roasted coffee beans using only two inputs: raw coffee beans and human labor. Let the revenue of the company be represented by:

$$R(b, h) = \frac{3}{2}b^{\frac{2}{3}}h^{\frac{1}{3}}$$

where b represents the amount of raw coffee beans in pounds (tons), h represents hours of human labor, and R is the revenue. If raw coffee beans cost \$200 per ton and human labor costs \$9 per hour, and *Starbucks* has \$10,000 available to spend on these inputs, what is the optimal amount of raw coffee and human labor (in hours) to use?