Applied Production Economcis

AGEC 317: Economic Analysis for Agribusiness Management
Instructor: Michael Black

We can now build and interpret simple and complicated models. It is now time to apply this technical skill to real-world problems. We start with the economics of *production*.

Production economics

Quick review:

- **Production** is the process of using inputs to create a good or service that has economic value to consumers.
- A **production function** is the mathematical relationship that describes a firm's ability to transform inputs into output.
- The **short-run** time horizon is any time period where there are some fixed costs.
- The **long-run** time horizon is any period where all costs are variable.

A **production function**:

$$q = f(x, y)$$

where q is output, x and y are inputs.
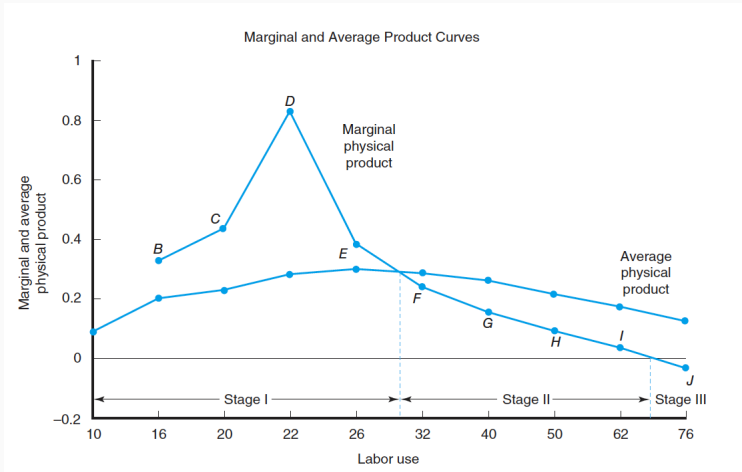
$$MPP_x = \frac{\partial q}{\partial x}$$

MPP_x is the *marginal physical product of x*: it measures the change in output for a unit increase in the x input.

Production economics

$$APP_x = \frac{q}{x}$$

APP_x is the *average physical product of x*: it is the ratio of total output of q to amount of x input.

Stages of Production:

Marginal and Average Product Curves

Production economics

Stages of Production (for a single input):

- Stage 1: MPP>APP
- Stage 2: APP>MPP *and* MPP>0
- Stage 3: MPP < 0

Operating in Stage 1 of production is not desirable, because increasing input will still, on average, result in increased output. That's good. Stage 3 is not good because increasing output will actually *decrease* output. Stage 2 is good. We want to operate somewhere in Stage 2.

The *elasticity of production* for an input x is:

$$\varepsilon_x = \frac{\partial q}{\partial x} \cdot \frac{x}{y} = MPP_x \cdot \frac{1}{APP_x} = \frac{MPP_x}{APP_x}$$

Production economics

The **Law of Diminishing Returns** rules in production: eventually, you'll get less "bang for the buck", and get to a point where adding inputs decreases output. Think about trying to use 4,000 lemons to create lemonade for a small lemonade stand. The 4,001st lemon isn't going to help you; it will just bury you in lemons and hurt production.

Production economics

Suppose we have production with two inputs: $q = f(x, y)$. What is the optimal level of production? More precisely, how do we maximize output?

$$q = f(x, y)$$
$$\frac{\partial q}{\partial x} = \frac{\partial f(\cdot)}{\partial x} = MPP_x = 0$$
$$\frac{\partial q}{\partial y} = \frac{\partial f(\cdot)}{\partial y} = MPP_y = 0$$
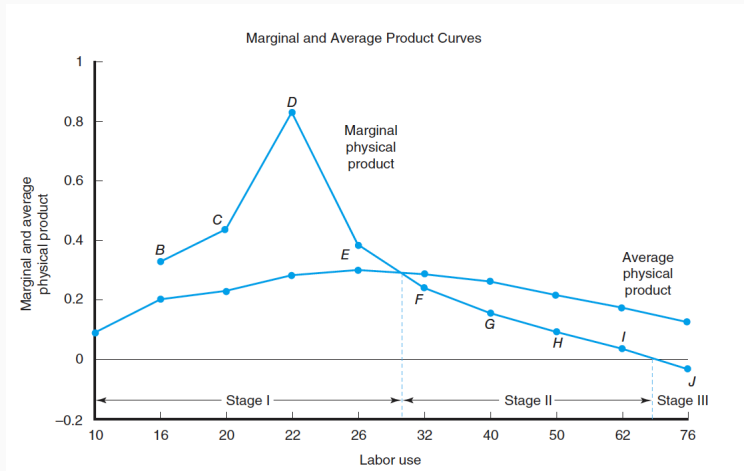
Without constraints, to maximize production we would set the marginal physical product of both inputs to zero.

Okay, but how do we know what the production function looks like? Everything depends on this.

It is up to us to estimate.

If production typically looks like this:

…what kind of model should we specify?

- A quadratic model allows us to estimate and upside down "U-shaped" curve! Perfect!
- We can also use a "log-log" model, like Cobb-Douglas.

Quadratic model:

$$q_i = \beta_0 + \beta_1 capital_i + \beta_2 capital_i^2 + \beta_3 labor_i + \beta_4 labor_i^2 + u_i$$

Production economics

Max-production with quadratic model:

$$q_i = \beta_0 + \beta_1 capital_i + \beta_2 capital_i^2 + \beta_3 labor_i + \beta_4 labor_i^2 + u_i$$

$$\frac{\partial q}{\partial capital} = \beta_1 + 2\beta_2 capital_i = 0$$

$$\frac{\partial q}{\partial labor} = \beta_3 + 2\beta_4 labor_i = 0$$

$$\Rightarrow captial_i^* = \frac{-\beta_1}{2\beta_2}$$

$$\Rightarrow labor_i^* = \frac{-\beta_3}{2\beta_4}$$

Log-log model:

$$q_i = AK^{\alpha}L^{\beta}$$
$$\Rightarrow \ln(q_i) = \ln(A) + \alpha \ln(K) + \beta \ln(L) + u_i$$

Production economics

Max-production with log-log model:

$$\ln(q_i) = \ln(A) + \alpha \ln(K) + \beta \ln(L) + u_i$$
$$q_i = AK^\alpha L^\beta$$
$$\frac{\partial q}{\partial K} = \alpha AK^{1-\alpha}L^\beta = 0$$
$$\frac{\partial q}{\partial L} = \beta AK^\alpha L^{1-\beta} = 0$$

The optimal solution now is a bit more complicated:

$$\alpha A K^{1-\alpha} L^\beta = \beta A K^\alpha L^{1-\beta}$$
$$\Rightarrow K^* = \frac{\beta}{\alpha} L$$
$$\Rightarrow L^* = \frac{\alpha}{\beta} K$$

Production economics

Of course, both of these examples are without constraints. If we add constraints, then we have a constrained optimization problem where we need to use a Lagrangian to solve.

Production economics

To summarize, we:

1. Observe data
2. Use OLS to predict an appropriate model
3. Use OLS regression results to form an optimization problem
4. Solve the optimization problem

Okay, but our goal is seldom to simply maximize output. We may want to minimize costs or maximize profit.

Costs and profit

- **Total value product** of inputs: $TVP = P \cdot f(x, y)$, where P is the price of the sold output.

- **Marginal value product** of input x: $MVP_x = P \cdot MPP_x$, where P is the price of the sold output.

- **Average value product** of input x: $AVP_x = P \cdot APP_x$, where P is the price of the sold output.

- **Total cost** function: $c = r_x x + r_y y + b$, where r_x is the per-unit cost of input x (same for y), and b is some fixed cost.

- **Profit**: $\pi = P \cdot q - r_x x - r_y y - b$, where q is the estimated production function.

Profit function with quadratic form:

$$\pi = P \cdot (\beta_0 + \beta_1 K_i + \beta_2 K_i^2 + \beta_3 L_i + \beta_4 L_i^2) - r_K K - r_L L - b$$

Where the red portion comes from an estimated econometric model.

Consider the production process for toilet paper...

Example

Inputs:

- Human labor
- Electricity to run machines
- Recycled paper
- Glue
- Bleach

$$\pi = P \cdot (\beta_0 + \beta_1 electricity_i + \beta_2 electricity_i^2 + \beta_3 paper_i + \beta_4 paper_i^2)$$
$$-r_{paper} paper - r_{electricity} electricity - b$$

Let's choose the optimal amount of electricity and recycled paper to use to maximize profit, *without any constraints*:

$$\frac{\partial \pi}{\partial electricity} = P(\beta_1 + 2\beta_2) - r_{electricity} = 0$$

$$\Rightarrow electricity^* = \frac{\frac{r_{electricity}}{P} - \beta_1}{2\beta_2}$$

Example

...we can find the same optimal input use for recycled paper as well. Now what if we have a budget constraint?

Now let's solve a real-world production problem.

Suppose you work for a large international agribusiness corporation: *Lentils-n-More* owns a large lentil operation in India. Your firm has run an experiment on some of its fields. Each field receives a randomly selected amount of nitrogen fertilizer, and a randomly selected amount of seed sachets to plant in the field. Some farms are allowed to operate totally independently, while others must report the entire growing process to *Lentils-n-More*.

Your goal is to determine the profit-maximizing level of nitrogen fertilizer and seed sachets to purchase for your India operation. Your secondary goal is to confirm that your India operation is in Stage II production, and not Stage I or III.

Data Description

- Yield: yield of lentils, in bushels per ha
- Nitrogen: amount of nitrogen fertilizer concentrate, in gallons
- SeedPop: the seed population, measured in number of seed sachets available locally
- Ownership: =1 if operation is monitored by your firm, =0 if no oversight

Solution: Step-by-step

- Estimate model that is quadratic in both inputs, plus interaction, plus dummy variable
- Use results and current prices to predict profit
- Solve for optimal by hand and substitute parameters, or use Solver in Excel