Forget to remember
Remember to forget

VLC = $\iiint$ Vision Learning Control $dVdLdC$

Long Short Term Memories and Gated Recurrent Units

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Some of the images and animations used here were originally designed by Adam Prügel-Bennett.

Recap: An RNN is just a recursive function invocation

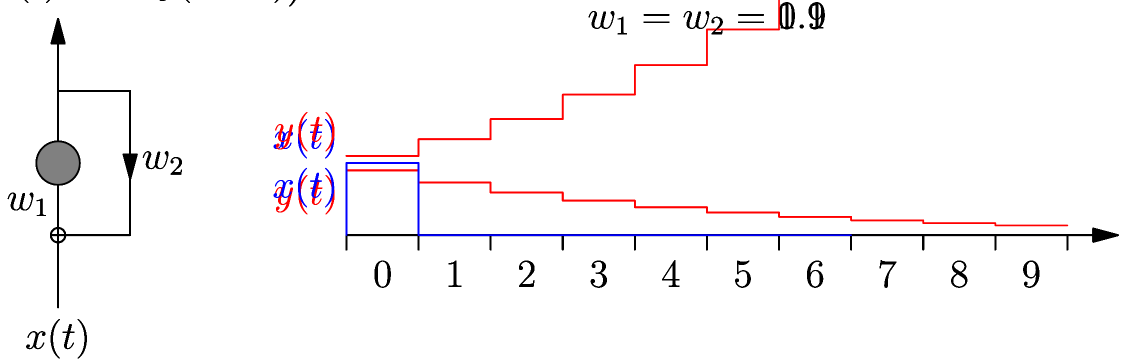
- $\boldsymbol{y}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{c}(t)|\boldsymbol{W})$
- and the state $\boldsymbol{c}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{c}(t-1)|\boldsymbol{W})$
- If the output $\boldsymbol{y}(t)$ depends on the input $\boldsymbol{x}(t-2)$, then prediction will be

$$\boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{g}(\boldsymbol{x}(t-1), \boldsymbol{g}(\boldsymbol{x}(t-2), c(t-2)|\boldsymbol{W})|\boldsymbol{W})|\boldsymbol{W})|\boldsymbol{W})$$

- it should be clear that the gradients of this with respect to the weights can be found with the chain rule
- The back-propagated error will involve applying $\boldsymbol{f}$ multiple times
- Each time the error will get multiplied by some factor a
- If $\boldsymbol{y}(t)$ depends on the input $\boldsymbol{x}(t-\tau)$ then the back-propagated signal will be proportional to $a^{\tau-1}$
- This either vanishes or explodes when τ becomes large

Vanishing and Exploding Gradients

$$y(t) = w_1\left(x(t) + w_2\, y(t-1)\right)$$



LSTM Architecture

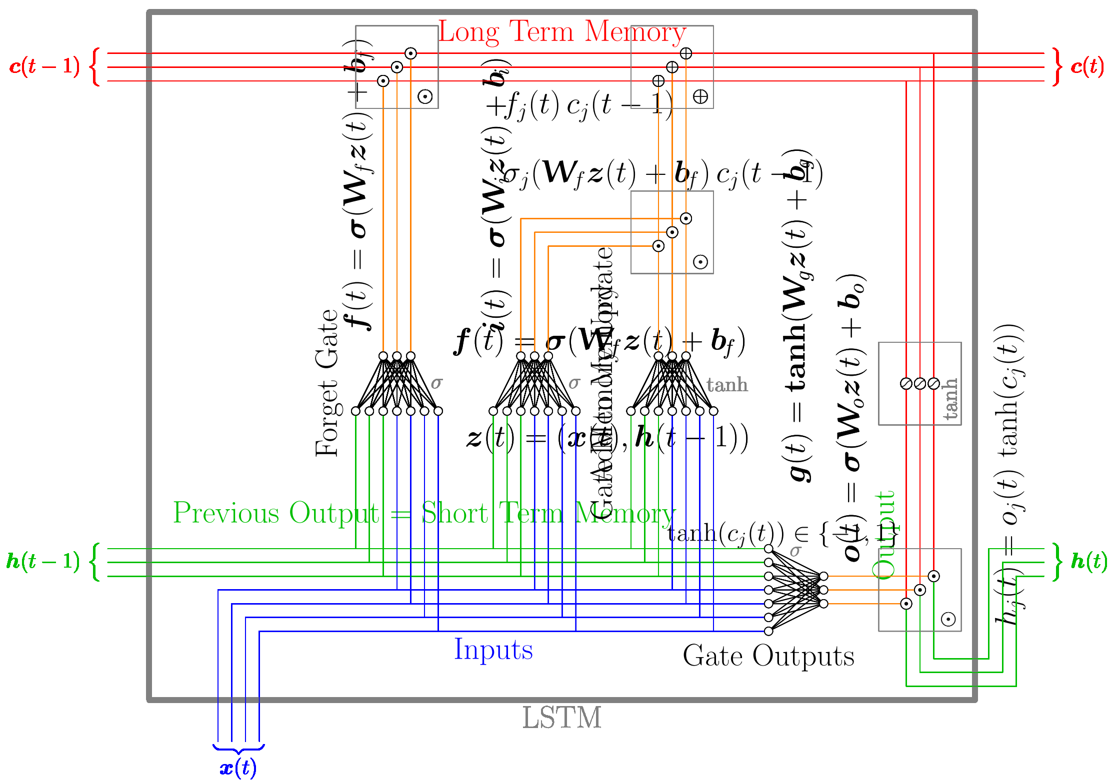
- LSTMs (long-short term memory) was designed to solve this problem
- Key ideas: to retain a 'long-term memory' requires

$$\boldsymbol{c}(t) = \boldsymbol{c}(t-1)$$

- Sometimes we have to forget and sometimes we have to change a memory
- To do this we should use 'gates' that saturate at 0 and 1
- Sigmoid functions naturally saturate at 0 and 1

LSTM Architecture

$$c_j(t) = f_j(t)\, c_j(t-1) + b_f + g_j(t)\, i_j(t) + b_i$$



Update Equations

Initially, for $t = 0$, $\boldsymbol{h}(0) = \boldsymbol{0}$

- Inputs $\boldsymbol{z}(t) = \big(\boldsymbol{x}(t), \boldsymbol{h}(t-1)\big)$
- Network updates ($\boldsymbol{W}_*$ and $\boldsymbol{b}_*$ are the learnable parameters)

$$\boldsymbol{f}(t) = \sigma(\boldsymbol{W}_f\,\boldsymbol{z}(t) + \boldsymbol{b}_f) \qquad \boldsymbol{i}(t) = \sigma(\boldsymbol{W}_i\,\boldsymbol{z}(t) + \boldsymbol{b}_i)$$
$$\boldsymbol{g}(t) = \mathbf{tanh}(\boldsymbol{W}_g\,\boldsymbol{z}(t) + \boldsymbol{b}_g) \qquad \boldsymbol{o}(t) = \sigma(\boldsymbol{W}_o\,\boldsymbol{z}(t) + \boldsymbol{b}_o)$$

- Long-term memory update

$$\boldsymbol{c}(t) = \boldsymbol{f}(t) \odot \boldsymbol{c}(t-1) + \boldsymbol{g}(t) \odot \boldsymbol{i}(t)$$

- Output $\boldsymbol{h}(t) = \boldsymbol{o}(t) \odot \mathbf{tanh}(\boldsymbol{c}(t))$

Training LSTMs

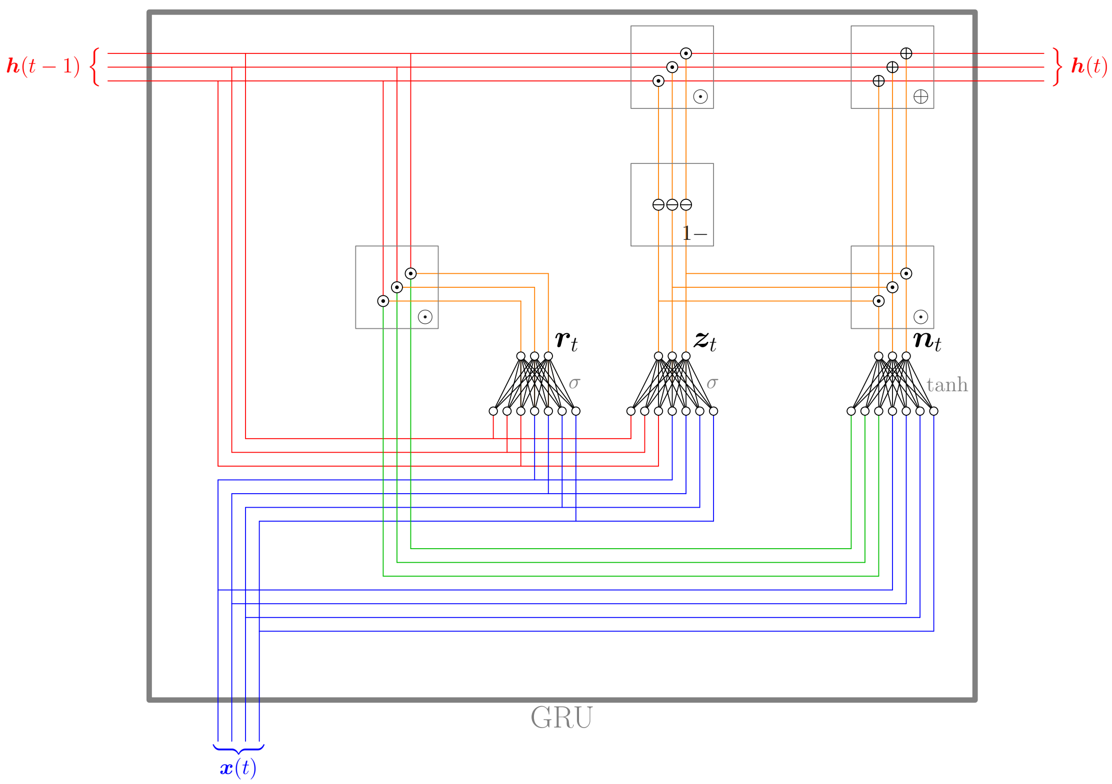
- We can train an LSTM by unwrapping it in time.
- Note that it involves four dense layers with sigmoidal (or tanh) outputs.
- This means that typically it is very slow to train.
- There are a few variants of LSTMs, but all are very similar. The most popular is probably the Gated Recurrent Unit (GRU).

LSTM Success Stories

- LSTMs have been used to win many competitions in speech and handwriting recognition.
- Major technology companies including Google, Apple, and Microsoft are using LSTMs as fundamental components in products.
- Google used LSTM for speech recognition on the smartphone, for Google Translate.
- Apple uses LSTM for the "Quicktype" function on the iPhone and for Siri.
- Amazon uses LSTM for Amazon Alexa.
- In 2017, Facebook performed some 4.5 billion automatic translations every day using long short-term memory networks[1].

[1] https://en.wikipedia.org/wiki/Long_short-term_memory

Gated Recurrent Unit (GRU)



Gated Recurrent Unit (GRU)

- $\boldsymbol{x}(t)$: input vector
- $\boldsymbol{h}(t)$: output vector (and 'hidden state')
- $\boldsymbol{r}(t)$: reset gate vector
- $\boldsymbol{z}(t)$: update gate vector
- $\boldsymbol{n}(t)$: new state vector (before update is applied)
- $\boldsymbol{W}$ and $\boldsymbol{b}$: parameter matrices and biases

Gated Recurrent Unit (GRU)

Initially, for $t = 0$, $\boldsymbol{h}(0) = \boldsymbol{0}$

$$
\begin{aligned}
\boldsymbol{z}(t) &= \sigma(\boldsymbol{W}_z(\boldsymbol{x}(t), \boldsymbol{h}(t-1)) + \boldsymbol{b}_z) \\
\boldsymbol{r}(t) &= \sigma(\boldsymbol{W}_r(\boldsymbol{x}(t), \boldsymbol{h}(t-1)) + \boldsymbol{b}_r) \\
\boldsymbol{n}(t) &= \tanh(\boldsymbol{W}_n(\boldsymbol{x}(t), r(t) \odot h(t-1)) + \boldsymbol{b}_h) \\
\boldsymbol{h}(t) &= (1 - \boldsymbol{z}(t)) \odot \boldsymbol{h}(t-1) + \boldsymbol{z}(t) \odot \boldsymbol{n}(t)
\end{aligned}
$$

Most implementations follow the original paper and swap $(1 - \boldsymbol{z}(t))$ and $(\boldsymbol{z}(t))$ in the $\boldsymbol{h}(t)$ update; this doesn't change the operation of the network, but does change the interpretation of the update gate, as the gate would have to produce a 0 when an update was to occur, and a 1 when no update is to happen (which is somewhat counter-intuitive)!

GRU or LSTM?

- GRUs have two gates (reset and update) whereas LSTM has three gates (input/output/forget)
- GRU performance on par with LSTM but computationally more efficient (less operations & weights).
- In general, if you have a very large dataset then LSTMs will likely perform slightly better.
- GRUs are a good choice for smaller datasets.