Forget to remember
Remember to forget

VLC = Vision Learning Control

Long Short Term Memories and Gated Recurrent Units

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Vision, Learning and Control
University of Southampton

Some of the images and animations used here were originally designed by Adam Prügel-Bennett.

Recap: An RNN is just a recursive function invocation

- $\boldsymbol{y}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{c}(t)|\boldsymbol{W})$
- and the state $\boldsymbol{c}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{c}(t-1)|\boldsymbol{W})$
- If the output $\boldsymbol{y}(t)$ depends on the input $\boldsymbol{x}(t-2)$, then prediction will be

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- If $\boldsymbol{y}(t)$ depends on the input $\boldsymbol{x}(t-\tau)$ then the back-propagated signal will be proportional to $a^{\tau-1}$
- This either vanishes or explodes when τ becomes large

Vanishing and Exploding Gradients

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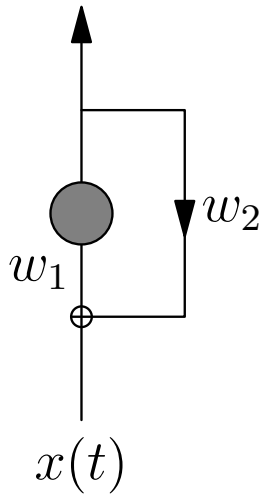

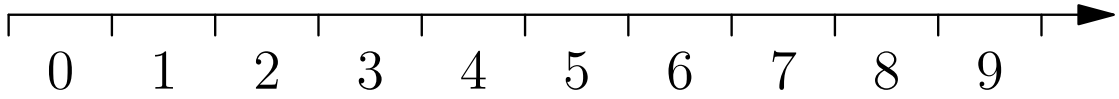

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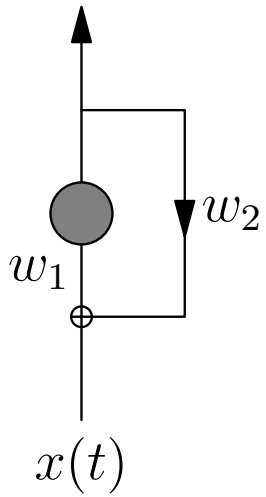

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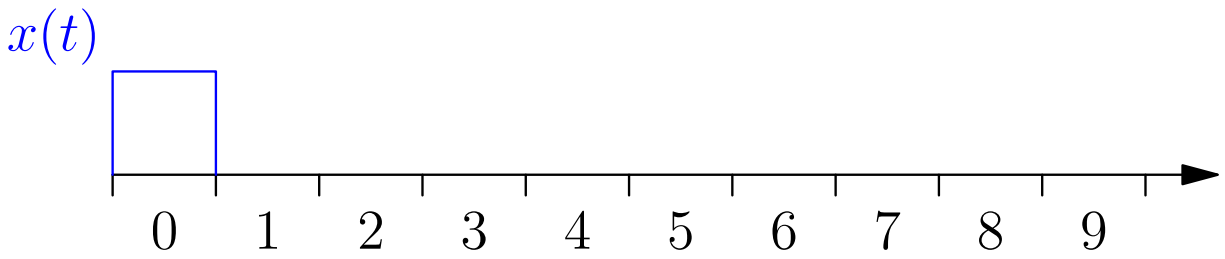

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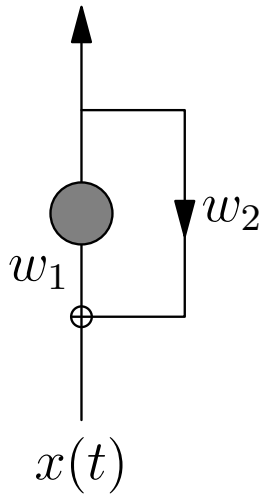

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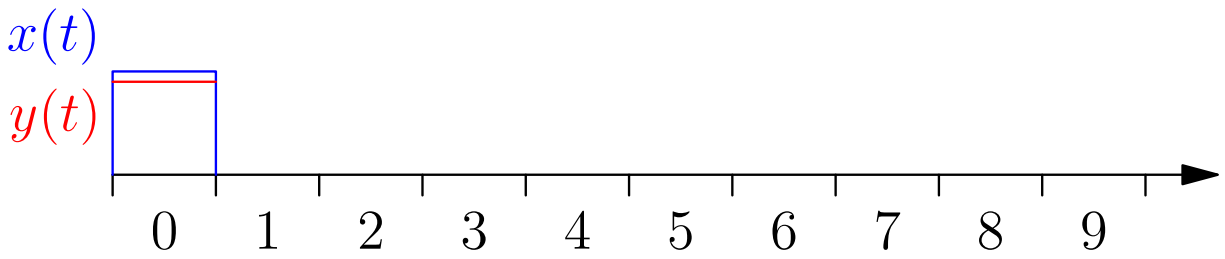

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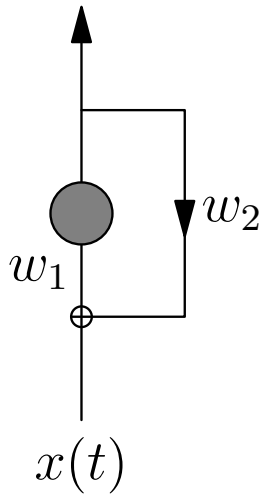

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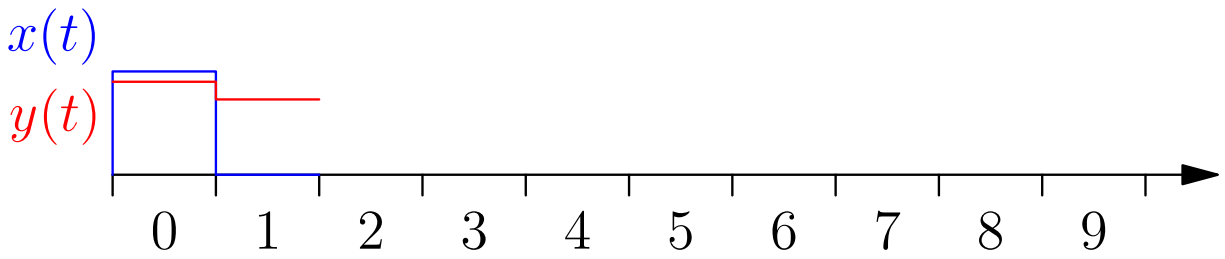

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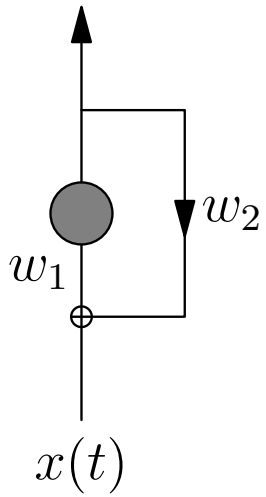

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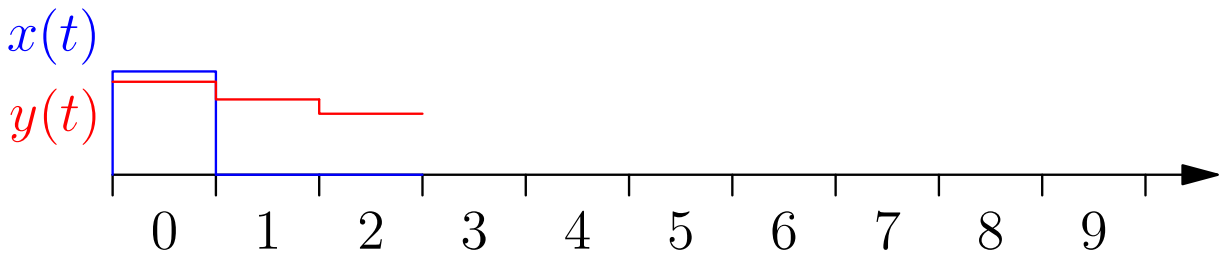

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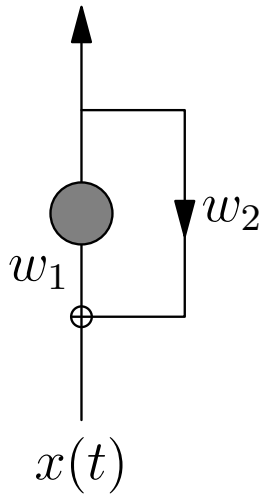

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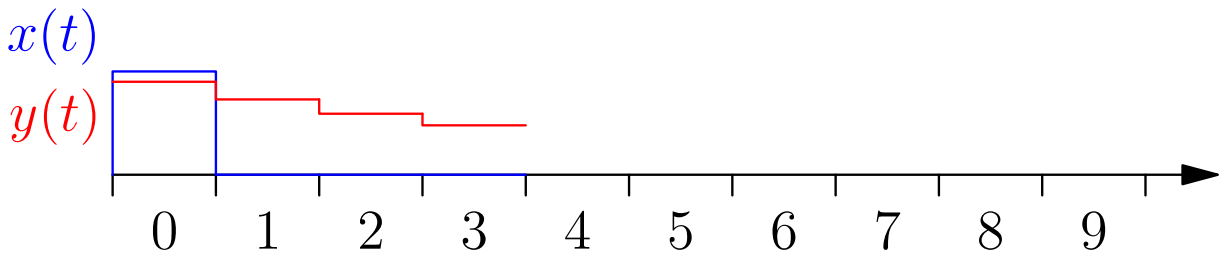

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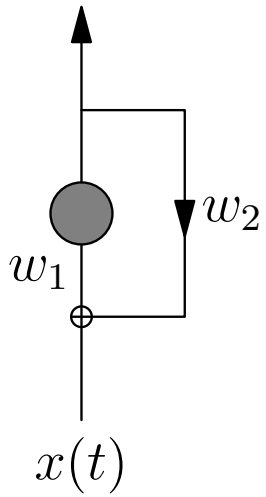

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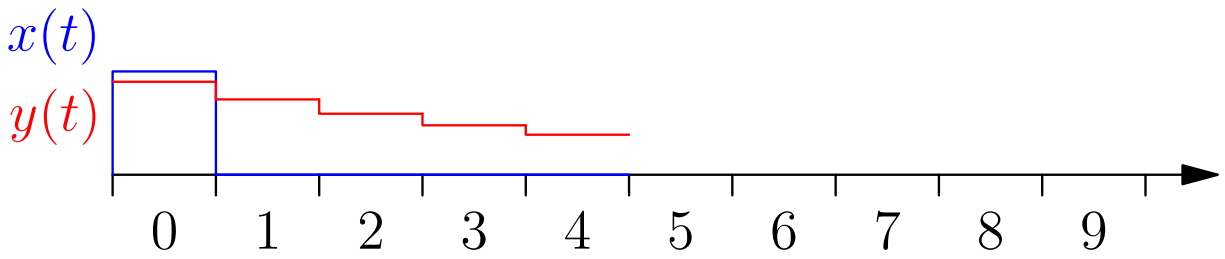

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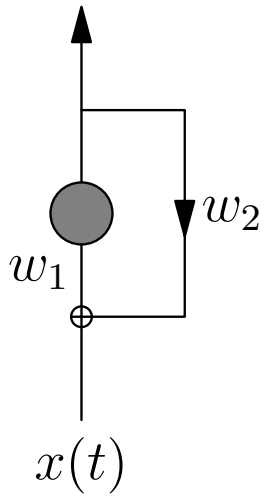

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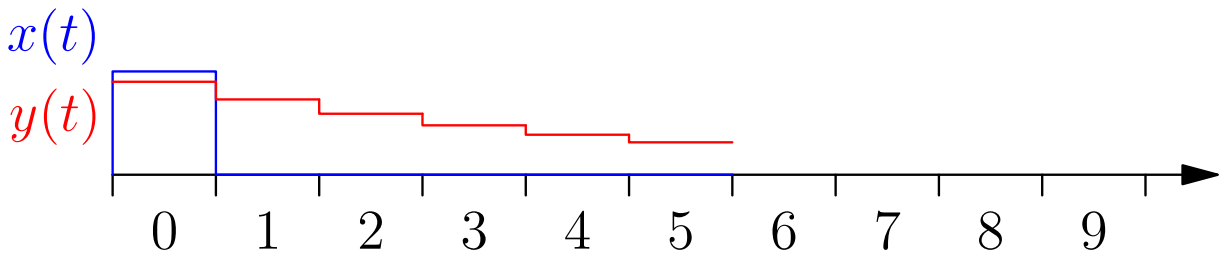

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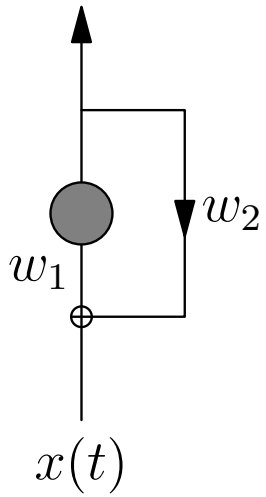

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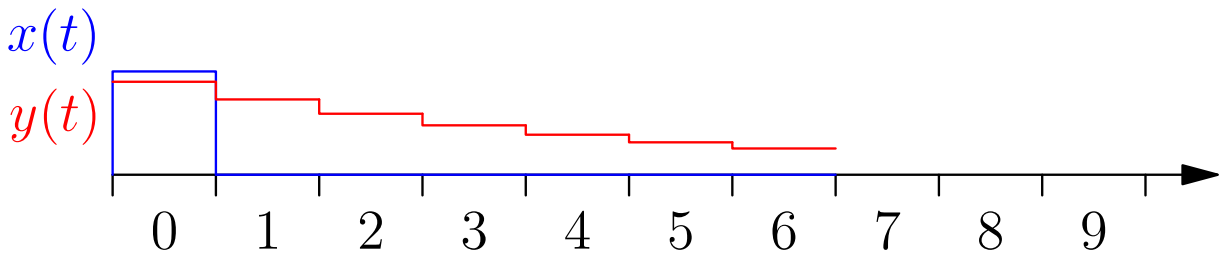

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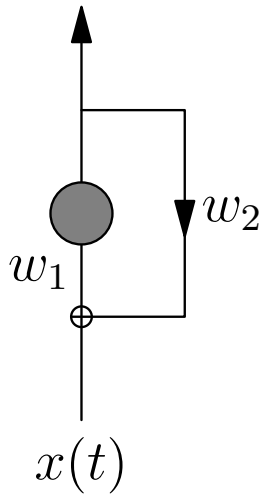

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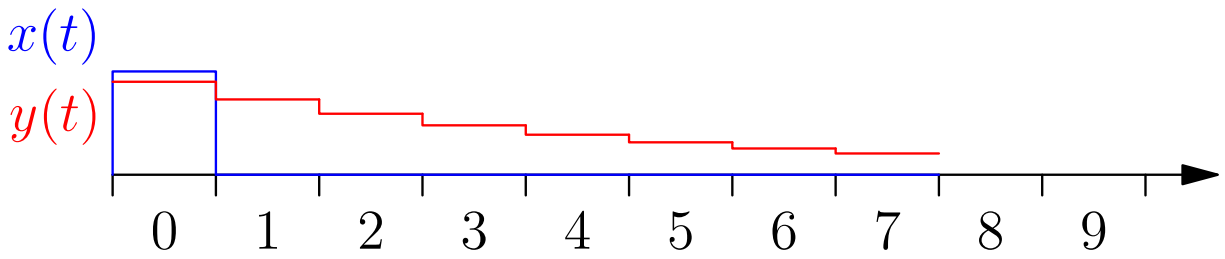

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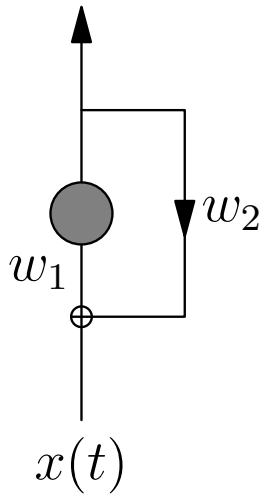

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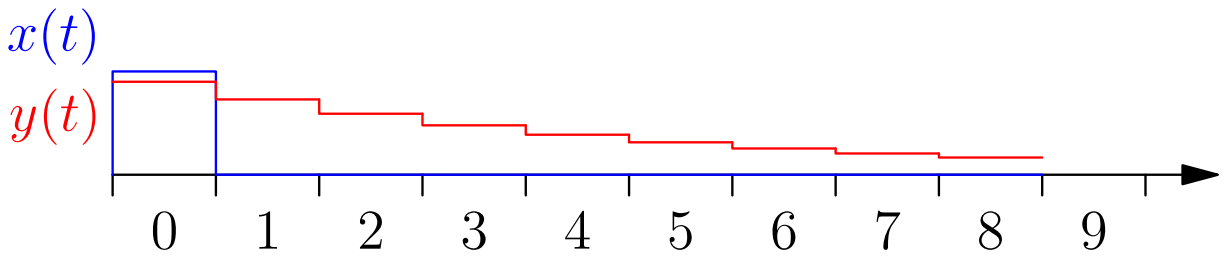

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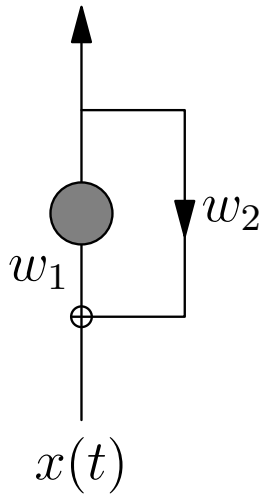

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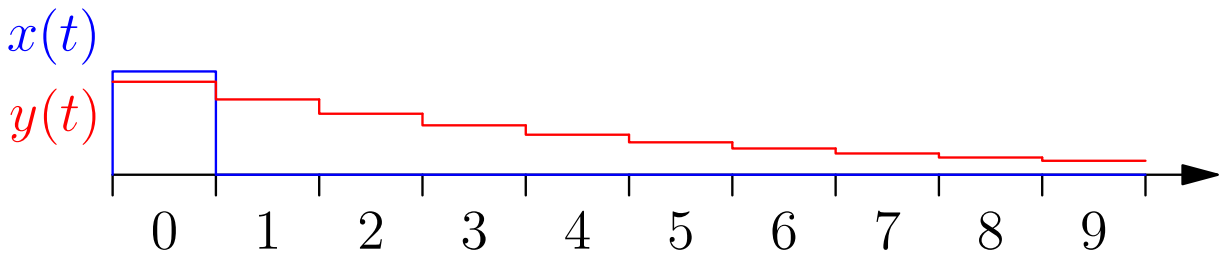

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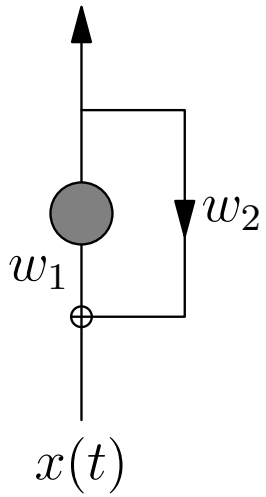

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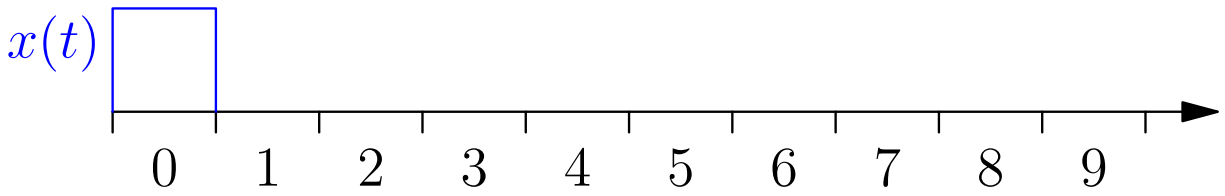

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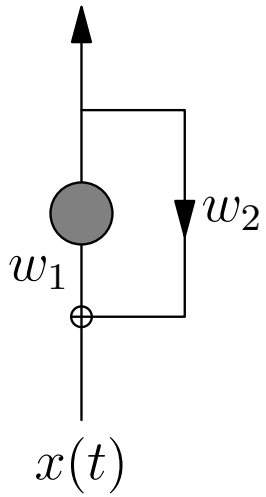

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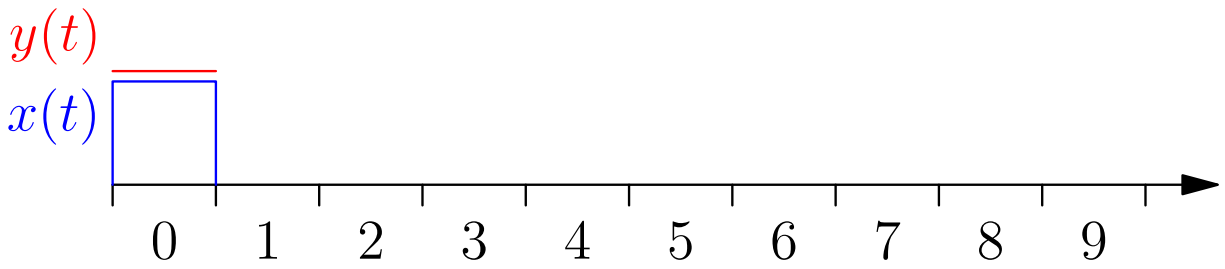

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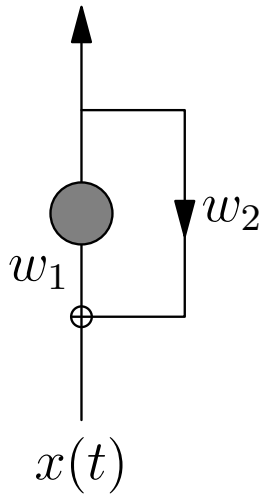

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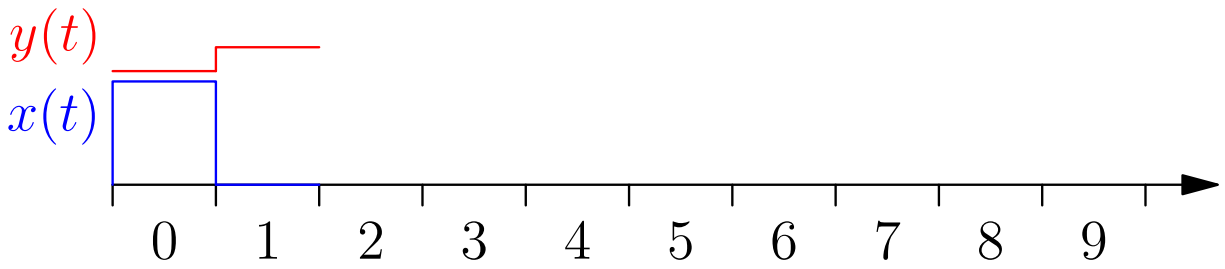

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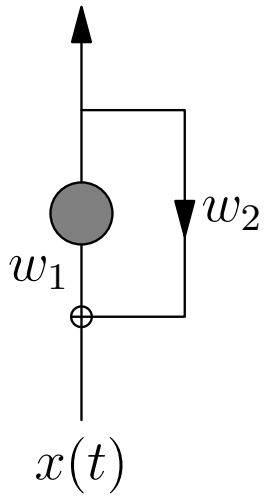

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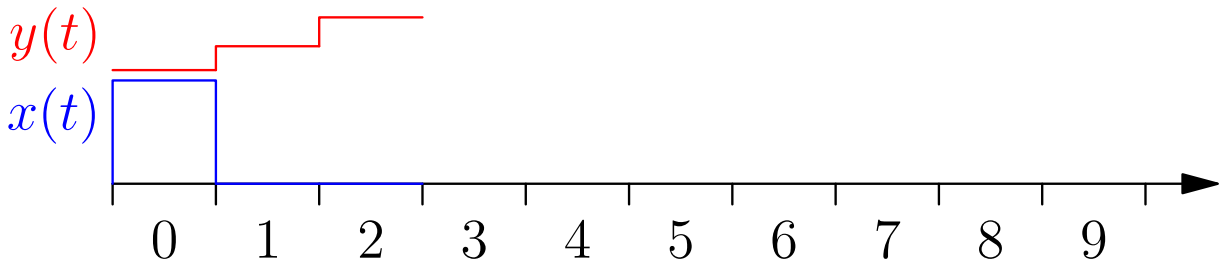

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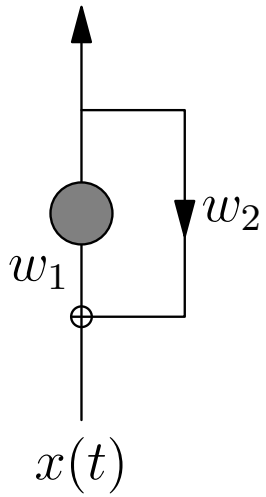

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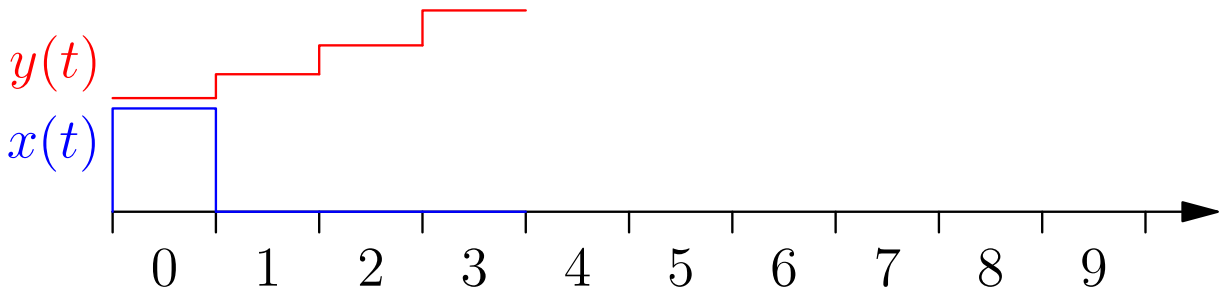

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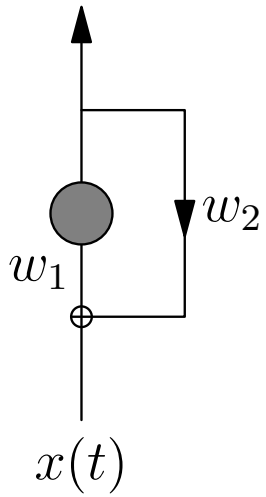

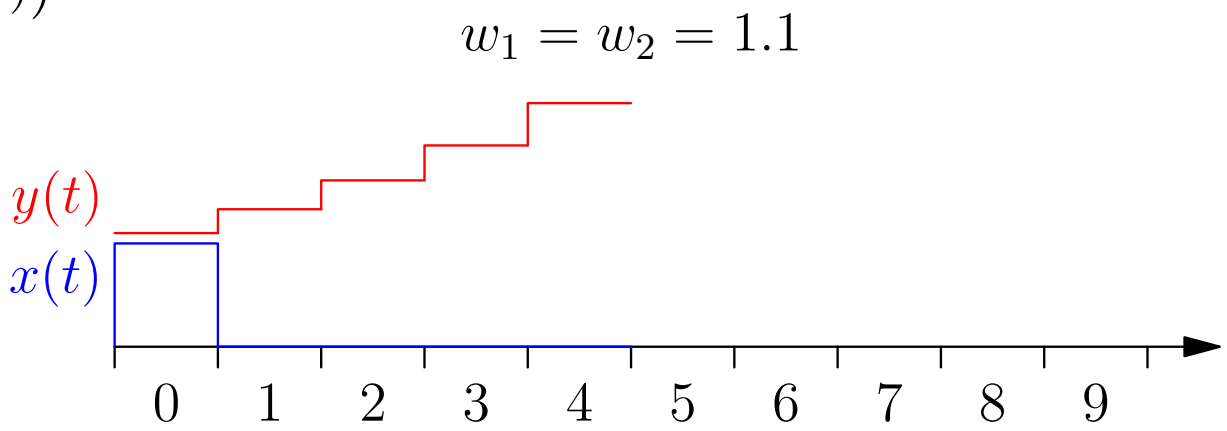

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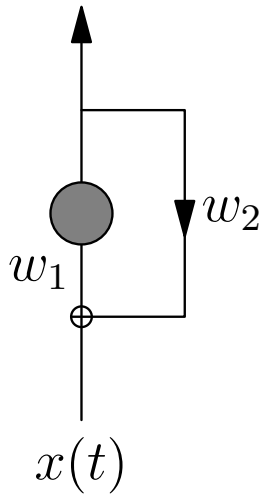

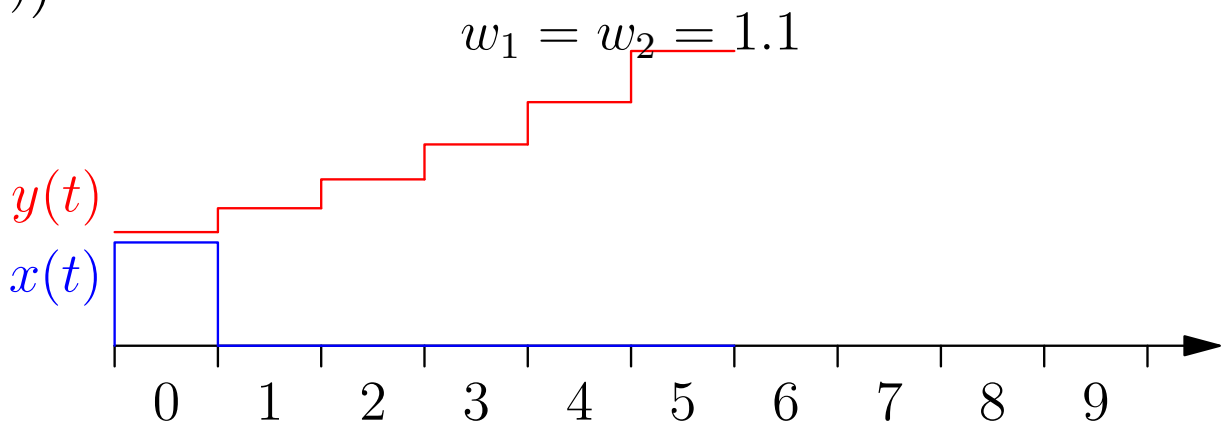

Vanishing and Exploding Gradients

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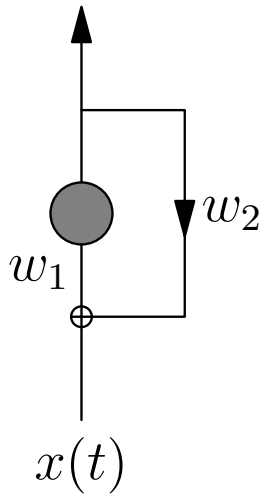

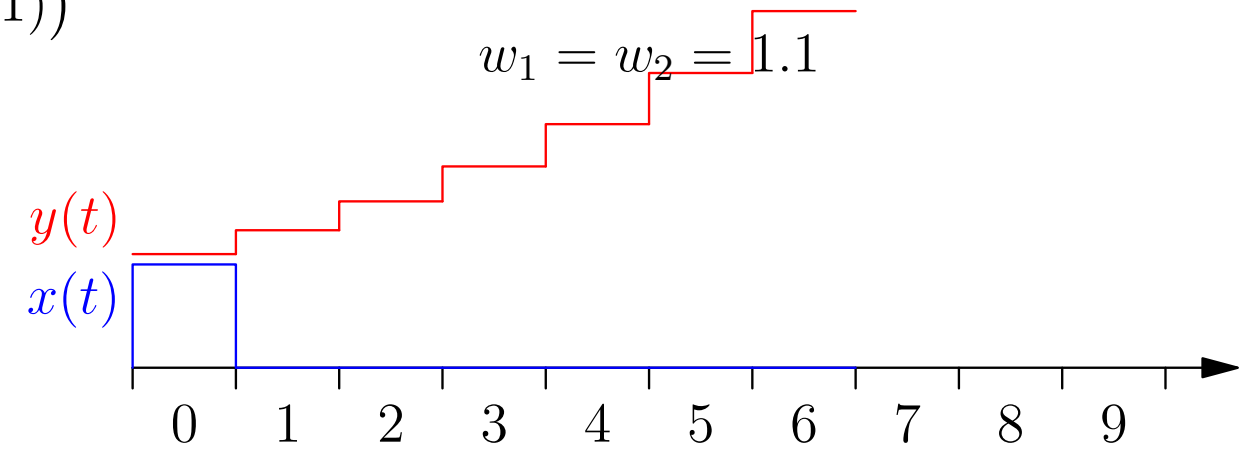

LSTM Architecture

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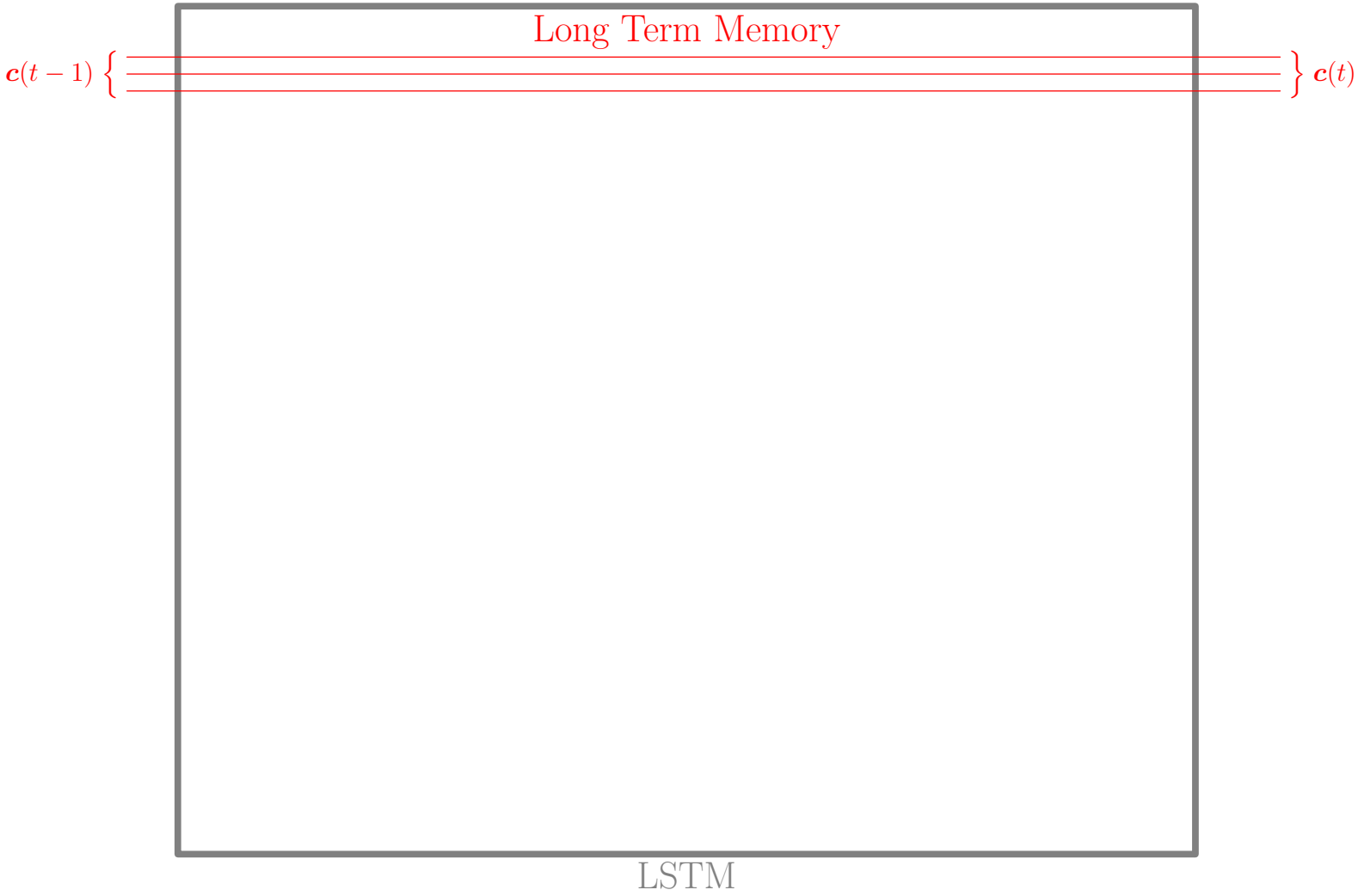

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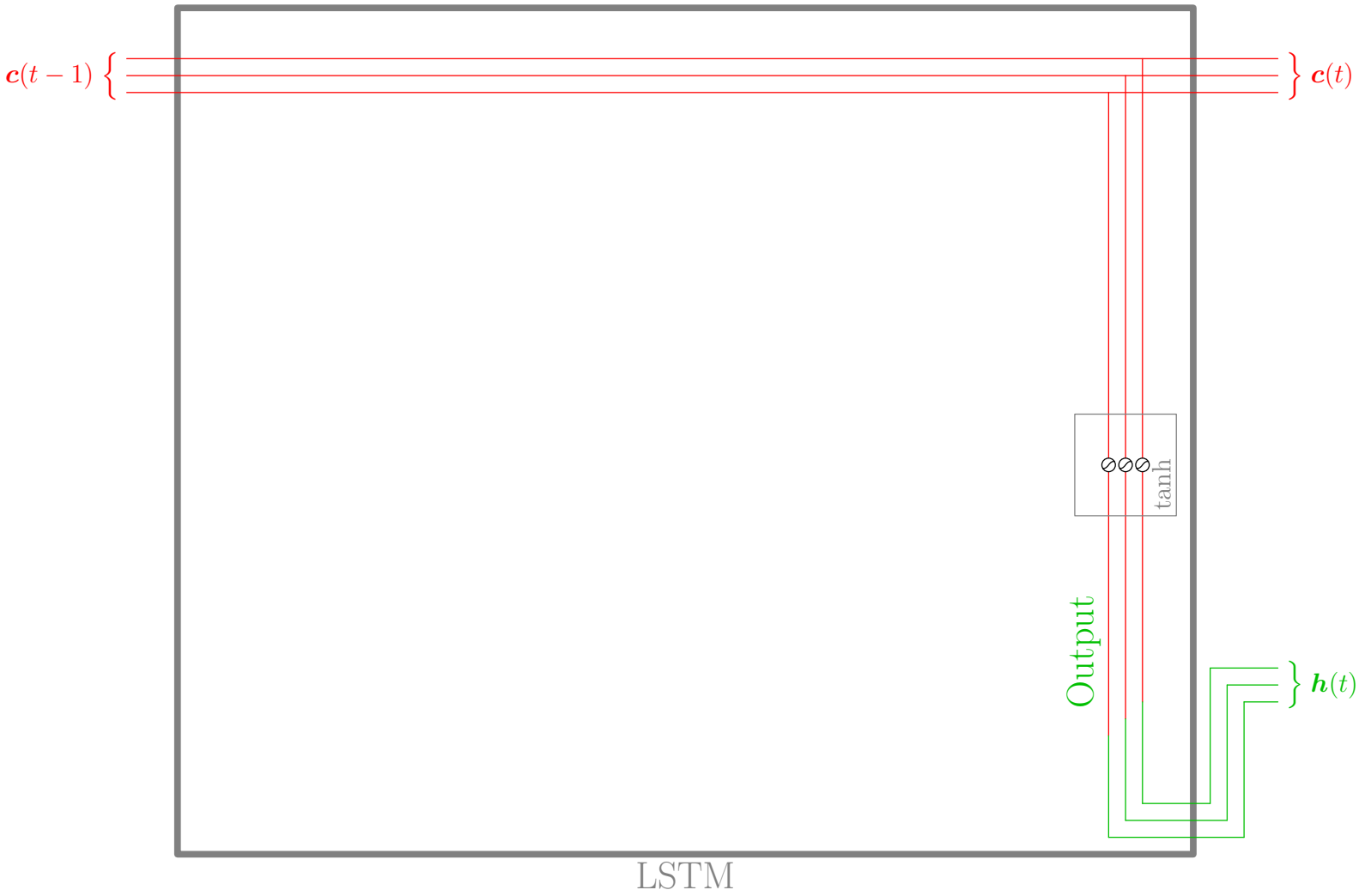

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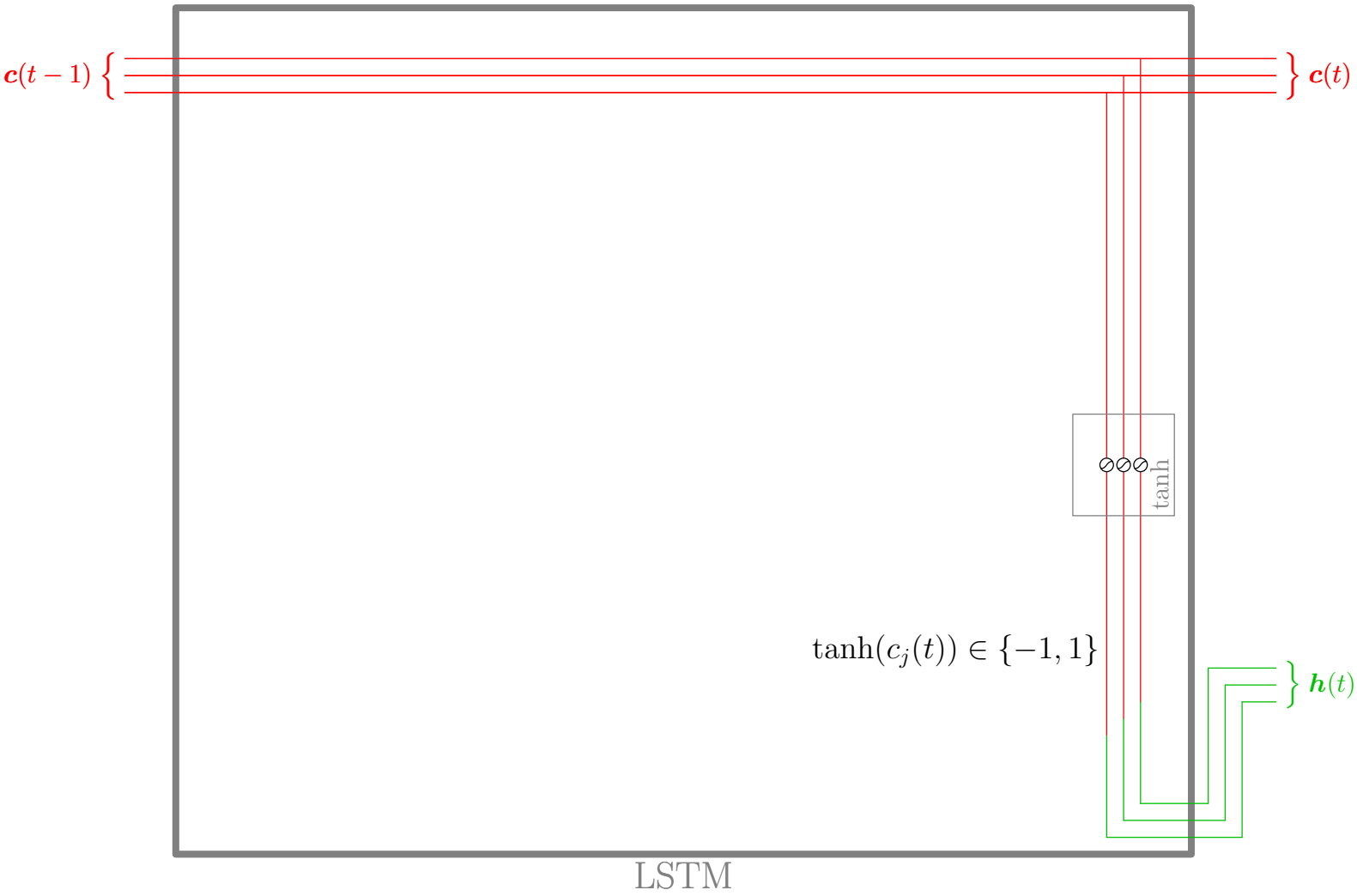

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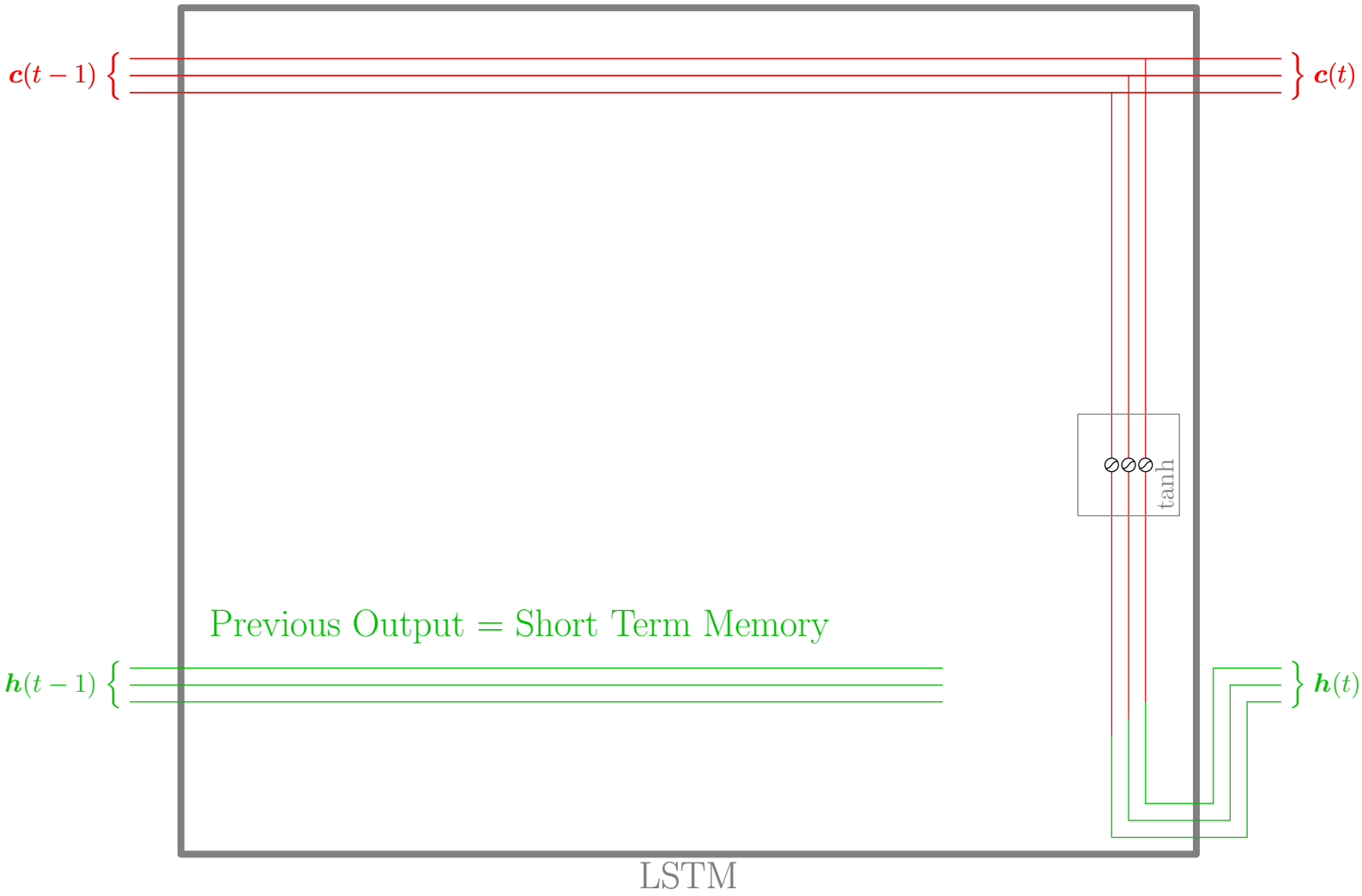

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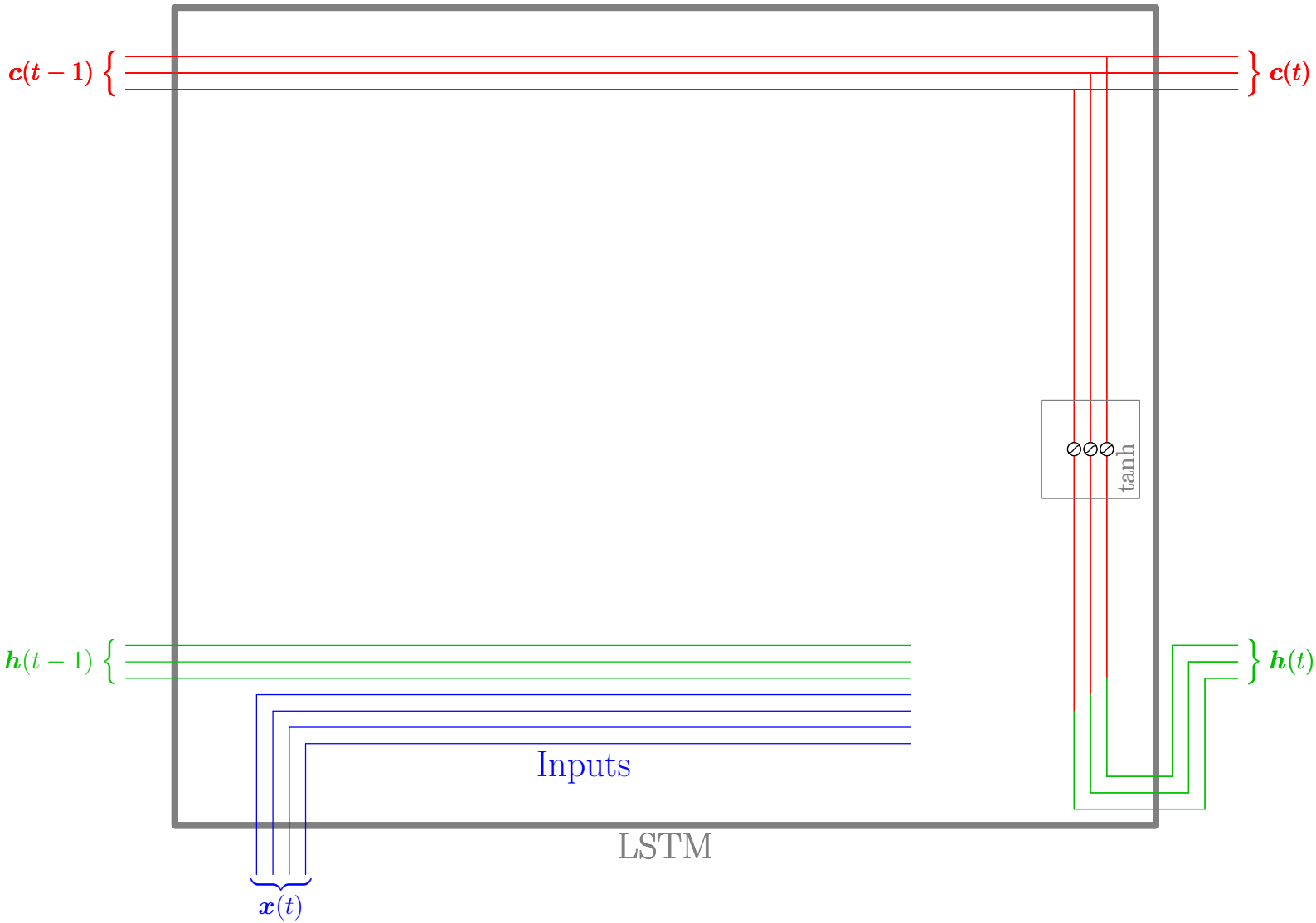

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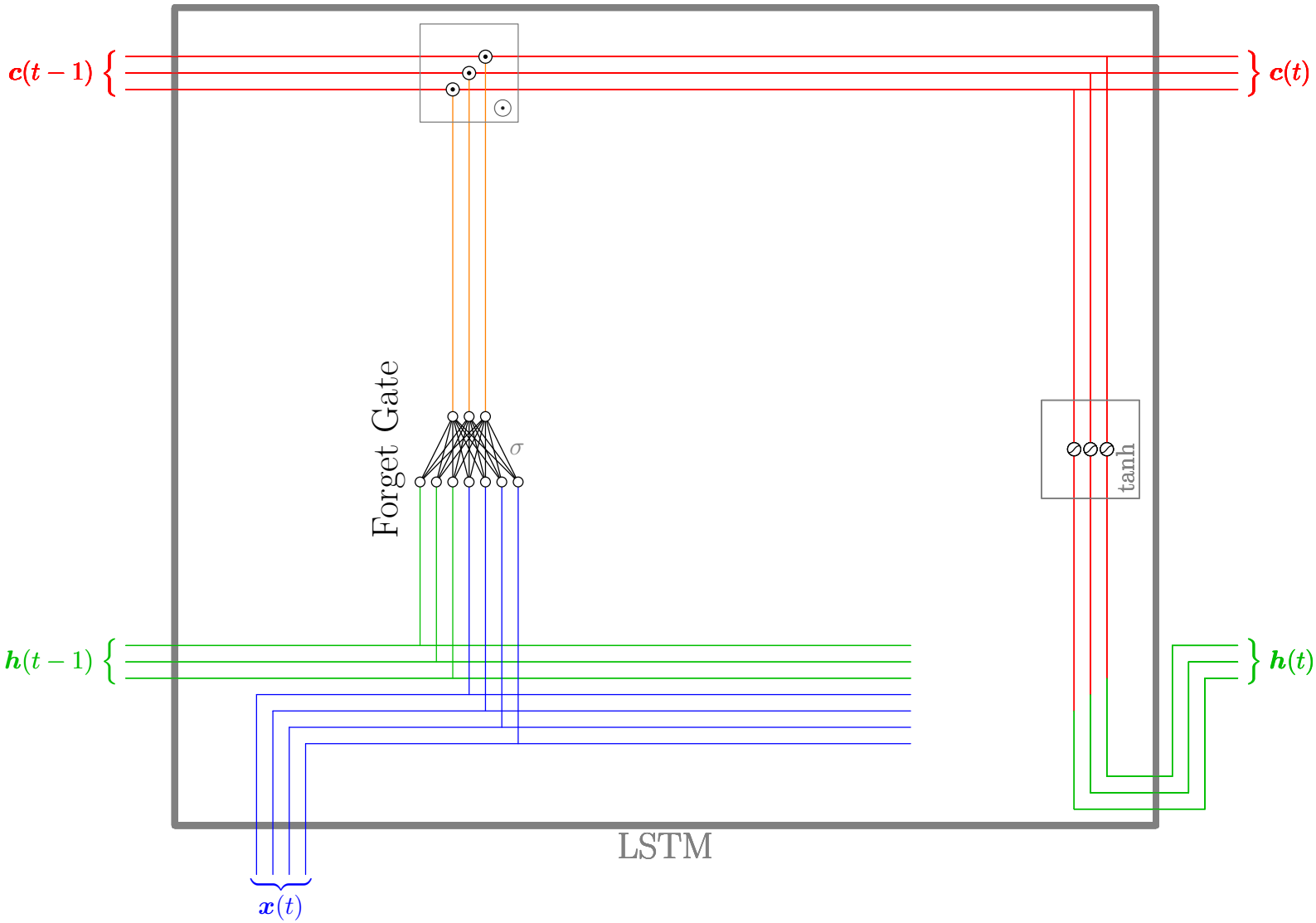

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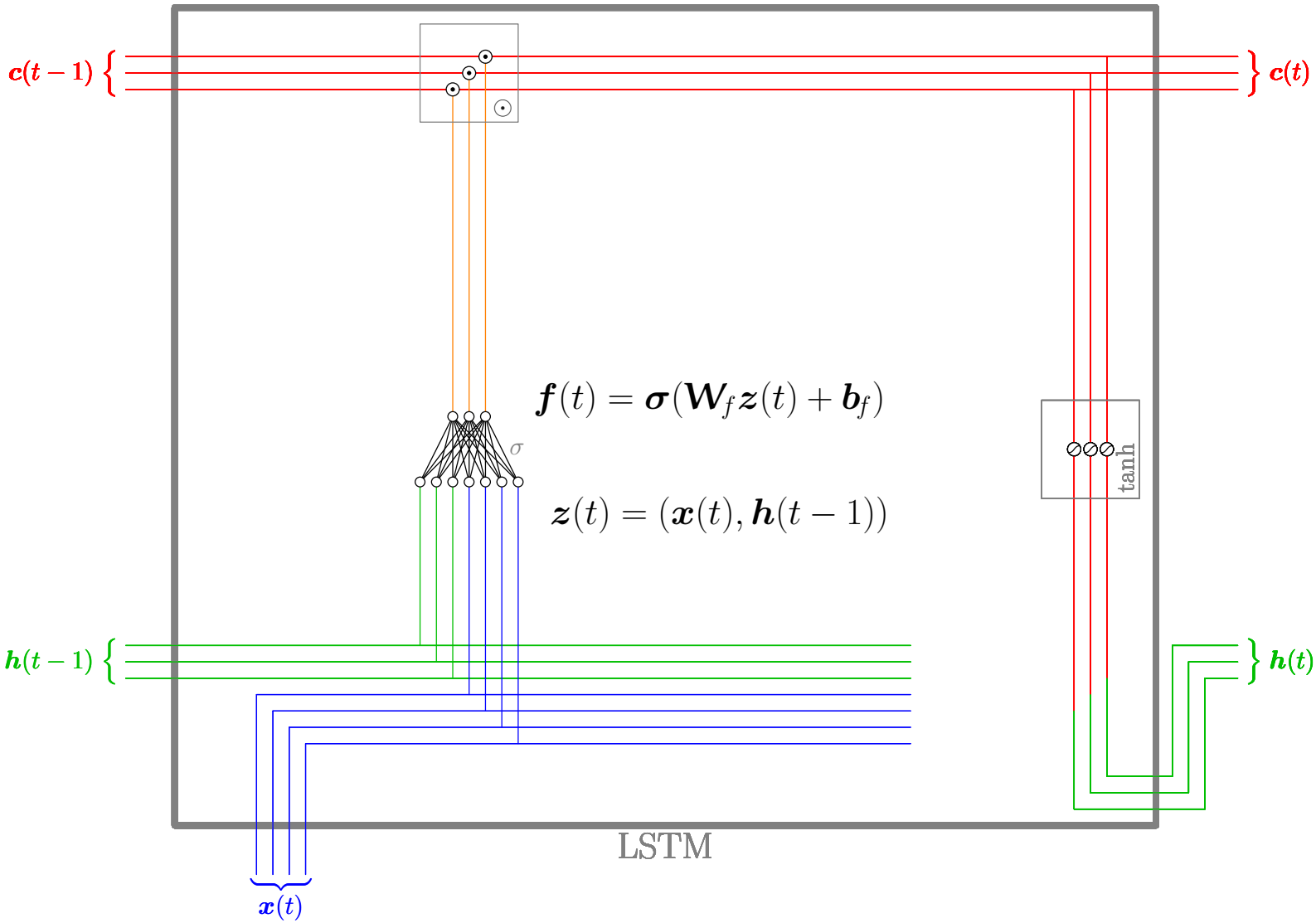

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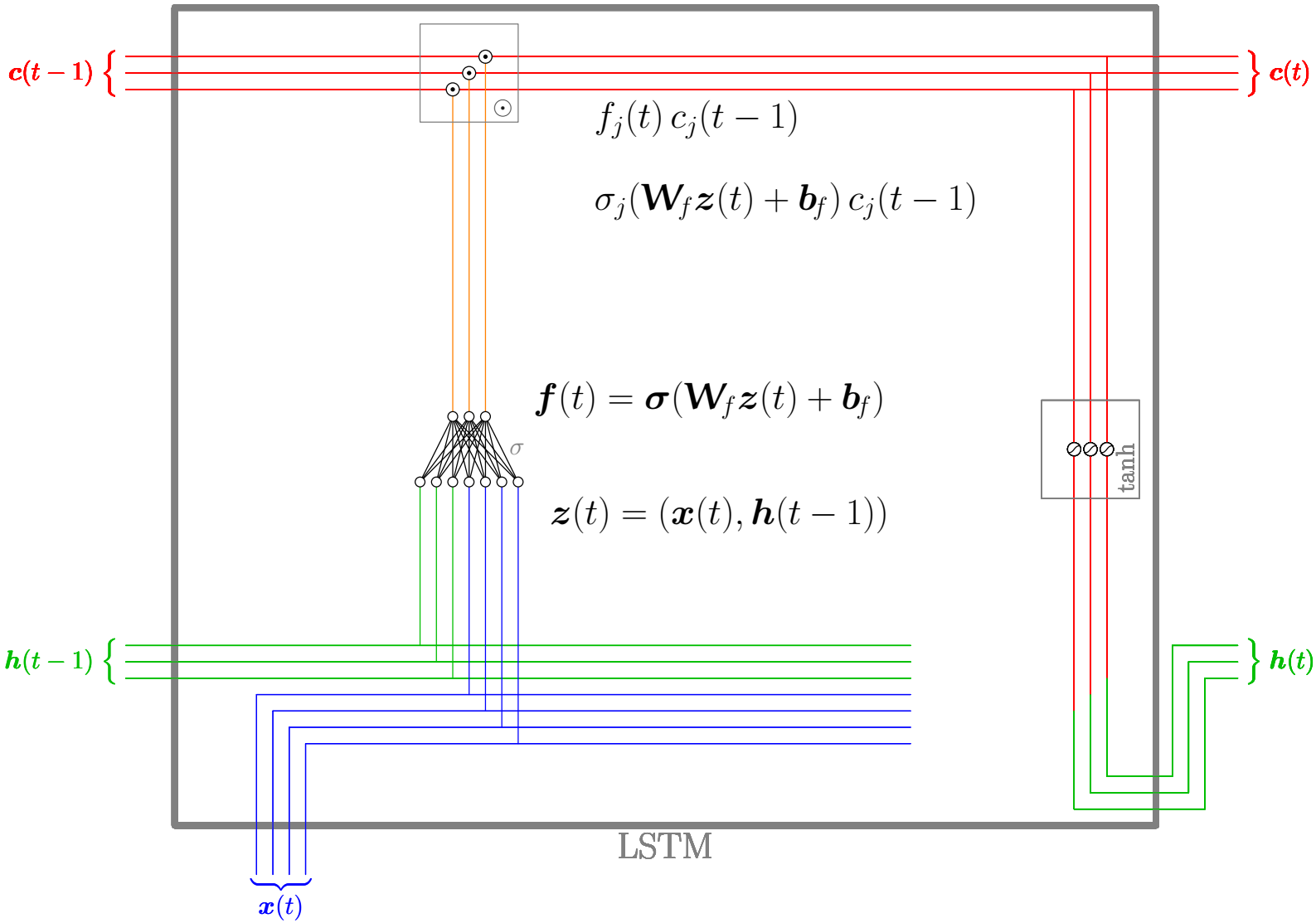

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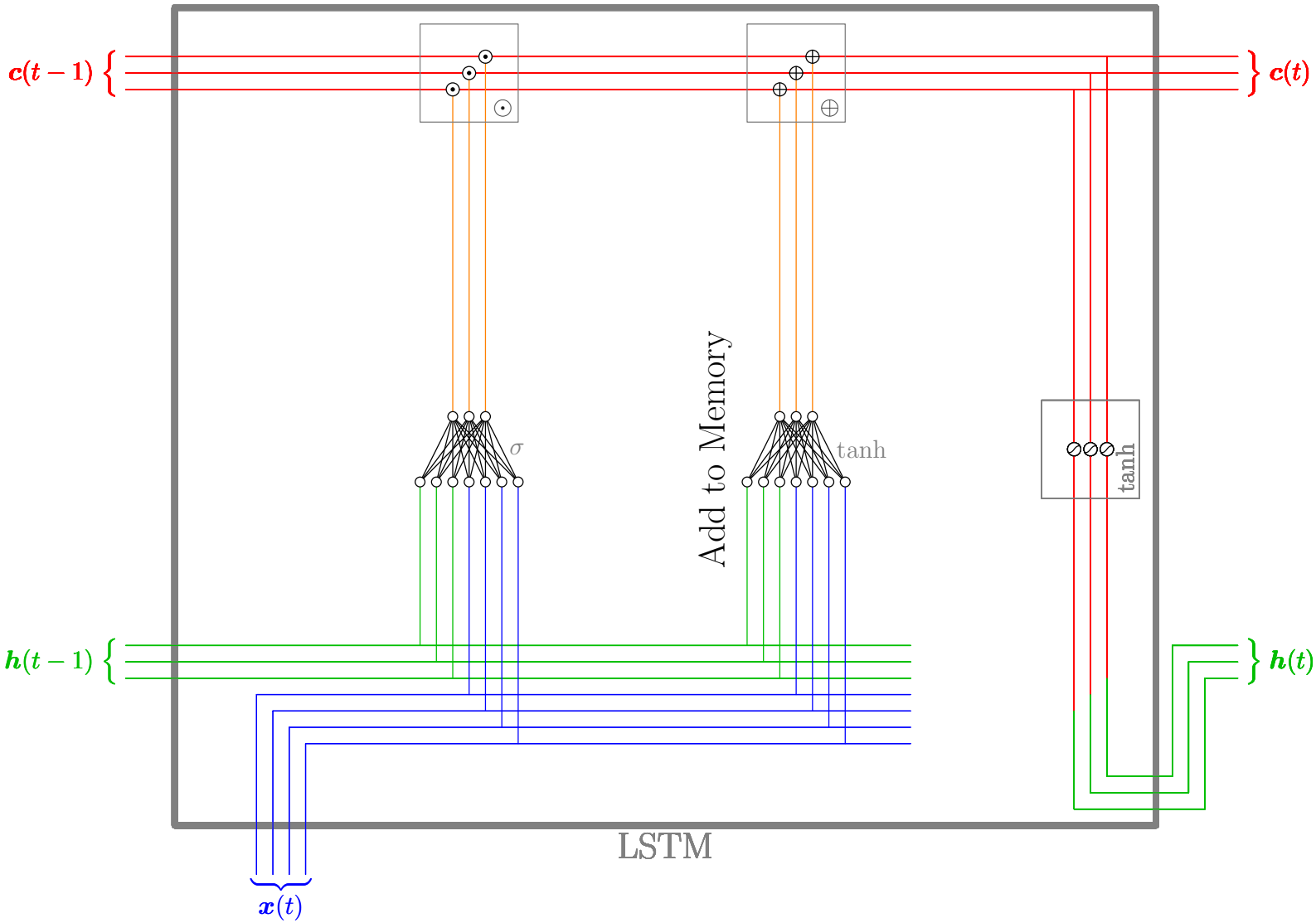

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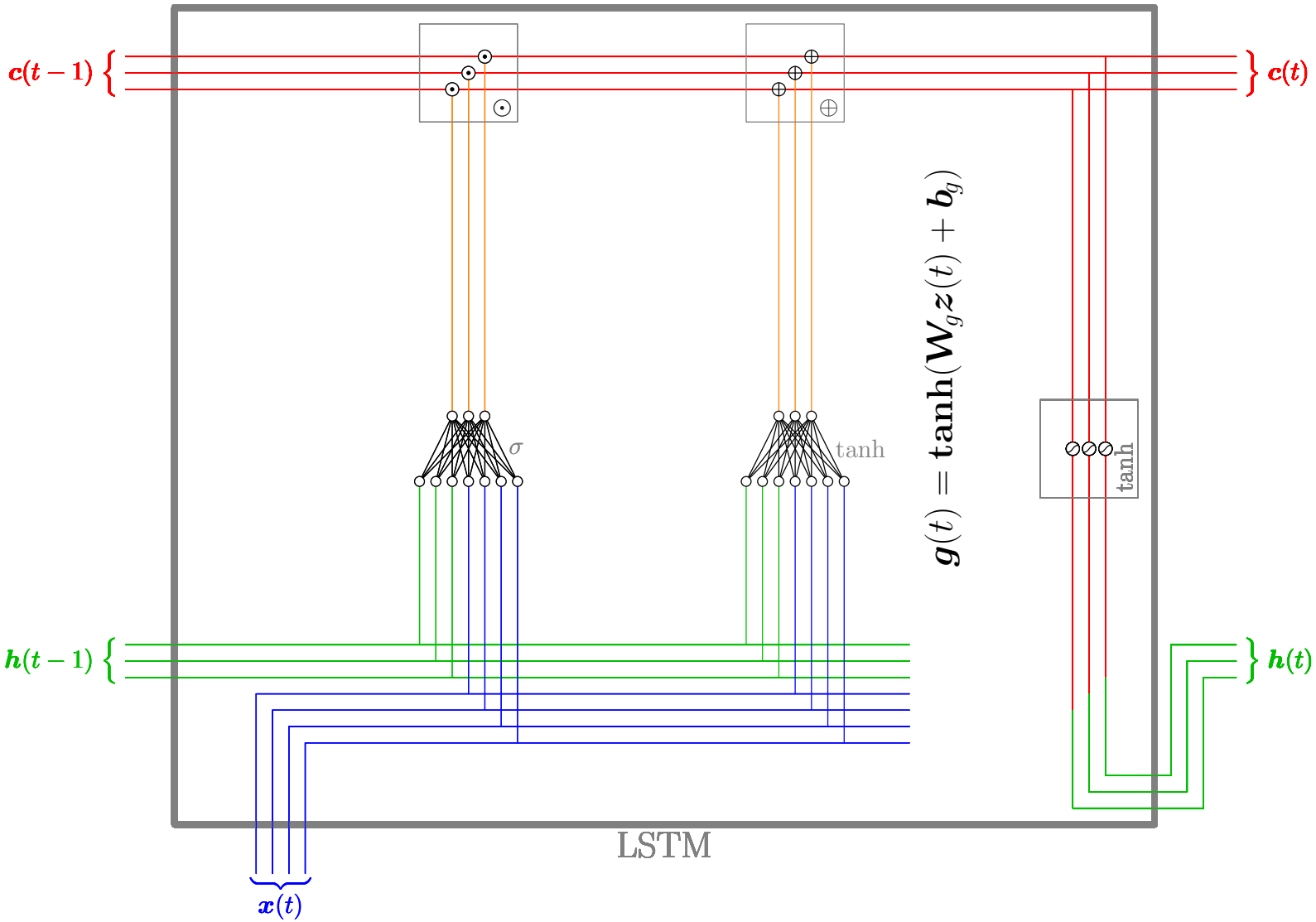

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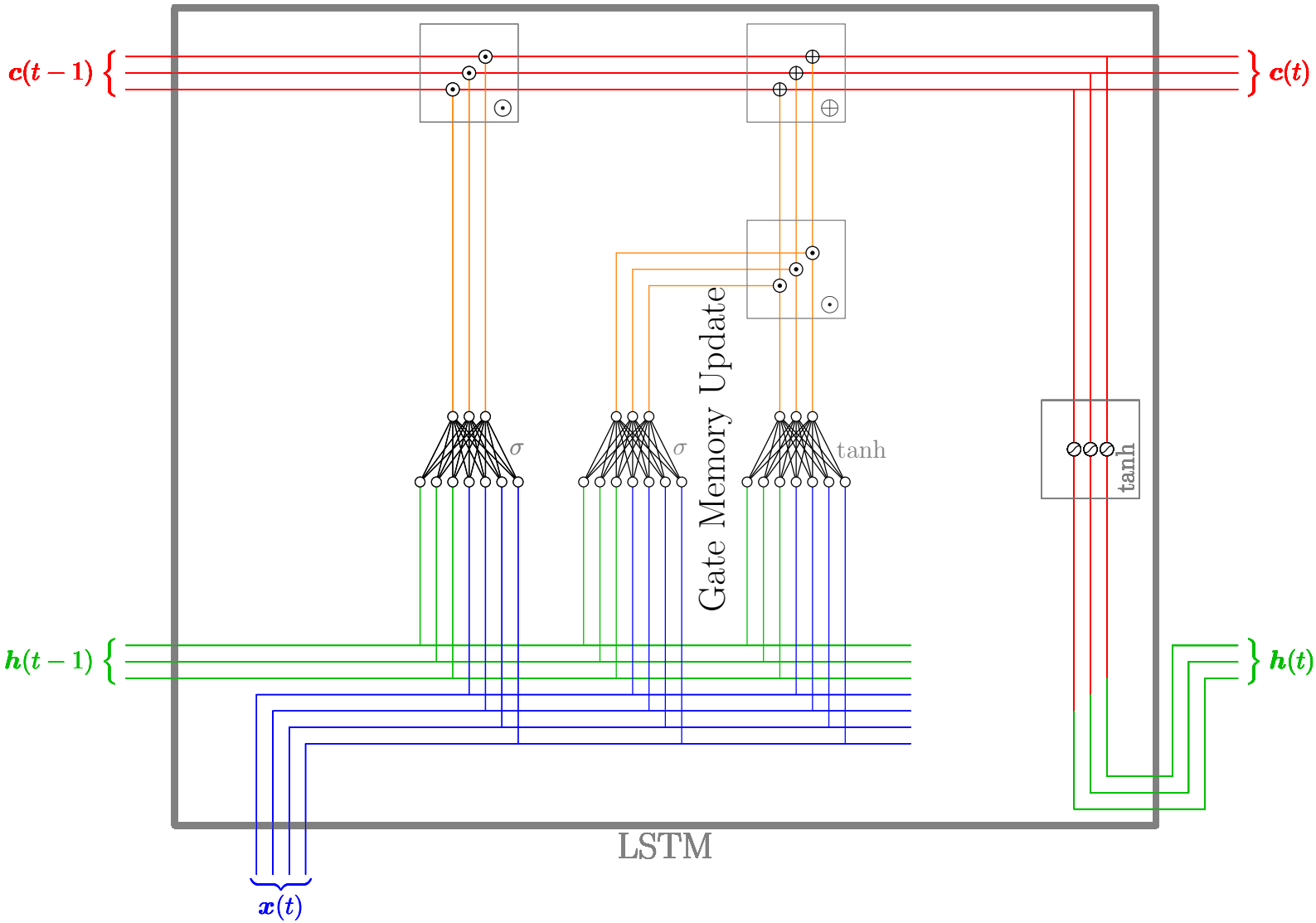

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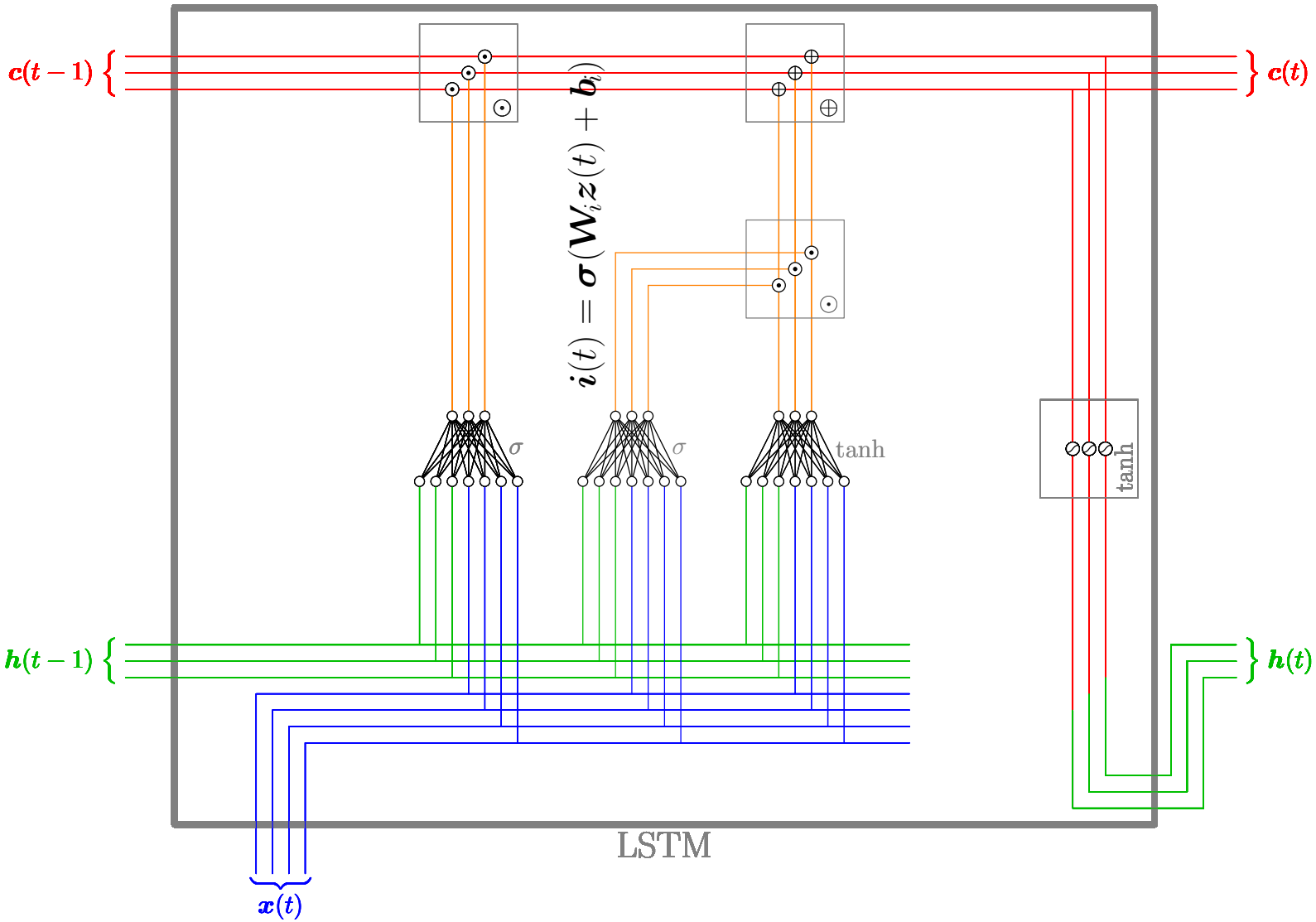

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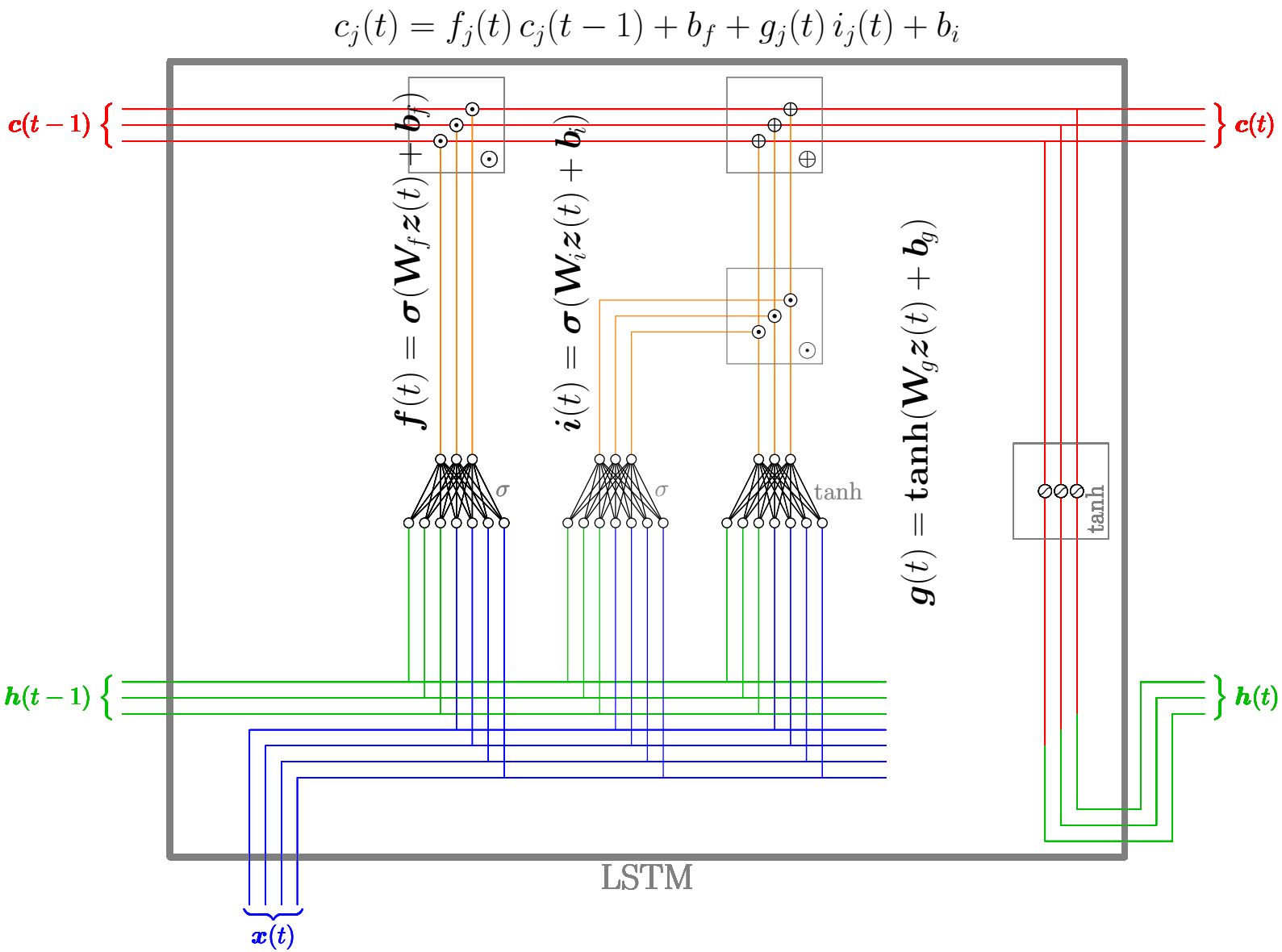

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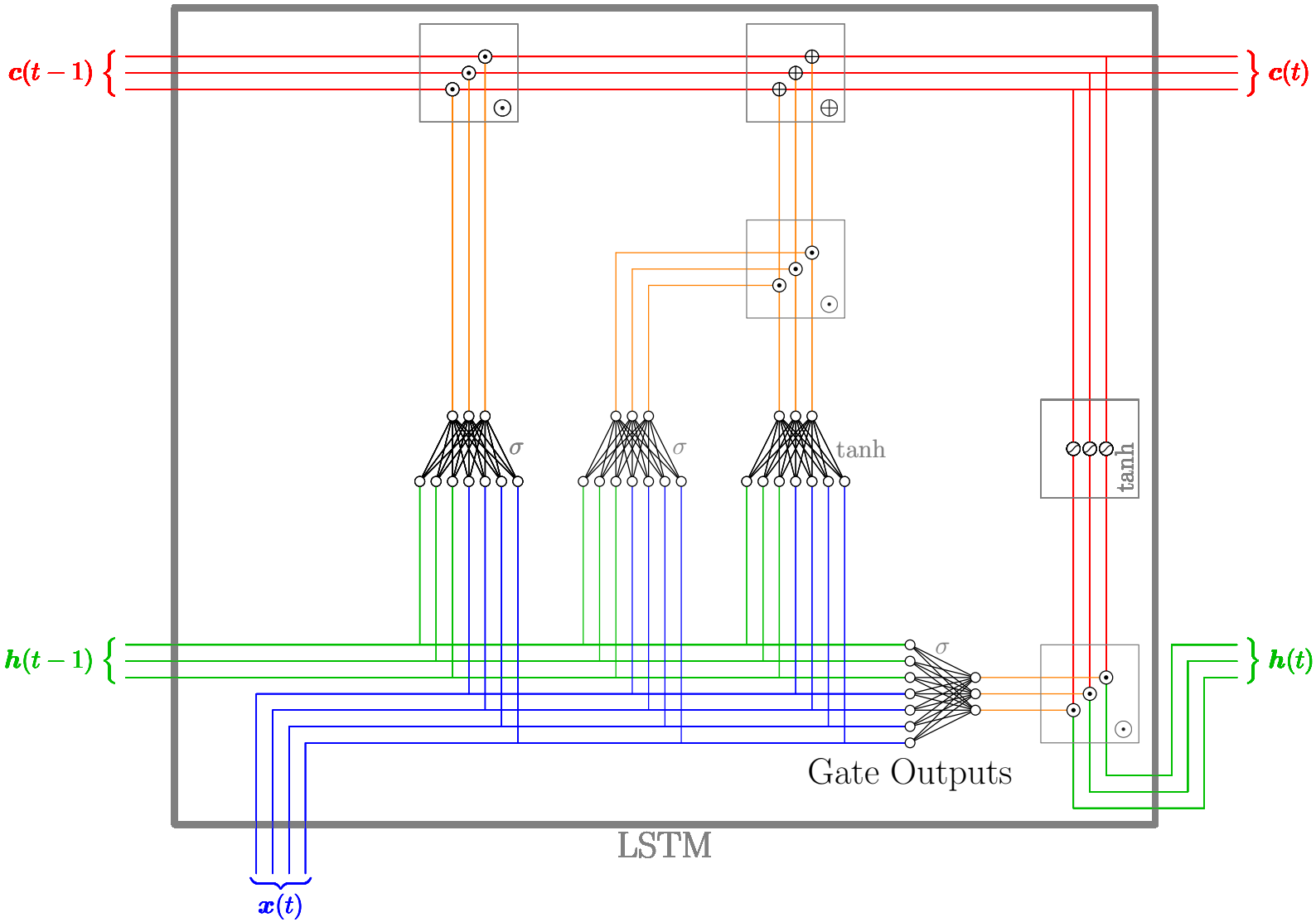

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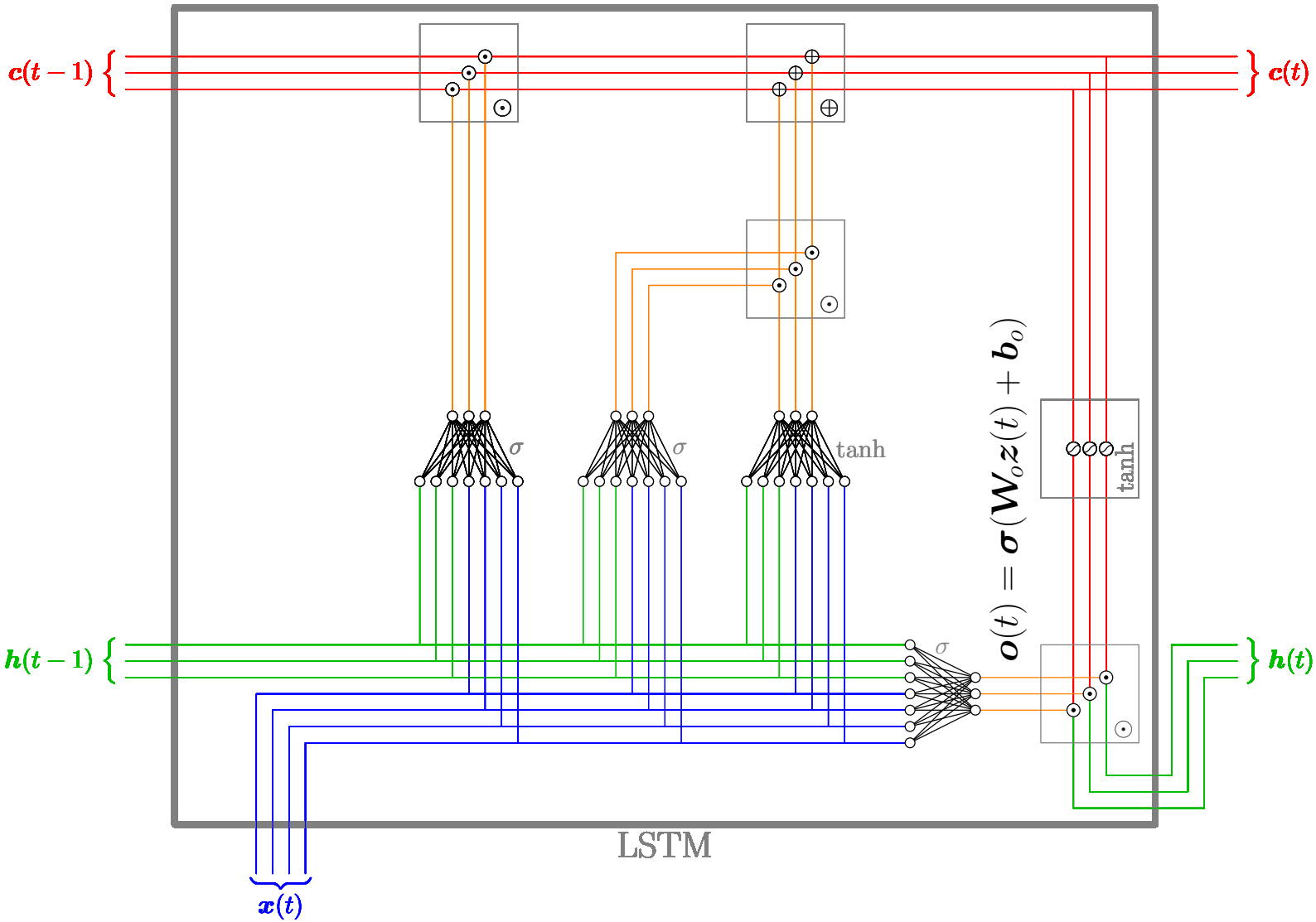

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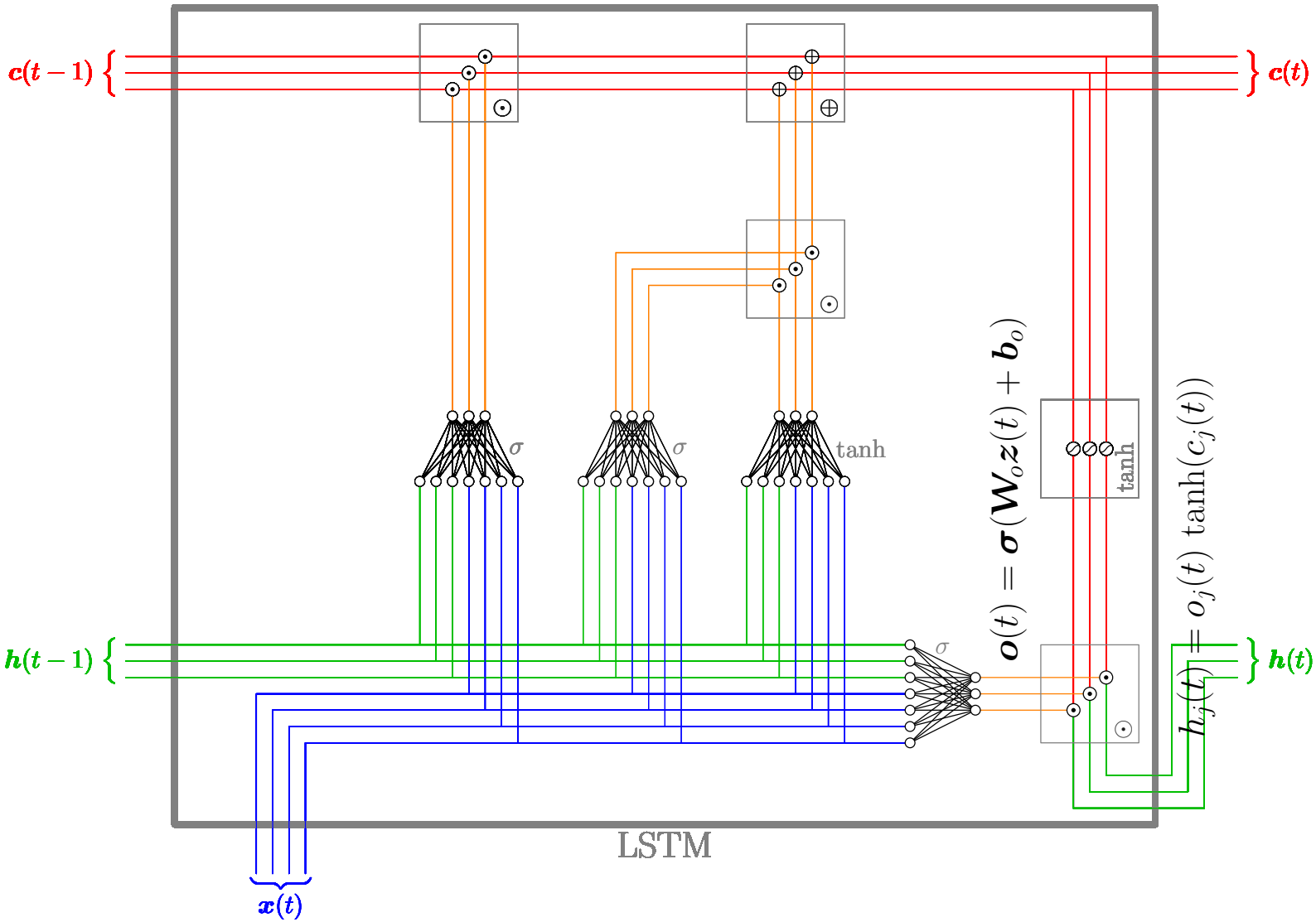

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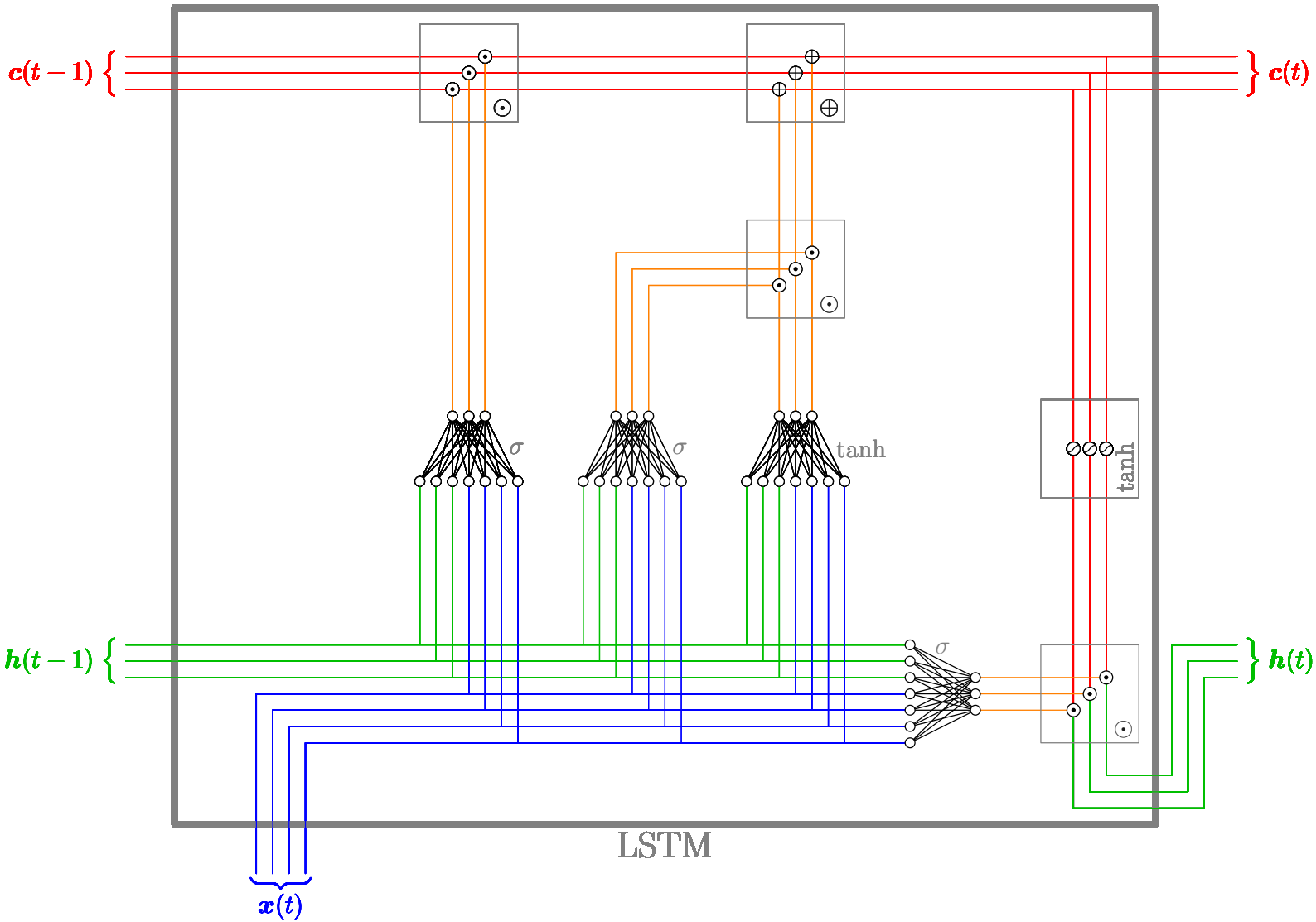

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Initially, for $t = 0$, $\boldsymbol{h}(0) = \mathbf{0}$

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- Output $\boldsymbol{h}(t) = \boldsymbol{o}(t) \odot \mathbf{tanh}(\boldsymbol{c}(t))$

Training LSTMs

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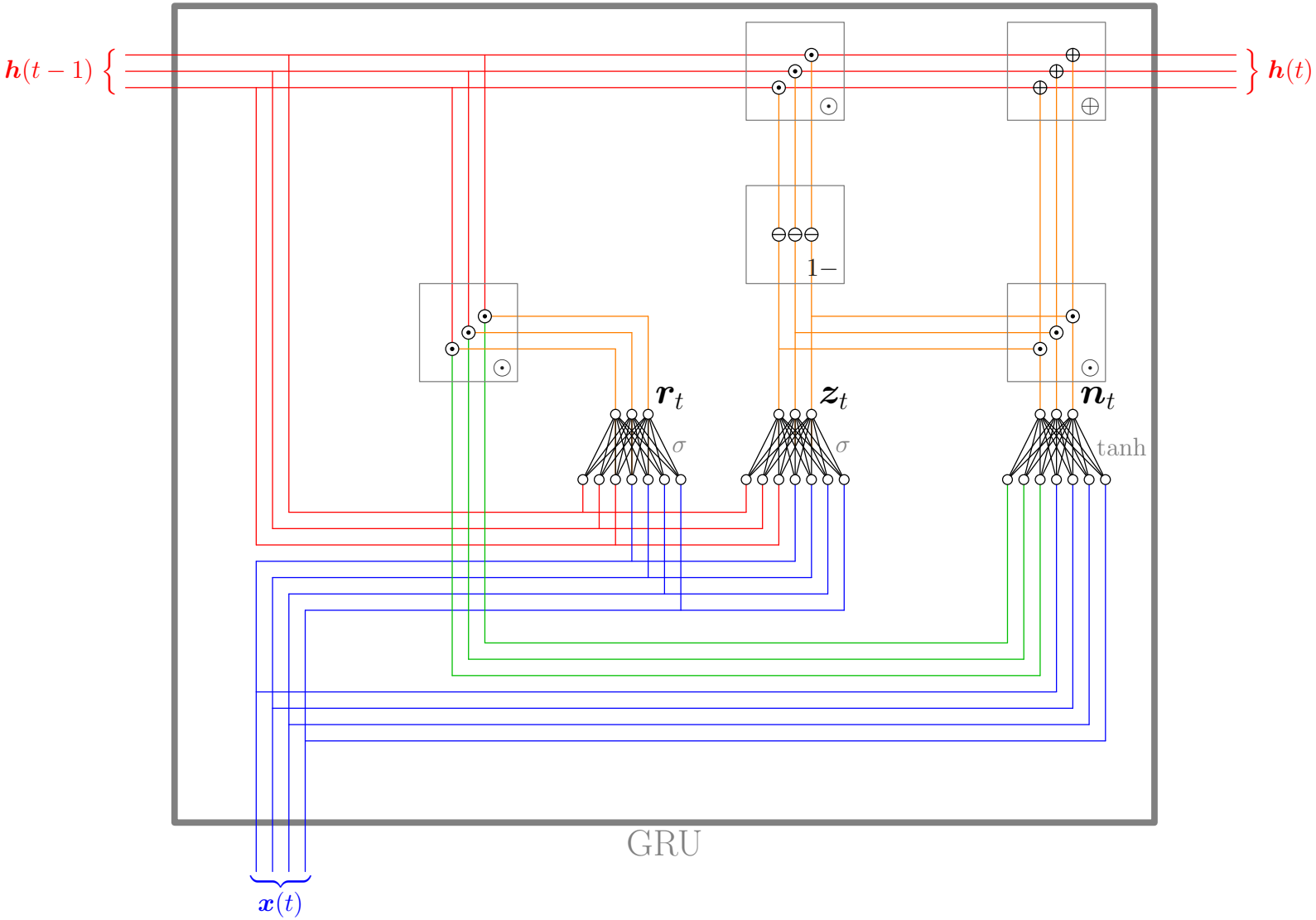
- We can train an LSTM by unwrapping it in time.
- Note that it involves four dense layers with sigmoidal (or tanh) outputs.
- This means that typically it is very slow to train.
- There are a few variants of LSTMs, but all are very similar. The most popular is probably the Gated Recurrent Unit (GRU).

LSTM Success Stories

- LSTMs have been used to win many competitions in speech and handwriting recognition.
- Major technology companies including Google, Apple, and Microsoft are using LSTMs as fundamental components in products.
- Google used LSTM for speech recognition on the smartphone, for Google Translate.
- Apple uses LSTM for the "Quicktype" function on the iPhone and for Siri.
- Amazon uses LSTM for Amazon Alexa.
- In 2017, Facebook performed some 4.5 billion automatic translations every day using long short-term memory networks[1].

[1] https://en.wikipedia.org/wiki/Long_short-term_memory

Gated Recurrent Unit (GRU)



Gated Recurrent Unit (GRU)

- $\boldsymbol{x}(t)$: input vector
- $\boldsymbol{h}(t)$: output vector (and 'hidden state')
- $\boldsymbol{r}(t)$: reset gate vector
- $\boldsymbol{z}(t)$: update gate vector
- $\boldsymbol{n}(t)$: new state vector (before update is applied)
- $\boldsymbol{W}$ and $\boldsymbol{b}$: parameter matrices and biases

Gated Recurrent Unit (GRU)

Initially, for $t = 0$, $\boldsymbol{h}(0) = \mathbf{0}$

$$\begin{aligned}
\boldsymbol{z}(t) &= \sigma(\boldsymbol{W}_z(\boldsymbol{x}(t), \boldsymbol{h}(t-1)) + \boldsymbol{b}_z) \\
\boldsymbol{r}(t) &= \sigma(\boldsymbol{W}_r(\boldsymbol{x}(t), \boldsymbol{h}(t-1)) + \boldsymbol{b}_r) \\
\boldsymbol{n}(t) &= \tanh(\boldsymbol{W}_n(\boldsymbol{x}(t), r(t) \odot h(t-1)) + \boldsymbol{b}_h) \\
\boldsymbol{h}(t) &= (1 - \boldsymbol{z}(t)) \odot \boldsymbol{h}(t-1) + \boldsymbol{z}(t) \odot \boldsymbol{n}(t)
\end{aligned}$$

Most implementations follow the original paper and swap $(1 - \boldsymbol{z}(t))$ and $(\boldsymbol{z}(t))$ in the $\boldsymbol{h}(t)$ update; this doesn't change the operation of the network, but does change the interpretation of the update gate, as the gate would have to produce a 0 when an update was to occur, and a 1 when no update is to happen (which is somewhat counter-intuitive)!

GRU or LSTM?

- GRUs have two gates (reset and update) whereas LSTM has three gates (input/output/forget)
- GRU performance on par with LSTM but computationally more efficient (less operations & weights).
- In general, if you have a very large dataset then LSTMs will likely perform slightly better.
- GRUs are a good choice for smaller datasets.