

Learn Latent Representations

Autoencoders and Self-supervised Learning

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Low Dimensional Representations

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- A few lectures ago this was particularly evident when when we looked at embedding models like word2vec which explicitly try to capture relationships in the data in a low dimensional 'latent' space.

Self-supervised Learning



Yann LeCun

30 April 2019 · 🌐



I now call it "self-supervised learning", because "unsupervised" is both a loaded and confusing term.

In self-supervised learning, the system learns to predict part of its input from other parts of its input. In other words a portion of the input is used as a supervisory signal to a predictor fed with the remaining portion of the input.

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- Let's now consider a different type self-supervised of task where we want to learn a model that learns to **copy** its input to its output.

- An **autoencoder** is a network that is trained to copy its input to its output

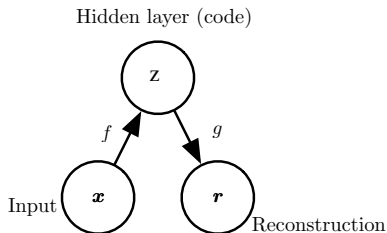
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Autoencoder constraints

- Clearly a linear autoencoder with a sufficient number of weights (e.g. if the dimension of \mathbf{z} was greater than or equal to that of \mathbf{x}) could learn set $g(f(\mathbf{x})) = \mathbf{x}$ everywhere, but this obviously wouldn't be useful!

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- In practice we apply *restrictions*¹ to stop this happening.
- The objective is to use these restrictions to force the autoencoder to learn useful properties of the data.

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Undercomplete Autoencoders

- Undercomplete autoencoders have $\dim(\mathbf{z}) \ll \dim(\mathbf{x})$.
- This forces the encoder to learn a *compressed representation* of the input.
- The representation will capture the most *salient* features of the input data.

Undercomplete Autoencoders — Linear

Consider the single-hidden layer linear autoencoder network given by:

$$h = \mathbf{W}_e \mathbf{x} + \mathbf{b}_e$$

$$r = \mathbf{W}_d \mathbf{z} + \mathbf{b}_d$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{z} \in \mathbb{R}^m$ and $m < n$.

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With the MSE loss, this autoencoder will learn to span the same subspace as PCA for a given set of training data.

Note that the autoencoder weights are not however constrained to be orthogonal (like they would be in PCA)

Undercomplete Autoencoders — deeper and nonlinear

- A linear autoencoder with a single hidden layer learns to map into the same subspace as PCA.

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 - Interestingly, a single hidden layer network with non-linear activations on the encoder (keeping the decoder linear) and MSE loss also just learns to span the PCA subspace²!

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 - Interestingly, a single hidden layer network with non-linear activations on the encoder (keeping the decoder linear) and MSE loss also just learns to span the PCA subspace²!
 - But, if you add more hidden layers with non-linear activations (to either the encoder, decoder or both) you can effectively perform a powerful non-linear generalisation of PCA

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Deep Autoencoders

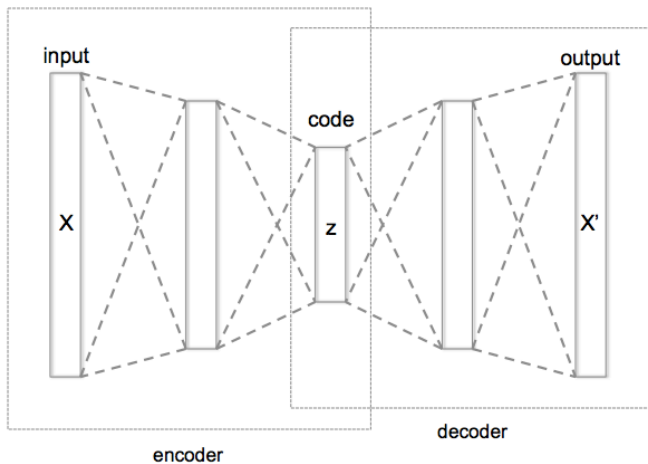


Image taken from wikipedia

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- Extreme example:
 - Consider a powerful encoder that maps \mathbf{x} to $\mathbf{z} \in \mathbb{R}^1$
 - Each training example $\mathbf{x}^{(i)}$ could e.g. be mapped to i .
 - The decoder just needs to memorise the training examples so that it can map back from i .

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- There is nothing stopping us using any other kinds of layers though...
- If we're working with image data, where we know that much of the structure is 'local', then using convolutions in both the decoder makes sense

Convolutional Autoencoder

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- Many ways to do this; let's look at two of them:
 - Denoising Autoencoders
 - Sparse Autoencoders

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 - E.g. by adding Gaussian noise.
- The loss is computed between the reconstruction (computed from the noisy input) against the original noise-free data.

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- A popular choice that you've seen before would be to use an l1 penalty $\Omega(\mathbf{z}) = \lambda \sum_i |h_i|$
 - this of course does have a slight problem... what is the derivative of $y = |x|$ with respect to x at $x = 0$?

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Beyond Deterministic Autoencoders: Stochastic Encoders and Decoders

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- What about the encoder - could we make that output $p(\mathbf{z}|\mathbf{x})$?