

**Yes,  
we GAN.**

# Deep Generative Modelling

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- What is generative modelling and why do we do it?
- Differentiable Generator Networks
- Variational Autoencoders
- Generative Adversarial Networks

# Generative Modelling and Differentiable Generator Networks

# Recap: Generative Models

- Learn models of the data:  $p(x)$
- Learn *conditional* models of the data:  $p(x|y = y)$
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    - The likelihood of data  $x$  is the weighted sum of the likelihood from each of the  $k$  Gaussians
    - Sampling can be achieved by sampling the categorical distribution of  $k$  weights followed by sampling a data point from the corresponding Gaussian

# Why do generative modelling?

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  - Probabilistic latent variable models like VAEs or topic models (PLSA, LDA, ...) for text
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- Make 'new' data
  - Make 'fake' data to use to train large supervised models?
  - 'Imagine' new, but plausible, things?

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  - Generative adversarial networks - A way to train generative models by optimizing them to fool a classifier
- **Common thread in recent advances is that the loss functions are end-to-end differentiable.**

# Differentiable Generator Networks: key idea

- We're interested in models that transform samples of latent variables  $\mathbf{z}$  to
  - samples  $x$ , or,
  - distributions over samples  $x$
- The model is a (differentiable) function  $g(\mathbf{z}, \theta)$ 
  - typically  $g$  is a neural network.

## Example: drawing samples from $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

- Consider a simple generator network with a single affine layer that maps samples  $\mathcal{N}(\mathbf{0}, \mathbf{I})$  to  $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ :

$$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \longrightarrow \boxed{g_{\boldsymbol{\theta}}(\mathbf{z})} \longrightarrow \mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

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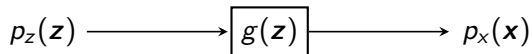
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- Note: Exact solution is  $\mathbf{x} = g_{\theta}(\mathbf{z}) = \boldsymbol{\mu} + \mathbf{L}\mathbf{z}$  where  $\mathbf{L}$  is the Cholesky decomposition of  $\boldsymbol{\Sigma}$ :  $\boldsymbol{\Sigma} = \mathbf{L}\mathbf{L}^{\top}$ , lower triangular  $\mathbf{L}$ .

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For any *invertible, differentiable, continuous*  $g$ :

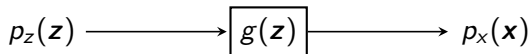
$$p_{\mathbf{z}}(\mathbf{z}) = p_{\mathbf{x}}(g(\mathbf{z})) \left| \det \left( \frac{\partial g}{\partial \mathbf{z}} \right) \right|$$

Which implicitly imposes a probability distribution over  $\mathbf{x}$ :

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Note: usually use an indirect means of learning  $g$  rather than minimise  $-\log(p(\mathbf{x}))$  directly

- Rather than use  $g$  to provide a sample of  $\mathbf{x}$  directly, we could instead use  $g$  to define a conditional distribution over  $\mathbf{x}$ ,  $p(\mathbf{x}|\mathbf{z})$ 
  - For example,  $g$  might produce the parameters of a particular distribution - e.g.:
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  - For example,  $g$  might produce the parameters of a particular distribution - e.g.:
    - means of Bernoulli
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- The distribution over  $\mathbf{x}$  is imposed by marginalising  $\mathbf{z}$ :  $p(\mathbf{x}) = \mathbb{E}_{\mathbf{z}} p(\mathbf{x}|\mathbf{z})$

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- In both cases ( $g$  generates samples and  $g$  generates distributions) we can use the reparameterisation tricks we saw last lecture to train models.
- Generating distributions:
  - + works for both continuous and discrete data
  - - need to specify the form of the output distribution
- Generating samples:
  - + works for continuous data
    - + discrete data is recently possible - we need the STargmax
  - + don't need to specify the distribution in explicit form

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- Generative modelling is more complex than classification because
  - learning requires optimizing intractable criteria
  - data does not specify both input  $\mathbf{z}$  and output  $\mathbf{x}$  of the generator network
  - learning procedure needs to determine how to arrange  $\mathbf{z}$  space in a useful way and how to map  $\mathbf{z}$  to  $\mathbf{x}$

# Variational Autoencoders

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- However, VAEs map the input into a distribution.
- VAEs are a combination of neural networks (AEs) and **graphical models**.

# Graphical Models (Background)

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- Graphical models are commonly used in probability theory, statistics—particularly Bayesian statistics—and machine learning.<sup>1</sup>

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# KL Divergence (Background)

- Kullback–Leibler divergence,  $D_{\text{KL}}(P \parallel Q)$ : a measure of how one probability distribution  $Q$  is different from a second, reference probability distribution  $P$ .<sup>2</sup>

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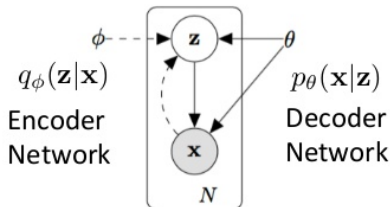
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- A simple interpretation of the divergence of  $P$  from  $Q$  is the expected excess surprise from using  $Q$  as a model when the actual distribution is  $P$ .
- While it is a distance, it is not a metric, the most familiar type of distance: it is asymmetric in the two distributions.

---

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## Variational Autoencoder



Minimize:  $D_{KL}[q_{\phi}(\mathbf{z}|\mathbf{x})||p_{\theta}(\mathbf{z}|\mathbf{x})]$

Intractable:  $p_{\theta}(\mathbf{z}|\mathbf{x}) = \frac{p_{\theta}(\mathbf{x}|\mathbf{z})p_{\theta}(\mathbf{z})}{p_{\theta}(\mathbf{x})}$



# Variational Autoencoders (VAEs)

The distance loss just defined is expanded as

$$\begin{aligned} D_{KL}(q_{\Phi}(\mathbf{z} | \mathbf{x}) \parallel p_{\theta}(\mathbf{z} | \mathbf{x})) &= \int q_{\Phi}(\mathbf{z} | \mathbf{x}) \log \frac{q_{\Phi}(\mathbf{z} | \mathbf{x})}{p_{\theta}(\mathbf{z} | \mathbf{x})} d\mathbf{z} \\ &= \int q_{\Phi}(\mathbf{z} | \mathbf{x}) \log \frac{q_{\Phi}(\mathbf{z} | \mathbf{x}) p_{\theta}(\mathbf{x})}{p_{\theta}(\mathbf{z}, \mathbf{x})} d\mathbf{z} \\ &= \int q_{\Phi}(\mathbf{z} | \mathbf{x}) \left( \log(p_{\theta}(\mathbf{x})) + \log \frac{q_{\Phi}(\mathbf{z} | \mathbf{x})}{p_{\theta}(\mathbf{z}, \mathbf{x})} \right) d\mathbf{z} \\ &= \log(p_{\theta}(\mathbf{x})) + \int q_{\Phi}(\mathbf{z} | \mathbf{x}) \log \frac{q_{\Phi}(\mathbf{z} | \mathbf{x})}{p_{\theta}(\mathbf{z}, \mathbf{x})} d\mathbf{z} \\ &= \log(p_{\theta}(\mathbf{x})) + \int q_{\Phi}(\mathbf{z} | \mathbf{x}) \log \frac{q_{\Phi}(\mathbf{z} | \mathbf{x})}{p_{\theta}(\mathbf{x} | \mathbf{z}) p_{\theta}(\mathbf{z})} d\mathbf{z} \\ &= \log(p_{\theta}(\mathbf{x})) + E_{\mathbf{z} \sim q_{\Phi}(\mathbf{z} | \mathbf{x})} \left( \log \frac{q_{\Phi}(\mathbf{z} | \mathbf{x})}{p_{\theta}(\mathbf{z})} - \log(p_{\theta}(\mathbf{x} | \mathbf{z})) \right) \\ &= \log(p_{\theta}(\mathbf{x})) + D_{KL}(q_{\Phi}(\mathbf{z} | \mathbf{x}) \parallel p_{\theta}(\mathbf{z})) - E_{\mathbf{z} \sim q_{\Phi}(\mathbf{z} | \mathbf{x})} (\log(p_{\theta}(\mathbf{x} | \mathbf{z}))) \end{aligned}$$

At this point, it is possible to rewrite the equation as

$$\log(p_{\theta}(\mathbf{x})) - D_{KL}(q_{\Phi}(\mathbf{z} | \mathbf{x}) \parallel p_{\theta}(\mathbf{z} | \mathbf{x})) = E_{\mathbf{z} \sim q_{\Phi}(\mathbf{z} | \mathbf{x})} (\log(p_{\theta}(\mathbf{x} | \mathbf{z}))) - D_{KL}(q_{\Phi}(\mathbf{z} | \mathbf{x}) \parallel p_{\theta}(\mathbf{z}))$$

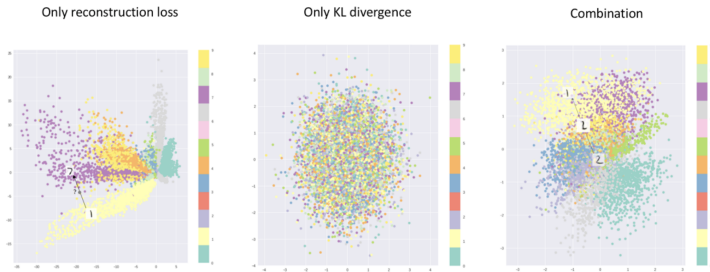
# Evidence Lower Bound (ELBO) Loss

$$L_{VAE}(\theta, \phi) = -\mathbb{E}_{z \sim q_{\phi}(z|x)} \log(p_{\theta}(x|z)) + D_{KL}(q_{\phi}(z|x) || p_{\theta}(z))$$

- We are trying to minimize the ELBO loss with respect to the model parameters.

# Why Autoencoder?

- The reconstruction term, forces each image to be unique and spread out.

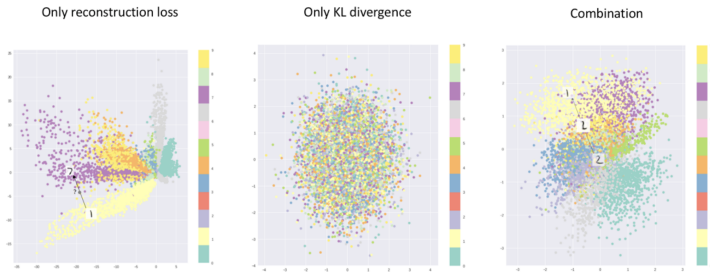


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<sup>4</sup>Figure taken from <https://towardsdatascience.com/intuitively-understanding-variational-autoencoders-1bfe67eb5daf>

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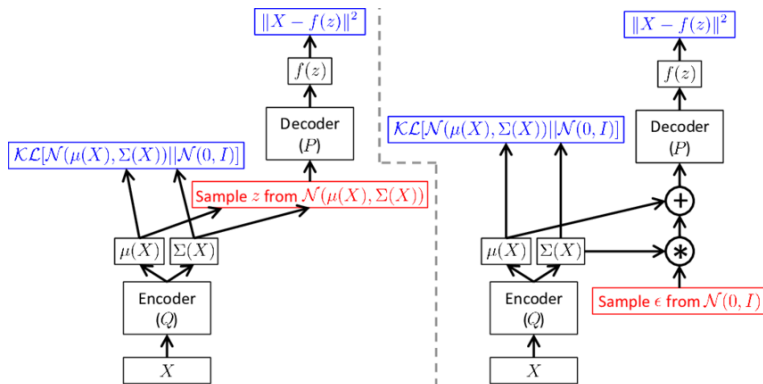
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- The KL term will push all the images towards the same prior.



4

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# Training Procedure

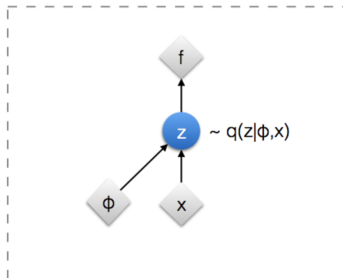


5

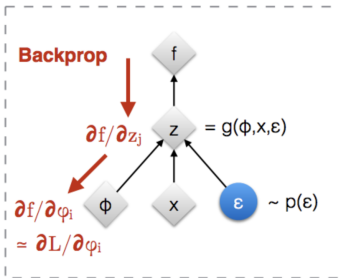
<sup>5</sup>Figure taken from Carl Doersch tutorial



# Reparametrization Trick Visualisation

Original form



Reparameterised form



 : Deterministic node  
 : Random node

[Kingma, 2013]  
[Bengio, 2013]  
[Kingma and Welling 2014]  
[Rezende et al 2014]

# VAE Models and Performance

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  - the distributions and network architecture just needs to be set accordingly
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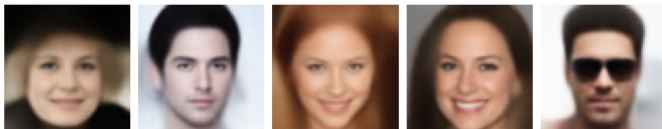
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- VAEs tend to only utilise a small subset of the dimensions of  $\mathbf{z}$

# Reconstructions Example

**Input**



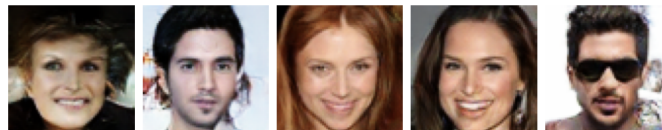
**VAE**



**VAE<sub>Dis<sub>l</sub></sub>**



**VAE/GAN**



# Generative Adversarial Networks

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- Both discriminator and generator are deep networks (differentiable functions)
- LeCun quote 'GANs, the most interesting idea in the last ten years in machine learning'

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## Aside: Adversarial Learning vs. Adversarial Examples

The approach of GANs is called adversarial since the two networks have *antagonistic* objectives.

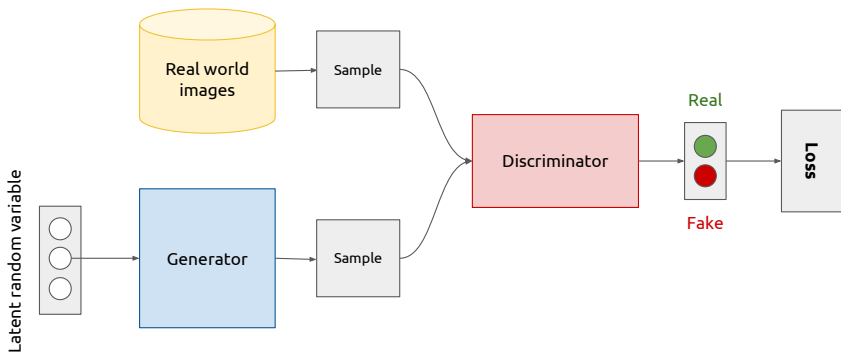
This is not to be confused with *adversarial examples* in machine learning.

See these two papers for more details:

<https://arxiv.org/pdf/1412.6572.pdf>

<https://arxiv.org/pdf/1312.6199.pdf>

# Generative adversarial networks (conceptual)



Picture Credit: Xavier Giro-i-Nieto

- The **generator**

$$\mathbf{x} = g(\mathbf{z})$$

is trained so that it gets a random input  $\mathbf{z} \in \mathbb{R}^n$  from a distribution (typically  $\mathcal{N}(\mathbf{0}, \mathbf{I})$  or  $\mathcal{U}(\mathbf{0}, \mathbf{I})$ ) and produces a sample  $\mathbf{x} \in \mathbb{R}^d$  following the data distribution as output (ideally). Usually  $n \ll d$ .

- The **discriminator**

$$y = d(\mathbf{x})$$

gets a sample  $\mathbf{x}$  as input and predicts a probability  $y \in [0, 1]$  (or real-valued logit of a Bernoulli distribution) determining if it is real or fake.

- Training a standard GAN is difficult and often results in two undesirable behaviours
  - Oscillations without convergence. No guarantee that the loss will actually decrease...
    - It has been shown that a GAN has saddle-point solution, rather than a local minima.
  - The **mode collapse** problem, when the generator models very well a small sub-population, concentrating on a few modes.
- Additionally, performance is hard to assess and often boils down to heuristic observations.

# Deep Convolutional Generative Adversarial Networks (DCGANs)

- Motivates the use of GANs to learn reusable feature representations from large unlabelled datasets.
- GANs known to be unstable to train, often resulting in generators that produce “nonsensical outputs”.
- Model exploration to identify architectures that result in **stable** training across datasets with higher resolution and deeper models.



# Architecture Guidelines for Stable DCGAN

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- Use LeakyReLU activation in the discriminator for all layers.

- Generative modelling is a massive field with a long history
- Differentiable generators have had a profound impact in making models that work with real data at scale
- VAEs and GANs are currently the most popular approaches to training generators for spatial data
- We've only scratched the surface of generative modelling
  - Auto-regressive approaches are popular for sequences (e.g. language modelling).
    - But also for images (e.g. PixelRNN, PixelCNN)
  - typically RNN-based
  - but not necessarily - e.g. WaveNet is a convolutional auto-regressive generative model