Process Sequences

VLC = $\iiint$ Vision Learning Control $dV\,dL\,dC$

Recurrent Neural Networks

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University of Southampton

A lot of the ideas in this lecture come from Andrej Karpathy's blog post on the Unreasonable Effectiveness of RNNs (http://karpathy.github.io/2015/05/21/rnn-effectiveness/). Many of the images and animations were made by Adam Prügel-Bennett.

Recurrent Neural Networks - Motivation

x:	Jon	and	Dani	gave	deep	learning	lectures
y:	1	0	1	0	0	0	0

Recurrent Neural Networks - Motivation

x:	$x^{(1)}$	...	$x^{(t)}$	...	$x^{(T_x)}$
x:	Jon	...	Dani	...	lectures
y:	$y^{(1)}$	...	$y^{(t)}$	...	$y^{(T_y)}$
y:	1	...	1	...	0

In this example, $T_x = T_y = 7$ but T_x and T_y can be different.

Recurrent Neural Networks

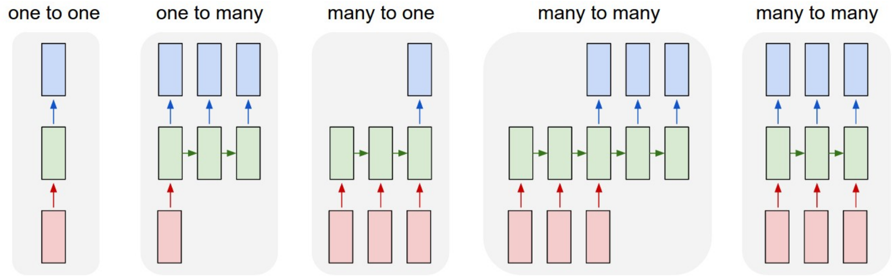

Image from http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Aside: One Hot Encoding

How can we represent individual words (or other discrete tokens)?

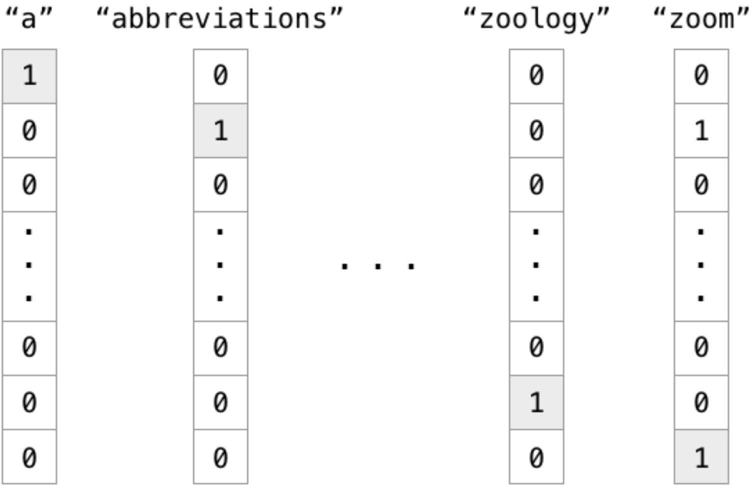

Image from https://ayearofai.com

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- To facilitate this we would like to add a feedback loop delayed in time

Recurrent Neural Networks

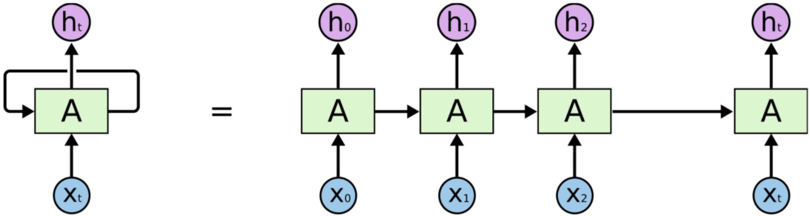

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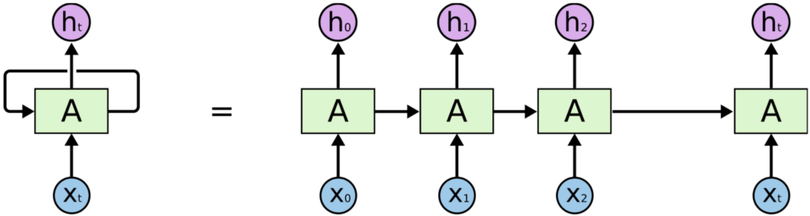
Image taken from `https://towardsdatascience.com`

Recurrent Neural Networks



- RNNs are a family of ANNs for processing sequential data
- RNNs have directed cycles in their computational graphs

Image taken from `https://towardsdatascience.com`

Recurrent Neural Networks

RNNs combine two properties which make them very powerful.

1. Distributed hidden state that allows them to store a lot of information about the past efficiently. This is because several different units can be active at once, allowing them to remember several things at once.
2. Non-linear dynamics that allows them to update their hidden state in complicated ways[1].

[1] Often said to be difficult to train, but this is not necessarily true - dropout can help with overfitting for example

Recurrent Neural Networks

RNNs are Turing complete in the sense they can simulate arbitrary programs[2].

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> If training vanilla neural nets is optimisation over functions, training recurrent nets is optimisation over programs.

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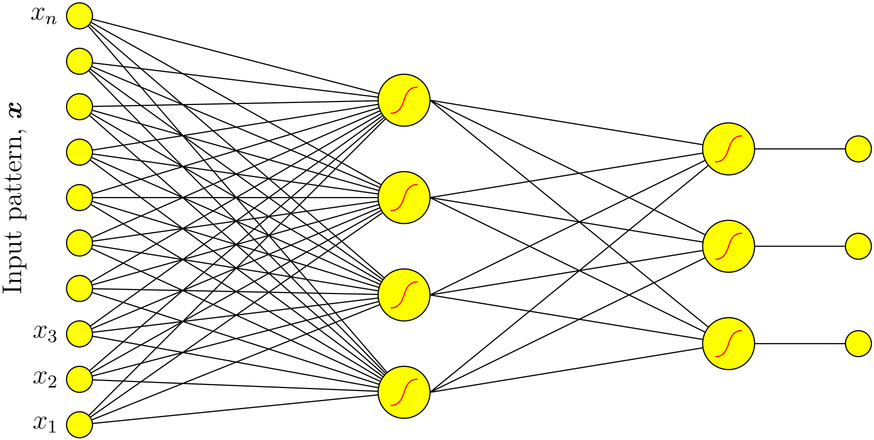
Recurrent Network



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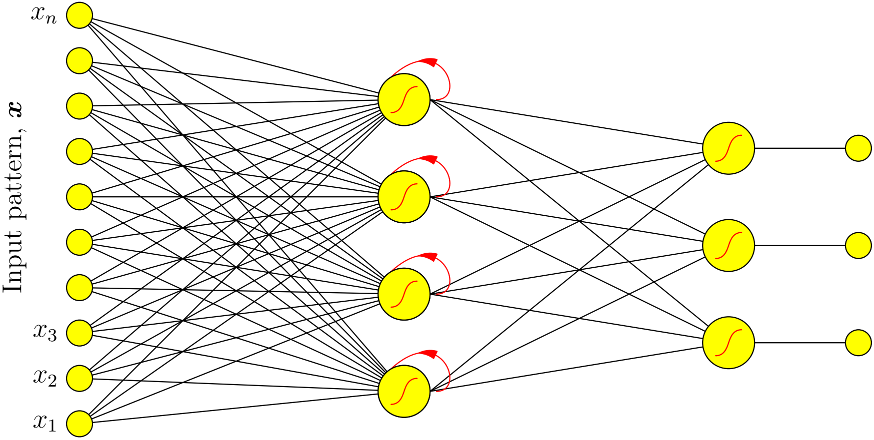

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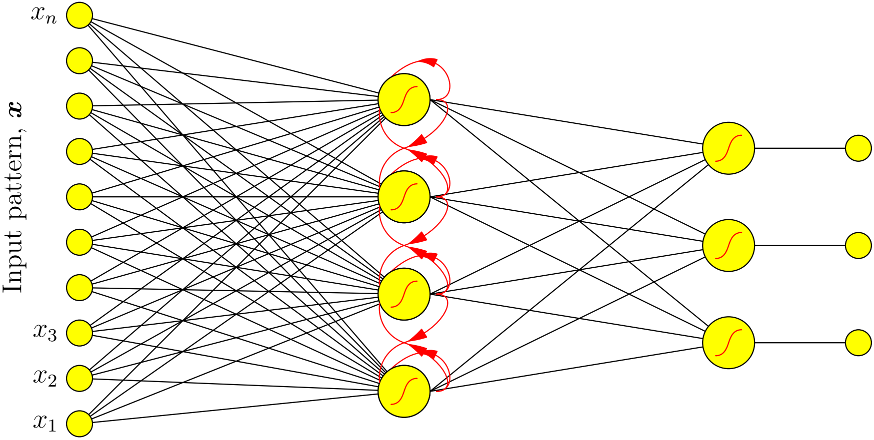

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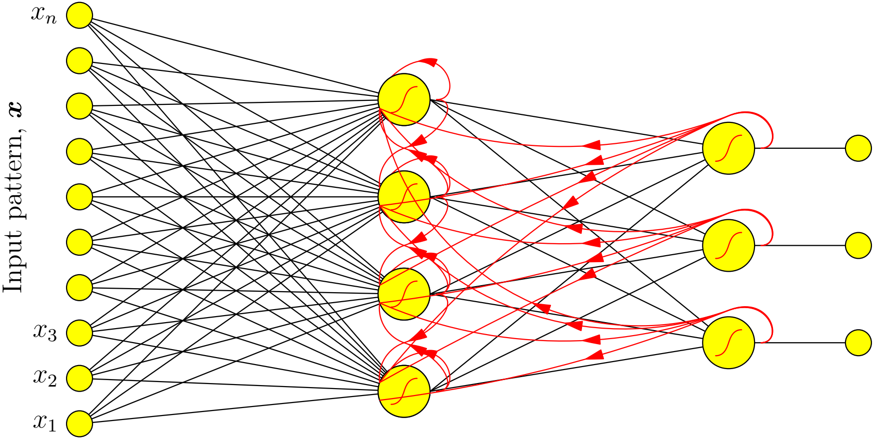

Training Recurrent Networks

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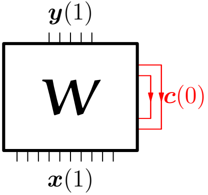

- Minimise an error (here MSE, but your choice):

$$E(\boldsymbol{W}) = \sum_{t=1}^{T} \left\| \boldsymbol{y}(t) - \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{c}(t-1) | \boldsymbol{W}) \right\|^2$$

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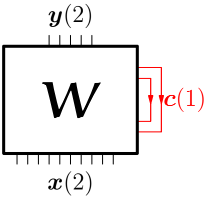

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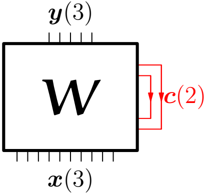

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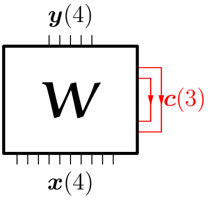

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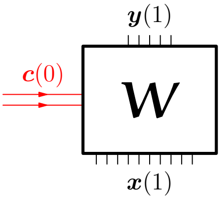

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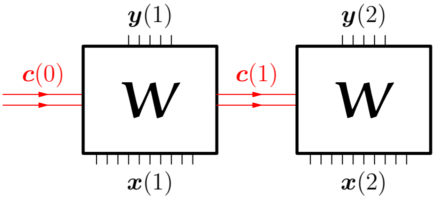

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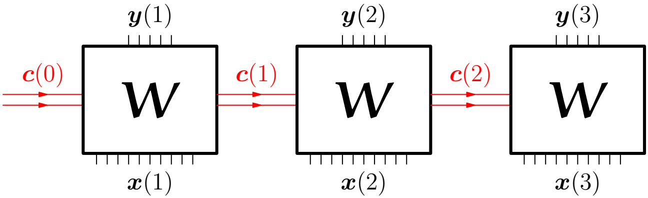

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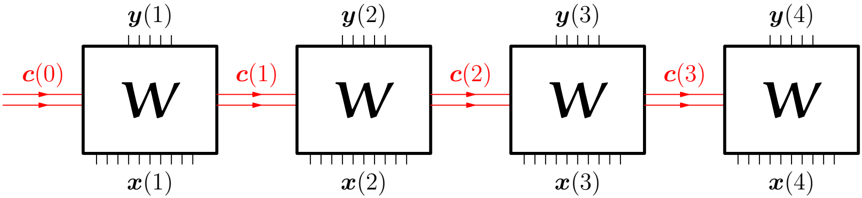

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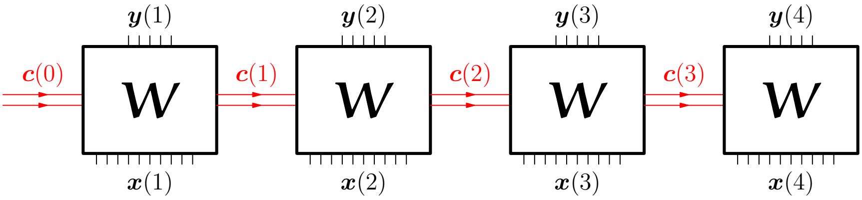

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- This is known as *back-propagation through time*

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- it should be clear that the gradients of this with respect to the weights can be found with the chain rule

What is the state update $g()$?

- It depends on the variant of the RNN!
  - Elman
  - Jordan
  - LSTM
  - GRU

Elman Networks ("Vanilla RNNs")

$$\boldsymbol{h}_t = \sigma_h(\boldsymbol{W}_{ih}\boldsymbol{x}_t + \boldsymbol{b}_{ih} + \boldsymbol{W}_{hh}\boldsymbol{h}_{t-1} + \boldsymbol{b}_{hh})$$
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- σ_h is usually tanh
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- the hidden state at time t is a summation of a projection of the input and a projection of the previous hidden state

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- The output of the inner RNN at time t is fed into the input of the outer RNN which produces the prediction y
- Also note: RNNs are most often not used in isolation - it's quite common to process the inputs and outputs with MLPs (or even convolutions)

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- We'll end with an example: an RNN that learns to 'generate' English text by learning to predict the next character in a sequence

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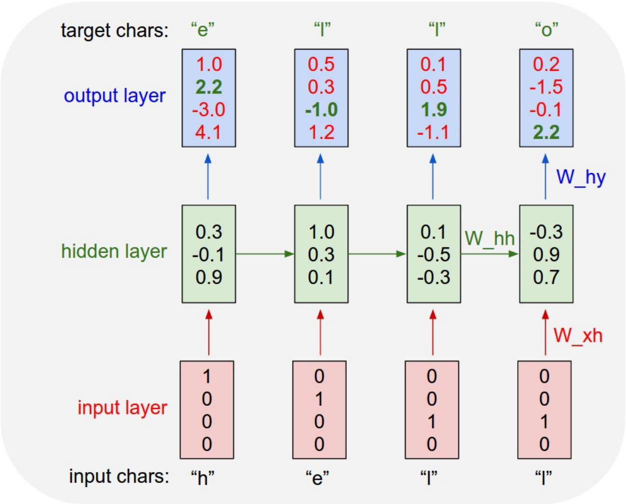

Image from `http://karpathy.github.io/2015/05/21/rnn-effectiveness/`

Training a Char-RNN

- The training data is just text data (e.g. sequences of characters)
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  - Overfitting could be a problem: dropout is very useful here

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- how do you 'sample' a character from the logits?
    - you could pick the most likely (maximum-likelihood solution), but this might lead to generated text with very low variance (it might be boring and repetitive)
    - you could treat the softmax probabilities defined by the logits as a categorical distribution and sample from them
        - you might increase the 'temperature', T, of the softmax to make the distribution more diverse (less 'peaky'): $q_i = \frac{\exp(z_i/T)}{\sum_j \exp(z_j/T)}$

A lot of the ideas in this lectu on the input c(t − 1), g (x i (x(t − 2), g (t − 1)—W)
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Onaafed a tre the sidisicters of to prediction couponet on the logits its venvows usts sevouvd be this in as useuled at on in the pan Lerate'atectsrray to paet inputs D = Pxxpraition the rople, the next
vog the state atite

- Sampled from a single layer RNN[3].

[3] LSTM, 128 dim hidden size, with linear input projection to 8-dimensions and output to the number of characters (84). Trained on the text of these slides for 50 epochs.