BB512/BB612 - Week III

```
suppressPackageStartupMessages(library(Biobase))
suppressPackageStartupMessages(library(dendextend))
suppressPackageStartupMessages(library(factoextra))
```

Clustering

Data

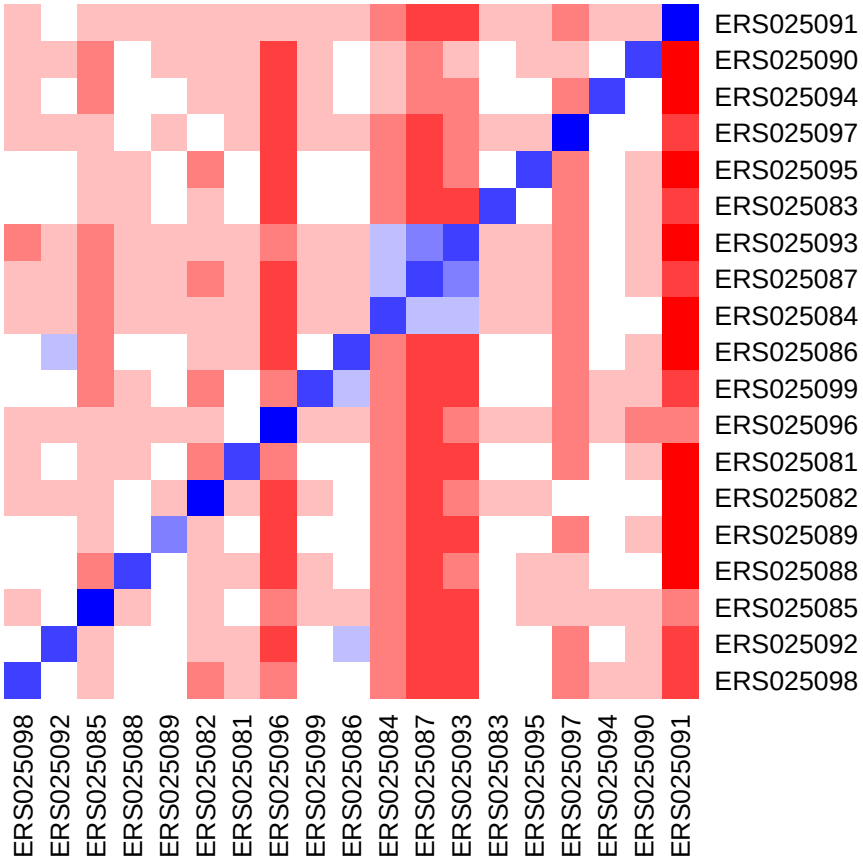
Once again, we'll use the `bodymap` expression dataset.

```
con <- url("http://bowtie-bio.sourceforge.net/recount/ExpressionSets/bodymap_eset.RData")
load(file = con)
close(con)

bm <- bodymap.eset

pdata <- pData(bm) # phenotype data
edata <- exprs(bm) # expression data
fdata <- fData(bm) # features data
```

```
edata <- edata[rowMeans(edata) > 5000, ]
edata <- log2(edata + 1)
```

Distances

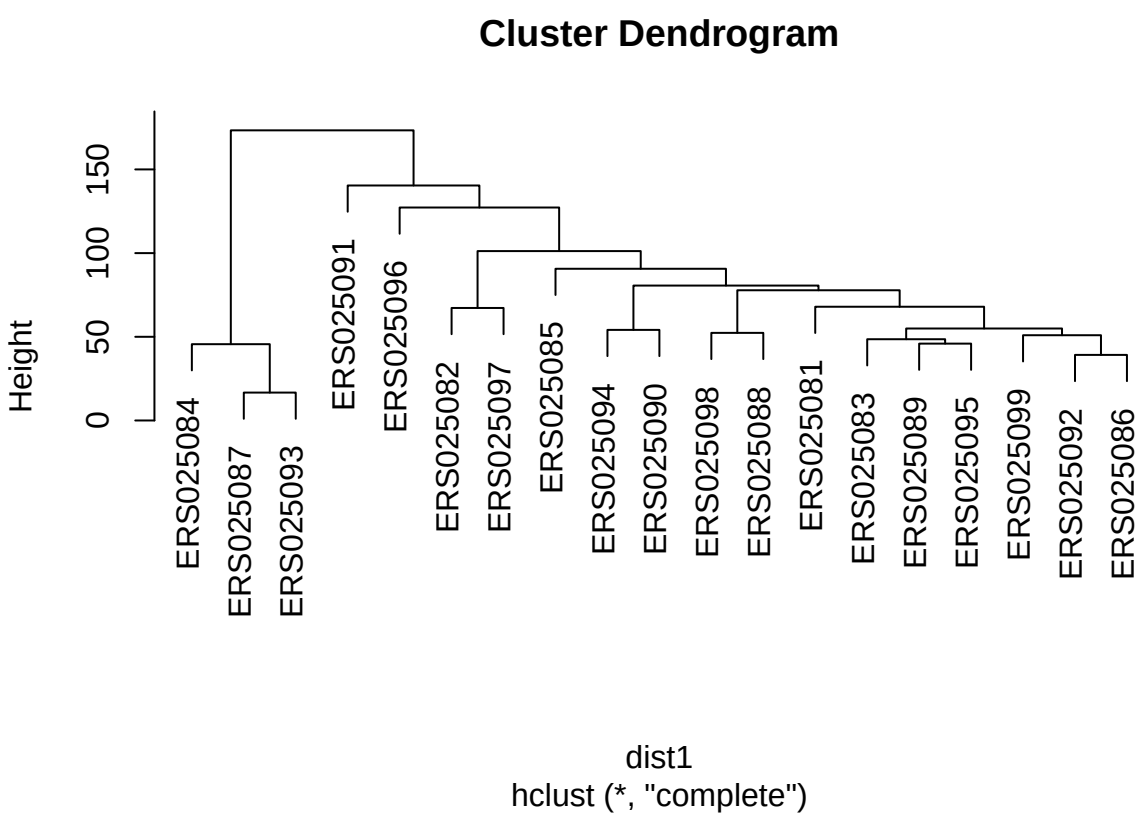
```
# By default calculates the euclidean distance between rows
dist1 = dist(t(edata))

## Look at distance matrix
colramp <- colorRampPalette(c("blue", "white", "red"))(9)
heatmap(as.matrix(dist1), col = colramp, Colv = NA, Rowv = NA)
```



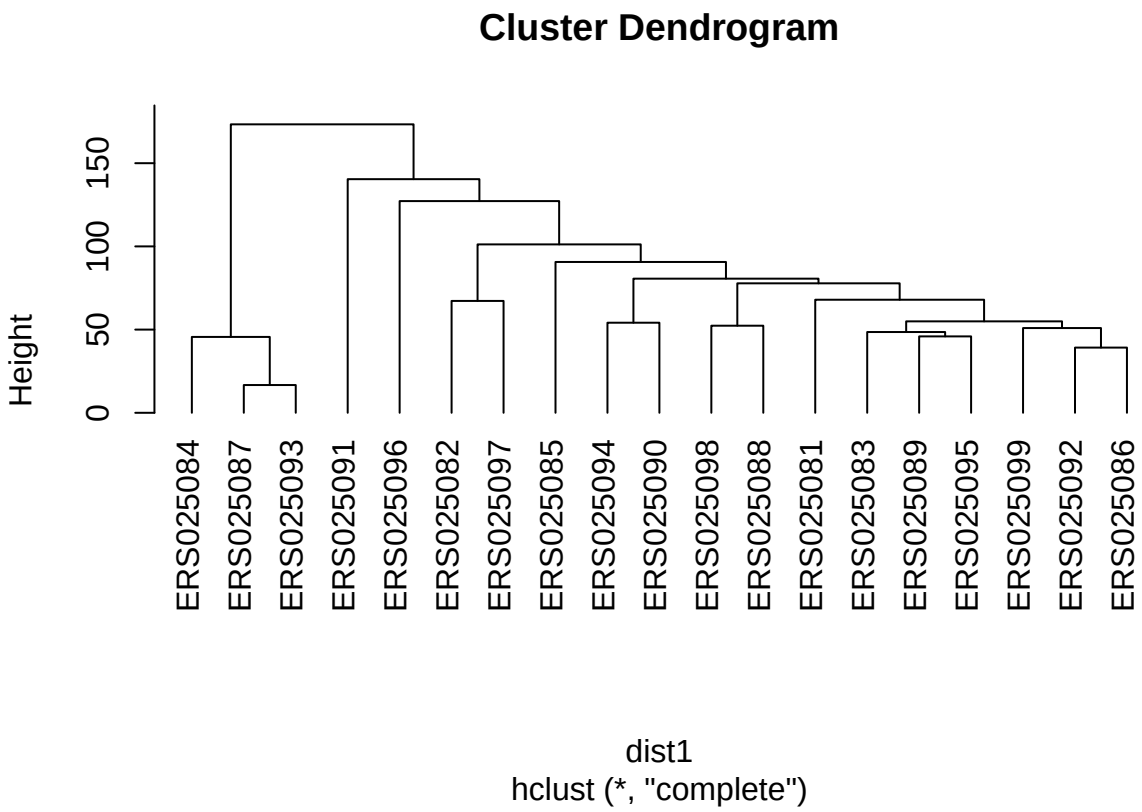
Hierarchical clustering

Here we use the distance we previously calculated to perform a hierarchical clustering and plot the dendrogram:

```
hclust1 <- hclust(dist1)
plot(hclust1)
```

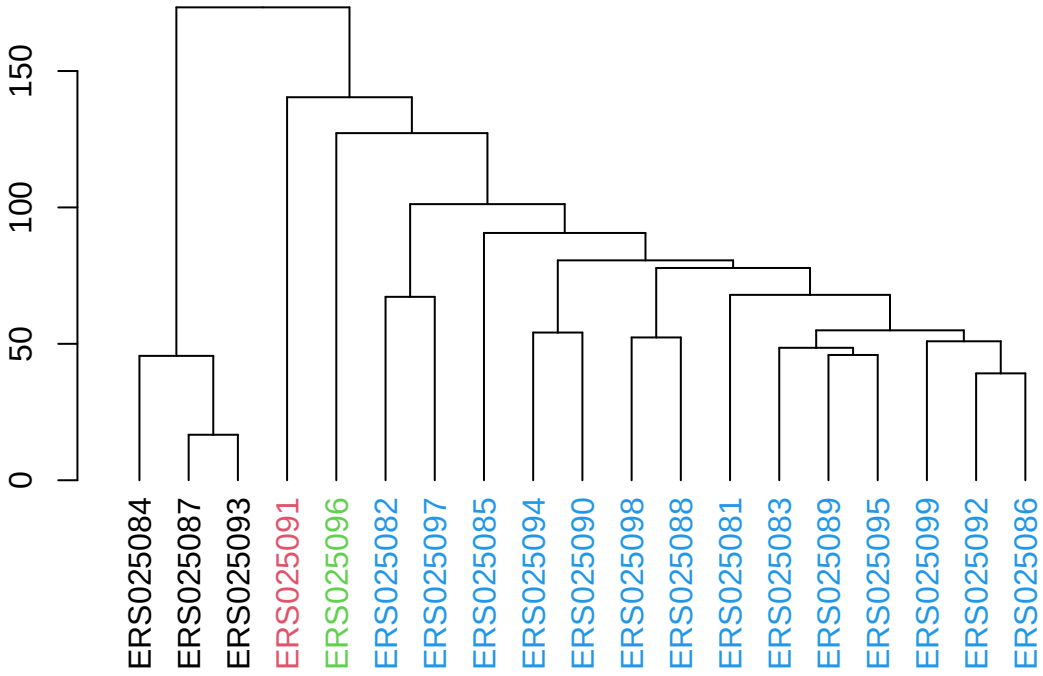



```
plot(hclust1, hang = -1)
```



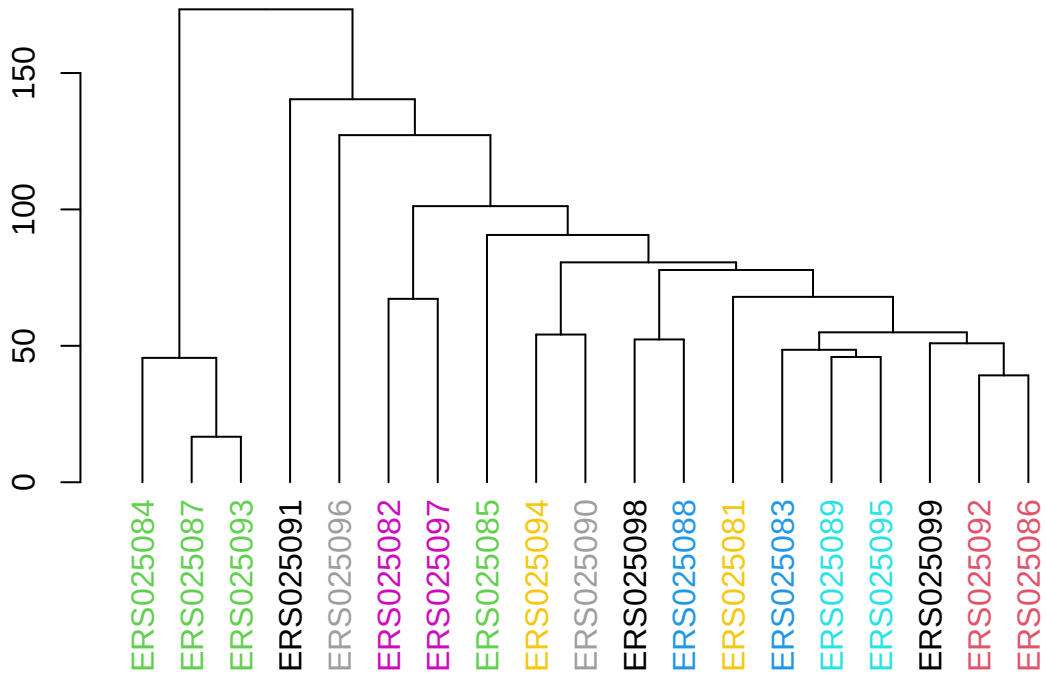
We can also color the dendrogram either into a fixed number of groups:

```
dend <- as.dendrogram(hclust1)
dend <- color_labels(hclust1,4,col=1:4)
plot(dend)
```



Or you can color them directly:

```
labels_colors(dend) <- as.numeric(pdata$tissue.type[match(labels(dend), pdata$sample.id)])
plot(dend)
```



K-means clustering

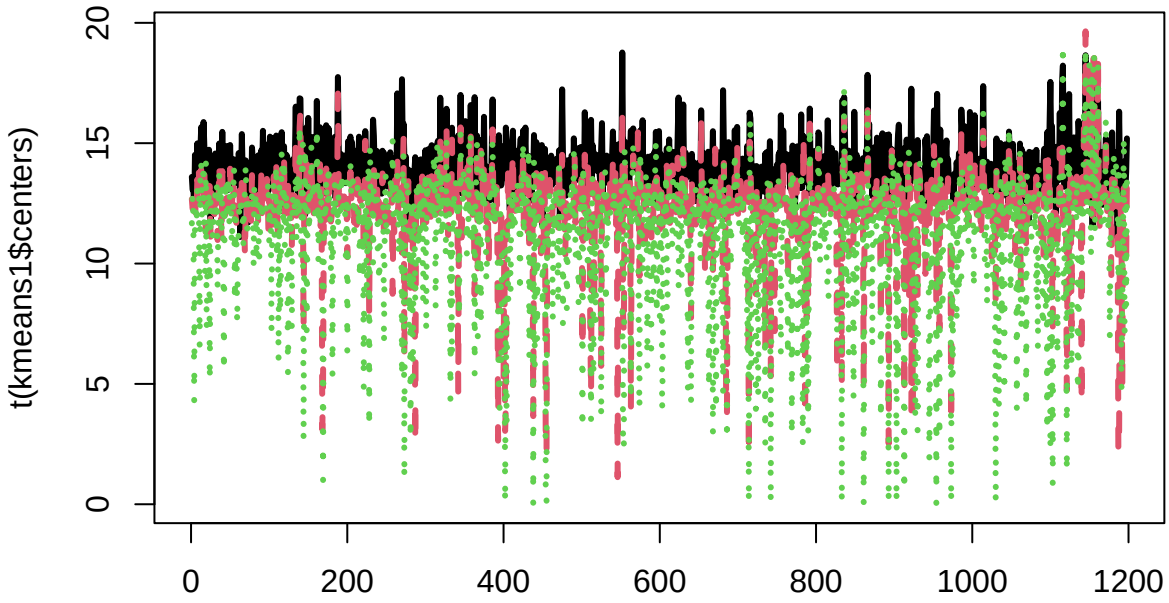
Now we can perform k-means clustering. By default, the rows are clustered. You can either input the cluster means (often unknown) or the number of clusters:

```
set.seed(123)
kmeans1 <- kmeans(t(edata), centers = 3)
names(kmeans1)
```

```
## [1] "cluster"      "centers"      "totss"        "withinss"     "tot.withinss"
## [6] "betweenss"    "size"         "iter"         "ifault"
```

Now we can look at the cluster centers:

```
matplot(t(kmeans1$centers), col = 1:3, type = "l", lwd = 3)
```

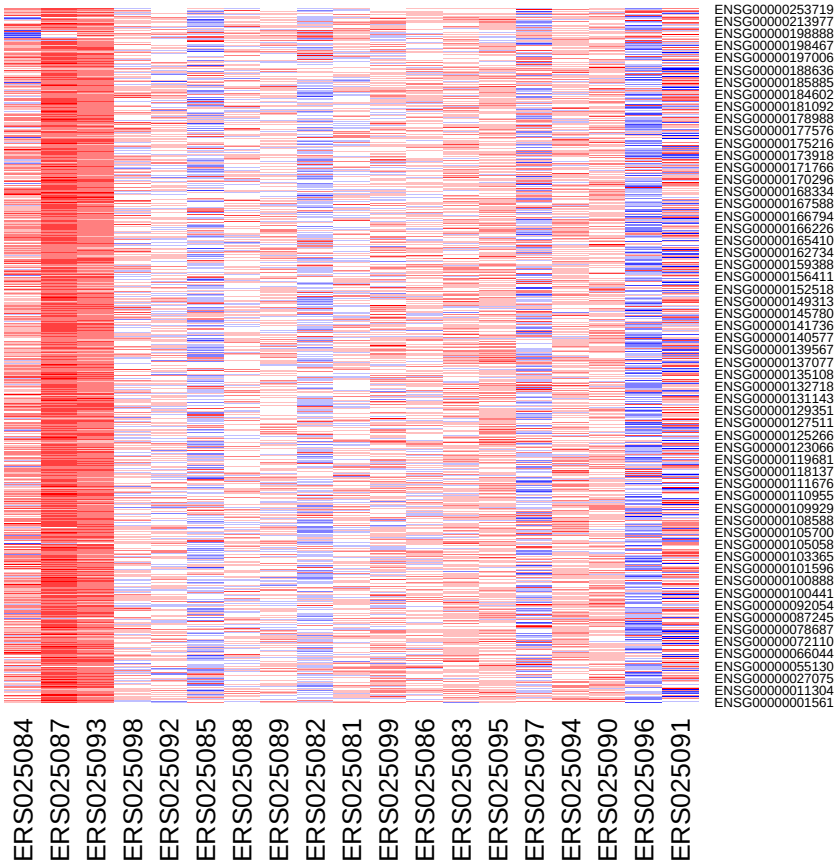


We can observe which points are assigned to which cluster

```
kmeans1$cluster
```

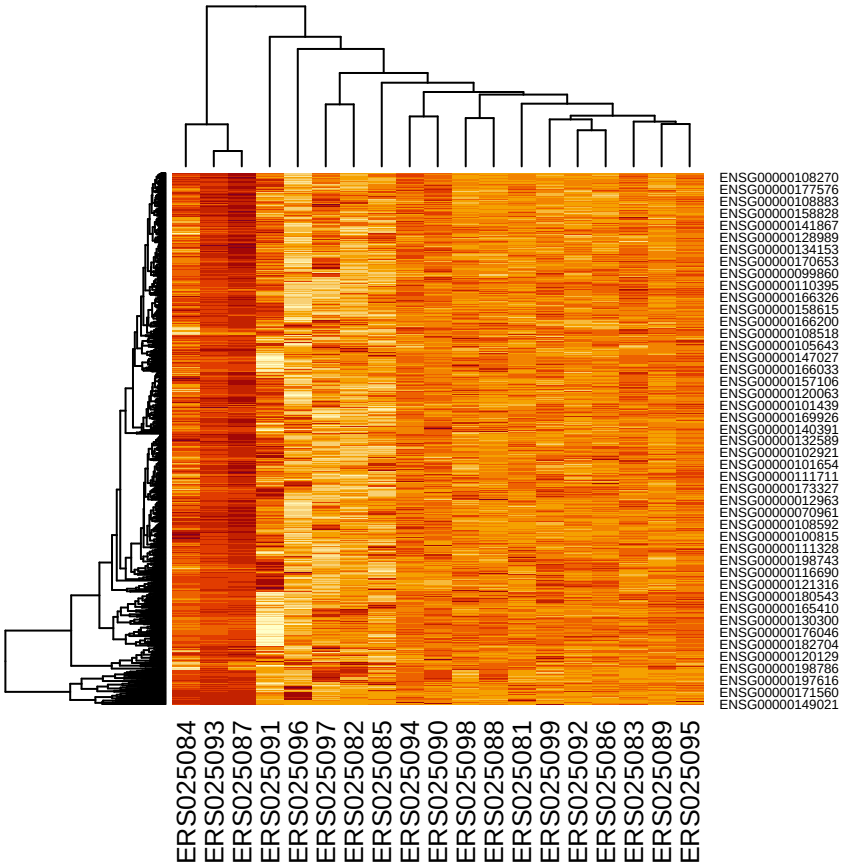
```
## ERS025098 ERS025092 ERS025085 ERS025088 ERS025089 ERS025082 ERS025081 ERS025096
##         2         2         2         2         2         2         2         3
## ERS025099 ERS025086 ERS025084 ERS025087 ERS025093 ERS025083 ERS025095 ERS025097
##         2         2         1         1         1         2         2         2
## ERS025094 ERS025090 ERS025091
##         2         2         3
```

```
heatmap(as.matrix(edata)[, order(kmeans1$cluster)], col = colramp, Colv = NA, Rowv = NA)
```



Biclustering

```
heatmap(as.matrix(edata))
```



Dimensionality Reduction

See this article for more detail.

Data

```
data(decathlon2)
decathlon2.active <- decathlon2[1:23, 1:10]
head(decathlon2.active)
```

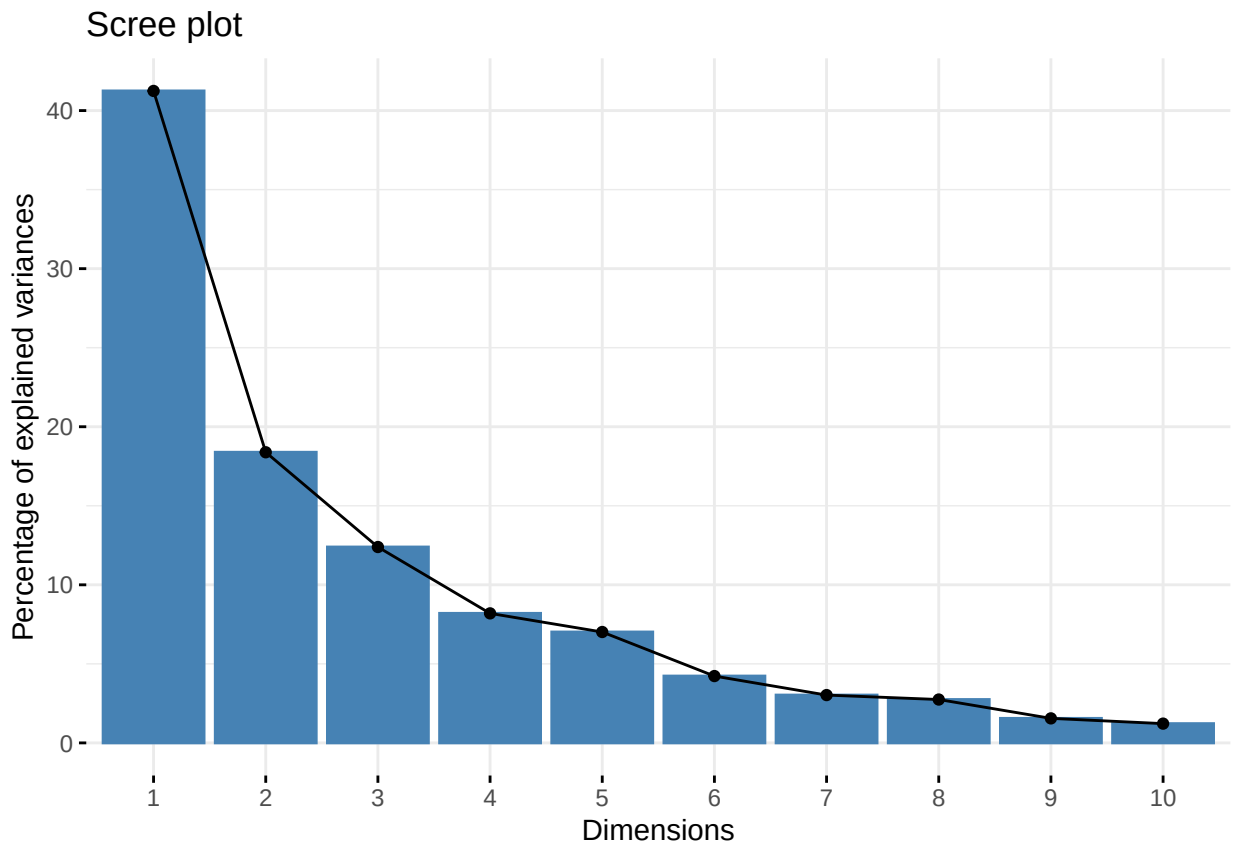
```
##           X100m Long.jump Shot.put High.jump X400m X110m.hurdle Discus
## SEBRLE    11.04      7.58    14.83      2.07 49.81        14.69  43.75
## CLAY      10.76      7.40    14.26      1.86 49.37        14.05  50.72
## BERNARD   11.02      7.23    14.25      1.92 48.93        14.99  40.87
## YURKOV    11.34      7.09    15.19      2.10 50.42        15.31  46.26
## ZSIVOCZKY 11.13      7.30    13.48      2.01 48.62        14.17  45.67
## McMULLEN  10.83      7.31    13.76      2.13 49.91        14.38  44.41
##           Pole.vault Javeline X1500m
## SEBRLE          5.02    63.19  291.7
## CLAY            4.92    60.15  301.5
## BERNARD         5.32    62.77  280.1
## YURKOV          4.72    63.44  276.4
## ZSIVOCZKY       4.42    55.37  268.0
## McMULLEN        4.42    56.37  285.1
```

Compute PCA

```
res.pca <- prcomp(decathlon2.active, scale = TRUE)
```

Visualize eigenvalues (scree plot). Show the percentage of variances explained by each principal component:

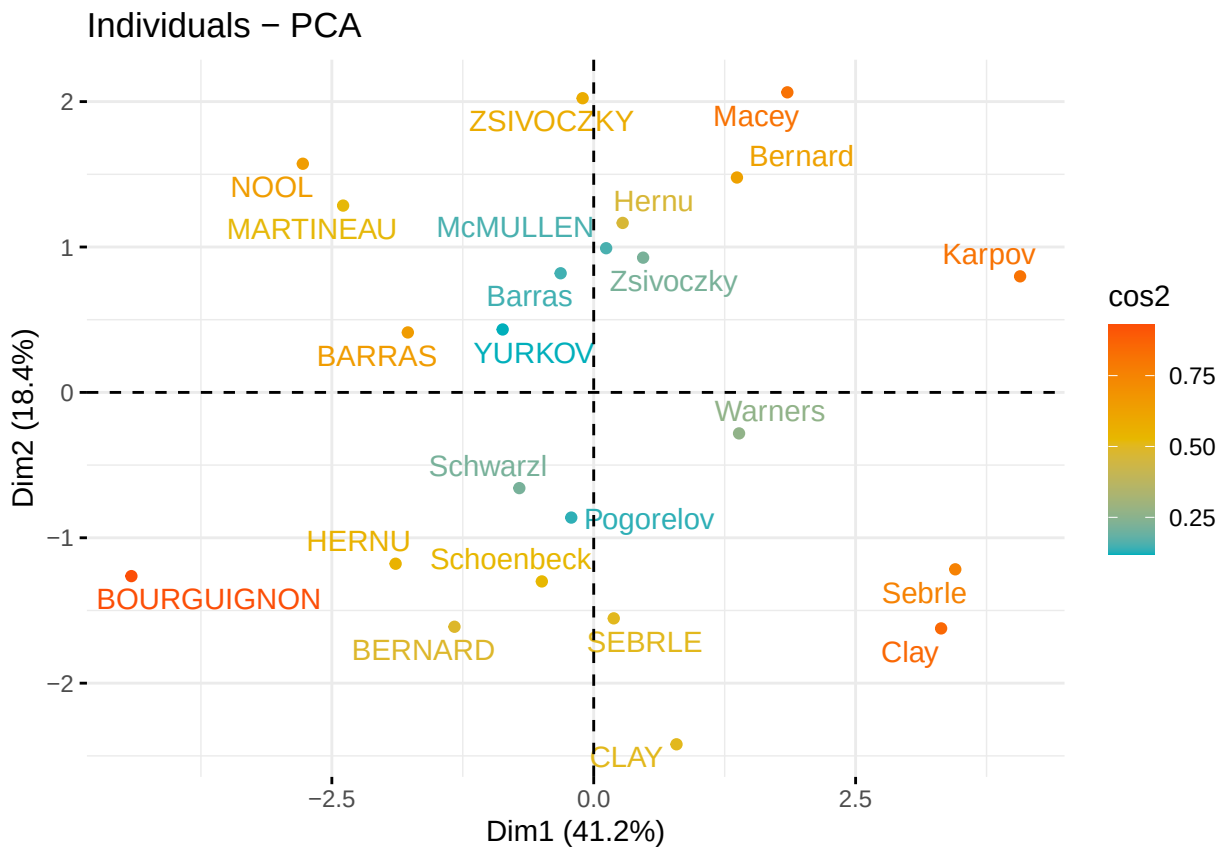
```
fviz_eig(res.pca)
```



Plots

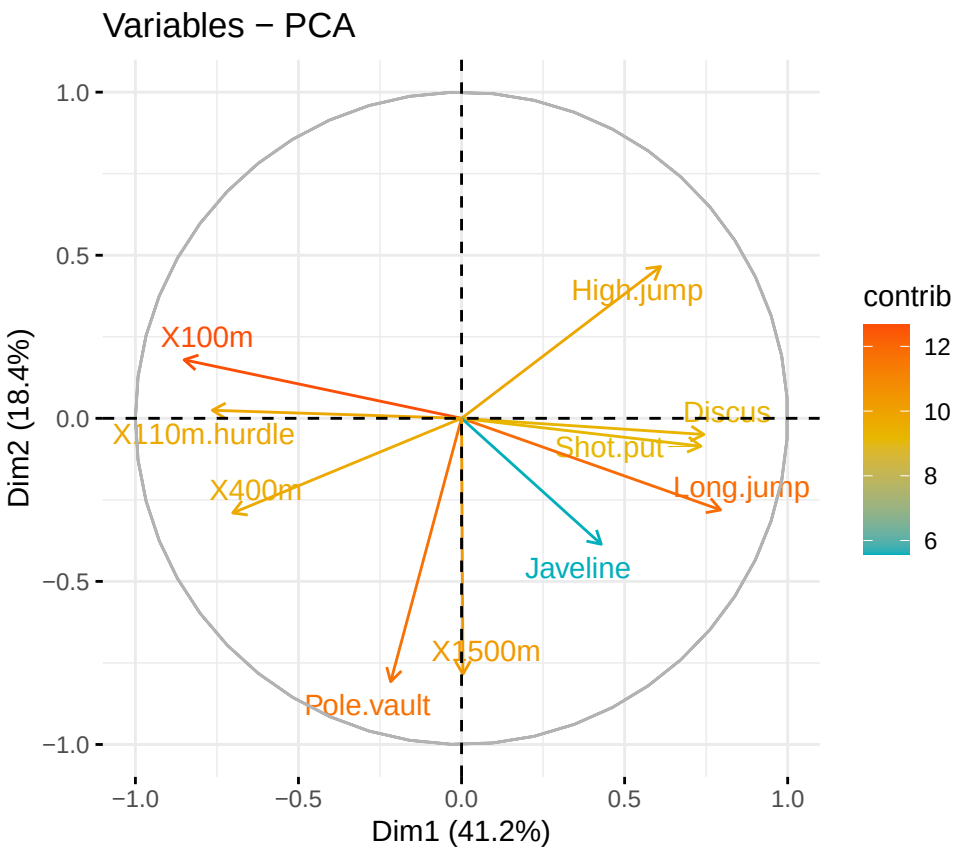
Graph of individuals. Individuals with a similar profile are grouped together.

```
fviz_pca_ind(res.pca,
             col.ind = "cos2", # Color by the quality of representation
             gradient.cols = c("#00AFBB", "#E7B800", "#FC4E07"),
             repel = TRUE     # Avoid text overlapping
             )
```



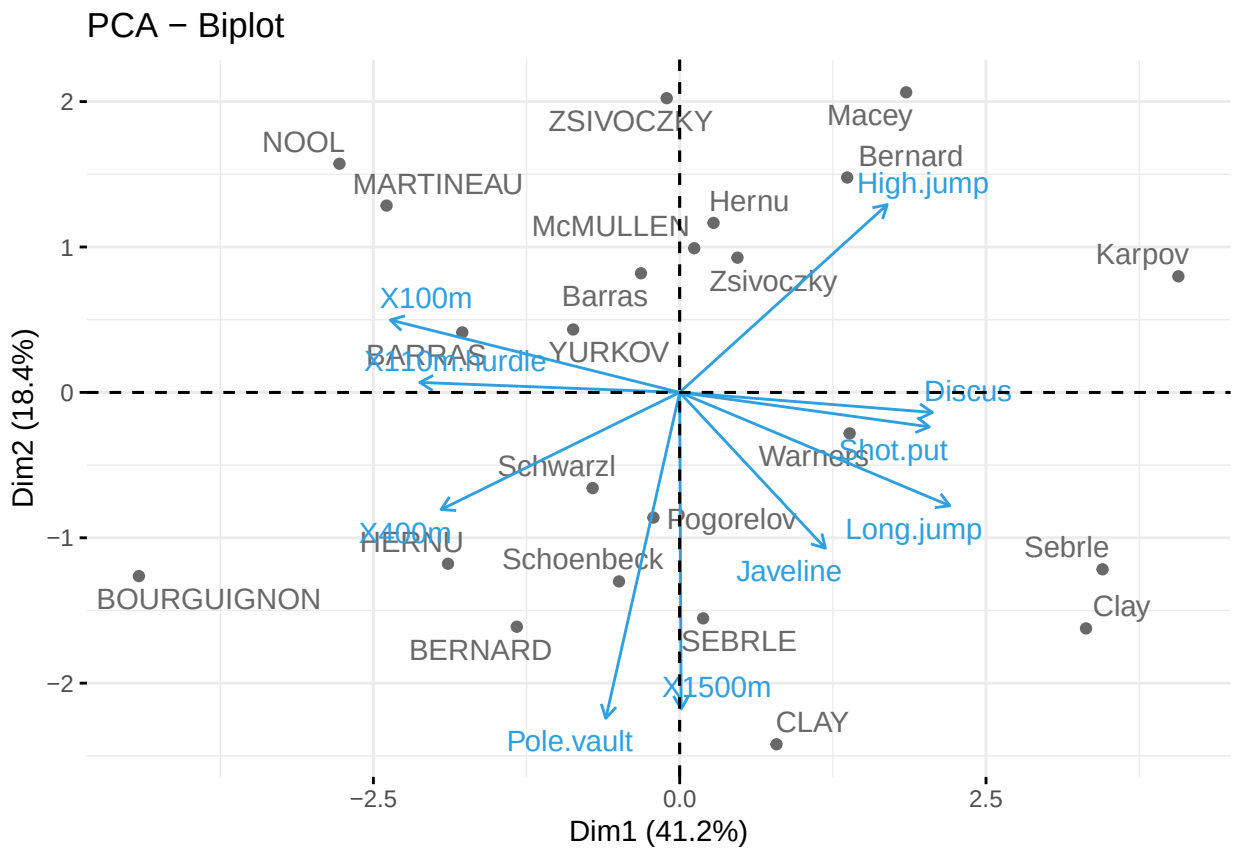
Graph of variables. Positive correlated variables point to the same side of the plot. Negative correlated variables point to opposite sides of the graph.

```
fviz_pca_var(res.pca,
             col.var = "contrib", # Color by contributions to the PC
             gradient.cols = c("#00AFBB", "#E7B800", "#FC4E07"),
             repel = TRUE     # Avoid text overlapping
             )
```



Biplot:

```
fviz_pca_biplot(res.pca, repel = TRUE,
                col.var = "#2E9FDF", # Variables color
                col.ind = "#696969"  # Individuals color
                )
```



Access to PCA results

```
# Eigenvalues
eig.val <- get_eigenvalue(res.pca)
eig.val
```

```
##        eigenvalue variance.percent cumulative.variance.percent
## Dim.1     4.12421          41.2421                       41.242
## Dim.2     1.83853          18.3853                       59.627
## Dim.3     1.23914          12.3914                       72.019
## Dim.4     0.81944           8.1944                       80.213
## Dim.5     0.70155           7.0155                       87.229
## Dim.6     0.42288           4.2288                       91.458
## Dim.7     0.30258           3.0258                       94.483
## Dim.8     0.27447           2.7447                       97.228
## Dim.9     0.15522           1.5522                       98.780
## Dim.10    0.12197           1.2197                      100.000
```

```
# Results for Variables
res.var <- get_pca_var(res.pca)
res.var$coord          # Coordinates
```

```
##                  Dim.1     Dim.2     Dim.3     Dim.4     Dim.5      Dim.6
## X100m        -0.8506257  0.179398 -0.301556  0.033573 -0.19444  0.0353748
## Long.jump     0.7941806 -0.280857  0.190547 -0.115390  0.23316 -0.0337279
## Shot.put      0.7339127 -0.085404 -0.517598  0.128468 -0.24881 -0.2397890
## High.jump     0.6100840  0.465214 -0.330085  0.144550  0.40270 -0.2846448
```

```
## X400m        -0.7016034 -0.290178 -0.283533  0.430826  0.10391 -0.0492900
## X110m.hurdle -0.7641252  0.024741 -0.448887 -0.016896  0.22422  0.0026324
## Discus        0.7432090 -0.049661 -0.176525  0.395009 -0.40824  0.1985449
## Pole.vault   -0.2172680 -0.807451 -0.094058 -0.338985 -0.22169 -0.3274645
## Javeline      0.4282266 -0.386109 -0.604124 -0.331735  0.19781  0.3620976
## X1500m        0.0042785 -0.784480  0.219471  0.448010  0.26325  0.0420502
##                   Dim.7      Dim.8     Dim.9     Dim.10
## X100m        -0.0913364 -0.1047169 -0.303064  0.0444180
## Long.jump    -0.1543308 -0.3973807 -0.051590  0.0297195
## Shot.put     -0.0098866  0.0243590  0.047787  0.2174519
## High.jump     0.0281575  0.0844056 -0.112138 -0.1335668
## X400m         0.2861060 -0.2335522  0.082160 -0.0341707
## X110m.hurdle -0.3700722 -0.0083447  0.161760 -0.0156299
## Discus       -0.1427256 -0.0395593  0.013362 -0.1725904
## Pole.vault   -0.0103932  0.0329149 -0.025769 -0.1372113
## Javeline      0.1335643  0.0528411 -0.040454 -0.0038543
## X1500m       -0.1113671  0.1944697 -0.102240  0.0628348
```

```
res.var$contrib        # Contributions to the PCs
```

```
##                   Dim.1     Dim.2    Dim.3     Dim.4   Dim.5      Dim.6
## X100m        1.7544e+01  1.750510  7.33866  0.137552  5.3893  0.2959153
## Long.jump    1.5293e+01  4.290416  2.93009  1.624859  7.7488  0.2690036
## Shot.put     1.3060e+01  0.396722 21.62043  2.014073  8.8244 13.5968587
## High.jump    9.0248e+00 11.771584  8.79289  2.549880 23.1155 19.1596070
## X400m        1.1936e+01  4.579930  6.48764 22.650906  1.5390  0.5745099
## X110m.hurdle 1.4158e+01  0.033293 16.26126  0.034837  7.1662  0.0016386
## Discus       1.3393e+01  0.134140  2.51474 19.041320 23.7558  9.3217465
## Pole.vault   1.1446e+00 35.461861  0.71395 14.023071  7.0051 25.3576223
## Javeline     4.4464e+00  8.108668 29.45318 13.429633  5.5776 31.0049644
## X1500m       4.4385e-04 33.472876  3.88716 24.493869  9.8784  0.4181336
##                   Dim.7    Dim.8    Dim.9   Dim.10
## X100m          2.757053  3.99520 59.17400  1.61756
## Long.jump      7.871594 57.53322  1.71468  0.72414
## Shot.put       0.032304  0.21619  1.47120 38.76769
## High.jump      0.262026  2.59566  8.10155 14.62649
## X400m         27.052747 19.87344  4.34897  0.95731
## X110m.hurdle 45.261635  0.02537 16.85794  0.20029
## Discus        6.732268  0.57017  0.11503 24.42174
## Pole.vault    0.035699  0.39472  0.42781 15.43559
## Javeline      5.895740  1.01730  1.05435  0.01218
## X1500m        4.098936 13.77873  6.73448  3.23701
```

```
res.var$cos2           # Quality of representation
```

```
##                   Dim.1      Dim.2     Dim.3      Dim.4    Dim.5      Dim.6
## X100m        7.2356e-01 0.03218366 0.0909363 0.00112716 0.037808 1.2514e-03
## Long.jump    6.3072e-01 0.07888063 0.0363080 0.01331475 0.054362 1.1376e-03
## Shot.put     5.3863e-01 0.00729386 0.2679075 0.01650412 0.061908 5.7499e-02
## High.jump    3.7220e-01 0.21642421 0.1089562 0.02089474 0.162167 8.1023e-02
## X400m        4.9225e-01 0.08420342 0.0803909 0.18561063 0.010797 2.4295e-03
## X110m.hurdle 5.8389e-01 0.00061211 0.2014998 0.00028547 0.050275 6.9295e-06
## Discus       5.5236e-01 0.00246620 0.0311611 0.15603223 0.166659 3.9420e-02
## Pole.vault   4.7205e-02 0.65197728 0.0088469 0.11491068 0.049144 1.0723e-01
## Javeline     1.8338e-01 0.14908037 0.3649662 0.11004781 0.039130 1.3111e-01
```

```
## X1500m       1.8305e-05 0.61540916 0.0481674 0.20071261 0.069302 1.7682e-03
##                   Dim.7      Dim.8      Dim.9     Dim.10
## X100m        8.3423e-03 1.0966e-02 0.09184808 1.9730e-03
## Long.jump    2.3818e-02 1.5791e-01 0.00266148 8.8325e-04
## Shot.put     9.7745e-05 5.9336e-04 0.00228355 4.7285e-02
## High.jump    7.9284e-04 7.1243e-03 0.01257498 1.7840e-02
## X400m        8.1857e-02 5.4547e-02 0.00675033 1.1676e-03
## X110m.hurdle 1.3695e-01 6.9634e-05 0.02616638 2.4429e-04
## Discus       2.0371e-02 1.5649e-03 0.00017855 2.9787e-02
## Pole.vault   1.0802e-04 1.0834e-03 0.00066403 1.8827e-02
## Javeline     1.7839e-02 2.7922e-03 0.00163652 1.4856e-05
## X1500m       1.2403e-02 3.7818e-02 0.01045305 3.9482e-03
```

```
# Results for individuals
res.ind <- get_pca_ind(res.pca)
res.ind$coord          # Coordinates
```

```
##                Dim.1    Dim.2     Dim.3     Dim.4       Dim.5     Dim.6
## SEBRLE       0.19121 -1.55413 -0.628369  0.082052  1.14261394 -0.463898
## CLAY         0.79012 -2.42042  1.356887  1.269843 -0.80684837  1.304200
## BERNARD     -1.32926 -1.61187 -0.196150 -1.920922  0.08234282 -0.400629
## YURKOV      -0.86941  0.43288 -2.473982  0.697238  0.39885841  0.102863
## ZSIVOCZKY   -0.10575  2.02336  1.304931 -0.099296 -0.19702411  0.895541
## McMULLEN     0.11856  0.99162  0.843558  1.312153  1.58587086  0.186573
## MARTINEAU   -2.39235  1.28492 -0.898168  0.373098 -2.24335159 -0.456664
## HERNU       -1.89105 -1.17846 -0.156410  0.891301 -0.12674125  0.436235
## BARRAS      -1.77446  0.41253  0.658177  0.228729 -0.23383670  0.090260
## NOOL        -2.77701  1.57268  0.607248 -1.555481  1.42418398  0.497164
## BOURGUIGNON -4.41373 -1.26358 -0.010037  0.666755  0.41915185 -0.082002
## Sebrle       3.45145 -1.21692 -1.678167 -0.808707 -0.02505307 -0.082793
## Clay         3.31622 -1.62329 -0.618404 -0.316799  0.56916459  0.777160
## Karpov       4.07036  0.79835  1.015017  0.313364 -0.79742596 -0.329581
## Macey        1.84846  2.06388 -0.979285  0.584691 -0.00021578 -0.197281
## Warners      1.38735 -0.28191  1.999696 -1.019598 -0.04054015 -0.556733
## Zsivoczky    0.47155  0.92674 -1.728155 -0.184831  0.40730299 -0.113832
## Hernu        0.27631  1.16573  0.170564 -0.848694 -0.68947954 -0.331684
## Bernard      1.36726  1.47804  0.831379  0.745316  0.85980165 -0.328066
## Schwarzl    -0.71028 -0.65843  1.040752 -0.927175 -0.28875680 -0.688916
## Pogorelov   -0.21435 -0.86106  0.297610  1.355603 -0.01505311 -1.593796
## Schoenbeck  -0.49532 -1.30005  0.103004 -0.249277 -0.64522571  0.161724
## Barras      -0.31589  0.81937 -0.861695 -0.589360 -0.77973894  1.174154
##                   Dim.7      Dim.8      Dim.9     Dim.10
## SEBRLE      -0.207960  0.0434606 -0.6593441  0.032732
## CLAY        -0.212919  0.6172406 -0.0601254 -0.317160
## BERNARD     -0.406438  0.7038560  0.1700833 -0.099081
## YURKOV      -0.324874  0.1149961 -0.1095240 -0.119697
## ZSIVOCZKY    0.088256 -0.2023413 -0.5231031 -0.348423
## McMULLEN     0.478284  0.2930900 -0.1056232 -0.393178
## MARTINEAU   -0.299755 -0.2916285 -0.2234177 -0.616405
## HERNU       -0.566100 -1.5294043  0.0061844  0.553680
## BARRAS       0.215941  0.6825831 -0.6692820  0.530854
## NOOL        -0.532057 -0.4333857 -0.1157778 -0.096221
## BOURGUIGNON -0.598337  0.5636199  0.5258140  0.058559
## Sebrle       0.010162 -0.0305858 -0.8472107  0.219704
## Clay         0.257509 -0.5806383  0.4097766 -0.616019
```

```
## Karpov      -1.363656  0.3453064  0.1930551  0.217219
## Macey       -0.269278 -0.3632195  0.3682603  0.212495
## Warners     -0.267394 -0.1094708  0.1802831  0.242084
## Zsivoczky    0.039912  0.5380398  0.5859662 -0.142717
## Hernu        0.443087  0.2472936  0.0669086 -0.208683
## Bernard      0.363579  0.0061653  0.2794887  0.320678
## Schwarzl     0.565686 -0.6870533 -0.0083588 -0.302115
## Pogorelov    0.783701 -0.0376237 -0.1305314 -0.036976
## Schoenbeck   0.857524 -0.2558507  0.5642223  0.296805
## Barras       0.945127  0.3655506  0.1022558  0.611867
```

```
res.ind$contrib        # Contributions to the PCs
```

```
##                  Dim.1    Dim.2      Dim.3     Dim.4      Dim.5     Dim.6
## SEBRLE        0.038543  5.71182  1.3854184  0.035722 8.0912e+00  2.212566
## CLAY          0.658141 13.85419  6.4600973  8.555688 4.0346e+00 17.488019
## BERNARD       1.862732  6.14413  0.1349983 19.578273 4.2021e-02  1.650198
## YURKOV        0.796863  0.44313 21.4755770  2.579391 9.8594e-01  0.108786
## ZSIVOCZKY     0.011788  9.68164  5.9748485  0.052314 2.4058e-01  8.245617
## McMULLEN      0.014817  2.32539  2.4967890  9.135317 1.5586e+01  0.357889
## MARTINEAU     6.033671  3.90441  2.8305267  0.738584 3.1189e+01  2.144098
## HERNU         3.769962  3.28422  0.0858386  4.215056 9.9551e-02  1.956559
## BARRAS        3.319420  0.40245  1.5199796  0.277585 3.3887e-01  0.083761
## NOOL          8.129889  5.84897  1.2938507 12.837611 1.2570e+01  2.541274
## BOURGUIGNON  20.537296  3.77576  0.0003535  2.358779 1.0888e+00  0.069136
## Sebrle       12.558386  3.50207  9.8814824  3.470062 3.8899e-03  0.070476
## Clay         11.593614  6.23152  1.3418281  0.532504 2.0076e+00  6.209728
## Karpov       17.466096  1.50726  3.6149142  0.521017 3.9409e+00  1.116805
## Macey         3.602071 10.07329  3.3648793  1.813875 2.8857e-07  0.400149
## Warners       2.029103  0.18794 14.0307128  5.515857 1.0186e-02  3.186736
## Zsivoczky     0.234419  2.03105 10.4789363  0.181262 1.0281e+00  0.133223
## Hernu         0.080488  3.21362  0.1020764  3.821705 2.9461e+00  1.131101
## Bernard       1.970755  5.16620  2.4252132  2.947374 4.5815e+00  1.106557
## Schwarzl      0.531848  1.02521  3.8005460  4.561193 5.1674e-01  4.879611
## Pogorelov     0.048438  1.75333  0.3107757  9.750343 1.4043e-03 26.116656
## Schoenbeck    0.258641  3.99690  0.0372269  0.329701 2.5801e+00  0.268906
## Barras        0.105195  1.58767  2.6053054  1.842960 3.7680e+00 14.174323
##                  Dim.7      Dim.8      Dim.9    Dim.10
## SEBRLE       0.6214264 2.9920e-02 12.1774773  0.038192
## CLAY         0.6514139 6.0351e+00  0.1012624  3.585689
## BERNARD      2.3736528 7.8477e+00  0.8103198  0.349945
## YURKOV       1.5165641 2.0948e-01  0.3360098  0.510721
## ZSIVOCZKY    0.1119233 6.4855e-01  7.6649198  4.327411
## McMULLEN     3.2870164 1.3608e+00  0.3125012  5.510535
## MARTINEAU    1.2911095 1.3472e+00  1.3981959 13.544029
## HERNU        4.6048508 3.7053e+01  0.0010713 10.927816
## BARRAS       0.6700383 7.3805e+00 12.5473316 10.045370
## NOOL         4.0676697 2.9753e+00  0.3754773  0.330034
## BOURGUIGNON  5.1442475 5.0321e+00  7.7445711  0.122236
## Sebrle       0.0014838 1.4819e-02 20.1055463  1.720638
## Clay         0.9528241 5.3406e+00  4.7035668 13.527082
## Karpov      26.7201581 1.8888e+00  1.0439883  1.681935
## Macey        1.0419105 2.0899e+00  3.7987679  1.609577
## Warners      1.0273842 1.8983e-01  0.9104224  2.089048
## Zsivoczky    0.0228890 4.5857e+00  9.6178522  0.726052
```

```
## Hernu        2.8210274 9.6873e-01  0.1253998  1.552343
## Bernard      1.8994490 6.0213e-04  2.1880713  3.665667
## Schwarzl     4.5981221 7.4775e+00  0.0019572  3.253579
## Pogorelov    8.8253226 2.2423e-02  0.4772688  0.048736
## Schoenbeck  10.5662728 1.0369e+00  8.9173029  3.140200
## Barras      12.8354176 2.1168e+00  0.2928927 13.345338
```

```
res.ind$cos2          # Quality of representation
```

```
##                 Dim.1    Dim.2      Dim.3     Dim.4      Dim.5      Dim.6
## SEBRLE      0.0075302 0.497473 8.1325e-02 0.0013867 2.6890e-01 0.04432413
## CLAY        0.0487012 0.457017 1.4363e-01 0.1257917 5.0785e-02 0.13269073
## BERNARD     0.1971998 0.289966 4.2940e-03 0.4118192 7.5673e-04 0.01791312
## YURKOV      0.0961098 0.023826 7.7823e-01 0.0618126 2.0228e-02 0.00134536
## ZSIVOCZKY   0.0015744 0.576419 2.3975e-01 0.0013882 5.4655e-03 0.11291769
## McMULLEN    0.0021754 0.152195 1.1014e-01 0.2664865 3.8926e-01 0.00538770
## MARTINEAU   0.4040139 0.116547 5.6946e-02 0.0098263 3.5526e-01 0.01472103
## HERNU       0.3992827 0.155062 2.7315e-03 0.0886999 1.7935e-03 0.02124788
## BARRAS      0.6162420 0.033307 8.4782e-02 0.0102391 1.0702e-02 0.00159445
## NOOL        0.4898725 0.157111 2.3424e-02 0.1536947 1.2884e-01 0.01570106
## BOURGUIGNON 0.8596981 0.070459 4.4460e-06 0.0196185 7.7531e-03 0.00029675
## Sebrle      0.6753806 0.083959 1.5967e-01 0.0370790 3.5585e-05 0.00038863
## Clay        0.6875929 0.164754 2.3911e-02 0.0062750 2.0254e-02 0.03776278
## Karpov      0.7836669 0.030148 4.8732e-02 0.0046448 3.0078e-02 0.00513797
## Macey       0.3634360 0.453082 1.0201e-01 0.0363630 4.9527e-09 0.00413977
## Warners     0.2556520 0.010556 5.3113e-01 0.1380811 2.1830e-04 0.04116898
## Zsivoczky   0.0450532 0.174014 6.0510e-01 0.0069217 3.3612e-02 0.00262538
## Hernu       0.0248243 0.441847 9.4591e-03 0.2341967 1.5457e-01 0.03577072
## Bernard     0.2893475 0.338133 1.0698e-01 0.0859802 1.1442e-01 0.01665864
## Schwarzl    0.1167214 0.100301 2.5060e-01 0.1988922 1.9291e-02 0.10980631
## Pogorelov   0.0078035 0.125920 1.5043e-02 0.3121016 3.8484e-05 0.43141622
## Schoenbeck  0.0670701 0.462046 2.9005e-03 0.0169874 1.1381e-01 0.00715008
## Barras      0.0189727 0.127651 1.4118e-01 0.0660431 1.1560e-01 0.26212975
##                  Dim.7      Dim.8      Dim.9     Dim.10
## SEBRLE      8.9075e-03 3.8903e-04 8.9541e-02 0.00022067
## CLAY        3.5365e-03 2.9721e-02 2.8201e-04 0.00784710
## BERNARD     1.8436e-02 5.5291e-02 3.2286e-03 0.00109565
## YURKOV      1.3420e-02 1.6814e-03 1.5252e-03 0.00182173
## ZSIVOCZKY   1.0967e-03 5.7645e-03 3.8527e-02 0.01709243
## McMULLEN    3.5406e-02 1.3296e-02 1.7267e-03 0.02392681
## MARTINEAU   6.3428e-03 6.0035e-03 3.5236e-03 0.02682120
## HERNU       3.5782e-02 2.6117e-01 4.2704e-06 0.03422887
## BARRAS      9.1262e-03 9.1187e-02 8.7667e-02 0.05515319
## NOOL        1.7982e-02 1.1931e-02 8.5149e-04 0.00058813
## BOURGUIGNON 1.5799e-02 1.4019e-02 1.2201e-02 0.00015133
## Sebrle      5.8544e-06 5.3038e-05 4.0694e-02 0.00273665
## Clay        4.1460e-03 2.1079e-02 1.0499e-02 0.02372642
## Karpov      8.7958e-02 5.6400e-03 1.7629e-03 0.00223183
## Macey       7.7127e-03 1.4033e-02 1.4425e-02 0.00480290
## Warners     9.4968e-03 1.5917e-03 4.3170e-03 0.00778411
## Zsivoczky   3.2275e-04 5.8653e-02 6.9568e-02 0.00412683
## Hernu       6.3835e-02 1.9884e-02 1.4556e-03 0.01415960
## Bernard     2.0460e-02 5.8834e-06 1.2091e-02 0.01591680
## Schwarzl    7.4036e-02 1.0921e-01 1.6165e-05 0.02111739
## Pogorelov   1.0431e-01 2.4041e-04 2.8937e-03 0.00023220
```

```
## Schoenbeck  2.0103e-01 1.7895e-02 8.7029e-02 0.02408269
## Barras      1.6984e-01 2.5407e-02 1.9881e-03 0.07118365
```