1 Main Wing Desing

Selection of the airfoil cross-section shape:

- Average chord length, $\bar{c}$
- Maximum thickness-to-chord, $\left(\frac{t}{c}\right)_{\max}$
- Aspect ratio, $A = \frac{b^2}{s}$
- Taper ratio, $\lambda = c_t/c_r$
- Sweep angle, M

1.1 Airfoil cross-section shape

It should be based

- Low drag at design lift-coefficient
- High maximum lift coefficient
- Gentle stalling behaviour
- Low pitching moment coefficient
- For the inboard sections, low drag at high lift is desirable in case of deflected flaps.
- The outboard sections should manifest a high lift coefficient.
- Sufficient volume should be available to enable storage or undercarriage and fuel.
- Smaller low subsonic speed aircraft could use the NACA five digit series with $(t/c)_{\max} \sim 14\%$.
- High subsonic speed a/c ($M_{\text{cruise}} \simeq 0.8$) such as commercial jet transport would use NACA supercritical airfoils.

1.2 Taper ratio

- Key designer for the selection of the taper ratio: minimize the amount of lift-induced drag.
- Minimum drag is obtained with an eliptical wing planform.
- **Approximation:** trapezoidal wing with taper ratio of 0.4.

1.3 Sweep angle selection

- Primary reason to add sweep: to increase the section critical Mach number

$$M_{\text{effective}} = M_\infty \cos\left(\Lambda_{LE}\right)$$

- However, it has disadvantages:
  - Lowering the lift through a lower effective dynamic pressure:

$$q_{\text{effective}} = q_\infty \cos^2\left(\Lambda\right)$$

  - And an incrase in the wing weight:

$$W_{\text{wing}} \propto [\tan(\Lambda)]^2$$

  - Large sweep angle results in **poorer** take-off and landing characteristics because lift-devices are **less effective**.

1.4 Wing Drag Estimation

Drag coefficient Can be written as a sum of a fixed and a variable contribution

$$C_D = \bar{C}_{D_P} + \frac{f_{\text{fix}}}{s} + \underbrace{\frac{C_L^2}{\pi A \varphi}}_{\text{Similar to the Oswald factor but includes the influence of the fuselage}}$$

Where

$$\underbrace{\bar{C}_{D_P}}_{\text{profile}} = \underbrace{\left(\bar{C}_{D_P}\right)_W}_{\text{wing}} + \underbrace{\left(\bar{C}_{D_P}\right)_h \frac{S_h}{S}}_{\text{horizontal tail plane}}$$

$$f_{\text{fix}} = \underbrace{\left(\bar{C}_{D_P} S\right)_f}_{\text{fuselage}} + \underbrace{\left(\bar{C}_{D_P} S\right)_v}_{\text{vertical tailplane}} + \underbrace{\left(\bar{C}_{D_P} S\right)_n}_{\text{nacelles}}$$

1.5 Drag to Lift Ratio

$$\frac{C_D}{C_L} = \left(C_{D_P} + \frac{f_{\text{fix}}}{S}\right)\frac{1}{C_L} + \frac{C_L}{\pi A \varphi}$$

In a climb with constant equivalent speed

$$q = \frac{1}{2}\gamma p M^2$$

with p being climb gradient.

$$\frac{C_D}{C_L} = \left(C_{D_P}\frac{P_0 S}{W} + \frac{f_{\text{fix}} P_0}{W}\right)\frac{1}{2}\gamma \delta M^2 + \frac{1}{\frac{1}{2}\gamma \delta M^2 \pi A \varphi}\frac{W}{P_0 S}$$

where δ is pressure ratio.

The optimal drag-to-lift ratio is

$$\left(\frac{C_D}{C_L}\right)_{\text{min}} = 2\sqrt{\frac{\bar{C}_{D_P}}{\pi}} + \frac{1}{2}\gamma P_0 \delta M^2 \frac{f_{\text{fix}}}{W}$$

and for the wing loading

$$\frac{W}{S} = \frac{1}{2}\gamma P_0 \delta M^2 \sqrt{C_{D_P}\pi A \varphi}$$

For typical values:

$$\begin{aligned} C_{D_P} &= 0.0095 \\ \varphi &= 0.95 \\ \underbrace{C_{L_{\text{des}}}}_{\text{design lift coefficent}} &= 0.17\sqrt{A} \end{aligned}$$

1.6 Aspect ratio

- Choice of aspect ratio given by two criteria
  - Induced drag at the cruise condition
  - Take-off performance and second-segment climb gradient after engine failure
- Since it is difficult to choose a value based on performance requirements
  - Span loading is used: $W/b^2 = (W/S)/A$

Span loading good parameter to choose when the take-off field length criteria is critical, because **it is a direct measure for the induced drag per a/c weight**.

$$\frac{1}{W}C_{D_i} = \frac{1}{W}\frac{C_L^2}{\pi A \rho} \Rightarrow \frac{D_i}{W} = \frac{C_L^2}{\pi W A \rho}\frac{1}{2}\rho V^2 = \frac{1}{\pi\rho}\frac{\frac{W}{b^2}}{\frac{1}{2}\rho V^2}$$

By Thorenbeek, we have

- High-subsonic jet propeller a/c

$$\frac{\frac{W}{b^2}}{\frac{1}{2}\rho V^2} = 0.18 - 0.2$$

- Twin-engined transport a/c

$$\frac{\frac{W}{b^2}}{\frac{1}{2}\rho V^2} < \left(1.45\frac{T_{\text{TO}}}{W_{\text{TO}}} - 0.215\right)$$

1.7 Planform Geometric Relations

Useful relations that apply to a trapezoidal shape, based on knowing:

- Wing area S
- Aspect ratio A
- Taper ratio λ
- Sweep angle M
- Wing span $b = \sqrt{SA}$
- Root chord

$$A = \frac{2b}{C_t + C_r} = \frac{2b}{C_r(\lambda + 1)} \Rightarrow C_r = \frac{2b}{A(1+\lambda)}$$

- Tip chord $C_t = \lambda C_r$
- Mean aerodynamic chord length $\bar{c}$
- Normalized spanwise location of the mean aerodynamic chord from the center span of the wing

$$\frac{\bar{y}}{b} = \frac{1}{b}\frac{1+2\lambda}{1+\lambda}$$

- Sweep angle at any x/c location of the wing

$$\Lambda_{x/c} = \arctan\left[\tan\Lambda_{\mathrm{CE}} - \frac{x}{c}\frac{xC_r}{b}(1-\lambda)\right]$$

- Sweep angle of the quarter-chord line

$$\Lambda_{c/4} = \arctan\left[\tan\Lambda_{\mathrm{CE}} - \frac{1}{4}\frac{2C_r}{b}(1-\lambda)\right]$$