

# Lab Sheet 6 Answers

Elliot Potts  
ep15449@my.bristol.ac.uk

March 13, 2017

Natural Semantics for Booleans:

$$\begin{array}{c}
 \frac{}{\langle \text{true}, \sigma \rangle \rightarrow \top} [\text{tt}_{\text{ns}}] \\
 \\
 \frac{}{\langle \text{false}, \sigma \rangle \rightarrow \perp} [\text{ff}_{\text{ns}}] \\
 \\
 \frac{\langle a_1, \sigma \rangle \rightarrow \text{lhs} \quad \langle a_2, \sigma \rangle \rightarrow \text{rhs}}{\langle a_1 = a_2, \sigma \rangle \rightarrow \begin{cases} \top & \text{if } \text{lhs} = \text{rhs} \\ \perp & \text{otherwise} \end{cases}} [\text{eq}_{\text{ns}}] \\
 \\
 \frac{\langle a_1, \sigma \rangle \rightarrow \text{lhs} \quad \langle a_2, \sigma \rangle \rightarrow \text{rhs}}{\langle a_1 \leq a_2, \sigma \rangle \rightarrow \begin{cases} \top & \text{if } \text{lhs} \leq \text{rhs} \\ \perp & \text{otherwise} \end{cases}} [\text{leq}_{\text{ns}}] \\
 \\
 \frac{\langle b, \sigma \rangle \rightarrow \text{val}}{\langle \neg b, \sigma \rangle \rightarrow \neg \text{val}} [\text{neg}_{\text{ns}}] \\
 \\
 \frac{\langle b_1, \sigma \rangle \rightarrow \perp}{\langle b_1 \wedge b_2, \sigma \rangle \rightarrow \perp} [\text{and}_{\text{ns}} \bullet] \\
 \\
 \frac{\langle b_1, \sigma \rangle \rightarrow \perp \quad \langle b_2, \sigma \rangle \rightarrow \text{rhs}}{\langle b_1 \wedge b_2, \sigma \rangle \rightarrow \text{rhs}} [\text{and}_{\text{ns}} \bullet] \\
 \\
 \frac{}{\langle \text{false}, \sigma \rangle \rightarrow \perp} \\
 \frac{}{\langle \neg(\text{false}), \sigma \rangle \rightarrow \top}
 \end{array}$$

### Evaluation of Factorial Program:

[illegible]

Lazy evaluation of conjunction:

$$\frac{\frac{\overline{\langle 5, \sigma \rangle} \rightarrow 5 \text{ [lit}_{\text{ns}}]}}{\langle x := 5, \sigma \rangle \rightarrow \sigma[x \mapsto 5] \text{ [ass}_{\text{ns}}]}} \quad \frac{\frac{\overline{\langle 2, \sigma[x \mapsto 5] \rangle} \rightarrow 2 \text{ [lit}_{\text{ns}}]}}{\langle y := 2, \sigma[x \mapsto 5] \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2] \text{ [ass}_{\text{ns}}]}} \quad \frac{\frac{\overline{\langle y, \sigma[x \mapsto 5][y \mapsto 2] \rangle} \rightarrow 2 \text{ [st}_{\text{ns}}]}}{\langle y \leq x, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow \top \text{ [leq}_{\text{ns}}]}} \quad \frac{\frac{\overline{\langle x, \sigma[x \mapsto 5][y \mapsto 2] \rangle} \rightarrow 5 \text{ [st}_{\text{ns}}]}}{\langle x = 5, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow \top \text{ [and}_{\text{ns}}, \bullet]}} \quad \frac{\frac{\overline{\langle 5, \sigma[x \mapsto 5][y \mapsto 2] \rangle} \rightarrow 5 \text{ [lit}_{\text{ns}}]}}{\langle 5, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow 5 \text{ [eq}_{\text{ns}}]}} \quad \frac{\overline{\langle 100, \sigma[x \mapsto 5][y \mapsto 2] \rangle} \rightarrow 100 \text{ [lit}_{\text{ns}}]}}{\langle z := 100, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2][z \mapsto 100] \text{ [ass}_{\text{ns}}]}} \quad \frac{\langle x := 5; y := 2, \sigma \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2] \text{ [comp}_{\text{ns}}]}}{\langle x := 5; y := 2; \text{if}(y \leq x \wedge x = 5) \text{ then } z := 100 \text{ else } z := -50, \sigma \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2][z \mapsto 100] \text{ [if}_{\text{ns}}^{\text{tt}}]}}$$

Strict evaluation of conjunction:

$$\frac{\overline{\langle 5, \sigma \rangle \rightarrow 5} [\text{lit}_{\text{ns}}]}{\overline{\langle x := 5, \sigma \rangle \rightarrow \sigma[x \mapsto 5]} [\text{ass}_{\text{ns}}]} \quad \frac{\overline{\langle 2, \sigma[x \mapsto 5] \rangle \rightarrow 2} [\text{lit}_{\text{ns}}]}{\overline{\langle y := 2, \sigma[x \mapsto 5] \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2]} [\text{ass}_{\text{ns}}]} \quad \frac{\overline{\langle x, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow 5} [\text{st}_{\text{ns}}]}{\overline{\langle x \leq y, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow \perp} [\text{and}_{\text{ns}}, \bullet]} \quad \frac{\overline{\langle y, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow 2} [\text{st}_{\text{ns}}]}{\overline{\langle x \leq y \wedge x = 5, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow \perp} [\text{and}_{\text{ns}}, \bullet]} \quad \frac{\overline{\langle -50, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow -50} [\text{lit}_{\text{ns}}]}{\overline{\langle z := -50, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2][z \mapsto -50]} [\text{ass}_{\text{ns}}]} \quad \frac{\overline{\langle x := 5; y := 2, \sigma \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2]} [\text{comp}_{\text{ns}}]}{\overline{\langle x := 5; y := 2; \text{if } (x \leq y \wedge x = 5) \text{ then } z := 100 \text{ else } z := -50, \sigma[x \mapsto 5][y \mapsto 2] \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2][z \mapsto -50]} [\text{if}_{\text{ns}}^{\text{ff}}, \text{comp}_{\text{ns}}]} \quad \frac{\overline{\langle x := 5; y := 2; \text{if } (x \leq y \wedge x = 5) \text{ then } z := 100 \text{ else } z := -50, \sigma \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2][z \mapsto -50]} [\text{comp}_{\text{ns}}]}{\overline{\langle x := 5; y := 2; \text{if } (x \leq y \wedge x = 5) \text{ then } z := 100 \text{ else } z := -50, \sigma \rangle \rightarrow \sigma[x \mapsto 5][y \mapsto 2][z \mapsto -50]} [\text{comp}_{\text{ns}}]}$$