Ellipses

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Equations are books written onto themselves.
You simply have never been taught how to read.

1 Acknowledgements

The following mentors have been instrumental to my individual success:

- Tyler Hartke
- Dr. Emmett Lentilucci
- Dr. William Furnas
- Dr. David Haefner

Thank you for your time and resources.

2 Forward

The purpose of this book is to teach an engineering approach to mathmatics.
And present mathmatical theory that has yet to be peer reviewed or measured.
Only time and measurements will confirm or deny my hypothesis.
Degenerate functions drive the generation of the number line.

3 Perspective

Humans have so much left to learn.
You can learn.
Never stop trying to learn.

4 Notation

Notation is not trivial.
Every equation is written with the up most attention to detail.
Underneath every formalized equation is a supplementary equation (#).
The supplementary equation describes how to read the proceeding equation.
Any refutations or suggestions to facilitate clarity is greatly appreciated.

5 Sequencing

The sequence of mathmatics utilized to develop equations is as follows:

Trigonometry - to establish elemental shapes.

Vectors - to give directions to elemental shapes.

Calculus - to propogate vectors spatially.

6 Resources and Release of Liability

As apart of the reproducable research effort, python code replicating the equations and textbook are provided. This document, along with its supplimentary resources, are provided free and open to the public to be used as a learning tool for anyone interested in furthering their understanding.

7 Numbers

Numbers represent quantities.

$$1, 2, 3 : \text{integers}$$

$$1.1, 2.1, 3.3 : \text{doubles/fractions}$$

$$\pi, \mathrm{e} : \text{irrationals}$$

8 Variables

$a, b, d :$ constants
i.e. holding any single number representing a magnitude

$x, y, z :$ axis
i.e. holding multiple numbers in a sampling configuration

9 Basis

$\square, \circ :$ A unit shape you use to quantify functions against.

10 Vector

$\uparrow :$ A basis with an established direction.

11 Operators

Used to propogate vectors spatially.

Multiplication
Used to establish a basis and direction.

a $\cdot$ b : constant $\cdot$ constant

$x.y$: axis . axis

$a{\cdot}x$: constant $\cdot$ axis

ax: constant axis

a^n, x^n : constant or axis multiplied by itself (i.e. power)

Addition
Used to propogate the vector to fill in the function.

$+$

$\int$: multiple addition actions (i.e. integral)

12 Calculus

Derivative

f'

i.e. the underlying functional form
i.e. the rate at which the functional form grows
i.e. the minimum distance between two points along a traveled path

Function

f : a function is an observed shape

Degenerate Function

A special type of function invarient with projection and differentiation.

Conservative

Synonymous with "closed form".

Another name for unity (i.e. 1).

Basis

$(dx, dy), (rd\theta, d\phi)$: establishment of the unit shape,
i.e. unit shape used to measure and quantify functions against.

Axis

x, r: A measuring stick used to quantify and describe functions.

Created by propogating the basis.

Domain

$(x, y, \theta, \phi), (\chi, \nu)$: the total shape established by propogating the basis.

Domains have to be bounded in order to be conservative.

(r, i, j) : Domains can be spread conservatively along rotative dimensions.

Changing the domain does not change the functional form of interest.

A domain cannot be shifted.

Coordinate System

A domain that can be shifted.

Parameterization

Breaking a function apart into components.

You can parameterize any functional from and it will pass the vertical line test.

Equation

$f(x) = g(x)$: Establishing the equivalency between two functions

Range

$f(x)_0^2$: Functional form along a singular segment of an axis.

$f(x = 1)$: Singular point on a functional form.

$f(x)|_0^2$: Total length of functional form between a singular segment on an axis.

13 Division

Division can only exist when discussing magnitudes not directions.

14 Square Roots

Square roots, as a functional form, only exist outside of unity.
Square roots can only exist when discussing magnitudes not directions.

$$\sqrt{50} = 10 \cdot \sqrt{2}$$
$$\sqrt{20} = 2 \cdot \sqrt{5}$$
$$\sqrt{10} = \sqrt{5} \cdot \sqrt{2}$$
$$\sqrt{6} = \sqrt{3} \cdot \sqrt{2}$$
$$\sqrt{1} = 1$$
$$\sqrt{.7} = \frac{\sqrt{7}}{\sqrt{10}} = 1/5 \cdot \sqrt{7/2} = 1/5 \cdot 1.870828693...$$
$$\sqrt{.5} = 1/2 \cdot \sqrt{2} = 1/2 \cdot 1.414213562...$$
$$\sqrt{.3} = 1/5 \cdot \sqrt{3/2} = 1/5 \cdot 1.224744871...$$

15 Degenerate Functions

Degenerate functions are invarient to projection and differentiation.

$$sin\theta$$
$$cos\theta$$
$$sin\theta cos\theta$$
$$e^{\theta} sin\theta$$
$$e^{\theta} cos\theta$$
$$e^{i\theta}$$

You are never going to be able to get smaller than a degenerate functional form.

16 Spatial Coordinate Systems

Visualizing space as a summation of closed elemental shapes.

16.1 1D Coordinate System

16.1.1 Cartesian

dx : elemental line

$x = \int_0^x dx$: unit line

16.2 2D Coordinate Systems

16.2.1 Cartesian

dx, dy : elemental square

$f(x, y) = \int_0^y \int_0^x dxdy$: unit square

16.2.2 Polar (Domain)

You need at least two dimensions in order to describe an angle.

dr: elemental radius

$d\theta$: elemental angle

$rd\theta$: elemental polar cap

$f(r, \theta) = \int_0^\theta \int_0^r drd\theta$: unit concentric circle

16.3 3D Coordinate Systems

16.3.1 Cartesian

dx, dy, dz : elemental cube

$f(x, y, z) = \int_0^z \int_0^y \int_0^x dxdydz$: unit cube

16.3.2 Spherical (Domain)

dr: elemental radius

$d\theta, d\phi$: elemental angles

$f(r, \theta, \phi) = \int_0^\phi \int_0^\theta \int_0^r drd\theta d\phi$: unit concentric sphere

17 Taylor Series #[1]

$$sin\theta = x - \frac{x^3}{3!} + \frac{x^5}{5!}...$$
$$cos\theta = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}...$$
$$e^\theta = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}...$$

The unit circle establishes the domain of polar based functions.

18 Euler Identity and Expodential Rules

$$e^{2a} = e^a e^a$$
$$e^{a+b} = e^a e^b$$
$$e^{a+ib} = e^a e^{ib}$$

Magnitudes are seperable from directions.

$$e^{i\theta} = cos\theta + isin\theta$$
$$e^{a+i\theta} = e^a e^{i\theta}$$

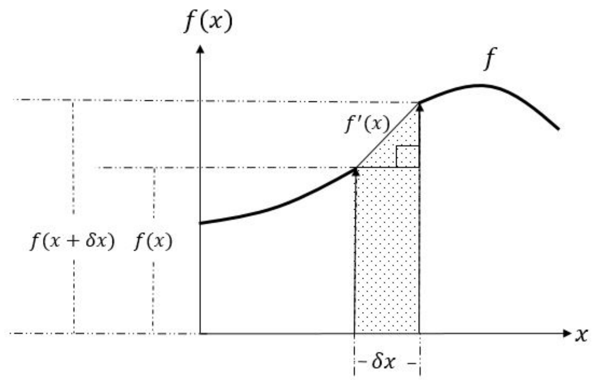
19 Small Angle Approximation

In the limit, $tan\theta = sin\theta$.
The approximation remains as an approximation.
However, in the limit, these two functional forms are equivalent.

$$sin\theta \approx \theta$$
$$tan\theta \approx \theta$$
$$cos\theta \approx 1 - \frac{\theta}{2}$$

20 Reimann Rectangles [2]

Built a tool formalizing the definition of a f'.
The f' is the minimum distance between two points along a predescribed path.



- Confining the f' with an upper and lower bound square wave $f(\chi)$.

$$f'(x) = lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \frac{df(x)}{dx}$$

- Subtraction does not exist.
- Division does not exist.
- Operations that DO NOT exist: -, $\div$, $\sqrt{}$
- Operations that DO exist: $+$, $\cdot$, ^{n}
- Square waves (χ) envelope the f'
- The minimum distance between two points on a euclidean plane is a line.

21 Leibniz Notation

Leibiz Notation is the standard form used to describe a derivative: $\frac{df(x)}{dx}$.

However, this leads to some confusion.

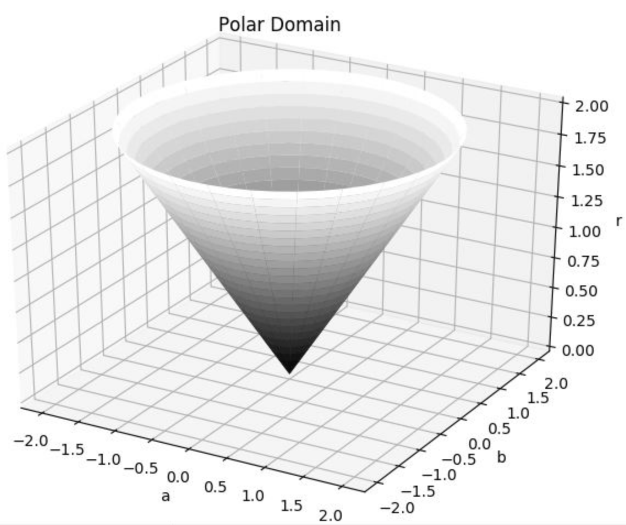
$$f(x) = \int_0^x f'(x)dx = \int_0^x \frac{df(x)}{dx}dx$$

Boundary Value Problems states that if you understand *df*
i.e. the underlying functional form of a function
you can integrate and plug in the correct boundary conditions to determine f.

The *df* underlying functional form is the derivative of a function.

$$^f(x) = \int_0^x df(x).dx$$

The coordinate system used to quantify the *df* is irrelevent to the f.
The notation provided above is for reading purposes only.
With the exception of Fourier, who was a proponent of this type notation.



22 Current Mathmatical Models Perimeter of an Ellipse [3]

The approximated equations for the perimeter of an ellipse are written below.

$$P \approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$$
$$P \approx \pi(3(a+b) - \sqrt{(3a+b)(a+3b)})$$
$$P \approx \pi(a+b)(1+\frac{3h}{10-\sqrt{4-3h}}), h = (\frac{a-b}{a+b})^2$$

The "perfect" answer is accepted is written below.

$$P \approx 4a\int_0^{\frac{\pi}{2}} \sqrt{1-(\frac{a^2-b^2}{a^2})sin^2\theta}\ d\theta$$

The "perfect" equation for the perimeter as a function of angle.

$$P(\theta)_0^{\frac{\pi}{2}} \approx 4a\sqrt{1-(\frac{a^2-b^2}{a^2})sin^2\theta}$$

Here is another generally accepted answer.

$$P(\theta)_0^{\frac{\pi}{2}} \approx \sqrt{a^2cos^2\theta + b^2sin^2\theta}$$

None satisfy the appropriate boundary conditions for all values of a and b.

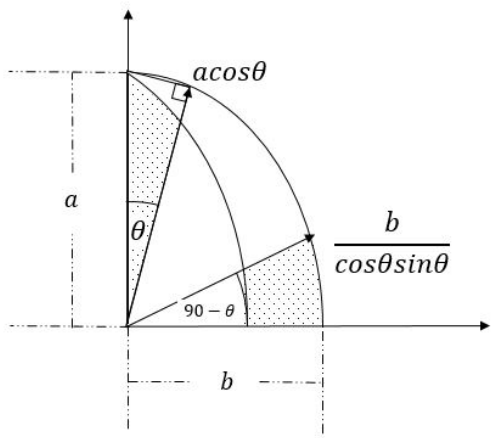
$$P(\theta = 0) = 0$$
$$P(\theta = 2\pi; a = b) = 2\pi r$$

You need at least 2 boundary conditions in order to determine the functional form of a polar based function because you need at least 2 dimensions in order to describe an angle.

23 Enveloping Ellipse [4]

Primary ellipse describing the envelope function.
The following relationship $cos^2\theta + (2sin(\frac{\theta}{2})cos(\frac{\theta}{2}))^2 = 1$ holds for any angle.

23.1 Area

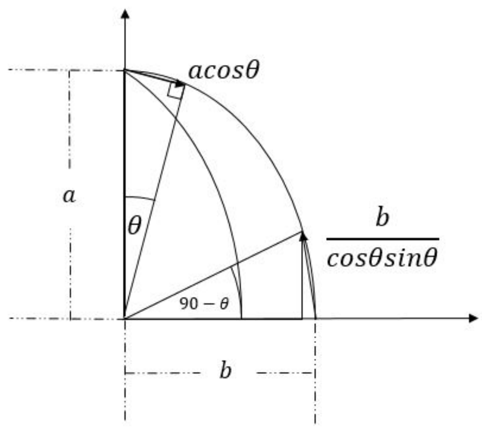


$$dA(\theta) = abcos\theta sin\theta$$
$$dA(\theta) = a \cdot b \cdot sin\theta.cos\theta$$

- Take length a and shrink it by $a \cdot cos\theta$ until you get to $90 - \theta$.
- Transform a into b by multiplying it by $cos(90 - \theta) = sin\theta$
- Grow b at the same rate to fill in the rest of the area missed by a.
- $dA(\theta)$ is the equation for the unit area of an enveloping ellipse.

23.2 Perimeter

Repeating the same procedure for the perimeter of an ellipse.



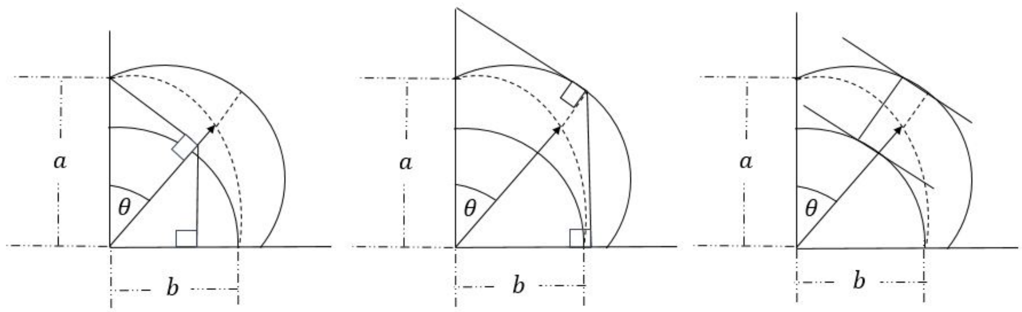
$$dP(\theta) = (a+b)cos\theta sin\theta$$
$$dP(\theta) = a \cdot cos\theta.sin\theta - (-b \cdot cos\theta.sin\theta)$$

24 Goldbauch Conjecture

In the limit, the square root equals the average value.
In the limit, $tan\theta = sin\theta$.
Showcases a fundamental minimization boundary condition: $f(x) = f'(x)$.

$$r^2 = ab$$
$$\sqrt{ab} = \frac{a+b}{2}$$

25 The Fourier Transform i.e. Reimann Ellipses#



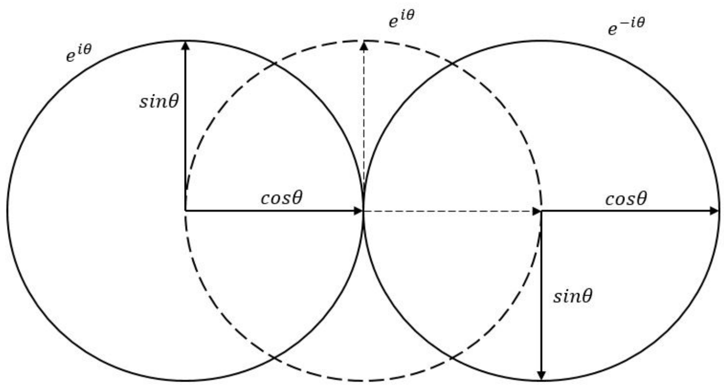
$$f'(\theta) = lim_{\delta\theta\to 0}\, \frac{a(cos\theta + sin\theta) - b(cos\theta + sin\theta)}{a(cos\theta + sin\theta) - b(cos\theta + sin\theta)} = abcos\theta sin\theta$$

- The Fourier Transform bounds the f' using ellipses.
- The minimum distance between two points on a circle is the arc length.
- The minimum distance between two points on an ellipse is the arc length.

Interim steps will be provided in subsequent sections.

$$F(\nu) = \int_0^\infty f(x)e^{-2\pi ix\nu}dx$$

26 Quadrinary Rotations Introductions



Polar functional forms $e^{i\theta}$ are shift invarient functional forms.
Directional component conventions are listed below.
These are consistent with Fourier convention and Euler′s formula.
These are consistent with the right hand rule.

$$e^{i\theta}(\uparrow\rightarrow) = cos\theta(\rightarrow) + sin\theta(\uparrow)$$
$$e^{-i\theta}(\downarrow\rightarrow) = cos\theta(\rightarrow) + isin(\theta)(\downarrow)$$

Add components along respective dimensions.

$$e^{i\theta}(\uparrow\rightarrow) + e^{-i\theta}(\downarrow\rightarrow) = cos\theta(\rightarrow)$$

For respective quadrinary orientation.

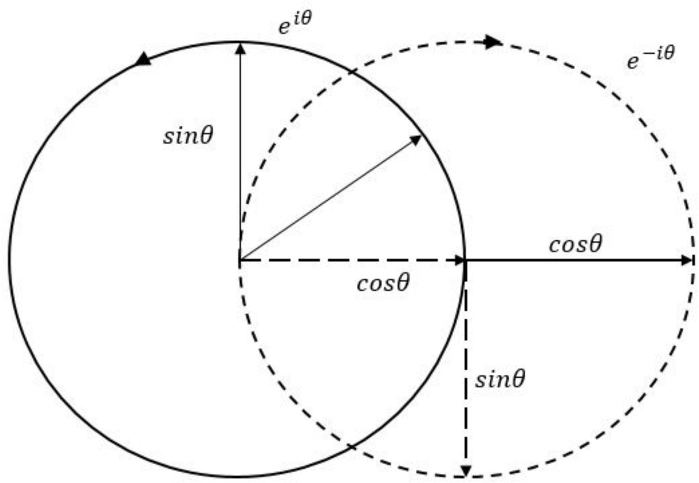
$$e^{i\theta}(\uparrow\rightarrow) - e^{-i\theta}(\downarrow\rightarrow) = sin\theta(\uparrow)$$

For each case, the functional form of $e^{i\theta}$ is invarient as a functional form.

$$e^{i\theta}(\uparrow\rightarrow) = cos\theta(\rightarrow) + sin\theta(\uparrow)$$

$e^{i\theta}$ establishes the path of travel.
You see, I have yet to apply the directional propogation.
No multiplication has occurred. I have only established the initial directions.

27 Quadrinary Rotations Depth and Projection



Machine drawing standard applied.
Dotted lines are indicative of depth.
Add the prospective components of these two quadrinary oriented systems.

$$e^{i\theta}(\uparrow\rightarrow) = cos\theta(\rightarrow) + sin\theta(\uparrow)$$

The projection ($\perp$) of $e^{i\theta}$ onto the imaginary axis.

$$e^{i\theta}_{\perp i}(\uparrow\rightarrow) = sin\theta(\uparrow)$$
$$e^{-i\theta}_{\perp i}(\downarrow\rightarrow) = isin(\theta)(\downarrow)$$

Require technical resource quadrinary number python module development.

28 Imaginary Numbers i.e. Rotative Numbers

Imaginary numbers are usually introduced using the following notation.

$$i = \sqrt{-1}$$

This notation greatly undermines the utility of imaginary numbers.
It is an extreme disservice to the mathmatical community.
We do not introduce 1 as $\sqrt{1}$.
It keeps both mathmeticians and engineers from using imaginary numbers.
i is indicative of a direction, not a magnitude.

$$\sqrt{1} = 1$$
$$\sqrt{-1} = -1$$
$$i = i^2$$

The engineering representation of i provides much more clarity.

$$i = -1$$

The following integrals become much more understandable in this manor.
You are changing the orientation of the components of a circle.
You are not changing the magnitude of the radius of a circle.

$$r = \int_0^2 dr = 2 ... \uparrow 1 + \uparrow 1$$
$$r = \int_0^{\pi/2} e^{i\theta} d\theta = i + 1 ... \downarrow 1 + \uparrow 1$$
$$r = \int_0^{\pi/2} cos\theta + sin\theta d\theta = 2 ... \uparrow 1 + \uparrow 1$$

The magnitude of the radius of a circle r is seperable from its direction of travel.

29 Eulers Formula

The definition of i above does not interfere with euler′s formula.

$$cos\theta = e_r^{i\theta} = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$sin\theta = e_i^{i\theta} = \frac{e^{i\theta} - e^{-i\theta}}{2}$$

The equation for the $sin\theta$ formula provided above can be simplified.

$$\frac{dcos\theta}{d\theta} = -sin\theta, \ \frac{dsin\theta}{d\theta} = cos\theta, \ \frac{de^{c\theta}}{d\theta} = ce^{c\theta},$$

Simplification provided below.

$$\frac{dcos\theta}{d\theta} = \frac{ie^{i\theta} - ie^{-i\theta}}{2}$$

Substitute in $i = -1$ into the relationship above.

$$\frac{dcos\theta}{d\theta} = \frac{e^{-i\theta} - e^{i\theta}}{2}$$

Showing equivalency with $-sin\theta$. To take a derivative of a real function, you need to apply an imaginary rotation to orient the derivative correctly.

$$-sin\theta = -(\frac{e^{i\theta} - e^{-i\theta}}{2}) = \frac{e^{-i\theta} - e^{i\theta}}{2}$$

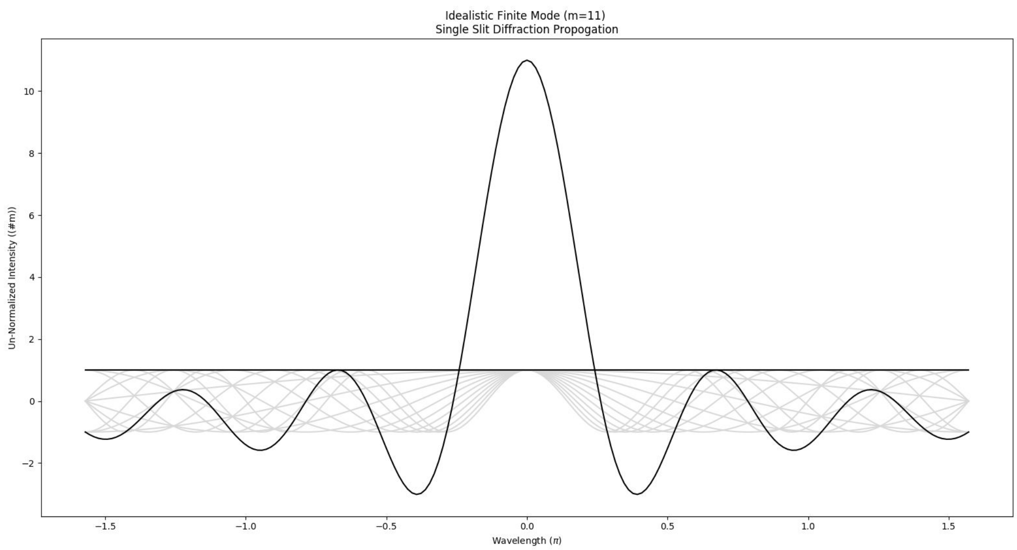
Repeating the process with $sin\theta$.

$$\frac{dsin\theta}{d\theta} = \frac{ie^{i\theta} + ie^{-i\theta}}{2}$$
$$\frac{dsin\theta}{d\theta} = \frac{-e^{i\theta} - e^{-i\theta}}{2}$$
$$\frac{dsin\theta}{d\theta} = -(\frac{e^{i\theta} + e^{-i\theta}}{2})$$
$$\frac{dsin\theta}{d\theta} = -cos\theta$$

Remember, $sin\theta$ is oriented perpendicular to $cos\theta$ on the imaginary axis.
To take a derivative of an imaginary function, you need to apply a quadrinary rotation to orient the derivative correctly.

$$\frac{dsin\theta}{d\theta} = cos\theta$$

30 Fourier Diffraction Theory Cosine Summation



Intensity profile of an idealistic infinite slit.
Packed into a constrained span in order to try to communicate a 3 dimensional configuration with a 2 dimensional plane.

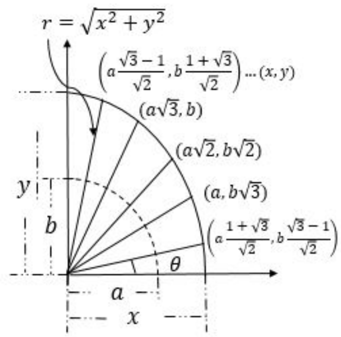
$$I(\theta)^{\frac{\pi}{2}}_{-\frac{\pi}{2}} = \int_1^m cos(m\theta) = cos(\theta) + cos(2\theta) + cos(3\theta)...cos(m\theta)$$

The path traveled by light in order to minimize the distance traveled along an elliptical path is characterized by the functional form describing the nodes.

$$p(\theta) = cos(m\theta) = -sin(m\theta - \frac{\pi}{2})$$

Require single slit experiment measurements for confirmation of conversion factor for curvillinear coordinate systems.

31 The Unit Circle [5] i.e Collapsed Orthogonal Ellipse Equal Components



$$1 = x^2 + y^2$$
$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$

Polar parameterization of the magnitude of a circle from $\theta|_0^{\frac{\pi}{2}}$.

$$x(\theta) = cos\theta$$
$$y(\theta) = sin\theta$$

The magnitude of the radius of a circle is constant.

$$r(\theta) = r = 1$$
$$1 = cos^2\theta + sin^2\theta$$

The arc length of the unit circle is the perimeter of the unit circle.

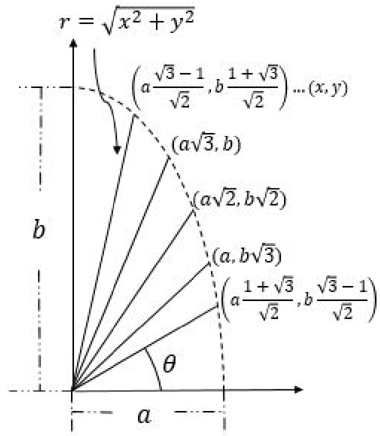
$$P(\theta) = \int_0^{\theta} r d\theta$$
$$\frac{\pi}{2} = \int_0^{\frac{\pi}{2}} d\theta$$

32 The Unit Ellipse i.e. Non-Orthogonal Dissimilar Components

The traditional equations specifically written based on the case of the collapsed orthogonal parameterization which subsitutes in the following relations:

$x(\theta) = cos\theta, y(\theta) = sin\theta$ and $a = a\sqrt{2}, b = b\sqrt{2}$ where $a = b = 1/2$.

However, orthogonal ellipses do not have to be collapsed, they are simply always assumed to be so: $x(\theta) = 2acos\theta, y(\theta) = 2bsin\theta$.



$$
\begin{aligned}
1 &= a^2x^2 + b^2y^2 \\
r^2 &= a^2x^2 + b^2y^2 \\
r &= \sqrt{a^2x^2 + b^2y^2}
\end{aligned}
$$

Alternative parameterization of the magnitude an orthogonal ellipse from $\theta|_0^{\frac{\pi}{2}}$. For the unit circle, $a = b = 1/2$.

$$
\begin{aligned}
x(\theta) &= 2acos\theta \\
y(\theta) &= 2bsin\theta
\end{aligned}
$$

Substitution into the formalized equations above.

$$1 = (2acos\theta)^2 + (2bsin\theta)^2$$
$$r^2 = 4a^2cos^2\theta + 4b^2sin^2\theta$$

Traditional equations above subsitute in the following relations as a and b.

$$a^2 = (\sqrt{2}a)^2, b^2 = (\sqrt{2}b)^2$$
$$x^2 = cos^2\theta, y^2 = sin^2\theta$$

Resulting in a very confusing notation for this particular equation.

$$r^2 = a^2x^2 + b^2y^2$$
$$r = \sqrt{a^2x^2 + b^2y^2}$$

For the remainder of this text, the following parameterization will be used:

$$x(\theta) = 2acos\theta$$
$$y(\theta) = 2bsin\theta$$

While maintaining the following magnitude relationship.

$$r = \sqrt{x^2 + y^2}$$

Which showcases the main problem with the current mathmatical model.
The equation of the perimeter is misrepresented as the radius.

$$r(\theta) = \sqrt{4a^2cos^2\theta + 4b^2sin^2\theta}$$

Where, again, you see the variable substitution as: $a = a\sqrt{2}, b = b\sqrt{2}$ is applied.

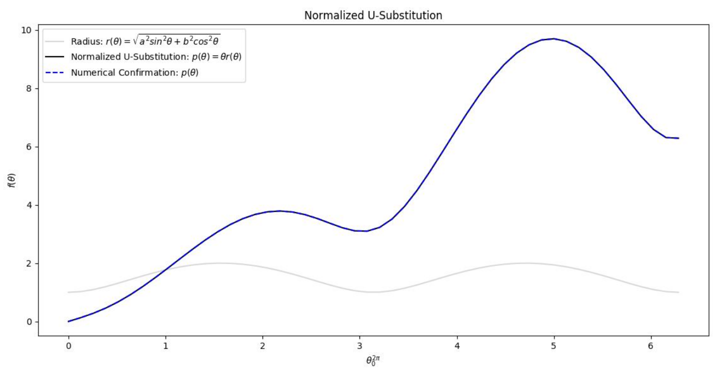
$$r(\theta) = \sqrt{a^2cos^2\theta + b^2sin^2\theta}$$

The integral of the radius of a circle or ellipse determines the perimeter.

$$P(\theta) = \int_0^\theta r(\theta)d\theta$$

There is a continuous method you can use to integrate this function.
However, it only applies to magnitudes.

33 Normalized U-Substitution



The radius of a circle is a constant magnitude.

$$P(\phi) = \int_0^{\phi} \sqrt{cos^2\theta + sin^2\theta} d\phi = r\phi$$

The perimeter as a function of angle for an ellipse.
The radius may be varying as a function of angle at every instance.
This does not mean that it is not seperable from its direction.
It does not matter how many times or in how many ways you take a dot product.
Magnitudes will always be seperable from directions.

$$P(\phi) = \int_0^{\phi} \sqrt{4a^2cos^2\theta + 4b^2sin^2\theta} d\phi = (\sqrt{4a^2cos^2\theta + 4b^2sin^2\theta})\ \phi = r(\theta)\phi$$

The equation above satisfies the appropriate boundary conditions.

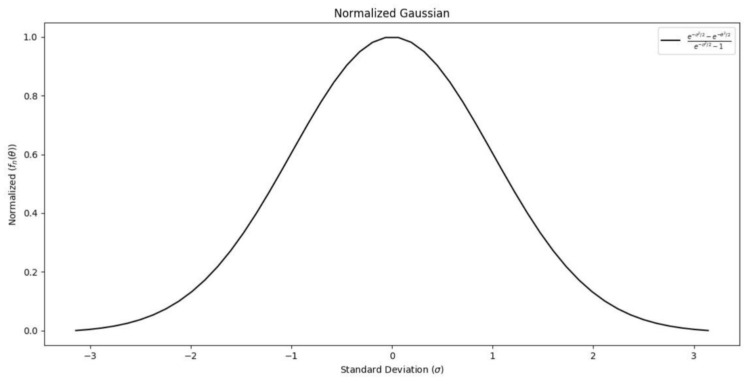
$$P(\phi = 0) = 0P(a = b) = 2\pi$$

The radius of an ellipse is a varying magnitude seperable from its direction.
Again, you cannot take the derivative of a magnitude.
The derivative of the perimeter must always equal the radius of that function.

$$r(\theta) = \frac{dP(\phi)}{d\phi}$$

As such, the radius does not need to be parameterized orthogonally.
You do not need to normalize your vectors orthogonally.
You can create a conservative non-orthogonal vector space.
Multivariate statistic techniques can be applied in a non-orthogonal basis.

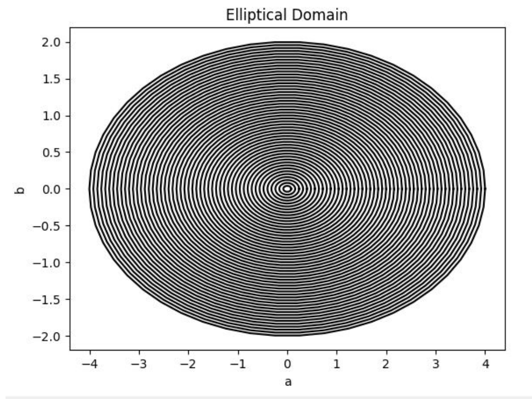
34 Normalized Gaussian



Normalized Gaussian Equation independent of standard deviation independent of correction offset can be written as follows.

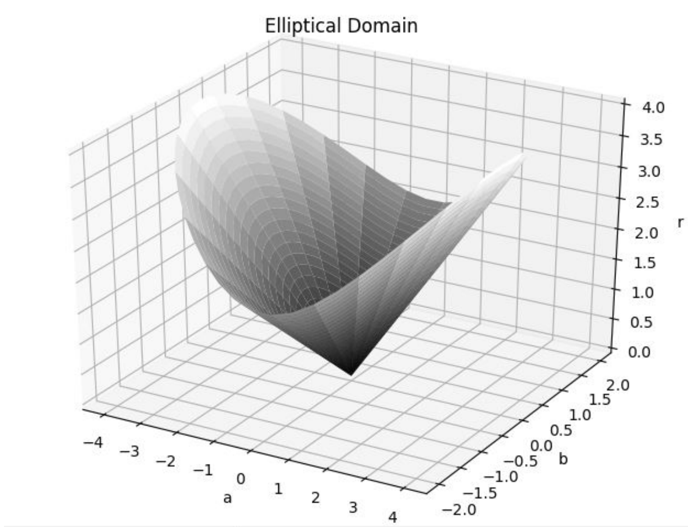
$$f(\theta)_{-\sigma}^{\sigma} = \frac{e^{-\frac{1}{2}\sigma^2} - e^{-\frac{1}{2}\theta^2}}{e^{-\frac{1}{2}\sigma^2} - 1}$$

35 Elliptical Domain



Domains are coordinate systems that cannot be shifted. Polar functional forms are shift invarient.

$$x(\theta, \phi) = \frac{1}{\pi} a\theta cos\phi$$
$$y(\theta, \phi) = \frac{1}{\pi} b\theta sin\phi$$
$$r = \sqrt{x^2 + y^2}$$



36 Perimeters

36.1 Square

$$P(x) = 4\int_0^x dx = 4x$$

36.2 Circle

$$P(\theta)|_0^{2\pi} = \int_0^\theta r d\theta = \int_0^\theta \int_0^r dr d\theta = r\theta|_0^{2\pi} = 2\pi r$$

36.3 Orthogonal Ellipse

$$P(\theta, \phi)|_0^{2\pi} = \int_0^\phi \sqrt{4a^2 cos^2\theta + 4b^2 sin^2\theta} d\phi = 2\pi r(\theta)$$

36.4 Enveloping Ellipse

$$P(\theta)|_0^{2\pi} = \frac{\pi}{2}(a+b)\int_0^\theta cos(\theta/4) sin(\theta/4) d\theta$$

Compute integral using u-substitution.

$$u = \frac{x}{4}, du = \frac{dx}{4}$$
$$\int_0^\theta cos(\theta/4) sin(\theta/4) d\theta = 4\int_0^u sin(u) cos(u) du$$
$$P(\theta)|_0^{2\pi} = \pi(a+b) sin^2(\theta/4)$$

Showing equivalency with r for good measure.

$$P(\theta) = 2\pi r sin^2(\theta/4)|_0^{2\pi}$$
$$P = 2\pi r = \pi(a+b)$$

37 Conclusion

I hope these equations save everyone about 10 years on their life span.
Human intellectual capacities are needed now more than ever.
You are not obsolete. Humans are not obsolete. Humans are far from useless.
We have all of the tools necessary to enter an age of Mathmatical Renaissance.
You are no longer tools limited when trying to develop calculus based equations.
Computers are assistive technologies, not replacement technologies.
Data does not mean anything if you do not understand how to interpret it.

38 Sources

[**1**] https://en.m.wikipedia.org/wiki/Taylor_series

[**2**] https://en.m.wikipedia.org/wiki/Riemann_sum

[**3**] https://www.mathsisfun.com/geometry/ellipse-perimeter.html

[**4**] https://en.m.wikipedia.org/wiki/Envelope_(waves)

[**5**] https://en.m.wikipedia.org/wiki/Unit_circle