## The Normal Distribution
- Investment management is easier when returns are normal
    - Standard deviation is a good measure of risk when returns are symmetric
    - If security returns are symmetric, portfolio returns will be as well
    - Future scenarios can be estimated using only the mean and the standard deviation
    - The dependence of returns across securities can be summarized using only pairwise correlation coefficients
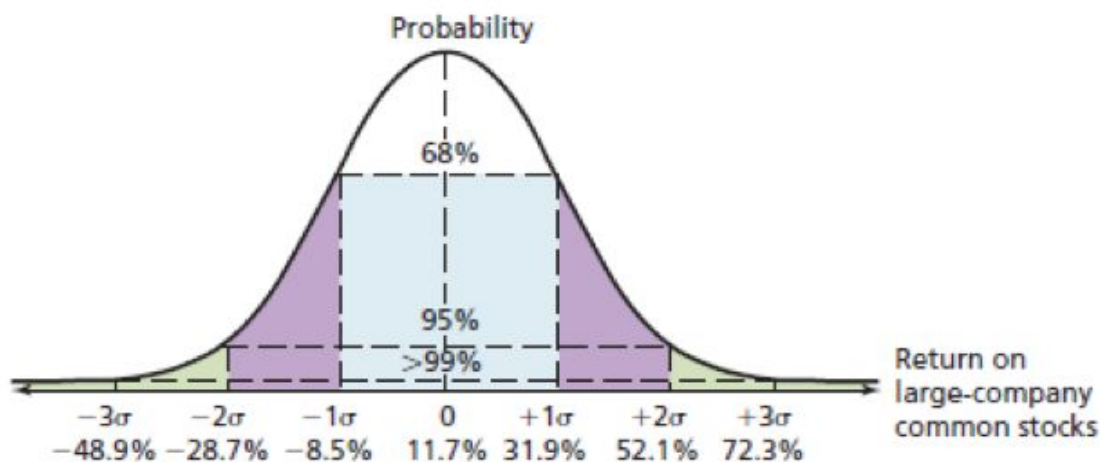
Normality and Risk Measure
- What if excess returns are not normally distributed
    - Standard deviation is no longer a complete measure of risk
   - Need to consider skewness and kurtosis
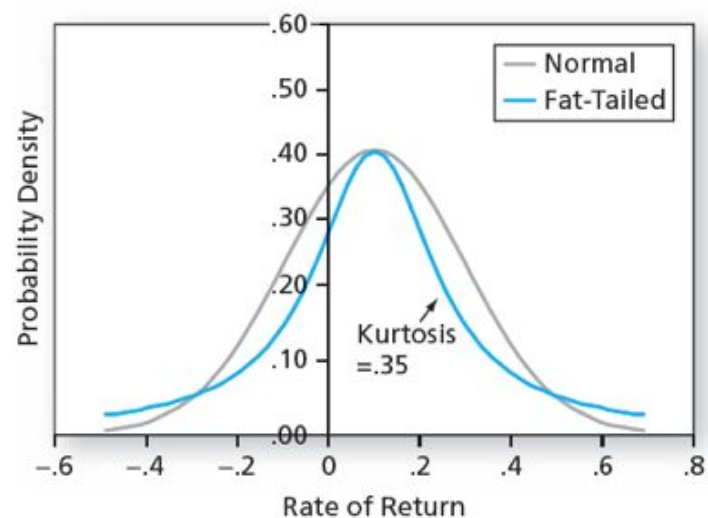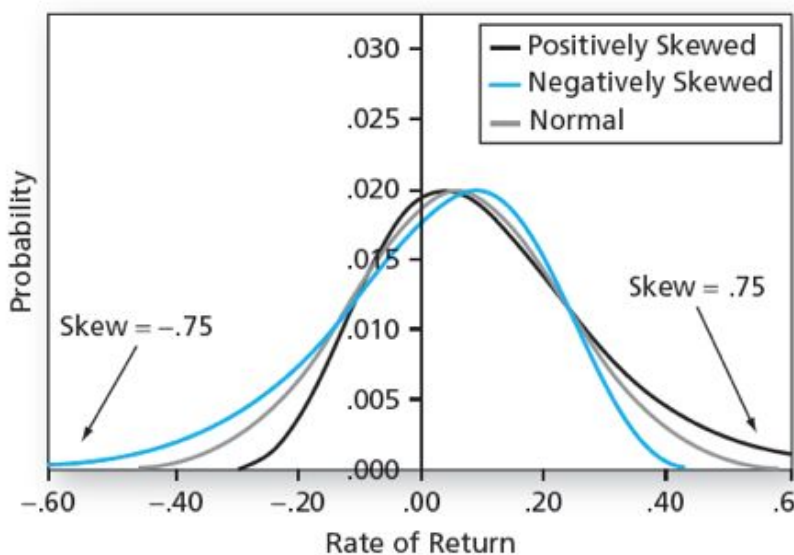
Other Risk Measures
1. Semi-variance: similar to variance but uses only deviations below the mean
2. Lower Partial Standard Deviation (LPSD): uses only deviations below the risk-free return
3. Sortino Ratio - The ratio of average excess returns to LPSD (replaces the Sharpe Ratio)
4. Value at Risk (VaR)
    - Loss corresponding to a very low percentile of the entire return distribution. Widely used by banks to measure their exposure to adverse events
        - For example, if 5% of the outcomes are below -30%, then -30% is the value at risk
5. Expected Shortfall (ES)
    - Also called conditional VaR (cVaR), focuses on the expected loss in the worst-case scenario (left tail of the distribution)
    - More conservative measure of downside risk than VaR, it takes the average of all of the worst-case scenario returns that are expected to occur 5% of the time

More on VaR: Value at Risk - calculating the worst-case scenarios
- 3 elements
    - Time period
    - Probability of occurrence
    - The dollar loss
        - Eg. 5% probability of a loss of 295k or more occurring in the next week
- 3 methods to estimate VaR
    - Use historical data to calculate the future expected loss
    - Assume a normal distribution for future returns
    - Use simulations to calculate future probabilities eg. Monte Carlo simulation

Monte Carlo Simulation
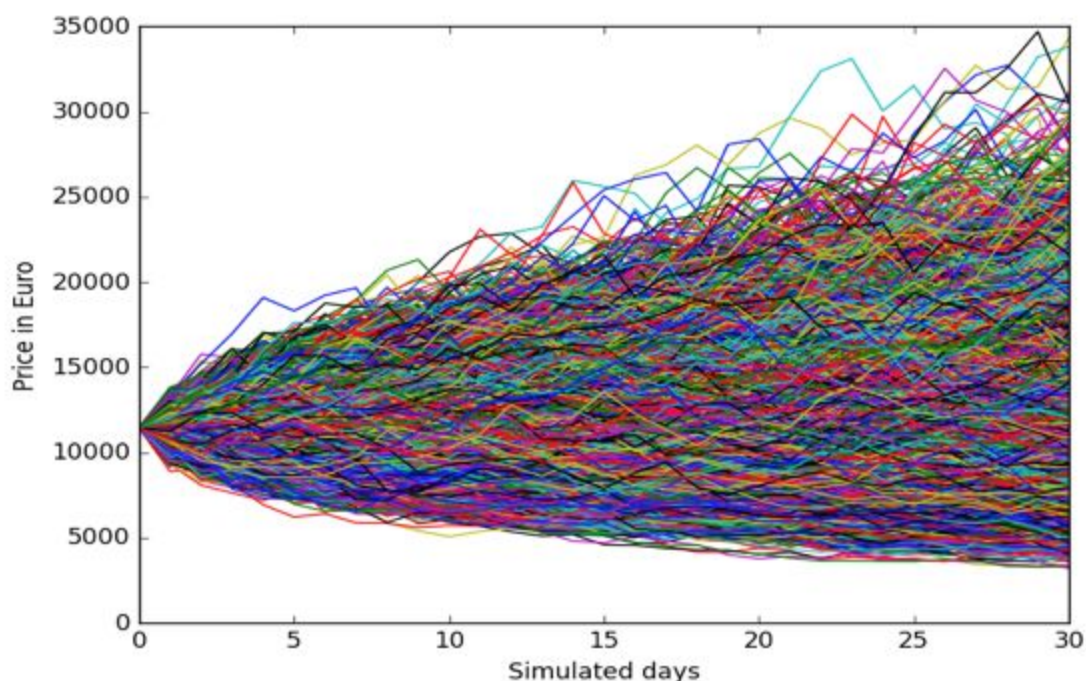
Most commonly used measure of risk
- Where the distribution of returns is symmetrical the ordering of portfolios will be the same using semi-variance or other measures of downside risk
- For well-diversified portfolios, symmetrical distribution is a reasonable assumption
- In normal market conditions variance is an appropriate measure and is in most cases used as a measure of dispersion

Risk Aversion
- A person is given the choice between two scenarios, one with a guaranteed payoff and one without
- In the guaranteed scenario, the person receives 1k
- In the uncertain scenario, a coin is flipped to decide whether the person receives 2k or 0
- The expected payoff for both scenarios is 1k meaning that an individual who was insensitive to risk would not care whether they took the guaranteed payment or the gamble

How do you measure risk aversion?
- Utility of an investment
- $U = E(r) - \frac{1}{2} A\sigma^2$
- A is greater than zero for a risk-averse investor, so any increase in risk reduces their utility. Note that the risk-free asset generates the same utility