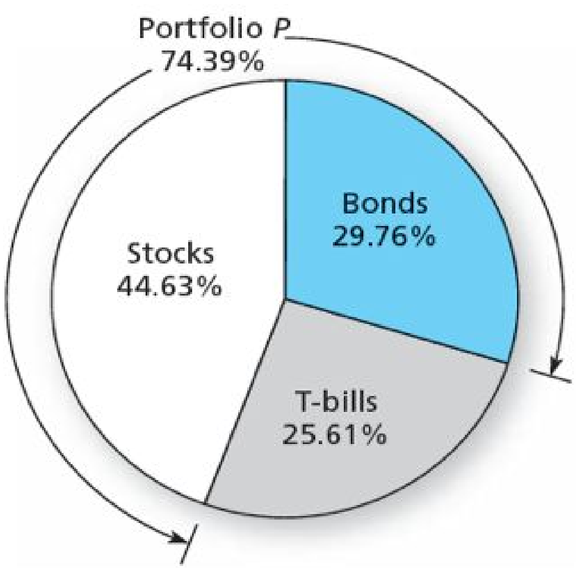
The Efficient Frontier with Riskless Lending and Borrowing

- We introduced a risk-free asset into our portfolio possibility set
- We can consider lending at a risk-free rate as investing in an asset with a certain outcome
- Borrowing can be considered selling such a security short; thus borrowing can take place at the risk-free rate

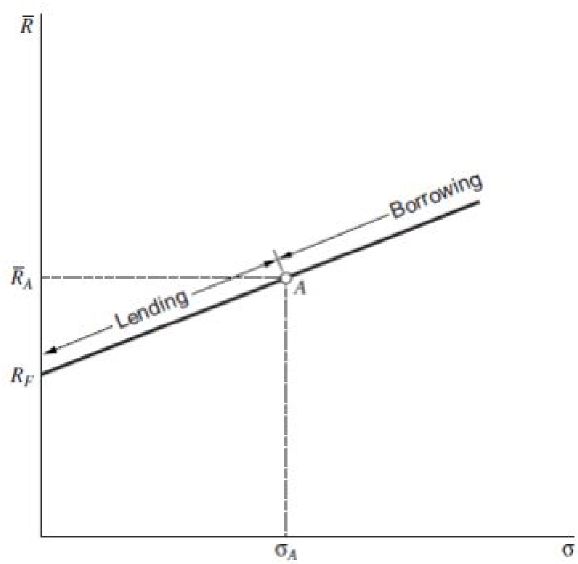
Capital Allocation Across Risky and Risk-Free Portfolios

- Asset allocation
    - The choice among broad asset classes represents a very important of portfolio construction
- The simplest way to control risk is to manipulate the fraction of the portfolio invested in risk-free assets versus the portion invested in the risky assets

The opportunity set of financial assets



Imagine we have just possibilities investing in a risk-free rate and investing in a risky portfolio



Imagine we invest x in a risky asset A and 1-x in a risk-free asset:

Risk free asset: $r_f = 7\%$ $\sigma_{rf} = 0\%$

Risky asset: $E(r_A) = 15\%$ $\sigma_A = 22\%$

The expected return on a portfolio P combining both assets, with x invested in the risky asset will be :

$$\mathbf{1.}\ \boldsymbol{E(r_P) = x(r_A) + (1 - x)r_f}$$
$$= 15x + 7(1 - x)$$

The risk of the complete portfolio:

$$\mathbf{2.}\ \boldsymbol{\sigma_P = \sigma_A x}$$
$$= 22x$$

Rearrange equations 1 and 2

$$1.\ E(r_P) = r_f + (r_A - r_f)x$$

$$2.\ x = \frac{\sigma_P}{\sigma_A}$$

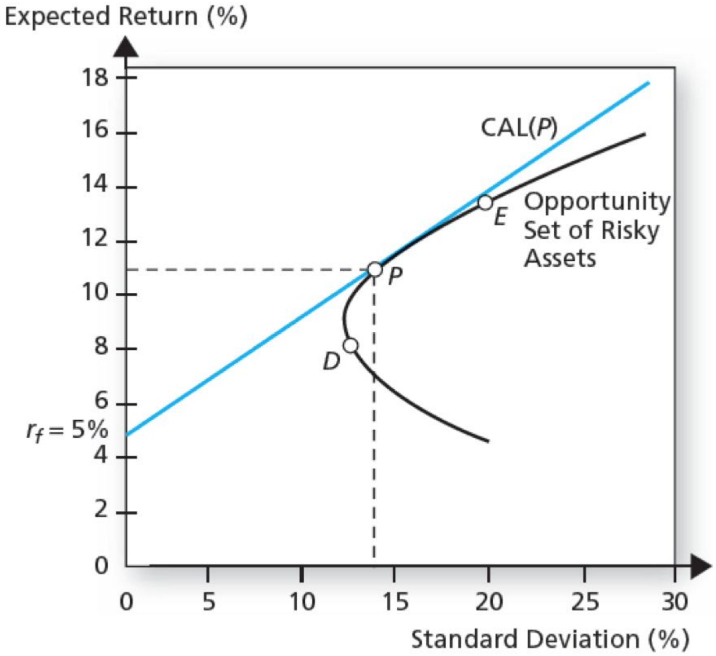
Substitute for x

$$E(r_P) = r_f + (r_A - r_f)\frac{\sigma_P}{\sigma_A}$$

Or $E(r_P) = r_f + \left(\frac{r_A - r_f}{\sigma_A}\right)\sigma_P$

The CAL has an Intercept of r_f *and a Slope of* $\frac{r_A - r_f}{\sigma_A}$

Combining the CAL and the Efficient Frontier



The tangency point of the CAL and the efficient frontier gives us the highest possible slope for the CAL

Tangency Point of the CAL to the Efficient Frontier

- The tangency point represents the highest slope for the CAL
- This is the highest possible reward-to-risk ratio and also happens to be the Sharpe ratio

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

The Capital Allocation Line

- Where should an investor lie on this line between a risk-free and a risky asset?
- This depends on the risk aversion or risk appetite of the investor

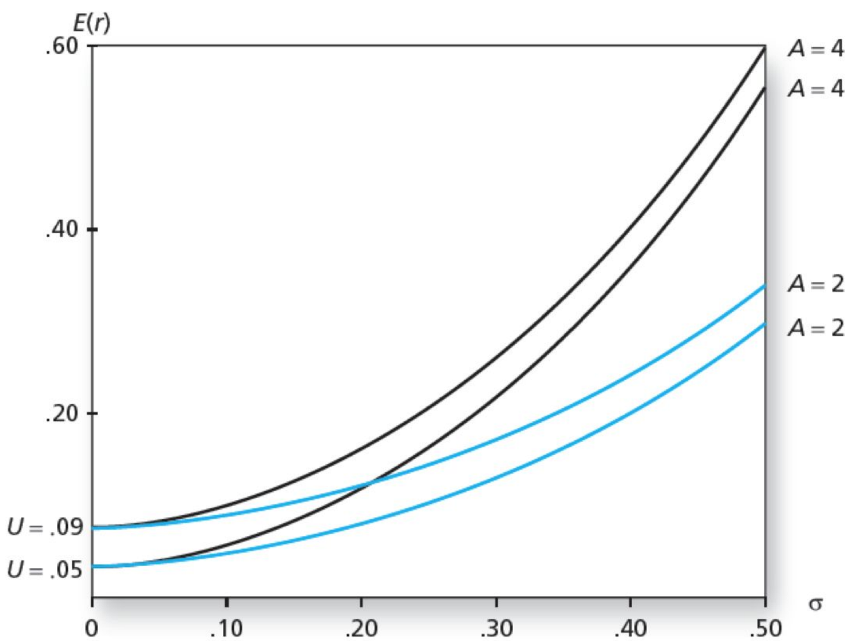
We need 3 elements to determine the optimal portfolio for an investor

1. The efficient frontier of all risky assets
2. The capital allocation line
3. An investor’s indifference curves

Indifference Curves

- The final step is to add investors indifference curves to establish where on the CAL their optimal investment will be
    - The indifference curve plots equal levels of utility for an investor
    - Given the choice an investor will prefer a portfolio on a higher indifference curve, offering a higher return for a given level of risk

Indifference Curves for $U = .05$ and $U = .09$ with $A = 2$ and $A = 4$

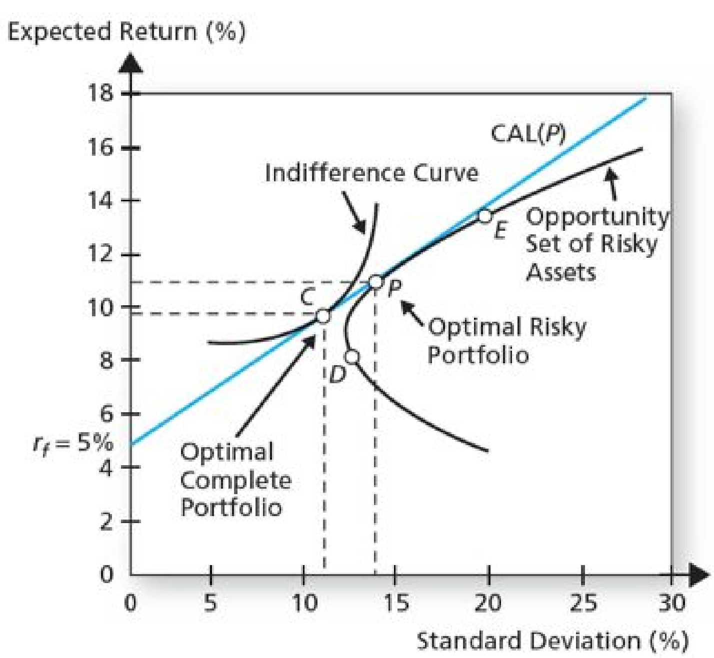


A is an index of the investor's risk aversion

U is the utility value

Note that with higher risk aversion (A=4), more return is required for more risk in order to keep utility constant

Determination of the Optimal Overall Portfolio - The Markowitz Model



The Capital Market Line - A Passive Strategy

- The CML is a capital allocation line formed by investing in two passive portfolios:
    1. Virtually risk-free short-term T-bills (or a money market fund)
    2. Fund of common stocks that mimic a broad market index
- It avoids any security analysis and devotes no resources to acquiring information on any individual stock or groups