How do we evaluate the opportunity set?

- We try to measure the expected risk and return of the opportunity set of risky assets
- We assume that investors like high return but don't like high risk = risk-aversion
- Several factors affect risk
- Returns should be consistent with risk

Rate of return: Single period

$$HPR = \frac{p1 - p0 + D1}{P0}$$

- HPR = holding period return
- P0 = begining price
- P1 = ending price
- D1 = dividend or income during period one

If future outcomes are equally likely

$$\overline{R}_i = \sum_{j=1}^{M} \frac{R_{ij}}{M}$$

If they are not equally likely

$$\overline{R}_i = \sum_{j=1}^{M} P_{ij} R_{ij}$$

Time Series Analysis of Past Returns

- We may base our future expectations of returns on past returns
- We use time series of realized returns to infer expected returns and standard deviations

Arithmetic or Mean Return

- The arithmetic or mean return is the simple average of all holding period returns
- For example, an asset has had a return of -50%, 35% and 27% over the past 3 years

$$\overline{R}_i = \frac{-50\% + 35\% + 27\%}{3} = 4\%$$

Geometric Mean Return

- The geometric mean return accounts for the compounding of returns

$$R_{Gi} = \sqrt[3]{(1-.50)\times(1+.35)\times(1+.27)} - 1 = -5\%$$

Arithmetic Averages vs. Geometric Averages

- The arithmetic average return answers the question: “What was your return in an average year over a particular period”
- The geometric average return answers the question “What was your average compound return per year over a particular period”
- For the purpose of forecasting future returns:
    - The arithmetic average is probably “too high” for long forecasts
    - The geometric average is probably “too low” for short forecasts
- When we talk about average returns, we generally are talking about arithmetic average returns

Average Returns

- Risk-free rate: The rate of returns on a riskless, ie. certain investment
- Risk premium: the extra return on a risky asset over the risk-free rate, i.e. the reward for bearing risk
- There is a reward, on average, forbearing risk

A Measure of Dispersion

- Not only is it necessary to have a measure of the average return, but it is also useful to have some measure of how much the outcomes differ from the average

Measuring Risk

- Variance is a common measure of return dispersion
- Standard deviation is the square root of the variance
    - The standard deviation is often called the volatility
    - Standard deviation is useful because it is in the same “units” as the average
- Normal distribution: Asymmetric, bell-shaped frequency distribution that can be described with only an average and a standard deviation

Measuring Risk of Expected Returns

$$\text{Variance}$$

$$\sigma_i^2 = \sum_{j=1}^{M}\left[P_{ij}\left(R_{ij} - \overline{R}_i\right)^2\right]$$

- The variance is either a simple average or weighted average of expected squared deviations
- These expectations can be formed using historical data or using predictions about the future

Measuring Risk of Observed Returns

$$\sigma_i^2 = \sum_{j=1}^{M}\frac{\left(R_{ij} - \overline{R}_i\right)^2}{M}$$

Standard Deviation = square root of the variance

The Sharpe Ration - The Reward to Volatility

- Excess return
    - The difference in any particular period between the actual rate of return on a risky asset and the actual risk-free rate
- Risk Premium
    - The difference between the expected return on a risky asset and the risk-free rate
- Sharpe Ratio

$$\frac{\text{Risk premium}}{\text{SD of excess returns}} \quad S = \frac{E(r) - r_f}{\sigma}$$