Single-Index Model

Mean

$$\overline{R}_i = \alpha_i + \beta_i R_M$$

2. 1. 1.

Unique
return

Market related
return

Variance

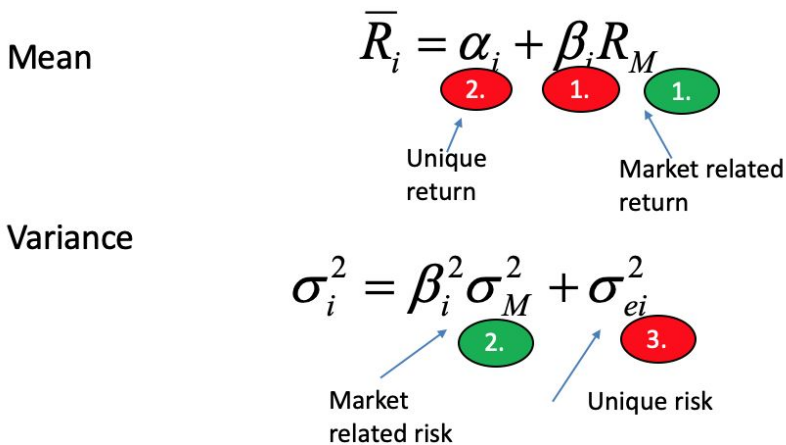$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{ei}^2$$

2. 3.

Market
related risk

Unique risk

What about Covariance/Correlation in the Single Index Model?
- The covariance between 2 assets is the product of their betas and the risk of the market index

$$\text{Cov}(r_i, r_j) = \beta_i \beta_j \sigma_M^2$$

- The correlation is the product of the correlation of each asset with the market index

$$Corr(r_{i,}r_j) = Corr(r_{i,}r_M) x\, Corr(r_{j,}r_M)$$

More on Covariance
- The only reason that assets move together is a common response to market movements
- The covariance between firm-specific surprises and the market return is zero

Way to Calculate Beta
- Regress the firm returns on the market returns
- Use the slope function in Excel
- Use the formula

$$Beta = Cov(r_i, r_M)/Var(r_M)$$

Interpretation of Beta
- Beta is a measure of how sensitive an asset's return is to the return on the market portfolio
- A positive beta indicates that the return of an asset moves in the same direction as the market, whereas a negative beta indicates that the return of an asset moves in the opposite of the market
- A risk-free asset's beta is 0 because its covariance with other assets is 0. Any assets correlation with itself is 1 so the beta of the market is 1
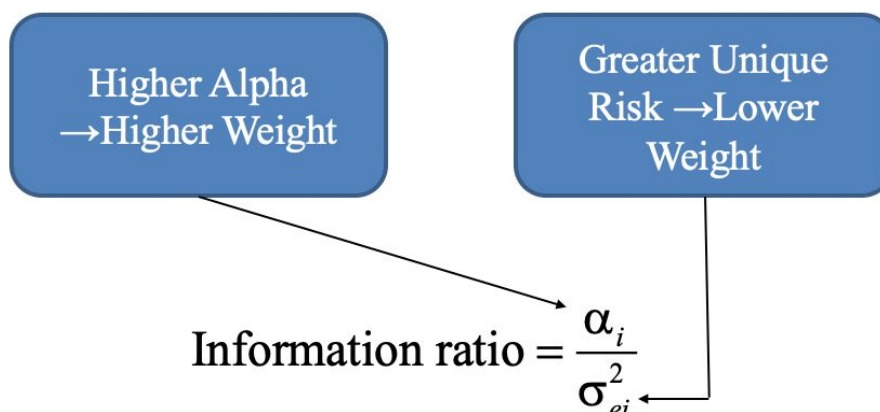
Estimating Beta from Historical Data
- Although beta estimates are important for forecasting future levels of risk, there is concern about their accuracy
- In general shorter periods of estimation (1 year) represent beta that are close to an asset's current level of systematic risk
- However, they can be less accurate than beta estimates measured over three to five years because they may be affected by special events in that short period
- Although longer period beta estimates are more accurate they may be a poor representation of future expectations especially if major changes in an asset have occurred
- Therefore, it is necessary to recognize that estimates of beta, whether obtained through calculation or regression analysis may or may not represent current or future levels of an asset's systematic risk
- The beta of a portfolio tends to be more stable as the changes in the risk of individual stocks tend to cancel out. Historical beta on portfolios are better predictors of future returns than betas on individual assets.

Portfolio Construction and the Single-Index Model
1. Use macroeconomic analysis to estimate the risk premium and risk of the market index
2. Use statistical analysis to estimate the historical alpha, beta and unique risk of all securities
3. Using your estimate of future beta, calculate the expected return of each security attributable only to the market return
4. Using security analysis, estimate the security-specific return for each asset, ie the alphas

What should the relative weight of securities in the portfolio be?

Higher Alpha →Higher Weight

Greater Unique Risk →Lower Weight

$$\text{Information ratio} = \frac{\alpha_i}{\sigma_{ei}^2}$$

The Information Ratio
- The information ratio measures the abnormal return risk added by the security to a well-diversifed portfolio. The larger the information ratio is, the more valuable the security
- Securities with a higher alpha should have a higher weight in the portfolio, and securities with a greater non-systematic risk should be given less weight
- The weight in each security should be proportional to the information ratio

Portfolio Construction and the Single-Index Model
- The Sharpe ratio of an optimal constructed risky portfolio will exceed that of the index portfolio (the passive strategy)

$$S_P^2 = S_M^2 + \left[\frac{\alpha_A}{\sigma(e_A)}\right]^2$$

The Information Ratio
- The contribution of the active portfolio depends on the ratio of its alpha to its residual standard deviation
- The information ratio measure the extra return we can obtain from security analysis

Is the Index Model Inferior to the Full-Covariance Model?
- Full Markowitz model may be better in principle, but
    - Using the full-covariance matrix invokes estimation risk of thousands of terms
    - Cumulative errors may result in a portfolio that is actually inferior to that derived from the single-index model
    - The single-index model is practical and divides risk and return into unique and systematic, or micro and macro risk

Are there drawbacks to the single-index model?
- This simple dichotomy - macro vs. micro risk
- Oversimplifies real-world uncertainty. For example, events may affect many firms within an industry without substantially affecting the broad economy
- Imagine that BP and shell are correlated, the single-index model will ignore this, whereas the Markowitz model will include this. The Markowitz model will place a smaller weighting on these assets, resulting in a portfolio with a lover variance
- The optimal portfolio derived from the single index model can be inferior where stocks are correlated and have large alpha values

- If many stocks have residual correlation (outside of the macro covariance), a multi-index model which includes additional factors to capture those extra sources of cross security correlation would be preferable