Index Models - Beyond the Markowitz Modle

- There are 2 drawbacks with the Markowitz model that we have just studied
    1. It requires a huge number of estimates to create the covariance matrix for all tradable risky assets
    2. It does not provide any guidance to forecasting the risk premium to construct the efficient frontier

Estimating Correlations between Multiple Assets

- How do you simplify the process of selecting the optimal portfolio? Although the theory makes sense, estimating the pairwise correlation between all stocks is hugely cumbersome
- There are many return-generating models which try to generate expected returns
- Not surprisingly the simplest ones are the most widely used. We will describe a number of them, from the simplest to the more complex
- Although the more complex ones are theoretically more accurate they are harder to implement as they need more estimates of the future

Estimating Correlations between Multiple Assets

- When the market goes up. Most stocks tend to increase in price and when the market goes down, most stocks tend to decrease in price
- One reason why security return might be correlated is because of a common response to market changes and a useful measure of this correlation might be obtained by relating the return on a stock to the return on a stock market index

General Formula for Return-Generating Models

Factor weights

Risk factors

$$E(R_i) - R_f = \sum_{j=1}^{k} \beta_{ij} E(F_j)$$

Return over the risk-free rate

$$E(R_i) - R_f = \sum_{j=1}^{k} \beta_{ij} E(F_j) = \beta_{i1}\left[E(R_m) - R_f\right] + \sum_{j=2}^{k} \beta_{ij} E(F_j)$$

All models contain return on the market portfolio as a key factor

The Single-Index Model

- The single index model - assumes that the co-movement between stocks is due to a single common influence or index

$$R_i = \alpha_i + \beta_i R_M + e_i$$

Where

- R_M is the excess market return, $R_M - R_F$
- R_i is the excess return of an asset, $R_i - R_F$

This equation can be estimated using historical returns by regressing historical asset returns on historical market index returns.

The Expected Return of the Single Index Model

$$E(R_i) = \alpha_i + \beta_i E(R_M)$$

Because E(e_i) = 0
Firm-specific surprises will cancel out over time.

There are 2 components to the asset's return, α_i is a nonmarket risk premium.

Confident managers will believe in their ability to find stocks with non-zero values of alpha.

The Expected Risk of the Single Index Model

There are also 2 components to the asset's risk

Total risk = Systematic risk + Firm-specific risk

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{ei}^2$$

Single-Index Model

Mean

$$\overline{R}_i = \alpha_i + \beta_i R_M$$

Unique return

Market related return

Variance

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{ei}^2$$

Market related risk

Unique risk

Advantages of the Single Index Model

- βi divides risk and return into a market-related term and a unique term,
- Far fewer terms to estimate
- The alpha value distills the incremental risk premium attributable to information developed from security analysis