How do we choose between 2 assets?
- How much return do we earn for a given level of risk?

Risk-adjusted return = $\dfrac{r}{\sigma}$

Sharpe ratio $\quad S = \dfrac{E(r) - r_f}{\sigma}$

Combining Assets
- So far we have considered the risk and return of a single asset. What happens when we combine assets into a portfolio?
- Let's start with the simplest case of 2 assets (both with risk >0)
- The expected return of a portfolio is the weighted average of the expected returns on the individual assets

Variance of Two Risky Assets
- Imagine we have 2 assets, with returns R_1 and R_2 and we invest X_1 of our savings into Asset 1 and X_2 into Asset 2
- The variance of the portfolio P comprised of the 2 assets where $X_1 + X_2 = 1$ will be:

$$\sigma_P^2 = E(R_p - \bar{R}_P)^2$$

$$= E\big[X_1(R_{1j} - \bar{R}_1) + X_2(R_{2j} - \bar{R}_2)\big]^2$$

Recall that $(X + Y)^2 = X^2 + Y^2 + 2XY$

$$\sigma_P^2 = E\Big[X_1^2(R_{1j} - \bar{R}_1)^2 + 2X_1X_2(R_{1j} - \bar{R}_1)(R_{2j} - \bar{R}_2) + X_2^2(R_{2j} - \bar{R}_2)^2\Big]$$

$$\sigma_P^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_{12}$$

This is why diversification works – it is called the covariance, average of deviation of A x deviation of B

If asset 1 and asset 2 increase together, on average, the covariance will be positive.

Covariance
- For 2 assets X and Y, the covariance of their returns is measured as

$$Cov(R_1, R_2) = \frac{\sum_{j=1}^{M}(R_{1j} - \overline{R_1})(R_{2j} - \overline{R_2})}{M - 1}$$

- Multiply the deviation of R_1 from its mean and the deviation of R_2 from its mean in each period
- What is the average product of the deviation? This can be positive or negative.

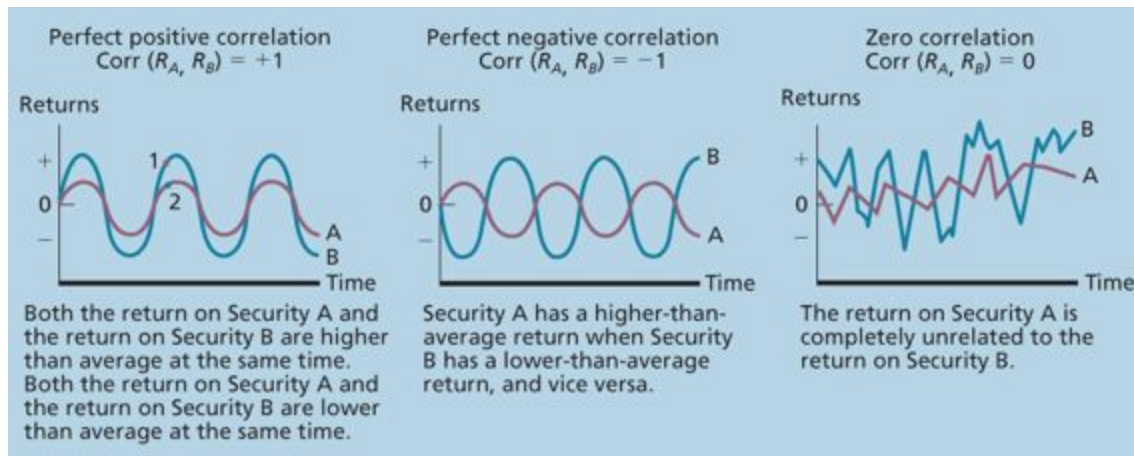**Covariance and the Correlation Coefficient**
- Covariance (sigma) is called the covariance
    - It is the product of the expected (or average) deviations of the assets and as the deviations are not squared, this product can be positive or negative
- Correlation: it is useful to standardized covariance. Covariance has the same properties as the covariance but has a range of -1 to +1 and is called the correlation coefficient and is often denoted by rho

Covariance and the Correlation Coefficient

$$\frac{\sigma_{12}}{\sigma_1\sigma_2} = \rho_{12}$$

Or rearrange…

$$\sigma_{12} = \rho_{12}\sigma_1\sigma_2$$

Covariance = correlation x stdev of asset 1 x stdev of asset 2

Perfect positive correlation
Corr $(R_A, R_B) = +1$

Returns

Both the return on Security A and the return on Security B are higher than average at the same time. Both the return on Security A and the return on Security B are lower than average at the same time.

Perfect negative correlation
Corr $(R_A, R_B) = -1$

Returns

Security A has a higher-than-average return when Security B has a lower-than-average return, and vice versa.

Zero correlation
Corr $(R_A, R_B) = 0$

Returns

The return on Security A is completely unrelated to the return on Security B.

Calculating Portfolio Risk

For a portfolio of two assets, A and B, the variance of the return on the portfolio is:

$$\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_{AB}$$

$$\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B p_{AB} \sigma_A \sigma_B$$

Where: x_A = portfolio weight of asset A

x_B = portfolio weight of asset B

Note: $x_A + x_B = 1$.

Return and Risk of a Two-Asset Portfolio
- As a US investor, you decide to hold a portfolio with 80% in the S&P 500 index and 20% index
- The expected return is 9.92% for the S&P 500 and 18.20% for the EM index
- The standard deviation is 16.21% for the S&P and 33.11% for the EM

- What will the portfolio's expected return and risk given that the covariance between the S&P 550 and the EM index
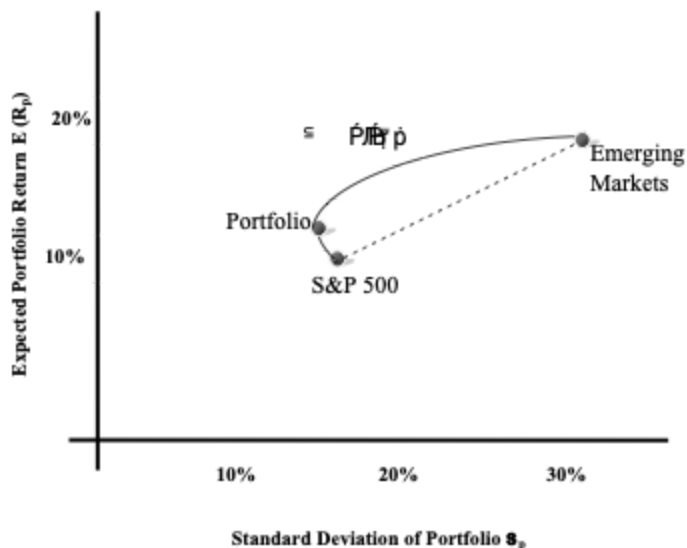
Return and Risk of a Two-Asset Portfolio

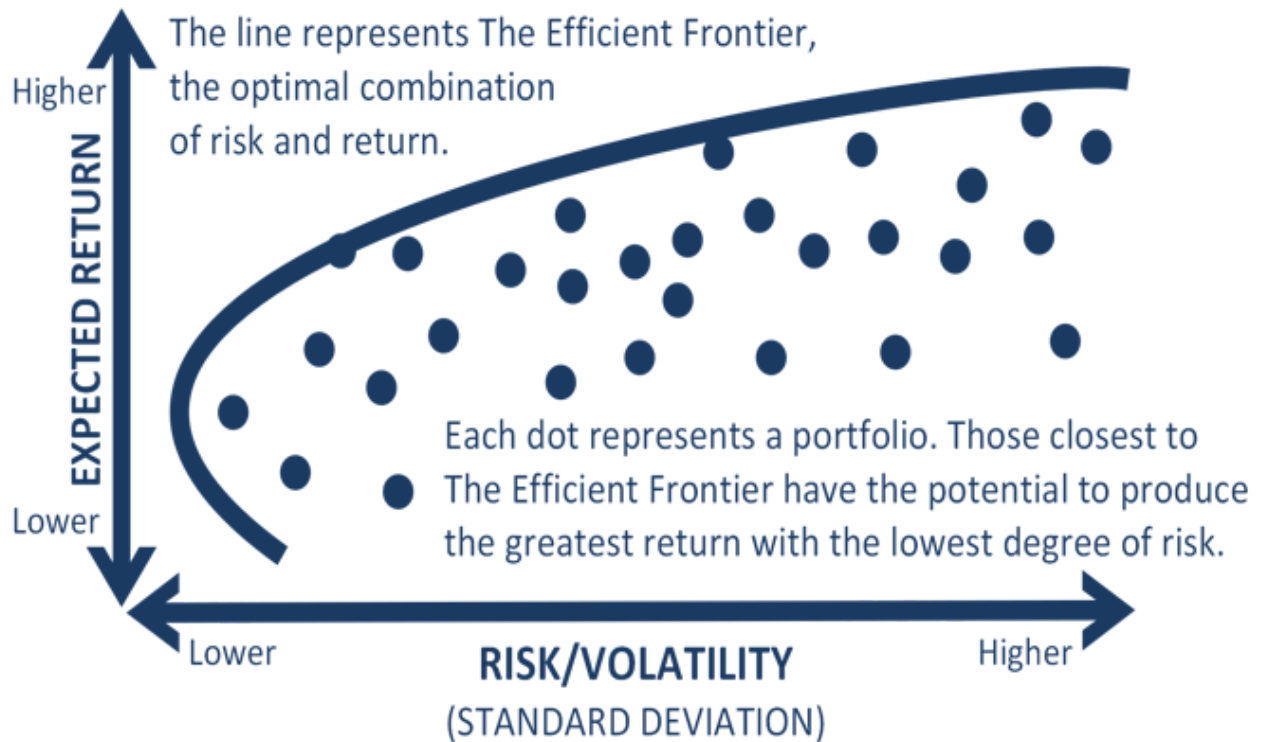*Portfolio Return* $$R_p = X_1R_1 + X_2R_2$$

$$R_P = 11.58\%$$

Portfolio Variance $$\sigma_P^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_{12}$$

$$\sigma_P = 15.10\%$$

What do you notice about the risk of the portfolio?

The line represents The Efficient Frontier, the optimal combination of risk and return.

Higher

Lower

EXPECTED RETURN

Each dot represents a portfolio. Those closest to The Efficient Frontier have the potential to produce the greatest return with the lowest degree of risk.

Lower   **RISK/VOLATILITY**   Higher
               (STANDARD DEVIATION)

Diversification

- The return of a portfolio is the weighted average of the returns of the assets in the portfolio
- The variance of a portfolio is less than the weighted average of the variance of the assets in the portfolio
- By combining assets you can achieve a higher risk-adjusted return or Sharpe ratio
- To what extent depends on the correlation between the assets

Portfolios of more than 2 assets

- It is relatively easy to calculate the portfolio return and variance of 2 or 3 assets
- When we have more multiple assets in a portfolio, it becomes computationally more difficult, so we will move to using excel
- To calculate portfolio risk we create a matrix of the variances and covariances between assets
- The weighted average of the variances and covariances is equal to the portfolio risk

Portfolios with multiple assets

$$\sigma_P^2 = \sum_{j=1}^{N}\left(X_j^2\sigma_j^2\right) + \sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N}\left(X_j X_k \sigma_{jk}\right)$$