Statement

10. Seja $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$ e $x^0 = (1, 0)^T$.

(a) Determine os autovalores/autovetores de A.

(b) Aplique o Método das Potências.

(c) Aplique o Método das Potências Inverso.

(d) Aplique o Método de Rayleigh.

(e) Analise os resultados obtidos.

Solution

a) $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \quad x^0 = (1 \;\; 0)^T$

$$Av = \lambda v \Rightarrow (A - \lambda I)v = 0$$

$$(A - I\lambda)v = \begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$p_A(\lambda) = (1-\lambda)^2 - 4 = 1 - 2\lambda + \lambda^2 - 4 = \lambda^2 - 2\lambda - 3 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2} = \frac{2 \pm 4}{2} \begin{array}{l} \nearrow 3 = \lambda_1 \\ \searrow -1 = \lambda_2 \end{array}$$

$$\begin{pmatrix} 1-3 & 1 \\ 4 & 1-3 \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \Rightarrow \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} -2v_1 + v_2 = 0 \\ 4v_1 - 2v_2 = 0 \end{cases} \Rightarrow v_2 = 2v_1$$

$$\begin{pmatrix} 1+1 & 1 \\ 4 & 1+1 \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \Rightarrow 2v_1 + v_2 = 0 \Rightarrow v_2 = -2v_1$$

Autopares: $\left(3, \begin{bmatrix} t \\ 2t \end{bmatrix}\right), \left(-1, \begin{bmatrix} t \\ -2t \end{bmatrix}\right)$

$\boxed{\text{Vamos usar } \|\cdot\|_\infty}$

b) $x^1 = \dfrac{Ax^0}{\|Ax^0\|} = \dfrac{\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\|Ax^0\|} = \dfrac{\begin{pmatrix} 1 \\ 4 \end{pmatrix}}{4} = \begin{pmatrix} 1/4 \\ 1 \end{pmatrix}$

$x^2 = \dfrac{Ax^1}{\|Ax^1\|} = \dfrac{\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1/4 \\ 1 \end{pmatrix}}{\|Ax^1\|} = \dfrac{\begin{pmatrix} 1/4 + 1 \\ 1 \quad 1 \end{pmatrix}}{\|Ax^1\|} = \dfrac{\begin{pmatrix} 5/4 \\ 2 \end{pmatrix}}{2} = \begin{pmatrix} 5/8 \\ 1 \end{pmatrix}$

$x^3 = \dfrac{Ax^2}{\|Ax^2\|} = \dfrac{\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 5/8 \\ 1 \end{pmatrix}}{\|Ax^2\|} = \dfrac{\begin{pmatrix} 5/8 \\ 5/2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\|Ax^2\|} = \dfrac{\begin{pmatrix} 13/8 \\ 7/2 \end{pmatrix}}{7/2} = \begin{pmatrix} 13/28 \\ 1 \end{pmatrix}$ → convergindo p/ $\begin{pmatrix} t \\ 2t \end{pmatrix}$

$r_3 = \dfrac{\begin{pmatrix} 13/28 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 13/28 \\ 1 \end{pmatrix}}{\begin{pmatrix} 13/28 \\ 1 \end{pmatrix}^T \begin{pmatrix} 13/28 \\ 1 \end{pmatrix}} = \dfrac{\begin{pmatrix} \frac{13}{28} + 4 & 13/28 + 1 \end{pmatrix}\begin{pmatrix} 13/28 \\ 1 \end{pmatrix}}{\left(\frac{13}{28}\right)^2 + 1} = \dfrac{\frac{13}{28}\left(\frac{13+112}{28}\right) + 1\left(\frac{13+28}{28}\right)}{\frac{169}{28^2} + 1} \approx 2{,}4$

c) Sabemos que é invertível, pois seus autovetores formam uma base em $\mathbb{R}^2$. $\Rightarrow \exists A^{-1}$

$$\begin{cases} x^{k+1} = \dfrac{A^{-1}x^k}{\|A^{-1}x^k\|} \\ y = A^{-1}x^k \Rightarrow x^{k+1} = y/\|y\| \\ \hookrightarrow Ay = x^k \end{cases}$$

$$\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} y_1 + y_2 = 1 \\ 4y_1 + y_2 = 0 \end{cases} \Rightarrow \begin{matrix} y_2 = -4y_1 \\ \begin{cases} y_1 = -1/3 \\ y_2 = 4/3 \end{cases} \end{matrix}$$

$$x^1 = \frac{\begin{pmatrix} -1/3 \\ 4/3 \end{pmatrix}}{4/3} = \begin{pmatrix} -1/4 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1/4 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} y_1 + y_2 = -1/4 \\ 4y_1 + y_2 = 1 \end{cases} \Rightarrow \begin{matrix} y_2 = 1 - 4y_1 \\ y_1 + 1 - 4y_1 = -1/4 \\ -3y_1 = -5/4 \end{matrix}$$

$$y_1 = 5/12 \Rightarrow y_2 = 1 - 4\frac{5}{12} = 1 - \frac{5}{3} = -2/3$$

$$x^2 = \frac{\begin{pmatrix} 5/12 \\ -2/3 \end{pmatrix}}{2/3} = \begin{pmatrix} 5/8 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 5/8 \\ -1 \end{pmatrix} \Rightarrow \begin{cases} y_1 + y_2 = 5/8 \\ 4y_1 + y_2 - 1 \end{cases} \Rightarrow \begin{matrix} y_2 = -1 - 4y_1 \\ y_1 - 1 - 4y_1 = 5/8 \\ -3y_1 = 13/8 \end{matrix}$$

$$y_1 = -13/24 \Rightarrow y_2 = -1 + 13/6 = 7/6$$

$$x^3 = \frac{\begin{pmatrix} -13/24 \\ 7/6 \end{pmatrix}}{7/6} = \begin{pmatrix} -13/28 \\ 1 \end{pmatrix}$$

$$\mu^3 = \frac{\begin{pmatrix} -13/28 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} -13/28 \\ 1 \end{pmatrix}}{} = \frac{\begin{pmatrix} -\frac{13}{28} + 4 & -\frac{13}{28} + 1 \end{pmatrix}\begin{pmatrix} -13/28 \\ 1 \end{pmatrix}}{} =$$