Statement

Solution



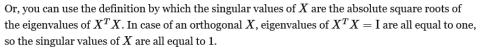
SVD of a matrix A is $A=U\Sigma V^T$, where U and V are orthogonal and Σ is nonnegative real diagonal.





Now, let X be orthogonal. Note that $X = U\Sigma V^T$, where U := X is orthogonal, $\Sigma := I$ is diagonal, and V := I is orthogonal. So, singular values are all equal to 1.







As for your second question, I don't think the statement is true. Let

$$X = \mathbf{I} = \begin{bmatrix} 1 \\ & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} & 1 \\ 1 & \end{bmatrix}.$$

If the set of the orthogonal matrix is convex, then $Z:=\frac{1}{2}(X+Y)$ is also orthogonal. But,

$$Z=rac{1}{2}(X+Y)=rac{1}{2}egin{bmatrix}1&1\1&1\end{bmatrix}$$

is singular, so it cannot be orthogonal. We can even check directly: $Z^TZ=Z
eq {
m I}$.

You can find a topic on the convex hull of the set of orthogonal matrices here.

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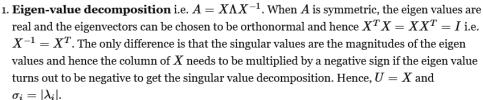
The SVD for a symmetric matrix $A=U\Sigma V^T$, where U and V are unitary matrices with $U=[u_1|u_2|\dots|u_n], V=[v_1|v_2|\dots|v_n]$ and Σ is a diagonal matrix with non-negative diagonal entries and $v_i=\pm u_i$



32

For a symmetric matrix the following decompositions are equivalent to SVD. (Well, almost equivalent if you do not worry about the signs of the vectors).







- 2. **Orthogonal decomposition** i.e. $A = PDP^T$, where P is a unitary matrix and D is a diagonal matrix. This exists only when matrix A is symmetric and is the same as eigen value decomposition.
- 3. **Schur decomposition** i.e. $A = QSQ^T$, where Q is a unitary matrix and S is an upper triangular matrix. This can be done for any matrix. When A is symmetric, then S is a diagonal matrix and again is the same as the eigen value decomposition and orthogonal decomposition.

I do not remember the cost for each of these operations i.e. I don't remember the coefficients before the leading order n^3 term. If my memory is right, the typical algorithm for orthogonal decomposition is slightly cheaper than the other two, though I cannot guarantee.

Reference