# 1 Summary

This discussion paper is in response to, “Measurement of Impulsive Thrust from a Closed Radio-Frequency Cavity in Vacuum” by authors Harold White, Paul March, James Lawrence, Jerry Vera, Andre Sylvester, David Brad, Paul Bailey and can be found here http://arc.aiaa.org/doi/10.2514/1.B36120.

After attempts to contact the primary author to discuss and obtain their data failed, an analysis of their methods was carried out using digitized data from their report and following their outlined methods. The sampled results produced a fairly accurate replication of their calculations and was then used as a basis to explore their models.

It was found that Eagleworks data did not fit their presented model well, so new models were explored in this paper. The final model chosen included two transients and an additional heating profile during power on. This new model fit the data's characteristics better and reproduced their results. As a result, the conclusion is their reported force was likely a thermal effect. However, since no thermal couples were used in the experiment, it is impossible to directly correlate the thermal changes with the displacement changes.

In addition a number of error terms were identified that were ignored in the report that in the worst case can total up to approximately 38-40 uN not including noise sources that were unquantified.

A large number of technical issues were found problematic in the paper and are summarized in a marked copy with comments that can be found on git hub along with the code supporting this paper: https://github.com/eric1600/eagleworks/

# 2 Simulating Eagleworks Numerical Models

Model verification was done using the data presented in “Figure 8” and the text describing their calculations. This code can be found at [https://github.com/eric1600/eagleworks/](https://github.com/eric1600/eagleworks/) and the following section was computed using “test4.py”. Figure A shows the recreated data for comparision.

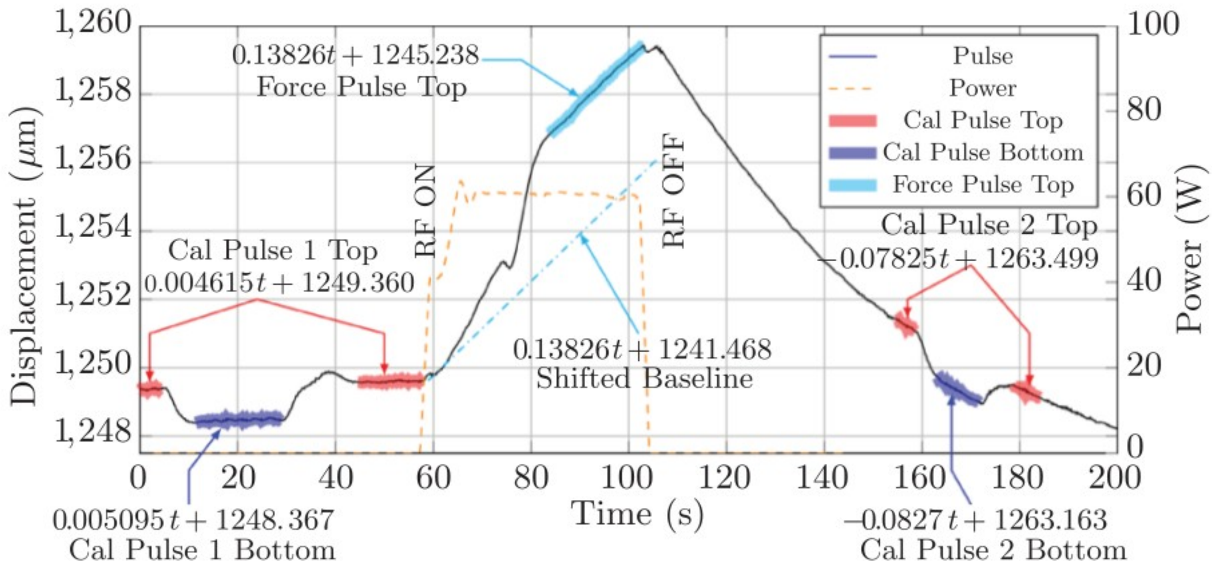


**Fig. 8 Force measurement procedure plot: the figure shows one of the 60 W forward thrust runs with the data annotated to indicate the sections used to determine the calibration pulse characteristics and the force pulse characteristics (Cal, calibration).**

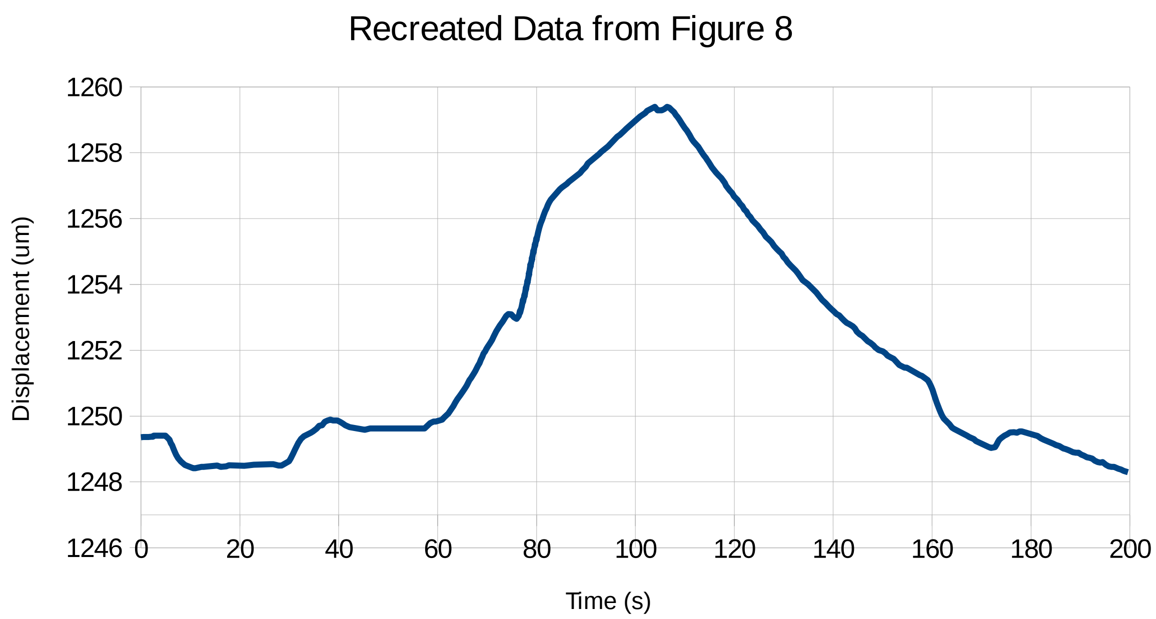


*Figure A: **Digitally extracted from Fig.8 graph and is shown recreated here.***

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Using the recreated data waveform shown in Figure A, the same time windows and methods were used to compare the code to the paper's results.

| Area | EW Slope | Calculated Slope | Slope Error | EW Intercept | Calculated Intercept | Intercept Error |
|---|---|---|---|---|---|---|
| Pulse 1 Top | 0.004615 | 0.0048089 | -0.000194 | 1249.360 | 1249.37297 | -0.01297 |
| Pulse 1 Bot | 0.005096 | 0.00472355 | 0.00037245 | 1248.367 | 1248.40259 | -0.03559 |
| Pulse 2 Top | -0.07825 | -0.0787747 | 0.0005247 | 1263.499 | 1263.65444 | -0.155439 |
| Pulse 2 Bot | -0.0827 | -0.08323702 | 0.000537 | 1263.163 | 1263.3130 | -0.150011 |
| Pulse Force | 0.13826 | 0.1376234 | 0.0006366 | 1245.238 | 1245.20805 | 0.0299464 |
| Shifted Eq. | | | | 1241.468 | 1241.53001 | -0.062001 |

***Table 1: Comparison of Eagleworks (EW) and Simulated Code Accuracy (Calculated)***

Table 1 shows that most calculations are are very low in the difference of values. The largest errors are around 0.16 um or ( ~ 4.5 uN using the calculated dx/df of 0.035495 computed using EW's method).

The final results show the theoretical impulse force calculated is:

- 3.678044 um or 103.6204 uN force and dx/df = 0.0354953532734
- This is a computational error difference of ~2.38 uN
- Accuracy would be improved only by having the actual EW data to work from.

Figure B shows the linear approximations plotted onto areas of the curves which can be compared to Figure 8.

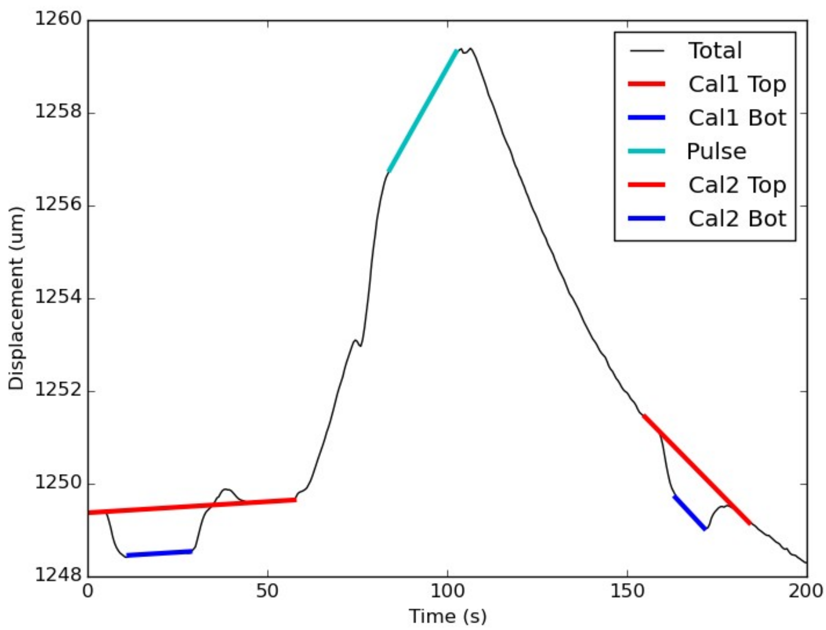


***Figure B: The curve fitted regions from test4 and shown in Table 1***

Note that the curve fits for the tops of the calibration pulses were computed exactly as done in the Eagleworks paper. The red lines for the tops of the calibration pulses plotted on Figure B are not shown piece-wise like the calculation, however in spite of the appearance on the plot, their method was duplicated exactly with gaps in the data during the bottom portion of the calibration pulse.

## 2.1 Experiments with test4.py using Eagleworks digitized data

### 2.1.1 Trial 1 – Remove Force in data using reciprocal pulse

Once the relative accuracy of the simulation was verified it is now possible to introduce signals to test the response of the numerical method. The first test is to try to remove the 106 uN force from the thermal data using their method and their impulse signal model shown in Figure 5 below:

Using a displacement of -3.7625 um or approximately 106uN produces the computed result of -0.07836 um or -2.208 uN force. This is very close to 0uN which means that we were able to remove the force signal using the superposition of the impulse model shown in dashed red in Figure 5.

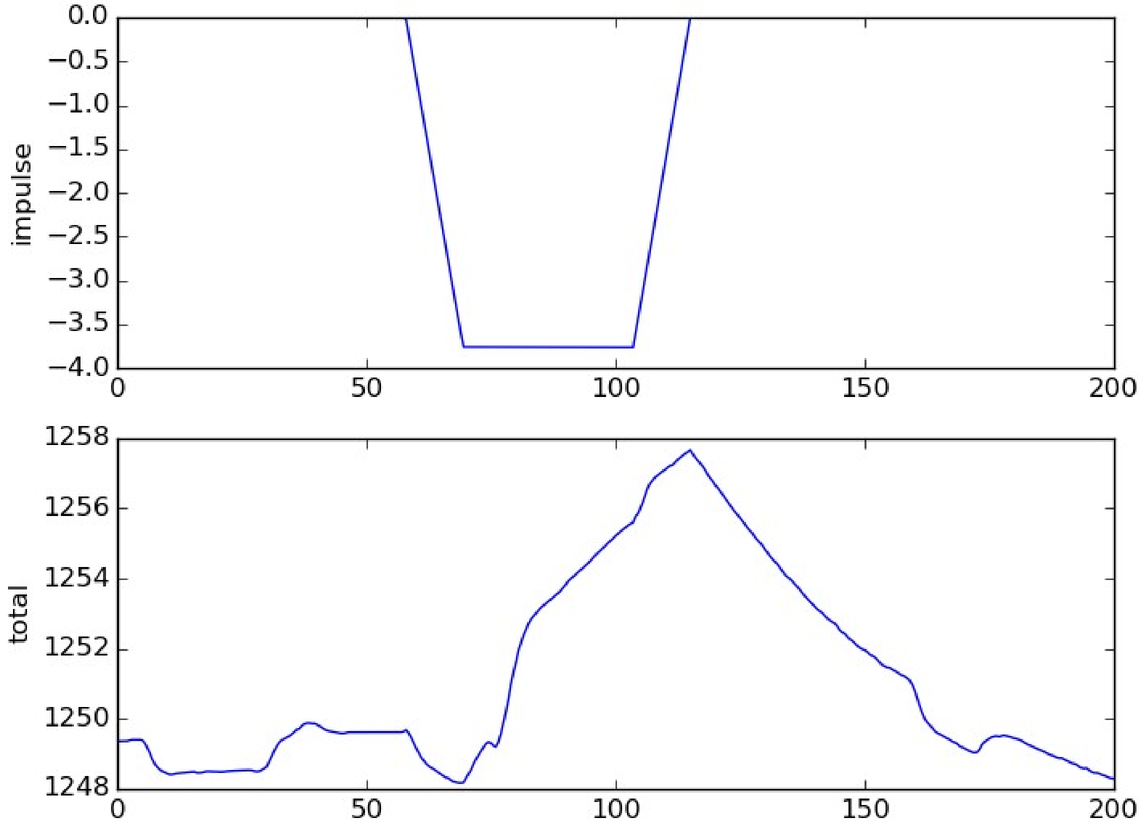


***Figure C: Negative Impulse waveform (top) was used to try to remove the signal from the data which is shown with the pulse subtracted (bottom)***

### 2.1.2 Trial 2 – Add force in to see if it adds correctly

In this test the force was set to 103.6204 uN or (3.7625 um) which should produce 103.6204+103.6204= 207.2408 uN

The simulated results were 7.4344 um or 209.4484 uN force for an error difference of only 2.2 uN assuming they should add linearly as proposed by Eagleworks.

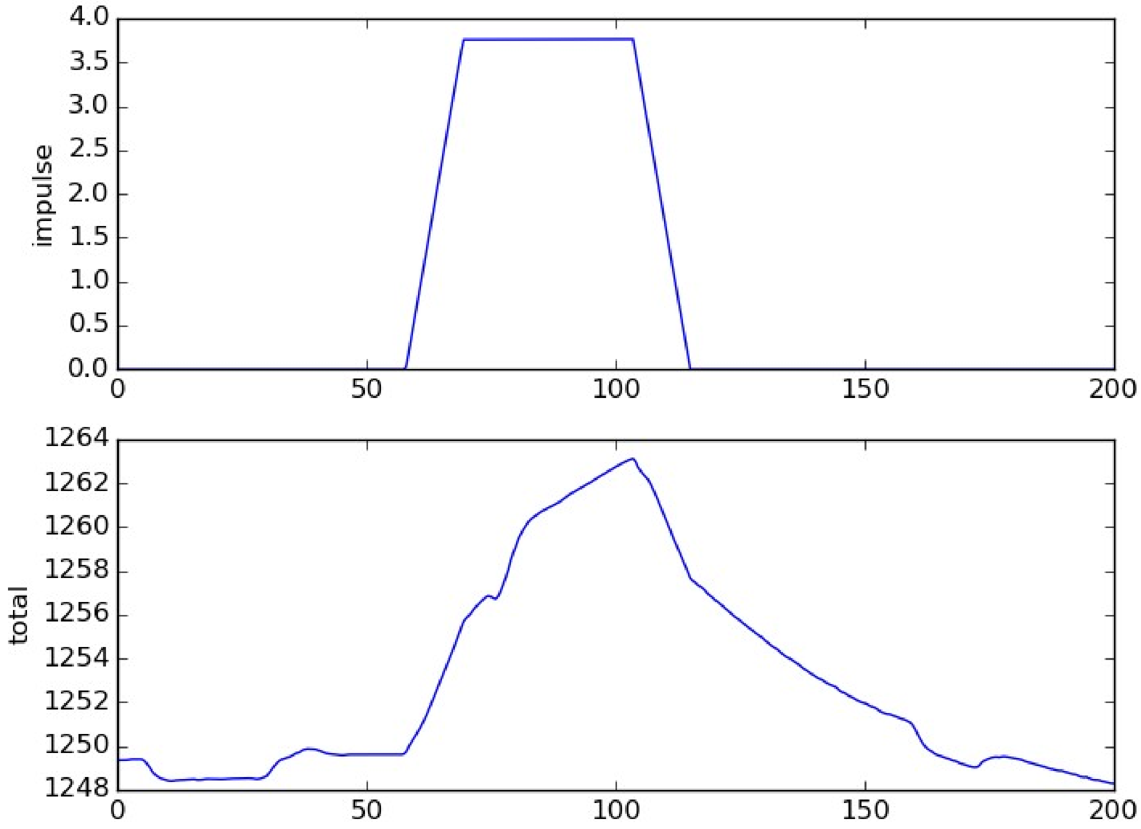


***Figure D: Inserting an additional 106 uN pulse into the measured data set***

## 2.2 Trial 1 & 2 Issues

Even though both key trial runs worked well, the resulting graphs do not look quite right. The thermal curve is dropping before the RF power or impulse is off. The reason for this is the width of the pulse is not well documented in the Eagleworks paper and is modeled based on their presentation shown in Figure 5 below.

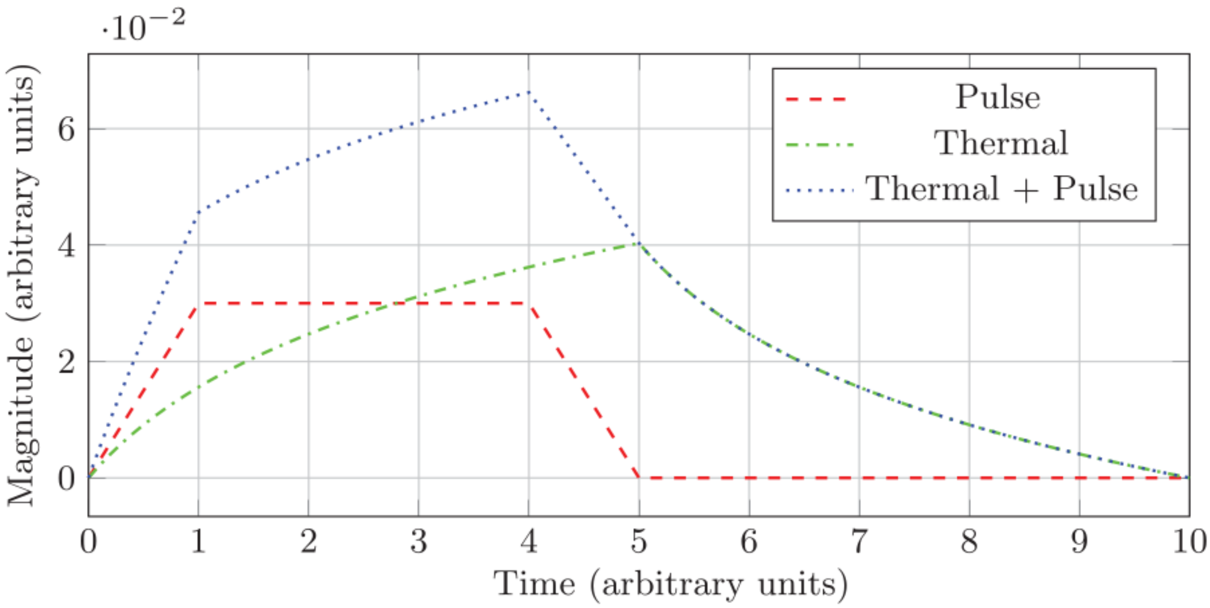


**Fig. 5 Superposition of signals: conceptual superposition of an impulsive thrust (red) and thermal drift (green) signal over an on/off power cycle on the torsion pendulum.**

- Do these signals slopes fit the measured data?
- If they do fit the measured data are they predictive of the force value if either the thermal or the force is modified?

## 2.3 Experiments with test5.py using Eagleworks Model and Data

Using Figure 5 from above, the curves were digitized and extracted as per the arbitrary time scale described. The raw data for this can be found in EW-data.ods in the code repository in the worksheet tab marked “signals”.

Using the models built in test5.py the following recreation of Figure 5 was done using the same time frames and scales to prove the models were accurate.

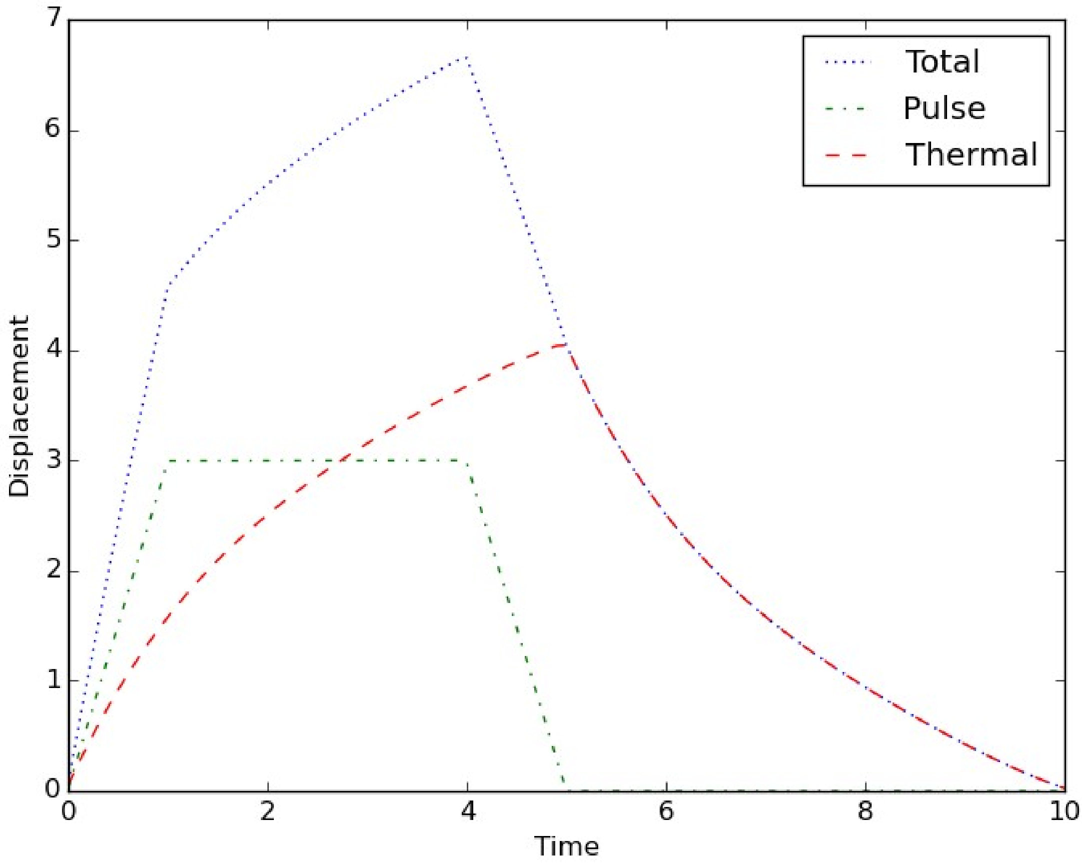


***Figure E: Recreated Numerical model built from Fig. 5 of Eagelworks paper***

Now that the signals correspond to the numerical model that is presented in order to test them against the data directly. Initial attempts to duplicate their experiment profiles using their model failed to find a good fit.

Manually adjusting the parameters the best fit was found visually

| **Force** | **Therm Peak** | **Pulse Rise** | **Pulse fall** | **Thermal start** | **Thermal center** | **Pearson** |
|---|---|---|---|---|---|---|
| 3.7625 | 7.75 | 66 | 110 | 65 | 118 | 0.97883 |

***Table 2: Manual Parameter Selection for Eagleworks Model***

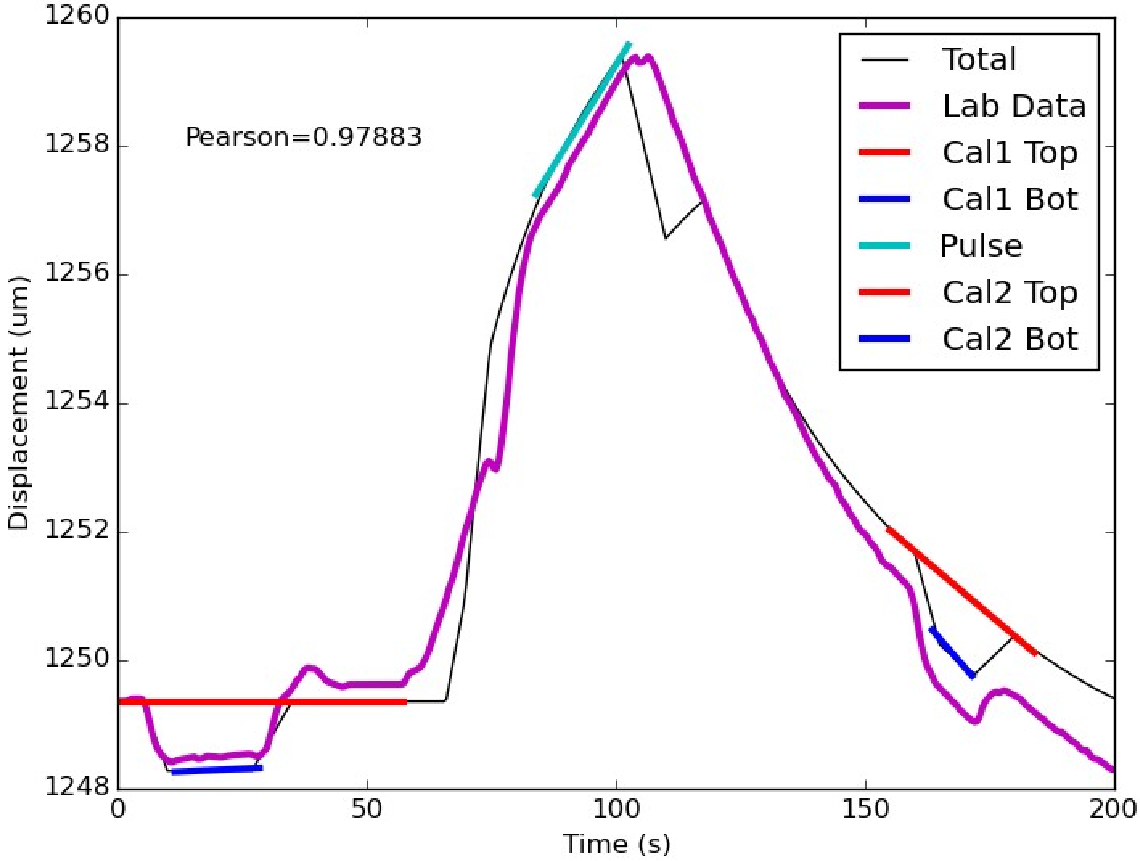


***Figure F: Adjusting Parameters Manually to Fit Eagleworks Model to Data***

As the above figure shows there are a number of error regions in this model. It also produced a large error in calculated force compared to Eagleworks measurements. The computed force using this model was 4.758 um or 128.770 uN (vs. 106 uN).

Since adjusting the parameters by hand is time consuming and non-optimal test5.py was modified to loop through a variety of settings and search for a better overall fit using the Pearson Coefficient.

## 2.3.1 Fit 1 Pearson Coefficient

There are a 6 basic variables to change in order to fit these theoretical curves to the measured data.

- Thermal peak
- Thermal rise start time
- Thermal fall time
- Pulse peak
- Pulse rise start time
- Pulse fall start time

As a measurement of how well these parameters are adjusted to fit the measured data a parameter known as Pearson's Coefficient. Pearson’s correlation coefficient is a statistical measure of the strength of a linear relationship between paired data. The coefficient ranges between 1 and -1 and the follow charts show how they relate to the data fit.

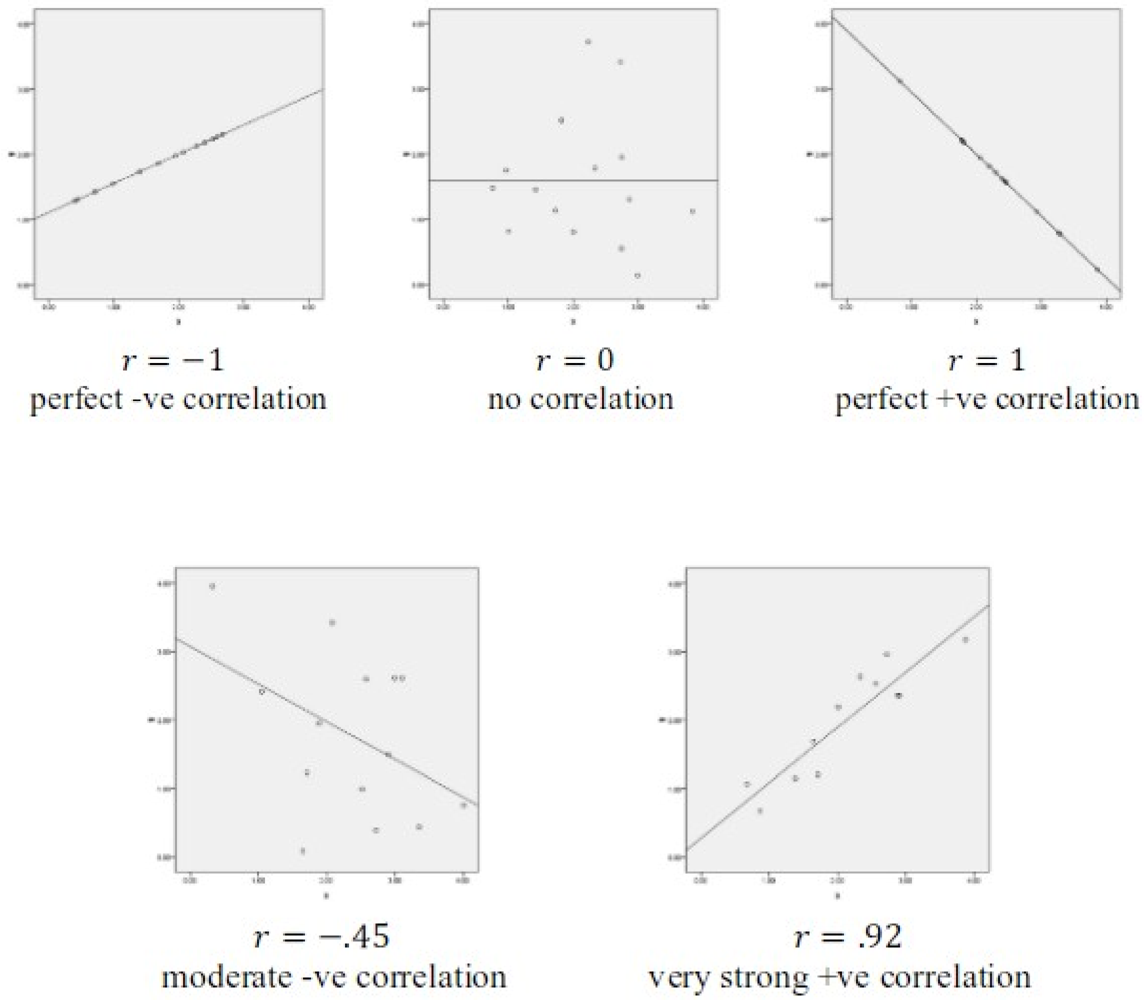


***Figure G: Comparison Examples of R for Pearson Correlation Coefficient values***

Using the Pearson correlation is a rough way to estimate how closely the two curves, the model and the measured data, correlate with each other.

Keeping the force pulse at 106uN, all other 5 variables were swept over about 3 million iterations to search for the best overall data fit.

| **Force** | **Therm Peak** | **Pulse Rise** | **Pulse fall** | **Thermal start** | **Thermal center** | **Pearson** |
|---|---|---|---|---|---|---|
| 3.7625 | 9.0 | 59 | 119.5 | 47 | 118 | 0.990875 |

***Table 3: Results of Pearson Optimization***

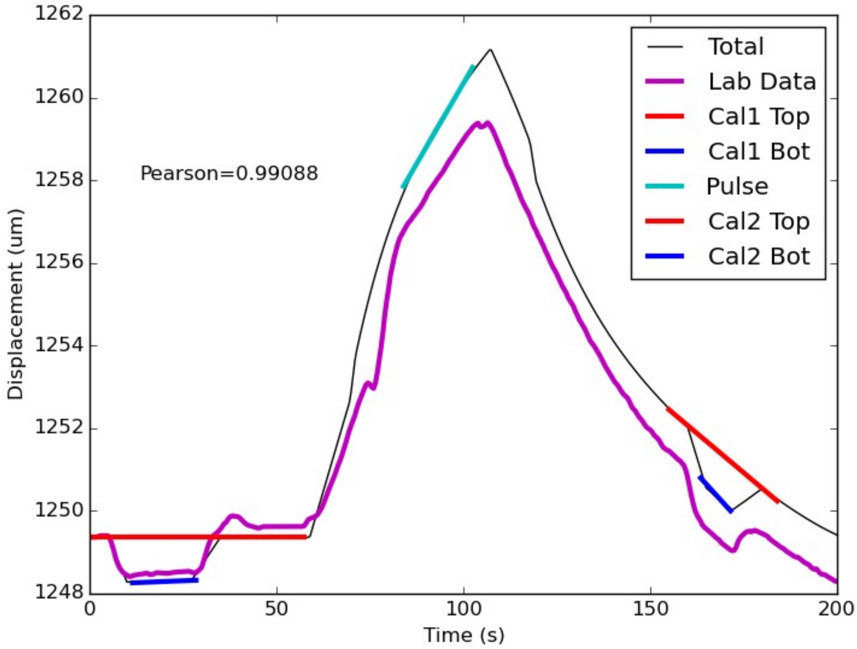


***Figure H: Pearson Curve Fit Contains Offset Errors***

The resulting curve fit shows the model doesn't fit the data very well. While the coefficient shows the trend lines are correctly tracking the data, there is a offset error due to the shape of the trailing curve where it tried to reduce the offset error the most.

This curve fit is poor in the areas where the calculations are made, so it is not usable for a model.

The results of the force calculation contained a large error. The expected result according to Eagleworks is 106 uN and what was computed using the Pearson Curve fit of their model was 4.61219 um or 124.29166 uN.

As in interesting side note, the error in the curve fits appear quite large, even more so than the previous curve fit, however the nature of the segmented calculations of force ignore many of these differences. Where in Figure H the force would expected to produce a larger error, it is 124 uN and is slightly closer to the 106 uN than in Figure F which was 129 uN.

## 2.3.2 Fit 2 Modified Measured Data with Zero Force Pulse

Another way to look at this problem is to take Eagleworks measured data and extract the force pulse. What should be left is strictly the thermal pulse. In fact using their force calculation method on the resulting curve produces -3.7625 uN as was shown in Trial 1 – Remove Force in data using reciprocal pulse.

The following figure compares the theoretical model with this Modified Measured Data (MMD) of zero force.

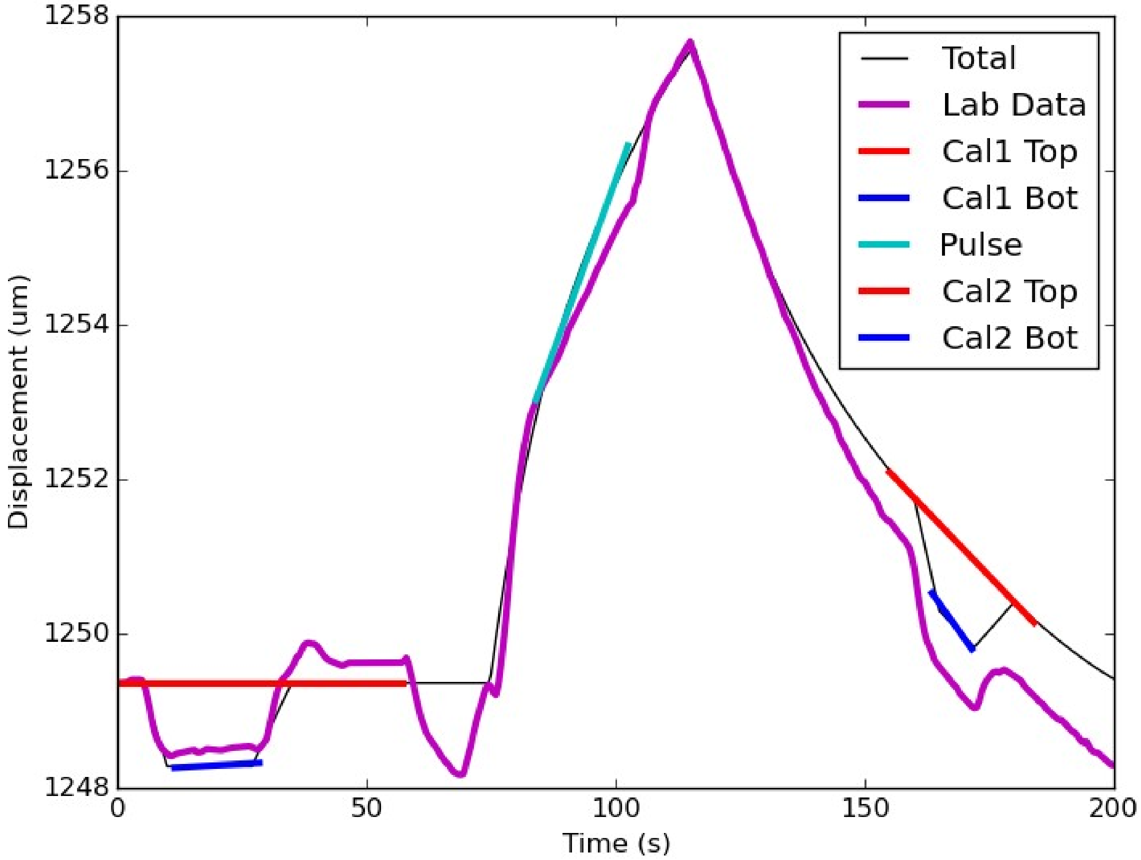


***Figure I: Modified Measured Data ("Lab Data") to Remove 106 uN Pulse Compared to Theoretical Model ("Total")***

In the case of Fit 2, we would expect the resulting force pulse to be zero, because it was removed from the model. However using the same techniques from Trial 1 which produced only -3.7625 uN, Fit 2 produces -0.77819 um or -21.0751 uN of force. This is 21 uN of force in the wrong direction.

The removal of the pulse also has some critical issues as seen in Figure I. Those negative going pulses during the on (60-70 seconds) and off (~110s) periods don't show up in any of the Eagleworks data. This is also an indicator that their method or assumptions of superposition of signals may not be correct. Their method for computation conveniently avoids the biggest of these transition regions (note the red, blue and teal lines where the data is extracted for computing the force).

Obviously there is still a problem with their model matching the cooling curve.

### 2.3.3 Fit 3 Remove All Pulse Signals From Data and Smooth Artifacts

Test7.py uses the following thermal curve extracted from EW data with the impulse force removed and the curve smoothed to remove artifacts. You can see how the model compares to Eagleworks Lab Data in the following figure.

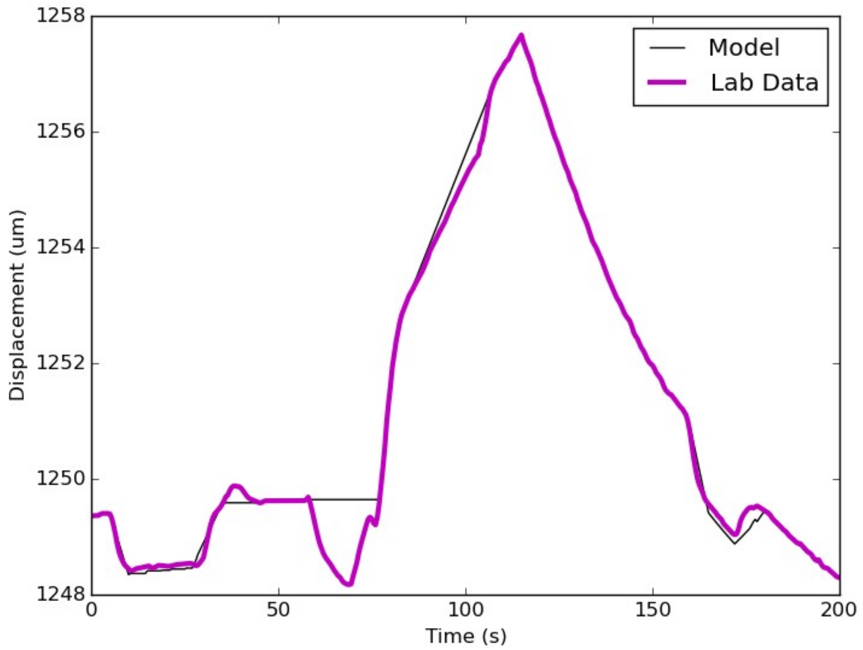


***Figure J: Smoothed Model compared to Lab Data (with Force Pulse Removed)***

Compare this to the figure of the thermal model plotted against the actual raw lab data in the following figure.

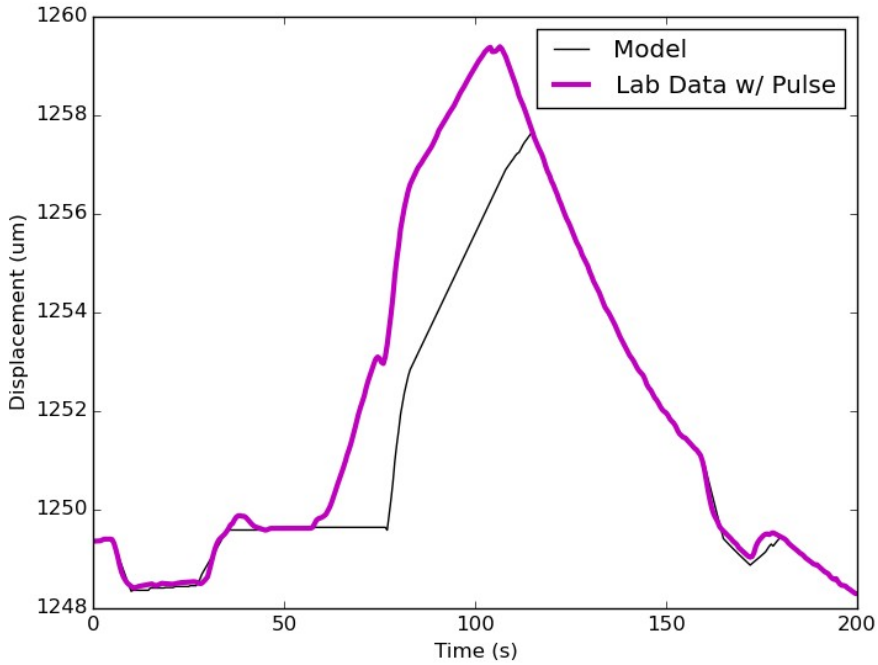


***Figure K: Thermal Model compared to Measured Lab Data***

### 2.3.4 Critical Problems With Eagleworks Thermal Model

Two key problems become apparent when looking at this thermal model that are unrealistic.

- It doesn't start heating until ~17 seconds after it is powered on
- It doesn't start cooling until ~15 seconds after it is powered down

These are unphysical assumptions that do not justify using this model. *The only way this model could be acceptable is if it was measured independently with other thermocouples which demonstrate a strong correlation between the thermal heating and the detected motion due to that heating.*

## 2.4 Testing Fit 3 Thermal Model against Lab Data

Fit 3 based on a smoothed model of the measured data with a subtracted force pulse is now tested against re-introducing the force pulse of 106 uN and comparing it to the Eagleworks data. The goal of this test is to determine how well the assumptions and computation methods work.

Now the model fits the data as theorized the investigation into how well the computational method works because it depends on the following compounding factors:

- Calibration Pulse variations
- Removing a theoretical EM drive force pulse from thermal noise
- Computation of the force using calibration pulse and selected slope areas and intercepts

## 2.5 Testing Thermal Model with 106 uN of force

Using Fit 3 for the thermal curve the simulation was run to try to recreate the lab data exactly using the superposition method as described in the Eagleworks paper and using all the some timing windows and computational methods. The following figure shows the signals used for each component and their composite result.

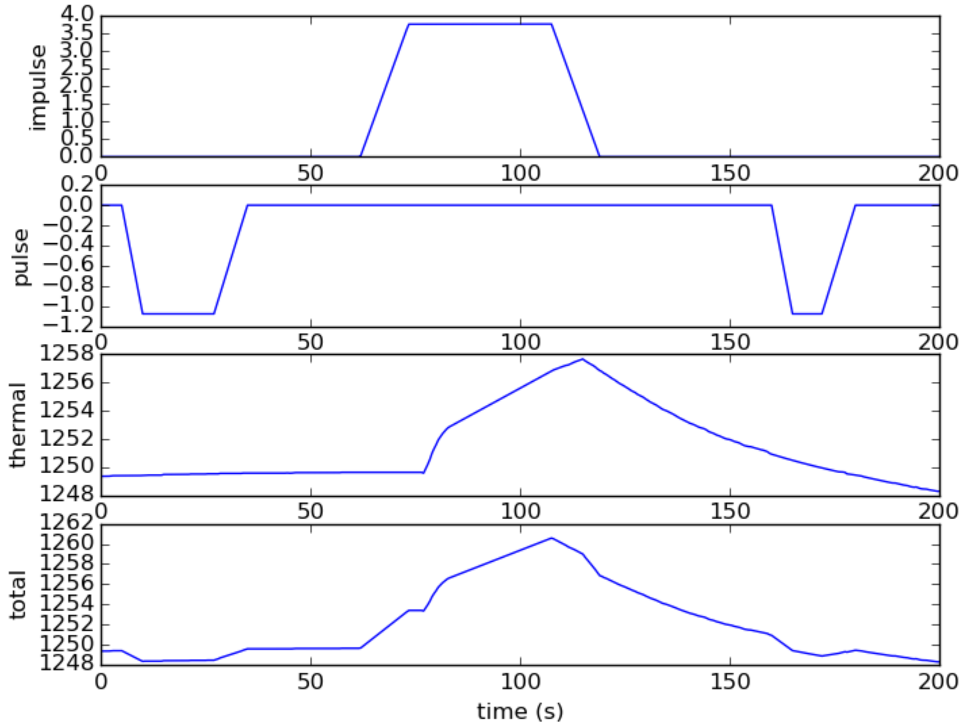


***Figure L: Using Fit 3 to Simulate 106 uN against Measured Lab Data of 106 uN***

And the following figure shows the comparison to the lab data as well as the segments used for computing the force results.

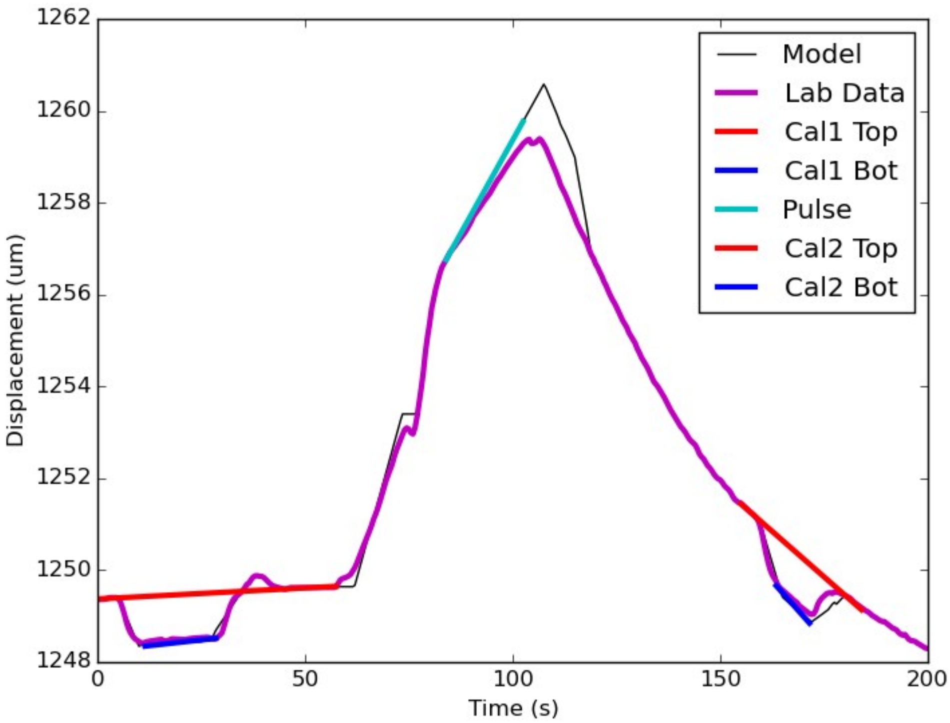


***Figure M: Fit 3 with 106uN Simulated Force Compared to Measured Data ("Lab Data")***

Two very interesting aspects stand out in this simulation. The first is the small shelf occurring at about 74.5 seconds which corresponds to the data well. And the second is the large peak difference while the EM drive impulse ramps down which isn't present in the lab data. The only way to correct this is by reshaping the assumptions about the superposition of the force model.

This model shows 3.05190887782 um and 79.7440312977 uN as the calculated force compared to 106 uN (or 103 uN as test4.py shows our model should achieve).

Part of this difference is due to the results from the calibration pulse calculations. Eagleworks model produces dx/df = 0.0354953532734 whereas the model yields dx/df = 0.0382713142057.

If the calibration pulse from the lab data is used the calculated pulse data is about 85.9805 uN. This is about 6uN of error observed in calibration due to drift (see Error Section 20).

### 2.5.1 Summary of Fit 3 Test

- Error for computing the theoretical force is between 20 uN and 27 uN
- Large errors are introduced in curve fitting because of incorrect superposition signal shape assumptions.
- Model is completely noise free for direct comparison
- Based on this the proposed model from Eagleworks does not correctly fit the observed data
- It appears the assumed EM Drive impulse force should taper off well before the RF amplifier is turned off.

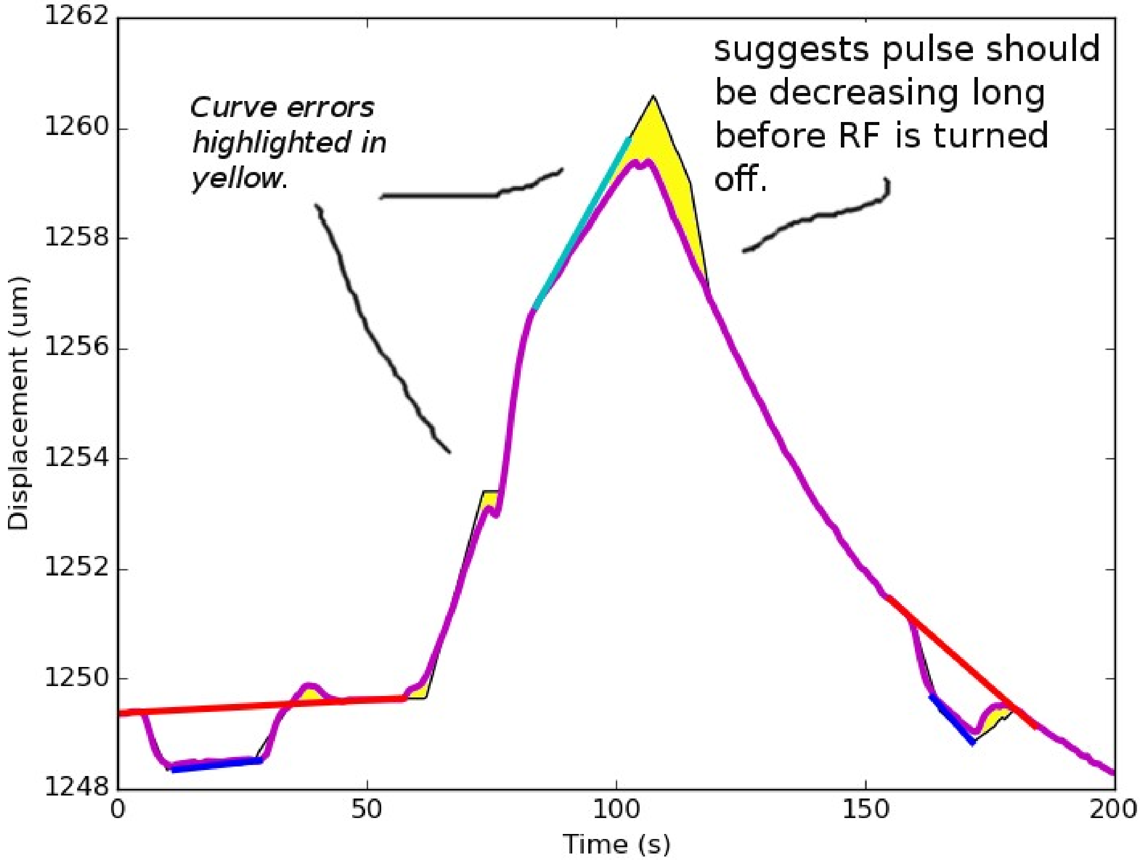


*Figure N: Errors Using a Pulse Type Force for Model Against Thermal Curve*

## 2.6 Fit 4 – Changing the Force Pulse Model

A more likely scenario is that when the power amplifier is first turned on there is a small transient. In addition rapid heating is likely to occur then taper off, followed by a small transient again. This rapid heating would add directly to the beginning of the test and appear like additional displacement.

The motivation for this type of signal is based on several observations of the data presented by Eagleworks. There are 2 transients evident in the data using their “Figure 7”.

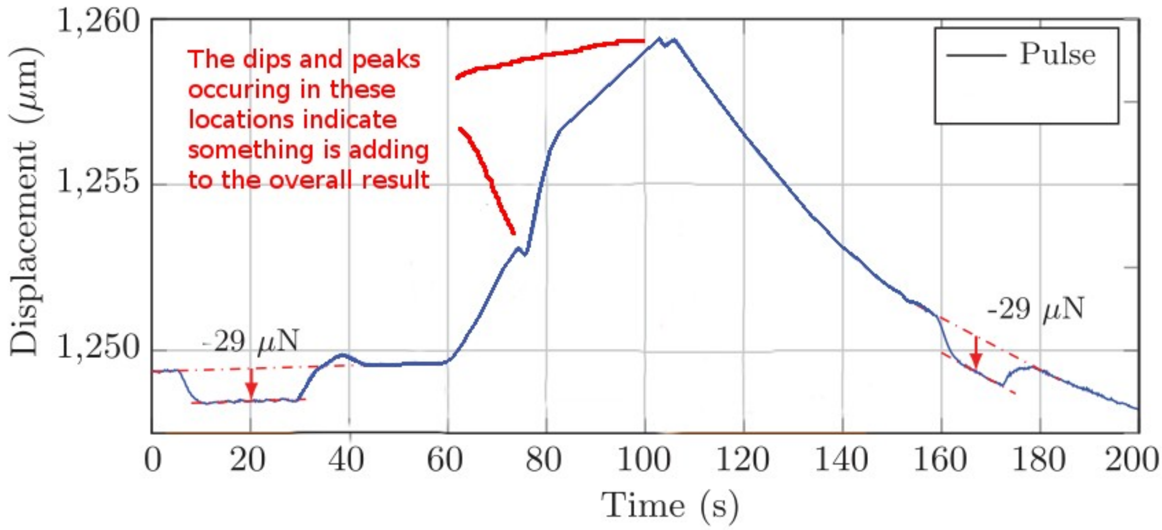


***Figure O: Transients in the Data***

Those transients and the change in slope between them indicate several things are changing during the application of the RF power. This could be more closely modeled using approximately 3 pulses with decaying contributions.

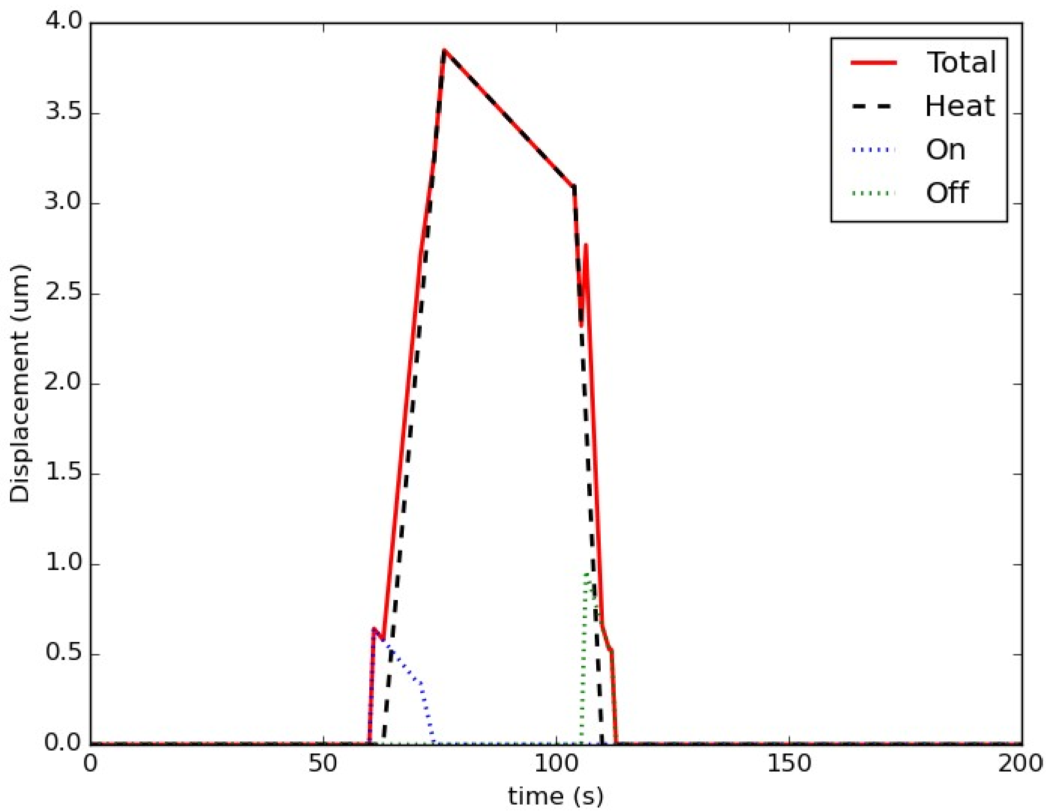


***Figure P: Fit 4 – New "Pulse Model"***

The beginning shows energy contributed by a transient labeled "On" followed by some rapid thermal heating labeled "Heat" and then a transient caused by shutting down the RF labeled "Off".

In the real system rapid changes in heat would be smoothed out due to the dampened response of the torsion balance system. However in the mathematical model, these edges are not filtered and will appear sharper than they would in the real system.

However this model fits the data quite well as seen in the composite plotted against the “Lab Data”.

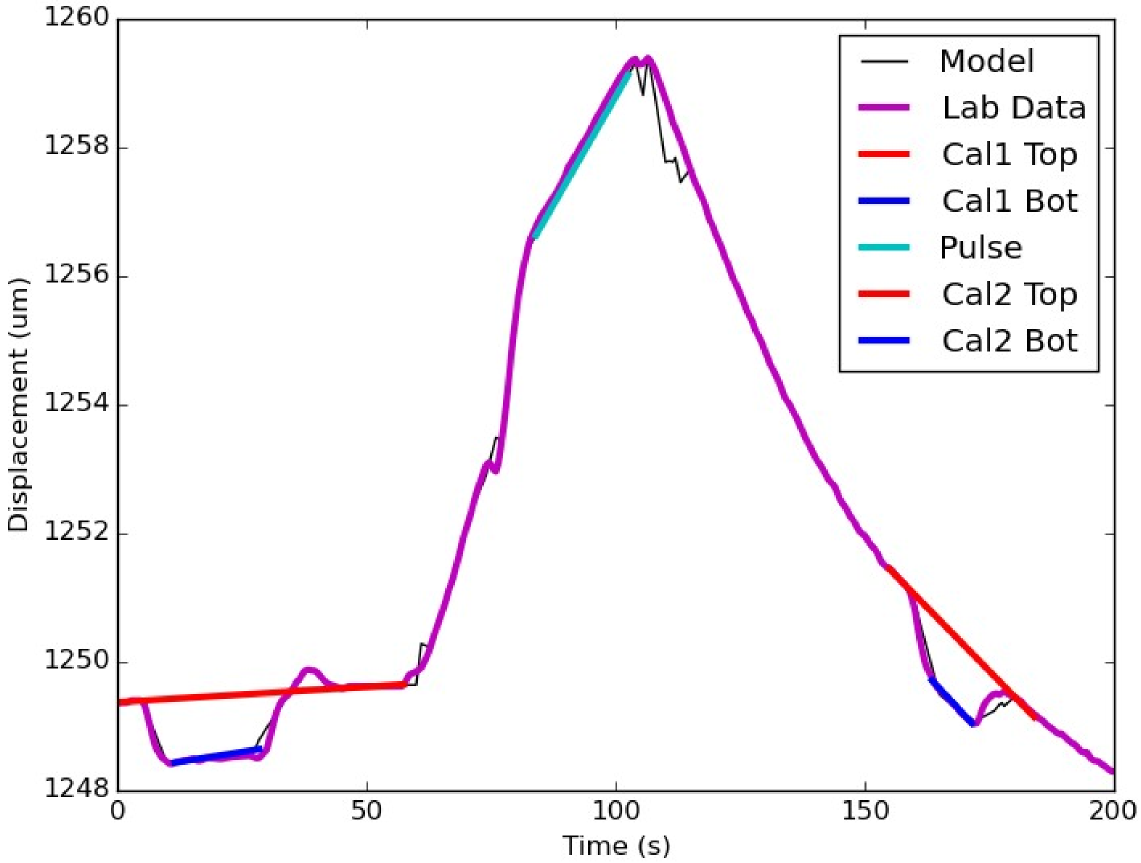


***Figure Q: Fit 4 Model Plotted Against the Lab Data***

It is important to remember that the spikes will appear more rounded because the measurement setup has a slower response time. Notice this model fits the two key irregularities well. The only problem is during the first part of the cooling. The model cools a little faster at first, however the fit is pretty accurate.

Critical in this model is to note the slope of the “Heat” curve because it is reducing over time. Often when an amplifier is heated the thermal change is extremely rapid and then slows as the surrounding materials begin to saturate with heat. Since Eagleworks failed to include any thermocouple data, it is difficult to determine exactly how to correlate the heat to the displacement, however it is more probable to find a rapid heat rise on power on which slowly reduces as the system approaches a steady state. The new model represents those physically known attributes rather than a novel new force.

Using this new model for heat, we can then preform the same calculations Eagelworks did and compare them.

| Area | EW Slope | Calculated Slope | Slope Error | EW Intercept | Calculated Intercept | Intercept Error |
|---|---|---|---|---|---|---|
| Pulse 1 Top | 0.004615 | 0.004809 | -0.00019 | 1249.360 | 1249.37297 | -0.01297 |
| Pulse 1 Bot | 0.005096 | 0.0123418 | -0.0072458 | 1248.367 | 1248.29170 | 0.075305 |
| Pulse 2 Top | -0.07825 | -0.079239 | 0.000989 | 1263.499 | 1263.72510 | -0.226102 |
| Pulse 2 Bot | -0.0827 | -0.0833204 | 0.000620 | 1263.163 | 1263.33656 | -0.173561 |
| Pulse Force | 0.13826 | 0.1347642 | 0.0034958 | 1245.238 | 1245.31099 | -0.072989 |
| Shifted Eq. | | | | 1241.468 | 1241.69885 | -0.23085 |

***Table 4: Summary of Fit 4 Error Terms Compared to Eagleworks Data***

The final "force calculation" computes 3.612136 um or 104.78709 uN force which is close to the 106 uN which is only a difference of 1.21291 uN. The following figure shows the computations and fit against Eagleworks Lab data.

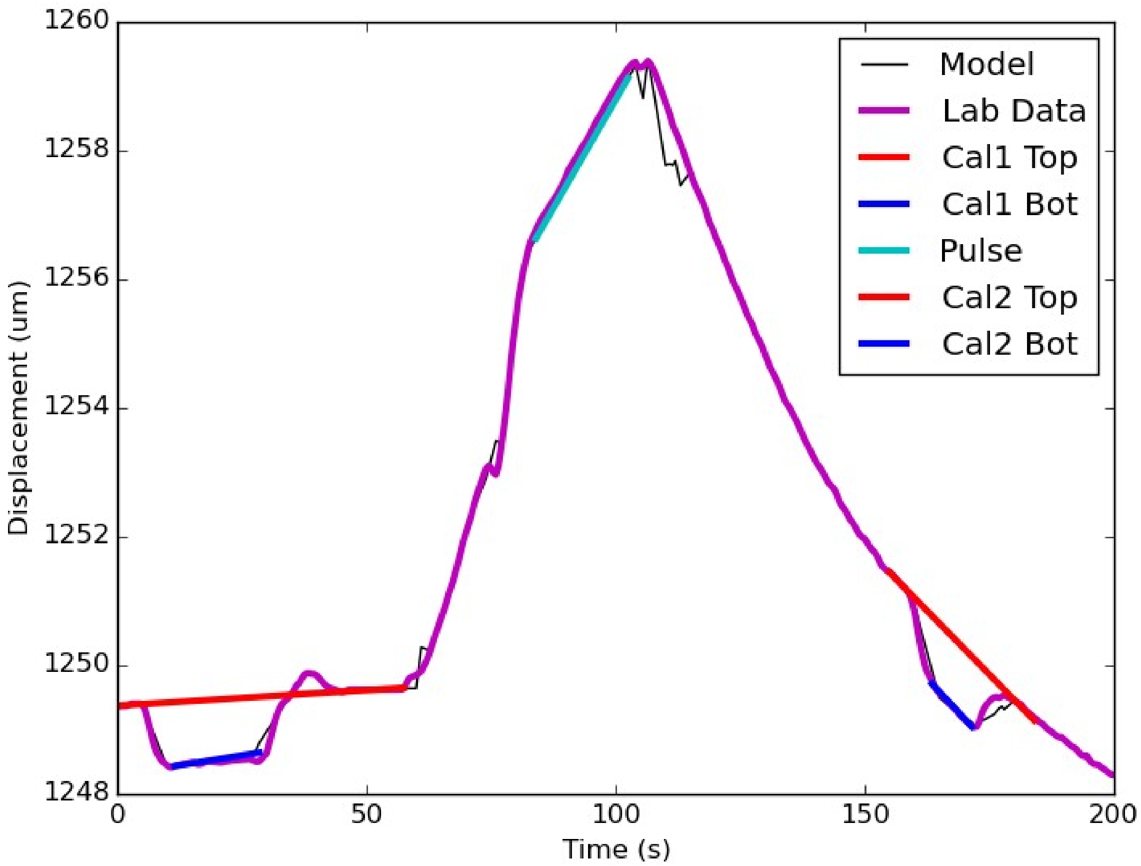


***Figure R: Calculation of Fit 4 ("Model") Against Eagleworks Data ("Lab Data")***

## 2.6.1 Discussion of Fit 4 Model

This model fits both the transient data as well as the slope for the force calculation. However it does not have any physical characteristics that one would expect from a constant force as being assumed in the original Eagleworks model. Visually the sharp edges would be smoothed by whatever dampening is in place in the test fixture. Not enough data was provided by Eagleworks to estimate the response times from an impulse force to preform accurate filtering. However some filtering was simulated by slowing the rise and fall times of the signals.

This model provided the best fit to the Eagleworks data (106 uN), which when simulated generated 103.6204 uN of thrust verses the Fit 4 model 104.78669 uN.

The model could be further refined to fit the data better and have less of a difference in the trailing portion of the cooling curve. However due to the lack of information about the system response makes this effort speculative at best.

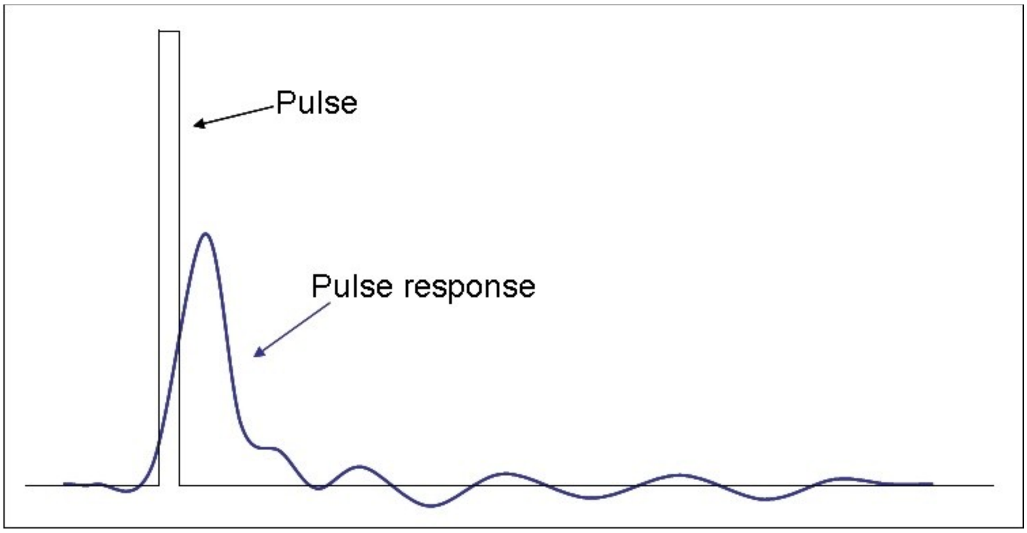


***Figure S: Example of Dampened System Responding to a Fast Transient***

Shaping the response requires a more knowledge of the test system to make the signals correspond to realistic values. Simulating this result numerically can have an infinite amount of variables if the system is not well characterized from the beginning.

# 3 Additional Error Sources

A number of unaccounted errors were found in the single 60W forward thrust test data that was presented in the paper.

## 3.1 Drift Due to Calibration Pulse

Noticeable deformation or drift in the laser displacement readings after calibration pulses. In Figure 8 specifically where all the computational numbers are available, it was found there was a 5.822 uN error.

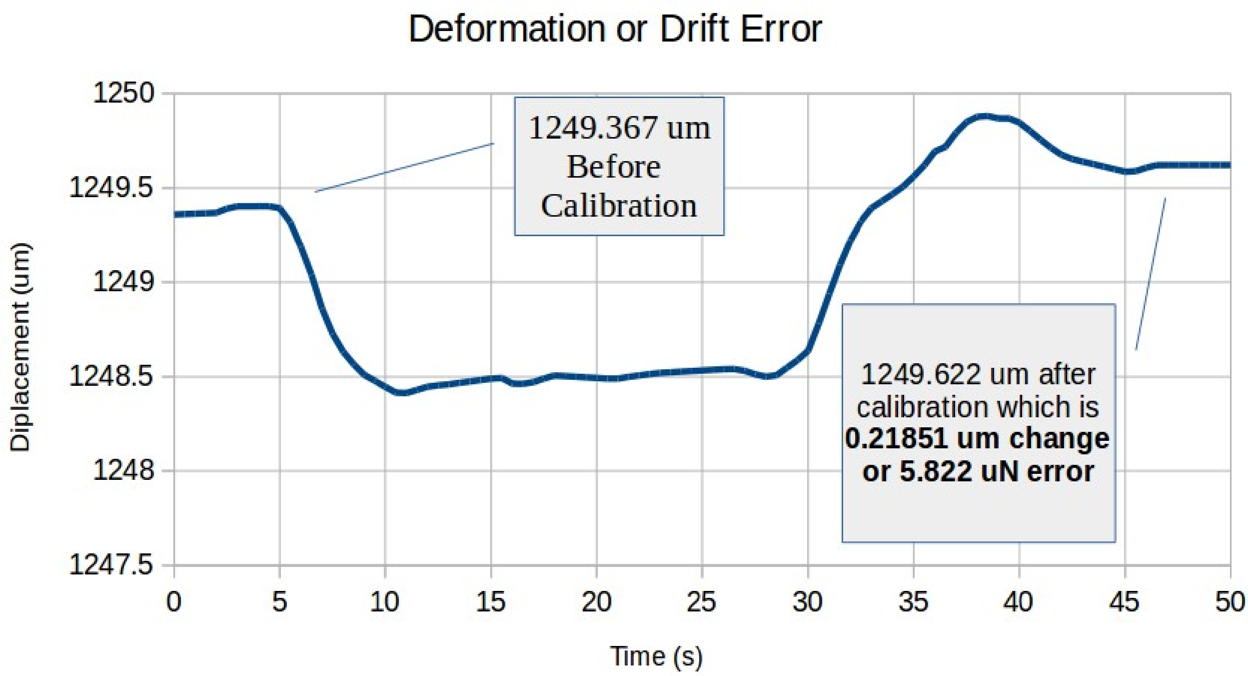


*Figure T: Displacement Drift Before and After Calibration Pulse*

## 3.2 Long Term Thermal Force Error or Deformation

An interesting problem seen with the dataset is that the during the cooling cycle the end of the test run shows less displacement than at the beginning of the test. Notice that in their Figure 8, the data starts at rest around 1248.360 and after the test is complete and has cooled, it rests at 1248.2964 which is a difference of -1.063542 um or about -29.9629 uN error. Is there an instability in the moment of inertia?

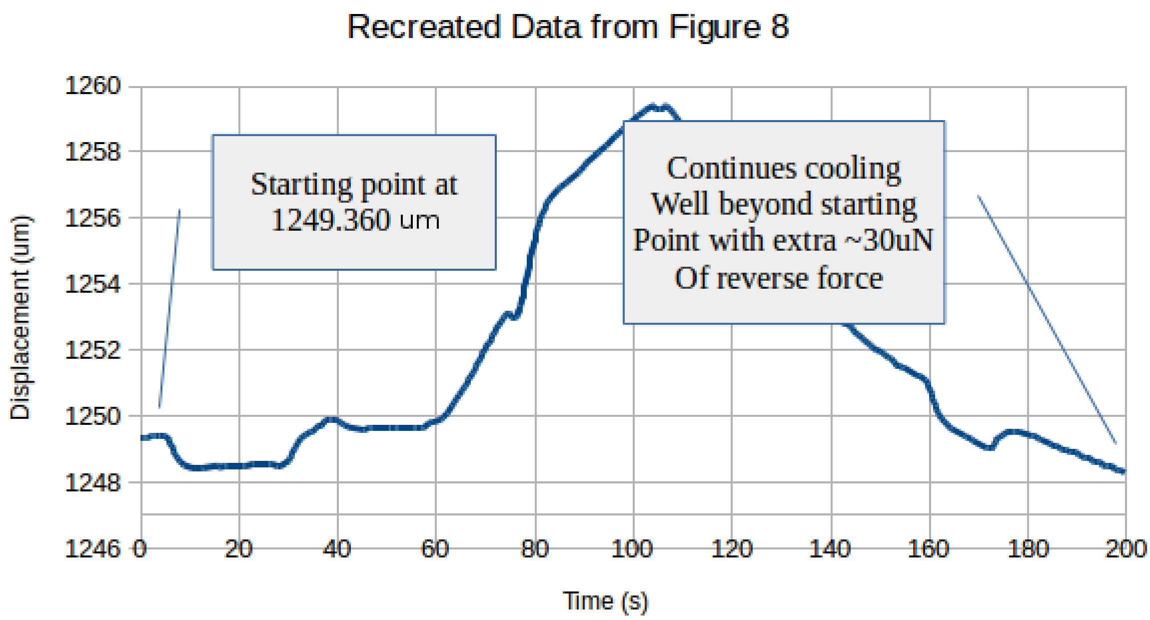


*Figure U: Test Stand Does Not Return To Neutral Position*

## 3.3 Error Due to Shifted Intercept Method

The method for computing how much the thermal noise is shifted upwards do to an impulse is described in the paper as follows:

> *The vertical axis intercept (1245.238) to this line is adjusted downward so that the line that represents the thermally* shifted baseline will roughly intersect with the optical displacement curve where the RF power is turned on. *This shifted baseline and its equation are shown on the plot: 0.13826t + 1241.468.*

This method is unclear. The goal is to find an equation that allows computing the force pulse contribution to the thermal curve. The first step is using the slope from the linear equation that fits the portion of the pulse curve (0.13826) then assuming it is shifted from the calibration pulse line by an intercept amount "b" and "x" represents the displacement:

$$x = 0.13826 * t + b \qquad \text{(um)}$$

where this equation intersects roughly with calibration pulse line

Since t and b are unknown at this point it is impossible to solve without picking a time (t). However since we know from his description that b=1241.468, t can be solved for to find t=59.052 seconds.

Using this intercept (1241.468) and the intercept value from the calibration pulse (1245.238) the force is computed by taking the difference.

Problems with this method:

- Not documented and could be applied randomly or in an uncontrolled manner when computing forces throughout Eaglework's paper.
- How was t=59.052 chosen?
- Error introduced using this "rough" method directly propagate into the result at a rate of 3.74 uN per second depending on what time is chosen as a reference for this calculation. See tables and graphs below.

| time (s) | shifted_b | force_dx (um) | Calc (um) | Calc (uN) | Error (um) | Error (uN) |
|---|---|---|---|---|---|---|
| 55.000 | 1242.068 | 3.763 | 3.140 | 88.459 | 0.623 | 17.541 |
| 56.000 | 1241.935 | 3.763 | 3.273 | 92.201 | 0.490 | 13.799 |
| 57.000 | 1241.803 | 3.763 | 3.406 | 95.943 | 0.357 | 10.057 |
| 58.000 | 1241.670 | 3.763 | 3.538 | 99.684 | 0.224 | 6.316 |
| 59.052 | 1241.530 | 3.763 | 3.678 | 103.620 | 0.084 | 2.379 |
| 60.000 | 1241.404 | 3.763 | 3.804 | 107.168 | -0.041 | -1.168 |
| 61.000 | 1241.271 | 3.763 | 3.937 | 110.909 | -0.174 | -4.910 |
| 62.000 | 1241.138 | 3.763 | 4.070 | 114.651 | -0.307 | -8.651 |
| 63.000 | 1241.006 | 3.763 | 4.202 | 118.393 | -0.440 | -12.393 |
| 64.000 | 1240.873 | 3.763 | 4.335 | 122.135 | -0.573 | -16.135 |
| 65.000 | 1240.740 | 3.763 | 4.468 | 125.876 | -0.706 | -19.877 |

***Table 5: Tabulation of Errors Due to Intercept Estimations***

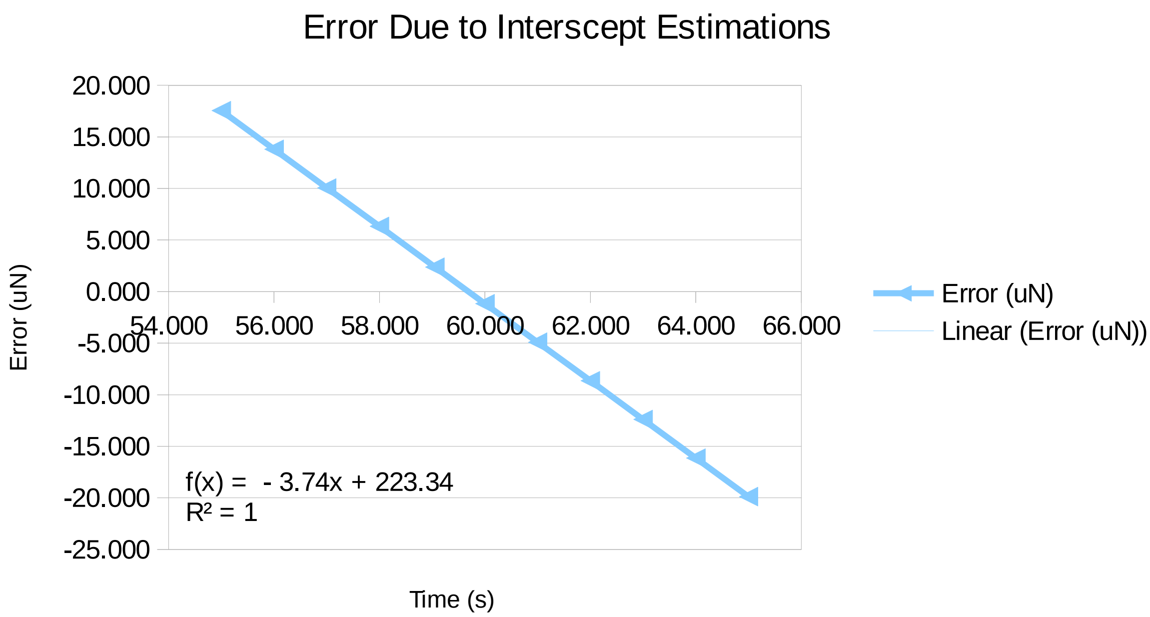


***Figure V: Errors Due to Intercept Estimations***

## 3.4 Force is Assumed Constant During Calibration

Calibration of the measured force depends entirely on there application of 200 V or 300 V pulses to the test instrument. However they do not quantify the bounds of errors on these pulses and their effective force changes during their example calculations.

From EW paper P. 4, the ration of dx/df (change in displacement per change in force applied) is computed based on their statement:

> *0.983 µm, which corresponds with the calibration pulse magnitude of 29 µN*

*which means dx/df = 0.0338965517 um/uN*

However, on P.5

> *two fitted linear equations is 1.078 µm, which corresponds with the calibration pulse magnitude of 29 µN*

*which means dx/df = 0.0371724137931 um/uN*

They reconciled these fairly dramatic differences (2.6 uN/um) without any explanation by just averaging them together. They also just state:

> *a leading calibration pulse, typically either 200 or 300 V equating to 29 or 66 µN, respectively*

Problems associated with this method are:

- The assumption that the *pulse is always 29 μN or 66 μN* and the variable they are measuring is displacement.
- Possibly every setup has s different amount of deflection due to unknown inconsistencies in their test system. However this is not discussed. How often are the different and by how much? Is every trial run different too?
- They could also be experiencing variations in induced force since the 29 or 66 uN is not evaluated anywhere.
- How stable is the moment of inertia?
- Is averaging them justified?

## 3.5 Error Propagations

A study was done to look at the effect of error propagation in the measurement method. Each test was run for 100 trials and the statistical results were examined.

### 3.5.1 Effects of Random Noise in System

First introducing random Gaussian noise of mean=0 and stdev=0.1 um (or about 2.8 uN). This produces a computed force with 100 trial runs with an average error from 106uN of 0.88 and about +11.4uN and -7.4uN for Fit 4 and +/- 9.7 uN for Lab Data. That's a 2.8 uN noise expanding to about 3.5 uN of error due to noise.

| **Parameter** | **Force (uN) Lab Data** | **Force (uN) Fit 4** |
|---|---|---|
| Stdev | 3.48870487 | 3.48849497 |
| Average | 1.45084932 | 0.886954095 |
| Min | -9.68203992 | -7.38287054 |
| Max | 9.59470394 | 11.3967604 |

***Table 6: Error Propagations in Lab Data and Fit 4 Simulations with Random Noise***

### 3.5.2 Effects of Noise Perturbation on Calibration Pulse 1

The magnitude of the calibration pulse 1 was varied by a random fixed offset containing the standard deviation of 0.1 um or ~2.8 uN. This produced a 5.13 uN error and -14.7 to -12.3 uN range.

| Parameter | Force (uN) Lab Data | Force (uN) Fit 4 |
|---|---|---|
| Stdev | N/A | 5.12887969 |
| Average | N/A | -0.17356093 |
| Min | N/A | -14.742519 |
| Max | N/A | 12.3245191 |

***Table 7: Error Propagations in Lab Data and Fit 4 Simulations with Random Noise on Cal Pulse 1 Force***

### 3.5.3 Effects of Noise Perturbation on Calibration Pulse 2

The magnitude of the calibration pulse 1 was varied by a random fixed offset containing the standard deviation of 0.1 um or ~2.8 uN. This produced a 4.6 uN error and -13.3 to -10.7 uN range.

## 3.6 Summary of Error Terms

Some terms could not be definitively estimated because the data was not recorded during the experiments. In these cases, factors of how they contribute to the final force measurements were given.

| Item | Eagleworks Quantified | Approximate Value |
|---|---|---|
| Calibration Pulse Drift | N/A | 5.822 uN |
| Long Term Test Drift | N/A | -30 uN |
| Force Calibration | 2.6 uN / um[1] | 2.6 uN/um |
| Intercept Estimations | N/A | 3.74 uN/s |
| Random Noise | N/A | Factor +/- 1.25 |
| Calibration Pulse 1 | N/A | Factor +/- 1.8 |
| Calibration Pulse 2 | N/A | Factor +/- 1.6 |

***Table 8: Summary of Observed Errors***

Note the force calibration error is a predominant problem in this simulation as the dx/df numbers vary widely both in the Eagleworks Paper and when computing it from calibration pulses in simulations. The assumption for all simulations and Eagleworks paper is 29 uN and 69 uN, however this was left unquantified and would play a large role in contributing to errors, however it was left untested by Eagleworks.

1 This critical value was only presented for one test and was not quantified or discussed or used as an additional error bound. It was simply averaged during each test.