# CS221 Fall 2015 Homework [logic]

## Problem 4: Logical Inference

**4a.**

New rules: $R1: \dfrac{P \rightarrow Q}{\neg P \vee Q}$ $R2: \dfrac{\neg P \vee Q}{P \rightarrow Q}$

Given knowledge base: $KB = \{(A \vee B) \rightarrow C, A\}$

Now, applying *R1*: $KB = \{\neg(A \vee B) \vee C, A\}$

KB in CNF is: $KB = \{\neg A \vee C, \neg B \vee C, A\}$

Now, applying *R2*: $KB = \{A \rightarrow C, \neg B \vee C, A\}$

And applying modus ponens: $\dfrac{A \rightarrow C, A}{C}$

Thus $KB = \{A \rightarrow C, \neg B \vee C, A, C\}$

**4b.**

Given: $KB = \{A \vee B, B \rightarrow C, (A \vee C) \rightarrow D\}$

Then:

$KB = \{A \vee B, \neg B \vee C, \neg(A \vee C) \vee D\}$

$KB = \{A \vee B, \neg B \vee C, (\neg A \wedge \neg C) \vee D\}$

$KB = \{A \vee B, \neg B \vee C, (\neg A \vee D), (\neg C \vee D)\}$

And, resolving: $A \vee B$ and $\neg A \vee D$

$KB = \{A \vee B, \neg B \vee C, (\neg A \vee D), (\neg C \vee D), \boldsymbol{B \vee D}\}$

And, resolving: $B \vee D$ and $\neg B \vee C$

$KB = \{A \vee B, \neg B \vee C, (\neg A \vee D), (\neg C \vee D), B \vee D, \boldsymbol{C \vee D}\}$

And, resolving: $\neg C \vee D$ and $C \vee D$

$KB = \{A \vee B, \neg B \vee C, (\neg A \vee D), (\neg C \vee D), B \vee D, C \vee D, \boldsymbol{D}\}$

Therefore $\boldsymbol{D}$ has been derived using the resolution rule.

# Problem 5: Logical Inference

**5b.**

Supposing we add the constraint that a number is not larger than itself, we can prove there is no finite, non-empty model for which the resulting set of 7 constraints is consistent with proof by contradiction.

Assume there is a finite model of $n$ elements that satisfies all the 7 constraints. Let this set of elements be $\{a, a+1, a+2, \ldots a+n\}$ where $a+1$ is the successor of $a$, $a+2$ is the successor of $a+1$ and so forth. The 6th constraint (defined in the problem) implies that $a+1$ is larger than $a$; $a+2$ is larger than $a+1$ and so forth. Due to the transitive property of "Larger" (the 6th constraint), the elements of the set are in a **total order**.

As a direct logical consequence, let's define the first conclusion:

**This means that $a+n$ is larger than any $a+i$ for $0 \leq i < n$**

Now, suppose we define the successor of $a+n$ as any other element $a+i$ in the set.

Then, as the next logical consequence, let's define the second conclusion:

**The constraints implies that $a+i$ should be larger than $a+n$**

From the first conclusion and second conclusion, we can state that $a+n$ should be larger than itself due to the **transitive property**. This contradicts the 7th constraint that no number is larger than itself.

Therefore, proof by contradiction states that a finite model can not satisfy all the 7 constraints.