The model used is the standard biofilm model.

$$
\begin{aligned}
M_t &= \nabla_x \left(D(M) \nabla_x M \right) + f(C) M \qquad (1)\\
C_t &= -g(C) M \qquad (2)
\end{aligned}
$$

$$D(M) = \delta \frac{M^\alpha}{(1-M)^\beta} \qquad (3)$$

$$f(C) = \mu \frac{C}{k+C} - \nu \qquad (4)$$

$$g(C, M) = y \frac{C}{k+C} \qquad (5)$$

The dimensions of the parameters and variables are in Tabel 1.

Variable/Parameter	Dimensions	Parameter Value
t	$[days]$	$-$
x	$[meters]$	$-$
M	$[-]$	$-$
C	$[\frac{grams}{meters^3}]$	$-$
δ	$[\frac{meters^2}{days}]$	10^{-12}
α	$[-]$	4
β	$[-]$	4
μ	$[days^{-1}]$	6
k	$[\frac{grams}{meters^3}]$	4
ν	$[days^{-1}]$	$\frac{1}{10}\mu$
y	$[\frac{grams}{meters^3 \cdot days}]$	$\frac{\mu M_0}{0.63}$
M_0	$[-]$	10000
C_0	$[\frac{grams}{meters^3}]$	30
L	$[meters]$	0.01

Table 1: List of parameters and their dimensions

For this system, it is better to nondimensionalize. To do this, the following variable changes are used:

$$
\begin{aligned}
x &= \chi L \implies \partial x = \partial \chi L \qquad (6)\\
t &= \frac{\tau}{\mu} \implies \partial t = \frac{1}{\mu} \partial \tau \qquad (7)\\
C &= \mathscr{C} C_0 \qquad (8)\\
\delta &= d \mu L^2 \qquad (9)\\
k &= \kappa C_0 \qquad (10)\\
\nu &= n \mu \qquad (11)\\
y &= \gamma \mu C_0. \qquad (12)
\end{aligned}
$$

This gives the system

$$\mu \cdot M_\tau = \frac{1}{L^2}\nabla_\chi \left(\hat{D}(M)\nabla_\chi M\right) + \hat{f}(\mathscr{C})M \tag{13}$$
$$C_0\mu \cdot \mathscr{C}_\tau = -\hat{g}(\mathscr{C})M \tag{14}$$

where

$$\hat{D}(M) = \mu L^2 d\frac{M^\alpha}{(1-M)^\beta} \tag{15}$$

$$\hat{f}(\mathscr{C}) = \mu\frac{\mathscr{C}C_0}{\kappa C_0 + \mathscr{C}C_0} - n\mu \tag{16}$$

$$\hat{g}(\mathscr{C}) = \gamma\mu C_0\frac{\mathscr{C}C_0}{\kappa C_0 + \mathscr{C}C_0} \tag{17}$$

This can be greatly simplified by cancelling out parameters when pluggin the functions into the system. This gives,

$$M_\tau = \nabla_\chi \left(d\frac{M^\alpha}{(1-M)^\beta}\nabla_\chi M\right) + \left(\frac{\mathscr{C}}{\kappa + \mathscr{C}} - n\right)M \tag{18}$$
$$C_\tau = -\gamma\frac{\mathscr{C}}{\kappa + \mathscr{C}}M \tag{19}$$

Now we can group together functions and get the non-dimensionalized system:

$$M_\tau = \nabla_\chi \left(D(M)\nabla_\chi M\right) + f(\mathscr{C})M \tag{20}$$
$$\mathscr{C}_\tau = -g(\mathscr{C})M \tag{21}$$

where

$$D(M) = d\frac{M^\alpha}{(1-M)^\beta} \tag{22}$$

$$f(\mathscr{C}) = \frac{\mathscr{C}}{\kappa + \mathscr{C}} - n \tag{23}$$

$$g(\mathscr{C}) = \gamma\frac{\mathscr{C}}{\kappa + \mathscr{C}} \tag{24}$$

with parameter values

$$\begin{aligned}
d =& \frac{\delta}{\mu L^2} \approx 1.667 \times 10^{-9} \\
\kappa =& \frac{k}{C_0} \approx 1.333 \times 10^{-1} \\
n =& \frac{\nu}{\mu} = 10^{-1} \\
\gamma =& \frac{y}{\mu C_0} = \frac{M_0}{0.63 \times C_0} \approx 529.1
\end{aligned} \tag{25}$$