The system

$$M_t = \nabla_x \left(D(M)\nabla_x M\right) + f(C,M)M \tag{1}$$
$$C_t = -yg(C)M \tag{2}$$

where

$$D(M) = \delta \frac{M^\alpha}{(1-M)^\beta} \tag{3}$$
$$f(C,M) = g(C) - \frac{y}{10} \tag{4}$$
$$g(C,M) = \frac{C}{k+C} \tag{5}$$

is solved on a rectangular region with length L and width λL with the following parameter values,

$$\begin{aligned} L &= 0.01 & C_0 &= 30 \\ \lambda &= \frac{1}{128} & M_0 &= 30 \\ \alpha &= 4 & \delta &= \frac{10^{-7}}{\mu L^2} \approx 10^-4 \\ \beta &= 4 & k &= \frac{4}{C_0} \approx 0.1333 \\ \mu &= 6 & \nu &= \frac{M_0}{0.63 \cdot C_0} \approx 1.59, \end{aligned} \tag{6}$$

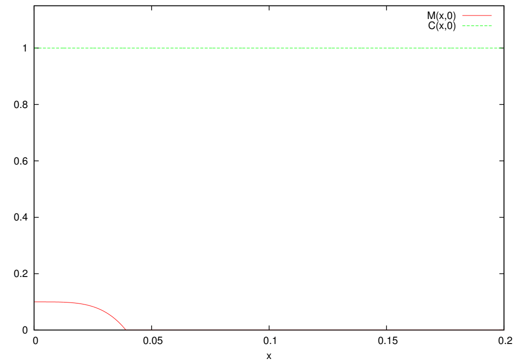
and with initial conditions

$$C = 1$$
$$M = \begin{cases} -(\frac{h}{d^4})x^4 + h & \text{if } x < 0.04 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

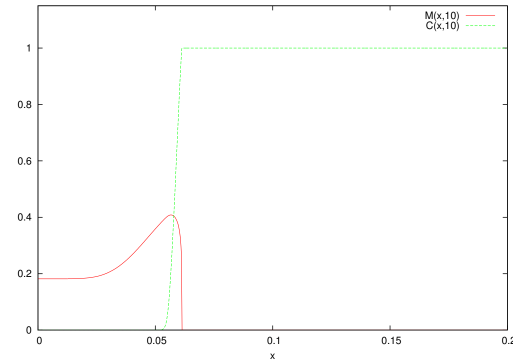
where $h = 0.1, d = \frac{5}{128}$, representing the height and depth of the inoculation site.

Using simulation code version 0.6 the possibilty of a travelling wave solution existing was examined. The following results were found using a finite difference method to solve M and trapizedral rule to solve C with $\Delta x = \frac{1}{8192}$ and $\Delta t = 10^-2$.

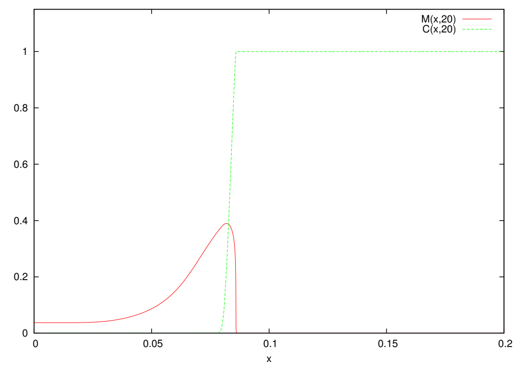
The solution to the above system can be seen in Figure 1. This solution seems to have characteristics of a travelling wave solution. This is because after $t = 70$, we can write the solution $M(x,t)$ as $M(x-ct)$, where c is the wavespeed of the solution. We can numerically approximate the value for c by looking at how fast the peak of the wave travels, Figure 2 shows that this value is $c \approx c^n = 1/440$. Now we can show that the solutions can be written as $M(x-ct)$ by shifting solutions of $M(x,t)$ by $\pm n_i c$, with $n_i \in \mathbb{Z}$, as seen in Figure 3.



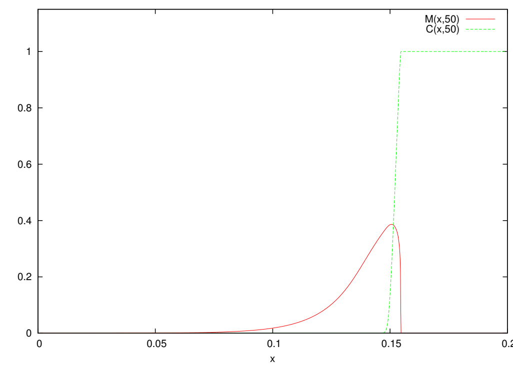
(a)



(b)



(c)



(d)

Figure 1: Solutions of $M(x,t)$ and $C(x,t)$ at (a) t = 0, (b) t = 10, (c) = 20, (d) = 50.

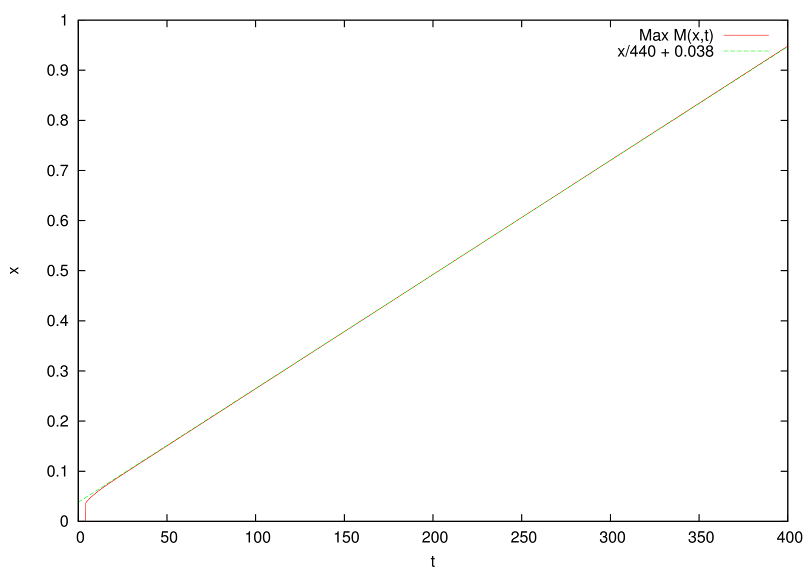


Figure 2: Max M value as a function of t. The red line is an approximation for the wavespeed, c.

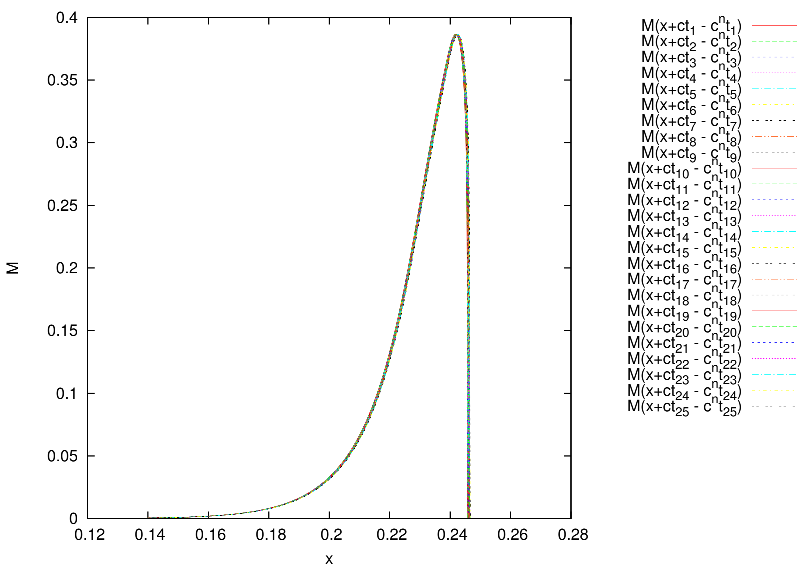


Figure 3: Solutions of $M(x+ct)$ at time $t_i = 1 \ldots 25$ each horizontally translated by $-n_i c^n$, where $n_i = 1 \ldots 25$ and $c^n = 1/440$